Working to Learn

George Georgiadis (Northwestern Kellogg)

with Drew Fudenberg (MIT) & Luis Rayo (Northwestern Kellogg)

Motivation

- Studies of contracting problems typically focus on relationships in which *labor is traded for money*
- In many relationships, **labor traded** *(at least partly)* **for knowledge**
- Such arrangements have been popular since the middle ages
  - *Austria:* ~ 40% of teens start apprenticeship after compulsory education
  - *Germany:* ~ 50% of young adults have completed an apprenticeship
  - *USA:* An apprenticeship (work experience) is required to be licensed as an engineer, doctor, accountant, lawyer (some states), etc...
- Often informal: research assistants, junior consultants, law interns, etc
- Very common for blue collar and white collar professions alike

Overview: Setting

- Dynamic agency model with four key ingredients:
  i. Principal trades knowledge (and money) for labor
  ii. Agent cannot commit to stay in the relationship after being trained
  iii. Knowledge transmission requires time and effort
    - Rate of knowledge transmission is subject to an upper bound
    - The principal can withhold knowledge (extending the apprenticeship)
  iv. Agent values consumption smoothing

Overview: Results

- Features of the optimal contract:
  - *During the early stages*, the learning constraint binds (*i.e.*, the agent is trained as fast as possible), and is paid a constant wage
  - *During the later stages*, the agent is trained at an artificially slow rate, and is paid a progressively higher wage
  - Slow-training phase exists even if agent has a lot of bargaining power
  - Effort decreases over time, but is always above the static first-best
- Consistent with anecdotal evidence from apprenticeships
  - *e.g.*, PhD students get a lot of training during first years + a stipend, but later on, are required to work as research and teaching assistants

Model (1/2)

- Continuous-time contracting game between a principal and an agent
- Agent combines *knowledge* X_t and effort a_t to generate flow output

$$y_t = f(X_t) + a_t$$

- Principal transfers *knowledge* to the agent at rate $z_t = \dot{X}_t$
  - Initially, $X_0 = \underline{X}$, and the principal has total *knowledge stock* $\overline{X}$
  - Knowledge transfer is subject to a learning constraint: $z_t \leq L(X_t, a_t)$
- At time 0, principal offers a *contract* $\mathcal{C} = \{w_t, a_t, z_t\}_{t \geq 0}$ specifying:
  - Wages $w_t \geq \underline{w}$,
  - contractible effort $a_t \in [0, \overline{a}]$, and
  - training rate $z_t \in [0,\ L(X_t, a_t)]$

Model (2/2)

- At any τ, the agent can walk away with his current knowledge, X_τ, and consume his output ($i.e.$, $y_t = f(X_\tau) + a_t$) in perpetuity.

- The agent's continuation payoff at t is

$$v_t = r \int_t^\infty e^{-r(s-t)} \left[u(c_s) - d(a_s) \right] ds, \text{ where } c_t = \begin{cases} w_t & \text{if } t \leq \tau \\ f(X_\tau) + a_t & \text{if } t > \tau \end{cases}$$

- The principal's payoff is

$$\Pi = r \int_0^\tau e^{-rt} \left[f(X_t) + a_t - w_t \right] dt$$

- At $t = 0$, agent has outside option $\underline{v}$ & principal has outside option 0
  - $\underline{v}$ is a measure of the agent's bargaining power at the contracting stage

Definitions

- *Definition:*

$$\eta(X) := \max_{a \in [0,\overline{a}]} r \int_0^\infty e^{-rt} \left[u\left(f(X) + a\right) - d(a) \right] dt$$

is the agent's exit payoff if he walks away with knowledge X.

  - We denote by $a(X)$ the corresponding effort given X.

- Normalize $u'(f(\overline{X}) + a(\overline{X})) = 1$

Assumptions

- *A.1.* f, u, and d are twice continuously differentiable with

$$f'(X) > 0 > f''(X)$$
$$u'(w) > 0 > u''(w)$$
$$d'(0) = 0 \text{ and } d''(a) > 0$$

- *A.2.* $L(X, a)$ is additively separable and strictly positive with

$$L_a \geq 0 \ , \ L_{XX} \leq 0 \ , \text{ and } L_{aa} \leq 0$$

- *A.3.* All functions have bounded first and second derivatives

- *A.4.* For all X and a,

$$L(X, a)\eta'(X)/r > \eta(X) - u(\underline{w}) + d(a)$$

  i.e., the principal can train the agent at a rate such that PC is slack

- *A.5.* There is a feasible contract giving principal nonnegative profit

Outline

- *Theorem 1.* Agent-first-best contract
  - Maximizes agent's payoff subject to principal's participation constraint
  - Assumes the agent can commit to not walk away
- *Theorem 2.* Optimal contract
  - Contract which maximizes the principal's payoff subject to constraints
- *Theorem 3.* Regulating apprenticeships
  - Suppose planner wishes to make the agent-first-best contract incentive compatible for the principal

A Benchmark: Agent-first-best Contract

- Consider the following benchmark problem:

$$\max \int_0^\infty e^{-rt} \left[u(w_t) - d(a_t) \right] dt$$

subject to

- the learning constraint

$$\dot{X}_t = z_t \leq L(X_t, a_t) \,,$$

- a credit balance constraint

$$\int_0^\infty e^{-rt} \left[f(X_t) + a_t - w_t \right] dt \geq 0 \text{ , and}$$

- the constraint $X_t \leq \overline{X}$. (Let's ignore the constraints on w_t and a_t)

- *Interpretation:* Agent can borrow (at interest rate r), can choose his training rate, and can commit to not walk away.

Agent-first-best Contract

Theorem 1. Agent-first-best contract

There exists an agent-optimal contract and a knowledge path X_t such that

- the agent's training rate is

$$z_t = \begin{cases} L(X_t, a_t) & \text{if } X_t < \overline{X}\text{ , and} \\ 0 & \text{if } X_t = \overline{X}\text{ ,} \end{cases}$$

- he receives a constant wage w^*, and
- his effort path satisfies

$$d'(a_t) = u'(w^*)\left[1 + \mu_t L_a(X_t, a_t)\right]$$

for some explicitly defined function $\mu_t > 0$.

- The agent *overworks* to relax learning constraint; *i.e.*, $a_t > a(X_t)$.

Agent-first-best Contract: Intuition for training rate (z_t)

- We have the following optimal control problem:

$$
\begin{aligned}
\max \quad & \int_0^\infty e^{-rt}\left[u(w_t) - d(a_t)\right] dt \\
\text{s.t.} \quad & \dot{X}_t = z_t \\
& z_t \le L(X_t, a_t) \\
& \int_0^\infty e^{-rt}\left[f(X_t) + a_t - w_t\right] dt \ge 0 \\
& X_0 = \underline{X} \quad \text{and} \quad X_t \le \overline{X}
\end{aligned}
$$

- Increasing X relaxes the credit balance constraint. Thus, in any optimal contract, the agent is trained at the maximum rate:

$$
z_t = \begin{cases} L(X_t, a_t) & \text{if } X_t < \overline{X} \text{ , and} \\ 0 & \text{otherwise} \end{cases}
$$

Agent-first-best Contract: Intuition for wage (w_t)

- We have the following optimal control problem:

$$\begin{aligned} \max \quad & \int_0^\infty e^{-rt} \left[u(w_t) - d(a_t) \right] dt \\ \text{s.t.} \quad & \dot{X}_t = z_t \\ & z_t \leq L(X_t, a_t) \\ & \int_0^\infty e^{-rt} \left[f(X_t) + a_t - w_t \right] dt \geq 0 \\ & X_0 = \underline{X} \quad \text{and} \quad X_t \leq \overline{X} \end{aligned}$$

- In any optimal contract, agent's consumption is constant; *i.e.*,

$$w_t = w^* \quad \text{for all } t$$

Agent-first-best Contract: Intuition for effort (a_t)

- We have the following optimal control problem:

$$\begin{aligned} \max \quad & \int_0^\infty e^{-rt} \left[u(w_t) - d(a_t) \right] dt \\ \text{s.t.} \quad & \dot{X}_t = z_t \\ & z_t \leq L(X_t, a_t) \\ & \int_0^\infty e^{-rt} \left[f(X_t) + a_t - w_t \right] dt \geq 0 \\ & X_0 = \underline{X} \quad \text{and} \quad X_t \leq \overline{X} \end{aligned}$$

- In choosing effort, agent trades off its marginal cost and the marginal benefit of (a) consuming more, and (b) relaxing learning constraint:

$$d'(a_t) = u'(w^*) + (\text{marg. benefit of increasing } L(X_t, a_t))$$

Back to the Original Problem

- We now characterize the optimal contract in the original problem:
- Phase 1: Resembles agent-optimal contract; *i.e.*, agent is
  - trained at the technologically constrained rate; *i.e.*, $z_t = L(X_t, a_t)$,
  - is paid a constant wage, and
  - is *overworked*.
- Phase 2: The agent
  - is trained at the slowest rate such that he doesn't walk away,
  - is paid a progressively increasing wage, and
  - exerts a progressively lower effort.

Agent's Payoff

- The agent's continuation payoff at t can be written as

$$v_t = r \int_t^{\tau} e^{-r(s-t)} \left[u(w_s) - d(a_s) \right] ds + e^{-r\tau} \eta(X_\tau)$$

i.e., flow payoffs are dictated by the contract until graduation date τ, at which moment the agent earns his exit payoff, $\eta(X_\tau)$.

- For $t < \tau$, v_t can equivalently be rewritten in differential form as

$$\dot{v}_t = r \left[v_t - u(w_t) + d(a_t) \right]$$

- Thus, a contract must satisfy
  - the *initial participation constraint* $v_0 \geq \underline{v}$, and
  - the *ongoing participation constraint* $v_t \geq \eta(X_t)$ for all $t \leq \tau$

Principal's Problem

- The principal chooses a contract $\{w_t, a_t, z_t\}_{t=0}^{\tau}$ to

$$\text{maximize } r\int_0^{\tau} e^{-rt}\left[f(X_t) + a_t - w_t\right] dt \quad \text{subject to}$$

  - the dynamic constraints

$$\dot{X}_t = z_t \text{ and}$$
$$\dot{v}_t = r\left[v_t - u(w_t) + d(a_t)\right],$$

  - the agent's learning constraint

$$z_t \leq L(X_t, a_t),$$

  - the agent's initial and ongoing participation constraints

$$v_0 \geq \underline{v} \quad \text{and} \quad v_t \geq \eta(X_t) \quad \text{for all } t \leq \tau,$$

  - the knowledge constraint $X_t \leq \overline{X}$,
  - the boundary condition $v_\tau = \eta(X_\tau)$,
  - and the constraints on the controls: $w_t \geq \underline{w}$, $a_t \in [0, \overline{a}]$, and $z_t \geq 0$

Towards a solution

- Because the problem is linear in z_t, we cannot pin down the optimal z_t using first-order conditions — we need an educated guess & verify

Conjecture 1: In an optimal contract,

- the learning constraint binds; *i.e.*, $z_t = L(X_t, a_t)$, or
- the participation constraint binds; *i.e.*, $v_t = \eta(X_t)$.

- *Intuitively*, if both constraints are slack, can increase z_t slightly so that all constraints are still satisfied and the principal is better off.
- Define the **zero-rent training rate**

$$\phi(X, w, a) = \frac{\eta(X) - u(w) + d(a)}{\eta'(X)/r}$$

  - If $v_t = \eta(X_t)$ and $z_t = \phi(X_t, w_t, a_t)$, then $v_{t+dt} = \eta(X_{t+dt})$

Towards a solution (Cont'd)

Conjecture 2: Optimal contract comprises two phases:

- *Phase 1* $[0, \theta)$*:* Learning constraint binds; *i.e.*, $z_t = L(X_t, a_t)$
- *Phase 2* (θ, τ)*:* Participation constraint binds; *i.e.*, $z_t = \phi(X_t, w_t, a_t)$

- If principal wants to train agent so fast that he earns rents, prefers to do so early & then slow down training to profit from productive agent
- Define θ to be the *junction* time between the two regimes.
- Given conjectured contract form, we can use a sufficiency theorem to characterize the optimal contract, and establish uniqueness.

Theorem 2: Unique Optimal Contract

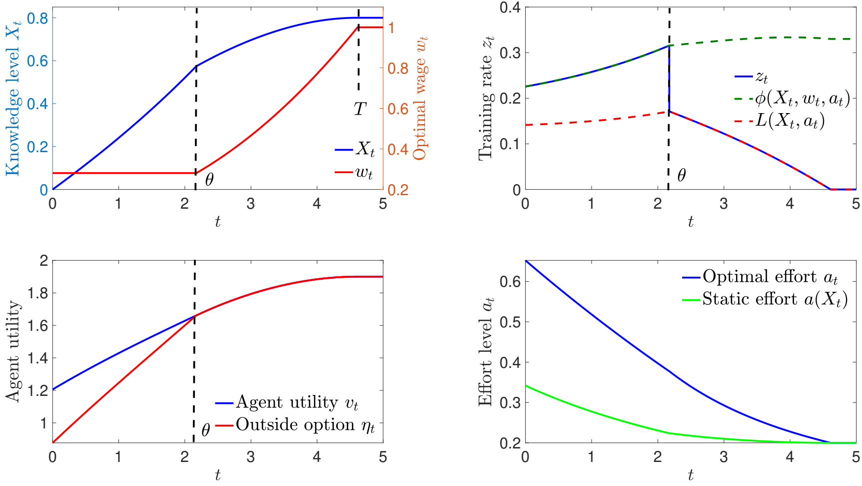
There exists times θ and T such that:

- *Phase 1.* For $t \in (0, \theta)$,
  - the agent is trained at technologically constrained rate $z_t = L(X_t, a_t)$,
  - receives a constant wage w_θ, and
  - effort satisfies $d'(a_t) = u'(w_\theta)\left[1 + \mu_t L_a(X_t, a_t)\right]$
- *Phase 2.* For $t \in (\theta, T)$,
  - the agent is trained at the zero-rent rate; *i.e.*, $z_t = \phi(X_t, w_t, a_t)$,
  - receives a non-decreasing wage w_t, and
  - effort satisfies $d'(a_t) = u'(w_t)$
- At T, the agent becomes fully trained (i.e., $X_T = \overline{X}$). Thereafter,

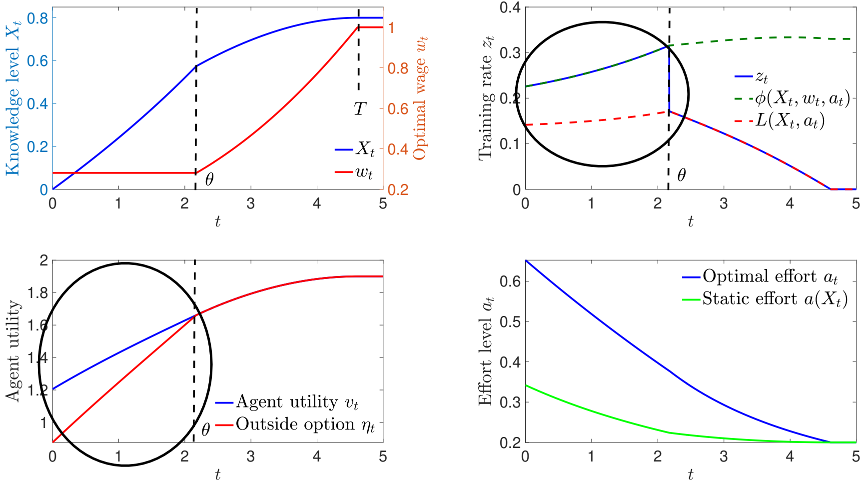
$$a_t = a(\overline{X}) \quad , \quad w_t = f(\overline{X}) + a(\overline{X}),$$

and he is indifferent between staying and walking away.

Optimal Contract: Illustration

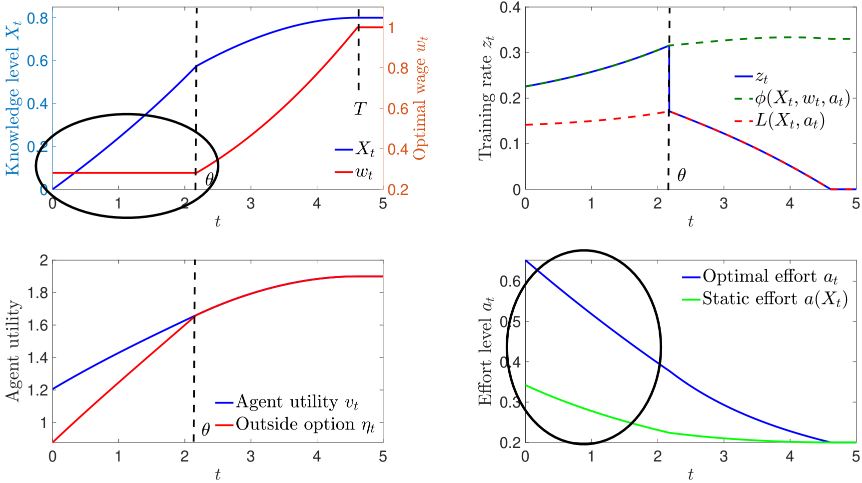


Optimal Contract: Illustration



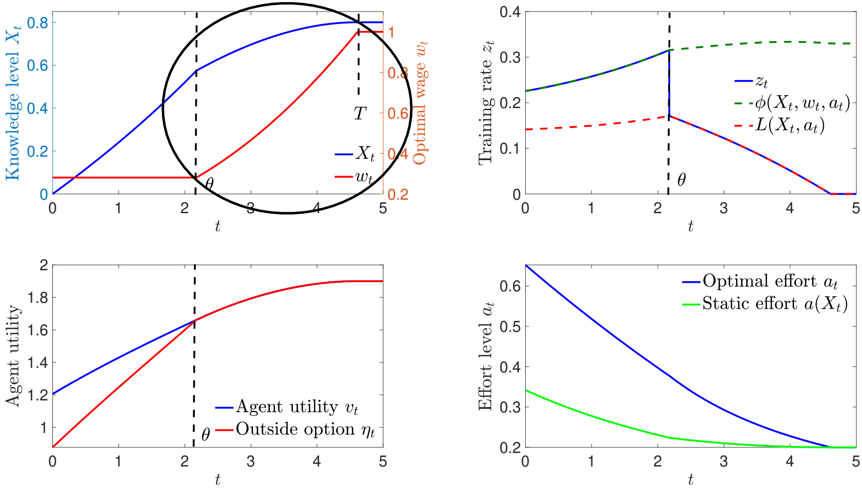
- During phase 1, the agent is trained as fast as the learning constraint allows, meanwhile earning rents ...

Optimal Contract: Illustration



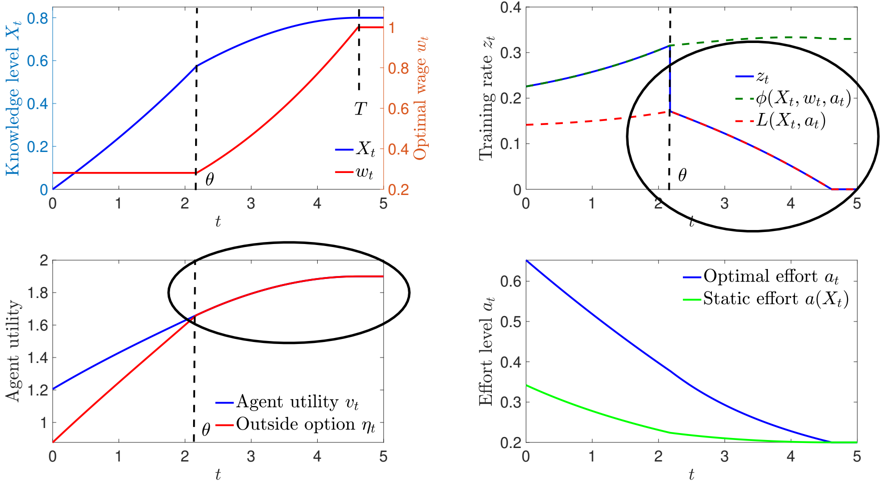
- ... he is paid a subsistence wage, and his effort is distorted upwards relative to the static effort level when there is no learning.

Optimal Contract: Illustration



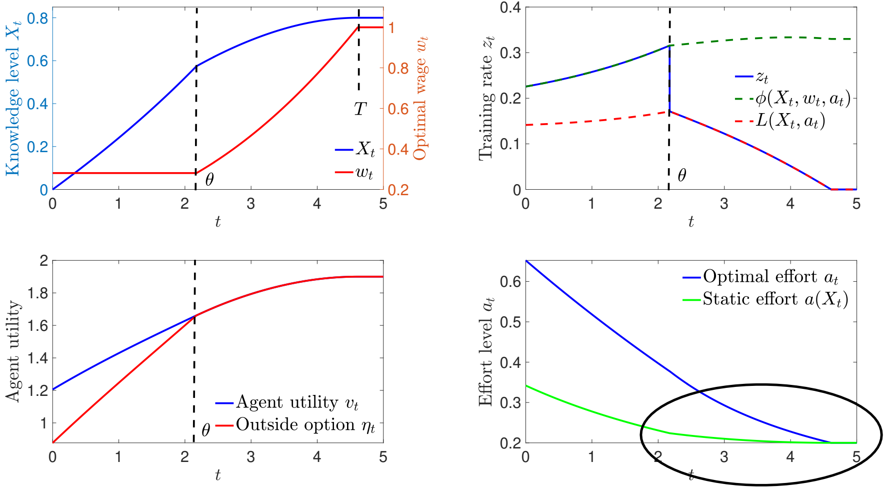
- During phase 2, the agent's wage increases towards his steady-state post-graduation earnings, ...

Optimal Contract: Illustration



- … he is trained just fast enough that he doesn't walk away,

Optimal Contract: Illustration



- and while his effort is still distorted upwards, this distortion vanishes as the apprenticeship nears its end

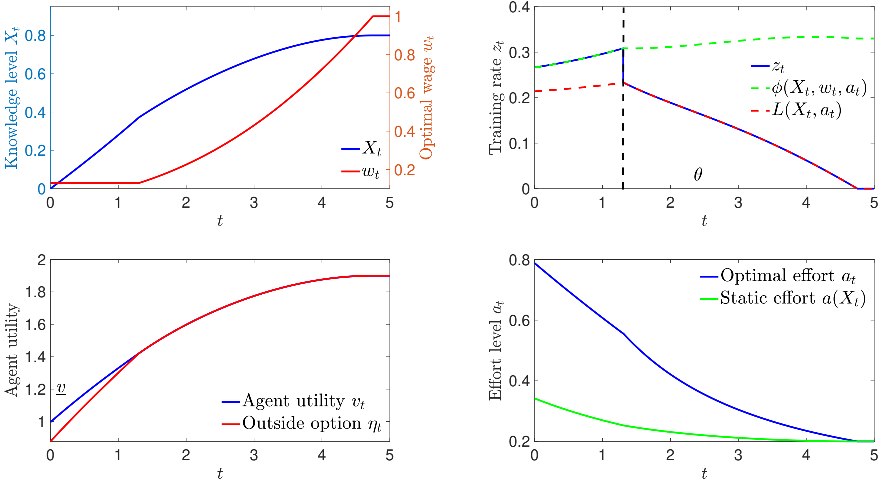
Intuition: Phase 2

- During phase 1, the agent's wages may be higher than the output he generates, placing him in the principal's debt
- Because the agent cannot commit to stay with the principal, phase 2 serves as an endogenous commitment device to repay this debt
    - This contract is preferred to one in which agent is trained faster but wages are more backloaded (since he values consumption smoothing)

Corollary 1: Phase 2 is non-empty if knowledge is sufficiently valuable

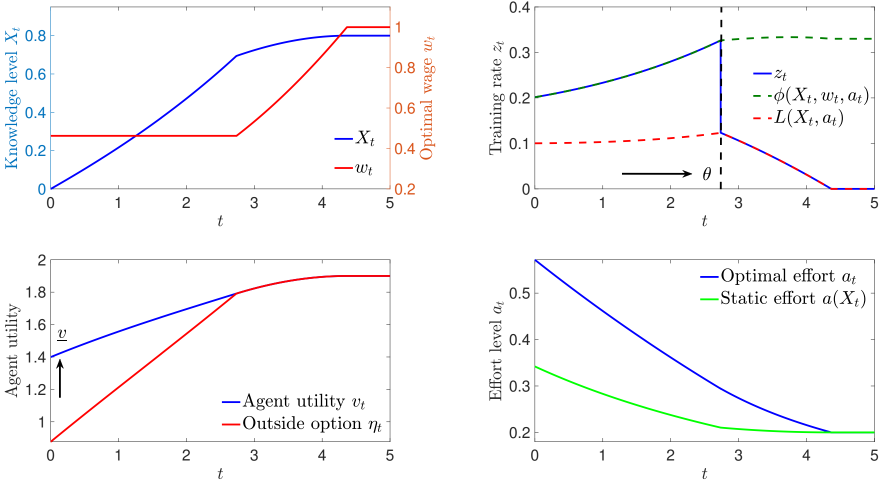
- *Example:* If output $y_t = \gamma f(X_t) + a_t$ and γ is sufficiently large, then phase 2 is non-empty; *i.e.*, $\theta < T$ (and threshold is independent of $\underline{v}$)

Comparative Statics: Agent's Outside Option



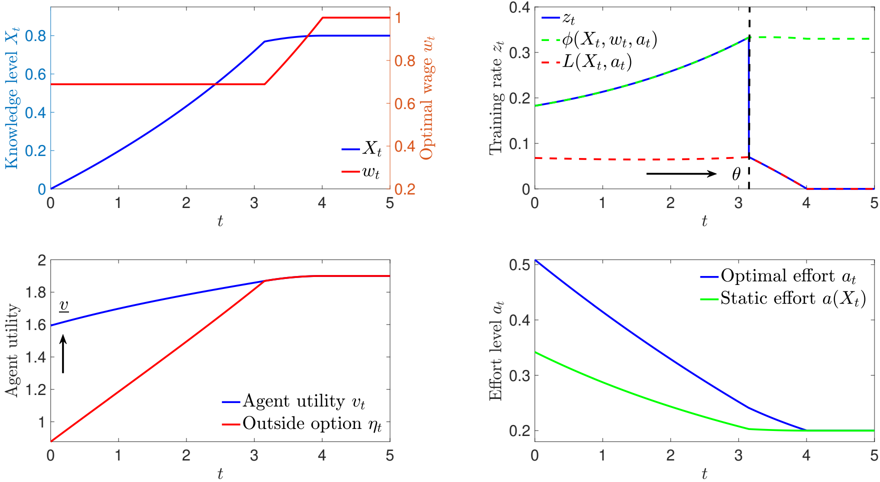
- When the agent's outside option $\underline{v}$ is small, phase 1 is relatively short, and the contract prescribes minimum subsistence wages

Comparative Statics: Agent's Outside Option



- As $\underline{v}$ increases, duration of phase 1 (*i.e.*, θ) grows, contract prescribes higher wages, grants more rents, and phase 2 becomes truncated

Comparative Statics: Agent's Outside Option



- Even as $\underline{v}$ grows large, phase 2 never disappears completely, as it is allows principal to collect "debt" while smoothing agent consumption

Optimal Regulation

- Suppose planner wishes to implement the agent-first-best contract:
  - Training at a rate such that the learning constraint binds
  - Constant wages; *i.e.*, perfect consumption smoothing
- Let w_t^* and a_t^* denote the corresponding wage and effort path

Theorem 3: Optimal regulation

- Suppose principal can retain the agent for as long as she wishes, but:
  - wage path must be *at least as large as* w_t^*, and
  - effort must be *no larger than* a_t^*.
- Then the principal optimally implements the agent-optimal contract.

- Rationale for certification requirements and non-compete clauses
- Must be accompanied by restrictions on min. wages and max. effort

Discussion

- Simple model to study the exchange of labor for knowledge
  - *Key ingredient:* Knowledge transmission subject to upper bound, and agent cannot commit to stay in relationship after acquiring knowledge
  - Optimal contract features a phase of fast training and constant, low wages, followed by period of artificially slow training and rising wages.
- Next steps:
  - Certification requirements
  - Regulating apprenticeships
  - Moral hazard (hidden effort)
  - Adverse selection (principal learns the ability of apprentice)