Working to Learn

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Motivation

- Studies of contracting problems typically focus on relationships in which labor is traded for money.
- In many relationships, labor traded (at least partly) for knowledge.
- Such arrangements have been popular since the middle ages.
  - Austria: ~40% of teens start apprenticeship after compulsory education.
  - Germany: ~50% of young adults have completed an apprenticeship.
  - USA: An apprenticeship (work experience) is required to be licensed as an engineer, doctor, accountant, lawyer (some states), etc...
- Often informal: research assistants, junior consultants, law interns, etc.
- Very common for blue collar and white collar professions alike.
Overview: Setting

- Dynamic agency model with four key ingredients:
  
  i. Principal trades knowledge (and money) for labor
  
  ii. Agent cannot commit to stay in the relationship after being trained
  
  iii. Knowledge transmission requires time and effort
      
      - Rate of knowledge transmission is subject to an upper bound
      - The principal can withhold knowledge (extending the apprenticeship)
  
  iv. Agent values consumption smoothing
Overview: Results

Features of the optimal contract:

- *During the early stages*, the learning constraint binds (*i.e.*, the agent is trained as fast as possible), and is paid a constant wage.

- *During the later stages*, the agent is trained at an artificially slow rate, and is paid a progressively higher wage.

- Slow-training phase exists even if agent has a lot of bargaining power.

- Effort decreases over time, but is always above the static first-best.

Consistent with anecdotal evidence from apprenticeships

- *e.g.*, PhD students get a lot of training during first years + a stipend, but later on, are required to work as research and teaching assistants.
Model (1/2)

- Continuous-time contracting game between a principal and an agent
- Agent combines knowledge $X_t$ and effort $a_t$ to generate flow output
  \[ y_t = f(X_t) + a_t \]
- Principal transfers knowledge to the agent at rate $z_t = \dot{X}_t$
  - Initially, $X_0 = X$, and the principal has total knowledge stock $\bar{X}$
  - Knowledge transfer is subject to a learning constraint: $z_t \leq L(X_t, a_t)$
- At time 0, principal offers a contract $C = \{w_t, a_t, z_t\}_{t \geq 0}$ specifying:
  - Wages $w_t \geq \underline{w}$,
  - contractible effort $a_t \in [0, \bar{a}]$, and
  - training rate $z_t \in [0, L(X_t, a_t)]$
At any $\tau$, the agent can walk away with his current knowledge, $X_\tau$, and consume his output (i.e., $y_t = f(X_\tau) + a_t$) in perpetuity.

The agent’s continuation payoff at $t$ is

$$v_t = r \int_t^\infty e^{-r(s-t)} [u(c_s) - d(a_s)] \, ds,$$

where

$$c_t = \begin{cases} w_t & \text{if } t \leq \tau \\ f(X_\tau) + a_t & \text{if } t > \tau \end{cases}$$

The principal’s payoff is

$$\Pi = r \int_0^\tau e^{-rt} [f(X_t) + a_t - w_t] \, dt$$

At $t = 0$, agent has outside option $v$ & principal has outside option 0.

$v$ is a measure of the agent’s bargaining power at the contracting stage.
Definitions

- **Definition:**
  \[
  \eta(X) := \max_{a \in [0, \bar{a}]} \ r \int_{0}^{\infty} e^{-rt} \left[ u(f(X) + a) - d(a) \right] dt
  \]

  is the agent’s exit payoff if he walks away with knowledge \( X \).

  - We denote by \( a(X) \) the corresponding effort given \( X \).

  - Normalize \( u'(f(\overline{X}) + a(\overline{X})) = 1 \)
Assumptions

- **A.1.** \( f, u, \) and \( d \) are twice continuously differentiable with

\[
\begin{align*}
    f'(X) &> 0 > f''(X) \\
u'(w) &> 0 > u''(w) \\
d'(0) &= 0 \text{ and } d''(a) > 0
\end{align*}
\]

- **A.2.** \( L(X, a) \) is additively separable and strictly positive with

\[
L_a \geq 0, \ L_{XX} \leq 0, \ \text{and} \ L_{aa} \leq 0
\]

- **A.3.** All functions have bounded first and second derivatives

- **A.4.** For all \( X \) and \( a \),

\[
L(X, a) \eta'(X)/r > \eta(X) - u(w) + d(a)
\]

i.e., the principal can train the agent at a rate such that PC is slack

- **A.5.** There is a feasible contract giving principal nonnegative profit
**Theorem 1.** Agent-first-best contract
- Maximizes agent’s payoff subject to principal’s participation constraint
- Assumes the agent can commit to not walk away

**Theorem 2.** Optimal contract
- Contract which maximizes the principal’s payoff subject to constraints

**Theorem 3.** Regulating apprenticeships
- Suppose planner wishes to make the agent-first-best contract incentive compatible for the principal
A Benchmark: Agent-first-best Contract

Consider the following benchmark problem:

\[
\text{max } \int_{0}^{\infty} e^{-rt} [u(w_t) - d(a_t)] \, dt
\]

subject to

- the learning constraint
  \[
  \dot{X}_t = z_t \leq L(X_t, a_t),
  \]

- a credit balance constraint
  \[
  \int_{0}^{\infty} e^{-rt} [f(X_t) + a_t - w_t] \, dt \geq 0,
  \]

- the constraint \( X_t \leq \overline{X}. \) (Let’s ignore the constraints on \( w_t \) and \( a_t \))

**Interpretation:** Agent can borrow (at interest rate \( r \)), can choose his training rate, and can commit to not walk away.
Agent-first-best Contract

Theorem 1. Agent-first-best contract
There exists an agent-optimal contract and a knowledge path $X_t$ such that

- the agent’s training rate is

$$z_t = \begin{cases} L(X_t, a_t) & \text{if } X_t < \overline{X}, \text{ and} \\ 0 & \text{if } X_t = \overline{X}, \end{cases}$$

- he receives a constant wage $w^*$, and

- his effort path satisfies

$$d'(a_t) = u'(w^*) \left[ 1 + \mu_t L_a(X_t, a_t) \right]$$

for some explicitly defined function $\mu_t > 0$.

- The agent overworks to relax learning constraint; i.e., $a_t > a(X_t)$.  

Agent-first-best Contract: Intuition for training rate ($z_t$)

- We have the following optimal control problem:

$$\max \quad \int_0^\infty e^{-rt} \left[ u(w_t) - d(a_t) \right] dt$$

s.t. 

$$\dot{X}_t = z_t$$

$$z_t \leq L(X_t, a_t)$$

$$\int_0^\infty e^{-rt} \left[ f(X_t) + a_t - w_t \right] dt \geq 0$$

$$X_0 = X \quad \text{and} \quad X_t \leq \bar{X}$$

- Increasing $X$ relaxes the credit balance constraint. Thus, in any optimal contract, the agent is trained at the maximum rate:

$$z_t = \begin{cases} 
L(X_t, a_t) & \text{if } X_t < \bar{X}, \text{ and} \\
0 & \text{otherwise}
\end{cases}$$
Agent-first-best Contract: Intuition for wage ($w_t$)

We have the following optimal control problem:

$$\max \int_0^\infty e^{-rt} [u(w_t) - d(a_t)] \, dt$$

s.t. $\dot{X}_t = z_t$

$$z_t \leq L(X_t, a_t)$$

$$\int_0^\infty e^{-rt} [f(X_t) + a_t - w_t] \, dt \geq 0$$

$$X_0 = X \quad \text{and} \quad X_t \leq \bar{X}$$

In any optimal contract, agent's consumption is constant; i.e.,

$$w_t = w^* \quad \text{for all} \ t$$
Agent-first-best Contract: Intuition for effort \((a_t)\)

- We have the following optimal control problem:

\[
\max \int_0^\infty e^{-rt} \left[ u(w_t) - d(a_t) \right] dt \\
\text{s.t.} \quad \dot{X}_t = z_t \\
\quad \quad z_t \leq L(X_t, a_t) \\
\quad \quad \int_0^\infty e^{-rt} \left[ f(X_t) + a_t - w_t \right] dt \geq 0 \\
\quad \quad X_0 = \underline{X} \quad \text{and} \quad X_t \leq \overline{X}
\]

- In choosing effort, agent trades off its marginal cost and the marginal benefit of (a) consuming more, and (b) relaxing learning constraint:

\[
d'(a_t) = u'(w^*) + (\text{marg. benefit of increasing } L(X_t, a_t))
\]
We now characterize the optimal contract in the original problem:

- **Phase 1:** Resembles agent-optimal contract; i.e., agent is
  - trained at the technologically constrained rate; i.e., \( z_t = L(X_t, a_t) \),
  - is paid a constant wage, and
  - is *overworked*.

- **Phase 2:** The agent
  - is trained at the slowest rate such that he doesn’t walk away,
  - is paid a progressively increasing wage, and
  - exerts a progressively lower effort.
Agent’s Payoff

- The agent’s continuation payoff at $t$ can be written as

$$v_t = r \int_t^\tau e^{-r(s-t)} [u(w_s) - d(a_s)] \, ds + e^{-r\tau} \eta(X_\tau)$$

i.e., flow payoffs are dictated by the contract until graduation date $\tau$, at which moment the agent earns his exit payoff, $\eta(X_\tau)$.

- For $t < \tau$, $v_t$ can equivalently be rewritten in differential form as

$$\dot{v}_t = r [v_t - u(w_t) + d(a_t)]$$

- Thus, a contract must satisfy
  - the *initial participation constraint* $v_0 \geq v$, and
  - the *ongoing participation constraint* $v_t \geq \eta(X_t)$ for all $t \leq \tau$
Principal’s Problem

- The principal chooses a contract \( \{ w_t, a_t, z_t \}_{t=0}^{\tau} \) to maximize

\[
    r \int_0^{\tau} e^{-rt} [f(X_t) + a_t - w_t] \, dt
\]

subject to

- the dynamic constraints

\[
    \dot{X}_t = z_t \quad \text{and} \quad \dot{v}_t = r [v_t - u(w_t) + d(a_t)] ,
\]

- the agent’s learning constraint

\[
    z_t \leq L(X_t, a_t) ,
\]

- the agent’s initial and ongoing participation constraints

\[
    v_0 \geq v \quad \text{and} \quad v_t \geq \eta(X_t) \quad \text{for all} \quad t \leq \tau ,
\]

- the knowledge constraint \( X_t \leq \bar{X} \),

- the boundary condition \( v_\tau = \eta(X_\tau) \),

- and the constraints on the controls: \( w_t \geq \underline{w} , \ a_t \in [0, \bar{a}] , \) and \( z_t \geq 0 \).
Towards a solution

- Because the problem is linear in $z_t$, we cannot pin down the optimal $z_t$ using first-order conditions — we need an educated guess & verify

Conjecture 1: In an optimal contract,
- the learning constraint binds; i.e., $z_t = L(X_t, a_t)$, or
- the participation constraint binds; i.e., $v_t = \eta(X_t)$.

- *Intuitively*, if both constraints are slack, can increase $z_t$ slightly so that all constraints are still satisfied and the principal is better off.

- Define the zero-rent training rate

$$
\phi(X, w, a) = \frac{\eta(X) - u(w) + d(a)}{\eta'(X)/r}
$$

- If $v_t = \eta(X_t)$ and $z_t = \phi(X_t, w_t, a_t)$, then $v_{t+dt} = \eta(X_{t+dt})$
Towards a solution (Cont’d)

Conjecture 2: Optimal contract comprises two phases:

- **Phase 1** \([0, \theta)\): Learning constraint binds; i.e., \(z_t = L(X_t, a_t)\)

- **Phase 2** \((\theta, \tau)\): Participation constraint binds; i.e., \(z_t = \phi(X_t, w_t, a_t)\)

- If principal wants to train agent so fast that he earns rents, prefers to do so early & then slow down training to profit from productive agent

- Define \(\theta\) to be the junction time between the two regimes.

- Given conjectured contract form, we can use a sufficiency theorem to characterize the optimal contract, and establish uniqueness.
Theorem 2: Unique Optimal Contract

There exists times $\theta$ and $T$ such that:

- **Phase 1.** For $t \in (0, \theta)$,
  - the agent is trained at technologically constrained rate $z_t = L(X_t, a_t)$,
  - receives a constant wage $w_\theta$, and
  - effort satisfies $d'(a_t) = u'(w_\theta) \left[ 1 + \mu_t L_a(X_t, a_t) \right]$.

- **Phase 2.** For $t \in (\theta, T)$,
  - the agent is trained at the zero-rent rate; i.e., $z_t = \phi(X_t, w_t, a_t)$,
  - receives a non-decreasing wage $w_t$, and
  - effort satisfies $d'(a_t) = u'(w_t)$.

At $T$, the agent becomes fully trained (i.e., $X_T = X$). Thereafter,

$$a_t = a(X) , \quad w_t = f(X) + a(X),$$

and he is indifferent between staying and walking away.
Optimal Contract: Illustration

Knowledge level $X_t$ and wage $w_t$ over time $t$.

Training rate $z_t$ and effort $a_t$ over time $t$.
During phase 1, the agent is trained as fast as the learning constraint allows, meanwhile earning rents …
... he is paid a subsistence wage, and his effort is distorted upwards relative to the static effort level when there is no learning.
During phase 2, the agent’s wage increases towards his steady-state post-graduation earnings, ...
... he is trained just fast enough that he doesn’t walk away,
and while his effort is still distorted upwards, this distortion vanishes as the apprenticeship nears its end.
Optimal Contract

Intuition: Phase 2

- During phase 1, the agent’s wages may be higher than the output he generates, placing him in the principal’s debt.
- Because the agent cannot commit to stay with the principal, phase 2 serves as an endogenous commitment device to repay this debt.
  - This contract is preferred to one in which agent is trained faster but wages are more backloaded (since he values consumption smoothing).

Corollary 1: Phase 2 is non-empty if knowledge is sufficiently valuable

- **Example:** If output \( y_t = \gamma f(X_t) + a_t \) and \( \gamma \) is sufficiently large, then phase 2 is non-empty; i.e., \( \theta < T \) (and threshold is independent of \( v \)).
When the agent’s outside option $v$ is small, phase 1 is relatively short, and the contract prescribes minimum subsistence wages.
As $v$ increases, duration of phase 1 (i.e., $\theta$) grows, contract prescribes higher wages, grants more rents, and phase 2 becomes truncated.
Even as $v$ grows large, phase 2 never disappears completely, as it allows principal to collect “debt” while smoothing agent consumption...
Optimal Regulation

- Suppose planner wishes to implement the agent-first-best contract:
  - Training at a rate such that the learning constraint binds
  - Constant wages; \( i.e., \) perfect consumption smoothing
- Let \( w_t^* \) and \( a_t^* \) denote the corresponding wage and effort path

**Theorem 3: Optimal regulation**

- Suppose principal can retain the agent for as long as she wishes, but:
  - wage path must be *at least as large as* \( w_t^* \), and
  - effort must be *no larger than* \( a_t^* \).
- Then the principal optimally implements the agent-optimal contract.

- Rationale for certification requirements and non-compete clauses
- Must be accompanied by restrictions on min. wages and max. effort
Discussion

- Simple model to study the exchange of labor for knowledge
  - **Key ingredient:** Knowledge transmission subject to upper bound, and agent cannot commit to stay in relationship after acquiring knowledge
  - Optimal contract features a phase of fast training and constant, low wages, followed by period of artificially slow training and rising wages.

- Next steps:
  - Certification requirements
  - Regulating apprenticeships
  - Moral hazard (hidden effort)
  - Adverse selection (principal learns the ability of apprentice)