

# Problem Set 3

Information Economics (Ec 515) · George Georgiadis

Due in class or by e-mail to [quel@bu.edu](mailto:quel@bu.edu) at 12:30, Monday, November 24

## Problem 1. Holdup Problem with Externalities

Consider a buyer-seller model where the seller can make a costly investment  $i$  in quality enhancement. Then respective payoff functions are

$$P - cq - i$$

for the seller, and

$$v(q, i) - P$$

for the buyer, where  $v_i, v_q, v_{qi} > 0$  and  $v_{qq}, v_{ii} < 0$ . In addition,  $P$  and  $c$  stand for price and marginal production cost. Prices and quantities are assumed to be contractible, while quality is observable but not contractible.

The timing of the game is as follows: In stage 0, the parties write an initial contract. In stage 1, the seller chooses  $i$  (which then becomes a sunk cost). In stage 2, with probability 0.5, the buyer can make an offer  $(P, q)$  while with the remaining probability 0.5 it is the seller who can make an offer  $(P, q)$ . Finally, in stage 3, the party who did not make the offer in the previous stage can either accept the offer or choose to stick to the initial contract.

1. What are the first-best levels of  $q$  and  $i$ ?
2. What are the equilibrium levels of  $q$  and  $i$  in the absence of an initial contract [or, equivalently, with an initial contract  $(P_0, q_0) = (0, 0)$ ]?
3. Suppose that the initial contract can consist of a single pair  $(P_0, q_0)$ . What will the equilibrium contract be, as well as the associated quality level?
4. Suppose now that the payoff functions are  $v(q) - P$  for the buyer and  $P - c(i)q - i$  for the seller [with  $c'(i) < 0$ ]. Show that choosing a single pair  $(P_0, q_0)$  as initial contract can achieve the first-best outcome.

## Problem 2. Information Disclosure

Consider a variation of the information disclosure model from class (see Module 13). An informed seller offers a good to two risk-neutral buyers who engage in Bertrand competition. The quality of the good  $\theta_i \in \{\theta_L, \theta_H\}$  is privately known by the seller, whereas the buyers have a prior belief that the quality is high; *i.e.*,  $\Pr\{\theta = \theta_H\} = \beta$ . Suppose that the seller cannot make a manifestly false claim (*i.e.*, she can disclose  $r_i = \{\theta_i\}$  or  $r_i = \{\theta_1, \theta_2\}$  but not  $r_i = \{\theta_j\}$  where  $\theta_j \neq \theta_i$ ), but she can choose to disclose nothing (*i.e.*,  $r_i = \emptyset$ ). If the seller decides to make a disclosure (*i.e.*,  $r_i \neq \emptyset$ ), then she incurs a cost  $c \geq 0$ . The buyer observes the disclosure (or lack thereof) and offers price  $p = \mathbb{E}[\theta_i | r_i]$ .

1. Suppose that  $c = 0$ . Characterize the equilibrium of this game.

2. Show that there some exists  $\bar{c} > 0$  such that for all  $c \leq \bar{c}$ , in equilibrium, the seller discloses her quality if and only if  $\theta_i = \theta_H$  (whereas she discloses nothing if  $\theta_i = \theta_L$ )
3. Show that for all  $c > \bar{c}$ , in equilibrium, the seller never discloses her quality.

### Problem 3. Screening

Consider the following monopoly screening problem: A government agency writes a procurement contract with a firm to deliver  $q$  units of a good. The firm has constant marginal cost  $c$ , so that its profit is  $P - cq$ , where  $P$  denotes the payment for the transaction. The firm's cost is private information and may be either high ( $c_H$ ) or low ( $c_L$ , with  $0 < c_L < c_H$ ). The agency's prior belief about the firm's cost is  $\Pr(c = c_L) = \beta$ , and it makes a take-it-or-leave-it-offer to the firm (whose default profit is zero).

1. If  $B(q)$  denotes the (concave) benefit to the agency of obtaining  $q$  units, what is the optimal contract for the agency?
2. Compare this second-best solution with the first-best one, obtained if costs are known by the agency. Discuss the results.
3. What would the first-best and second-best solutions be if  $c$ , instead of taking two values, were uniformly distributed on  $[\underline{c}, \bar{c}]$ , with  $0 < \underline{c} < \bar{c}$ ?

### Problem 4. Signaling

A firm has a project requiring an investment of 20 at  $t = 0$  for a sure return of 30 at  $t = 1$ . There is no discounting. The investment cost has to be raised from the financial market. Assume that a new equity issue is proposed. Potential new investors are uncertain about the value of the firm's assets in place:  $A \in \{50, 100\}$  with  $\Pr[A = 100] = 0.1$ .

1. Suppose that investors believe that both types of firms invest. What fraction of the firm's equity has to be issued to new investors? What are the payoffs to existing shareholders if they undertake the project? Are these beliefs reasonable?
2. Suppose that investors believe that only bad firms issue new equity. Same questions.
3. Suppose now that shareholders commit at  $t = 0$  to a wasteful advertising campaign at  $t = 1$  after the project return is realized. The advertising expenditure is an irreversible action on the part of the firm that results in a drop in profits of  $K$ . The size of the expenditure is a choice variable. Show that a good firm can signal its type through such expenditures. Discuss.

### Bonus Problem. Information Disclosure

Consider the following stylized example of trading by two risk-neutral market makers, each of them small enough that the *direct* influence of their trade on the stock prices is negligible. It is common knowledge that one of the traders will become informed about the true underlying value of a stock with probability  $\beta$ , while the other always remains uninformed. Assume each market maker can trade up to one unit twice in succession. The question is, should the market makers be required to disclose their first trade before they initiate their second trade? The question is considered at an ex-ante stage where both market makers are identical and neither knows who will become informed (if any). So, at stage 0, the market makers must decide whether to introduce mandatory disclosure of trades at stage 1.

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Prior to stage 1, the stock is worth either 110 or 90, with equal probabilities, so that the stock price is 100. At the beginning of stage 1, one market maker becomes informed about the value of the stock with probability  $\beta$ , while the other remains uninformed. For the first trade, assume the uninformed market maker buys with probability 0.5 and sells with probability 0.5. Following the first trade, there may be mandatory or voluntary disclosure. Then a second trade can be initiated by both informed and uninformed market makers.

1. What are the ex post payoffs of the informed and uninformed traders under mandatory disclosure and no mandatory disclosure, respectively?
2. When is mandatory disclosure better from an ex ante perspective than no mandatory disclosure?
3. When there is no mandatory disclosure, when is there voluntary disclosure? Discuss.

*Note:* In computing the parties' payoffs, you must take into account that the market will optimally update its price depending on the market makers' trades.