Projects and Team Dynamics

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Motivation

- Teamwork & projects are central in firms and partnerships.
- 66% of Fortune 1000 corporations engage $>$ 20% of their workforce in teams.
  Source: Lazear and Shaw (2007); 1996 survey.
- Empirical literature: adoption of teamwork has increased productivity in manufacturing & service firms.
  Source: Ichniowski and Shaw (2003)
- Teams are especially useful for tasks that will result in a defined deliverable (a.k.a projects).
  Source: Harvard Business School Press (2004)

Motivation (Cont'd)

- Extensive literature studying team and free-rider problems.
  - incl. 1^{st} issue of the *AER*: Coman (1911).
- Little is known about dynamic problems in which agents collaborate to complete a project.
- In particular:
  - What is the effect of the group size to agents' incentives?
  - *Principal's Problem:* Optimal team size and incentive contracts?
    - Reward agents upon reaching intermediate milestones?
    - Symmetric or asymmetric compensation?

Objectives

- Develop a dynamic model of collaboration on a project.

- **Key features:** The project
  1. progresses gradually at rate that depends on the agents' efforts ;
  2. it is completed once its state reaches a pre-specified threshold ; and
  3. it generates a payoff upon completion.

Examples:

- *Within firms:* new product development, consulting projects.
- *Across firms:* R&D joint ventures

Overview of Results: Part I

Agent's Problem:

- Characterize the equilibrium.
  - Agents work harder the closer the project is to completion.

- *Main Result*: Individual and Aggregate Effort vs. Team Size.
  - Bigger teams work harder than smaller ones (both individually and on aggregate) **iff** project is sufficiently far from completion.

    (*Result holds both when $V_n = V$, and when $V_n = \frac{V}{n}$.*)

- Optimal Partnership Size.

Overview of Results: Part II

Introduce a Manager:

1. *Symmetric Contracts:*
    - Optimal contract rewards the agents only upon completion.
    - Characterize optimal budget and team size.
    - Dynamically change the team size as the project progresses.
2. *Asymmetric Contracts: (2 agents)*
    - Reward upon reaching different milestones.
    - Reward asymmetrically upon completion.

Related Literature

- Moral Hazard in Teams:
  - Holmström (1982), Legros and Matthews (1993), and others.
  - Bonatti and Hörner (2011)
- Dynamic Contribution Games:
  - Admati and Perry (1991) and Marx and Matthews (2000)
  - Yildirim (2006) and Kessing (2007)

My Contributions:

1. Tractable & natural framework for dynamic contribution games.
2. Novel comparative static about (total) effort *vs.* team size.
3. Insights for team design & contracting in projects.

Model Setup

- Team comprises of n agents. Agent i
  - is risk neutral and discounts time at rate $r > 0$;
  - *privately* exerts effort $a_{i,t}$ *at cost* $c(a) = \frac{1}{p+1} a^{p+1}$ *(*$p > 0$*) ;*
  - receives lump-sum V_i upon completion of the project.
- Project starts at $q_0 < 0$, it evolves according to

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t} \right) dt + \sigma dW_t \, ,$$

and it is completed at the first time τ such that $q_\tau = 0$.

- Assume Markov Perfect strategies.
  - *i.e.*, efforts at t depend only on q_t.

Building Blocks: Agents' Payoff Functions

- Agent i's problem at t:

$$J_{i,t} = \max_{a_{i,s}} \mathbb{E}\left[e^{-r(\tau-t)} V_i - \int_t^{\tau} e^{-r(s-t)} c\left(a_{i,s}\right) ds \mid q_t \right]$$

- Hamilton-Jacobi-Bellman Equation:

$$rJ_i\left(q\right) = \max_{a_i} \left\{ -c\left(a_i\right) + \left(\sum_{j=1}^{n} a_j \right) J_i'\left(q\right) + \frac{\sigma^2}{2} J_i''\left(q\right) \right\}$$

subject to the boundary conditions

$$\lim_{q \to -\infty} J_i\left(q\right) = 0 \quad \text{and} \quad J_i\left(0\right) = V_i \;\; \text{for all } i.$$

Building Blocks: Agents' Payoff Functions (Cont'd)

- First-order condition: $a_i^p = J_i'(q)$
  - *Guess* (and verify later) that $J_i'(\cdot) \geq 0$ so that FOC binds.

$$\implies a_i(q) = \left[J_i'(q)\right]^{1/p}$$

- A MPE must satisfy the system of ODE

$$rJ_i(q) = -\frac{1}{p+1}\left[J_i'(q)\right]^{\frac{p+1}{p}} + \sum_{l=1}^{n}\left[J_l'(q)\right]^{\frac{1}{p}} J_i'(q) + \frac{\sigma^2}{2}J_i''(q)$$

subject to the set of boundary conditions.

Markov Perfect Equilibrium (MPE)

Theorem 1:

1. A MPE exists and $J_i'(q) > 0$ for all i and q.
    - If $p \in (0,1)$, then also need $\int_0^\infty \frac{s\,ds}{r\sum_{i=1}^n V_i + ns^{\frac{p+1}{p}}} > \sum_{i=1}^n V_i$.
2. If agents are symmetric, then the equilibrium is symmetric.
3. Eq'm is unique with n symmetric or 2 asymmetric agents.
4. $a_i'(q) > 0$ for all i and q.

Some Intuition

- Why $a_i'(q) > 0$?
  - Deterministic case with 1 agent: Discounted reward $= e^{-r\tau}V$.
  - Marg. benefit of bringing completion time forward$= \underbrace{re^{-r\tau}V}_{\downarrow \text{ in } \tau}$.

$a_i'(q) > 0$ implies that efforts are strategic complements (across time).

- Unlike standard models of free-riding. So what?
  - Agent's trade off:

$$(\text{marg. effort cost}) = \begin{pmatrix} \text{marg. benefit of progress} \\ \text{marg. benefit of influencing future efforts} \end{pmatrix}$$

  - Implications for the effect of team size to incentives.

Sketch of the Proof of Theorem 1

- Existence & Uniqueness Proof: Apply Hartman (1960).
  - Need to show that $|J_i(q)|$ and $|J_i'(q)|$ are bounded $\forall q$.
  - *Challenge*: showing that $|J_i'(q)| \leq \bar{A}$ for all q.

- $J_i(q) > 0$: Project is completed in finite time even w/o effort.

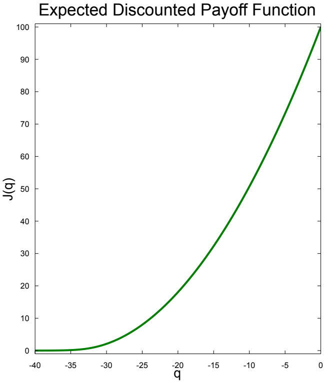
- $J_i'(q) > 0$: Suppose there exists z such that $J_i'(z) = 0$.
  - Then $rJ_i(z) = \frac{\sigma^2}{2} J_i''(z) > 0 \Rightarrow z$ is a strict local min.
  - Hence $J_i(\cdot)$ has a local max $\hat{z} \in (-\infty, z)$.
  - $J_i'(\hat{z}) = 0$ and $J_i''(\hat{z}) \leq 0$ implies $J_i(\hat{z}) \leq 0$ `.
  - Therefore, $J_i'(q) > 0$ for all q.

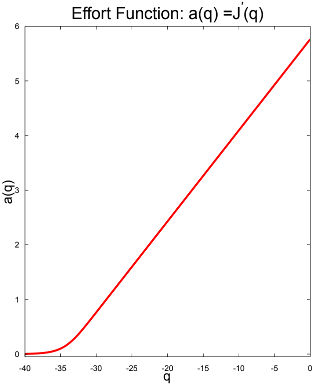
- A similar approach using the envelope theorem shows that $J_i''(q) > 0$, so that $a_i'(q) > 0$ for all q.

Illustration of the Agent's Payoff and Effort Functions

- *Example*: Quadratic effort costs ($p = 1$) & symmetric agents.

$$rJ(q) = \frac{2n-1}{2}\left[J'(q)\right]^2 + \frac{\sigma^2}{2}J''(q)$$





Comparative Statics

Proposition 1: Consider a group of n symmetric agents.

(i) If $V_1 > V_2$, then other things equal, $a_1(q) > a_2(q)$ for all q.

(ii) If $r_1 > r_2$, then other things equal, $a_1(q) \leq a_2(q)$ iff $q \leq \Theta_r$.

(iii) If $\sigma_1 > \sigma_2$, then other things equal, $a_1(q) \geq a_2(q)$ if $q \leq \Theta_{\sigma,1}$ and $a_1(q) \leq a_2(q)$ if $q \geq \Theta_{\sigma,2}$.

- Less patient agents have more to gain from earlier completion.
  - But bringing the completion time forward is costly.
  - Benefit > Cost iff project is sufficiently close to completion.

- Higher volatility $\sigma \implies$ project more likely to be completed either earlier (*upside*), or later (*downside*) than expected.
  - If $q \leq \Theta_{\sigma,1}$, then $J_i(q) \simeq 0$ so that *downside* is negligible.
  - On the other hand, *upside* diminishes as $q_t \to 0$.

Robustness

- Theorem 1 and the main result continue to hold if
  1. Project is deterministic: $\sigma = 0$.
  2. Agents can abandon project and collect outside option $\bar{u} > 0$.
  3. Project is inhomogeneous; *i.e.*, σ depends (smoothly) on q.
  4. Effort affects both drift and variance of the process.
  5. Synergies or coordination costs so that (total effort) $\gtrless \sum_i a_{i,t}$.
- If project generates a flow payoff $h(q)$ (in addition to V_i):
  - Effort profile $a_i(q)$ is hump-shaped in q.
  - Team size comparative static continues to hold.

Team Size Effects: Introduction

- How do the agents' rewards depend on the team size?
    1. *Public Good Allocation*: ea. agent's reward independent of n.
    2. *Budget Allocation*: ea. agent's reward is equal to $\frac{V}{n}$.

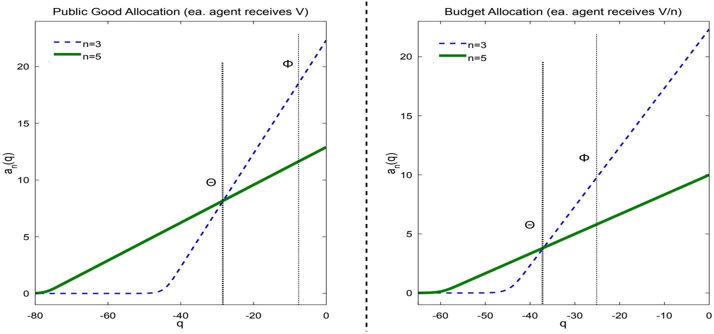
Team Size Effects: Main Result

Theorem 2: Consider a *big* (m) and a *small* (n) team. $(m > n)$

Under both allocations, $\exists$ thresholds Θ and $\Phi > \Theta$ such that

(A) $a_m(q) \geq a_n(q)$ iff $q \leq \Theta$, and

(B) $m\, a_m(q) \geq n\, a_n(q)$ iff $q \leq \Phi$.



The Free-riding Effect: Intuition

- In a larger team, incentives to free-ride are stronger:
  - Fix strategies & consider an agent's *dilemma* to $\downarrow$ effort by ϵ:
    1. He saves $\varepsilon c'(a_t)\,dt$ in effort cost; but
    2. At $t + dt$, the project is εdt farther from completion.
  - In eq'm, he will carry out only $\frac{1}{n}$ of this *lost* progress.
- Gain from shirking $= \varepsilon c'(a_t)\,dt$ increases in q.
  - $c'(\cdot)$ is increasing, and in eq'm, $a(q)$ increases in q.
- Therefore, the *free-riding effect becomes stronger with progress.*
  - $\lim_{q \to -\infty} c'(a(q)) = 0$: free-riding effect diminishes as $q \to -\infty$.

The Encouragement Effect: Intuition

- Assume $\sigma = 0$ and fix the agents' strategies.
    - If team size $n \nearrow 2n$, then completion time $\tau \searrow \frac{1}{2}\tau$.

- *Recall:* ea. agent's discounted reward $= V_n e^{-r\tau}$.
    - Marg. benefit of bringing completion time forward $= rV_n e^{-r\tau}$.

- Measure of encouragement effect:

$$\frac{V_{2n}}{V_n} e^{\frac{r\tau}{2}}$$

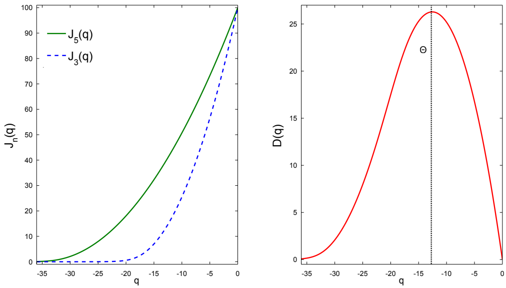
    - *The encouragement effect becomes weaker with progress.*

- Under *budget allocation*, $n \nearrow 2n$ also implies that $\frac{V_{2n}}{V_n} = \frac{1}{2}$.
    - Encouragement effect > 0 as long as τ is sufficiently large.

Proof of Theorem 2

Statement A under *public good allocation*.

- Observe: $J_m(-\infty) = J_n(-\infty) = 0$ and $J_m(0) = J_n(0) = V$.
  - Define $D(q) = J_m(q) - J_n(q)$ and note $D(-\infty) = D(0) = 0$.



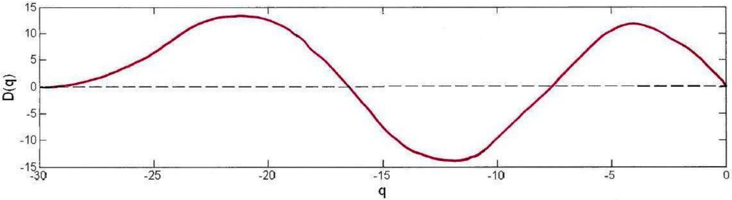
- *Objective:* Show that $D'(q) \geq 0$ iff $q \leq \Theta$.
  - $\because$ $a_n(q) = [J_n'(q)]^{1/p}$, this implies $a_m(q) \geq a_n(q)$ iff $q \leq \Theta$.

Proof of Theorem 2

- Either $D(\cdot) \equiv 0$, or it has at least one interior extreme point.
  - There exists some z such that $D'(z) = 0$. Then

$$rD(z) = \underbrace{(m-n)\left[J_n'(z)\right]^{\frac{p+1}{p}}}_{>0} + \frac{\sigma^2}{2} D''(z)$$

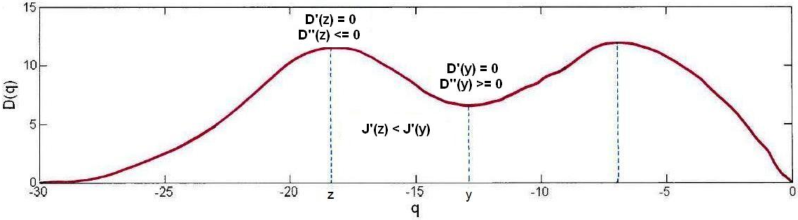
1. If $D(\cdot) \equiv 0$, then $D''(\cdot) \equiv 0$, which is a contradiction.
   - Therefore, $D(\cdot)$ has at least one interior extreme point.



2. Suppose $z = \min$.
   - Then $D''(z) \geq 0 \Longrightarrow D(z) > 0$.
   - Therefore, $D(q) \geq 0$ for all q.

Proof of Theorem 2

- *Claim:* $D(\cdot)$ has a exactly one extreme point which is a max.



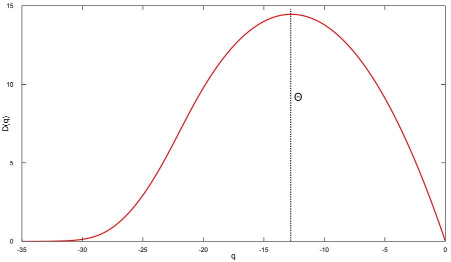
- Suppose not. Then $\exists$ a local max z and a local min $y > z$.
  - $D''(z) \leq 0 \leq D''(y)$ and $J_n'(z) < J_n'(y)$.

$$\begin{aligned} \Rightarrow rD(z) \quad &= (m-n)\left[J_n'(z)\right]^{\frac{p+1}{p}} + \frac{\sigma^2}{2} D''(z) \\ &< (m-n)\left[J_n'(y)\right]^{\frac{p+1}{p}} + \frac{\sigma^2}{2} D''(y) = \quad rD(y) \end{aligned}$$

  - Contradicts the facts that $y = \min$ while $z = \max$.

Proof of Theorem 2

- Thus $D(\cdot)$ has exactly one extreme point which is a max.



  - Recall: $D(q) = J_m(q) - J_n(q)$ and $a_n(q) = [J_n'(q)]^{1/p}$.
  - Therefore, $a_m(q) \geq a_n(q)$ iff $q \leq \Theta$.

- For **statement B**, def. $\bar{D}(q) = m^p J_m(q) - n^p J_n(q)$ instead.
  - Note: $m\, a_m(q) \geq n\, a_n(q)$ iff $\bar{D}'(q) \geq 0$. (Same approach.)

Interiorness of the Thresholds

- Θ **is generally always interior.** (*Individual Effort*)
  - Under public good allocation, $D(-\infty) = D(0) = 0$.
    - Therefore, Θ is guaranteed to be interior in this case.
  - Under budget allocation, $D(0) = J_m(q) - J_n(q) < 0$.
    - So it is possible that $D'(\cdot) \leq 0$ and $\Theta = -\infty$.
    - Numerical analysis indicates that Θ is always interior.
- Φ **needs not always be interior.** (*Aggregate Effort*)
  - Guaranteed to be interior only under budget allocation, if effort costs are (at most) quadratic.
  - Otherwise, possible $\Phi = 0$: larger teams always work harder.
  - *Numerically*, Φ is interior as long as effort costs *not too convex*.

Partnership Formation

- Optimal partnership size maximizes $J_n(q_0)$.
  - Partnership composition is finalized before agents begin to work.

Proposition 3.

1. *Public good allocation*: Optimal partnership size $n^* = \infty$.
2. *Budget allocation*: n^* increases in project size $|q_0|$.

- **Public good allocation:** "size of pie" is nV.
  - Larger team $\Rightarrow$ smaller share of work for each agent.

- **Budget allocation:** a new member $\downarrow$ everyone's reward.
  - Agents will increase team size only if the gain from sharing the effort among a bigger group is sufficiently large.

Manager's Problem: Setup

- Risk-neutral manager hires n agents to undertake a project.
- The manager values the project at U and discounts time at rate r.
- At $t = 0$, she commits to a set of
  - milestones $Q_1 < .. < Q_K = 0$; and
  - rewards $\{V_{i,k}\}_{i=1,k=1}^{n,K}$ attached to each milestone.

  (Agent i is paid $V_{i,k}$ upon reaching Q_k for the first time.)
- *Objective:* Choose the team size, the set of milestones and rewards to maximize her expected discounted profit.

Manager's Problem & Optimal Symmetric Contract

- The profit function satisfies an ordinary differential equation.

Theorem 3: Characterization of the manager's problem

- A solution to the manager's problem exists.
- It is unique with n symmetric or 2 asymmetric agents.

Theorem 4.

The optimal symmetric scheme rewards agents only upon completion.

- By backloading payments, manager can provide same incentives early on (via continuation utility), and stronger incentives later on.

- Manager's problem reduces to choosing budget B and team size n.

Optimal Budget & Team Size

Proposition 4: *Optimal budget B.*

- Suppose the manager employs n agents and contracts are symmetric.
- Her optimal budget B increases in the project length $|q_0|$.

- Larger project requires more effort $\Rightarrow$ stronger incentives.

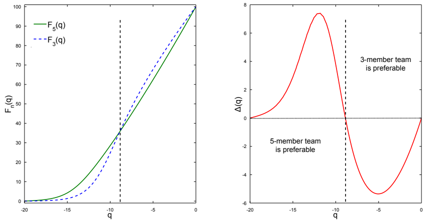
Proposition 5: *Optimal team size n.*

- Suppose manager has a fixed budget B and contracts are symmetric.
- Her optimal team size n increases in the project length $|q_0|$.

- Larger team is preferable if
  - benefit from harder work while project is far from completion,
  - outweighs loss from more free-riding when close to completion.

Proof of Proposition 5

Lemma: Fix $m > n$. Then $F_m(q_0) \geq F_n(q_0)$ iff $q_0 \leq T_{m,n}$.



- Let $\Delta(q) = F_m(q) - F_n(q)$ and note $\Delta(-\infty) = \Delta(0) = 0$.
  - Either $\Delta(\cdot) \equiv 0$ or $\Delta(\cdot)$ has an int. global extreme point.

- *Proof Approach:*
  1. Cannot be the case that $\Delta(\cdot) \equiv 0$.
  2. Any extreme point $z \leq [\geq] \Phi$ must satisfy $\Delta(z) \geq [\leq] 0$.
  3. Conclude that $\Delta(\cdot)$ may cross 0 at most once from above.

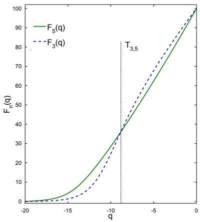
Proof of Proposition 5

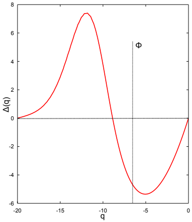
There exists at least one extreme point z s.t $\Delta'(z) = 0$. Then

$$r\Delta(z) = \underbrace{[ma_m(z) - na_n(z)]}_{\geq 0 \text{ iff } z \leq \Phi} \underbrace{F_n'(z)}_{>0} + \frac{\sigma^2}{2}\Delta''(z)$$

- If $\Delta(\cdot) \equiv 0$, then $\Delta''(\cdot) \equiv 0$, which leads to a contradiction.
- Now consider an extreme point $z \leq \Phi$:
  - If $z = \min$, then $\Delta''(z) \geq 0$, and hence $\Delta(z) \geq 0$.
  - Therefore, any extreme point $z \leq \Phi$ must satisfy $\Delta(z) \geq 0$.
- Next, consider an extreme point $z \geq \Phi$:
  - If $z = \max$, then $\Delta''(z) \leq 0$, and hence $\Delta(z) \leq 0$.
  - Therefore, any extreme point $z \geq \Phi$ must satisfy $\Delta(z) \leq 0$.

Proof of Proposition 5





- We know that:
  1. Any extreme point $z \leq \Phi$ must satisfy $\Delta(z) \geq 0$.
  2. Any extreme point $z \geq \Phi$ must satisfy $\Delta(z) \leq 0$.
- Therefore, $\Delta(\cdot)$ crosses 0 at most once, from above.
- Comparative static: n^* increases in project length $|q_0|$.
  - Apply Monotonicity Thm of Milgrom and Shannon (1994).

An Example of Dynamic Team Size Management

- Consider the following *retirement* contract:
  - The manager employs 2 (identical) agents.
  - She picks an R such that one agent is retired at $q_t = R$.
  - Agent i receives V_i upon completion of the project.

  (The V_i's are chosen such that agents are indifferent at R).

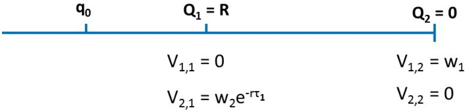
Proposition 6.

- Suppose that effort costs are quadratic and $|R| \leq T_1$.
- Then this contract is *beneficial* iff $|q_0| < \Theta_R$.

- Interpretation: If $|q_0| = |R|$, then optimal team size $= 1$.
- Once one agent is retired, the other exerts first-best effort.
- While they collaborate, aggregate effort is lower.

Implement with an Asymmetric Contract

Consider the following asymmetric contract w/ one intermediate milestone:



Proposition 6: preferable to symmetric contract iff $|q_0| < \Theta_R$.

- Enables the manager to dynamically decrease the team size.

Remark: In general, the optimal contract is asymmetric.

- *Negative result:* Optimal contracting requires $n + 1$ state variables.

Symmetric vs. Asymmetric Compensation

Proposition 7.

- Suppose $n = 2$, $c(a) = \frac{a^2}{2}$ and agents rewarded only upon compl'n.
- Asymmetric contract is *preferable* if $|q_0|$ is sufficiently short.
  - *i.e.*, $\forall \epsilon \in [0, B]$, $\left\{\frac{B+\epsilon}{2}, \frac{B-\epsilon}{2}\right\} \succcurlyeq \left\{\frac{B}{2}, \frac{B}{2}\right\}$ iff $|q_0| \leq T_\epsilon$.

- **Extreme Case:** $V_1 = B$ and $V_2 = 0$.
  - This contract is preferable iff $|q_0| \leq T_1$.

- **Intermediate Cases:** A *full-time* agent and a *part-time* one.
  - Full-time agent cannot free-ride much on the part-time agent.

- *Takeaway:* Asymmetric pay can mitigate free-riding.

Current Research

1. Project size is endogenous and manager has limited commitment.
   - Manager's has incentives to extend the project as it progresses.

   Georgiadis, Lippman and Tang (RAND, forthcoming)

2. Incorporate deadlines and imperfect observability of the state q_t.

3. Test the effects of n and observability of q_t in the laboratory.
   joint with F. Ederer and S. Nunnari.

4. Endogenous project size: voting among n heterogeneous agents.
   joint with R. Bowen and N. Lambert.

5. A group of agents extract a common resource over time.
   - Better off if agents do not observe the amount of resource remaining.

   joint with T. Palfrey.

Directions for Future Research

- Characterize the optimal contract. *Intuitively:*
    - Optimal contract will be asymmetric ; and
    - ea. agent will be rewarded at the end of his involvement in project.
    - But each agent's reward will depend on the path of q_t.
    - What if agents can imperfectly observe ea. other's effort choices?
- Incorporate asymmetric information.
    1. Agents are uncertain about the production technology (*learning*).
    2. Agents are uncertain about their peers' preferences (*signaling*).