Flexible Moral Hazard Problems

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Overview

Classic moral hazard model:

- Effort is either binary, or belongs to an interval.
- Main result: contracts are motivated by informativeness.
- Consequently, contracts are monotone only under MLRP.

Current paper:

- Allow agent to choose *any* output distribution.
- Contracts pinned down by an output-by-output FOC.
- Monotone costs $\implies$ monotone contracts.
- In particular: Informativeness plays no role.

Two Examples

Common Setup for Examples

A principal (she) contracts with an agent (he).

- Compact set $X \subset \mathbb{R}$ of possible outputs.
- Principal offers agent a (bounded) contract: $w : X \to \mathbb{R}$.
- Agent can opt out and get u_0.
- If opts in, agent covertly chooses $\alpha \in \mathcal{A} \subseteq \Delta(X)$.
- Effort costs: $C : \mathcal{A} \to \mathbb{R}_+$, continuous, increasing in FOSD.
- Payoffs:

$$\text{Principal: } x - w \qquad \text{Agent: } u(w) - C(\alpha).$$

u: strictly increasing, differentiable, unbounded, concave.

Standard Binary Effort Model

$$X = [L, H], \qquad \mathcal{A} = \{\alpha_l, \alpha_h\}.$$

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$$\min_{w(\cdot)} \int w(x)\alpha_h(\mathrm{d}x) \quad \text{s.t.} \quad \text{IR and IC.}$$

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The FOC from this cost minimization problem is:

$$\frac{1}{u'(w(x))} = \lambda + \mu\left[1 - \frac{f_l(x)}{f_h(x)}\right]$$

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So: w is monotone $\iff$ MLRP holds.

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Implications:

- Cost minimization is trivial: min $w(L)$ s.t. IR.
- IC contracts are monotone:

$$w(H) = u^{-1}(u \circ w(L) + C'(\alpha)) \geq u^{-1}(u \circ w(L)) = w(L).$$

Model

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- Limited liability: $w(\cdot) \geq 0$.
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Assumption. (smoothness) C is **Gateaux differentiable**: every α admits a continuous $k_\alpha : X \to \mathbb{R}$ s.t.

$$\lim_{\epsilon \downarrow 0} \frac{1}{\epsilon} \left[C(\alpha + \epsilon(\beta - \alpha)) - C(\alpha) \right] = \int k_\alpha(x)\, (\beta - \alpha)\, (\mathrm{d}x)$$

for all $\beta \in \mathcal{A}$.

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(if X is finite: C smooth $\iff$ differentiable, which holds a.e.)

First-Order Approach

Lemma. For a bounded $v : X \to \mathbb{R}$, and $\alpha \in \mathcal{A}$,

$$\alpha \in \arg\max_{\beta \in \mathcal{A}} \left[\int v(x)\beta\,(\mathrm{d}x) - C(\beta) \right]$$

if and only if

$$\alpha \in \arg\max_{\beta \in \mathcal{A}} \left[\int v(x)\beta(\mathrm{d}x) - \int k_\alpha(x)\beta(\mathrm{d}x) \right]$$

(the "only if" direction also works if C is not convex)

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An equivalent way of writing the above condition is:

$$x^* \in \text{argmax}_{x \in [0,1]}[xv - xc'(x^*)].$$

The lemma generalizes the second formulation.

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Fix $\beta \in \mathcal{A}$, and for every $\epsilon \in [0, 1]$, define $\beta_\epsilon := \alpha + \epsilon(\beta - \alpha)$.

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$$0 \geq \frac{1}{\epsilon}\left[\int v(x)(\beta_\epsilon - \alpha)(\mathrm{d}x)\right] - \frac{1}{\epsilon}\left[C(\beta_\epsilon) - C(\alpha)\right]$$

(because $\alpha \in \operatorname{argmax}_{\beta \in \mathcal{A}} \left[\int v(x)\beta(\mathrm{d}x) - C(\beta)\right]$)

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(by definition of β_ϵ)

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&= \int v(x)(\beta - \alpha)(\mathrm{d}x) - \frac{1}{\epsilon}\left[C(\beta_\epsilon) - C(\alpha)\right] \\
&\xrightarrow{\epsilon \to 0} \int v(x)(\beta - \alpha)(\mathrm{d}x) - \int k_\alpha(x)(\beta - \alpha)(\mathrm{d}x).
\end{aligned}
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(by Gateaux differentiability)

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Divide by ϵ, and take $\epsilon \to 0$,

$$C(\beta) - C(\alpha) \geq \frac{1}{\epsilon}\left[C(\epsilon\beta + (1-\epsilon)\alpha) - C(\alpha)\right] \to \int k_\alpha(x)(\beta - \alpha)(\mathrm{d}x).$$

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- Effort costs $C : \mathcal{A} \to \mathbb{R}_+$: convex, increasing, smooth.
- Limited liability: $w \geq 0$.
- Feasible contracts: $W = \{w : X \to \mathbb{R}_+ : \text{bounded}\}$.
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u: increasing, continuous, unbounded & $u(0) = 0$.

Characterization of IC

A contract-distribution pair $(w, \alpha) \in W \times \mathcal{A}$ is **IC** if and only if

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Proposition. (w, α) is IC if and only if a $m \in \mathbb{R}$ exists such that

$$w(x) \leq u^{-1}(k_\alpha(x) + m)$$

for all x, and with equality α-almost surely.

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(iii) Every α can be implemented with a monotone contract (since C is FOSD monotone, k_α is increasing).

Profit Maximization

The Principal's Problem

Let $w_\alpha := w_{m^*_\alpha, \alpha}$ be the cost minimizing wage implementing α.

The principal's problem is:

$$\max_{\alpha \in \mathcal{A}} \left[\int x \alpha(\mathrm{d}x) - \int w_\alpha(x) \alpha(\mathrm{d}x) \right].$$

Additional Assumptions

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2nd Order Differentiability. Every α admits a continuous function $h_\alpha : X \times X \to \mathbb{R}$ such that for

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon} \left[k_{\alpha+\epsilon(\beta-\alpha)}(\cdot) - k_\alpha(\cdot) \right] = \int h(\cdot, y)(\beta - \alpha)(\mathrm{d}y),$$

where convergence is in the supnorm.

(for finite X: equivalent to twice differentiability).

Principal First Order Condition

Define the function:

$$\chi_\alpha(x) = \int \frac{h_\alpha(x, y)}{u' \circ w_\alpha(y)} \alpha(\mathrm{d}y).$$

Theorem.

A profit maximizing α^* exists. Moreover, α^* must solve

$$\max_{\alpha \in \mathcal{A}} \int [x - w_{\alpha^*}(x) - \chi_{\alpha^*}(x)] \, \alpha(\mathrm{d}x).$$

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(ii) If π_α is strictly *quasiconvex* $\forall\alpha$, then supp $\alpha^* \subseteq \{L, H\}$.

(iii) If $w_\alpha + \chi_\alpha$ is a non-affine & analytic $\forall\alpha$, α^* is discrete.

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We showed that in flexible moral hazard problems:

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Also obtained results about principal optimality.

- First order approach is valid.
- Optimality of single, binary, and discrete distributions.

Related Literature

- Flexible models with specific functional forms:
  - CARA utility, monetary effort costs, finite X: Holmstrom and Milgrom (1987).
  - Mean-measurable costs: Diamond (1998), Barron, Georgiadis, and Swinkels (2020).
  - f-Divergence costs, finite X: Hebert (2018), Bonham (2021), Mattsson and Weibull (2022), Bonham and Riggs-Cragun (2023).
- Flexible Monitoring: Georgiadis and Szentes (2020), Mahzoon, Shourideh, and Zetlin-Joines (2022), Wong (2023).
- Robust contracting: Carroll (2015), Antic (2022), Antic and Georgiadis (2022), Carroll and Walton (2022).

Thanks!