# Problem Set 2

Information Economics (Ec 515) · George Georgiadis

*Due in class or by e-mail to quel@bu.edu at 12:30, Tuesday, October 14*

**Problem 1.**

A principal employs an agent. Both parties are risk-neutral and have outside option 0. The agent chooses non-negative effort levels $\{a_1, a_2\}$ to produce outputs

$$q_1 = a_1 + \epsilon_1$$
$$q_2 = a_2 + \epsilon_2$$

where $\epsilon_1, \epsilon_2 \sim N(0, \sigma^2)$ and $\epsilon_1$ and $\epsilon_2$ are independent. The agent's cost of effort is given by $c(a_1, a_2) = \frac{a_1^2 + a_2^2}{2} + k a_1 a_2$, where $k \in [-1, 1]$. The principal's profit is given by

$$q_1 + \phi q_2 - w(q_1, q_2)$$

where $\phi > 0$ and $w$ denotes the wage paid to the agent.

1. Characterize the first-best outcome of this game.

2. Suppose that the principal offers a linear contract of the form $w(q_1, q_2) = \alpha + \beta_1 q_1 + \beta_2 q_2$. Characterize the optimal linear contract.

Now suppose that the principal cannot observe $q_2$ (and hence the contract cannot depend on $q_2$).

3. Characterize the optimal linear contract.

4. Compare the optimal contracts in parts 2 and 3. Explain the intuition behind the differences.

**Problem 2.**

A risk-neutral principal employs two risk-averse agents. The agents (indexed by $i \in \{1, 2\}$) choose non-negative effort levels $\{a_1, a_2\}$ to produce outputs

$$q_1 = a_1 + \varepsilon_1 + \delta \varepsilon_2$$
$$q_2 = a_2 + \varepsilon_2 + \delta \varepsilon_1$$

where $\epsilon_1, \epsilon_2 \sim N(0, \sigma^2)$ and $\epsilon_1$ and $\epsilon_2$ are independent. Given effort level $a_i$ and outputs $\{q_1, q_2\}$, agent $i$'s utility is

$$u(a_i, q_1, q_2) = -e^{-r\left[w_i(q_1, q_2) - \frac{c}{2} a_i^2\right]}$$

where $w_i(q_1, q_2)$ is the wage paid to agent $i$.[1] The principal's profit is given by

$$q_1 + q_2 - w_1(q_1, q_2) - w_2(q_1, q_2)$$

1. Characterize the first best outcome of this game.

2. Suppose that the principal offers a linear contract to each agent of the form $w_1(q_1, q_2) = \alpha_1 + \beta_1 q_1 + \gamma_1 q_2$ and $w_2(q_1, q_2) = \alpha_2 + \beta_2 q_2 + \gamma_2 q_1$. Characterize the optimal linear contract.

3. How does the optimal contract depend on $\delta$? Provide some intuition for the sign of $\beta_i$ and $\gamma_i$.

4. Compare the optimal contract from part 2 to the first best outcome in part 1. What do you observe when $\delta \in \{0, 1\}$? Can you explain this intuitively?

**Problem 3.**

A (risk-neutral) municipal government considers funding an investment project put forward by an association (also risk-neutral). The cost of the project is known, but the government is unsure about its social value, and its assessment is at odds with that of the association. Specifically, if the project is of "good quality", then its social value (net of the cost of the project) as assessed by the government is $\theta_G > 0$, while the association would derive a private benefit $v_G > 0$ from seeing it go through. If instead the project is of "bad quality", then its net social value is $\theta_B < 0$, but the association would derive a private benefit $v_B$, higher than $v_G$, if it went through. The association knows the quality of the project, while the government's (common-knowledge) belief is $\Pr(v_B) = \beta$.

In the absence of information, the government is ready to fund the project, since we assume $\beta\theta_B + (1 - \beta)\theta_G > 0$. However, since taxation is distortionary, the government has net value $\lambda > 0$ for each unit of revenue raised from the association. However, the government would be unwilling to allow a bad-quality project to go through even if it were able to charge the association for its full private benefit; that is, we assume $\theta_B + \lambda v_B < 0$.

Assume the government has access to a (risk-neutral) "expert" who, when the project is of bad quality, manages to obtain an (unfalsifiable) proof of this fact with probability $p$, but observes "nothing" with probability $(1 - p)$; "nothing" is also observed with probability 1 when the project is of good quality. The expert starts with no financial resources and can therefore only be rewarded, not punished. The association is assumed to observe when the expert obtains a proof of bad quality, while the government has to be "alerted" by the expert.

1. Derive first the optimal scheme for the government when it cannot rely at all on the expert.

2. What is the optimal scheme when the government can rely on the expert and when collusion between the expert and the association is impossible because the expert is "honest"?

3. What is the optimal scheme when the government can rely on the expert but the expert is "self-interested" and the association can promise the expert a side payment for not alerting the government when he obtains a proof of bad quality? Assume the collusion technology is such that, for every unit of money the association pays, the expert only collects an equivalent of $k < 1$ units of money.

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[1] Note that if $\delta \neq 0$, then their outputs are correlated, and hence the principal may find it optimal to reward each agent based on the other agent's output.

4. What is the optimal scheme when the government is unsure about the prospect for collusion, because it believes that with probability $\gamma$ the expert is "honest" and with probability $1 - \gamma$ it is "self-interested"?

*Hint:* The government imposes a "tax" $T_i$ when the association reports the project to be $i \in \{G, B\}$. In part 1, you are asked to characterize the optimal taxes $\{T_G, T_B\}$.

**Problem 4.**

This problem involves a comparison of long-term vs. short-term wage contracts in an efficiency wage setting. Suppose workers and firms live two periods instead of one. Workers have utility $y - e$ per period, where $y \geq 0$ is income and $e \in \{0, 1\}$ is effort. Firms cannot directly observe effort, but if it is equal to zero, then they have a chance $q$ of detecting this. This chance $q$ is chosen by the firm at a cost $c(q)$, where $c' > 0$, $c'' > 0$. The outside option of the worker is $u$ each period; assume that for any contract the firm offers, the participation constraint of the worker binds.

1. A short term contract lasts one period. It consists of a wage paid in case shirking is not detected (0 is paid if it is) and a monitoring probability $q_s$. Write the incentive compatibility constraint, argue that it binds, and then write an expression for the total monitoring costs incurred by a firm if it offers two consecutive short term contracts.

2. A long term contract with deferred compensation lasts two periods. In the first, if shirking is detected (with probability $q_1$), 0 is paid and the contract is not renewed; the worker then seeks a short term contract in the labor market (assume he can always find one). If shirking is not detected, the worker is not paid anything, but the worker is allowed to continue with the firm. In the second period, if shirking is detected (probability $q_2$), 0 is paid; if not, the deferred compensation $W$ is paid.

   (a) Write the incentive compatibility condition that must hold in the second period, assuming the worker worked in the first period.

   (b) Do the same assuming the worker shirking in the first period but escaped detection.

   (c) Show that if $q_2$ satisfies IC in case (i), it does so in case (ii) and write an expression for it in terms of $u$.

   (d) Write the incentive compatibility constraint for the first period, assuming that the worker will have the incentive to exert effort in the second period regardless of what he does in the first period; use this to derive an expression for $q_1$ in terms of $u$.

3. Compare $q_1$ and $q_2$. Interpret.

4. Write down expressions for the total monitoring cost (per worker) under short- and long-term contracts. Show that the long-term contract is less costly in terms of monitoring. Provide some intuition for this result.

5. How does the benefit of the long term contract (the cost of short term less that of long term) depend on $u$? Provide some intuition.

6. Suppose that the outside option $u$ of the workers increases. What do you expect will happen to the prevalence of long term contracts?