Project Design with Limited Commitment and Teams

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Introduction

- Dynamic contribution games to a project are ubiquitous in practice.
- Main features:
  1. Positive externalities $\Longrightarrow$ there is scope for free-riding ;
  2. contributions accumulate over time (*i.e.*, non-stationary game) ; and
  3. a certain goal must be reached before a payoff is generated.
- For example:
  - New product development
  - Consulting projects
  - Film production
  - Startup Companies

Introduction (Cont'd)

- Substantial literature studying this class of games.
  - *But*, the goal is given exogenously.

- A central decision involves deciding the requirements that must be fulfilled for the project to be deemed *complete*.
  - **Trade off:** Value of additional features vs. associated cost.

Objective:

Analyze a dynamic contribution game in which

- a group of agents collaborate to complete a project, and
- a manager chooses its features (*i.e.*, its size).

Limited Commitment

Managers are often unable to commit to the requirements in advance.

1. Because they are difficult to contract on.
   - *e.g.*, if the project involves innovation, quality, design.

2. Due to the asymmetry in bargaining power.
   - *e.g.*, employer - employee relationship.

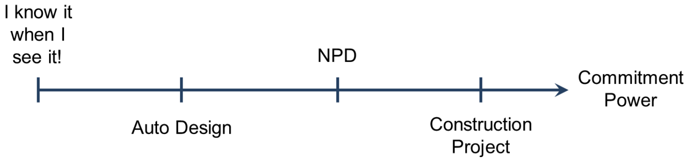
Example: Steve Jobs and the development of the 1^{st}-generation iPod

1. "Steve doesn't think it's loud enough."
2. "The menu's not coming up fast enough."

Source: Ben Knauss, Snr. Manager at PortalPlayer - co-developer of the iPod.

Limited Commitment (Cont'd)

- How to model *limited commitment* ?
  - **Idea:** The manager cannot commit too far ahead.
  - *Commitment power* measures how far ahead she can commit.



Outline of the Results (1 / 2)

- Tractable characterization of the dynamic contribution game.
  - Closed form; both Markovian and non-Markovian equilibria.

Main Result:

Manager has incentives to extend the project as it progresses.

- Intuition:
  - Manager trades off cost of waiting vs value of larger project.
  - Eq'm effort increases as the project progresses (due to discounting).
  - So the marg. cost of waiting decreases with progress.

Outline of the Results (2 / 2)

Implication #1

Smaller commitment power $\Longrightarrow$ Larger project.

- In anticipation, agents reduce their effort (*ratchet effect*).

Implication #2

Optimal delegation of the project requirements to the agents.

- Agents prefer a smaller project, but have time-consistent preferences.

Lastly:

Contracting on Project Duration vs. Project Size

Related Literature

- Dynamic Contribution Games:
  - *DT:* Admati & Perry (1991), Marx & Matthews (2000),..
  - *CT:* Kessing (2007), Georgiadis (2013),..
- Incomplete Contracting and Optimal Delegation:
  - Grossman & Hart (1986), Hart & Moore (1990), Tirole (1999),..
  - Dessein (2002), Alonso & Matouschek (2008),..
- Corporate Culture, Social Identity, and Eq'm Selection:
  - Kreps (1990)
  - Akerlof & Kranton (2000, 2005)

The Model

- A manager employs n identical agents to undertake a project.
  - All parties discount time at rate $r > 0$.
  - At time t, agent i *privately* exerts effort $a_{i,t}$ *at cost* $\frac{a_{i,t}^2}{2}$.
- Project starts at $q_0 = 0$, and it progresses according to

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t} \right) dt \, .$$

- If project is *completed* at $q_\tau = Q$, then it generates payoff Q.
  - The manager receives $(1-\beta)\,Q$, and ea. agent receives $\frac{\beta}{n}Q$.
  - Q : 1-D parameter that *captures* the project's requirements.

Agents' Problem

- At time t, agent i's discounted payoff is equal to

$$\Pi_{i,t}(q_t ; Q) = \max_{\left\{a_{i,s}\right\}_{s\geq t}} \left[e^{-r(\tau - t)} \frac{\beta}{n} Q - \int_t^{\tau} e^{-r(s-t)} \frac{a_{i,s}^2}{2} ds \right]$$

- We consider Markov Perfect equilibria. (*Will relax this later)*

- Problem is stationary, and $\Pi_i(q ; Q)$ satisfies

$$r\Pi_i(q ; Q) = \max_{a_i} \left\{ -\frac{a_i^2}{2} + \left(\sum_{j=1}^{n} a_j \right) \Pi_i'(q ; Q) \right\}$$

subject to $\Pi_i(Q ; Q) = \frac{\beta Q}{n}$.

Characterization of Markov Perfect Equilibria

Proposition 1: Characterization of MPE

- For any Q, strategies in the unique project-completing MPE satisfy

$$a(q;Q) = \frac{r\left[q - C(Q)\right]}{2n-1}\mathbf{1}_{\{Q<\Theta\}}.$$

- If $Q \geq \frac{2\beta}{r}$, then there exists an eq'm in which $a(q;Q) = 0$ for all q.

- $a(q;Q)$ increases in q.
  - Reward looms larger, the closer the project is to completion.

- We focus on the *project-completing* equilibrium.
  - Optimal Q satisfies this condition.

Non-Markovian Strategies

- What if effort at t is allowed to depend on $\{q_s\}_{s \leq t}$?

Consider the following trigger strategy:

- For some fixed $k \in (1, n]$, suppose that each agent solves

$$a(q\,;Q) \in \arg\max_a \left\{ a\,k\,\Pi'(q\,;Q) - \frac{a^2}{2} \right\}$$

as long as no agent has so far deviated from this strategy.

- After a deviation, all agents revert to Markov eq'm ($k = 1$).

- Social Identity Theory: Agents can behave as *insiders* or *outsiders*.
  - k can be interpreted as a measure of group cohesiveness.

- Inertia Strategies by Bergin and MacLeod (1993).

Characterization of (Non-Markov) Public Perfect Equilibria

Proposition 2: Characterization of Perfect Public Equilibria (PPE)

- For all $k \in (1, n]$, there exists a PPE in which each agent follows the aforementioned trigger strategy.
- Along the eq'm path, each agent's effort level satisfies

$$a(q; Q) = \frac{r\left[q - C(Q)\right]}{2n - k} \mathbf{1}_{\{Q < \Theta_k\}}.$$

- Agents work harder the closer they are to completion.
- For any q, ea. agent is strictly better off the higher k is.

Manager's Problem

- For given Q, the manager's discounted profit satisfies

$$rW(q;Q) = [n\,a(q;Q)]\,W'(q;Q)$$

subject to $W(Q;Q) = (1-\beta)\,Q$.

- Using the agents' equilibriumm strategy, we have

$$W(q;Q) = \underbrace{(1-\beta)\,Q}_{\text{net profit}} \underbrace{\left[\frac{q-C(Q)}{Q-C(Q)}\right]^{\frac{2n-k}{n}} \mathbf{1}_{\{Q<\Theta_k\}}}_{\text{present discounted value}}.$$

Main take-away so far:

- Agents work harder as the project progresses.

Endogenizing the Project Size Q

Benchmark Case: Full Commitment

- At time 0, manager chooses $Q_{FC} = \arg\max_Q \{W(0;Q)\}$.

$$\Longrightarrow Q_{FC} = \frac{\beta}{r}\frac{k(2n-k)}{2n}\left(\frac{4n}{4n-k}\right)^2$$

Modeling the Commitment Problem

- At q_t, manager can commit only to project sizes $Q \in [q_t, q_t + y]$.
  - We interpret y as her *commitment power*.

- Game form with limited commitment (y is public info):
  - At every moment, ea. agent forms a belief $\tilde{Q}$ about the project size, and chooses his strategy to maximize his discounted payoff.
  - Solution concept: Nash equilibrium (w/ pure strategies).

- **Second Benchmark Case:** $y = 0$ (*I know it when I see it*)

$$Q_{NC} = \frac{\beta}{r} \frac{2kn}{2n - k} > Q_{FC}$$

An Auxiliary Problem

- Suppose the manager commits to her optimal Q at $q_t = x$.
  - Let $Q_x = \arg\max_Q \{W(x;Q)\}$. It follows that

$$Q_x = \gamma\left(\sqrt{\delta} + \sqrt{\delta + \frac{x}{4n-k}}\right)^2 .$$

Main Result: Q_x increases in x

The manager has incentives to extend the project as it progresses.

- Manager trades larger payoff from bigger Q *vs* longer wait.
  - Effort increases with progress ⇒ marg. waiting cost decreases.
  - Hence, the optimal Q increases in x.

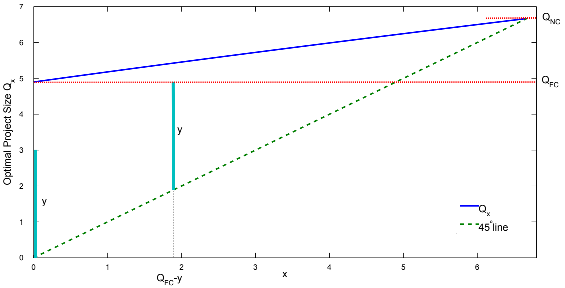
  (*Note*: The manager does not internalize cost of effort.)

An Illustration of the Manager's Commitment Problem

- If $y \geq Q_{FC}$, then the manager will commit to Q_{FC} at time 0.

Suppose that $y < Q_{FC}$. At q_0, the manager can

1. Commit to y immediately ; or
2. Wait until $x = Q_{FC} - y$ so that she can commit to Q_{FC}.
   - But at $x = Q_{FC} - y$, $Q_x > Q_{FC}$: It's *déjà vu* all over again!



Optimal Q with Limited Commitment

- Assume that $y < Q_{FC}$.
- Suppose the manager finds it optimal to commit at (some) x.
  - Then she must be indifferent between:
    1. Committing to $Q = x + y$ immediately ; and
    2. Waiting to commit to $Q = x + y + \varepsilon$ at $x + \varepsilon$.

    i.e., $W(x, x+y) \simeq W(x, x+x+\varepsilon)$.
- Therefore, q must satisfy: $\frac{\partial}{\partial Q} W(x, Q)\Big|_{Q=x+y} = 0$

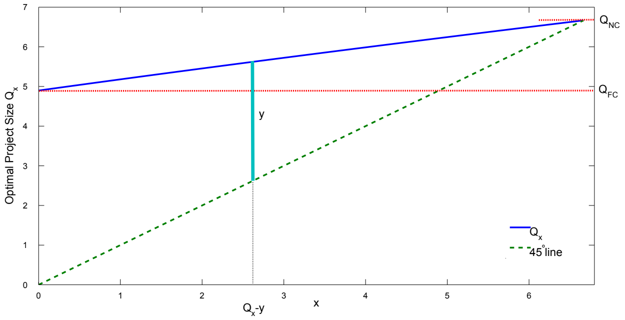
  i.e., manager will commit to Q_x at $x = Q_x - y$.

  (where $Q_x = \arg\max_Q \{W(x; Q)\}$)

Optimal Q with Limited Commitment (Cont'd)

Proposition 3: Suppose that $y < Q_{FC}$.

The manager finds it optimal to commit to Q_x at $x = Q_x - y$.



- Manager will commit as soon as *height* of the wedge equals y.
  - If y is smaller, then manager will commit later and to a larger project.

A "Ratchet Effect"

- Recall that this is a complete information game.
- Anticipating the manager's commitment problem, if y is smaller,
  1. the agents work less ; and
  2. the manager worse off.
- Manager might delegate decision rights over Q to the agents.

The Agents' Optimal Project Size

- Suppose that the project size is chosen by the agents.
- At q_t, the agents will choose $Q_A = \arg\max_Q \{\Pi(q_t; Q)\}$

$$\implies Q_A = \frac{\beta}{r}\frac{k(2n-k)}{2n}.$$

  - Q_A is independent of y, *but* $Q_A < Q_{x(y)}$ for all y.
  - Intuitively, because the agents also trade off effort cost.

Optimal Delegation

- Delegation is optimal iff $\underbrace{W(0;Q_A)}_{\text{independent of y}} > \underbrace{W(0;Q_{x(y)})}_{\text{increases in y}}$.

Proposition 4: Optimal Delegation.

- Manager is better off delegating iff $y < \theta$, where $\theta > 0$.

- If $y = 0$, then the manager always prefers to delegate.

- Typically, delegation arises due to asymmetric information.
  - *e.g.*, Aghion and Tirole (1997), Dessein (2002).
  - Here, it arises solely due to moral hazard !

"Punishing" the Manager

- Can agents induce manager to choose smaller Q by lowering k ?
  - *e.g.*, by deviating to trigger a switch to the MPE ($k = 1$).

1. If a switch is *permanent* (and manager knows this), then *yes*.
   - Agents cannot induce manager to complete project at any $Q < Q_y|_{k=1}$.
   - *Claim*: Agents trigger a switch to MPE at $q_t = \min\left\{Q^A, Q_y|_{k=1}\right\}$.
   - Manager finds it optimal to complete the project *immediately*.
2. If agents can coordinate back to original k w/o delay, then *no*.

Contracting on Project Duration

- Manager might delegate decision rights over Q, but impose a deadline.
  - Clearly, it is optimal to commit to a deadline at time 0.

Proposition: Symmetric MPE with a deadline

Derivation

1. Given a deadline T, there exists a symmetric MPE in which

$$a_t = \frac{\beta}{n} e^{\frac{rn(t-T)}{2n-1}} \;;$$

2. If the manager chooses deadline optimally, then $Q_{T^*} = \frac{\beta}{r} \frac{2n-1}{3n-1}$;

3. $Q_{T^*} \leq Q^A$ and $T^* \leq \tau\left(Q^A\right)$.

Contracting on Project Duration vs Project Size

- Contract on project duration iff $\underbrace{W(0; T^*, Q_{T^*})}_{\text{independent of y}} > \underbrace{W\left(0; Q_{x(y)}\right)}_{\text{increases in y}}$.
  - Similar to optimal delegation result: Inequality holds iff $y < \Phi$.

- The manager might prefer to contract on duration even if $y = \infty$.

Proposition: Suppose manager has full commitment power (*i.e.*, $y = \infty$).
Other things equal, $W(0; Q_{T^*}, T^*) > W(0; Q_{FC})$ iff $n > N_{crit}$ or $k < k_{crit}$.

- Contracting on project duration is optimal *(i)* if the team is sufficiently large, or *(ii)* if free-ridering is too severe.

Socially Efficient Outcome

- **Case** 1: Social planner chooses Q but cannot control efforts.
  - For any y, the social planner will choose $Q_A < Q_y^{SP} < Q_{x(y)}$.
  - Social planner also has incentives to extend the project.
    - Q_y^{SP} decreases in commitment power y.
    - Commitment problem perseveres (but is mitigated).
- **Case 2**: Social planner chooses Q and effort levels.
  - Commitment problem disappears.
  - Because the social planner internalizes effort cost.

Robustness Tests : 5 Extensions

1. Production synergies / team coordination costs ;
   - Total effort is greater / smaller than sum of individual efforts.
2. Agents' compensation is independent of Q ;
   - *e.g.*, projects in which the budget is fixed.
3. A new project is initiated as soon as the previous is completed ;
   - *e.g.*, Employees often hired for multiple projects.
4. Manager incurs per-unit time cost to keep the project *going ;*
   - Her incentives to extend the project are mitigated.
5. Project evolves stochastically.
   - $dq_t = \left(\sum_{i=1}^{n} a_{i,t}\right) dt + \sigma dW_t$.

- In all 5 cases, main results continue to hold.

Main Take-aways

1. Tractable model to analyze a dynamic contribution game.
   - Capture interaction between a manager and a group of agents.
   - Endogenous project size and limited commitment power.
2. Manager has incentives to extend the project as it progresses.
   - She chooses a larger project, the smaller her commitment power.
3. Delegating choice of Q is optimal w/o sufficient commitment power.

What's Next ?

- Optimal Contract
- Deadlines
- Incomplete Information

Derivation of Game with Deadline

Return

- Agent i's Hamiltonian

$$H_{i,t} = -e^{-rt}\frac{a_{i,t}^2}{2} + \lambda_{i,t}\left(\sum_{j=1}^{n} a_{j,t}\right),$$

and his terminal value function $\phi_T = \frac{\beta Q}{n} e^{-rT}$.

- Necessary (and it turns out sufficient) conditions for a MPE:
  1. Optimality equation:
  $$\frac{dH_{i,t}}{da_{i,t}} = 0$$
  2. Adjoint equation:
  $$\dot{\lambda}_{i,t} = -\frac{dH_{i,t}}{dq}$$
  3. Transversality condition:
  $$\lambda_{i,T} = \frac{d\phi_T}{dQ}$$