Module 12: Holdup Problem

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Standard Holdup Problem

- Canonical model by Hart and Moore (1988)
- 2 contracting parties: A buyer and seller can trade a quantity $q \in [0,1]$ at a price P.
- Buyer's valuation v and seller's production cost c are uncertain when contracting takes place and can be influenced by investments

$$\begin{aligned} \text{Buyer: } v &\in \{v_L, v_H\} \text{ and } \Pr(v_H) = j \text{ at cost } \psi(j) \\ \text{Seller: } c &\in \{c_L, c_H\} \text{ and } \Pr(c_L) = i \text{ at cost } \phi(i) \end{aligned}$$

- For example:
  - Buyer invests in marketing to increase price that he can sell the good at.
  - Seller invests in modern infrastructure to reduce production cost.
- Ex-post payoff levels are

$$\begin{aligned} \text{Buyer: } &\quad vq - P - \psi(j) \\ \text{Seller: } &\quad P - cq - \phi(i) \end{aligned}$$

- *Timing:*
  1. The buyer and the seller contract.
     - Contract specifies quantity q to be traded at price P.
  2. Each party simultaneously chooses his investment level i and j.
  3. Both parties learn state of nature $\theta = (v, c)$.
  4. The contract is executed (possibly after renegotiation).

First Best

- Assume that

$$c_H > v_H > c_L > v_L$$

- *Ex-post efficient* level of trade is

  - $q = 1$ if $\theta = (v_H, c_L)$
  - $q = 0$ otherwise

- The total expected surplus is given by

$$\max_{i,j} \{ij\,(v_H - c_L) - \psi\,(j) - \phi\,(i)\}$$

so the first best investment levels satisfy

$$\begin{aligned} i^{fb}\,(v_H - c_L) &= \psi'\left(j^{fb}\right) \\ j^{fb}\,(v_H - c_L) &= \phi'\left(i^{fb}\right) \end{aligned}$$

Nash Equilibrium

- θ is observable to both parties ex-post, but it is not contractable ex-ante, nor are the investment levels i and j.

- Assume that ex-post bargaining gives each party half of the surplus.

- The buyer solves

$$\max_j \left\{\frac{1}{2} i^* j\,(v_H - c_L) - \psi\,(j)\right\}$$

while the seller solves

$$\max_i \left\{\frac{1}{2} i\,j^*\,(v_H - c_L) - \phi\,(i)\right\}$$

- So in equilibrium, they choose

$$\frac{1}{2} i^*\,(v_H - c_L) = \psi'\,(j^*) \qquad \text{and} \qquad \frac{1}{2} j^*\,(v_H - c_L) = \phi'\,(j^*)$$

- Clearly, $i^* < i^{fb}$ and $j^* < j^{fb}$, due to "moral-hazard-in-teams".

- Solutions ?

  - Can we formulate an optimal long-term contract *independent of* θ that mitigates underinvestment?

Default Options

- Define level of trade $\tilde{q}$ such that

$$\tilde{q}\left(c_H - c_L\right) = \phi'\left(i^{fb}\right)$$

- Consider the following contractual mechanism (after the state of nature θ is revealed):

  1. Buyer makes a take-it-or-leave-it offer (P, q).
  2. Seller accepts (P, q), or rejects it, in which case $\tilde{q}$ is traded at price $\tilde{P}$.
     - $\tilde{P}$ chosen to share the ex-ante surplus according to bargaining weights.

- We will show that this mechanism implements first best!

- Buyer will offer (P, q) such that seller is indifferent between accepting the offer and rejecting it.

- Seller always expects to obtain the default option payoff so he solves

$$\max_i \left\{\tilde{P} - ic_L\tilde{q} - (1-i)\,c_H\tilde{q} - \phi(i)\right\}$$

  First order condition: $\tilde{q}\left(c_H - c_L\right) = \phi'(i)$, so that $i = i^{fb}$.

- Buyer maximizes

$$\max_j \left\{ \underbrace{i^{fb} j\left(v_H - c_L\right)}_{\text{total surplus}} - \underbrace{\left[\tilde{P} - i^{fb}c_L\tilde{q} - \left(1 - i^{fb}\right)c_H\tilde{q}\right]}_{\text{seller's payoff}} - \psi(j) \right\}$$

  First order condition: $i^{fb}\left(v_H - c_L\right) = \psi'(j)$, so that $j = j^{fb}$.

- *Lesson:* By choosing $\tilde{q}$ appropriately, it is possible to induce $\left(i^{fb}, j^{fb}\right)$.

- *Comments:*

- Investment efficiency for the buyer since he's the residual claimant ...
- ... but why is there investment efficiency for the seller who has no bargaining power at all?
- Incentive to invest comes from availability of default option, which becomes more attractive when cost is c_L and this can be influenced through i.

References

Che Y.K. and Hausch D.B., (1999), "Cooperative Investments and the Value of Contracting", *American Economic Review.*

Ederer F., (2011), Lecture Notes.

Hart O. and Moore J., (1988), "Incomplete Contracts and Renegotiation", *Econometrica.*