Module 10: Relational Contracts

Information Economics (Ec 515) · George Georgiadis

- Insofar, output has been assumed to be
  - observable and *verifiable; i.e.*, it can be enforced by a court of law ;
  - or not observable at all (by the agent).
- Both are extreme assumptions!
  - Performance evaluation is often based on "soft information".
- Suppose that output is observable to both parties, but not verifiable.
- Parties can use repeated interaction to ensure that the firm pays for good performance.

A Simple Model of Discretionary Bonuses

- Time $t \in \mathbb{N}$.
- Output is binary: $x_t \in \{H, L\}$.
- In each period t, agent chooses effort $a_t = \Pr\{x_t = H\} \in [0, 1]$.
  - Cost of effort $c\frac{a_t^2}{2}$, where $c < H - L$.
- Compensation consists of base salary s and bonus b that principal "promises" to pay if $x_t = H$.
  - Both parties observe x_t, but it cannot be contracted on, so the principal can renege on her promise to pay b.
- Both parties are risk neutral and discount rate is $\delta \leq 1$.
  - The larger δ is, the more the parties "care" about the future.

Timing: In each period t

1. Principal offers a compensation package $\{s, b\}$.
   - $\{s, b\}$ is chosen to maximize her expected discounted profit.
2. Agent accepts or rejects it in favor of alternative employment $\bar{U} = L$.
3. If the agent accepts, he exerts effort $a_t \in [0, 1]$ at cost $c\frac{a_t^2}{2}$.
   - The principal does not observe the agent's effort choice.
4. Both parties observe x_t.
5. If $x_t = H$, the principal chooses whether to pay the agent bonus b.

First best outcome:

- Total surplus:

$$S(a_t) = \underbrace{(1 - a_t)L + a_t H}_{\text{expected payoff}} - \underbrace{c\frac{a_t^2}{2}}_{\text{effort cost}}$$

- Principal chooses effort: $a_t \in \max\left\{L + a(H - L) - c\frac{a^2}{2}\right\}$
  - First order condition: $(H - L) - ca > 0$ for all $a \in [0, 1] \Longrightarrow a^{fb} = 1$.
  - Pays salary $s = L$ and $b = 0$.

One-shot Game (no repetition):

- Principal will renege on promise to pay bonus b.
- Agent will choose effort $a = 0$ and produce output $x_t = L$.
- Principal will pay salary $s = L$, earning 0 profit.

Repeated Game

- The equilibrium of the one-shot game is also an equilibrium of the repeated game.
  - Can we construct another equilibrium where agent chooses higher effort?
- WLOG we can assume that the agent uses a grim-trigger strategy.

- This is the worst possible penalty here (Abreu, Pearce and Stachetti, JET, 1984).
- If the principal reneges on promise, then the agent chooses $a = 0$ forever after (giving the principal profit 0).

◦ Given contract $\{s, b\}$, if agent believes that principal will pay bonus:

$$a_t \in \arg\max_a \left\{ s + ab - c\frac{a^2}{2} \right\}$$

- First order condition: $a^*(b) = \frac{b}{c}$ (for $b \leq c$).

◦ Agent will accept contract if $s + a^*(b)\, b - c\frac{(a^*(b))^2}{2} \geq L$.

- Principal will offer minimum salary that agent will accept: $s + a^*(b)\, b = L - \frac{b^2}{2c}$

◦ Then the principal's expected profit (per period) is

$$V(b) = \underbrace{L + a^*(b)(H - L)}_{\text{net profit}} - \underbrace{[s + a^*(b)\, b]}_{\text{payroll cost}} = \frac{b(H-L)}{c} - \frac{b^2}{2c}$$

◦ Will the principal choose to pay the bonus if $x_t = H$? Yes if

$$\begin{aligned} (H - s - b) + \delta V(b) &\geq (H - s) + \delta 0 \\ \Longleftrightarrow b &\leq \delta V(b) \\ \Longleftrightarrow b &\leq 2\left(H - L - \frac{c}{\delta}\right) \end{aligned}$$

- The cost of paying the bonus is $\$b$.
- The cost of not paying the bonus is $\$\,\delta V(b)$.

◦ Principal solves

$$\begin{aligned} \max_b \quad & \frac{b(H-L)}{c} - \frac{b^2}{2c} \\ \text{s.t.} \quad & b \leq 2\left(H - L - \frac{c}{\delta}\right) \end{aligned}$$

- First order condition: $\frac{(H-L)-b}{c} > 0$ for all $b \leq c$.
- Solution: $b^* = \min\left\{c,\, 2\left(H - L - \frac{c}{\delta}\right)\right\}$
- Observe that b^* increases in δ.

Lessons Learned

- In a long-term relationship, the principal has a reputation to protect.
- By reneging on her promise to pay the bonus, she looses her reputation, and the agent will not exert effort in the future.
- The value of reputation increases in her patience (*i.e.*, in δ).
- Principal would like to promise bonus $b = c$ (to induce first best effort), but this promise is not credible unless she is patient enough.

References

Baker G., Gibbons R., and Murphy K. J., (1994), "Subjective performance measures in optimal incentive contracts", *Quarterly Journal of Economics.*

Board S., (2011), Lecture Notes.

Ederer F., (2011), Lecture Notes.

Gibbons R., (2005), "Incentives Between Firms (and Within)", *Management Science.*

Levin J., (2003), "Relational Incentive Contracts", *American Economic Review.*