Optimal Project Design

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Motivation

- Rents due to agency problems is key determinant of economic welfare
- Determinants of these frictions are usually part of model description
 - In adverse selection models, distribution of types typically exogenous
 - In moral hazard models, production technology taken as given
- If an agent's payoff depends on agency frictions, then he is likely to take actions to generate these frictions optimally.

This Paper.

Revisit standard principal-agent model under moral hazard to understand how an agent might gain by designing the production technology optimally.

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Model

- Players. Risk-neutral principal & risk-averse, cash constrained agent
 - Today: Agent will be risk-neutral (results generalize to any concave u)
- Timing.
 - i. Agent chooses a "project" $c : \Delta([0,1]) \to \mathbb{R}_+$; *i.e.*, a map from every output distribution with support on [0,1] to a (nonnegative) cost.
 - ii. Principal offers a wage scheme $w : [0,1] \rightarrow \mathbb{R}_+$
 - iii. Agent chooses an "action" $F \in \Delta([0,1])$
 - iv. Output $x \sim F$ and payoffs are realized
- Payoffs.
 - Agent: $\mathbb{E}_{F}[w(x)] c(F)$
 - Principal: $\mathbb{E}_{F}[x w(x)]$
 - Both players have outside option 0

Applications

- An entrepreneur (agent) seeks funding from a VC (principal)
- Before contracting, the entrepreneur must develop a business plan, specifying various aspects of his production function
- Conceivable he has at least some flexibility in choosing the biz plan.
- If VC has a lot of bargaining power, the entrepreneur benefits from putting forward a biz plan that exacerbates moral hazard problem.
- Remark: Abstract away from constraints in the agent's flexibility.
- More broadly, employees can often influence aspects of production function (*e.g.*, assignment of projects, goals, evaluation metrics, etc

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Some Intuition

- Efficient outcome. Agent sets c(F) = 0 for all F
- Principal responds by offering wage 0 and implementing $F(x) = \mathbb{I}_{\{x=1\}}$
 - *i.e.*, while outcome is efficient, the agent is left with no rents!
- *Mechanism.* Agent chooses the project to make the moral hazard problem *severe*, which will enable him to extract rents.

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Problem Formulation

• Principal. Given project c, she solves:

$$\max_{w(\cdot),F} \mathbb{E}_{F}[x - w(x)]$$

s.t. $\mathbb{E}_{F}[w(x)] - c(F) \ge \mathbb{E}_{\widetilde{F}}[w(x)] - c(\widetilde{F}) \text{ for all } \widetilde{F}$
 $w(x) \ge 0 \text{ for all } x$
 $F \in \Delta([0,1])$

Denote the optimal contract by w^c and implemented action by F^c .

• Agent. Chooses the optimal project by solving:

$$\max \mathbb{E}_{F^c}[w^c(x)] - c(F^c)$$

s.t. $c: \Delta([0,1]) \to \mathbb{R}_+$

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() Optimal project is *coarse*: all feasible actions generate binary output

• Binary projects effectively restrict the contracting space, making it more expensive for the principal to motivate the agent.

Action space is rich: Optimal (binary) project comprises

- continuum of zero-cost actions where project succeeds with some prob.
- a high cost action which guarantees success
- a spectrum of actions in between.
- Inefficiency: Maximal output realized in equilibrium at bloated costs
- *Rents*: The agent extracts all rents
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Plan of Attack

- Theorem 1: Show it suffices to restrict attention to binary projects
 - Given an arbitrary project, we construct a new project such that

c(F) < 1 iff $supp(F) = \{0, 1\}$, and the agent is (weakly) better off.

- This dramatically reduces the dimensionality of the problem so that:
 - In Stage 1, the agent assigns a cost $C(p) \ge 0$ to each $p = \Pr\{x = 1\}$
 - In Stage 2, the principal offers a bonus contract $w(x) = b\mathbb{I}_{\{x=1\}}$
 - In Stage 3, agent chooses p at a cost C(p)

• Theorem 2: Characterize the optimal project (in closed form)

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Properties of an Optimal Project

Theorem 1.

- For any project c, there exists another project, \widetilde{c} , such that
 - i. $\widetilde{c}(F) < 1$ if and only if $supp(F) = \{0, 1\}$ (*i.e.*, output is binary), and
 - ii. the principal optimally implements $F(x) = \mathbb{I}_{\{x=1\}}$ (*i.e.*, x = 1 w.p 1),

which gives the agent a (weakly) larger expected payoff.

- Informativeness principle: if the signal is made less informative, then incentivizing the agent becomes more expensive
- Output x acts as a signal about the agent's action
- Binary distribution provides the least informative signal among all distributions with the same mean

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Binary Projects: Proof

- Fix a c & suppose principal offers w^* , implementing F^* (w/ mean μ^*)
- Construct a new project \tilde{c} : For each $\mu \in [0,1]$, define

$$B_{\mu} = (1 - \mu) + \mu \mathbb{I}_{\{x=1\}}$$
 and
 $\widetilde{c}(B_{\mu}) = \inf \{c(F) : \mathbb{E}_{F}[x] = \mu\}$

i.e., B_{μ} is a distribution with support $\{0, 1\}$ and mean μ , and we assign it the cost of the cheapest distribution in c with same mean.

Given *c*, wolog, the principal offers a bonus contract w(x) = bI_{x=1}, or equivalently, a linear contract w(x) = bx.

Binary Projects: Proof

• Consider the problem of implementing any action at max profit

$$\Pi(F) = \sup_{w(\cdot) \ge 0} \left\{ \mathbb{E}_F[x - w(x)] : F \text{ is } \mathsf{IC} \right\}, \text{ and}$$
$$\widetilde{\Pi}(B_\mu) = \sup_{b \in [0,1)} \left\{ (1 - b)\mu : B_\mu \text{ is } \mathsf{IC} \right\},$$

in the original and the new project, c and \tilde{c} , respectively.

- Lemma 1: For any F such that $\mathbb{E}_F[x] = \mu$, $\widetilde{\Pi}(B_{\mu}) \leq \Pi(F)$.
 - *i.e.*, implementing B_{μ} is less profitable than an F with same mean.
 - Suppose the principal were restricted to linear contracts in c. Then: $\Pi_{lin}(F) = \widetilde{\Pi}(B_{\mu}) \text{ for all } F \text{ with mean } \mu.$
 - Absent this restriction, her profit is weakly larger; *i.e.*, $\Pi(F) \ge \prod_{lin}(F)$.

Binary Projects: Proof

- Define $B^* = B_{\mu^*}$ and $b^* = \mathbb{E}_{F^*}[w^*(x)]/\mu^* < 1$
- If $w(x) = b^* \mathbb{I}_{\{x=1\}}$ implements B^* , then:
 - It makes the same expected payment to the agent as w^* .
 - **2** It generates profit equal to $\Pi(F^*)$ for the principal.
- If b^* does not implement B^* , adjust cost $\tilde{c}(B^*) = \inf_{\mu} \{b^* \mu c(B_{\mu})\}$
- Lemma 2: Principal cannot implement B^* with any $b < b^*$.
 - Suppose B^* can be implemented by some $b < b^*$
 - If $\tilde{c}(B^*)$ was adjusted, this contradicts the above definition of $\tilde{c}(B^*)$.
 - If $\tilde{c}(B^*)$ was not, then the premise contradicts Lemma 1.

Binary Projects: Proof

- By assumption, F^* is optimal in c; *i.e.*, $\Pi(F^*) \ge \Pi(F)$ for all F
- By Lemma 1, $\widetilde{\Pi}(B_{\mu}) \leq \Pi(F)$ for any F with mean μ
- By construction, $\widetilde{\Pi}(B^*) = \Pi(F^*)$, and therefore,

 $\widetilde{\Pi}(B^*) \ge \widetilde{\Pi}(B_\mu)$ for all μ

i.e., the principal optimally implements B^* in \tilde{c} .

- Also by construction, agent is weakly better off relative to $\{c, w^*\}$.
- If $\mu^* = 1$, then the proof is complete.

Binary Projects: Proof

• Suppose $\mu^* < 1$. Since b^* implements B^* , the following IC is satisfied

$$b^*\mu^* - \widetilde{c}(B^*) \ge b^*\mu - \widetilde{c}(B_\mu)$$
 for all μ .

- Observation: This constraint is slack for all $\mu > \mu^*$.
 - If not, b^* implements $B_{\mu'}$ for some $\mu' > \mu^*$ giving principal bigger profit
- Therefore, wolog, we can adjust $\tilde{c}(B_{\mu}) = \infty$ for all $\mu > \mu^*$.
- Multiply bonus b^* , costs and success prob. $\Pr{x = 1}$ by $1/\mu^* > 1$.
 - Payoffs are scaled up and IC constraints are unchanged.
- **Summary:** New project comprises only actions with support {0,1}, principal optimally implements *x* = 1 w.p. 1, and agent is better off.

Implication

- By Theorem 1, it suffices to restrict attention to:
 - Actions such that

$$x = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases}$$

- Cost function $C(p) \ge 0$ such that principal optimally implements p = 1
- Bonus contracts $w(x) = b\mathbb{I}_{\{x=1\}}$ for some $b \ge 0$ to be chosen.
- We will solve the problem using backward induction

Heuristic Characterization – Stage 2

• Fix a cost function $C(\cdot)$. Then the principal solves

$$\begin{array}{l} \max \ p(1-b) \\ \text{s.t.} \ pb - C(p) \ge \widetilde{p}b - C(\widetilde{p}) \quad \text{for all } \widetilde{p} \in [0,1] \\ p \in [0,1] \quad \text{and} \quad b \ge 0 \end{array}$$

• Guess that *C* is twice differentiable and convex. Then we can replace the agent's IC constraint with its first-order condition:

$$b = C'(p)$$

and rewrite the principal's problem as

$$\pi \coloneqq \max_{p} p \left[1 - C'(p) \right]$$

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Heuristic Characterization – Stage 1

The agent solves

$$\max_{C(\cdot)\geq 0} p^* b - C(p^*)$$

s.t $p^* \left[1 - C'(p^*)\right] \geq p \left[1 - C'(p)\right]$ for all p (IC_P)

where $p^* = 1$ by Theorem 1, and $b = C'(p^*)$ from the agent's FOC.

• Using that $C'(1) = 1 - \pi$, we can rewrite this maximization program as

$$\max 1 - \pi - \int_0^1 C'(q) dq$$

s.t. $C'(p) \ge 1 - \frac{\pi}{p}$ for all $p < 1$
 $C(\cdot) \ge 0$ and $\pi \in [0, 1]$

Heuristic Characterization – Stage 1 (Continued)

• Step 1: For (any) fixed π , we solve

$$\max_{C(\cdot)\geq 0} 1 - \pi - \int_0^1 C'(p) dp$$

s.t. $C'(p) \geq 1 - \frac{\pi}{p}$ for all $p < 1$

• Objective decreases in C'(p) and constraint imposes lower bound. So

$$C'(p) = \left[1 - \frac{\pi}{p}\right]^2$$

• Step 2: Plugging $C'(\cdot)$ into the agent's objective, we solve

$$\max_{\pi \in [0,1]} \{ -\pi \ln \pi \} = \frac{1}{e} \text{ and } \pi^* = \frac{1}{e};$$

i.e., the principal's, as well as the agent's payoff is 1/e.

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Characterization

Theorem 2. Optimal Project

• There exists an optimal project in which the agent chooses

$$C'(p) = \begin{cases} 0 & \text{if } p \le 1/e \\ 1 - \frac{1}{pe} & \text{if } p > 1/e \end{cases}$$

- The principal offers bonus contract with b = 1 1/e
- Each player obtains payoff equal to 1/e
- The agent chooses a convex cost function s.t any p ≤ 1/e is costless, while larger p's are progressively more expensive and the principal is is indifferent across any bonus contract with b ∈ [0, 1 – 1/e].

• Principal's profit $\pi = 1/e$, and agent captures all rents for p > 1/e.

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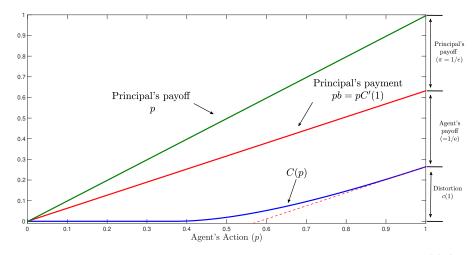
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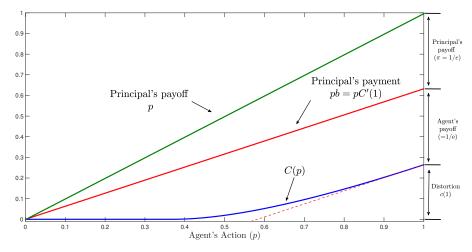
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Graphically



• To capture rents, agent commits to rent seeking activity costing C(p).

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Payoff pairs implementable by an arbitrary binary project

• Insofar, we have assumed the agent can choose any cost function

 $c: \Delta([0,1]) \to \mathbb{R}_+$

- Suppose the agent is constrained and must choose among a subset of these cost functions.
- Q: Can we make any predictions regarding surplus allocation?
- Let V (c) = {π*, U*} be the set of equilibrium payoffs for given c, and define the payoff possibility set:

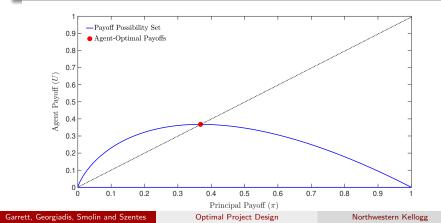
$$\mathcal{P} = \bigcup_{c:\Delta([0,1])\to\mathbb{R}_+} V(c).$$

Payoff pairs implementable by an arbitrary binary project

Theorem 3. Payoff Possibility Set

The payoff possibility set is

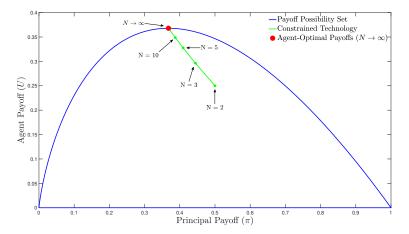
$$\mathcal{P} = \operatorname{co}\left(\left\{\pi, -\pi \log \pi\right\} : \pi \in [0, 1]\right).$$



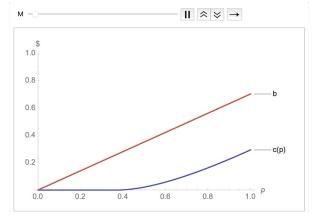
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Bounded Project Complexity

- Suppose the agent can choose a project with at most N actions.
- By Theorem 1, wolog, he chooses $p_i \in [0,1]$ and $C(p_i) \ge 0$ for each i

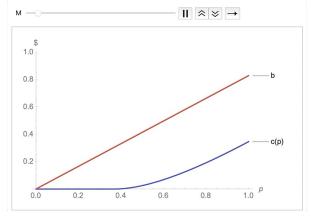


- Suppose agent can choose output distributions with support [-M, 1].
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



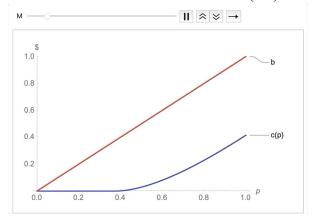
• When M = 0, $C(\cdot)$ and b are given in Theorem 2.

- Suppose agent can choose output distributions with support [-M, 1].
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



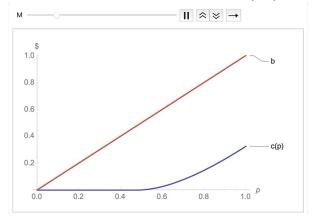
• As $M \uparrow$, both $C(\cdot)$ and b are shifted upwards.

- Suppose agent can choose output distributions with support [-M, 1].
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



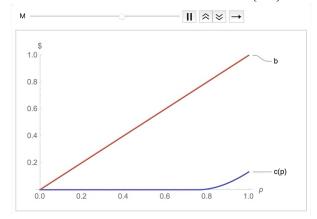
• For M sufficiently large, b = 1, and agent extracts all surplus.

- Suppose agent can choose output distributions with support [-M, 1].
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



• As $M \uparrow$ further, $C(\cdot)$ is shifted downwards, decreasing distortion.

- Suppose agent can choose output distributions with support [-M, 1].
- Suffices to focus on binary projects s.t $F(x) = \mathbb{I}_{\{x=1\}}$ is implemented.



• As $M \to \infty$, b = 1 and $C(\cdot) \to 0$ leading to efficiency.

Related Literature (Incomplete List)

- Principal-agent models:
 - Informativeness principle: Holmström (1979), Chaigneau et al. (2019)
 - Limited liability: Innes (1990), Poblete and Spulber (2012)
 - Gaming / multitasking: Carroll (2015), Barron et al. (2020)
 - Endogenous monitoring technology: Georgiadis and Szentes (2020)
- Sequential mechanism design:
 - Krähmer and Kovác (2016)
 - Bhaskar et al. (2019)
 - Condorelli and Szentes (2020)

Discussion

- We consider an agency model of moral hazard in which production technology is endogenous and chosen by the agent.
- The agent optimally designs a project with binary output such that the principal is indifferent between b* and any smaller bonus, enabling him to extract all rents.
- *Potential implication.* Promoting more flexibility for workers to design their job as an alternative to regulation (*e.g.*, minimum wages)