Module 6: Principal-Agent Problem with Subjective Evaluations

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- In the basic moral hazard problem, the output q is (i) observable by all parties, and (ii) verifiable by an external observer (*e.g.*, a court of law).
- What if it is not observable to the agent ?

Model:

- Agent takes action $a \in \{H, L\}$.
- Cost of effort $c(H) > c(L) = 0$.
- Output $q \sim f(q \mid a)$ and $q \in \{q_1, .., q_N\}$.
- Strict MLR: $\frac{f(q_i \mid H)}{f(q_i \mid L)} > \frac{f(q_{i-1} \mid H)}{f(q_{i-1} \mid L)}$ for all i.
- Principal wishes to implement $a = H$.
- q is private information of the principal.
- Principal is risk neutral; agent is risk averse and has utility $u(\cdot)$.

- Principal's Problem:

$$\begin{aligned}
\max_{w(\cdot),\, t(\cdot)} \quad & \mathbb{E}\left[q - t(q) \mid H\right] \\
\text{s.t.} \quad & \mathbb{E}\left[u(w(q)) \mid H\right] - c(H) \geq \bar{u} && \text{(agent IR)} \\
& \mathbb{E}\left[u(w(q)) \mid H\right] - c(H) \geq \mathbb{E}\left[u(w(q)) \mid L\right] && \text{(agent IC)} \\
& q - t(q) \geq q - t(\tilde{q}) \quad \forall \tilde{q} && \text{(principal IC)} \\
& t(q) \geq w(q) && \text{(feasibility)}
\end{aligned}$$

- Notation:
  - $t(q)$: what the principal pays given output q.
  - $w(q)$: what the agent is paid given output q.
- *Claim:* $t(q) = constant\ for\ all\ q$.

Proof.

- Pick any q_i and q_j. The principal's IC implies that

$$\left.\begin{array}{l} q_i - t(q_i) \geq q_i - t(q_j) \\ q_j - t(q_j) \geq q_j - t(q_i) \end{array}\right\} \Longrightarrow \left.\begin{array}{l} t(q_i) \leq t(q_j) \\ t(q_j) \leq t(q_i) \end{array}\right\} \Longrightarrow t(q_i) = t(q_j)$$

- Therefore, $t(q) = constant\ for\ all\ q$.

□

- *Claim:* $w(q) < t(q)$ for some q (*i.e.*, the principal must "burn" money in some states).

Proof.

- Suppose not. *Because* $t(q) = constant$ for all q, it must be that $w(q) = constant\ for\ all\ q$.
- But then the agent's IC cannot be satisfied (because $c(H) > 0$).

□

- Write the Lagrangian:

$$\begin{aligned} L &= \sum_i [q_i - t(q_i)] f(q_i \mid H) + \lambda \left[\sum_i u(w(q_i)) f(q_i \mid H) - c(H) - \bar{u} \right] \\ &\quad + \mu \left\{ \sum_i u(w(q_i)) \left[1 - \frac{f(q_i \mid L)}{f(q_i \mid H)} \right] f(q_i \mid H) - c(H) \right\} \\ &\quad + \sum_i \psi(q_i) [t(q_i) - w(q_i)] \end{aligned}$$

- *Claim:* $w(q) < t(q)$ if and only if $q = q_1$.

Proof.

- Because $w(q) < t(q)$ for some q^*, by complementary slackness, $\psi(q^*) = 0$.
- Take FOC w.r.t w and evaluate it at q^*:

$$u'(w(q))\left[\lambda + \mu\left(1 - \frac{f(q^* \mid L)}{f(q^* \mid H)}\right)\right] f(q^* \mid H) = 0 \tag{1}$$

- Strict MLR $\Longrightarrow$ (1) can hold only for one state.
  - Therefore, $\psi(q) > 0$ for all $q \neq q^*$.
- For states $q_i \neq q^*$:

$$u'(w(q_i))\left[\lambda + \mu\left(1 - \frac{f(q_i \mid L)}{f(q_i \mid H)}\right)\right] f(q_i \mid H) = \psi(q_i) > 0 \tag{2}$$

- Strict MLR, (1) and (2) $\Longrightarrow q^* = q_1$.

□

- Optimal contract:

$$w(q) = \begin{cases} w_H & \text{if } q \geq q_2 \\ w_L & \text{if } q = q_1 \end{cases} \quad \text{and } t(q) = w_H \text{ for all } q\,.$$

  - Principal pays a constant wage, and he "burns" money in the lowest state.
  - *Intuition:* The principal punishes only in the lowest state, which is the state where a deviation (to $a = L$) is most likely to have occurred.
- Determining wages:
  - Let $p(a) = \Pr\{q \geq q_2 \mid a\}$.
  - Two equations:

(Agent's IR) $u(w_H)\,p(H) + u(w_L)[1 - p(H)] - c(H) = \bar{u}$

(Agent's IC) $u(w_H)\,p(H) + u(w_L)[1 - p(H)] - c(H) = u(w_H)\,p(L) + u(w_L)[1 - p(L)]$

  - These two equations pin down w_H and w_L.
- Implications:

- Compressed wage schedule.
- Most workers get some (subjective) performance evaluation.
- Worker gets punished if his performance is too bad.

∘ Intuition:

- Principal must pay the same amount irrespective of output.
- She wants to minimize the amount of wasted resources

$$\sum_i \left[t\left(q_i\right) - w\left(q_i\right)\right] f\left(q_i \mid H\right)$$

- This is best done by punishing in the lowest state, which is the state where a deviation is most likely to have occurred.

∘ What if the principal could sell the money "burned" when $q = q_1$ in an ex-ante contract?

- Firm pays out $t = \max_q w\left(q\right) = w_H$.
- Agent gets $w\left(q\right)$.
- 3^{rd} party gets $w_H - w\left(q\right)$ and pays $\mathbb{E}\left[w_H - w\left(q\right) \mid H\right] = \left(w_H - w_L\right)\left[1 - p\left(H\right)\right]$ up-front.

  i.e., principal can obtain additional profit $\left(w_H - w_L\right)\left[1 - p\left(H\right)\right]$.

References

Board S., (2011), Lecture Notes.

MacLeod B., (2003), "Optimal Contracting with Subjective Evaluation", *American Economic Review.*