Optimal Contracts with a Risk-Taking Agent

George Georgiadis

Joint with Daniel Barron and Jeroen Swinkels (Northwestern Kellogg)

Kellogg School of Management, Northwestern University

Risk-Taking & the Financial Crisis

- *"History is littered with examples of firms that got what they paid for."*
- A particularly insidious form of gaming is by taking on (left-tail) risk.
  - Difficult to prevent via monitoring & enormously costly
- In 2009, Fed chairman Ben Bernanke stated that:

  "compensation practices at some banking organizations have led to misaligned incentives and excessive risk-taking, contributing to bank losses and financial instability"

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Gaming via Risk-Taking is Widespread

- **Portfolio managers** adjust the riskiness of their investments
  Chevalier & Ellison (1997), de Figueiredo et al. (2016)
- **Executives** may cut maintenance to meet earnings targets
  Repenning & Henderson (2015), Garicano & Rayo (2016) and references
- **Entrepreneurs** pursue unproven technologies or incremental progress
  Vereshchagina and Hopenhayn (2008)
- **Salespeople** can accelerate or delay sales to meet their quotas
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What is this paper about?

Optimal contracting when

the agent can game the contract it by *gambling*

Framework

- **Model in a Nutshell.** Canonical principal-agent framework, in which:
  1. The agent chooses effort, which determines *intermediate output*.
  2. He gambles by choosing *any* mean-preserving spread of interm. output.
  3. This mean-preserving spread determines final, contractible output.
- **Mechanism.** The agent gambles to game convex incentives.
  - His payoff equals the *concave closure* of his utility under the contract.
- **Prop. 1.** Optimal contract makes agent's utility *concave* in output.
  - "Classic" principal-agent problem + *no-gaming constraint*

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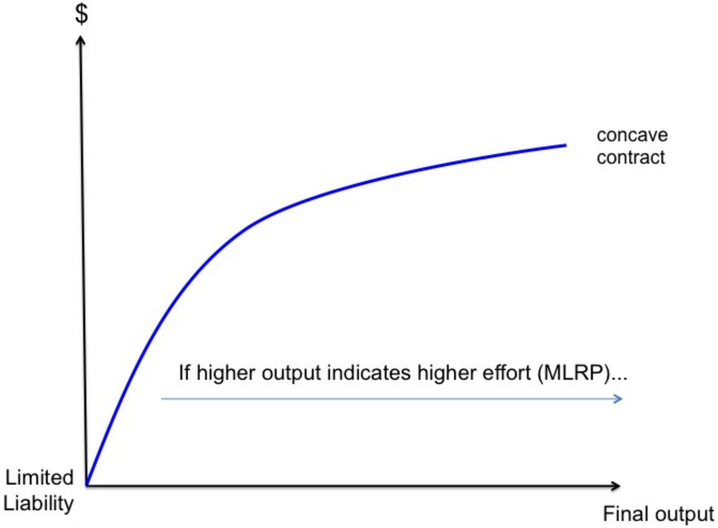

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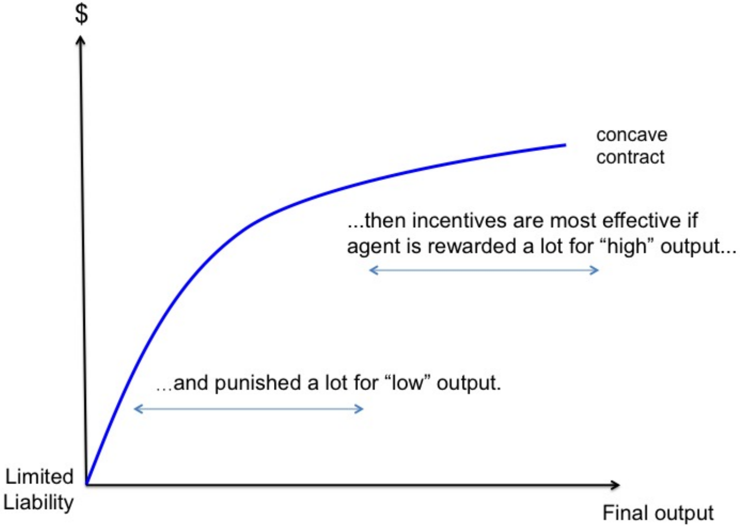

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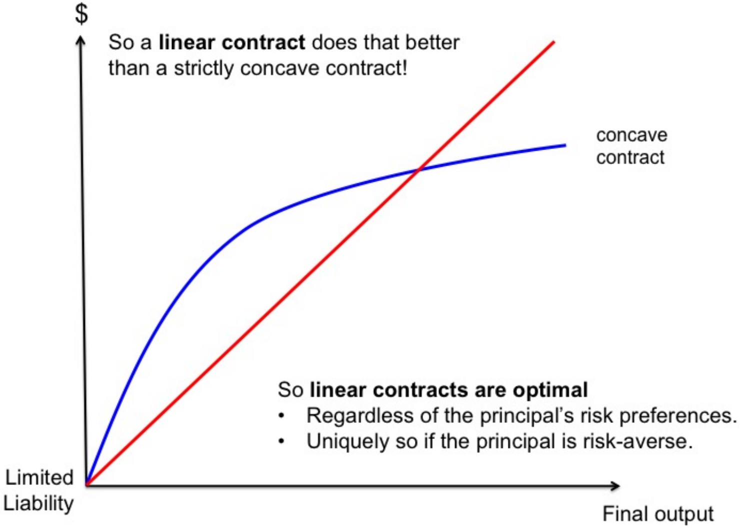

Rough Intuition: Risk-averse Agent

- No-gaming constraint: agent's *utility* must be concave in output.
  - If agent is risk-averse, optimal incentives *may* be *strictly concave*.
- We develop a set of tools to characterize the optimal contract.
  - Where no-gaming *binds*, contract makes agent's utility *linear* in output.
  - Where it is *slack*, contract resembles "classic" optimal contract.
- Two notable cases:
  1. If LL is slack, under mild conditions, optimal incentives are linear below a threshold, and coincide with the "classic" contract above threshold.
  2. If IR is slack, optimal incentives are linear below a threshold.

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Extensions and Reinterpretations

- Three extensions with risk-neutral players.

1. What if agent gambles *before* the exogenous uncertainty is resolved?
    - Leads to different but related constraints on contracts.
    - A linear contract is optimal under mild conditions.
2. What if gambling is costly?
    - Tools extend (for our formulation of risk-taking costs)
    - Optimal contract is convex, and converges to linear as costs vanish.
3. What if the agent can game by shifting output over time?
    - Reinterpret model as intertemporal gaming of stationary contracts.
    - A linear contract is optimal.

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Related Literature

- Large empirical literature on risk-taking and intertemporal gaming.
  - Brown, Harlow & Starks (1996), Chevalier & Ellison (1997), Brav et al. (2005), Matta & Beamish (2008), Rajan (2010), de Figueiredo et al. (2014), Repenning & Henderson (2015), Shue & Townsend (2017)
  - Oyer (1998), Larkin (2014), Repenning & Henderson (2015)
- Some theory on agents using risk-taking to game contracts:
  - **Diamond (1998)** and Garicano & Rayo (2016)
  - Palomino & Pratt (2003)
  - DeMarzo et al. (2014), Hébert (2015), Makarov & Plantin (2016)
- Theory on the optimality of simple (linear) contracts:
  - **Holmström & Milgrom (1987)** and Edmans & Gabaix (2011)
  - Chassang (2013), **Carroll (2015)**, Antic (2016)

Model

- Set of possible outcomes $\mathcal{Y} = \left[\underline{y}, \bar{y}\right]$.
- Players:
  - Weakly risk-averse principal
  - Weakly risk-averse agent with liability M and outside option u_0
- Timing:
  1. Principal offers a contract $s(y)$.
  2. Agent accepts / rejects contract, and chooses effort a.
  3. Intermediate output $x \sim F(\cdot|a)$ is privately observed by the agent.
     - F satisfies strict MLRP, some regularity conditions, and $\mathbb{E}_{F(\cdot|a)}[x] = a$.
  4. Agent chooses distribution $G_x \in \Delta(\mathcal{Y})$ subject to $\mathbb{E}_{G_x}[y] = x$.
  5. Final output $y \sim G_x$ is realized and the agent is paid $s(y)$.
- Payoffs:
  - Principal: $\pi(y - s(y))$, where $\pi'' \leq 0 < \pi'$.
  - Agent: $u(s(y)) - c(a)$, where $u'' \leq 0 < u'$ and $c', c'' > 0$.

Problem Formulation and a Simplifying Result

- Principal solves the following constrained maximization problem:

$$\max_{a,\, G\in\mathcal{G}, v(\cdot)} \quad \mathbb{E}_{F(\cdot|a)}\left[\mathbb{E}_{G_x}\left[\pi\left(y-u^{-1}\left(v\left(y\right)\right)\right)\right]\right] \qquad (\text{Obj}_F)$$

$$\text{s.t.} \quad a, G \in \arg\max_{\tilde{a},\, \tilde{G}\in\mathcal{G}} \left\{\mathbb{E}_{F(\cdot|\tilde{a})}\left[\mathbb{E}_{\tilde{G}_x}\left[v\left(y\right)\right]\right] - c(\tilde{a})\right\} \qquad (\text{IC}_F)$$

$$\mathbb{E}_{F(\cdot|a)}\left[\mathbb{E}_{G_x}\left[v\left(y\right)\right]\right] - c(a) \geq u_0 \qquad (\text{IR}_F)$$

$$v\left(\cdot\right) \geq u(-M)\,, \qquad (\text{LL}_F)$$

where $v(y) = u(s(y))$, $G = \{G_x\}_{x\in\mathcal{Y}}$.

Proposition 1:

- Suppose $\{a, G, v(\cdot)\}$ solves (Obj_F) - (LL_F).
- Then $\left\{a, G^{Degenerate}, v^c(\cdot)\right\}$, where v^c denotes concave closure of v, satisfies (IC_F) - (LL_F), and gives the principal weakly higher profit.

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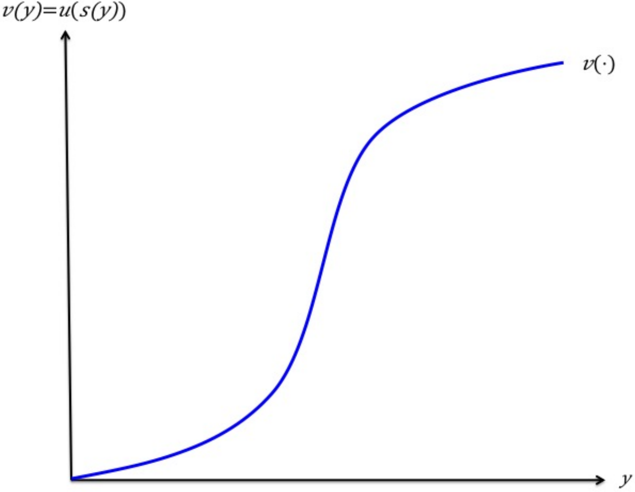
Intuition for Concave Incentives



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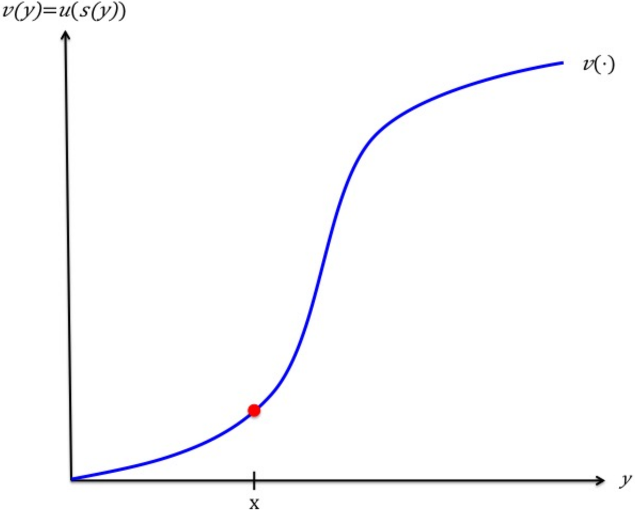

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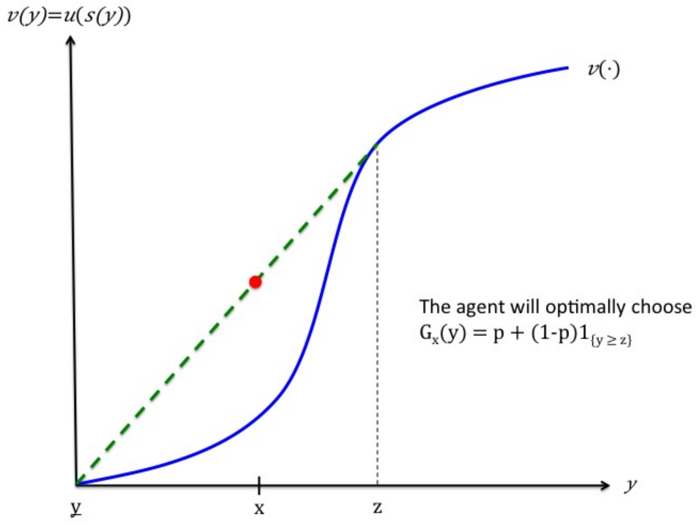

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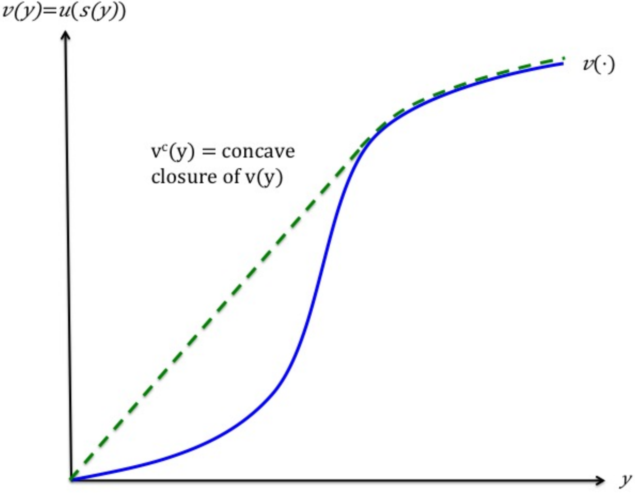

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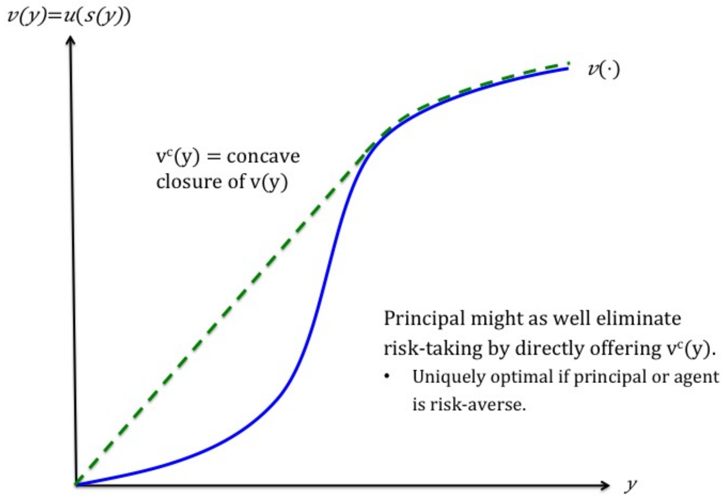

Reformulating the Problem

- Therefore, the principal's problem can be simplified to:

$$\max_{a,v(\cdot)} \quad \mathbb{E}_{F(\cdot|a)}\left[\pi\left(y - u^{-1}(v(y))\right)\right] \qquad \text{(Obj)}$$

$$\text{s.t.} \quad a \in \arg\max_{\tilde{a}} \left\{\mathbb{E}_{F(\cdot|\tilde{a})}\left[v(y)\right] - c(\tilde{a})\right\} \qquad \text{(IC)}$$

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$$v(y) \geq \underline{u} \text{ for all } y \in \mathcal{Y} \qquad \text{(LL)}$$

$$v(\cdot) \text{ weakly concave.} \qquad \text{(Conc)}$$

Lemma 1:

- Fix $a \geq 0$ and assume $\underline{u} > -\infty$. A profit-maximizing contract exists.
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Risk-Neutral Agent: A Linear Contract is Optimal

- Define

$$s_a^L(y) = c'(a)(y - \underline{y}) - \underbrace{\min\left\{M, c'(a)(a - \underline{y}) - c(a) - u_0\right\}}_{\text{constant to satisfy (IR) and (LL)}}$$

  - Cheapest *linear contract* that satisfies (IC), (IR) and (LL) for effort a.

- Define a^{FB} such that $c'(a^{FB}) = 1$. This is the first-best effort level.

Proposition 2: Risk-neutral agent

- Assume $u(s) \equiv s$.
- If a^* is optimal, then $a^* \leq a^{FB}$ and $s_{a^*}^L(\cdot)$ is optimal.

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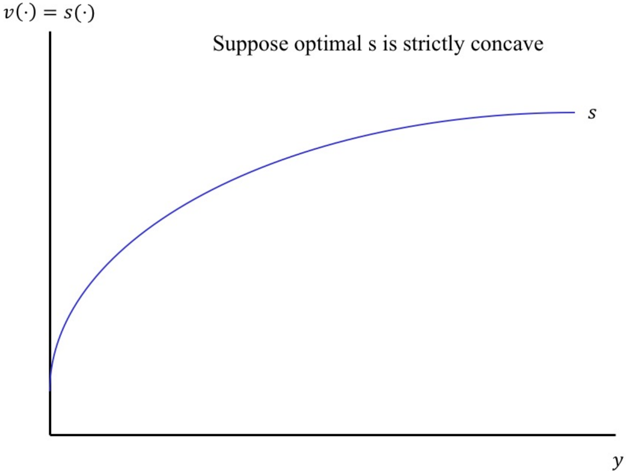

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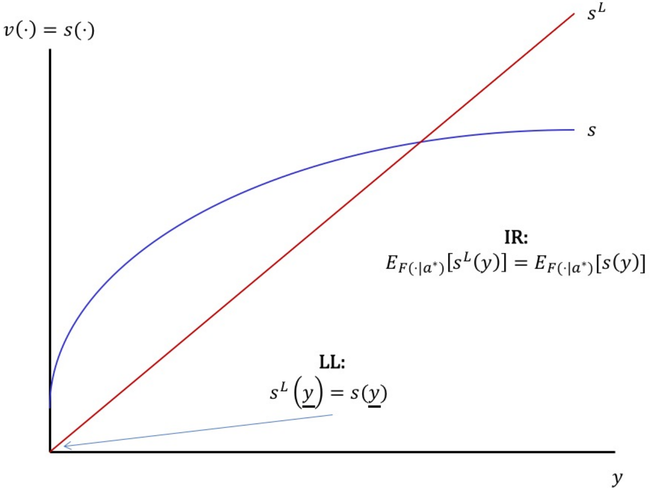
$v(\cdot) = s(\cdot)$
s^L
s
IR:
$E_{F(\cdot|a^*)}[s^L(y)] = E_{F(\cdot|a^*)}[s(y)]$
LL:
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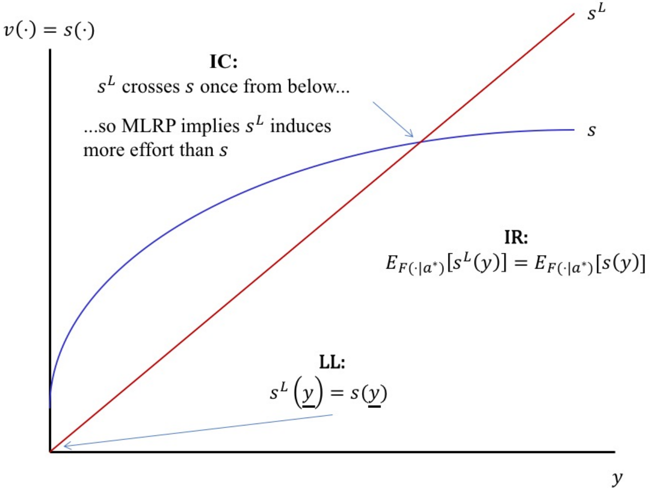

$v(\cdot) = s(\cdot)$
IC:
s^L crosses s once from below...
...so MLRP implies s^L induces more effort than s
s^L
s
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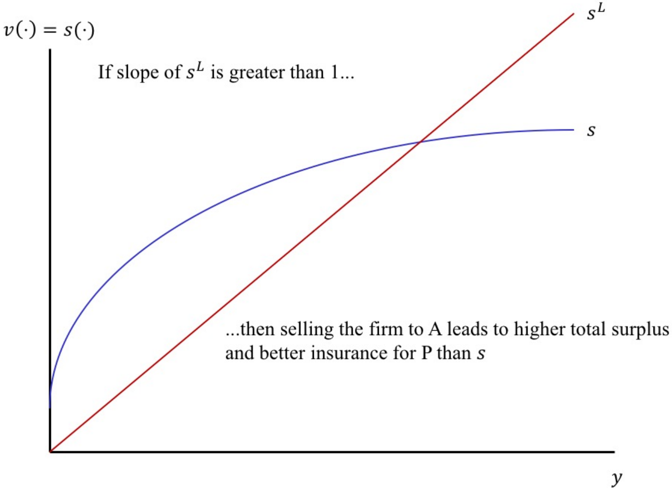
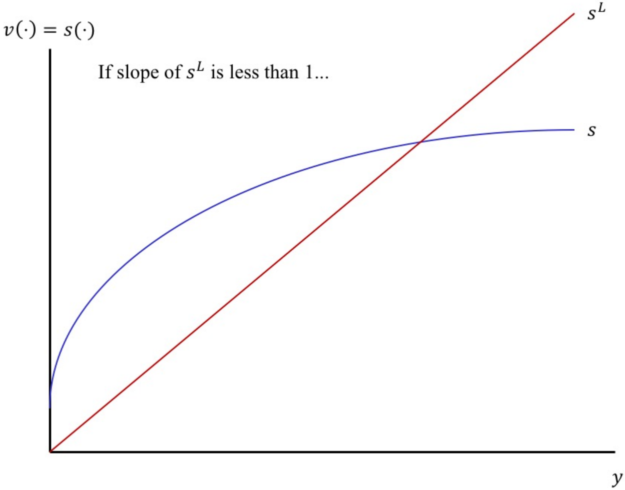

$v(\cdot) = s(\cdot)$
If slope of s^L is greater than 1...
s^L
s
...then selling the firm to A leads to higher total surplus and better insurance for P than s
y

Intuition



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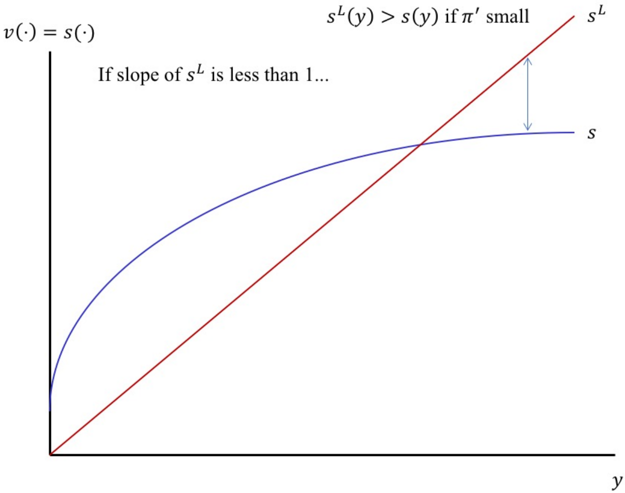
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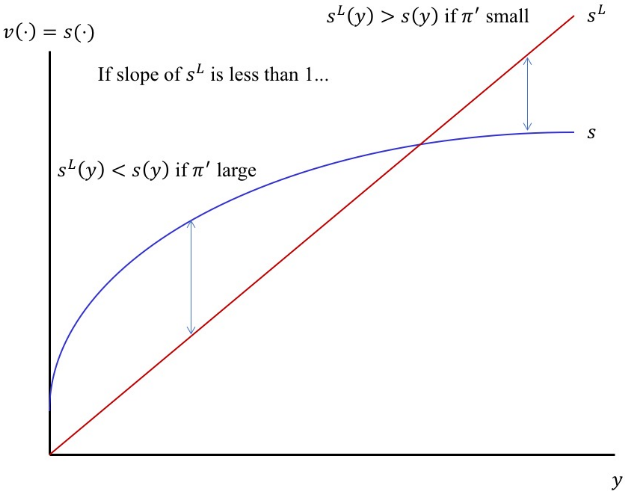
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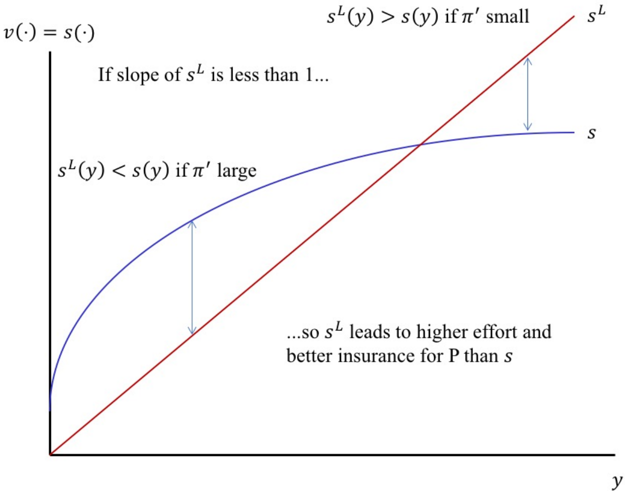
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...so s^L leads to higher effort and
better insurance for P than s
y

If the Principal is loves risk...

- Rajan (2011) argues that anticipating bailouts, financial institutions may have had an incentive to encourage excessive risk-taking.

Corollary 1.

- Assume $\pi(\cdot)$ is strictly increasing with concave closure $\pi^c(\cdot)$.
- If a^* is optimal, then $a^* \leq a^{FB}$, and $s^L_{a^*}(\cdot)$ is optimal.

- The principal wants the agent to choose

$$G_x^P \in \Delta(\mathcal{Y}) \text{ such that } \mathbb{E}_{G_x^P}\left[\pi\left(y - s(y)\right)\right] = \pi^c\left(x - s(x)\right).$$

- From Prop. 2, $s^L_{a^*}(\cdot)$ maximizes $\pi^c(y - s(y))$ subject to (IC)-(Conc).
- Given $s^L_{a^*}(\cdot)$, the agent is indifferent across all G.

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- From Prop. 2, $s^L_{a^*}(\cdot)$ maximizes $\pi^c(y - s(y))$ subject to (IC)-(Conc).
- Given $s^L_{a^*}(\cdot)$, the agent is indifferent across all G.

If the Principal is loves risk...

- Rajan (2011) argues that anticipating bailouts, financial institutions may have had an incentive to encourage excessive risk-taking.

Corollary 1.

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Problem Formulation with a Risk Averse Agent

- Assume the principal is risk-neutral.
- Contract $v(\cdot)$ implements $a \geq 0$ at max. profit / min. cost if it solves:

$$\begin{aligned} \min_{v(\cdot)} \quad & \mathbb{E}_{F(\cdot|a)}\left[u^{-1}(v(y))\right] & \\ \text{s.t.} \quad & \mathbb{E}_{F(\cdot|a)}[v(y)] - c(a) \geq u_0 & \text{(IR)} \\ & a \in \arg\max_{\tilde{a}} \mathbb{E}_{F(\cdot|\tilde{a})}[v(y)] - c(\tilde{a}) & \text{(IC)} \\ & v(y) \geq \underline{u} \text{ for all } y \in \mathcal{Y} & \text{(LL)} \\ & v(\cdot) \text{ weakly concave.} & \text{(NG)} \end{aligned}$$

- Replace (IC) with weaker condition that local incentives are slack:

$$\int v(y) f_a(y|a)\,dy \geq c'(a) \qquad \text{(IC-FOC)}$$

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Building Blocks

- Ignoring (LL) and (NG) for now, we can write the Lagrangian as

$$L(\lambda, \mu) = \min_{v(\cdot)} \int \left[u^{-1}(v(y)) - \lambda v(y) - \mu v(y) \frac{f_a(y|a)}{f(y|a)} \right] f(y|a) dy + \ldots,$$

where λ and μ are shadow values on (IR) and (IC-FOC).

- Differentiating with respect to $v(y)$ yields

$$n(y) \triangleq \underbrace{\frac{1}{u'\left(u^{-1}(v(y))\right)}}_{\triangleq \rho^{-1}(v(y))} - \lambda - \mu \underbrace{\frac{f_a(y|a)}{f(y|a)}}_{\triangleq l(y|a)}$$

  - *Note:* $n(y)$ can be interpreted as net cost of marginally increasing $v(y)$.
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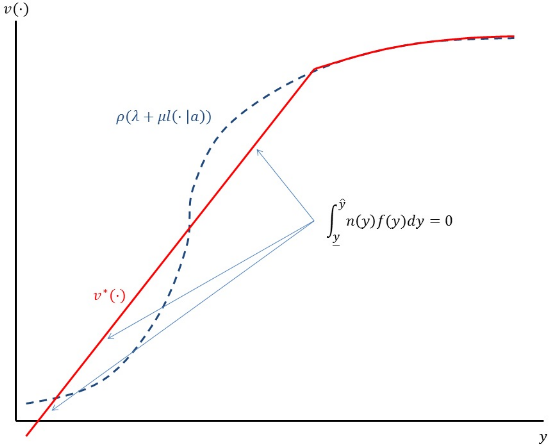
Rough Intuition for our Characterization (LL slack)



- If for some y, setting $n(y) = 0$ violates (NG), then $v(y)$ locally linear.
- Linear segments are "ironed" in the sense that $\mathbb{E}[n(y)] = 0$ on interval.
- Outside linear segments, (NG) is slack, and $n(y) = 0$ at such y.

Implication #1 of No-Gaming Constraint – (LL) Slack

Proposition 3.

- Let $v^*(\cdot)$ implement a at max. profit, and assume (LL) is slack.
- Suppose $\rho(\lambda + \mu l(\cdot|a))$ is convex for $y < y_I$, and concave otherwise.
- Then there exist $\hat{y} > y_I$, $\underline{v}$, and $\alpha > 0$ such that

$$v^*(y) = \begin{cases} \underline{v} + \alpha(y - \underline{y}) & \text{if } y < \hat{y}, \text{ and} \\ \rho(\lambda + \mu l(y|a)) & \text{otherwise,} \end{cases}$$

where $\int_{\underline{y}}^{\hat{y}} n(y) f(y|a) dy = 0$.

- $\rho(\lambda + \mu l(\cdot|a))$ is convex-concave $\forall \lambda$ and μ if, for example,
  - $l_y(\cdot|a)$ is strictly log-concave, and
  - $u(w) = \log w$, or for a range of utilities that exhibit HARA.

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Proposition 4.

- Let $v^*(\cdot)$ implement a at max. profit, and assume (IR) is slack.
- Then $v^*(\cdot)$ is linear on $[\underline{y}, y_0]$, where $l(y_0|a) = 0$.

- For any y such that $l(y|a) < 0$, principal wants to reduce pay.
  - If $v(\cdot)$ is strictly concave, then it is profitable to *flatten* it.
- Cannot rule out linear segments on $(y_0, \overline{y}]$.

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Risk-Taking before Intermediate Output is Realized

Summary

- What if the agent gambles *before* exogenous uncertainty is resolved?

Timing:

1. Principal offers a contract $s(y)$.
2. Agent chooses effort $a \geq 0$ and distribution $G \in \Delta(\mathcal{Y})$ s.t $\mathbb{E}_G[x] = a$.
3. Outcome of gamble $x \sim G$, and final output $y \sim F(\cdot|x)$ are realized.
   - $F(\cdot|x)$ satisfies strict MLRP in x and $\mathbb{E}_{F(\cdot|x)}[y] = x$.

- Can interpret x as profitability conditional on economic conditions, and $F(\cdot|x)$ as capturing residual uncertainty.
- Intuitively, a more dispersed G leads to more "risky" output.

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- For fixed $s(\cdot)$, denote agent's expected pay conditional on x by

$$V_s(x) = \mathbb{E}_{F(\cdot|x)}[s(y)]$$

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Results

- The principal's problem can be written as:

$$\begin{aligned} \max_{a\,,\,s(\cdot)\geq -M} \quad & a - V_s^c(a) \\ \text{s.t.} \quad & a \in \arg\max_{\tilde{a}} \left\{ V_s^c(\tilde{a}) - c(\tilde{a}) \right\} \\ & V_s^c(a) - c(a) \geq u_0 \end{aligned}$$

and note that $s(\cdot)$ determines $V_s(\cdot)$, which in turn determines $V_s^c(\cdot)$.

Proposition 6.

- For optimal effort a^*, $s(y) = c'(a^*)\left(y - \underline{y}\right) + \textit{constant}$ is optimal

- If principal could choose $V_s^c(\cdot)$ directly, this boils down to baseline problem with a degenerate $F(\cdot|a)$. Hence, a linear $V_s^c(\cdot)$ is optimal.

- But $\mathbb{E}_{F(\cdot|x)}\left[y\right] = x$ ensures that $V_s^c(\cdot)$ is linear iff $s(\cdot)$ is linear.

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Costly Gaming

- Identifying ways to game a given contract often requires effort.
- Assume that to choose distribution G_x, the agent incurs cost

$$\mathbb{E}_{G_x}\left[d\left(y\right)\right] - d\left(x\right)$$

  where $d\left(\cdot\right)$ is some smooth, increasing, convex function.
  - *Example:* If $d(y) = y^2$, then agent's cost equals the variance of G_x.
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Optimal Contract

- Conditional on the realization of the final output y, payoffs are:

$$\text{Agent:} \quad \underbrace{s(y) - d(y)}_{\equiv \tilde{v}(y)} - \underbrace{\left(c(a) - \mathbb{E}_{F(\cdot|a)}[d(x)]\right)}_{\equiv \tilde{c}(a) \text{ (assume incr. \& convex)}}$$

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- Principal and agent contract during $[0, 1]$ and do not discount time.
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- Agent will exploit convex incentives by bunching output, and concave incentives by smoothing output over time.
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- Agent will exploit convex incentives by bunching output, and concave incentives by smoothing output over time.
- Equivalently, one can think of the agent choosing

$$G_x(y) = \text{fraction of time for which } y_x(t) \leq y \text{ s.t. } \mathbb{E}_{G_x}[y] = x$$

  - By Lemma 1, agent will optimally choose G_x s.t $\mathbb{E}_{G_x}[s(y)] = s^c(x)$.

Proposition 8.

- This problem coincides with the original risk-taking problem.
- Hence, if a^* is optimal, then $a^* \leq a^{FB}$ and $s^L_{a^*}(\cdot)$ is optimal.

Takeaways

- Tractable model of gaming by risk taking.
- Linear contracts are optimal if the agent is risk neutral.
- Characterization if the agent is risk averse.
- Why might risk-taking occur?
  - Principal may be unable to commit, or might benefit from risk-taking
  - Competition?
  - Dynamics?