Module 5: Welfare Analysis of Horizontal Mergers

Market Organization & Public Policy (Ec 731) · George Georgiadis

- A merger is the combining of two or more firms.
- A merger is called *horizontal* when it occurs among firms in the same industry.
  - *e.g.*, recent merger between Chrysler and Fiat, or American Airlines and US Airways.
  - In contrast to vertical mergers / agreements; *e.g.*, when a firm merges with one of its suppliers.
  - In this section, the term *mergers* refers to horizontal mergers only.
- Mergers have two effects:
  1. They reduce competition, and may help firms raise prices (*e.g.*, recall that in both Cournot and Bertrand competition, the equilibrium price decreases in the number of firms).
  2. May increase efficiency (*e.g.*, reduce production costs due to economics of scale).
- In evaluating mergers, there are 2 possible objectives: Permit a merger iff
  - it does not decrease consumer surplus (*i.e.*, it will not lead to a price increase).
  - it does not decrease social surplus (*i.e.*, consumer surplus + firms' profits)

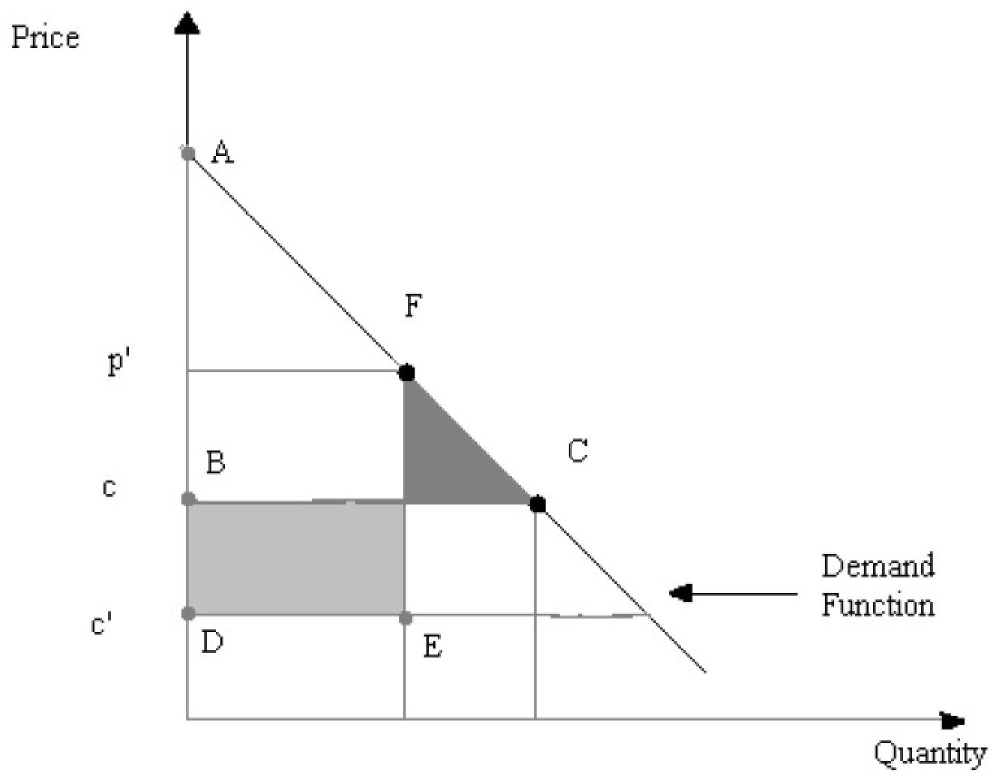
Williamson Trade-off

- The central issue in the evaluation of (horizontal) mergers is the trade off between
  - productivity improvements arising from a merger ; and
  - any reduction in competition.
- Consider an initially competitive market, with a price $p = c$.

- After the merger, the marginal cost falls to c' and the price rises to p'.
- Aggregate social surplus before the merger is given by the area ABCA.
- Aggregate social surplus after the merger is given by the area ADEFA.
- Which area is larger involves a comparison between the area of the shaded triangle and the area of the shaded rectangle.

∘ Williamson's main point was that it does not take a large decrease in cost for the area of the rectangle to exceed that of the triangle.

  - put simply, "rectangles tend to be larger than triangles".



∘ *Remarks:*

1. A critical part of the above argument involves the assumption that the pre-merger price is competitive (*i.e.*, $p = c$). If not, then we would be comparing a rectangle to a trapezoid, and even small increases in price can cause significant reductions in welfare (because rectangles do not tend to be larger than trapezoids).

2. This analysis seeks to maximize aggregate surplus. If the objective is to maximize consumer surplus (as current U.S. law does), then a merger should be allowed if and only if the efficiencies are enough to ensure that price does not increase.

Farrell and Shapiro (AER 1990)

- Analysis of the Welfare Effects of a Merger
- Setup:
  - N firms engage in Cournot competition.
  - Demand is given by $P(X)$, where $X = \sum_i x_i$ and x_i is the output of firm i.
  - Firm i has cost $c_i(x)$, where $c_i' > 0$.
- *Assumptions:*
  - (A1): $P'(X) < 0$ for all X; *i.e.*, price is decreasing in quantity.
  - (A2): $P'(X) + x_i P''(X) < 0$ for all x_i and X; *i.e.*, an increase in rivals' output $X - x_i$ lowers firm i's marginal revenue (and so firm i will reduce its quantity).
  - (A3): $c_i''(x_i) > P'(X)$ for all x_i and X.

Preliminaries:

- Firm i chooses its output $\hat{x}_i$ by solving

$$\hat{x}_i = \arg\max_{x_i} \{x_i P(x_i + y_i) - c_i(x_i)\}$$

  where $y_i = \sum_{j \neq i} x_j$.

- FOC:

$$\hat{x}_i P'(\hat{X}) + P(\hat{X}) - c_i'(\hat{x}_i) = 0$$

  where $\hat{X} = \sum_i \hat{x}_i$.

- *Result 1:* What is the effect of a change in rivals' aggregate output y_i on firm i's output x_i ?
  - Re-write the FOC as:

$$x_i(y_i) P'(x_i(y_i) + y_i) + P(x_i(y_i) + y_i) - c_i'(x_i(y_i)) = 0$$

and differentiate w.r.t y. Then

$$R_i = x_i'(y_i) = -\frac{P'(\hat{X}) + x_i P''(\hat{X})}{2P'(\hat{X}) + x_i P''(\hat{X}) - c_i''(\hat{x}_i)}$$

- Using (A2) and (A3), we get $R_i \in (-1, 0)$.
- *Interpretation:* If rivals jointly expand production, firm i contracts, but by less than its rivals' expansion.
- *A change of variable:* Using that $dx_i(1+R_i) = R_i(dx_i + dy_i) = R_i dX$, define $\lambda_i = -\frac{dx_i}{dX}$, where $\lambda_i = -\frac{R_i}{1+R_i} > 0$.

◦ Firms 1 and 2 contemplate a merger. Two questions:

1. Under what conditions are cost improvements sufficiently great for a merger to reduce price?
2. Can the fact that a proposed merger is profitable for the merging parties be used to help examine whether said merger increases aggregate surplus?

Question #1: Necessary and sufficient conditions for a merger to increase consumer surplus.

◦ Firm i chooses its output $\hat{x}_i$ by solving

$$\hat{x}_i = \arg\max_{x_i} \{x_i P(X) - c_i(x_i)\}$$

◦ Letting $\hat{X}$ be the aggregate pre-merger output, the first-order conditions of firms 1 and 2 are

$$\begin{aligned} \hat{x}_1 P'(\hat{X}) + P(\hat{X}) - c_1'(\hat{x}_1) &= 0 \\ \hat{x}_2 P'(\hat{X}) + P(\hat{X}) - c_2'(\hat{x}_2) &= 0 \end{aligned}$$

- Suppose that $\hat{x}_1 \geq \hat{x}_2 > 0$.

◦ Adding the two FOCs yields

$$(\hat{x}_1 + \hat{x}_2) P'(\hat{X}) + 2P(\hat{X}) - c_1'(\hat{x}_1) - c_2'(\hat{x}_2) = 0 \tag{1}$$

◦ Suppose that the merged firm's cost function is $c_M(\cdot)$.

- The merged firm chooses

$$x_M = \arg\max_x \{xP(X) - c_M(x)\}$$

- *Brute-force* approach:
  - Compute the new equilibrium outputs and compare the new aggregate output to $\hat{X}$.
  - Won't work! (not tractable)
- *Externalities* approach:
  - Fix the rivals' aggregate output $\hat{X}_{-12}$, and examine whether $x_M \gtrless \hat{x}_1 + \hat{x}_2$.
  - If $x_M > \hat{x}_1 + \hat{x}_2$, then by *Result 1*, the new aggregate output $X_M > \hat{X}$, and so the price will fall.
- The merged firm's best response to $\hat{X}_{12}$ is greater than $\hat{x}_1 + \hat{x}_2$ if and only if

$$\begin{aligned}
(\hat{x}_1 + \hat{x}_2) P'(\hat{X}) + P(\hat{X}) - c'_M(\hat{x}_1 + \hat{x}_2) &> 0 \\
\iff P(\hat{X}) - c'_M(\hat{x}_1 + \hat{x}_2) &> \left[P(\hat{X}) - c'_1(\hat{x}_1)\right] + \left[P(\hat{X}) - c'_2(\hat{x}_2)\right]
\end{aligned}$$

  i.e., price will fall iff M's markup would be greater than the sum of the pre-merger markups of firms 1 and 2 at the pre-merger outputs.

- The assumption $\hat{x}_1 \geq \hat{x}_2$ implies that $c'_1(\hat{x}_1) \leq c'_2(\hat{x}_2)$, so that this can happen only if

$$c'_M(\hat{x}_1 + \hat{x}_2) < c'_1(\hat{x}_1) \tag{2}$$

  i.e., for the price to fall, the merged firm's marginal cost at the pre-merger joint output of the merging firms must be below the marginal cost of the more efficient merger partner.

- From this condition, we can see that some kinds of mergers can never reduce price.
  1. A merger that reduces fixed but not marginal costs.
     - Suppose $c_1(x) = c_2(x) = F + cx$, and $c_M(x) = F_M + cx$, where $F_M < 2F$.
     - By (2), we know that this merger cannot reduce price.

2. A merger that involves no synergies (*i.e.*, one whose only efficiencies involve a reallocation of output across firms).
   - *Example:* $c_M(x) = \min_{x_1, x_2} \{c_1(x_1) + c_2(x_2) \;:\; x_1 + x_2 = x\}$
   - Assume convex production costs. Then $\{x_1, x_2\}$ will be chosen s.t. $c_1'(x_1) = c_2'(x - x_1)$.
   - So $c_M'(x) \in (c_1'(\hat{x}_1), c_2'(\hat{x}_2))$ where $x = \hat{x}_1 + \hat{x}_2$.

- If the post-merger price falls, then absent other considerations, it enhances consumer surplus, and therefore, should not be blocked.

Question #2: Sufficient conditions for a merger to increase aggregate surplus.

- Suppose that the merger does increase price.
  - Under what conditions does it nevertheless increase aggregate surplus?
- Suppose that firms in set I contemplate merging, and let $X_I = \sum_{i \in I} x_i$.
- *Outline of the approach:*
  - In general, a merger changes all firms' output in equilibrium.
  - Consumers only care about the net effect on aggregate output ΔX.
  - Rivals only care about the change in eq'm output by the merging firms ΔX_I.
  - In examining the welfare effects of a merger, we can treat ΔX_I as exogenous, and ask what is its effect on the other firms' profits and on consumer surplus (denote this by E).
  - We will decompose ΔX_I into the integral of the effects of infinitesimal changes dX_I that make up ΔX_I.

- Consider the effect of a small reduction in the output X_I, say $dX_I < 0$ and the accompanying reduction in aggregate output $dX < 0$.
  - *Note:* If price is to increase, then aggregate output must decrease, and because $R_i \in (-1, 0)$, it must be the output of the merging firms that falls.

- Let dx_i and dp be the corresponding changes in firm i's output (for $i \neq I$) and price.

- Assume that the proposed merger is profitable for the merging firms. We will derive a *sufficient condition* for the merger to increase aggregate surplus, based on the externality of the merger on nonparticipants.

- The welfare of non-participants is given by

$$E = \underbrace{\int_{P(X)}^{\infty} x(s)ds}_{\text{consumer welfare}} \quad + \quad \underbrace{\sum_{i \neq I} [x_i P(X) - c_i(x_i)]}_{\text{profits of non-merging firms}} \tag{3}$$

- If a (privately profitable) merger increases E, then it also increases aggregate surplus.

- A merger reduces the overall output in the market: $dX = dX_I + \sum_{i \neq I} dx_i < 0$. What is the effect of a small change in X to E?
  - We will study the effect of a "differential" price-increasing merger.

- Totally differentiating (3) yields

$$dE = -\hat{X}P'(\hat{X})dX + \sum_{i \neq I} \hat{x}_i P'(\hat{X})dX + \sum_{i \neq I} \left[P(\hat{X}) - c_i'(\hat{x}_i)\right] dx_i$$

  - *First term:* welfare loss of consumers.
  - *Second term:* welfare gain of the non-merging firms due to price increase.
  - *Third term:* change in non-merging firms' profits due to production reshuffling.

- Recall that each firm's first order condition satisfies $\hat{x}_i P'(\hat{X}) + P(\hat{X}) - c_i'(\hat{x}_i) = 0$. Using this equality and that $\hat{X} = \hat{X}_I + \sum_{i \neq I} \hat{x}_i$, we can write

$$\begin{aligned} dE &= -\hat{X}_I P'(\hat{X})dX + \sum_{i \neq I} \left[-\hat{x}_i P(\hat{X})\right] dx_i \\ &= P'(\hat{X})dX \left(\sum_{i \neq I} \lambda_i \hat{x}_i - \hat{X}_I \right) \\ &= \underbrace{P'(\hat{X})\hat{X}dX}_{>0} \left(\sum_{i \neq I} \lambda_i s_i - s_I \right), \end{aligned}$$

where s_i is firm i's pre-merger market share ($i.e.$, $s_i = \frac{\hat{x}_i}{\hat{X}}$) and $\lambda_i = -\frac{dx_i}{dX} > 0$.

- *Proposition 4:* Consider an infinitesimal reduction in X_I by a subset of *insider* firms. Then the net welfare effect on *outsider* firms ($i.e.$, $i \notin I$) and on consumers $dE \geq 0$ if and only if $s_I \leq \sum_{i \neq I} \lambda_i s_i$.

- *Interpretation:* An "infinitesimal" merger is welfare enhancing only if the merging firms have sufficiently small market share.

- *Key Insight:*

  - A reduction in X_I increases the non-merging firms' profits and decreases consumer welfare.
  - If the non-merging firms did not respond to the reduction in X_I, then $\lambda_i = 0$, and the external effect of the merger would be negative ($i.e.$, rivals would benefit but consumers would lose by more).
  - Proposition 4 shows that many output-reducing mergers benefit the non-merging firms more than they hurt consumers.

- Are we done?

  - No! We need to extrapolate this result to show that $\Delta E > 0$; $i.e.$,

$$\Delta E = \int_{x_I^{initial}}^{x_I^{final}} \frac{dE}{dX_i} dX_I = \int_{x_I^{final}}^{x_I^{initial}} \left(\sum_{i \neq I} \lambda_i \hat{x}_i - \hat{X}_I \right) \left[-P'(\hat{X}) \right] \frac{dX}{dX_I} dX_I \geq 0$$

  - This will be a sufficient condition for a privately profitable merger to increase aggregate welfare.

- *Proposition 5:* Suppose that initial (joint) market share $s_I \leq \sum_{i \neq I} \lambda_i s_i$, and that $[P'', P''', c_i''] \geq 0$ and $c_i''' \leq 0$. Then if the merger is profitable and would raise the market price, it would also raise aggregate welfare.

Proof.

- First, we show that $\frac{d[\lambda_i x_I]}{dX} \leq 0$ for all $i \notin I$.

  - Write $d[\lambda_i x_i] = \lambda_i dx_i + x_i d\lambda_i$ and using $dx_i = -\lambda_i dX$, we have $d[\lambda_i x_i] = -\lambda_i^2 dX + x_i d\lambda_i$.
  - Note that we can think of $\lambda_i = \frac{-P'(X) + x_i P''(X)}{c_i'(x_i) - P'(X)}$ as a function of X and x_i.

- So we have $\frac{d[\lambda_i x_i]}{dX} = -\lambda_i^2 + x_i \left(\frac{\partial \lambda_i}{\partial X} - \lambda_i \frac{\partial \lambda_i}{\partial x_i} \right)$ and substituting for λ_i and its partial derivatives yields

$$\begin{aligned} (c_i'' - P')^2 \frac{d[\lambda_i x_i]}{dX} &= -(P' + x_i P'')^2 - x_i^2 \underbrace{P'''(c_i'' - P')}_{\geq 0} + x_i \underbrace{c_i''' \frac{(P' + x_i P'')^2}{c_i'' - P'}}_{\leq 0} \\ &\quad - x_i \underbrace{P''(c_i'' + P' + 2x_i P'')}_{\geq 0} \end{aligned}$$

- So the RHS ≤ 0, and hence $\frac{d[\lambda_i x_I]}{dX} \leq 0$.

◦ Since an output-reducing merger involves a reduction in X_I, an infinitesimal merger's effect on $\left(\sum_{i \neq I} \lambda_i \hat{x}_i - \hat{X}_I \right)$ will then be positive.

- *i.e.*, after an infinitesimal merger that benefits rivals, a further infinitesimal merger benefits them by even more.

◦ Because $-P'(\hat{X}) > 0$ and $\frac{dX}{dX_I} > 0$, it follows that $\Delta E \geq 0$.

□

Examples:

1. *Linear Demand and Constant Marginal Costs.*

   ◦ Suppose that $P(X) = \alpha - \beta X$ and $c_i(x) = c\,x$ for all i (*i.e.*, symmetric firms).

   ◦ Can show that $\lambda_i = 1$ for all i.

   ◦ Then the sufficient condition for a merger to be welfare enhancing is

$$s_I \leq \sum_{i \notin I} s_i$$

   ◦ *Note:* A merger is welfare enhancing if the pre-merger market shares of the merging firms $\leq 50\%$.

2. *Linear demand and Quadratic Costs*

   ◦ Suppose that $P(X) = \alpha - X$ and $c_i(x) = \frac{x}{2k}$ for all i. (A merger of two firms results in a merged firm with $2k$ units of capital.)

- Can show that $\lambda_i = \frac{x_i}{p} = \frac{s_i}{\epsilon}$ for all i, where ϵ is the demand elasticity at the equilibrium quantity.
- Then the sufficient condition for a merger to be welfare enhancing is

$$s_I \leq \frac{1}{\epsilon} \sum_{i \neq I} s_i^2$$

- *Note:* A merger is more likely to be welfare damaging if demand is more elastic (*i.e.*, if ϵ is large).
  - Intuitively, because with elastic demand, markups are small, and so little welfare benefit can be had from their increased output.

References

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