Problem Set 1

Information Economics (Ec 515) · George Georgiadis

Due in Lingfeng Que's mailbox (Room 436, 270 Bay State Road), or by e-mail to quel@bu.edu at 12:30, Monday, September 22

Problem 1.

Consider the following "portfolio choice" problem. The investor has initial wealth w and utility $u(x) = \ln(x)$. There is a *safe asset* (such as a US government bond) that has net real return of zero. There is also a *risky asset* with a random net return that has only two possible returns, R_1 with probability q and R_0 with probability $1-q$. Let A be the amount invested in the risky asset, so that $w - A$ is invested in the safe asset.

1. Find A as a function of w. Does the investor put more or less of his portfolio into the risky asset as his wealth increases?
2. Another investor has the utility function $u(x) = -e^{-x}$. How does her investment in the risky asset change with wealth?
3. Find the coefficients of absolute risk aversion $r(x) = -\frac{u''(x)}{u'(x)}$ for the two investors. How do they depend on wealth? How does this account for the qualitative difference in the answers you obtain in parts (1) and (2)?

Problem 2.

You have an opportunity to place a bet on the outcome of an upcoming race involving a certain female horse named Bayes: if you bet x dollars and Bayes wins, you will have $w_0 + x$, while if she loses you will have $w_0 - x$, where w_0 is your initial wealth.

1. Suppose that you believe the horse will win with probability p and that your utility for wealth w is $\ln(w)$. Find your optimal bet as a function of p and w_0.
2. You know little about horse racing, only that racehorses are either winners or average, that winners win 90% of their races, and that average horses win only 10% of their races. After all the buzz you've been hearing, you are 90% sure that Bayes is a winner. What fraction of your wealth do you plan to bet?
3. As you approach the betting window at the track, you happen to run into your uncle. He knows rather a lot about horse racing: he correctly identifies a horse's true quality 95% of the time. You relay your excitement about Bayes. "Don't believe the hype," he states. "That Bayes mare is only an average horse." What do you bet now (assume that the rules of the track permit you to receive money only if the horse wins)?

Problem 3.

If an individual devotes a units of effort in preventative care, then the probability of an accident is $1 - a$ (thus, effort can only assume values in $[0, 1]$). Each individual is an expected utility maximizer with utility function $p \ln(x) + (1-p) \ln(y) - a^2$, where p is the probability of an accident, x is wealth if there is an accident, and y is wealth if there is no accident. If there is no insurance, then $x = 50$, while $y = 150$.

1. Suppose first there is no market for insurance. What level of a would the typical individual choose? What would her expected utility be?
2. Assume that a is verifiable. What relationship do you expect to prevail between x and y in a competitive insurance market? What relationship do you then expect to prevail between x and a?
3. Derive the value of a, x and y that maximize the typical customer's expected utility. What is the value of this maximized expected utility?
4. Suppose that a is not verifiable. What would happen (*i.e.*, what would the level of a and expected utility be) if the same contract (*i.e.*, same x and y values) as in (3) were offered by competitive firms? Do you expect this would be an equilibrium?
5. Under the non-verifiability assumption, what relationship must prevail between x, y, and a? Use this relationship along with the assumption of perfect competition to derive a relationship between x and a that contracts offered by insurers must have. Finally, find the level of a that maximizes the expected utility of the typical consumer, and find that level of expected utility.
6. Summarize your answers by ranking the levels of a and the expected utilities for each of the cases in (1), (3), (4) and (5). What do you notice?

Problem 4.

An agent can work for a principal. The agent's effort, a affects current profits, $q_1 = a + \varepsilon_{q_1}$, and future profits, $q_2 = a + \varepsilon_{q_2}$, where ε_{q_t} are random shocks, and they are i.i.d with normal distribution $N(0, \sigma_q^2)$. The agent retires at the end of the first period, and his compensation cannot be based on q_2. However, his compensation can depend on the stock price $P = 2a + \varepsilon_P$, where $\varepsilon_P \sim N(0, \sigma_P^2)$. The agent's utility function is exponential and equal to

$$-e^{-\eta\left[t - c\frac{a^2}{2}\right]}$$

where t is the agent's income, while his reservation utility is $\bar{t}$.[1] The principal chooses the agent's compensation contract $t = w + fq_1 + sP$ to maximize her expected profit, while accounting for the agent's IR and IC constraints.

1. Derive the optimal compensation contract $t = w + fq_1 + sP$.
2. Discuss how it depends on σ_P^2 and on its relation with σ_q^2. Offer some intuition?

[1]Reservation utility $\bar{t}$ means that the agent's IR constraint requires that $\mathbb{E} - e^{-\eta\left[t - c\frac{a^2}{2})\right]} \geq -e^{-\eta\bar{t}}$.

Problem 5.

Two agents can work for a principal. The output of agent i ($i = 1, 2$), is $q_i = a_i + \varepsilon_i$, where a_i is agent i's effort level and ε_i is a random shock. The ε_i's are independent of each other and normally distributed with mean 0 and variance σ^2. In addition to choosing a_2, agent 2 can engage in a second activity b_2. This activity does not affect output directly, but rather reduces the effort cost of agent 1. The interpretation is that agent 2 can *help* agent 1 (but not the other way around). The effort cost functions of the agents are

$$\psi_1 (a_1, b_2) = \frac{1}{2} (a_1 - b_2)^2$$

and

$$\psi_2 (a_2, b_2) = \frac{1}{2} a_2^2 + b_2^2.$$

Agent 1 chooses her effort level a_1 only after she has observed the level of help b_2. Agent i's utility function is exponential and equal to

$$-e^{[-\eta(w_i - \psi_i(a_i, b_2))]}$$

where w_i is the agent's income. The agent's reservation utility is -1, which corresponds to a reservation wage of 0. The principal is risk neutral and is restricted to linear incentive schemes. The incentive scheme for agent i is

$$w_i = z_i + v_i q_i + u_i q_j$$

1. Assume that a_1, a_2, and b_2 are observable. Solve the principal's problem by maximizing the total expected surplus with respect to a_1, a_2, and b_2. Explain intuitively why $a_1 > a_2$.
2. Assume from now on that a_1, a_2, and b_2 are not observable. Solve again the principal's problem. Explain intuitively why $u_1 = 0$.
3. Assume that the principal cannot distinguish whether a unit of output was produced by agent 1 or agent 2. The agents can thus engage in *arbitrage*, claiming that all output was produced by one of them. Assume that they will do so whenever it increases the sum of their wages. Explain why the incentive scheme in part 2 above leads to arbitrage. What additional constraint does arbitrage impose on the principal's problem? Solve this problem, and explain intuitively why $u_1 > 0$.