Module 1: Pricing Behavior

Market Organization & Public Policy (Ec 731) · George Georgiadis

Monopoly Pricing

- Consider a monopolist facing demand curve $D(p)$, where $D'(p) < 0$.
  - *i.e.*, if the price is p, then demand for the good will be equal to $q = D(p)$.
  - Write $P(q)$ to denote the inverse demand function; *i.e.*, $p = D^{-1}(q)$.
- The cost of producing q units of the good is $c(q)$, where $c'(q) > 0$.
- The monopolist wants to choose the price to maximize his profit. So he solves:

$$\max_p \{pD(p) - c(D(p))\}$$

- First order condition:

$$\begin{aligned}
\underbrace{D(p) + pD'(p)}_{\text{marginal revenue}} &= \underbrace{c'(D(p))\,D'(p)}_{\text{marginal cost}} \\
\Longrightarrow p - c'(D(p)) &= -\frac{D(p)}{D'(p)} \\
\Longrightarrow \frac{p - c'(D(p))}{p} &= \frac{1}{\epsilon} \qquad (1)
\end{aligned}$$

where $\epsilon = -\frac{pD'(p)}{D(p)}$ denotes the demand elasticity at price p.

  - Demand elasticity: % change in demand in response to a 1% price reduction.
  - We usually denote this price p^m.
  - Equation (1) tells us that the relative markup (*i.e.*, the ratio between the profit margin and the price), also called the *Lerner index*, is inversely proportional to the demand elasticity.

- *Note:* We assume that $D(\cdot)$ and $c(\cdot)$ are such that the monopolist's objective function is concave in p, so that the FOC is sufficient for a maximum.
  - *i.e.*, we assume that $2D'(p) + pD''(p) - c''(D(p))[D'(p)]^2 \leq 0$.

Cournot Competition

- Same setup as above with two changes:
  1. n instead of single firm compete in the market.
  2. Each firm i chooses a quantity q_i to produce, and the market price is determined by $p = P\left(\sum_{i=1}^{n} q_i\right)$.
- Firm i chooses q_i by solving

$$\max_{q_i} \left\{ q_i P\left(q_i + Q_{-i}\right) - c\left(q_i\right) \right\}$$

- First order condition:

$$\begin{aligned} P(Q) + qP'(Q) &= c'(q) \\ \Longrightarrow q &= -\frac{P(Q) - c'(q)}{P'(Q)} \end{aligned}$$

  Assuming symmetry (*i.e.*, $q = \frac{Q}{n}$), we obtain (in equilibrium):

$$\begin{aligned} \frac{Q}{n} &= -\frac{P(Q) - c'\left(\frac{Q}{n}\right)}{P'(Q)} \\ \Longrightarrow \frac{P(Q) - c'\left(\frac{Q}{n}\right)}{P(Q)} &= -\frac{Q\,P'(Q)}{n\,P(Q)} \end{aligned}$$

  - Recall that $P(Q) = D^{-1}(Q)$. Then $P'(Q) = [D^{-1}(Q)]' = \frac{1}{D'(D^{-1}(Q))} = \frac{1}{D'(p)}$.
  - Therefore, $\frac{Q\,P'(Q)}{P(Q)} = \frac{D(p)\frac{1}{D'(p)}}{p} = \frac{D(p)}{p\,D'(p)}$, and the equilibrium price satisfies

$$\frac{p - c'\left(\frac{D(p)}{n}\right)}{p} = \frac{1}{n\epsilon} \qquad (2)$$

    where $\epsilon = -\frac{p\,D'(p)}{D(p)}$

- Remarks:

1. *Sanity check:* When $n = 1$, the price in (2) coincides with the monopoly price.
2. As n increases, the relative markup and the profit of each firm decreases. (In fact, the total profit of all firms decreases with n.)
3. At the limit as $n \to \infty$, the price equals marginal cost (*perfect competition)*.

Bertrand Competition

- Two firms compete in a market.
- Each firm:
  - has constant marginal cost (so that $c(q) = cq$); and
  - faces market demand function $q = D(p)$.
- Firm i sets a price p_i to maximize its equilibrium profit

$$\Pi_i(p_i, p_j) = (p_i - c)\, D_i(p_i, p_j)$$

  where

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if} \quad p_i < p_j \\ \frac{1}{2} D(p_i) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

- Interpretation:
  - If a firm undercuts the other firm's price, then it captures the entire market.
  - If both firms set the same price, then each captures half of the market.
- *Claim:* The unique equilibrium of this game has both firms charging the competitive price: $p_1^* = p_2^* = c$.

Proof.

- Suppose that $p_1^* > p_2^* > c$.
  - Then firm 1 has no demand, and its profit is 0.
  - If instead firm 1 sets $p_1^* = p_2^* - \epsilon > c$, then it obtains the entire demand $D(p_2^* - \epsilon)$, and has a positive profit margin of $p_2^* - \epsilon - c > 0$.
  - Therefore, setting p_1^* cannot be optimal.

- Now suppose that $p_1^* = p_2^* > c$.
  - The profit of firm i is $\frac{1}{2}D(p_i^*)(p_i^* - c) > 0$.
  - If firm i reduces its price to $p_i^* - \epsilon$, then its profit becomes $D(p_i^* - \epsilon)(p_i^* - \epsilon - c)$, which is greater for small ϵ.
  - Therefore, both firms setting some $p^* > c$ cannot be optimal either.
- Lastly, suppose that $p_1^* > p_2^* = c$.
  - Then firm 2, which makes no profit, could raise its price slightly, still supply all the demand, and make a positive profit - a contradiction.
- Therefore, in the unique equilibrium, it must be that $p_1^* = p_2^* = c$.

$\square$

- *Takeaway:* Even with (only) two competing firms, firms price at marginal cost, and they do not make profits.
  - *Note:* Result extends to $n > 2$ competing firms.
  - This suggests that even a duopoly is enough to restore perfect competition.
  - We call this the *Bertrand paradox.* (Tough to believe!)

Solutions to the Bertrand Paradox:

1. Capacity constraints.
   - Suppose that firms can product at most γ units, where $D(c) > \gamma$.
   - Is $p_1^* = p_2^* = c$ still an equilibrium?
     - Suppose that firm 2 sets $p_2 > c$. Then firm 1 faces demand $D(c)$, but can only satisfy up to γ.
     - In this case firm 1 makes 0 profit, while firm 2 makes a positive profit. Therefore, this is not an equilibrium.
   - Characterizing the equilibrium of this game requires assumptions about how consumers are rationed.
2. Product differentiation.

- Bertrand analysis assumes that the firms' products are perfect substitutes.
- This creates a pressure on price, which is relaxed when the products are not exactly identical.

3. Temporal dimension.
   - Bertrand analysis assumes that the firms set prices simultaneously.
   - In the real world, firms can observe their competitors' prices and react.
   - Think of a dynamic environment where firms set $p_1 = p_2 > c$. Does any one firm have an incentive to set $p_i < p_j$?
     - Not clear! Must trade off the benefit of capturing all the market share "today", and making no profits in the future.
     - This is called "tacit collusion".

References

Tirole J., (1988), *The Theory of Industrial Organization*, MIT Press.