# Module 2: Portfolio Choice & Insurance

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- Suppose an investor has initial wealth $w$ and utility $u(\cdot)$.
  - Assume that $u' > 0 > u''$.
- Investor chooses between two assets:
  - safe asset, which yields a return $1 + r$ for every dollar invested with probability 1 ($r > 0$).
  - risky asset, which yields a return of $1 + R$ for every dollar invested
  - $R$ takes values $R_1 < R_2 < ... < R_n$, with probabilities $p_1, ...., p_n$.
- Assume that: $R_1 < r$ and $\mathbb{E}[R] = \sum_{i=1}^{n} p_i R_i > r$.

- *Investor's choice:* how much to invest in each asset.
- Let $A$ be the amount of money she invests in the risky asset.
- Then, the investor's expected return is given by

$$A \sum_{i=1}^{n} p_i (1 + R_i) + (w - A)(1 + r).$$

- Investor's problem:

$$\max_{A} \sum_{i=1}^{n} p_i u (A (1 + R_i) + (w - A)(1 + r))$$

- First order conditions:

$$\sum_{i=1}^{n} p_i u' (A^* (1 + R_i) + (w - A^*)(1 + r))(R_i - r) = 0.$$

- Second order conditions?
  - Always satisfied! (Explain.)
- There is a unique $A^*$ that solves this equation.
  - If $\mathbb{E}[R] = \sum_{i=1}^{n} p_i R_i > r$, then $A^* > 0$.
  - Proof by contradiction.
- If $u$ has DARA, then $A$ is increasing in $w$.

## An Application: Insurance Problem

- A consumer buying insurance.
- Two possible outcomes (states of nature): good (G) and bad (B).
- *Example:*
  - Good outcome: your house doesn't burn down.
  - Bad outcome: your house burns down.
- Consumer's income:
  - Income at state G: $y$.
  - Income at state B: $y - L$.
  - Assume $y > L$.
- G occurs with probability $p \in (0, 1)$ (B with prob. $1 - p$)
- Consumer can buy coverage (insurance).
- If consumers buys coverage $C$:
  - He receives $C$ from insurance company if outcome is $B$.
  - He receives 0 from insurance company if outcome is $G$.
- Getting coverage $C$ costs $\pi C$ ($\pi$ = cost of coverage).
- Consumer chooses coverage level $C$.

- Consumer's income if he purchases coverage $C$:
  - Income at state G: $y_G = y - \pi C$
  - Income at state B: $y_B = y - L - \pi C + C = y - L + (1-\pi)C$
- Note that $\pi < 1$: otherwise, the consumer will not purchase insurance.

- What effect does a marginal increase in $C$ have on $y_G$ and $y_B$?

$$\begin{aligned} \frac{\partial y_G}{\partial C} &= -\pi \\ \frac{\partial y_B}{\partial C} &= 1-\pi \Rightarrow \\ \frac{\partial y_B}{\partial C} \Big/ \frac{\partial y_G}{\partial C} &= -\frac{1-\pi}{\pi} \end{aligned}$$

  - By marginally increasing $C$, you trade off $1-\pi$ units of consumption in bad state by $\pi$ units of consumption in good state.
  - $\frac{1-\pi}{\pi}$ is called the "price-ratio".

- Given $(y_G, y_B)$, the consumer's utility is:

$$U(y_G, y_B) = pu(y_G) + (1-p)\,u(y_B).$$

- Assume that $u' > 0$ and $u'' < 0$: The consumer likes more income and he is risk averse.
- Consumer's problem

$$\max_C \{pu(y-\pi C) + (1-p)\,u(y - L - \pi C + C)\}.$$

- First order conditions:

$$pu'(\underbrace{y - \pi C^*}_{y_G^*})(-\pi) + (1-p)\,u'(\underbrace{y - L + (1-\pi)C^*}_{y_B^*})(1-\pi) = 0.$$

- This implies that:

$$(1-p)\,u'(y_B^*)(1-\pi) = pu'(y_G^*)\,\pi \Rightarrow \frac{1-\pi}{\pi} = \frac{pu'(y_G^*)}{(1-p)\,u'(y_B^*)}.$$

- *Solution:* MRS = Price ratio
- Second order conditions?
- Suppose $1 - p \geq \pi$. Then, $C^* > 0$.
  - If price of insurance is lower than probability of bad outcome, consumer will buy coverage.

**Example: Suppose** $u(y) = \ln y$, **so** $u'(y) = \frac{1}{y}$**.**

- In this case:

$$\begin{aligned} \frac{1-\pi}{\pi} &= \frac{pu'(y_G)}{(1-p)u'(y_B)} \\ &= \frac{py_B}{(1-p)y_G}. \end{aligned}$$

- Since $y_G = y - \pi C$ and $y_B = y - L - \pi C + C$,

$$\frac{1-\pi}{\pi} = \frac{p}{1-p}\frac{y - L - \pi C + C}{y - \pi C}.$$

- This equation pins down the level of coverage $C$. Solving for $C$ yields:

$$C = \frac{\frac{y}{\pi}(1 - \pi - p) + pL}{1 - \pi}.$$

- Assuming $1 - \pi - p > 0$, level of coverage is increasing in $L$ and $y$.

- What about changes in $p$?

$$\frac{\partial C}{\partial p} = \frac{L - \frac{y}{\pi}}{1-\pi} = \frac{\pi L - y}{\pi(1-\pi)}.$$

  - Since $y > L$, agent buys less coverage as $p$ increases.
  - As $p$ increases, bad state becomes less likely.

- What about changes in $\pi$?

$$\frac{\partial C}{\partial \pi} = \frac{\partial}{\partial \pi}\left(\frac{\frac{y}{\pi}(1 - \pi - p) + pL}{1 - \pi}\right).$$

– This derivative is negative:

– As price of coverage increases, agent buys less coverage.

**Perfect Competition in the Insurance Sector**

- What are the profits of the insurance company?

  – Insurance company earns $\pi C$ on the consumer.

  – Insurance company pays consumer an amount $C$ with probability $1-p$.

- Total expected profits: $\pi C-(1-p)C=(\pi+p-1)C$.

  – Suppose that there is perfect competition in the insurance sector.

  – Profits of the insurance company are zero, so $\pi=1-p$.

- Recall FOCs:

$$\frac{1-\pi}{\pi}=\frac{pu'(y_G^*)}{(1-p)u'(y_B^*)}$$

- Under perfect competition in the insurance sector, $\pi=1-p$, so

$$1=\frac{u'(y_G^*)}{u'(y_B^*)}\Rightarrow u'(y_G^*)=u'(y_B^*)\Rightarrow y_G^*=y_B^*.$$

- Consumer insures perfectly!

$$\begin{aligned} y_G &= y-\pi C=y-L-\pi C+C=y_B\Rightarrow \\ C &= L. \end{aligned}$$

# References

Mas-Colell, Whinston and Green, (1995), *Microeconomic Theory*, Oxford University Press.

Ortner J., (2013), Lecture Notes.