Optimal Incentives under Moral Hazard: From Theory to Practice

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Motivation

- Imagine you have to design an employee performance pay plan.
- If you know all payoff-relevant parameters (i.e., agent preferences, production function, etc), you can find optimal contract (in principle).
- Otherwise, agency theory gives us guiding principles (trade-offs, CS)

This paper: How to improve an existing PPP?

1. What information do you need?
2. And how should you use that information?
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2. And how should you use that information?
Framework: Static agency model with a risk-averse agent

- Principal knows only distribution of output following $w_A(\cdot)$ and $w_B(\cdot)$.
- Goal: Find a new contract that raises profits as much as possible.

Key Lemma:
If the principal knows the agent’s marginal utility for money, then she can predict the distribution of output corresponding to any* contract.

Then, the principal can find the optimal perturbation.

(*) Formally, (i) $w_B$ must provide infinitesimally different incentives than $w_A$, and (ii), predictions apply only to such contracts.

Demonstrate application using dataset from DellaVigna and Pope.
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Related Literature

- **Agency theory**: Mirrlees (1976), Holmström (1979), etc...

- **Agency problems — Applications**:  
  - Gibbons (1998), Murphy (1999), Prendergast (2002), etc...

- **Agency problems — Empirics**:  
  - Lazear (2000), Shearer (2004), Bandiera et al. (2007, 2009), etc...
  - Chiappori & Salanie (2002)

- **Sufficient statistics**:  
  - Monopoly pricing: Lerner (1934), Tirole (1988), etc...
  - Optimal taxation: Saez (2001), Golosov et al. (2014), etc...
  - General framework: Chetty (2009)
Model

- Principal-agent model with the following timing:
  1. Principal offers a contract $w(\cdot)$.
  2. Agent observes $w(\cdot)$ and chooses effort $a(w) \in \mathbb{R}$.
  3. Output $x \sim f(\cdot|a(w))$ and payoffs are realized. (Normalize $\mathbb{E}[x|a] = a$.)

- Preferences:
  - Agent’s utility: $\int v(w(x))f(x|a)dx - c(a)$
  - Principal’s profit: $\pi(w) := ma(w) - \int w(x)f(x|a)dx$.

- Information:
  - Principal knows (only) $f(\cdot|a)$ and $f_a(\cdot|a)$ for $a \in \{a(w_A), a(w_B)\}$.
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- Principal knows (only) $f(\cdot|a)$ and $f_a(\cdot|a)$ for $a \in \{a(w_A), a(w_B)\}$.

- Agent knows all payoff-relevant parameters.
In the canonical formulation (Holmström, 1979), the principal solves

\[
\max_{w(\cdot), a} \int [mx - w(x)] f(x|a) \, dx
\]

s.t.

\[
\int v(w(x)) f(x|a) \, dx - c(a) \geq u \quad \text{(IR)}
\]

\[
\int v(w(x)) f_a(x|a) \, dx = c'(a) \quad \text{(IC)}
\]

To do so, she must know \(v(\cdot), u, c(a),\) and \(f(\cdot|a)\) for all \(a\).

In our setting, she only knows \(f(\cdot|a)\) and \(f_a(\cdot|a)\) for \(a \in \{a_1, a_2\}\).
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Agent’s Problem

- **A1:** Optimal effort \( a(w) \) satisfies the first-order condition

\[
\int_v w(x) f_a(x|a(w)) dx = c'(a(w)) \tag{IC}
\]

- Suppose \( w(\cdot) \) is replaced by (some other) contract \( w(\cdot) + \theta t(\cdot) \).

- Define the directional (Gateaux) derivative

\[
D a(w, t) := \left. \frac{d a(w + \theta t)}{d \theta} \right|_{\theta=0} = \lim_{\theta \to 0} \frac{a(w + \theta t) - a(w)}{\theta},
\]

interpreted as the MC of \( a \) when \( w \) perturbed in the direction of \( w + t \).

- Using (IC), we can rewrite \( D a(w, t) \) as

\[
D a(w, t) = \frac{\int t v'(w) f_a dx}{c''(a(w)) - \int v(w) f_{aa} dx}.
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Principal’s Problem

- If $w(\cdot)$ is replaced by (some) $w(\cdot) + \theta t(\cdot)$, then the principal’s profit

$$\pi(w + \theta t) \approx \pi(w) + \theta \mathcal{D}\pi(w, t),$$

where $\mathcal{D}\pi(w, t)$ is the derivative of $\pi(w)$ in direction of $w + t$, and

$$\mathcal{D}\pi(w, t) := \frac{d\pi(w + \theta t)}{d\theta} \bigg|_{\theta=0} = \left( m - \int w f_a dx \right) \mathcal{D}a(w, t) - \int tf dx$$

- A2: Principal chooses $t(\cdot)$ to maximize $\mathcal{D}\pi(w_A, t)$ subject to giving the agent at least as much expected utility as $w_A$.

- Using (IC) and the envelope condition, we can write the constraint as

$$\int tv'(w_A) \tilde{f} dx \geq 0$$
Principal’s Problem

- If $w(\cdot)$ is replaced by (some) $w(\cdot) + \theta t(\cdot)$, then the principal’s profit
  \[ \pi(w + \theta t) \simeq \pi(w) + \theta \mathcal{D} \pi(w, t), \]
  where $\mathcal{D} \pi(w, t)$ is the derivative of $\pi(w)$ in direction of $w + t$, and
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- If \( w(\cdot) \) is replaced by (some) \( w(\cdot) + \theta t(\cdot) \), then the principal’s profit

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\pi(w + \theta t) \simeq \pi(w) + \theta D\pi(w, t),
\]

where \( D\pi(w, t) \) is the derivative of \( \pi(w) \) in direction of \( w + t \), and

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- Using (IC) and the envelope condition, we can write the constraint as

\[
\int tv'(w_A)\widehat{f} dx \geq 0
\]
Principal’s Problem (Cont’d)

The principal solves

\[
\max_{t \text{ u.s.c}} \left( m - \int w_A \hat{f}_a \, dx \right) \mathcal{D}_a(w_A, t) - \int t \hat{f} \, dx
\]  
\[
\text{s.t } \int t v'(w_A) \hat{f} \, dx \geq 0
\]  
\[
\int |t|^p \, dx \leq 1
\]

where \( p \in \{2, 3, \ldots\} \) is a “smoothing parameter”.

Given a solution to (P), the principal should replace \( w_A(\cdot) \) with

\[
w(x) = w_A(x) + \theta t(x),
\]

where \( \theta > 0 \) is an appropriately chosen step size.

To solve (P), principal must know \( v' \) and \( \mathcal{D}_a(w_A, t) \) for all feasible \( t \)!
The principal solves

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s.t

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To solve (P), principal must \textit{know} \( v' \) and \( \mathcal{D}a(w_A, t) \) for all feasible \( t \)!
Key Lemma

Recall that
\[ Da(w, t) = \frac{\int tv'(w)f_a dx}{c''(a(w))} - \int \nu(w)f_{aa} dx \]

Lemma 1. For any (upper semi-continuous) \( t \):
\[ Da(w_A, t) = \frac{Da(w_A, w_B - w_A)}{\int (w_B - w_A)\nu'(w_A)\hat{f_a} dx} \int tv'(w_A)\hat{f_a} dx \]
\[ = DM(w_A, t) \]

Perturbation leads to a change in the agent’s marginal incentives, \( DM(w_A, t) \), which is predictable given \( \nu' \) and \( \hat{f_a} \). Locally,
\[ Da(w_A, t) = \text{const} \times DM(w_A, t) \]
where \( \text{const} = \frac{Da(w_A, w_B - w_A)}{DM(w_A, w_B - w_A)} \).

If the principal takes a stance on \( \nu' \), then she can solve (P).
Key Lemma

- Recall that
  \[ D_a(w, t) = \frac{\int tv'(w)f_a \, dx}{c''(a(w)) - \int v(w)f_{aa} \, dx} \]

Lemma 1. For any (upper semi-continuous) \( t \):

\[ D_a(w_A, t) = \frac{D_a(w_A, w_B - w_A)}{\int (w_B - w_A)v'(w_A)\widehat{f}_a \, dx} \left( \int tv'(w_A)\widehat{f}_a \, dx \right) \frac{\widehat{D} M(w_A, t)}{\widehat{D} M(w_A, w_B - w_A)} \]

- Perturbation leads to a change in the agent’s marginal incentives, \( \mathcal{D} M(w_A, t) \), which is predictable given \( v' \) and \( \widehat{f}_a \). Locally,

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  where \( \text{const} = \frac{D_a(w_A, w_B - w_A)}{\mathcal{D} M(w_A, w_B - w_A)} \).

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Key Lemma

- Recall that
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**Lemma 1.** For any (upper semi-continuous) \( t \):

\[ D_a(w_A, t) = \frac{D_a(w_A, w_B - w_A)}{\int (w_B - w_A)v'(w_A)\widehat{f}_a dx} \left( \int tv'(w_A)\widehat{f}_a dx \right) / D_M(w_A, t) \]

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where \( \text{const} = \frac{D_a(w_A, w_B - w_A)}{D_M(w_A, w_B - w_A)} \).

- If the principal *takes a stance on* \( v' \), then she can solve (P).
Main Result

Optimal Perturbation

- The principal’s problem can be rewritten as

$$\max_{t \text{ u.s.c}} \mu^* \int t v'(w_A) \hat{f}_a dx - \int t \hat{f} dx$$

s.t. $$\int t v'(w_A) \hat{f} dx \geq 0$$

$$\int |t|^p dx \leq 1$$

where $$\mu^*$$ is a constant that depends on $$w_A$$, $$w_B$$, $$v'$$, and $$\hat{f}_a$$.

Main Proposition.

i. The optimal perturbation (when $$p = 2$$)

$$t^* = C \times \left[ \lambda^* + \mu^* \frac{\hat{f}_a}{\hat{f}} - \frac{1}{v'(w_A)} \right]$$

ii. $$w_A$$ is locally optimal iff $$\lambda + \mu^* \frac{\hat{f}_a}{\hat{f}} = \frac{1}{v'(w_A)}$$, where $$\lambda = \int \frac{\hat{f}}{v'(w_A)} dx$$. 
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ii. \(w_A\) is locally optimal iff \(\lambda + \mu^* \frac{\hat{f}_a}{\hat{f}} = \frac{1}{v'(w_A)}\), where \(\lambda = \int \hat{f} / v'(w_A) dx\).
Extensions

1. **Bounded payments.** Assume that $w_A(x) + \theta t(x) \in [\underline{w}, \overline{w}]$
   - New constraints are linear, so principal’s problem remains convex.

2. **Heterogeneous abilities.** Assume that the principal offers a common contract to multiple agents who have heterogeneous effort costs.
   - Principal must classify the agents into types $\phi$, and estimate $\Pr\{\phi\}$, $\overline{f}_\phi$, $\overline{f}_a$\phi$, and $\mathcal{D}_a(\overline{w}, \overline{t})$ for each $\phi$.
   - Can induce selection by imposing participation for subset of types.

3. **Multidimensional effort.** Assume agent’s effort $a \in \mathbb{R}^N$ at cost $c(a)$
   - e.g., effort towards quantity & quality, or selling different products.
   - Principal must have output data for $K \geq (N + 3)/2$ contracts.
4. **Parametric contract classes.** Assume the principal restricts attention to contracts of the form $w_{\alpha}$, where $\alpha$ is a vector of parameters.

- Find optimal perturbation direction $z$. *(New contract: $w_{\alpha+\theta z}$)*
- Same informational requirements as general case.

5. **Other sources of incentives.** (Promotion, firing threat, prestige, etc)

- Results hold verbatim if the agent’s IC constraint can be written as
  \[
  \int v(w)f_a \, dx + I(a(w)) = c'(a(w)),
  \]
  where $I(a)$ denotes marginal benefit of effort due to *other incentives*.
- **Key:** Additive separability and $I(\cdot)$ not directly dependent on $w$.

6. **Multiplicatively separable utility.** Agent’s payoff $u(\omega, a) = v(\omega)c(a)$

- **Example:** Agent’s utility satisfies CARA.
- Principal must take a stance on $v$ (instead of $v'$).
Goal: Illustrate how to apply the methodology presented

Dataset from DellaVigna and Pope (REStud, 2017)

Real-effort experiment on M-Turk: Subjects press a-b keys for 10 min

7 treatments with different monetary incentives:

<table>
<thead>
<tr>
<th>Contract (in ¢)</th>
<th>Mean effort ($\overline{x}$)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1(x) = 100$</td>
<td>1521</td>
<td>540</td>
</tr>
<tr>
<td>$w_2(x) = 100 + 0.001x$</td>
<td>1883</td>
<td>538</td>
</tr>
<tr>
<td>$w_3(x) = 100 + 0.01x$</td>
<td>2029</td>
<td>558</td>
</tr>
<tr>
<td>$w_4(x) = 100 + 0.04x$</td>
<td>2132</td>
<td>566</td>
</tr>
<tr>
<td>$w_5(x) = 100 + 0.10x$</td>
<td>2175</td>
<td>538</td>
</tr>
<tr>
<td>$w_6(x) = 100 + 40 I_{{x\geq 2000}}$</td>
<td>2136</td>
<td>545</td>
</tr>
<tr>
<td>$w_7(x) = 100 + 80 I_{{x\geq 2000}}$</td>
<td>2188</td>
<td>532</td>
</tr>
</tbody>
</table>
Assuming subjects are identical and $v'(\omega) = \omega^{-\rho}$, perform 3 exercises:
1. Given data for any two treatments, predict effort in the other five

   i. Linear extrapolation using Lemma 1

   ii. Nonlinear extrapolation using \( \log a(w) = \beta + \epsilon \log M(w) \), where

   \[
   M(w) := \int v(w(x)) f_a(x|a(w_A)) dx
   \]

   - Agent FOC implies \( M(w) = c'(a) \). (i) and (ii) implicitly assume \( f_a(\cdot|a) \) is constant in \( a \), and \( c(\cdot) \) has constant elasticity 1 and \( \epsilon \), respectively.

2. Given any 2 treatments, characterize the optimally perturbed contract

   - Pin down the step size \( \theta \) using the nonlinear extrapolation above.

3. Use all 7 treatments to structurally estimate the model

   - Compare optimal contract to perturbed contract derived above.

   - Estimate profit of perturbed contract & compare to optimal contract.
Data Exercises

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ii. Nonlinear extrapolation using \( \log a(w) = \beta + \epsilon \log M(w) \), where

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   i. Linear extrapolation using Lemma 1
   
   ii. Nonlinear extrapolation using $\log a(w) = \beta + \epsilon \log M(w)$, where
   
   $$M(w) := \int v(w(x)) f_a(x|a(w_A)) dx$$
   
   - Agent FOC implies $M(w) = c'(a)$. (i) and (ii) implicitly assume $f_a(\cdot|a)$ is constant in $a$, and $c(\cdot)$ has constant elasticity 1 and $\epsilon$, respectively.

2. Given any 2 treatments, characterize the optimally perturbed contract
   
   - Pin down the step size $\theta$ using the nonlinear extrapolation above.

3. Use all 7 treatments to structurally estimate the model
   
   - Compare optimal contract to perturbed contract derived above.
   
   - Estimate profit of perturbed contract & compare to optimal contract.
Data Exercises

1. Given data for any two treatments, predict effort in the other five
   
i. Linear extrapolation using Lemma 1
   
ii. Nonlinear extrapolation using $\log a(w) = \beta + \epsilon \log M(w)$, where

   $$M(w) := \int v(w(x)) f_a(x|a(w_A)) dx$$

   - Agent FOC implies $M(w) = c'(a)$. (i) and (ii) implicitly assume $f_a(\cdot|a)$ is constant in $a$, and $c(\cdot)$ has constant elasticity 1 and $\epsilon$, respectively.

2. Given any 2 treatments, characterize the optimally perturbed contract
   
   - Pin down the step size $\theta$ using the nonlinear extrapolation above.

3. Use all 7 treatments to structurally estimate the model
   
   - Compare optimal contract to perturbed contract derived above.
   
   - Estimate profit of perturbed contract & compare to optimal contract.
Exercise 1: Predicting effort

![Graph showing effort prediction given treatments 3 and 6 (\(\rho = 0.2\))](image)

- **True**
- **Linear extrap.**
- **Nonlinear extrap.**

- Piece Rate
- 0 Piece Rate
- 0.1c PR
- 1c PR
- 4c PR
- 10c PR
- 40c Bonus
- 80c Bonus

- Effort

- Treatment
- 1500
- 1700
- 1900
- 2100
- 2300
- 2500
Exercise 1: Predicting effort

Effort prediction given treatments 2 and 3 ($\rho = 0.2$)

- **True**
- **Linear extrap.**
- **Nonlinear extrap.**
Exercise 2: Estimated pdf’s

- Suppose firm has data for treatments 3 and 4 (1¢ and 4¢ piece-rate)
- First, estimate \( f(\cdot|a(w_3)) \) and \( f(\cdot|a(w_4)) \) using kernels, and compute

\[
f_a(x|a(w_3)) \approx f_a(x|a(w_3)) \approx \frac{f(x|a(w_4)) - f(x|a(w_3))}{a(w_4) - a(w_3)}\]

![Graph showing estimated pdf's](image-url)
Exercise 2: Optimally *Perturbed* Contract

Optimally Perturbed Contracts given treatments 3 and 4 (assuming $m = 0.2$ and $\rho = 0.9$)

$$w_{\theta=1k}(x)$$
$$w_{\theta=2.5k}(x)$$
$$w_{\theta=3k}(x)$$

Projected Profit

$x$

$\theta$
Exercise 2: Optimally *Perturbed* Contract

Optimally Perturbed Contract assuming CRRA utility and $m = 0.2$

- $\rho = 0.5$
- $\rho = 0.6$
- $\rho = 0.7$
- $\rho = 0.8$
- $\rho = 0.9$
- "Simple" contract
Exercise 3: Optimal Contract

Assumptions: CRRA utility with coefficient 0.7 and $m = 0.2$

- Optimal Contract
- Optimally Perturbed Contract
- "Simple" Contract

Approximately the same profit!
Summary & Future Work

- Framework for using agency theory to address an empirical question.
  - How to improve an existing performance pay plan?
  - What information do you need to do so?
  - Next steps: Further test methodology with additional experiments

- Other questions:
  - Optimal experimentation (ratchet effects, behavioral constraints)?
  - Extend to other settings?
  - Other optimization algorithms (more efficient / robust?)