Module 16: Signaling

Information Economics (Ec 515) · George Georgiadis

- Players with private information can take some action to "signal" their type.
  - Taking this action would distinguish them from other types.
- Privately informed agents credibly convey information about themselves to another party.
- Spence (QJE, 1973): Job Market Signaling.
  - Studied signaling in the labor market.
  - Workers can signal their type by obtaining education.
  - *In equilibrium:* high ability workers get more education, so employers can learn the agent's type by observing their education level.
  - Signal is credible: only high ability workers will be willing to get more years of education.

Setup

- Two types of workers: $\theta_H > \theta_L$ (high ability and low ability)
  - A worker of type θ produces output which is worth θ to the employer.

  There is a fraction $\lambda \in (0,1)$ of low type-workers in the market.
- A worker can obtain education level e, which is perfectly observable.
  - Assume that e does not affect the worker's productivity θ.
- Cost of getting education $e \geq 0$ for a type θ worker is $c(e,\theta)$.
- We assume that:
  - $c(0,\theta) = 0,\ c_e(e,\theta) > 0,\ c_{ee}(e,\theta) \geq 0$ for $\theta \in \{\theta_L, \theta_H\}$.

- $c_e(e,\theta_H) < c_e(e,\theta_L)$ for all e.
- Second condition implies that getting more education is more costly for low ability workers than for high ability workers.

◦ *Goal:* Can high types credibly signal to employers their quality by getting more education than low types.

◦ If a type θ worker gets a wage w and education e, her utility is

$$w - c(e,\theta).$$

◦ Assume that

1. the outside option of both types of workers is zero ; and
2. there is perfect competition among employers (and so they earn zero profits in expectation).

Benchmark

◦ Suppose first that workers cannot get education.

- In this case there is no signaling.

◦ Equilibrium in this case:

$$w = \mathbb{E}[\theta] = \lambda\theta_L + (1-\lambda)\theta_H$$

- All workers accept this wage (recall that outside option is 0).
- Firms earn zero profits.
- High types get a low wage, and low types get a high wage (relative to their productivity).

Education

◦ Suppose now that workers can get education to signal their type.

◦ We consider the following game:

- The worker first learns her type. (Worker is type θ_H w.p $1-\lambda$ and type θ_L w.p λ.)

  - After learning her type, worker chooses education level $e \geq 0$.
  - All firms then observe e but not θ. Then they make wage offers to the worker.
  - Worker accepts (at most) one offer.

- In equilibrium, a worker who chooses e cannot gain by choosing a different $e' \neq e$.

- Let $\mu(e) = \Pr(\theta = \theta_H | e)$.
  - $\mu(e)$ is the belief of the firms when they observe a worker with education level e.
  - These beliefs are derived from the workers' strategies.

- Let $w(e)$ be the wage that firms offer when they see a worker with education e.
  - Zero profit condition implies that $w(e) = \mu(e)\theta_H + [1 - \mu(e)]\theta_L$.
  - Wages depend solely on beliefs.
  - If $\mu(e)$ is not constant in e, then workers with different education level will get different wages.

- Let $e(\theta)$ be the education level chosen by a worker with type θ.

- Two types of equilibria:
  - Separating equilibria: $e(\theta_H) \neq e(\theta_L)$.
  - Pooling equilibria: $e(\theta_H) = e(\theta_L)$.

- We will focus mainly on separating equilibria (but we will discuss pooling equilibria at the end).

Separating equilibria

- Recall that $e(\theta_H) \neq e(\theta_L)$ and $\mu(e) = \Pr(\theta = \theta_H | e)$.

- In a separating equilibrium, $\mu(e(\theta_H)) = 1$ and $\mu(e(\theta_L)) = 0$.
  - If firms observe $e(\theta_i)$, they know that worker has type θ_i.
  - Thus, workers signal their types through their education level.

- This implies that $w(e(\theta_H)) = \theta_H$ and $w(e(\theta_L)) = \theta_L$.

- This follows since $w(e) = \mu(e)\,\theta_H + [1 - \mu(e)]\,\theta_L$.

◦ We shall show that $e(\theta_L) = 0$.

- Towards a contradiction, suppose that $e(\theta_L) > 0$.
- If a type θ_L worker deviates and chooses $e = 0$, she gets wage

$$w(0) = \mu(0)\,\theta_H + [1 - \mu(e)]\,\theta_L \geq \theta_L$$

- The utility she gets from doing so is

$$w(0) - c(0, \theta_L) \geq \theta_L - c(0, \theta_L) > \theta_L - c(e(\theta_L), \theta_L),$$

so this worker strictly prefers to choose $e = 0$ than to choose $e = e(\theta_L) > 0$.

◦ Therefore, in a separating equilibrium, it must be that $e(\theta_L) = 0$.

◦ What about $e(\theta_H)$?

◦ This education level has to satisfy two constraints:

$$\left.\begin{array}{l} \theta_H - c(e(\theta_H), \theta_L) \leq \theta_L - c(0, \theta_L) = \theta_L \\ \theta_H - c(e(\theta_H), \theta_H) \geq \theta_L - c(0, \theta_H) = \theta_L \end{array}\right\}$$

- First constraint guarantees that a worker with type θ_H prefers to get education level $e(\theta_H)$ than $e(\theta_L)$.
- Second constraint guarantees that a worker with type θ_L prefers to get education level $e(\theta_L)$ than $e(\theta_H)$.

$$\Longrightarrow c(e(\theta_H), \theta_L) \geq \theta_H - \theta_L \geq c(e(\theta_H), \theta_H)$$

- Recall that $c(e, \theta_L) > c(e, \theta_L)$ for all e.

◦ There is a range $[\underline{e}, \overline{e}]$ of values of $e(\theta_H)$ that satisfy these two inequalities:

- $\underline{e}$ is such that $c(\underline{e}, \theta_L) = \theta_H - \theta_L$.

- $\overline{e}$ is such that $c(\overline{e}, \theta_H) = \theta_H - \theta_L$.

- Any $e(\theta_H) \in [\underline{e}, \overline{e}]$ can be supported as an equilibrium.
- One thing left: what values does $\mu(e)$ take for $e \neq \{e(\theta_H), e(\theta_L)\}$?
  - We cannot derive these beliefs from the workers actions.
  - If employers observe $e \neq \{e(\theta_H), e(\theta_L)\}$, what should they believe?
- Recall that $w(e) = \mu(e)\theta_H + [1 - \mu(e)]\theta_L$.
- For this to be an equilibrium, both types of workers must prefer to take their equilibrium actions:
  - type θ_H must prefer to get education $e(\theta_H)$ than any $e \neq e(\theta_H)$.
  - type θ_L must prefer to get education $e(\theta_L)$ than any $e \neq e(\theta_L)$.
- We specify $\mu(e)$ so that workers don't want to deviate:
  - Different ways in which we can do this.
  - *Idea:* "punish" workers with $e \neq \{e(\theta_H), e(\theta_L)\}$ by making $\mu(e)$ small.

Pooling Equilibria

- In a pooling equilibrium, $e(\theta_H) = e(\theta_L) = e^*$.
  - All workers get the same education level.
- Zero profits by firms imply that $w(e^*) = (1 - \lambda)\theta_H + \lambda\theta_L$.
  - This implies that $\mu(e^*) = 1 - \lambda$.
- In an equilibrium, no type of worker must benefit from choosing $e \neq e^*$.
  - Need to specify $\mu(e)$ so that neither high types nor low types have a profitable deviation.

References

Bolton and Dewatripont, (2005), *Contract Theory*, MIT Press.

Ortner J., (2013), Lecture Notes.