# Module 3: Natural Monopolies

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## Market Entry and Monopoly

- Consider the following two period game:
  - In t = 1, a "large" number of identical firms *sequentially* decide whether to pay an entry fee F > 0 to enter the market.
  - In t = 2, the firms that entered, engage in Cournot competition.
- Assume that
  - each firm has product cost c(q) = cq; and
  - the inverse demand function  $P(Q) = \alpha \beta Q$ .
- From the previous section, we know that if n firms have entered, in t = 1, each will set quantity

$$q = -\frac{P\left(Q\right) - c'\left(q\right)}{P'\left(Q\right)} = \frac{\alpha - \beta Q - c}{\beta}$$

and using that Q = nq yields that  $q = \frac{a-c}{\beta(n+1)}$ .

- This corresponds to price  $P(Q) = \frac{a+cn}{n+1}$ .
  - Observe that it decreases in n, and converges to c as  $n \to \infty$ .
- Each firm's profit is then

$$\pi_n = q \left(\alpha - \beta n q\right) - cq = \frac{1}{\beta} \left(\frac{\alpha - c}{n+1}\right)^2$$

- Observe that  $n\pi_n$  decreases (monotonically) with n.

• Suppose n firms have already entered the market. Will the next firm choose to enter or not?

- Yes, if  $\pi_{n+1} \ge F$ - No, if  $\pi_{n+1} < F$ 

• Therefore, the equilibrium number of firms that will enter the market (denote  $n^*$ ) is the largest n such that  $\pi_n \geq F$ , or equivalently

$$n^* = \left\lfloor \frac{\alpha - c}{\sqrt{\beta F}} - 1 \right\rfloor$$

- If  $F > \frac{(\alpha - c)^2}{9\beta}$ , then  $n^* = 1$ , and we have a monopoly. - As  $F \to 0$  (*i.e.*, as entry costs vanish),  $n^* \to \infty$  (perfect competition).

• Questions:

- What if firms engage in Bertrand competition?
- What if firms decide whether to enter simultaneously?

# Monopoly Regulation

- How can a regulator restore the social optimum?
- $\circ$  Suppose that a regulator taxes monopoly output at rate t.
  - -i.e., if the monopolist sets price p, then consumers must pay p + t.
- $\circ$  The monopolist chooses p by solving

$$\max_{p} \left\{ pD(p+t) - c\left(D(p+t)\right) \right\}$$

• First order condition:

$$D(p+t) + D'(p+t)(p-c') = 0$$
  
$$\implies [D(p+t) - tD'(p+t)] + D'(p+t)(p+t-c') = 0$$

• To restore the social optimum, the price faced by consumers (i.e., p + t) must equal marginal cost c'.

- Therefore, we must set  $t = \frac{D(p+t)}{D'(p+t)}$ .
  - Denoting the competitive price by  $p_c$ , we can re-write  $t = -\frac{p_c}{\epsilon}$ , where  $\epsilon = -\frac{p_c D'(p_c)}{D(p_c)}$ .
- Observe that because D' < 0, t < 0; *i.e.*, the regulator must subsidize the monopolist. (Somewhat paradoxical!)
- Intuition:
  - The problem with monopoly pricing is that it induces consumers to consume too little.
  - In order to achieve efficiency, we must induce them to consume more, which requires to subsidize the good.
- *Problems:* Determining the proper subsidy requires that the regulator knows (i) the demand elasticity of the monopolist, and (ii) his entire cost curve.
  - Demand information can be obtained through sampling, but this is potentially expensive and inaccurate if the monopolist supplies only a few customers.
  - Cost information is harder to extract, because the monopolist will be reluctant to release accurate estimates of its cost structure.

### Regulating a Monopolist with Unknown Costs

#### Baron and Myerson (Ecta, 1982)

- Setting where the firm has a privately known cost parameter; *i.e.*, its cost is  $c(q, \theta)$ , where  $c_q > 0$  and  $c_{\theta} > 0$ .
- $\circ\,$  Regulator can choose (i) the price p that the firm can charge, and (ii) a subsidy s to be paid to the firm.
  - Solution Approach: the firm is asked to report  $\tilde{\theta}$ , and receives  $p\left(\tilde{\theta}\right)$  and  $s\left(\tilde{\theta}\right)$ .
- Application of the revelation principle.

#### Laffont and Tirole (JPE, 1986)

- Argue that accounting costs are usually observable to the regulator.
- Study a problem with both moral hazard and adverse selection.

#### Setup

- Natural monopolist has exogenous cost parameter  $\theta \in \{\theta_L, \theta_H\}$ . (Define  $\Delta \theta = \theta_H \theta_L > 0$ .)
  - Assume that  $\theta$  is private information of the monopolist.
  - The regulator has beliefs over  $\theta$ : Pr { $\theta = \theta_L$ } =  $\beta$ .
- Production cost:  $c = \theta e$ , where e stands for "effort" (e.g., investment in cost reduction).
  - Effort has cost  $\psi(e) = \frac{e^2}{2}$ .
  - Assume that c is contractible.
- The objective of the regulator is to choose the smallest payment P = c + s such that the firm produces the good.
- The payoff of the firm is:  $P c \psi(e) = P (\theta e) \frac{e^2}{2}$  (if it chooses to produce).

#### First Best

- Suppose that the regulator knows  $\theta$ .
- The regulator's problem then is:

$$\min_{P,e} \quad P \\ \text{s.t.} \quad P - (\theta - e) - \frac{e^2}{2} \ge 0$$

- This problem has solution:  $e^* = 1$  and  $P^* = \theta + \frac{1}{2}$ .
  - Let  $s = P c = P \theta + e$  denote the subsidy. Observe that  $s^* = \frac{3}{2}$  (independent of  $\theta$ ).
  - $P(\theta = \theta_H) > P(\theta = \theta_L)$ : If the regulator does not know  $\theta$ , then the firm would like "convince" the regulator that  $\theta = \theta_H$  to elicit a larger payment.

#### Adverse Selection

- The regulator would like to design a "menu" of contracts  $\{s_L, c_L\}$  and  $\{s_H, c_H\}$  such that a firm with cost parameter  $\theta_i$  will choose contract  $\{s_i, c_i\}$  and exert effort  $e_i = \theta_i - c_i$ .
  - Implemented using a price  $P_i = s_i + c_i$ .
  - Then the firm's payoff is  $P_i c_i(e_i) \frac{e_i^2}{2} = s_i \frac{e_i^2}{2}$ .
- The regulator solves the following problem:

$$\min_{s_L,e_L,s_H,e_H} \quad \beta \left( s_L - e_L \right) + \left( 1 - \beta \right) \left( s_H - e_H \right) + \left[ \beta \theta_L + \left( 1 - \beta \right) \theta_H \right]$$
s.t. 
$$s_L - \frac{e_L^2}{2} \ge 0 \quad (IR_L)$$

$$s_H - \frac{e_H^2}{2} \ge 0 \quad (IR_H)$$

$$s_L - \frac{e_L^2}{2} \ge s_H - \frac{(e_H - \Delta \theta)^2}{2} \quad (IC_L)$$

$$s_H - \frac{e_H^2}{2} \ge s_L - \frac{(e_L + \Delta \theta)^2}{2} \quad (IC_H)$$

- The first two inequalities are participation constraints.
- The next two are incentive constraints: if a firm with  $\theta_L$  reports  $\theta_H$ , then it must exert effort  $e = \theta_H - c_L = \Delta \theta - e_L$ .
- First best has the same effort level and the same subsidy for both types ( $\theta_L$  and  $\theta_H$ ), but a higher actual cost for  $\theta = \theta_H$ .
  - Incentive problem arises, because the efficient type  $(i.e., \tilde{\theta} = \theta_L)$  wants to mimic the inefficient type to collect the same subsidy while expending only effort  $e^* - \Delta \theta$ , and achieving actual cost  $c_H$ .
- Claim: The  $(IR_L)$  and  $(IC_H)$  are obsolete, while  $(IR_H)$  and  $(IC_L)$  bind.
  - $-s_H \frac{e_H^2}{2} \ge 0 \Longrightarrow s_H \frac{(e_H \Delta \theta)^2}{2} \ge 0 \Longrightarrow s_L \frac{e_L^2}{2} \ge 0$ , so  $(IR_L)$  is obsolete.
  - Add  $(IC_H)$  and  $(IC_L)$  to find  $e_H \leq e_L + \Delta \theta$ . So  $s_H \frac{e_H^2}{2} > s_L \frac{e_H^2}{2} \geq s_L \frac{(e_L + \Delta \theta)^2}{2}$ , so  $(IC_H)$  is obsolete.
  - Suppose  $(IR_H)$  is slack. Then decrease  $s_H$  to increase the regulator's payoff until it binds. Note also that this relaxes  $(IC_L)$

- Suppose  $(IC_L)$  is slack. Then decrease  $s_L$  to increase the regulator's payoff until it binds.
- Therefore:

$$s_H - \frac{e_H^2}{2} = 0$$
 and  $s_L - \frac{e_L^2}{2} = s_H - \frac{(e_H - \Delta\theta)^2}{2}$  (1)

• Re-writing the objective function using these equalities yields

$$\min\left\{\beta\left(\frac{e_{L}^{2}}{2} - e_{L} + \frac{e_{H}^{2}}{2} - \frac{(e_{H} - \Delta\theta)^{2}}{2}\right) + (1 - \beta)\left(\frac{e_{H}^{2}}{2} - e_{H}\right)\right\}$$

• First-order conditions:

$$- e_L = 1 = e^*$$
  
-  $e_H = 1 - \frac{\beta}{1-\beta}\Delta\theta < e^*$   
- *Note:* Assume that  $\frac{\beta}{1-\beta}\Delta\theta < 1$ .

- We can now solve for the subsidies  $s_L$  and  $s_H$  using (1).
- Intuition:
  - The "low" type would like to imitate the "high" type, but not vice verse.
  - So for IC, mechanism gives inefficient incentives to the "high" type and lower his payoff to make it undesirable to the "low" type to imitate him.
  - Give efficient incentives to the "low" type.

## References

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