Achieving Efficiency in Dynamic Contribution Games

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Motivation

- Team problems are ubiquitous in modern economies.
  - But such environments are susceptible to the free-rider problem.
  - Olson (1965), Hardin (1968), Alchian and Demsetz (1972), ...
- Collaboration is often geared towards achieving a particular objective.
  - *i.e.*, towards completing a "project".
  - *Examples:* startups, joint R&D and new product development projects
- We study a model of dynamic contributions to a joint project.
  - Construct mechanism that implements efficiency as outcome of MPE.

Our Framework

- Model in a nutshell:
  1. Ea. of n agents continually chooses his (costly) effort level $a_{i,t}$;
  2. project progresses at a rate that depends on $\sum_i a_{i,t}$; and
  3. it generates a payoff once a prespecified amount of progress is made.
- *Stepping-stone:* Effort increases with progress.
  - Because the agents discount time & are rewarded upon completion.
  - *Implication:* efforts are strategic complements (across time).
- *Free-rider problem:* two kinds of inefficiency.
  - Ea. agent gets fraction of project's payoff $\Rightarrow$ effort is inefficiently low.
  - Pos. externalities + str. complementarity $\Rightarrow$ agents front-load effort.

Outline

- Efficient mechanism specifies:
  - Flow payments that increase with progress to *kill* front-loading.
  - Terminal rewards that make each agent the residual claimant.
  - Magnitude of flow payments chosen such that budget is balanced.
- *Relevance:* Mechanism resembles the incentives structure in startups.
  - Flow payments = salary differential btw market rate and actual salary.
  - As startup value increases, the value of outside option increases.
  - Initial employees receives shares (*i.e.*, residual claimants).
- Other Applications:
  1. Dynamic Extraction of an Exhaustible Common Resource.
  2. Strategic Experimentation.
  - More broadly, applicable to any dynamic game with externalities.

Related Literature

- **Moral Hazard in Teams:** Free-rider problem and restore efficiency
  - Holmström (1982): Budget breaker
  - Legros & Matthews (1993): Mixed strategies and unlimited liability
- **Dynamic Contribution / Experimentation Games:**
  - Admati & Perry (1991) ; Marx & Matthews (2000)
  - Georgiadis et. al. (2014) ; Georgiadis (2015)
  - Bonatti & Hörner (2011) ; Halac et. al. (2015)
- **Dynamic VCG:** Efficiency in dynamic games with private information
  - Bergemann and Välimäki (2010) ; Athey and Segal (2013)

Model Setup

- Team comprises of n agents. Agent i
  - is risk neutral and has cash reserves w_i ;
  - discounts time at rate $r > 0$;
  - *privately* exerts effort $a_{i,t}$ at cost $c_i(a)$, where $c_i', c_i'', c_i''' \geq 0$; and
  - receives lump-sum $\alpha_i V$ upon completion of the project.
- Project starts at $q_0 = 0$, it evolves according to

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t} \right) dt \, ,$$

  and it is completed at the first time τ such that $q_\tau = Q$.
- Assume Markov strategies, *i.e.*, efforts at t depend only on q_t.

Building Blocks: Agents' Payoff Functions

- Agent i's discounted payoff at t:

$$J_i(q_t) = e^{-r(\tau - t)} \alpha_i V - \int_t^\tau e^{-r(s-t)} c_i(a_{i,s})\, ds$$

- Can write recursively as

$$rJ_i(q) = \max_{a_i} \left\{ -c_i(a_i) + \left(\sum_{j=1}^n a_j \right) J_i'(q) \right\}$$

subject to the boundary condition $J_i(Q) = \alpha_i V$.

- First order condition:

$$\underbrace{c_i'(a_i)}_{\text{marg. cost}} = \underbrace{J_i'(q)}_{\text{marg. benefit}}$$

  - Write $a_i(q) = f_i(J_i'(q))$, where $f_i(\cdot) = c_i'^{-1}(\max\{0, \cdot\})$.

Markov Perfect Equilibrium: Characterization

- In a MPE, each agent's payoff function satisfies

$$rJ_i(q) = -c_i\left(f_i\left(J_i'(q)\right)\right) + \left[\sum_{j=1}^{n} f_j\left(J_j'(q)\right)\right] J_i'(q)$$

subject to $J_i(Q) = \alpha_i V$ for all i.

Proposition 1. MPE Characterization

1. The game has a unique project-completing MPE if Q is *small enough*.
2. Payoffs satisfy $J_i(q) > 0$, $J_i'(q) > 0$, and $J_i''(q) > 0$ for all i and q.

- $J_i''(q) > 0 \implies a_i'(q) > 0$, *i.e.*, effort increases with progress.
  - Because agents discount time & are rewarded only upon completion.
  - Efforts are strategic complements in this game.

First-Best Outcome

- If efforts are chosen by a social planner, then her payoff satisfies

$$r\bar{S}(q) = \max_{a_1,..,a_n} \left\{ -\sum_{i=1}^{n} c_i(a_i) + \left(\sum_{i=1}^{n} a_i \right) \bar{S}'(q) \right\}$$

subject to $\bar{S}(Q) = V$.

- First order condition: $c_i'(a_i) = \bar{S}'(q) \Longrightarrow \bar{a}_i(q) = f_i\left(\bar{S}'(q)\right)$.

Proposition 2. First-Best Characterization

1. The social planner's problem admits a unique solution.
2. $\bar{S}(q) > 0$, $\bar{S}'(q) > 0$, and $\bar{S}''(q) > 0$ for all q, if Q is *small enough*.

- Assume the project is socially desirable, *i.e.*, Q is *small enough*.

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Comparison: MPE vs First-Best

▸ Details

- *Two sources of inefficiency:* In the MPE,
  1. the agents exert less effort ; and
  2. they front-load their effort

  relative to first-best outcome.

1. In the first-best (MPE), incentives are driven by V $(\alpha_i V < V)$.
   - Because the agents ignore their externality on the other agents.
2. Ea agent has incentives to front-load effort (*i.e.*, work harder early on) to induce others to raise future efforts, which renders him better off.
   - Due to positive externalities and strategic complementarity.
   - *Formally:* discounted marginal cost of effort (*i.e.*, $e^{-rt} c_i'(a_{i,t})$) $\downarrow$ in t.

Outline

- We consider the set of mechanisms that specify:
  - Upfront payment $P_{i,0} \geq 0$ for ea. agent i due before work commences.
  - Schedule of flow payments $h_i(q_t) \geq 0$ due while project is in progress.
  - Reward $\hat{V}_i$ upon completion of the project.

  ⋆ Payments are placed in a savings account accruing interest at rate r.
- *Objectives:* Efficiency, budget balance, individual rationality.
- Roadmap:
  1. Assume $w_i = \infty$ for all i and characterize efficient mechanism.
  2. Assume $w_i < \infty$ and provide conditions for implementability.
  3. Characterize optimal mechanism when conditions not satisfied.
  4. Extend the model to incorporate uncertainty & time-dependence.
  5. Other applications: resource extraction & strategic experimentation.

Incentivizing Efficient Actions

- Given set of flow payments $\{h_i(q)\}_{i=1}^n$, agent i's payoff satisfies

$$r\hat{J}_i(q) = \max_{a_i}\left\{-c_i(a_i) + \left(\sum_{j=1}^n a_j\right)\hat{J}_i'(q) - h_i(q)\right\}$$

  - FOC: $c_i'(a_i) = \hat{J}_i'(q) \Longrightarrow a_i(q) = f_i\left(\hat{J}_i'(q)\right)$
  - First-best FOC: $c_i'(a_i) = \bar{S}'(q)$

- For efforts to be efficient, we need: $\hat{J}_i'(q) = \bar{S}'(q)$ for all i and q.

- Therefore, $\hat{J}_i(q) = \bar{S}(q) - p_i$, where p_i is a constant TBD.

  - Upon completion, agent i must receive $\hat{J}_i(Q) = V - p_i$.

Flow Payments & Budget Balance

- Using $\hat{J}_i(\cdot)$ and $\bar{S}(\cdot)$, we back out agent i's flow payment schedule

$$h_i(q) = \sum_{j \neq i} c_j\left(f_j\left(\bar{S}'(q)\right)\right) + rp_i\,.$$

- Each agent i's ex-ante discounted payoff is

$$\hat{J}_i(0) - P_{i,0} = \bar{S}(0) - \left(p_i + P_{i,0}\right)\,.$$

- Budget balance requires that

$$\begin{aligned}\sum_{i=1}^{n}\left[\bar{S}(0) - \left(p_i + P_{i,0}\right)\right] &= \bar{S}(0) \\ \Longrightarrow \sum_{i=1}^{n}\left(P_{i,0} + p_i\right) &= (n-1)\,\bar{S}(0)\end{aligned}$$

Budget Balance

- How to pin down $\{P_{i,0},\, p_i\}$?
- Total discounted cost of payments along the equilibrium path

$$e^{-r\bar{\tau}}\left[(n-1)\,V-\sum_{i=1}^{n}p_i\right]$$

  - Minimized when $\sum_{i=1}^{n}p_i=(n-1)\,\bar{S}\,(0)$

- Henceforth, we set:
  - $P_{i,0}=0$ for all i
  - $\{p_i\}$ such that

$$\sum_{i=1}^{n}p_i=(n-1)\,\bar{S}\,(0)\quad\text{and}\quad\hat{J}_i\,(q)=\bar{S}\,(q)-p_i\geq 0$$

An Efficiency Result

Proposition 4. Efficient Mechanism

Suppose that ea. agent i makes flow payments given by

$$h_i(q) = \sum_{j \neq i} c_j\left(f_j\left(\bar{S}'(q)\right)\right) + rp_i\,,$$

and receives $V - p_i$ upon compl'n, where $\sum_{i=1}^{n} p_i = (n-1)\,\bar{S}(0)$. Then:

- $\exists$ a MPE in which ea. agent i exerts the efficient effort $\bar{a}_i(q)$ $\forall q$.
- $h_i'(q) > 0$ for all i and q, *i.e.*, flow payments are increasing in q.
- The agents' total discounted payoff is equal to FB discounted payoff.

- Ea. agent is (essentially) made a full residual claimant.
- $h_i(\cdot)$ increases at a rate s.t ea. agent's benefit from front-loading effort is offset by the cost associated with larger future flow payments.

A Problem (and a Solution)

- Mechanism in Proposition 4 is budget balanced on the eq'm path.
- Off the equilibrium path, if an agent deviates at some t, then the amount in the savings account at τ may be $\gtrless (n-1)V - \sum_{i=1}^{n} p_i$.
  - Unless 3^{rd} party can pay ea. agent $V - p_i$, incentives will be affected.
- *Lemma:* Impossible to achieve budget-balance both on and off the equilibrium path. Details

Corollary:

Consider previous mechanism, except upon completion, ea. agent receives $\min\{V - p_i, \beta_i (V + H_\tau)\}$. Efficiency properties of Prop. 4 are preserved.

- With strict budget balance, agents have incentives to shirk, let the balance grow, and collect a bigger reward upon completion.
- Capping ea. agent i's reward at $V - p_i$ eliminates these incentives.

Uncertainty and Time Dependence

Details

- Assume that the project progresses according to

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t} \right) dt + \sigma dW_t \, .$$

- Consider mechanisms that depend on both q and t.
  - *i.e.*, flow payments $h_i(t, q)$ and terminal rewards $g_i(\tau)$.

Proposition 8:

For $h_i(t, q)$ and $g_i(\tau)$ as characterized, the following hold:

1. There exists a MPE in which ea. agent exerts efficient effort level;
2. the agents' ex-ante discounted payoff = first-best payoff; and
3. the mechanism is ex-ante budged-balanced.

- With (Brownian) uncertainty, the agents must have unlimited cash, and there must be a 3^{rd} party to balance the budget.

Dynamic Extraction of Common Resource

- **Model:**
  - n agents extract a common resource. Stock is depleted according to

$$dq_t = -\sum_{i=1}^{n} a_{i,t} dt$$

  where $q_0 > 0$, and game ends when $q_\tau = 0$.
  - Ea. agent obtains flow utility $u_i(a_{i,t})$ from extracting at rate $a_{i,t}$.
- **Efficient mechanism** specifies that ea. agent receives subsidy

$$s_i(q) = \sum_{j \neq i} u_j(\bar{a}_j(q)) - rp_i,$$

where $\bar{a}_j(q)$ is the efficient extraction level.
  - $s_i'(q) > 0$, *i.e.*, subsidy decreases as resource becomes scarcer.
  - Budget-balance via "participation" fee $P_{i,0}$ and choice of constant p_i.

Strategic Experimentation

Model: Keller, Rady and Cripps (2005) ; Bonatti & Hörner (2011)

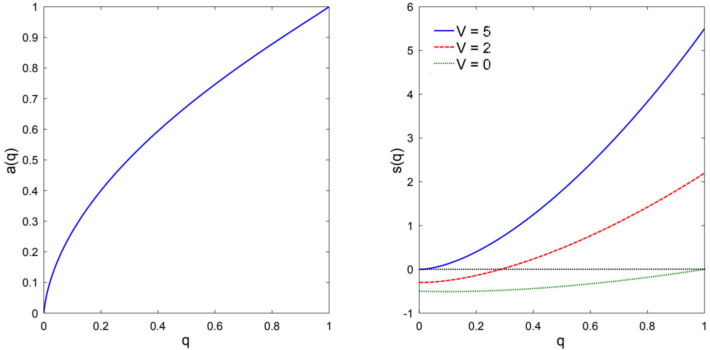
- Two arms: a *safe* arm with $\mathbb{E}\,[\text{flow payoff}] = 0$, and a *risky* arm.
    - If the risky arm is *bad*, then it never yields any payoff.
    - If it is *good*, then lump-sum payoff $= 1$ arrives $\sim$ *Poi* (exp. level).
- Ea. agent continuously chooses exp. level $a_{i,t}$ at flow cost $c_i\,(a_{i,t})$.
- *Belief Dynamics:*
    - Agents hold prior belief $q_0 = \Pr\{\text{risky arm is good}\}$.
    - As long as no arrival occurs, belief evolves according to

$$dq_t = -q_t\,(1 - q_t)\left(\sum_{i=1}^{n} a_{i,t}\right) dt$$

    - Following a breakthrough, belief jumps to 1 and stays there forever.
    - Assume that agents hold common belief p_t at all times.

Strategic Experimentation (Cont'd)

- Mechanism specifies for each agent:
  - prize V_i if he is the first to achieve a breakthrough, and
  - subsidy $s_i(q) = \sum_{k \neq i} \left[q(1+V_i)\bar{a}_k(q) - c_k(\bar{a}_k(q)) \right] + r(V_i - \gamma_i)$.
- Budget-balance (ex-ante) via "participation" fee $P_{i,0}$ and choice of V_i.



Wrapping Up

- We consider a dynamic contribution game, and propose a mechanism that induces agents to always choose the efficient actions in a MPE.
  - Specifies flow payments to eliminate front-loading incentives.
  - Terminal rewards that make ea. agent residual claimant.
  - Mechanism resembles features in incentive structures at startups.
- Mechanism can be adapted to other dynamic games w/ externalities.
  1. Dynamic extraction of a common resource
  2. Strategic experimentation
- Future work:
  - Optimal mechanism with uncertainty & cash constraints.
  - Efficient mechanism for generic dynamic game with externalities.

Comparison: MPE vs First-Best (1/3)

- *Two sources of inefficiency:* In the MPE,
  1. the agents exert less effort ; and
  2. they front-load their effort relative to first-best outcome.

Proposition 3:

- Assume the agents are symmetric, *i.e.*, $c_i(\cdot) = c_j(\cdot)$ and $\alpha_i = \alpha_j$.
- Eq'm effort is inefficiently low, *i.e.*, $a_i(q) < \bar{a}_i(q)$ for all i and $q > \underline{q}$.

- In equilibrium, incentives are driven by $\alpha_i V < V$.
  - Because agents ignore their externality on the other agents.

Comparison: MPE vs First-Best (2/3)

- To demonstrate the front-loading effect, use the maximum principle of optimal control (which is equivalent to the HJB approach).
- Social Planner's Hamiltonian:

$$\bar{H}_t = -\sum_{i=1}^{n} e^{-rt} c_i\left(a_{i,t}\right) + \lambda_t^{fb}\left(\sum_{i=1}^{n} a_{i,t}\right)$$

- Optimality and adjoint equations are

$$\frac{d\bar{H}_t}{da_{i,t}} = 0 \quad \text{and} \quad \dot{\lambda}_t^{fb} = -\frac{dH_t}{dq},$$

  or equivalently,

$$e^{-rt} c_i'\left(\bar{a}_{i,t}\right) = \lambda_t^{fb} \quad \text{and} \quad \dot{\lambda}_t^{fb} = 0$$

- Therefore, $e^{-rt} c_i'\left(\bar{a}_{i,t}\right) =$ constant for all t in the first-best outcome.
- *Intuition:* Efficient to smooth efforts over time (because $c_i'' > 0$).

Comparison: MPE vs First-Best (3/3)

Return

- Agent i's Hamiltonian (MPE):

$$H_{i,t} = -e^{-rt} c_i\left(a_{i,t}\right) + \lambda_{i,t}\left(\sum_{j=1}^{n} a_{j,t}\right)$$

- Optimality equation: $e^{-rt} c_i'\left(a_{i,t}\right) = \lambda_{i,t}$

- Adjoint equation: $\dot{\lambda}_{i,t} = -\sum_{j \neq i} \frac{\lambda_{i,t} \dot{a}_{j,t}}{\sum_{l=1}^{n} a_{l,t}} < 0$
  - Inequality follows because $\lambda_{i,t} > 0$ and $a_i'\left(q\right) > 0 \Rightarrow \dot{a}_{j,t} > 0$.

- So $e^{-rt} c_i'\left(a_{i,t}\right)$ decreases in t, *i.e.*, in MPE, agents front-load effort.

- *Intuition:* Each agent has incentives to front-load effort to induce others to raise future efforts (which renders him better off).

Strict Budget Balance

- Let

$$H_t = \sum_{i=1}^{n} \int_0^t e^{r(t-s)} h_i(q_s)\, ds$$

denote the balance in savings account at time t.

Lemma:

Suppose that ea. agent i receives $\beta_i (V + H_\tau)$ upon completion. Then:

1. $\nexists$ no flow payment functions $h_i(\cdot)$ that lead to the efficient outcome.
2. Optimal mechanism is equivalent to mechanism w/ $\sum_{i=1}^{n} h_i(q) = 0$.

- Takeaways:
  - If we want efficiency, we must give up strict budget balance.
  - If we require $h_i \geq 0$ for all i, then mechanism becomes *powerless*.
- Second best?

Second Best

Return

- Turns out a small modification suffices to ensure that
  1. the mechanism is budget balanced on the eq'm path ; and
  2. it never results in a budget deficit (but possibly a budget surplus).

Corollary:

Consider the previous mechanism, except upon completion, ea. agent receives $\min\left\{V - p_i,\, \beta_i\left(V + H_\tau\right)\right\}$, where $\beta_i = \frac{V - p_i}{V + (n-1)\left[V - \bar{S}(0)\right]}$.

- There exists a MPE in which ea. agent exerts the efficient effort level.
- The agents' total discounted payoff is equal to FB discounted payoff.

- With strict budget balance, agents have incentives to shirk, let the balance grow, and collect a bigger reward upon completion.
- Capping ea. agent i's reward at $V - p_i$ eliminates these incentives.

Limited (but Sufficient) Cash Reserves

- Assume $h_i(\cdot)$'s are cash payments (as opposed to foregone income).
- Also assume that each agent has cash reserves $w_i < \infty$ at $t = 0$.
- Two challenges:
  1. Mechanism must specify what if an agent runs out of cash.
  2. Efficient mechanism may not be implementable.

Limited Cash Reserves: Implementability Condition

Proposition 5:

- The mechanism in Corollary is implementable iff $\exists\ \{p_i\}_{i=1}^n$ s.t

$$\sum_{i=1}^{n} p_i = (n-1)\,\bar{S}(0) \qquad \text{(BB)}$$

$$\int_0^{\bar{\tau}} e^{-rt} \sum_{j \neq i} c_j\left(f_j\left(\bar{S}'(q_t)\right)\right) dt + p_i\left(1 - e^{-r\bar{\tau}}\right) \leq w_i \qquad \text{(Cash)}$$

$$0 \leq \bar{S}(0) - p_i \qquad \text{(IR)}$$

- If agents symmetric, implementable iff $w_i > e^{-r\bar{\tau}}\left(\frac{n-1}{n}\right)\left[V - \bar{S}(0)\right]$.

- Interpretation:

  C.2. Each agent must have sufficient cash to make payments.

  C.3. Each agent's IR constraint must be satisfied.

Limited Cash Reserves (Cont'd)

- Assume that an agent who runs out of cash, receives 0 upon completion, while others' rewards are unchanged.
- Define $I_i(q) = \mathbf{1}_{\{\text{agent i has had cash at every } \tilde{q} < q\}}$

Proposition 6:

- Suppose that ea. agent i makes flow payments $h_i(q)\, I_i(q)$, and receives $\min\{V - p_i,\, \beta_i(V + H_\tau)\}\, I_i(Q)$ upon completion.
- Assume that the conditions of Prop. 5 are satisfied.
- Then the resulting mechanism implements the efficient outcome.

- *Intuition:*
  - Mechanism must punish agent who runs out of cash.
  - But his "share" cannot be distributed to the other agents.

Cash Constraints

- Efficient mechanism is implementable iff $w_i \geq \underline{w}_i$ for each i.
- What if the agents don't have sufficient cash reserves?
- Assuming symmetry, for arbitrary w, we characterize the mechanism that maximizes ex-ante total surplus.
- Optimal Mechanism:
  - Agents make payments while they still have cash.
  - Upon completion, they share balance in account + project payoff.
  - Similar properties as the efficient mechanism.

Uncertainty and Time Dependence: Setup

- We extend our model by
  1. incorporating uncertainty in the evolution of the project:

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t} \right) dt + \sigma dW_t \text{ , and}$$

  2. considering mechanisms that depend on both q and t.
     - *i.e.*, flow payments $h_i(t, q)$ and terminal rewards $g_i(\tau)$.

- Assume $w_i = \infty$ for all i.

First-Best Outcome

- The planner's problem satisfies

$$r\bar{S}(q) = \max_{a_1,..,a_n} \left\{ -\sum_{i=1}^{n} c_i(a_i) + \left(\sum_{i=1}^{n} a_i \right) \bar{S}'(q) + \frac{\sigma^2}{2} \bar{S}''(q) \right\}$$

subject to $\lim_{q \to -\infty} \bar{S}(q) = 0$ and $\bar{S}(Q) = V$.

- FOC: $c_i'(a_i) = \bar{S}'(q)$

- We know from Georgiadis (2015) that
  1. the problem admits a unique solution ; and
  2. effort increases with progress, *i.e.*, $a_i'(q) > 0$.

Analysis

- Agent i's discounted payoff satisfies

$$rJ_i - J_{t,i} = \max_{a_i} \left\{ -c_i\left(a_i\right) + \left(\sum_{j=1}^{n} a_j\right) J_{q,i} + \frac{\sigma^2}{2} J_{qq,i} - h_i \right\}$$

subject to $\lim_{q\to-\infty} J_i\left(t,q\right) = 0$ and $J_i\left(t,Q\right) = g_i\left(t\right)$.

- FOC: $c_i'\left(a_i\right) = J_{q,i}\left(t,q\right)$

- Efficiency requires $J_{q,i}\left(t,q\right) = \bar{S}'\left(q\right)$ for all i, t, q.

- Therefore,

$$J_i\left(t,q\right) = \bar{S}\left(q\right) - p_i\left(t\right), \text{ where } p_i\left(t\right) = V - g_i\left(t\right)$$

Analysis (Cont'd)

- We now back out the flow payment functions

$$h_i(t,q) = \sum_{j \neq i} c_j\left(f_j\left(\bar{S}'(q)\right)\right) + r\left(V - g_i(t)\right) + g_i'(t)$$

- Budget balance requires that

$$\sum_{i=1}^{n} J_i(0,0) = \bar{S}(0) \Longrightarrow \sum_{i=1}^{n} g_i(0) = nV - (n-1)\bar{S}(0)$$

- We pick $g_i(\cdot)$ such that $\mathbb{E}\left[h_i(t,q)\right] = 0$, and so we solve

$$0 = \sum_{j \neq i} \mathbb{E}\left[c_j\left(f_j\left(\bar{S}'(\bar{q}_t)\right)\right)\right] + r\left(V - g_i(t)\right) + g_i'(t)$$

subject to $g_i(0)$, where $\sum_{i=1}^{n} g_i(0) = nV - (n-1)\bar{S}(0)$.

Efficient Mechanism

Return

Proposition 8

Suppose ea. agent pays flow $h_i(t, q)$ and receives $g_i(\tau)$ upon completion.

1. There exists a MPE in which ea. agent exerts efficient effort level;
2. the agents' ex-ante discounted payoff = first-best payoff; and
3. the mechanism is ex-ante budged-balanced.

- With uncertainty, to achieve efficiency,
  1. the agents must have unlimited cash; and
  2. there must be a 3^{rd} party to balance the budget.

Corollary

- Suppose that $\sigma = 0$, *i.e.*, the project is deterministic.
- The efficient mechanism specifies 0 flow payments on the first-best path, and each agent receives $\alpha_i V$ upon completion.