Module 10: Relational Contracts

Information Economics (Ec 515) · George Georgiadis

- Insofar, output has been assumed to be
  - observable and *verifiable; i.e.*, it can be enforced by a court of law ;
  - or not observable at all (by the agent).
- Both are extreme assumptions!
  - Performance evaluation is often based on "soft information".
- Suppose that output is observable to both parties, but not verifiable.
- Parties can use repeated interaction to ensure that the firm pays for good performance.

A Simple Model of Discretionary Bonuses

- Time $t \in \mathbb{N}$.
- Output is binary: $x_t \in \{H, L\}$.
- In each period t, agent chooses effort $a_t = \Pr\{x_t = H\} \in [0, 1]$.
  - Cost of effort $c\frac{a_t^2}{2}$, where $c < H - L$.
- Compensation consists of base salary s and bonus b that principal "promises" to pay if $x_t = H$.
  - Both parties observe x_t, but it cannot be contracted on, so the principal can renege on her promise to pay b.
- Both parties are risk neutral and discount rate is $\delta \leq 1$.
  - The larger δ is, the more the parties "care" about the future.

Timing: In each period t

1. Principal offers a compensation package $\{s, b\}$.
   - $\{s, b\}$ is chosen to maximize her expected discounted profit.
2. Agent accepts or rejects it in favor of alternative employment $\bar{U} = L$.
3. If the agent accepts, he exerts effort $a_t \in [0, 1]$ at cost $c\frac{a_t^2}{2}$.
   - The principal does not observe the agent's effort choice.
4. Both parties observe x_t.
5. If $x_t = H$, the principal chooses whether to pay the agent bonus b.

First best outcome:

- Total surplus:

$$S(a_t) = \underbrace{(1 - a_t) L + a_t H}_{\text{expected payoff}} - \underbrace{c\frac{a_t^2}{2}}_{\text{effort cost}}$$

- Principal chooses effort: $a_t \in \max\left\{L + a(H - L) - c\frac{a^2}{2}\right\}$
  - First order condition: $(H - L) - ca > 0$ for all $a \in [0, 1] \Longrightarrow a^{fb} = 1$.
  - Pays salary $s = L$ and $b = 0$.

One-shot Game (no repetition):

- Principal will renege on promise to pay bonus b.
- Agent will choose effort $a = 0$ and produce output $x_t = L$.
- Principal will pay salary $s = L$, earning 0 profit.

Repeated Game

- The equilibrium of the one-shot game is also an equilibrium of the repeated game.
  - Can we construct another equilibrium where agent chooses higher effort?
- WLOG we can assume that the agent uses a grim-trigger strategy.

- This is the worst possible penalty here (Abreu, Pearce and Stachetti, JET, 1984).
- If the principal reneges on promise, then the agent chooses $a = 0$ forever after (giving the principal profit 0).

◦ Given contract $\{s, b\}$, if agent believes that principal will pay bonus:

$$a_t \in \arg\max_a \left\{ s + ab - c\frac{a^2}{2} \right\}$$

- First order condition: $a^*(b) = \frac{b}{c}$ (for $b \leq c$).

◦ Agent will accept contract if $s + a^*(b)\, b - c\frac{(a^*(b))^2}{2} \geq L$.

- Principal will offer minimum salary that agent will accept: $s + a^*(b)\, b = L - \frac{b^2}{2c}$

◦ Then the principal's expected profit (per period) is

$$V(b) = \underbrace{L + a^*(b)(H - L)}_{\text{net profit}} - \underbrace{[s + a^*(b)\, b]}_{\text{payroll cost}} = \frac{b(H - L)}{c} - \frac{b^2}{2c}$$

◦ Will the principal choose to pay the bonus if $x_t = H$? Yes if

$$\begin{aligned} (H - s - b) + \delta V(b) &\geq (H - s) + \delta 0 \\ \iff b &\leq \delta V(b) \\ \iff b &\leq 2\left(H - L - \frac{c}{\delta}\right) \end{aligned}$$

- The cost of paying the bonus is $\$b$.
- The cost of not paying the bonus is $\$\,\delta V(b)$.

◦ Principal solves

$$\begin{aligned} \max_b \quad & \frac{b(H - L)}{c} - \frac{b^2}{2c} \\ \text{s.t.} \quad & b \leq 2\left(H - L - \frac{c}{\delta}\right) \end{aligned}$$

- First order condition: $\frac{(H-L)-b}{c} > 0$ for all $b \leq c$.
- Solution: $b^* = \min\left\{c,\, 2\left(H - L - \frac{c}{\delta}\right)\right\}$
- Observe that b^* increases in δ.

Lessons Learned

- In a long-term relationship, the principal has a reputation to protect.
- By reneging on her promise to pay the bonus, she looses her reputation, and the agent will not exert effort in the future.
- The value of reputation increases in her patience (*i.e.*, in δ).
- Principal would like to promise bonus $b = c$ (to induce first best effort), but this promise is not credible unless she is patient enough.

References

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