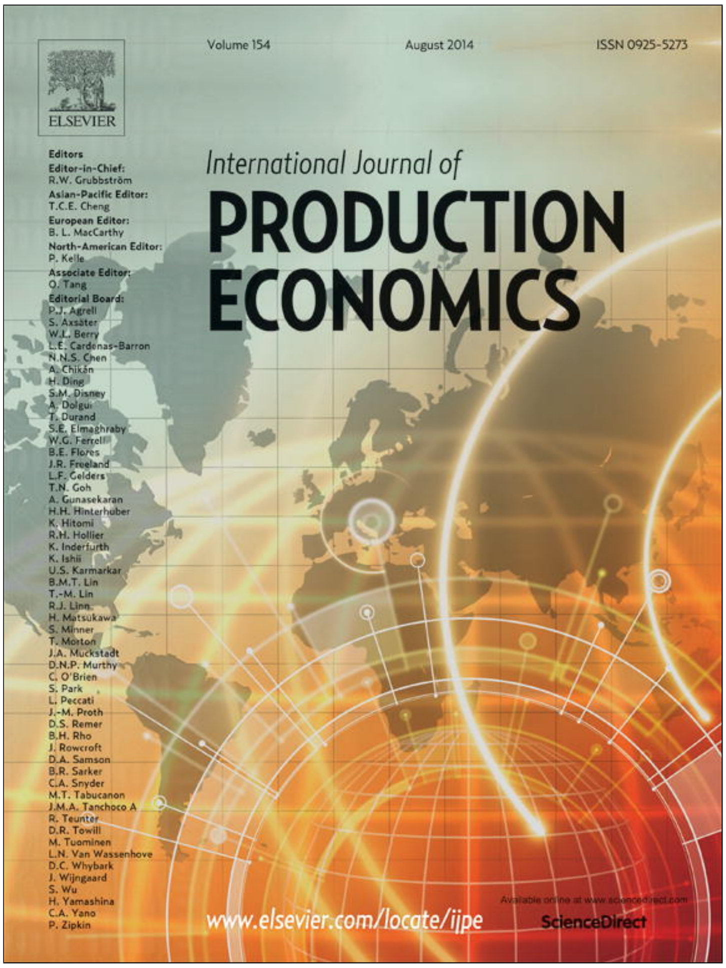
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Optimal reservation policies and market segmentation

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ABSTRACT

When operating in a market with heterogeneous customers, a service firm (e.g., a car rental company or a hotel) needs to manage its capacity so as to maximize its revenue. To gauge the potential demand, a service firm often allows each customer to reserve a unit of service in advance. However, to avoid the loss associated with "no-shows", service firms may require a non-refundable deposit. To determine an optimal reservation policy with a non-refundable deposit, we consider the case in which the market is divided into four segments (high vs. low valuation and high vs. low show-up probability). When customer demand and the firm's capacity are large so that they can be approximated by *continuous* values, we determine the optimal reservation policy analytically, and we establish analytical conditions under which the firm should discriminate against (i.e., price out) certain customer segments. For the case when customer demand and the firm's capacity are finite so that they take on *discrete* values, we find that some of the insights obtained from the "continuous" case continue to hold especially when the firm's capacity is large. However, the key difference is that in the former case, the firm discriminates mostly based on customers' valuation, whereas in the latter case it discriminates mostly based on customers' show-up probability.

1. Introduction

Pricing is a challenging decision for a service firm to make when operating in a heterogeneous market of uncertain size that is composed of customers with different (private) valuations and show up probabilities. In the service industry (e.g., a car rental company or an airline), "no shows" can be a significant problem: 10–15% of passengers do not show up to claim their reserved seats in the airline industry, 25% of guests do not show up for their reserved rooms in the hotel industry, and no-show rates can be as high as 30% in the car rental industry (Rothstein, 1974, 1985; USA Today, 1998; Campell, 2009).[1]

When dealing with potential no shows under uncertain customer demand, there is no known optimal mechanism that maximizes a firm's expected revenue. This observation has motivated us to examine a specific class of mechanisms (reservation policies with and without non-refundable deposits) in this paper. Reservation policies with non-refundable deposits are commonly observed in practice. Consider the car rental industry. Until 2009, customers could reserve rental cars without paying any no-show fees (i.e., non-refundable deposits were not required). As a way to discriminate against customers with low show up probabilities, Avis decided to charge "no show" fees (or non-refundable deposits) for customers who do not show up for their reserved rental in late 2009 (Campell, 2009). However, Enterprise Rent-A-Car decided not to impose no-show fees because such fees may discourage customers from making reservations (Hibbard, 2010). While Avis and Enterprise have different views on the issue of non-refundable deposits, they both adopt a "single-option" reservation policy; i.e., a single rental price and deposit for each category of rental cars. However, Hertz offers a "two-option" reservation policy: (1) a lower rental price with a non-refundable deposit, and (2) a higher rental price with no required deposit. Besides the car rental industry, many hotels such offer a similar two-option reservation policy: a lower rate with fully pre-paid, non-refundable deposit, and a higher rate without any required deposit. Clearly, Hertz's two-option policy is intended to segment the market so as to improve its revenue by extracting more surplus from the different segments.

Different reservation policies adopted by different firms in the travel industry have motivated us to develop a model to determine the optimal reservation policy for a firm operating in a *heterogeneous market* with asymmetric information. We consider the case when the

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[1] Reasons associated with not showing up may include change of plans and unforeseeable events.

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market is composed of an uncertain number of rational and risk-neutral customers, each of whom belongs to one of the following four segments: high or low valuation, and high or low show-up probability.[2] For example, business travelers and special event travelers tend to have high valuation, while leisure travelers have low valuation. The show-up probability captures the ex ante likelihood that the event will actually take place as described in Png (1989). For instance, planned events (e.g., conference meetings, weddings) tend to have high ex ante show-up probabilities, while unconfirmed events (e.g., business meetings) have low ex ante show up probabilities.

When each customer has private information about her valuation and show-up probability, we present a model in which a firm needs to develop an effective reservation policy to discriminate against (i.e., price out) certain market segments. We shall focus on the case when the firm offers a "single-option" reservation policy comprising: (1) the retail price; (2) the non-refundable deposit that a customer needs to pay when making a reservation; and (3) the firm's booking capacity (i.e., the firm may *overbook* as a way to hedge against no shows). We also extend our analysis to study "multi-option" reservation policies in Section 5. Because a firm may overbook, there is no guarantee that it can serve all customers who show up for their reserved service. In the event when a reservation is not honored due to "overbooking", most service firms will typically accommodate customers by offering them an alternative service and/or some compensation (Hibbard, 2010).

Our primary goal is to find the optimal single-option reservation policy for a firm with limited capacity serving a heterogeneous market with four customer segments (high vs. low valuation *and* high vs. low show-up probability). Because there are different customer segment combinations, our approach is based on two basic steps. First, we determine the optimal booking capacity and the optimal reservation policy and the corresponding optimal expected revenue for any given customer segment combination. Second, by comparing the optimal expected revenue associated with different segment combinations, we identify the optimal customer segment combination that generates the highest expected revenue for the firm.

Because our approach involves pair-wise comparisons of optimal expected revenue associated with different segment combinations, it is difficult to establish basic intuition about the optimal reservation policy and the corresponding optimal market segments that the firm should serve. For this reason, we first consider the case when the customer demand and the firm's capacity are large so that they can be approximated by*continuous* values. For this case, we determine the optimal reservation policy analytically, and we establish the conditions under which the firm should discriminate (i.e., price out) customers with low valuation (regardless of their show-up probability), and the conditions under which the firm should discriminate customers with both low valuation and low show-up probability. Then we consider the case when customer demand and the firm's capacity are finite so that they take on *discrete* values. We find some of the insights obtained from the "continuous" case continue to hold when the firm's capacity is large. Although no closed form expressions for the optimal reservation policy are available when the firm's capacity is small, our numerical analysis suggests that entertaining the segments with high show-up probability can be beneficial to the firm. Therefore, when developing a reservation policy with non-refundable deposit, a firm should take customer valuations, show up probabilities, capacity, and market size into consideration.

To our knowledge, our paper is the first attempt to examine reservation policies with non-refundable deposits by incorporating a framework that includes capacity constraints, customer heterogeneity along two dimensions (i.e., valuation and show up probability), demand uncertainty, overbooking, reservation deposits, as well as multiple reservation options. Therefore, the primary contributions of this paper are (a) a model that captures the essence of a firm with limited capacity serving a heterogeneous market comprising customers with private information about their valuations and show up probabilities; and (b) an approach for determining optimal reservation policies that are intended to discriminate against certain market segments. A key finding of this paper is that, when the firm's capacity is sufficiently large, the optimal single-option reservation policy tends to discriminate against customers with low valuation but not necessarily customers with low show up probability. As a way to differentiate customers with low show up probability, the firm may consider offering a reservation policy with multiple options. Other findings include (i) for any target segment, the optimal retail price increases and the optimal deposit decreases in the firm's capacity; (ii) full deposit policies are optimal when the firm targets only customers with high show up probability; and (iii) when operating in a market with four customer segments, the optimal reservation policy is either a single-option or a two-option policy.

This paper is organized as follows. Section 2 provides a brief review of related literature. Section 3 presents the base model for the case when the firm offers a single-option reservation policy. In Section 4, we extend our analysis to the case in which the firm offers a multiple option reservation policy and we show that the optimal reservation policy is either a single-option or a two-option policy. We conclude in Section 5 with a discussion of the limitations of our model and potential future research topics. To streamline our presentation, all proofs are given in Appendix A.

2. Literature review

Recognizing the fact that unused capacity has no salvage value in the service industry, we witness an increasing research interest in revenue management recently. Many researchers have examined different selling mechanisms to segment the market so that a firm can extract surplus from different customer segments. Specifically, operations management researchers have explored various selling mechanisms that involve opaque and probabilistic selling (Jerath et al., 2010; Fay and Xie, 2010), strategic stockouts (Liu and Van Ryzin, 2008), partial inventory information (Yin et al., 2009), reservations (Alexandrov and Lariviere, 2012; Elmaghraby et al., 2009; Png, 1989), and cancellable reservations (Xie and Gerstner, 2007). Because this stream of research does not deal with the issue of reservation policies with non-refundable deposits, it is fundamentally different from our paper. Due to page limitation, we shall refer the reader to Bitran and Caldentey (2003), Netessine and Tang (2009), Philips (2005), and Talluri and Van Ryzin (2004) for comprehensive reviews of this research stream.

In the economics literature, Courty and Li (2000) studies price discrimination in a setting where customers are uncertain about their actual valuation of the service (i.e., their show up probability). By considering incentive compatible menu contracts, the authors show that

[2] When dealing with market heterogeneity, it is common to analyze a 2-segment model to obtain tractable results (e.g., Png, 1989; Biyalogorsky and Gerstner, 2004; Elmaghraby et al., 2009). In our case, the same approach can be used to analyze a heterogeneous market with any discrete number of segments. However, the analysis becomes very tedious.

Table 1
A heterogeneous market with different valuations and show up probabilities.

Probability	$(\beta)\ V_H$	$(1-\beta)\ V_L$
$(\alpha)\ \psi_H$	HH	HL
$(1-\alpha)\ \psi_L$	LH	LL

with two classes of customers, "business" customers (i.e., those with higher expected valuation and lower show up probability) find it optimal to reserve a no-deposit ticket at a higher price, while the optimal ticket for "leisure" customers (i.e., those with lower expected valuation) may range from a no-deposit to a full deposit ticket. Akan et al. (2009) examines a similar environment where customers have private information about the distribution of their valuation and the time when their valuation will be realized. By focusing on the case in which "business" customers learn their valuation later than "leisure" customers, the authors show that the optimal selling mechanism is a menu of tickets with different cancellation deadlines and cancellation penalties. A key simplifying feature of these papers is that the seller has *unlimited capacity*. However, when the seller has limited capacity, the optimal selling mechanism remains unknown especially because the analysis becomes intractable. Instead of examining cancellable reservations with unlimited capacity, our model deals with different issues: non-cancellable reservation policies, non-refundable deposits, and limited capacity.

Our paper is related to a research stream in revenue management that deals with the issues of reservations and overbooking. In the reservations literature, Png (1989) is one of the first papers to examine a monopolistic airline which, in order to increase its capacity utilization, takes customer reservations, and he shows that overbooking is an effective strategy to reduce the risk of unused capacity. Instead of imposing a penalty for "no shows", Biyalogorsky et al. (1999) and Gallego et al. (2008) examine a situation in which the market is composed of customers with low and high valuations. The firm accepts customer reservations by offering a lower price to the low valuation customers (leisure travelers) who arrive in the first period. However, the firm reserves the right to recall (i.e., cancel) these reservations so that it can sell the recalled units at a higher price to the high valuation customers (business travelers) arriving in the second period. They show how "callable" reservations can enable the firm to reduce the risk of unused capacity and to obtain a higher expected profit. A similar concept is studied by Biyalogorsky and Gerstner (2004), who show that a firm can increase its expected profit by offering a pricing arrangement that is contingent on whether the seller is able to obtain a higher price within a specified period. Instead of restricting to the case in which only callable units are available in the first period and non-callable units are available in the second period, Elmaghraby et al. (2009) examine a situation in which the firm offers both callable and non-callable units at different prices at any point in time. They show that a firm can obtain an even higher profit by offering customers both options. Their result is due to the fact that when both options are available at any point in time, customers feel the competitive pressure to purchase the non-callable units at a higher price. In this paper, we examine a different selling mechanism under which customers can make reservations with non-refundable deposits. To the best of our knowledge, our paper is the first to examine reservation policies with non-refundable deposits by incorporating a framework that includes capacity constraints, customer heterogeneity along two dimensions (i.e., valuation and show up probability), demand uncertainty, overbooking, reservation deposits, as well as multiple reservation options.

3. Single-option reservation policies: the model

A service firm has m units to be consumed by customers in period 2. The firm starts selling its limited capacity in a heterogeneous market with A "potential" customers at the beginning of period 1. To ease our exposition of the base model, we assume that all potential customers are present in the system at the beginning of period 1, and no new customers will arrive at the beginning of period 2 (i.e., no walk-in customers). We discuss how the model can be extended to incorporate the issues of walk-in customers in Section 6.

The market is divided into four distinct segments (Table 1) according to two customer-centric elements: valuation and show-up probability. Each customer has a high (ex ante) show-up probability (ψ_H) with probability $\alpha > 0$ and a low show-up probability (ψ_L) with probability $1-\alpha$. Similarly, each customer has a high valuation for the service (V_H) with probability $\beta > 0$, and a low valuation (V_L) with probability $1-\beta$.[3] The actual show up process of each customer is captured by a Bernoulli process with the "success probability" being her show up probability ψ. For example, consider a customer who has an unconfirmed business meeting that will take place at the beginning of period 2 with an ex ante probability ψ. As the conduct of this meeting is realized according to the Bernoulli process at the end of period 1, the customer will show up at the beginning of period 2 only if the meeting is "on". An alternative interpretation of this setup is that each customer has initial valuation V_j, and her valuation in period 2 remains unaltered (i.e., equals V_j) with probability ψ_i and it drops to 0 with probability $(1-\psi_i)$, where $i, j \in \{\mathrm{L}, \mathrm{H}\}$.

Given the market size A and the customer characteristics (i.e., show-up probability and valuation), the firm determines its "single-option" reservation policy P at the beginning of period 1 that involves three decisions: (a) the retail price r; (b) the upfront non-refundable reservation deposit d, where $0 \le d \le r$; and (c) the booking capacity n, where $n \ge m$ (i.e., the firm may overbook to hedge against no-shows). Therefore, for any single-option reservation policy $P=(r, d, n)$, the firm announces (r, d) and accepts up to n reservations, where the booking capacity n is not announced publicly. We extend our model to the case in which the firm offers policies with multiple reservation options in Section 5. In our model, all accepted reservations are non-cancellable; however, our model can be extended to incorporate the issue of cancellable reservations. We discuss this in Section 6.

For any reservation policy $P=(r, d, n)$, each customer utilizes her private information (i.e., her ex ante show-up probability ψ and her valuation V) to decide whether to (attempt to) reserve a unit of capacity with the firm (or not) at the beginning of period 1. For any policy $P=(r, d, n)$, each customer needs to decide whether to make an attempt to reserve with the firm at the beginning of period 1 or leave the

[3] Our model and our analysis can be extended to the case when the show-up probability and the corresponding probability (i.e., ψ and α) depend on the customer valuation. However, such an extension will increase the complexity of the analysis. To ease our exposition and to obtain tractable results, we shall focus on the case when the show-up probability is independent of customer valuation.

system. Consider a customer who belongs to segment $\{ij\}$, and has show-up probability ψ_i, $i \in \{\text{L, H}\}$ and valuation V_j, $j \in \{\text{L, H}\}$. This customer will obtain a surplus $(V_j - r)$ with probability ψ_i from showing up for her reserved service in period 2, and incur a loss $-d$ with probability $(1-\psi_i)$ from not showing up (because cancellations are not allowed).[4] Therefore, if she attempts to reserve, then her expected surplus $\pi_{ij}(r,d)$ satisfies

$$\pi_{ij}(r,d) = -(1-\psi_i)d + \psi_i(V_j - r) \quad \text{for } i \in \{\text{L, H}\} \text{ and } j \in \{\text{L, H}\}. \tag{3.1}$$

Hence, assuming that each customer has outside option 0, every customer in segment $\{ij\}$ will attempt to reserve if $\pi_{ij}(r,d) \geq 0$ and accept the outside option otherwise.

For those customers with reservations, they show up at the beginning of period 2 simultaneously according to independent Bernoulli trials. Because cancellations are not allowed, there are three possible outcomes in period 2: (a) the firm earns the non-refundable deposit d from each customer who does not show up for her reserved service; (b) the firm earns the retail price r from each customer who shows up and the service is available; and (c) the firm refunds the deposit d and incurs a penalty c for each customer who shows up and the service is not available (due to overbooking), where c captures the cost for arranging an equivalent or better service provided to ensure customer satisfaction. Motivated by the airline industry, where the compensation schemes for passengers who are involuntarily denied boarding due to overbooking are regulated by the US Department of Transportation, we assume that this penalty cost is exogenously given.[5] Similarly, in the car rental industry, firms typically arrange an equivalent service (e.g., a rental car from another branch of the firm or a different firm) for customers who are denied service due to overbooking to ensure customer satisfaction (Hibbard, 2010).

3.1. Timing of reservations

For any policy $P = (r,d,n)$, the customer's expected surplus $\pi_{ij}(r,d)$ is given in (3.1). By considering the outside option 0, a customer will obtain a non-negative surplus $\max\{\pi_{ij}(r,d), 0\} \geq 0$ for $i \in \{\text{L, H}\}$ and $j \in \{\text{L, H}\}$ if she makes her reservation decision (reserve or accept the outside option) in period 1. This observation makes one wonder if a customer should postpone her reservation decision and purchase a unit at the "walk-in" price r_W to be announced in period 2 (i.e., the firm does not pre-commit its walk-in price in advance).[6] When there are no new customer arrivals occur in period 2, we argue that there is no incentive for a customer to postpone her reservation decision in period 1.

Let us begin by assuming that some customers decide to postpone their reservation decisions. If no units are available for sale in period 2, then each of these customers who postponed their reservation decisions and then show up in period 2 will get nothing. Now suppose that there are units available for sale at the beginning of period 2. Knowing each of these customers who shows up in period 2 has valuation either V_L or V_H, it is optimal for the firm to set its "walk-in" price $r_W = V_L$ or V_H.[7] By anticipating that the firm will set its walk-in price $r_W \geq V_L$, any customer with low valuation V_L who postponed her reservation decision cannot earn a surplus that is greater than $\max\{\pi_{iL}(r,d), 0\}$, $i = \{\text{L, H}\}$. Hence, there is no incentive for low valuation customers to postpone their reservation decisions.

Knowing that low valuation customers will not postpone their reservation decisions, only high valuation customers might consider postponing their reservation decisions. This observation suggests that the firm will set its walk-in price $r_W = V_H$ in period 2. As high valuation customers can anticipate the walk-in price $r_W = V_H$, any customer with high valuation V_H who postponed her reservation decision cannot earn a surplus that is greater than $\max\{\pi_{iH}(r,d), 0\}$, $i = \{\text{L, H}\}$. Hence, there is no incentive for high valuation customers to postpone their reservation decisions either. Combining the above observations, we can conclude that there is no incentive for any customer to postpone her reservation decision. Hence, it suffices to consider the case in which each customer either attempts to reserve or leaves the system by accepting the outside option in period 1.

3.2. Admissible target segments

By considering the customer's expected surplus $\pi_{ij}(r,d)$ given in (3.1) for $i,j \in \{\text{L, H}\}$, the firm can anticipate the reservation behavior of customers in each segment $\{ij\}$. Hence, the firm can use the following approach to determine an optimal single-option reservation policy when selling in a heterogeneous market with four segments: LL, LH, HL and HH. First, for any selected "target" segment, we determine the optimal reservation policy that would discriminate against those customers who do not belong to the chosen target segment, and compute the corresponding optimal expected revenue. Then we can determine the optimal target segment by choosing the segments that yield the highest expected revenue. Once the optimal market segment is identified, we can retrieve the optimal reservation policy accordingly.

For a market with four segments, the aforementioned approach involves the evaluation of the firm's optimal revenue associated with 15 different "potential" target segments; namely, {LL}, {LH}, {HL}, {HH}, {LL, HL}, {LL, LH}, {LL, HH}, {LH, HL}, {LH, HH}, {HL, HH}, {LL, LH, HL}, {LL, LH, HH}, {LL, HL, HH}, {LH, HL, HH}, and {LL, LH, HL, HH}. At first glance, the analysis of this approach appears to be very tedious. However, many of these potential target segments can be *ignored* because they are "inadmissible" in the sense that there do not exist $0 \leq d \leq r$ so that all customers in the target segment will reserve and all customers in other segments will not reserve. For example, observe from (3.1) that, for any given $0 \leq d \leq r$, $\pi_{LH} \geq \pi_{LL}$, which implies that segment {LL} alone is inadmissible. To elaborate, suppose that there exists a reservation policy P that supports segment {LL} only so that $\pi_{LL} \geq 0$. Because $\pi_{LH} \geq \pi_{LL} \geq 0$, this policy will also

[4] Notice that a customer with valuation V_j will obtain surplus $(V_j - r)$ even if the service is unavailable when she shows up. This is because the firm will arrange an equivalent or better service to ensure customer satisfaction under such circumstances. This assumption is consistent with common practice.

[5] See http://airconsumer.ost.dot.gov/reports/index.htm for details.

[6] The assumption that the firm is unable to pre-commit to future prices is consistent with common practice in the travel industry: firms generally do not pre-announce future prices. In a different context, Gallego and Sahin (2010) examine the value of refundable ticket under the assumption that the firm pre-commits to its future price.

[7] To see why, suppose that the firm sets $r_W < V_L$. As long as $r_W \leq V_L$, both high and low valuation customers will purchase the service in period 2. Hence, the firm can increase its expected revenue by increasing its walk-in price to equal V_L. Similarly, suppose that the firm sets $V_L < r_W < V_H$. As long as $V_L < r_W < V_H$, only high valuation customers will purchase the service. Hence, the firm can increase its expected revenue by increasing its walk-in price to equal V_H.

entice all customers in segment {LH} to reserve as well, so we can conclude that there is no policy that supports segment {LL} only. By using the same logical argument repeatedly, we obtain the following result:

Lemma 1. *In a four-segment market (LL, LH, HL and HH), there are only five admissible target segments* {HH}, {LH, HH}, {HL, HH}, {LH, HL, HH}, *and* {LL, LH, HL, HH} *to consider.*

For notational convenience, we label these five distinctive admissible target segments as: $TS^{(1)} = \{\text{HH}\}$, $TS^{(2)} = \{\text{LH, HH}\}$, $TS^{(3)} = \{\text{HL, HH}\}$, $TS^{(4)} = \{\text{LH, HL, HH}\}$, and $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$. Therefore, it remains to determine the optimal reservation policy for each of the target segments $TS^{(t)}$, where $t \in \{1, 2, 3, 4, 5\}$.

3.3. Notation

In preparation, let us introduce the following notation. For any policy $P^{(t)} = (r, d, n)$ that "supports" target segment $TS^{(t)}$, let $A^{(t)}$ be the number of customers who belong to the target segment $TS^{(t)}$. Due to the booking capacity $n^{(t)}$, the number of reservations that the firm will accept is $R^{(t)} \triangleq \min\{n^{(t)}, A^{(t)}\}$. Conditional on the number of reservations $R^{(t)}$, the number of customers who will show up for the reserved service is $(S^{(t)}|R^{(t)})$, where $(S^{(t)}|R^{(t)})$ is a Binomial random variable with parameters $(R^{(t)}, \varpi^{(t)})$, and $\varpi^{(t)}$ is the expected conditional show up probability for customers who belong to target segment $TS^{(t)}$.

For example, consider $TS^{(4)} = \{\text{LH, HL, HH}\}$. Any policy $P^{(4)} = (r, d, n)$ that supports $TS^{(4)}$ must discriminate against segment {LL} so that $\pi_{LL}(r, d) \le 0$, $\pi_{LH}(r, d) \ge 0$, $\pi_{HL}(r, d) \ge 0$ and $\pi_{HH}(r, d) \ge 0$. From Table 1, there are $A^{(4)} = [\alpha + (1-\alpha)\beta]A$ customers in target segment $TS^{(4)}$. Because the booking capacity is $n^{(4)}$, the number of accepted reservations $R^{(4)} = \min\{n^{(4)}, A^{(4)}\}$. Hence, the number of customers who shows up $(S^{(4)}|R^{(4)}) \sim \text{Binomial}(R^{(4)}, \varpi^{(4)})$. By considering Table 1 along with the fact that for $TS^{(4)} = \{\text{LH, HL, HH}\}$, $\varpi^{(4)}$ satisfies

$$\varpi^{(4)} = \frac{\alpha\psi_H + (1-\alpha)\beta\psi_L}{\alpha + (1-\alpha)\beta}. \tag{3.2}$$

By noting that $TS^{(1)} = \{\text{HH}\}$, $TS^{(2)} = \{\text{LH, HH}\}$, $TS^{(3)} = \{\text{HL, HH}\}$, and $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$. Using the same approach, one can check from Table 1 that $\varpi^{(1)} = \varpi^{(3)} = \psi_H$, and $\varpi^{(2)} = \varpi^{(5)} = \alpha\psi_H + (1-\alpha)\psi_L$. Also, we can determine other quantities for all other admissible target segments t=1, 2, 3, 5. We omit the details.

4. Analysis

Recall from Section 3.2 and Lemma 1 that we can determine optimal single-option reservation policy by using the following approach: (a) determine the optimal reservation policy and the optimal expected revenue associated with each of the five admissible target segments; (b) select the optimal admissible target segment that yields the highest expected revenue; and (c) retrieve the "global" optimal reservation policy that corresponds to the optimal target segment. Because this approach involves pair-wise comparisons of expected revenue associated with those five admissible target segments, it is difficult to establish basic intuition about the optimal reservation policy and the corresponding optimal market segments that the firm should serve. For this reason, we first consider the case when the customer demand and the firm's capacity are asymptotically large so that they can be approximated by *continuous* values in Section 4.1. For this case, we determine the optimal reservation policy analytically, and we establish the conditions under which the firm should discriminate (i.e., price out) customers with low valuation (regardless of their show-up probability), and the conditions under which the firm should discriminate customers with both low valuation and low show-up probability. In Section 4.2, we consider the case when customer demand and the firm's capacity are finite so that they take on *discrete* values. We find some of the insights obtained from the "continuous" case continue to hold when the firm's capacity is large. Although no closed form expressions for the optimal reservation policy are available when the firm's capacity is small, our numerical analysis suggests that the firm should not discriminate the segment with both low valuation and high show-up probability especially when V_L is sufficiently high.

4.1. Large deterministic customer demand and large capacity

Consider the case in which (i) the total number of potential customers A is deterministic; and (ii) both A and the firm's capacity m grow asymptotically large. To determine the optimal reservation policy $P^{(t)} = (r, d, n)$ for each target segment $TS^{(t)}$, we first determine the optimal booking capacity n and then we find the optimal retail price and the optimal deposit (r, d).

For any policy $P^{(t)} = (r, d, n)$ that "supports" target segment $TS^{(t)}$, all $A^{(t)}$ customers in target segment $TS^{(t)}$ would like to reserve. Due to the booking capacity $n^{(t)}$, the number of accepted reservations in period 1 is equal to $R^{(t)} \triangleq \min\{n^{(t)}, A^{(t)}\}$ and the number of customers who shows up in period 2 is equal to $S^{(t)}$, where $S^{(t)} \sim \text{Binomial}(R^{(t)}, \varpi^{(t)})$ and $\varpi^{(t)}$ is given in Section 3.3 for $t = 1, 2, \ldots, 5$. As A and m grow *large*, $A^{(t)}$ and $R^{(t)}$ also become large, and by applying the *strong law of large numbers*, it follows that $S^{(t)}/R^{(t)}$ converges to $\varpi^{(t)}$ almost surely. This implies that the number of customers who shows up in period 2 $S^{(t)} = R^{(t)}\varpi^{(t)}$. In this case, the firm can maximize its revenue by fully utilizing its capacity so that $m = S^{(t)} = R^{(t)}\varpi^{(t)}$. In other words, it is optimal for the firm to accept $R^{(t)}$ reservations, where $R^{(t)} = m/\varpi^{(t)}$. By noting that $R^{(t)} \triangleq \min\{n^{(t)}, A^{(t)}\}$, we can conclude that the optimal booking capacity $n^{(t)} = m/\varpi^{(t)}$.[8] By using the expected show up probability $\varpi^{(t)}$ for each target segment $TS^{(t)}$ as given in Section 3.3, we can determine the optimal booking capacity $n^{(t)} = m/\varpi^{(t)}$ for $t = 1, 2, \ldots, 5$.

[8] Due to the law of large numbers, the firm can estimate the number of customers who will show up for their reserved service. As a result, the firm will never incur any bumping costs.

4.1.1. Optimal expected revenue and optimal reservation policy for each target segment

Given the optimal booking capacity $n^{(t)} = m/\varpi^{(t)}$, we now determine the optimal retail price and deposit (r, d) for each target segment $TS^{(t)}$, where $t = 1, 2, \ldots, 5$. To avoid repetition, we illustrate our analysis for $TS^{(1)}$ and present the rest in Appendix A.

Recall from Section 3.3 that $TS^{(1)} = \{\text{HH}\}$ so that $A^{(1)} = \alpha\beta A$ and $\varpi^{(1)} = \psi_H$. Hence, the optimal booking capacity $n^{(1)} = m/\psi_H$, the optimal number of reservations $R^{(1)} = \min\{n^{(1)}, A^{(1)}\} = \min\{m/\psi_H, \alpha\beta A\}$, and the firm's expected revenue associated with any (r, d) satisfies

$$\Pi^{(1)} = [d + (r-d)\psi_H]\left[\min\left\{\frac{m}{\psi_H}, \alpha\beta A\right\}\right].$$

However, for any policy (r, d) that supports $TS^{(1)} = \{\text{HH}\}$ and discriminates other customer segments so that only customers in segment $\{\text{HH}\}$ will reserve. By examining the customer's expected surplus $\pi_{ij}(r, d)$ given in (3.1), it is easy to check that $\pi_{LL}(r, d) \le \pi_{HL}(r, d) \le \pi_{HH}(r, d)$ for any (r, d). Hence, it is sufficient for the firm to select (r, d) such that $\pi_{HH}(r, d) = 0$ and $\pi_{LH}(r, d) \le 0$. It is to check from (3.1) that (r, d) must satisfy

$$d = \frac{\psi_H}{1-\psi_H}(V_H - r) \ge \frac{\psi_L}{1-\psi_L}(V_H - r).$$

By substituting $d = (\psi_H/(1-\psi_H))(V_H - r)$ into the firm's expected revenue given above, we can conclude that when serving target segment $TS^{(1)} = \{\text{HH}\}$ only, the firm's optimal expected revenue satisfies

$$\Pi^{(1)} = V_H[\min\{\alpha\beta\psi_H A, m\}],$$

where the optimal reservation policy

$$P^{(1)} = (r^{(1)}, d^{(1)}; n^{(1)}) = \left(r^{(1)}, \frac{\psi_H}{1-\psi_H}(V_H - r^{(1)});\ \frac{m}{\psi_H}\right)$$

with $r^{(1)}$ satisfying $V_H \ge r^{(1)} \ge 0$.

By using the exact same approach, we determine the firm's optimal expected revenue and the optimal reservation policy for other target segment $TS^{(t)}$, $t = 1, .., 5$, as follows:

Proposition 1. *When the total number of potential customers A is large and deterministic and when the firm's capacity m is large, the firm's optimal expected revenue and the optimal reservation policy for each target segment $TS^{(t)}$ can be described as follows:*

1. *When $t=1$, $TS^{(1)} = \{\text{HH}\}$, $\Pi^{(1)} = V_H[\min\{\alpha\beta\psi_H A, m\}]$, and $P^{(1)} = (r^{(1)}, d^{(1)}; n^{(1)}) = (r^{(1)}, (\psi_H/(1-\psi_H))(V_H - r^{(1)}); m/\psi_H)$, where $\psi_H V_H \le r^{(1)} \le V_H$.*
2. *When $t=2$, $TS^{(2)} = \{\text{LH, HH}\}$, $\Pi^{(2)} = V_H[\min\{[\alpha\beta\psi_H + (1-\alpha)\beta\psi_L]A, m\}]$, and $P^{(2)} = (r^{(2)}, d^{(2)}; n^{(2)}) = (V_H, 0; m/\varpi^{(2)})$.*
3. *When $t=3$, $TS^{(3)} = \{\text{HL, HH}\}$, $\Pi^{(3)} = V_L[\min\{\alpha\psi_H A, m\}]$, and $P^{(3)} = (r^{(3)}, d^{(3)}; n^{(3)}) = (r^{(3)}, (\psi_H/(1-\psi_H))(V_L - r^{(3)}); m/\psi_H)$, where $0 \le r^{(3)} \le (\psi_H V_L - \psi_L V_H + \psi_L\psi_H(V_H - V_L))/(\psi_H - \psi_L)$ and $(\psi_H/(1-\psi_H))(V_L - r^{(3)}) \ge \psi_L\psi_H(V_H - V_L)/(\psi_H - \psi_L)$.*
4. *When $t=4$, $TS^{(4)} = \{\text{LH, HL, HH}\}$ and we have two cases to consider.*

   (a) *If $\psi_L V_H \ge \psi_H V_L$, then $\Pi^{(4A)} = \psi_H V_L[\min\{[\alpha\psi_H + (1-\alpha)\beta\psi_L]A, m\}]$, and $P^{(4A)} = (r^{(4A)}, d^{(4A)}; n^{(4A)}) = (\psi_H V_L, \psi_H V_L; m/\varpi^{(4)})$.*

   (b) *If $\psi_L V_H < \psi_H V_L$, then*

   $$\Pi^{(4B)} = \frac{\alpha\psi_H V_L + (1-\alpha)\beta\psi_L V_H}{\alpha\psi_H + (1-\alpha)\beta\psi_L}[\min\{[\alpha\psi_H + (1-\alpha)\beta\psi_L]A, m\}]$$

   and

   $$P^{(4B)} = (r^{(4B)}, d^{(4B)}; n^{(4B)}) = \left(\frac{\psi_H V_L - \psi_L V_H + \psi_L\psi_H(V_H - V_L)}{\psi_H - \psi_L}, \frac{\psi_L\psi_H(V_H - V_L)}{\psi_H - \psi_L}; \frac{m}{\varpi^{(4)}}\right).$$

5. *When $t = 5$, $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$, $\Pi^{(5)} = V_L[\min\{[\alpha\psi_H + (1-\alpha)\psi_L]A, m\}]$, and $P^{(5)} = (r^{(5)}, d^{(5)}; n^{(5)}) = (V_L, 0; m/\varpi^{(5)})$.*

Proposition 1 suggests that, depending on the target segment, the optimal reservation policy can take the form of "no deposit" (when serving target segments 2 and 5), "partial deposit" (when serving target segments 1, 3, and 4(b)), or "full deposit" (when serving target segment 4(a)).

4.1.2. Optimal target segment

We now identify the "global" optimal target segment that yields the highest expected revenue. By examining the firm's expected revenue $\Pi^{(t)}$, $t = 1, .., 5$, given in Proposition 1, it is easy to check that $TS^{(1)}$ and $TS^{(3)}$ can never be the optimal target segment, while $TS^{(4)}$ might be optimal only in case B (i.e., if $\psi_L V_H < \psi_H V_L$). Therefore, by using the expected show-up probabilities $\varpi^{(2)} = \varpi^{(5)} = \alpha\psi_H + (1-\alpha)\psi_L$ and $\varpi^{(4)}$ given in (3.2), we can compare the optimal revenue $\Pi^{(2)}$, $\Pi^{(4)}$ (in Case B), and $\Pi^{(5)}$ to determine the optimal target segment as follows:

Proposition 2. *When the total number of potential customers A is large and deterministic and when the firm's capacity m is large, the optimal target segment can be characterized as follows:*

1. *Suppose $(\psi_L V_H \ge \psi_H V_L)$. If $V_L/V_H \le \beta \max\{\varpi^{(5)}A/m, 1\}$, then target segment $TS^{(2)} = \{\text{LH, HH}\}$ is optimal; otherwise, target segment $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$ is optimal.*
2. *Suppose $(\psi_L V_H < \psi_H V_L)$. Then the following table summarizes the conditions under which any one of the three candidate target segments is optimal.*

Optimal target segment	$\frac{A}{m} \le \frac{1}{\tilde{w}}$	$\frac{A}{m} > \frac{1}{\tilde{w}}$
{LH, HH}	$\frac{V_L}{V_H} \le \beta$	$\frac{V_L}{V_H} \le \max\left\{1, \beta\left[\frac{w\tilde{w}\frac{A}{m} - (1-\alpha)\psi_L}{\alpha\psi_H}\right]\right\}$
{LH, HL, HH}	$\beta < \frac{V_L}{V_H} \le \max\left\{\beta, \frac{(1-\alpha)\beta\psi_L\frac{A}{m}}{1-\alpha\psi_H\frac{A}{m}}\right\}$	$\frac{V_L}{V_H} > \max\left\{1, \beta\left[\frac{w\tilde{w}\frac{A}{m} - (1-\alpha)\psi_L}{\alpha\psi_H}\right]\right\}$
{LL, LH, HL, HH}	$\frac{V_L}{V_H} > \max\left\{\beta, \frac{(1-\alpha)\beta\psi_L\frac{A}{m}}{1-\alpha\psi_H\frac{A}{m}}\right\}$	*never optimal*

where $w = \alpha\psi_H + (1-\alpha)\psi_L \equiv \varpi^{(2)} = \varpi^{(5)}$, $\tilde{w} = \alpha\psi_H + (1-\alpha)\beta\psi_L \equiv \varpi^{(4)}[\alpha + (1-\alpha)\beta]$, *and* $w > \tilde{w}$.

Given the optimal target segment as specified in Proposition 2, we can use Proposition 1 to determine the corresponding optimal reservation policy.

The intuition behind the result stated in Proposition 2 can be explained as follows. First, let us consider statement 1 when $\psi_L V_H \ge \psi_H V_L$ so that $\psi_H V_H \ge \psi_L V_H \ge \psi_H V_L \ge \psi_L V_L$. In this case, the expected value of serving a high valuation customer is always higher than that of a low valuation customer. Hence, it is optimal for the firm to discriminate low valuation customers by serving only high valuation customers (i.e., target segment $TS^{(2)} = \{\text{LH, HH}\}$), or to target all segments (i.e., $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$). In addition, if V_H is sufficiently higher than V_L so that $V_L/V_H \le \beta \max\{\varpi^{(5)}A, m\}$, then it is optimal for the firm to serve high value customers in target segment $TS^{(2)} = \{\text{LH, HH}\}$ only.

Next, let us consider statement 2 when $\psi_L V_H < \psi_H V_L$. First, let us consider the case when the potential customer demand per-unit of capacity is *small* such that $A/m \le 1/\tilde{w}$, then it is optimal for the firm to serve high value customers in target segment $TS^{(2)} = \{\text{LH, HH}\}$ only when V_H is sufficiently higher than V_L so that $V_L/V_H \le \beta$, while it should also entertain low valuation customers with high show-up probability (i.e., segment {HL}) if $\beta < V_L/V_H \le \max\{\beta, ((1-\alpha)\beta\psi_L A/m)/(1-\alpha\psi_H A/m)\}$, and it should target all segments (i.e., $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$) if $V_L/V_H > \max\{\beta, ((1-\alpha)\beta\psi_L A/m)/(1-\alpha\psi_H A/m)\}$ (i.e., if V_L is sufficiently high). Second, when the potential customer demand per-unit of capacity is *large* such that $A/m > 1/\tilde{w}$, it is optimal for the firm to serve high value customers in target segment $TS^{(2)} = \{\text{LH, HH}\}$ if $V_L/V_H \le \max\{1, \beta[(w\tilde{w}A/m - (1-\alpha)\psi_L)/\alpha\psi_H]\}$ (i.e., if V_H is high enough); otherwise, the firm should target segment $TS^{(3)} = \{\text{LH, HL, HH}\}$. Because the potential customer demand per-unit of capacity is *large*, the firm can afford to be selective and should never target all segments; i.e., choose $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$).

By considering the case when (i) the total number of potential customers A is *large* and *deterministic*, and (ii) the firm's capacity m is *large*, where A/m is bounded by a finite number, we obtain closed form expressions for the optimal reservation policy in Proposition 1 and show that, depending on the optimal target segment, no deposit, partial deposit, or full deposit can be optimal. In addition, we establish conditions for a target segment to be optimal for a firm to serve in Proposition 2, and we generate basic insights about the impact of show up probability, customer valuation, and market size affect the optimal target segment. In the next subsection, we shall consider the case when (i) the total number of potential customers A is *finite* and *stochastic*; and (ii) the firm's capacity m is *finite* and to examine if the insights obtained in this subsection would continue to hold.

4.2. Finite and stochastic customer demand and finite capacity

We now examine the case when the total number of potential customers A is *finite* and *stochastic* and the firm's capacity m is *finite* so that they take on *discrete* values (i.e., integers). To capture market uncertainty and to obtain tractable results, we assume A follows a Poisson distribution with rate λ. By using the splitting property of the Poisson distribution, we can check from Table 1 that $A^{(t)}$; i.e., the number of customers in target segment $TS^{(t)}$, satisfies: $A^{(1)} \sim Poi(\lambda[\alpha\beta])$, $A^{(2)} \sim Poi(\lambda[\beta])$, $A^{(3)} \sim Poi(\lambda[\alpha])$, $A^{(4)} \sim Poi(\lambda[\alpha + (1-\alpha)\beta])$, and $A^{(5)} \sim Poi(\lambda)$.

To determine the optimal reservation policy $P^{(t)} = (r, d, n)$ for each target segment $TS^{(t)}$, we use the same approach as in Section 4.1: we first determine the optimal booking capacity n and then we find the optimal retail price and the optimal deposit (r, d).

For any policy $P^{(t)} = (r, d, n)$ that "supports" target segment $TS^{(t)}$, all $A^{(t)}$ customers in target segment $TS^{(t)}$ would like to reserve. Due to the booking capacity $n^{(t)}$, the number of accepted reservations in period 1 is equal to $R^{(t)} \triangleq \min\{n^{(t)}, A^{(t)}\}$. Condition on the random variable $R^{(t)} = k$, the number of customers who shows up in period 2 is equal to $S^{(t)} = j$, where $(S^{(t)} = j | R^{(t)} = k) \sim \text{Binomial}(k, \varpi^{(t)})$ and $\varpi^{(t)}$ is given in Section 3.3 for $t = 1, 2, \ldots, 5$. In this case, the firm earns non-refundable deposits $(k-j)d$ from those $(k-j)$ "no show" customers, earns full retail prices $\min\{j, m\}r$ from those customers who show up and are served by the firm, and incurs an overbooking penalty $\max\{j-m, 0\}c$ for those customers who show up without being served by the firm due to overbooking, where c is the overbooking penalty. Because $\mathbb{E}(S^{(t)} = j | R^{(t)} = k) = k\varpi^{(t)}$, one can check that the firm's expected revenue associated with any target segment $TS^{(t)}$ satisfies:

$$\Pi^{(t)}(r, d, n; m) = \sum_{k=0}^{n} p_k \left\{ k(1-\varpi^{(t)})d + \sum_{j=0}^{k} \Pr(j|k)[\min\{j, m\}r - \max\{j-m, 0\}c] \right\}, \tag{4.1}$$

where $p_k = \Pr\{A^{(t)} = k\}$ for $k < n$ (i.e., the probability that target segment $TS^{(t)}$ has $k < n$ customers, and $p_n = \Pr\{A^{(t)} = n\}$ (or $p_{\ge n} = \sum_{k=n}^{\infty} \Pr\{A^{(t)} = k\}$) denotes the probability that $TS^{(t)}$ has n (or more than n) customers, respectively. Also, we denote $\Pr(j|k) = \Pr(S^{(t)} = j | R^{(t)} = k)$ that follows the distribution Binomial $(k, \varpi^{(t)})$.

By conducting marginal analysis on (4.1) with respect to n, we can establish the first order condition to show that the optimal booking capacity $n^{(t)*}_{(r,d)}$ for any given (r,d) that supports target segment $TS^{(t)}$ satisfies:

$$n^{(t)*}_{(r,d)} = \begin{cases} n^* & \text{if } \Pi^{(t)}(r,d,n^*;m) \geq \Pi^{(t)}(r,d,n^*+1;m) \\ n^*+1 & \text{otherwise} \end{cases},$$

where n^* satisfies:[9]

$$\sum_{j=0}^{m-1}(m-j)[\Pr(j|n+1)-\Pr(j|n)] \geq \frac{\varpi^{(t)}c-(1-\varpi^{(t)})d}{r+c}. \tag{4.2}$$

Observe from (4.2) that the optimal booking capacity $n^{(t)*}_{(r,d)}$ is complex and it depends on c, r and d. Also, recall from Section 4.1.2 that the optimal booking capacity $n^{(t)} = m/\varpi^{(t)}$ is independent of c, r and d for the case when A is deterministic and large and when m is large. Hence, we can conclude that, when dealing with finite and stochastic customer demand and finite capacity, the firm needs to understand the impact of its pricing strategy $(r,\ d)$ on the booking capacity n.

4.2.1. Optimal expected revenue and optimal reservation policy for each target segment $TS^{(t)}$

Because of the complexity of (4.2), there is no simple closed form expression for the optimal booking capacity n^*. For this reason, let us consider a first determine the optimal deposit $d^{(t)*}$ and the optimal retail price $r^{(t)*}$ for any target segment $TS^{(t)}$ and any booking capacity n in this section, and then we search for the optimal booking capacity numerically associated with the optimal deposit $d^{(t)*}$ and the optimal retail price $r^{(t)*}$ in the next section. By considering the firm's expected revenue given in (4.1), we now determine the optimal deposit $d^{(t)*}$ and the optimal retail price $r^{(t)*}$ for any target segment $TS^{(t)}$ and any booking capacity n. To avoid repetition, we shall present the analysis associated with target segment $TS^{(4)} = \{\text{LH, HL, HH}\}$, and we shall summarize the optimal pricing and deposit for all 5 target segments in Theorem 1. We shall show that for any booking capacity n, the optimal deposit $d^{(t)*}(n)$ and the optimal retail price $r^{(t)*}(n)$ for each target segment $TS^{(t)}$ can be determined by solving a linear program with a single decision variable. By noting that the boundary points associated with this decision variable are independent of the booking capacity n, there exist at most two candidate optimal policies, say, $(r_1^{(t)}, d_1^{(t)})$ and $(r_2^{(t)}, d_2^{(t)})$. Consequently, we can apply (4.2) to determine the optimal booking capacity associated with each candidate optimal policy. By using (4.1) to compare the expected revenue associated with these two candidate optimal policies, one can then obtain the optimal policy $P^{(t)*} = (r^{(t)*}, d^{(t)*}, n^{(t)*}_{(r^{(t)*},d^{(t)*})})$ and the firm's optimal expected revenue $\Pi^{(t)*}$ for $t=1,2,3,4,5$. Finally, we can determine the optimal target segment t^* by selecting the segment that yields the highest expected revenue, and then retrieve the corresponding optimal policy P^* associated with the optimal target segment.

To avoid repetition, we shall analyze the case when the target segment $TS^{(4)} = \{\text{LH, HL, HH}\}$. The analysis for other target segments follows exactly the same way. For any booking capacity n, using (3.1) and (4.1), the optimal deposit $d^{(4)*}$ and the optimal retail price $r^{(4)*}$ associated with a reservation policy that "supports" target segment $TS^{(4)}$ can be determined by solving the following problem:

$$\max_{r,d}\{\Pi^{(4)}(r,d,n;m) : \pi_{HH}(r,d) \geq 0, \pi_{HH}(r,d) \geq 0, \pi_{HH}(r,d) \geq 0, \pi_{HH}(r,d) \leq 0, r \geq d \geq 0\} \tag{4.3}$$

In order to determine the optimal policy analytically, we now analyze those two cases examined earlier in Section 4.1.2 for the deterministic demand case. Specifically, for Case A when $\psi_L V_H \geq \psi_H V_L$, we have $\psi_H V_H \geq \psi_L V_H \geq \psi_H V_L \geq \psi_L V_L$ so the expected value of serving a high valuation customer is always higher than that of a low valuation customer.

4.2.2. Case A: $\psi_L V_H \geq \psi_H V_L$

To simplify our exposition, we shall drop the superscript (4) in this section. For any policy that supports target segment $TS^{(4)} = \{\text{LH, HL, HH}\}$, it follows from (3.1) that the condition $\psi_L V_H \geq \psi_H V_L$ implies that any *feasible* policy $(r,\ d)$, $r \geq d \geq 0$ for problem (4.3) that will allow the firm to discriminate segment {LL} will satisfy $\pi_{HH}(r,d) \geq \pi_{LH}(r,d) \geq \pi_{HL}(r,d) \geq 0 \geq \pi_{LL}(r,d)$. In this case, it is optimal for the firm to set(r,d) to extract the entire surplus from segment $\{HL\}$ so that $\pi_{HL}(r,d)=0$. By considering (3.1) for the case when $i=H$ and $j=L$,

$$\pi_{HL}(r,d) = 0 \Longleftrightarrow d = \frac{\psi_H(V_L - r)}{1-\psi_H}. \tag{4.4}$$

Observe from (4.4) that the optimal deposit d is linearly decreasing in the retail price r, and that the constraint $0 \leq d \leq r$ can be rewritten as $\psi_H V_L \leq r \leq V_L$. By substituting (4.4) into (4.1) the firm's expected revenue can be written as

$$\Pi(r,d,n;m) = \sum_{k=0}^{n} p_k \left\{ r\left[\sum_{j=0}^{k}\min\{j,m\}\Pr(j|k) - k(1-\varpi)\frac{\psi_H}{1-\psi_H}\right] + k(1-\varpi)\frac{\psi_H V_L}{1-\psi_H} - c\sum_{j=m}^{k}(j-m)\Pr(j|k)\right\} \tag{4.5}$$

Observe from (4.5) that the firm's expected revenue is a linear function of the retail price r for any given booking capacity n. Therefore we can determine the optimal price r^* by solving the following linear program (and then retrieve the optimal deposit d^* from (4.4)).

$$\max\left\{\sum_{k=0}^{n} p_k \left[\sum_{j=0}^{k}\min\{j,m\}\Pr(j|k) - k(1-\varpi)\frac{\psi_H}{1-\psi_H}\right] r :\ \psi_H V_L \leq r \leq V_L \right\} \tag{4.6}$$

[9] While we are unable to prove that $\sum_{j=0}^{m-1}(m-j)[\Pr(j|n)-\Pr(j|n+1)]$ (i.e., the LHS of (4.2)) decreases in n, our extensive numerical analysis suggests that this term is indeed decreasing in n so that $\Pi^{(t)}(r,d,n;m)$ is concave in n. This observation enables us to make a conjecture that the LHS of (4.2) is decreasing in n. Combine this conjecture with the fact that the RHS of (4.2) (i.e., $(\varpi^{(t)}c-(1-\varpi^{(t)})d)/(r+c)$) is increasing in c, we can conclude that the optimal booking capacity $n^{(t)*}_{(r,d)}$ is decreasing in c for any $(r,\ d)$ that supports target segment $TS^{(t)}$. This result is intuitive: the firm should decrease its booking capacity when the overbooking penalty c increases.

The optimal solution to (4.6) hinges upon the sign of $\sum_{k=0}^{n} p_k[\sum_{j=0}^{k} \min\{j,m\}\Pr(j|k) - k(1-\varpi)\psi_H/(1-\psi_H)]$. In preparation, we establish the following Lemma.

Lemma 2. *For any $x \geq \varpi$ and $m < \infty$, the function $f(x) < 0$ for any booking capacity n, where.*

$$f(x) = \sum_{k=0}^{n} p_k \left[\sum_{j=0}^{k} \min\{j,m\}\Pr(j|k) - k(1-\varpi)\frac{x}{1-x} \right]. \tag{4.7}$$

By noting that $\psi_H \geq \varpi^{(4)}$ (where $\varpi^{(4)}$ is given in (3.2)), it follows from Lemma 2 that $f(\psi_H) < 0$, so that the optimal retail price that solves (4.6) is $r = \psi_H V_L$. By substituting this into (4.4), it follows that $d = \psi_H V_L$. Also note that the optimal price and deposit is independent of the booking capacity n.

This result suggests that, when serving segment $TS^{(4)}$ and when $\psi_L V_H \geq \psi_H V_L$, it is optimal for the firm to charge a "full" non-refundable deposit so that $d^* = r^* = \psi_H V_L$. The corresponding reservation policy extracts the entire surplus from segment HL, entices the other 2 segments {HH, LH} to reserve, and deters segment {LL} from reserving. Given the fact that (r^*, d^*) is independent of n, we can apply (4.2) to determine the corresponding optimal booking capacity $n^*_{(r^*,d^*)}$. Hence, when $\psi_L V_H \geq \psi_H V_L$, , the optimal policy $P^{(4)*}$ associated with the target segment $TS^{(4)} = \{\text{LH, HL and HH}\}$ is a full deposit policy that satisfies

$$P^* = (r^*, d^*, n^*_{(r^*,d^*)}) = (\psi_H V_L, \psi_H V_L, n^*_{(\psi_H V_L, \psi_H V_L)}).$$

It is interesting to note that, even though the analysis for the optimal booking capacity cannot be written in simple closed form expression when customer demand is stochastic, the optimal retail price and the optimal deposit $d^* = r^* = \psi_H V_L$ is the same as in the case when customer demand is deterministic as shown Proposition 1 (statement 4 case A).

4.2.3. Case B: $\psi_L V_H < \psi_H V_L$

When $\psi_L V_H < \psi_H V_L$, it is no longer true that $\pi_{LH}(r,d) \geq \pi_{HL}(r,d)$ for any (r, d) that supports target segment $TS^{(4)}$. However, it follows from (3.1) that $\pi_{LH}(r,d) \geq \pi_{HL}(r,d)$ if and only if $r \geq d + (\psi_H V_L - \psi_L V_H)/(\psi_H - \psi_L)$. This observation motivates us to consider two scenarios: (a) $r \geq d + (\psi_H V_L - \psi_L V_H)/(\psi_H - \psi_L)$; and (b) $r < d + (\psi_H V_L - \psi_L V_H)/(\psi_H - \psi_L)$ for Case B. By using the same approach as presented in the previous subsection, we can establish the following result:

Lemma 3. *When serving target segment $TS^{(4)}$ and when $\psi_L V_H < \psi_H V_L$, the optimal reservation policy (r^*, d^*, n^*) satisfies*

$$P^{(4)*} = \begin{cases} (\psi_L V_H, \psi_L V_H, n^*_{(\psi_L V_H, \psi_L V_H)}) & \text{if } \Pi(\psi_L V_H, \psi_L V_H, n^*_{(\psi_L V_H, \psi_L V_H)}) \geq \Pi(r^{(4)*}, d^{(4)*}, n^*_{(r^{(4)*}, d^{(4)*})}), \\ \left(r^{(4)*} = \dfrac{\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, d^{(4)*} = \dfrac{\psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, n^*_{(r^{(4)*}, d^{(4)*})} \right) & \text{otherwise.} \end{cases}$$

Observe from Lemma 3 that the optimal reservation policy $(r^{(4)*}, d^{(4)*})$ stated on the top is the same as in the case when customer demand is deterministic as shown in Proposition 1 (statement 4 case A), while the one stated on the bottom is the same as in the case when customer demand is deterministic as shown in Proposition 1 (statement 4 case B). Therefore, even though the optimal booking capacity is complex for the case when customer demand is stochastic, the optimal reservation policy is relatively simple.

By combining the results obtained in Cases A and B, we can obtain the optimal reservation policy $P^{(4)*}$ associated with target segment $TS^{(4)}$. Specifically, when targeting segment $TS^{(4)}$, it is optimal for the firm to issue a full deposit policy when $\psi_L V_H \geq \psi_H V_L$. However, when $\psi_L V_H < \psi_H V_L$, Lemma 3 reveals that the optimal reservation policy is either a partial deposit or a full deposit policy. Also, by using the same approach, we can determine the optimal reservation policy $P^{(t)*}$ for the remaining target segments $TS^{(t)}$, $t = 1,2,3,5$ in the following Theorem.

Theorem 1. *The optimal reservation policy $P^{(t)*}$ that supports target segment $TS^{(t)}$ satisfies:*

1. *For target segment $TS^{(1)} = \{\text{HH}\}$, $P^{(1)*} = (\psi_H V_H, \psi_H V_H, n^*_{(\psi_H V_H, \psi_H V_H)})$.*
2. *For target segment $TS^{(2)} = \{\text{LH, HH}\}$, the optimal policy satisfies:*

   (i) *If $\psi_L V_H \geq \psi_H V_L$, then*
   $$P^{(2)*} = \begin{cases} (V_H, 0, n^*_{(V_H,0)}) & \text{if } \Pi(V_H, 0, n^*_{(V_H,0)}) \geq \Pi(\psi_L V_H, \psi_L V_H, n^*_{(\psi_L V_H, \psi_L V_H)}) \\ (\psi_L V_H, \psi_L V_H, n^*_{(\psi_L V_H, \psi_L V_H)}) & \text{otherwise.} \end{cases}$$

   (ii) *If $\psi_L V_H < \psi_H V_L$, then*
   $$P^{(2)*} = \begin{cases} (V_H, 0, n^*_{(V_H,0)}) & \text{if } \Pi(V_H, 0, n^*_{(V_H,0)}) \geq \Pi(r^{(3)*}, d^{(3)*}, n^*_{(r^{(3)*}, d^{(3)*})}) \\ \left(\dfrac{\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, \dfrac{\psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, n^*_{(r^{(3)*}, d^{(3)*})} \right) & \text{otherwise.} \end{cases}$$

3. *For target segment $TS^{(3)} = \{\text{HL, HH}\}$, $P^{(3)*} = (\psi_H V_L, \psi_H V_L, n^*_{(\psi_H V_L, \psi_H V_L)})$ when $\psi_L V_H < \psi_H V_L$.*[10]

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[10] If $\psi_L V_H \geq \psi_H V_L$, then for any $r \geq d \geq 0$ such that $\pi_{HH}(r,d) \geq 0$ and $\pi_{HL}(r,d) \geq 0$, it follows that $\pi_{LH}(r,d) \geq 0$. Hence if $\psi_L V_H \geq \psi_H V_L$., then this target segment is not admissible.

4. *For target segment* $TS^{(4)} = \{\text{LH, HL, HH}\}$, *the optimal policy satisfies*

   (i) *If* $\psi_L V_H \ge \psi_H V_L$, *then* $P^{(4)*} = (\psi_H V_L, \psi_H V_L, n^*_{(\psi_H V_L, \psi_H V_L)})$.

   (ii) *If* $\psi_L V_H < \psi_H V_L$, *then*

$$P^{(4)*} = \begin{cases} (\psi_L V_H, \psi_L V_H, n^*_{(\psi_L V_H, \psi_L V_H)}) & \text{if } \Pi(\psi_L V_H, \psi_L V_H, n^*_{(\psi_L V_H, \psi_L V_H)}) \ge \Pi(r^{(4)*}, d^{(4)*}, n^*_{(r^{(4)*}, d^{(4)*})}) \\ \left(\dfrac{\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, \dfrac{\psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, n^*_{(r^{(4)*}, d^{(4)*})} \right) & \text{otherwise.} \end{cases}$$

5. *For target segment* $TS^{(5)} = \{\text{LL, HL, LH, HH}\}$, *the optimal policy satisfies*

$$P^{(5)*} = \begin{cases} (V_L, 0, n^*_{(V_L, 0)}) & \text{if } \Pi(V_L, 0, n^*_{(V_L, 0)}) \ge \Pi(\psi_L V_L, \psi_L V_L, n^*_{(\psi_L V_L, \psi_L V_L)}) \\ (\psi_L V_L, \psi_L V_L, n^*_{(\psi_L V_L, \psi_L V_L)}) & \text{otherwise.} \end{cases}$$

Theorem 1 asserts that, depending on the target segment $TS^{(t)}$, the optimal reservation policy may involve no deposits, partial deposits, or full deposits. For instance, when the target segment involves only customers with high show up probability (i.e., when the firm serves target segments $TS^{(1)}$ or $TS^{(3)}$, Theorem 1 reveals that full deposit policies are optimal.

Even though the determination of the optimal booking capacity is quite complex, the optimal reservation policy (r^*, d^*) stated Theorem 1 possesses a similar structure as Proposition 1 for the deterministic demand case in Section 4.1.1. Specifically, consider the case when $\psi_L V_H \ge \psi_H V_L$, and note that the no deposit policy for target segment $TS^{(2)}$ in statement 2 above, the full deposit policy for target segment $TS^{(4)}$ in statement 4 above, and the no deposit policy for target segment $TS^{(5)}$ in statement 5 above are consistent with the results as stated in Proposition 1. However, the optimal reservation policy for other target segments are different than those stated in Proposition 1.

Given the optimal policy $P^{(t)*}$ stated in Theorem 1, we can apply (4.1) to determine the firm's corresponding optimal expected revenue $\Pi^{(t)*}$ for each target segment $TS^{(t)}$. However, in order to identify the optimal target segment, one needs to make pair-wise comparisons of optimal expected revenues associated with different target segments. While the analytical comparison of different revenue functions is intractable, we are able to conduct analytical comparison for the case when the overbooking penalty c is sufficiently large (so that $n^* = m$) and when $c=0$ (so that $n^* \to \infty$). Before we make such comparison, let us establish Corollary 1 first.

Corollary 1. *If the overbooking penalty* $c \to \infty$ (*so that* $n^* = m$) *or when* $c=0$ (*so that* $n^* \to \infty$), *then the optimal reservation policy* $P^{(t)*}$ *that supports target segment* $TS^{(t)}$ *for* $t = 2, 4, 5$ *can be described as follows. There exist thresholds* $m_\tau^{(t)}(c)$ *so that*

1. *For target segment* $TS^{(2)} = \{\text{LH, HH}\}$, *the optimal reservation policy satisfies*

$$P^{(2)*} = \begin{cases} (V_H, 0, n^*_{(V_H, 0)}) & \text{if } m \ge m_\tau^{(2)}(c), \quad \text{and} \\ (\psi_L V_H, \psi_L V_H, n^*_{(\psi_L V_H, \psi_L V_H)}) & \text{otherwise.} \end{cases}$$

2. *For target segment* $TS^{(4)} = \{\text{LH, HL, HH}\}$, *if* $\psi_L V_H < \psi_H V_L$, *the optimal reservation policy satisfies*

$$P^{(4)*} = \begin{cases} \left(\dfrac{\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, \dfrac{\psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, n^*_{(r^{(4)*}, d^{(4)*})} \right) & \text{if } m \ge m_\tau^{(4)}(c), \quad \text{and} \\ (\psi_L V_H, \psi_L V_H, n^*_{(\psi_L V_H, \psi_L V_H)}) & \text{otherwise.} \end{cases}$$

3. *For target segment* $TS^{(5)} = \{\text{LL, HL, LH, HH}\}$, *the optimal reservation policy satisfies*

$$P^{(5)*} = \begin{cases} (V_L, 0, n^*_{(V_L, 0)}) & \text{if } m \ge m_\tau^{(5)}(c), \quad \text{and} \\ (\psi_L V_L, \psi_L V_L, n^*_{(\psi_L V_L, \psi_L V_L)}) & \text{otherwise.} \end{cases}$$

Observe from the three statements stated in Corollary 1 that, if the overbooking penalty c is either sufficiently large or sufficiently small, then the optimal deposit $d^{(t)*}$ is (weakly) decreasing in the firm's capacity m. By combining this observation with (4.4), we can conclude that the optimal retail price $r^{(t)*}$ is (weakly) increasing in the firm's capacity m. The intuition behind this result can be explained as follows. First, for any target segment (e.g., $t=2$, 5) that involves customers with both high and low show up probabilities, the firm can extract more surplus from customers with high show up probability (i.e., $\psi_H r$) relative to customers with low show up probability (i.e., $\psi_L \cdot r$) by offering policies with zero deposit but a higher retail price.[11] This explains why the "no deposit" policy is optimal when the firm's capacity m exceeds a certain threshold when serving target segment $TS^{(2)}$ or $TS^{(5)}$. (This result is consistent with Proposition 1 for the case when customer demand is deterministic: the no deposit policy is optimal when the firm serves target segment $TS^{(2)}$ or $TS^{(5)}$.) However, as m decreases, the "no deposit" policy exposes the firm to the risk of losing revenue due to no shows. To hedge against this risk, it is optimal for the firm to increase its deposit.

[11] The amount of (expected) surplus that a firm can extract by using policy (r, d) from a customer with private information (ψ, V) is equal to $(1-\psi)d + \psi r$.

4.2.4. Optimal target segment t^*: asymptotic and numerical analysis

In this section, we examine the impact of the firm's capacity m on the optimal reservation policy. Observe from Theorem 1 that the optimal target segment t^* satisfies

$$t^* = \arg\max\{\Pi^{(t)}(r^{(t)*}, d^{(t)*}, n^{(t)*}_{(r^{(t)*}, d^{(t)*})}) : \ t \in \{1,2,3,4,5\}\}. \tag{4.8}$$

However, because (4.8) is intractable for any finite capacity m, we first determine t^* *analytically* for the case when $m \to \infty$. Then we conduct our analysis *numerically* for any finite value of m. By applying Theorem 1, we establish the following result.

Proposition 3. *Consider the case when the firm's capacity $m \to \infty$. If $V_L/V_H \le \beta$, then target segment $TS^{(2)} = \{\text{LH, HH}\}$ is optimal and the corresponding optimal reservation policy $(r^{(2)*}, d^{(2)*}) = (V_H, 0)$; otherwise, target segment $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$ is optimal and the corresponding optimal reservation policy $(r^{(5)*}, d^{(5)*}) = (V_L, 0)$.*

When the firm's capacity $m \to \infty$, Proposition 3 asserts that the firm should serve only segments with high valuation (i.e., $TS^{(2)} = \{\text{LH, HH}\}$) when the low valuation is sufficiently low (i.e., $V_L \le \beta V_H$). Otherwise, the firm should lower the retail price to entice reservations from all four market segments. Notice that Proposition 3 is consistent with the optimal target segment as stated in statement 1 of Proposition 2 for the case when customer demand is deterministic and when $\psi_L V_H \ge \psi_H V_L$. However, unlike statement 2 of Proposition 2, Proposition 3 reveals that when customer demand is stochastic, serving target segment $TS^{(4)} = \{\text{LH, HL, HH}\}$ is never optimal.

As $m \to \infty$, Proposition 3 reveals that it is optimal for the firm to charge zero deposit and the highest retail price that the target segment can bare. Next, observe from both statements in Proposition 3 that, when the firm's capacity is sufficiently large, it is optimal for the firm to serve segments that involve both high and low show up probabilities. These observations and Proposition 2 enable us to form the following conclusion: it is never optimal to discriminate against customers with low show up probability when the firm's capacity is sufficiently large.

We now determine the optimal target segment t^* and the corresponding optimal policy (r^*, d^*, n^*) numerically for the case when m takes on finite and small values. In our numerical experiments, we vary the ratio of show up probabilities ψ_L/ψ_H and the ratio of valuations V_L/V_H from 0.5 to 1 (where we fix $\psi_H = 0.95$ and $V_H = 100$), and we set $\lambda = 10$, $c = 100$, $\alpha = 0.3$, and $\beta = 0.6$. First, to examine the impact of the capacity m on the optimal target segment and the corresponding optimal reservation policy, we vary m from 1 to 20. For illustrative purposes, we report our results for the case when $m = 3$ in Fig. 1(a) and for the case when $m = 6$ in Fig. 1(b).

When the capacity m is sufficiently large, Proposition 3 implies that the firm should charge no deposit, serve target segment {LH, HH} when $V_L/V_H \le \beta$ and target segment {LL, HL, LH, HH} when $V_L/V_H > \beta$. In our numerical experiments, Proposition 3 continues to hold when $m \gtrsim \lambda = 10$. However, when the capacity m is small; i.e., when $m < 10$, Fig. 1 suggests that, depending on the ratios ψ_L/ψ_H and V_L/V_H, different target segments and different reservation policies can be optimal.

To begin, consider Fig. 1(b) that deals with the case when $m = 6$. Observe from Fig. 1(b) that the underlying structure of the optimal policy as stated in Proposition 3 continues to hold: the firm should charge no deposit, serve target segment {LH, HH} when V_L/V_H is sufficiently small, and serve target segment {LL, HL, LH, HH} when V_L/V_H is sufficiently large. However, when V_L/V_H is medium, it is intuitive to expect the firm to focus on target segment {HL, LH, HH} that is "between" {LH, HH} and {LL, HL, LH, HH}. It is worth noting that the wedge in which the optimal target segment is {HL, LH, HH} shrinks as m increases. There is a less intuitive result we would like to explain. Consider the case when the firm "expands" its target segment from {LH, HH} to {HL, LH, HH} by "admitting" an additional segment {HL} with low valuation. To do so, the firm needs to reduce its retail price. However, due to the presence of segment LH that involves customers with low show up probability, the firm should charge either full or partial deposit so as to hedge against the risk of losing revenue due to no shows. This observation, as shown in Fig. 1(b), is consistent with the result as stated in Corollary 1 that calls for deposit as the firm's capacity is below a certain threshold.

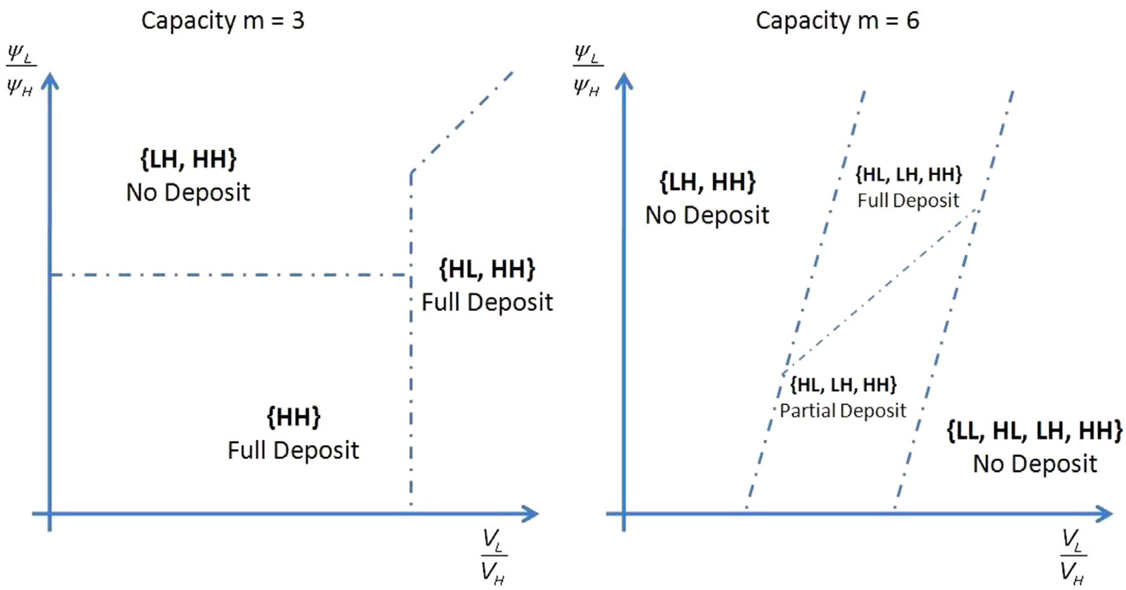

Fig. 1. Optimal target segments and optimal reservation policies.

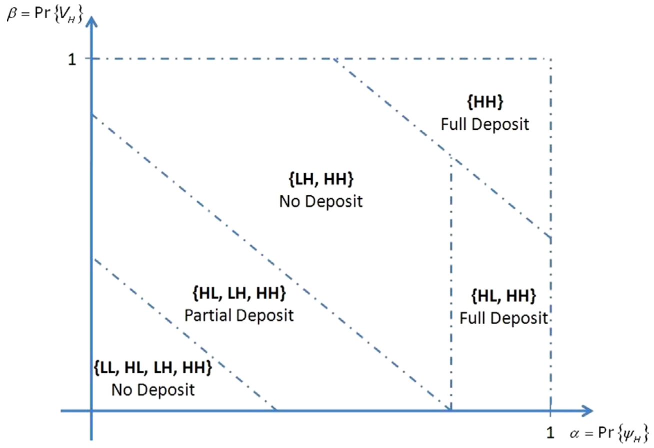

Fig. 2. Optimal target segments and optimal reservation policies when $m=5$.

Next, consider Fig. 1(a) that deals with the case when $m=3$. When ψ_L/ψ_H and V_L/V_H are small and when the firm has very limited capacity, segment HH is appealing because its customer has sufficiently higher valuation and higher show-up probability. Hence, it is intuitive that the firm is better off by focusing on one segment HH by charging a high retail price $\psi_H V_H$ with full deposit (as stated in the first statement of Theorem 1). Next, when V_L/V_H is sufficiently large so that $V_L \approx V_H$, the firm should focus on only customers with high show-up probability by capturing segment HL in addition to segment HH by charging $\psi_H V_L$ with full deposit (as stated in the second statement of Theorem 1). Finally, when ψ_L/ψ_H is large so that $\psi_L \approx \psi_H$ and V_L/V_H is small, it is intuitive that the firm should focus on customers with high valuation by expanding its target segment from {HH} to include segment {LH}. Also, to extract more surplus from customers with high show up probability relative to customers with low show up probability, we can use the same argument to explain Corollary 1 to show that it is optimal for the firm to charge a high retail price V_H with no deposit. This observation is also consistent with the first statement stated in Proposition 2.

Next, to investigate the effect of the size of each market segment (captured by α and β, where $\alpha \triangleq \Pr\{\psi_H\}$ and $\beta \triangleq \Pr\{V_H\}$) on the optimal target segment t^* and the corresponding optimal policy (r^*, d^*), we vary the capacity m from 1 to 20 and we vary the probabilities α and β from 0 to 1. In our numerical experiments, we set $\lambda = 10$, $c=100$, $\psi_H = 0.9$, $\psi_L = 0.7$, $V_H = 100$, $V_L = 75$. Fig. 2 depicts the result for the case when $m=5$.

First, consider the case when $\alpha \triangleq \Pr\{\psi_H\}$ and $\beta \triangleq \Pr\{V_H\}$ are small so that the size of the high valuation segment and the size of the high show up probability segment are small. In this case, the firm needs to expand its target segment by serving all four segments to ensure sufficient capacity utilization. However, to serve all four segments without lowering the price too much, statement 5 of Theorem 1 and statement 3 of Corollary 1 suggest that the firm should charge the highest possible price without imposing any deposit (i.e., $r^* = V_L$ and $d^* = 0$). This intuition is verified in the lower left hand corner of Fig. 2. Next, when α is sufficiently large as depicted in the right hand side of Fig. 2, the firm can afford to focus on the segments with high show up probabilities only (i.e., {HH} or {HL, HH}) and charge full deposit as stated in Theorem 1. By using the same logic, we can use Theorem 1 and Corollary 1 to explain the intuition behind the results associated with the case when α and β are medium so that {HL, LH, HH} is the optimal target segment, and when α is medium and β is large, so that {LH, HH} is the optimal target segment.

5. Multiple option reservation policies

In Section 4 we have examined a single-option reservation policy $P=(r,d,n)$ that "supports" the target segment $TS^{(t)}$ and discriminates against customers who do not belong to the target segment. As a way to further differentiate customer classes and to extract additional surplus from the different customer classes within the target segment, the firm may consider offering multiple reservation options. Multiple reservation options can be mutually beneficial, because they offer customers more flexibility and they enable the firm to extract more surplus. For instance, it is common practice in the hotel industry to offer two different reservation options: (1) a lower rate with full deposit (i.e., non-cancellable reservation with non-refundable deposit) and (2) a higher rate with no deposit required. This two-option policy can segment customers with different show-up probabilities because customers with high (low) show-up probability would prefer option (1) (option (2)).

By considering the case when the firm offers multiple reservation options, we are interested in examining the following questions: (i) How many options should the firm offer? (ii) How would a multiple-option policy segment the target segment $TS^{(t)}$? In preparation, let us define a multi-option policy as *non-redundant* if each reservation option is preferred by at least one market segment. In other words, a multi-option policy is *redundant* when it can be replaced by another policy with fewer options. Hence, in a market with four segments, the firm does not need to consider any policy with more than four reservation options. Upon closer examination, it is sufficient for the firm to consider no more than two options.

Lemma 4. *In a market with four segments (i.e.,* HH, HL, LH, LL*), it is optimal for the firm to offer no more than two non-redundant reservation options. Also, any optimal two-option policy has the following property: high show up probability customers will reserve under one option, and low show up probability customers will reserve under the other option.*

In view of the fact that we have examined single-option reservation policies in Section 3, Lemma 4 suggests that it suffices to analyze two-option policies. In addition, by using the property that any non-redundant two-option policy is intended to differentiate customers with different show up probabilities, it suffices to consider target segments $TS^{(2)} = \{\text{LH, HH}\}$, $TS^{(4)} = \{\text{LH, HL, HH}\}$, and $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$. We now determine the optimal two-option policy $P = (r_1, d_1; r_2, d_2)$. For brevity and to avoid repetition, we shall present the analysis for the case when the target segment is $TS^{(2)} = \{\text{LH, HH}\}$.[12] Also, without loss of generality, we design the options so that option 1 (r_1, d_1) is preferred by segment HH and option 2 (r_2, d_2) is preferred by segment LH.

When offering multiple reservation options, the firm needs to (i) establish a rule for allocating its booking capacity n to reservations under different options (especially when the total number of reservation requests under different options exceeds the firm's booking capacity) and (ii) develop a rule for allocating its physical capacity m to customers who show up to claim their reservation under different options (especially when the total number of show ups under different options exceeds the firm's physical capacity). In our model formulation, we allow any arbitrary rule for allocating the firm's booking capacity to reservation requests under the two options. To do so, we let p_i and q_{k-i} denote the probabilities that the firm accepts i reservations under option 1, and $(k-i)$ reservations under option 2, respectively. However, we do need to consider an allocation rule for allocating physical capacity m to customers who show up. There are several ways to allocate the firm's physical capacity to customers who show up for their reserved service including (1) the firm prioritizes customers who reserved under one option over the other option[13] and (2) the firm allocates capacity m_1 and $m - m_1$ for customers who reserve under option 1 and option 2, respectively.[14] To illustrate, we shall focus our analysis on the allocation rule (1), and we shall show that when serving target segment $TS^{(2)} = \{\text{LH, HH}\}$, it is optimal for the firm to prioritize LH customers over HH customers.

For the case when reservations under option 2 are prioritized over option 1, one can use (4.1) to show that the firm's expected revenue under the two-option policy can be written as

$$\begin{aligned} \Pi((r_1, d_1), (r_2, d_2), n) = & \sum_{k=0}^{n} \sum_{i=0}^{k} p_i q_{k-i} \{ i(1-\psi_H) d_1 + (k-i)(1-\psi_L) d_2 \\ & + \sum_{j_1=0}^{i} \sum_{j_2=0}^{k-i} \Pr(j_1|i) \cdot \Pr(j_2|k-i) \cdot [-\max\{j_1 + j_2 - m, 0\} c \\ & + \min\{j_1, (m-j_2)^+\} r_1 + \min\{j_2, m\} r_2] \} \end{aligned} \tag{5.1}$$

where $(\bullet)^+ = \max\{\bullet, 0\}$, and j_1 and j_2 denote the number of customers from segment HH and LH who show up with reservations made under option 1 and option 2, respectively. Notice that $(j_1|i) \sim \text{Binomial}(i, \psi_H)$ and $(j_2|k-i) \sim \text{Binomial}(k-i, \psi_L)$. Also, observe that the prioritization of customers who show up with a reservation made under option 2 is formalized in (5.1) as follows: given j_2 customers who show up with option 2 reservations, the firm has sufficient capacity to serve $\min\{j_2, m\}$ of these customers. Then the firm has remaining capacity of $(m - j_2)^+$ units to serve customers who reserved under option 1. (Similarly, for the case when option 1 reservations are prioritized over option 2 reservations, one can write the corresponding expected revenue by replacing the last line in (5.1) by $\min\{j_1, m\} r_1 + \min\{j_2, (m - j_1)^+\} r_2$.)

By considering the property stated in Lemma 4, any *non-redundant* two-option policy $P = (r_1, d_1; r_2, d_2)$ that serves target segment $TS^{(2)} = \{\text{LH, HH}\}$ must satisfy the following incentive compatibility (IC) and individual rationality (IR) constraints:

$$\begin{array}{ll} \pi_{HH}(r_1, d_1) \geq \pi_{HH}(r_2, d_2) & (\text{IC}_{\text{HH}}) \\ \pi_{HH}(r_1, d_1) \geq 0 & (\text{IR}_{\text{HH}}) \\ \pi_{LH}(r_2, d_2) \geq \pi_{LH}(r_1, d_1) & (\text{IC}_{\text{LH}}) \\ \pi_{LH}(r_2, d_2) \geq 0 & (\text{IR}_{\text{LH}}) \\ \pi_{HL}(r_1, d_1) \leq 0, \quad \pi_{HL}(r_2, d_2) \leq 0 & (\text{IR}_{\text{HL}}) \\ \pi_{LL}(r_1, d_1) \leq 0, \quad \pi_{LL}(r_2, d_2) \leq 0 & (\text{IR}_{\text{LL}}) \end{array}$$

The above constraints ensure that HH customers would prefer to reserve under option 1, that LH customers prefer to reserve under option 2, and that LL and HL customers will not reserve. To avoid trivial cases and to ease our exposition, we shall assume throughout the remainder of this section that $\psi_L/\psi_H \geq V_L/V_H$. By considering the above constraints, we analyze the characteristics of the optimal reservation policies associated with two different prioritization schemes in the following Lemma, and we show that the firm can obtain a higher expected revenue when option 2 reservations are prioritized over option 1 reservations.

Lemma 5. *When serving target segment* $TS^{(2)} = \{\text{LH, HH}\}$ *and when* $\psi_L/\psi_H \geq V_L/V_H$, *the optimal two-option reservation policy* $P^* = (r_1^*, d_1^*; r_2^*, d_2^*)$ *has the following properties:*

1. *Constraints* (IC_{HH}) *and* (IR_{LH}) *are binding; i.e.,* $\pi_{HH}(r_1^*, d_1^*) = \pi_{HH}(r_2^*, d_2^*)$ *and* $\pi_{LH}(r_2^*, d_2^*) = 0$;
2. *For any retail prices*
$$(r_1, r_2), \; d_1^*(r_1, r_2) = \frac{\psi_L(V_H - r_2)}{1-\psi_L} + \frac{\psi_H(r_2 - r_1)}{1-\psi_H} \quad \text{and} \quad d_2^*(r_1, r_2) = \frac{\psi_L(V_H - r_2)}{1-\psi_L}$$
3. *The optimal retail prices* (r_1^*, r_2^*) *satisfy*
$$r_1^* \leq r_2^* \leq V_H \text{ and } \frac{\psi_L(1-\psi_H)V_H + (\psi_H - \psi_L) r_2^*}{(1-\psi_L)} \leq r_1^* \leq \frac{\psi_L(1-\psi_H)V_H + (\psi_H - \psi_L) r_2^*}{(1-\psi_L)\psi_H}.$$

[12] The optimal two-option policy for the other two target segments $TS^{(4)} = \{\text{LH, HL, HH}\}$ and $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$ can be determined in a similar manner. The details are omitted.

[13] Because we assume all customers show up simultaneously at the beginning of period 2, the firm can serve the customers who reserve under one option with higher priority. Then, if there is any leftover capacity, the firm can serve customers who reserve under the other option with lower priority.

[14] This allocation scheme is less desirable because it can lead to a situation where the firm incurs overbooking penalty for customers who reserved under one option, while having unused capacity for customers who reserved under the other option.

4. *It is optimal for the firm to prioritize customers from segment LH who made their reservations under option* 2 *over customers from segment HH who made their reservations under option* 1.

Lemma 5 reveals that constraints (IC_{HH}) and (IR_{LH}) are binding under the optimal two-option reservation policy. The underlying intuition behind this result can be explained as follows. To maximize its expected revenue, the firm attempts to extract as much surplus as possible from customers in the target segment. Because each customer can select the most favorable option to attempt to reserve, and because for any $0 \le d \le r$ such that customers in both HH and LH segments find it optimal to reserve, HH customers enjoy at least as great expected surplus as LH customers (i.e., $\pi_{HH}(r,d) \ge \pi_{LH}(r,d)$), the optimal policy is such that HH customers enjoy an expected surplus no greater than that of the alternative option (i.e., $\pi_{HH}(r_2,d_2)$), and LH customers enjoy an expected surplus no greater than that of the outside option (i.e., 0). By noting that the second statement in Lemma 5 resembles (4.4) and by using the same approach as presented in Section 3.3.1 for the single-option case, we can determine the optimal two-option reservation policy when the target segment is $TS^{(2)} = \{\text{LH, HH}\}$ as follows:

Proposition 4. *When the target segment* $TS^{(2)} = \{\text{LH, HH}\}$ *and when* $\psi_L/\psi_H \ge V_L/V_H$, *the optimal two-option reservation policy* P^* *satisfies*

$$P^* = (r_1^*, d_1^*; r_2^*, d_2^*, n^*) = \begin{cases} P^A & \text{if } \Pi(P^A) \ge \Pi(P^B), \\ P^B & \text{otherwise,} \end{cases}$$

where $P^A = (\psi_H V_H, \psi_H V_H; V_H, 0, n^*_{P^A})$, $P^B = (\psi_L V_H, \psi_L V_H; \psi_L V_H, \psi_L V_H, n^*_{P^B})$, *and* $n^*_{P^A}$ *and* $n^*_{P^B}$ *denote the corresponding optimal booking capacity.*[15]

Proposition 3 states that, for the target segment $TS^{(3)} = \{\text{LH, HH}\}$, the optimal two-option reservation policy takes on one of the following forms: (a) the first option (r_1^*, d_1^*), which is preferred by customers in segment HH, is a full deposit policy; and (b) the second option (r_2^*, d_2^*), which is preferred by customers in segment LH, is either a no deposit policy or a full deposit policy. By exploring the optimal two-option policies for target segments $TS^{(4)} = \{\text{LH, HL, HH}\}$ and $TS^{(5)} = \{\text{LL, LH, HL, HH}\}$, one finds that the reservation option preferred by customers with high show up probability always involves a (weakly) higher deposit than the option preferred by customers with low show up probability.

6. Discussion

We have presented a model of reservation policies that is intended to help a service firm with limited capacity maximize its expected revenue when operating in a heterogeneous market comprising of customers with different valuations and show up probabilities. By considering subproblems associated with different target segments, we have shown how the optimal reservation policy for any given target segment can be determined by solving a linear program. We have determined the optimal target segment analytically for the case when the firm's capacity is “large”, and numerically for any capacity level. Our analysis generates three main insights. First, contrary to popular belief (see Campell, 2009), when the firm's capacity is sufficiently large, discriminating customers with low show up probability is suboptimal. Instead, the firm should discriminate only against customers with low valuation. Second, given any target segment, the optimal price increases and the optimal deposit decreases in the firm's capacity. The intuition is as follows: as the firm's capacity increases, it can estimate the proportion of customers who will show up to claim the reservations more accurately, and overbook accordingly. Noting that the role of deposits is to discriminate against customers with low show-up probability, as the firm's capacity increases, this tool becomes less necessary, and the firm finds it optimal to reduce the deposit and raise the price. At the limit when the firm's capacity is sufficiently large, no deposit policies are always optimal, and the firm should target all customers if the low valuation is sufficiently high, and only customers with high valuation otherwise. Intuitively, this is because by the law of large numbers, the firm can precisely estimate the proportion of customers who show up to claim their reservation. Third, for any finite capacity level, when the firm targets only customers with high show up probability, full deposit policies are optimal. Otherwise, depending on the setting, no deposit, partial deposit, or full deposit policies may be optimal.

We have extended our base model to the case when the firm offers a multiple-option policy so as to extract more surplus. We have shown that, when serving a market with four customer classes, the firm does not need to offer more than two reservation options. For the case in which the firm targets customers with high valuation, we have shown that the optimal two-option policy has the following properties: (1) a full deposit option to attract customers with high show up probability and (2) a second option with a lower deposit and a higher retail price to attract customers with low show up probability. In view of the length of this paper, we have omitted the details of our extensions that deal with issues of cancellable reservations and walk-in customers. The reader is referred to an unpublished manuscript (Georgiadis and Tang, 2010) for details.

As an initial attempt to examine optimal reservation policies with non-refundable deposits for the case when a firm with limited capacity operating in an uncertain and heterogeneous market (i.e., customer demand, show-ups, customer valuation, and show-up probability), this paper opens up several opportunities for new research. Three limitations of the model presented in our paper are (i) the model consists of only two discrete periods, (ii) all parties are risk neutral, and (iii) the firm operates in a monopolistic environment. First, an interesting extension is to allow customers to arrive dynamically over time and allow the firm to implement dynamic pricing. Second, it would be of interest to study how the optimal policy will be affected if customers are risk-averse. Another interesting extension would study the impact of competition on the optimal reservation policy, and the role of special relationships between customers and firms (see Lim, 2009 for a related problem). For example, firms often provide customers with incentives to repeat purchase, such as frequent-flier

[15] For any given reservation allocation rule, the optimal booking capacity for two-option policies can be determined by modifying Proposition 1. We omit the details.

programs in the airline industry. An interesting research question that arises is how can firms use such programs to attract loyal customers in a competitive environment.

Appendix A. Proofs

Proof of Proposition 1. Recall that the result for target segment $TS^{(1)} = \{HH\}$ is shown in Section 4.1.1. This proof is organized in four parts.

Part I: $TS^{(2)} = \{LH, HH\}$:

The measure of customers who will attempt to reserve is now $A^{(2)} = \beta A$, and because a portion $\varpi^{(2)} = \alpha\psi_H + (1-\alpha)\psi_L$ of the reserved customers will actually show up, the firm will set its booking capacity to $n = m/\varpi^{(2)}$. As a result, the firm's expected profit will be

$$\Pi^{(2)} = [d + (r-d)\varpi^{(2)}] \min\left\{\beta A, \frac{m}{\varpi^{(2)}}\right\},$$

and it will choose (r, d) such that $\pi_{LH}(r,d) = 0$ and $\pi_{HL}(r,d) \le 0$. Hence (r, d) must satisfy

$$d = \frac{\psi_L}{1-\psi_L}(V_H - r) \ge \frac{\psi_H}{1-\psi_H}(V_L - r),$$

and substituting this into the firm's expected profit yields

$$\Pi^{(2)} = \frac{\psi_L(1-\varpi^{(2)})V_H + (\varpi^{(2)} - \psi_L)r}{1-\psi_L} \min\left\{\beta A, \frac{m}{\varpi^{(2)}}\right\}.$$

Observe that $\varpi^{(2)} > \psi_L$, which implies that $\Pi^{(3)}$ increases in r subject to

$$d = \frac{\psi_L}{1-\psi_L}(V_H - r) \ge \frac{\psi_H}{1-\psi_H}(V_L - r).$$

As a result setting $r = V_H$ and $d = 0$ is optimal. Substituting this into the firm's expected profit, we have

$$\Pi^{(2)} = V_H \min\{\beta[\alpha\psi_H + (1-\alpha)\psi_L]A, m\}.$$

Part II: $TS^{(3)} = \{HL, HH\}$:

The measure of customers who will attempt to reserve is $A^{(3)} = \alpha A$, and because a proportion ψ_H of the reserved customers will actually show up, the firm will set its booking capacity to $n = m/\psi_H$. As a result, the firm's expected profit will be

$$\Pi^{(3)} = [d + (r-d)\psi_H]\min\left\{\alpha A, \frac{m}{\psi_H}\right\},$$

and it will choose (r, d) such that $\pi_{HL}(r,d) = 0$ and $\pi_{LH}(r,d) \le 0$. Hence (r, d) must satisfy

$$d = \frac{\psi_H}{1-\psi_H}(V_L - r) \ge \frac{\psi_L}{1-\psi_L}(V_H - r),$$

and substituting this into the firm's expected profit yields

$$\Pi^{(3)} = V_L \min\{\alpha\psi_H A, m\},$$

and any

$$r \le \frac{\psi_H V_L - \psi_L V_H + \psi_L\psi_H(V_H - V_L)}{\psi_H - \psi_L} \quad \text{and} \quad d \ge \frac{\psi_L\psi_H(V_H - V_L)}{\psi_H - \psi_L}$$

satisfying $d = (\psi_H/(1-\psi_H))(V_L - r)$ is optimal.

Part III: $TS^{(4)} = \{LH, HL, HH\}$:

The measure of customers who will attempt to reserve is now $A^{(3)} = [\alpha + (1-\alpha)\beta]A$, and because a portion $\varpi^{(4)} = (\alpha\psi_H + (1-\alpha)\beta\psi_L)/(\alpha + (1-\alpha)\beta)$ of the reserved customers will actually show up, the firm will set its booking capacity to $n = m/\varpi^{(4)}$. As a result, the firm's expected profit will be

$$\Pi^{(4)} = [d + (r-d)\varpi^{(4)}]\min\left\{[\alpha + (1-\alpha)\beta]\,A, \frac{m}{\varpi^{(4)}}\right\}.$$

To proceed we must distinguish two cases:

- Case A: $\psi_L V_H \ge \psi_H V_L$

  In this case, the firm will choose (r, d) such that $\pi_{HL}(r,d) = 0$ and $\pi_{LH}(r,d) \ge 0$. Hence (r, d) must satisfy

$$d = \frac{\psi_H}{1-\psi_H}(V_L - r) \le \frac{\psi_L}{1-\psi_L}(V_H - r).$$

  By substituting this into the firm's expected profit we obtain

$$\Pi^{(4)} = \frac{\psi_H(1-\varpi^{(4)})V_L + (\varpi^{(4)} - \psi_H)r}{1-\psi_H} \min\left\{[\alpha + (1-\alpha)\beta]\,A, \frac{m}{\varpi^{(4)}}\right\}.$$

Observe that $\varpi^{(4)} < \psi_H$, which implies that $\Pi^{(4)}$ decreases in r subject to $d = (\psi_H/(1-\psi_H))(V_L - r)$, which implies that $r = d = \psi_H V_L$. Note that $\psi_H V_L < (\psi_L/(1-\psi_L))(V_H - r)$, so that $\pi_{LH}(r,d) \geq 0$. Substituting this into the firm's expected profit yields

$$\Pi^{(4)} = \psi_H V_L \min\{[\alpha\psi_H + (1-\alpha)\beta\psi_L]A, m\}.$$

- Case B: $\psi_L V_H < \psi_H V_L$

  In this case, the firm will choose $(r,\ d)$ such that $\pi_{LH}(r,d) = 0$ and $\pi_{HL}(r,d) \geq 0$. Hence $(r,\ d)$ must satisfy $d = (\psi_L/(1-\psi_L))(V_H - r) \leq (\psi_H/(1-\psi_H))(V_L - r)$. By substituting this into the firm's expected profit we obtain

  $$\Pi^{(4)} = \frac{\psi_L(1-\varpi^{(4)})V_H + (\varpi^{(4)} - \psi_L)r}{1-\psi_L} \min\left\{[\alpha + (1-\alpha)\beta]A, \frac{m}{\varpi^{(4)}}\right\}.$$

  Observe that $\varpi^{(4)} > \psi_L$, which implies that $\Pi^{(4)}$ increases in r subject to $d = (\psi_L/(1-\psi_L))(V_H - r) \leq (\psi_H/(1-\psi_H))(V_L - r)$. The latter inequality implies that $r \leq V_L - d(1-\psi_L)/\psi_L$, so that the optimal pair $(r,\ d)$ must satisfy

  $$r = \frac{\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L} \quad \text{and} \quad d = \frac{\psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}.$$

  Substituting this back into the firm's expected profit yields

  $$\Pi^{(4)} = \frac{\alpha\psi_H V_L + (1-\alpha)\beta\psi_L V_H}{\alpha\psi_H + (1-\alpha)\beta\psi_L} \min\{[\alpha\psi_H + (1-\alpha)\beta\psi_L]A, m\}.$$

Part IV: $\text{TS}^{(5)} = \{\text{LL, LH, HL, HH}\}$:

The measure of customers who will attempt to reserve is now $A^{(3)} = A$, and because a portion $\varpi^{(5)} = \alpha\psi_H + (1-\alpha)\psi_L$ of the reserved customers will actually show up, the firm will set its booking capacity to $n = m/\varpi^{(5)}$. As a result, the firm's expected profit will be

$$\Pi^{(5)} = [d + (r-d)\varpi^{(5)}] \min\left\{A, \frac{m}{\varpi^{(5)}}\right\},$$

and it will choose $(r,\ d)$ such that $\pi_{LL}(r,d) = 0$. Hence $(r,\ d)$ must satisfy $d = (\psi_L/(1-\psi_L))(V_L - r)$, and substituting this into the firm's expected profit yields

$$\Pi^{(5)} = \frac{\psi_L(1-\varpi^{(5)})V_L + (\varpi^{(5)} - \psi_L)r}{1-\psi_L} \min\left\{A, \frac{m}{\varpi^{(5)}}\right\}.$$

Observe that $\varpi^{(5)} > \psi_L$, which implies that $\Pi^{(5)}$ increases in r subject to $d = (\psi_L/(1-\psi_L))(V_L - r)$. As a result setting $r = V_L$ and $d=0$ is optimal. Substituting this into the firm's expected profit, we have

$$\Pi^{(5)} = V_L \min\{[\alpha\psi_H + (1-\alpha)\psi_L]\, A, m\}. \qquad \square$$

Proof of Proposition 2. For the purposes of this proof, we define $w = \alpha\psi_H + (1-\alpha)\psi_L$ and $\tilde{w} = \alpha\psi_H + (1-\alpha)\beta\psi_L$, and note that $w > \tilde{w} > \beta w$. The proof is organized in two parts.

Part I: $\psi_L V_H \geq \psi_H V_L$

There are three cases to consider: (i) $A/m \leq 1/w$, (ii) $1/\varpi < A/m \leq 1/\beta w$, and (iii) $A/m > 1/\beta w$, each of which is considered separately.

1. Case $(A/m \leq 1/w)$: Then $\beta w A/m < wA/m \leq 1$, so that $\Pi^{(2)} = V_H \beta w A$ and $\Pi^{(5)} = V_L w A$. Therefore {LH, HH} is the optimal target segment if and only if $V_L/V_H \leq \beta$.
2. Case $(1/\varpi < A/m \leq 1/\beta w)$: Then $\beta w A/m < 1 \leq wA/m$, so that $\Pi^{(2)} = V_H \beta w A$ and $\Pi^{(5)} = V_L$. Therefore {LH, HH} is the optimal target segment if and only if $V_L/V_H \leq \beta w A/m$.
3. Case $(A/m > 1/\beta w)$: Then $1 < \beta w A/m < wA/m$, so that $\Pi^{(2)} = V_H > V_L = \Pi^{(5)}$; i.e., {LH, HH} is always the optimal target segment.

Therefore, {LH, HH} is the optimal target segment if and only if $V_L/V_H \leq \beta \max\{\varpi A/m, 1\}$.

Part II: $\psi_L V_H < \psi_H V_L$

There are now four cases: (i) $A/m \leq 1/w$, (ii) $1/w < A/m \leq 1/\tilde{w}$, (iii) $1/\tilde{w} < A/m \leq 1/\beta w$, and (iv) $A/m > 1/\beta w$, each of which is considered separately.

1. Case $(A/m \leq 1/w)$: In this case, $\Pi^{(2)} = V_H \beta w A$, $\Pi^{(4)} = [\alpha\psi_H V_L + (1-\alpha)\beta\psi_L V_H]A$, and $\Pi^{(5)} = V_L w A$. It follows that $\Pi^{(2)} \geq \Pi^{(4)} \geq \Pi^{(5)}$ if and only if $V_L/V_H \leq \beta$. As a result, {LH, HH} is the optimal target segment if $V_L/V_H \leq \beta$, while {LL, LH, HL, HH} is the optimal target segment otherwise.
2. Case $(1/w < A/m \leq 1/\tilde{w})$: In this case, $\Pi^{(2)} = V_H \beta w A$, $\Pi^{(4)} = [\alpha\psi_H V_L + (1-\alpha)\beta\psi_L V_H]A$, and $\Pi^{(5)} = V_L m$. It follows that $\Pi^{(2)} \geq \Pi^{(4)}$ iff $V_L/V_H \leq \beta$, $\Pi^{(2)} \geq \Pi^{(5)}$ iff $V_L/V_H \leq \beta w A/m$, and $\Pi^{(4)} \geq \Pi^{(5)}$ iff $V_L/V_H \leq ((1-\alpha)\beta\psi_L A/m)/(1-\alpha\psi_H A/m)$. Moreover, it is easy to check that $((1-\alpha)\beta\psi_L A/m)/(1-\alpha\psi_H A/m) > \beta w A/m > \beta$. Therefore, if $V_L/V_H \leq \beta$, then {LH, HH} is the optimal target segment. If $\beta < V_L/V_H \leq ((1-\alpha)\beta\psi_L A/m)/(1-\alpha\psi_H A/m)$, then {LH, HL, HH} is the optimal target segment. Finally, if $V_L/V_H > ((1-\alpha)\ \beta\psi_L A/m)/(1-\alpha\psi_H A/m)$, then {LL, LH, HL, HH} is the optimal target segment.
3. Case $(1/\tilde{w} < A/m \leq 1/\beta w)$: In this case, $\Pi^{(2)} = V_H \beta w A$, $\Pi^{(4)} = ((\alpha\psi_H V_L + (1-\alpha)\beta\psi_L V_H)/(\alpha\psi_H + (1-\alpha)\beta\psi_L))m$, and $\Pi^{(5)} = V_L m$. It follows that $\Pi^{(4)} > \Pi^{(5)}$, which implies that {LL, LH, HL, HH} cannot be the optimal target segment. Carrying out the algebra yields that $\Pi^{(2)} \geq \Pi^{(4)}$, so that {LH, HH} is the optimal target segment if and only if $V_L/V_H \leq \beta[(w\tilde{w}A/m - (1-\alpha)\psi_L)/\alpha\psi_H]$. Otherwise, {LH, HL, HH} is the optimal target segment. Note that $1 \leq [(w\tilde{w}A/m - (1-\alpha)\psi_L)/\alpha\psi_H] < 1/\beta$.

4. Case $(A/m > 1/\beta w)$: In this case, $\Pi^{(2)} = V_H m$, $\Pi^{(4)} = ((\alpha\psi_H V_L + (1-\alpha)\beta\psi_L V_H)/(\alpha\psi_H + (1-\alpha)\beta\psi_L))m$, and $\Pi^{(5)} = V_L m$. It is easy to check that $\Pi^{(2)} > \Pi^{(4)} > \Pi^{(5)}$, which implies that {LH, HH} is the optimal target segment.

By noting that $((1-\alpha)\beta\psi_L A/m)/(1-\alpha\psi_H A/m) \le \beta$ if $A/m \le 1/w$, it follows that we can combine cases 1 and 2 to claim that if $A/m \le 1/\tilde{w}$, then {LH, HH} is the optimal target segment if $V_L/V_H \le \beta$, while {LH, HL, HH} is the optimal target segment if $\beta < V_L/V_H \le \max\{\beta, ((1-\alpha)\beta\psi_L A/m)/(1-\alpha\psi_H A/m)\}$, and {LL, LH, HL, HH} is the optimal target segment if $V_L/V_H > \max\{\beta, ((1-\alpha)\beta\psi_L A/m)/(1-\alpha\psi_H A/m)\}$.

By noting that $\beta[(w\tilde{w}A/m - (1-\alpha)\psi_L)/\alpha\psi_H] > 1$ if $A/m > 1/\beta w$, it follows that we can combine cases 3 and 4 to claim that if $A/m > 1/\tilde{w}$, then {LH, HH} is the optimal target segment if $\max\{1, \beta[(w\tilde{w}A/m - (1-\alpha)\psi_L)/\alpha\psi_H]\}$, while {LH, HL, HH} is the optimal target segment otherwise.

This completes the proof. □

Proof of Lemma 2. Letting the physical capacity $m \to \infty$ (and because $n \ge m$ it follows that $n \to \infty$), we have that

$$f(x; m, n \to \infty) \triangleq \sum_{k=0}^{\infty} p_k \left\{ \sum_{j=0}^{k} j \cdot \Pr\{j|k\} - k(1-\varpi)\frac{x}{1-x} \right\} = \sum_{k=0}^{\infty} p_k \left\{ k\varpi - k(1-\varpi)\frac{x}{1-x} \right\} = \lambda \frac{\varpi - x}{1-x}$$

where λ is the rate of the Poisson process described by $\{p_k\}_{k=0}^{\infty}$. As a result, $f(x; m, n \to \infty) \le 0$ iff $x \ge \varpi$. However observe that $\sum_{j=0}^{k} \min\{j, m\} \Pr(j|k)$ is strictly increasing in m for any $k \ge m$ and because $n \ge m$, the LHS of (4.7) is strictly increasing in m. As a result, it follows that for any finite capacity $m \in \mathbb{N}$, $\sum_{j=0}^{k} \min\{j, m\} \Pr(j|k) < k\varpi$. Then, the term in brackets in (4.7) is negative $\forall x \ge \varpi$. This completes the proof. □

Proof of Lemma 3. First, let us consider scenario (a). Because $\psi_L/\psi_H < V_L/V_H$, $\pi_{HH}(r,d) \ge \pi_{LH}(r,d) \ge \pi_{HL}(r,d) \ge 0 \ge \pi_{LL}(r,d)$ if and only if

$$r \ge d + \frac{\psi_H V_L - \psi_L V_H}{\psi_H - \psi_L} \tag{A.1}$$

Hence, we can use the same argument as presented in Section 3.3.1 to show that the optimal deposit d will satisfy (4.4) and (A.1). By substituting (4.4) into (A.1), and by using the same approach as presented in Section 3.3.1, it follows that the optimal retail price $r^*(a)$ is the solution to the following linear program:

$$\max_r \left\{ \sum_{k=0}^{n} p_k \left[\sum_{j=0}^{k} \min\{j, m\} \Pr(j|k) - k(1-\varpi)\frac{\psi_H}{1-\psi_H} \right] r : \frac{\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L} \le r \le V_L \right\}$$

Because $\psi_H \ge \varpi$, we apply Lemma 2 to the above linear program to show that the optimal retail price $r^*(a) = (\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L))/(\psi_H - \psi_L)$, and then apply (4.4) to show that the optimal deposit $d^*(a) = \psi_L \psi_H (V_H - V_L)/(\psi_H - \psi_L)$. Finally, we can use (4.2) to retrieve the corresponding optimal booking capacity $n^*_{(r^*(a), d^*(a))}$. By noting that $d^*(a) < r^*(a)$, we conclude that $P^*(a)$ is a "partial" deposit policy.

Now let us consider scenario (b). In this scenario, $\pi_{HH}(r,d) \ge \pi_{HL}(r,d) \ge \pi_{LH}(r,d) \ge 0 \ge \pi_{LL}(r,d)$ when $\psi_L/\psi_H < V_L/V_H$ and $r \le d + (\psi_H V_L - \psi_L V_H)/(\psi_H - \psi_L)$. Hence, it is optimal for the firm to select (r, d) to extract the entire surplus from segment LH so that $\pi_{LH}(r,d) = 0$, which implies that (r, d) must satisfy:

$$d = \frac{\psi_L (V_H - r)}{1 - \psi_L} \quad \text{and} \quad 0 \le r \le d + \frac{\psi_H V_L - \psi_L V_H}{\psi_H - \psi_L}. \tag{A.2}$$

By using (4.1) and (A.2), the problem of determining the optimal price $r^*(b)$ in scenario (b) reduces to the following linear program:

$$\max_r \left\{ \sum_{k=0}^{n} p_k \left[\sum_{j=0}^{k} \min\{j, m\} \Pr(j|k) - k(1-\varpi)\frac{\psi_L}{1-\psi_L} \right] r : \psi_L V_H \le r \le \frac{\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L} \right\}$$

Observe from (3.2) that $\psi_L \le \varpi^{(4)}$. Hence, we cannot apply Lemma 2 as before. However, it is easy to check from the above linear program that the optimal retail price lies on one of the boundary points. First, consider the case when $r^*(b) = (\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L))/(\psi_H - \psi_L)$, then the resulting partial deposit policy is the same as the optimal policy in scenario (a). Second, if $r^*(b) = \psi_L V_H$, then it follows from (A.2) that the optimal deposit $d^*(b) = \psi_L V_H$. Finally, the optimal booking capacity $n^*(l)$, $l \in \{a, b\}$ can be retrieved from (4.2). Because $d^*(b) = r^*(b)$, we can conclude that the optimal policy $P^*(a)$ for the former case is a "partial" deposit policy and the optimal policy $P^*(b)$ for the latter case is a "full" deposit policy, where

$$P^*(a) = \left(\frac{\psi_H V_L - \psi_L V_H + \psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, \frac{\psi_L \psi_H (V_H - V_L)}{\psi_H - \psi_L}, n^*_{(r^*(a), d^*(a))} \right)$$

and

$$P^*(b) = (\psi_L V_H, \psi_L V_H, n^*_{(r^*(b), d^*(b))})$$

Hence, in scenario (b), policy $P^*(a)$ is the optimal policy if $\Pi(P^*(a)) \ge \Pi(P^*(b))$ and policy $P^*(b)$ is the optimal policy, otherwise. This completes the proof. □

Proof of Corollary 1. We shall prove the result for the case when $t=2$. The proof is similar for the remaining cases (i.e., when $t=4, 5$). To facilitate exposition, we drop the superscript (t). Observe that the optimal booking capacity $n^* = m$ if the penalty $c \to \infty$, and $n^* \to \infty$ if $c=0$. Next, consider Lemma 2 and fix $x = \psi_L$ in (4.7).

First, suppose that $c \to \infty$ so that $n^* = m$. Then it is easy to check that $f(\psi_L) = \sum_{k=0}^{m} k p_k ((\varpi - \psi_L)/(1-\psi_L)) \ge 0$, because $\varpi \ge \psi_L$. As a result, $r^{(2)*} = V_H$ for any capacity m. Next, suppose that $c=0$, which implies that $n^* \to \infty$. Then (4.7) suggests that $f(\psi_L)$ is strictly increasing in m. As a result, there exists some threshold capacity $m_\tau^{(2)}$ such that $f(\psi_L) \ge 0$ if and only if $m \ge m_\tau^{(2)}(c)$. As a result, $r^{(2)*} = V_H$ if $m \ge m_\tau^{(2)}(c)$, and $r^{(2)*} = \psi_L V_H$ otherwise. For the case in which $c \to \infty$, the threshold $m_\tau^{(2)}(c) = 1$. This completes the proof. □

Proof of Proposition 3. By applying Theorem 1 for the case in which $m \to \infty$ and $m \le n \to \infty$, one obtains:

- For $TS^{(1)} = \{HH\}$); $\Pi^{(1)*} = \lambda\alpha\beta\psi_H V_H$, where $0 \le d^{(1)*} \le r^{(1)*}$ such that $(1-\psi_H)d^{(1)*} + \psi_H r^{(1)*} = \psi_H V_H$.
- For $TS^{(2)} = \{LH, HH\}$; $\Pi^{(2)*} = \lambda[\alpha\psi_H + (1-\alpha)\psi_L]\beta V_H$, where $r^{(2)*} = V_H$ and $d^{(2)*} = 0$.
- For $TS^{(3)} = \{HL, HH\}$; $\Pi^{(3)*} = \lambda\alpha\psi_H V_L$, where $0 \le d^{(3)*} \le r^{(3)*}$ such that $(1-\psi_H)d^{(3)*} + \psi_H r^{(3)*} = \psi_H V_L$.
- For $TS^{(4)} = \{LH, HL, HH\}$;
  - If $\psi_H/\psi_L < V_H/V_L$, then $\Pi^{(4a)*} = \lambda[\alpha + (1-\alpha)\beta]\psi_H V_L$, where $r^{(4a)*} = \psi_H V_L$ and $d^{(4a)*} = \psi_H V_L$.
  - If $\psi_H/\psi_L \ge V_H/V_L$, then $\Pi^{(4b)*} = \lambda[\alpha\psi_H V_L + (1-\alpha)\beta\psi_L V_H]$, where
  $$r^{(4b)*} = \frac{\psi_H V_L - \psi_L V_H + \psi_L\psi_H(V_H - V_L)}{\psi_H - \psi_L} \quad \text{and} \quad d^{(4b)*} = \frac{\psi_L\psi_H(V_H - V_L)}{\psi_H - \psi_L}.$$
- For $TS^{(5)} = \{LL, LH, HL, HH\}$; $\Pi^{(5)*} = \lambda[\alpha\psi_H + (1-\alpha)\psi_L]V_L$, where $r^{(5)*} = V_L$ and $d^{(5)*} = 0$.

Observe that $\Pi^{(2)*} > \Pi^{(1)*}$, $\Pi^{(4a)*} > \Pi^{(3)*}$, and $\Pi^{(4b)*} > \Pi^{(3)*}$, which establishes that neither {HL, HH}, nor {HH} can be the optimal target segment. Now let us examine the conditions under which the optimal target segment is {LH, HL, HH}. First suppose that $\psi_H/\psi_L < V_H/V_L$. Then it follows that $\Pi^{(4a)*} > \Pi^{(5)*} \Longleftrightarrow \psi_H/\psi_L > 1/\beta$ and $\Pi^{(4a)*} > \Pi^{(2)*} \Longleftrightarrow V_L - \beta V_H > (1-\alpha)\beta/\alpha \cdot (\psi_L V_H - \psi_H V_L)/\psi_H$. Because $\psi_L V_H - \psi_H V_L > 0$, a necessary condition for $\Pi^{(4a)*} > \Pi^{(2)*}$ is that $V_H/V_L < 1/\beta$, which implies that $\psi_H/\psi_L > 1/\beta > V_H/V_L$. However this contradicts the assumption that $\psi_H/\psi_L < V_H/V_L$, and hence {LH, HL, HH} cannot be the optimal target segment if $\psi_H/\psi_L < V_H/V_L$. Now suppose that $\psi_H/\psi_L \ge V_H/V_L$. Then it follows that $\Pi^{(4a)*} > \Pi^{(5)*} \Longleftrightarrow V_H/V_L > 1/\beta$ and $\Pi^{(4a)*} > \Pi^{(2)*} \Longleftrightarrow V_H/V_L < 1/\beta$, which again leads to a contradiction. By noting that $\Pi^{(2)*} \ge \Pi^{(5)*}$ if and only if $V_L/V_H \le \beta$, the proof is complete. □

Proof of Lemma 4. Consider any pair (r, d). By using (3.1) it follows that $\pi_{LH}(r,d) - \pi_{LL}(r,d) = \psi_L(V_H - V_L) > 0$ and $\pi_{HH}(r,d) - \pi_{HL}(r,d) = \psi_H(V_H - V_L) > 0$. Because the expected surplus of an HH (LH) customer is equal to the expected surplus of an HL (LL) customer plus a constant, it follows that given a set of reservation options $(r_1, d_1; \ldots; r_n, d_n)$, each HL customer will find it optimal to reserve under the same option as each HH customer; say (r_1, d_1) (or not reserve at all). Similarly, each LL customer will find it optimal to reserve under the same option as each LH customer; say (r_2, d_2) (or not reserve at all). As a result, (i) high show up probability customers are better off reserving under (r_1, d_1), while low show up probability customers are better off reserving under (r_2, d_2), and (ii) the remaining $n-2$ reservation options are redundant. □

Proof of Lemma 5. First, it is easy to check that (IR_{LH}) and (IR_{HH}) render (IR_{LL}) redundant. By using (3.1) one can re-write (IC_{HH}), (IR_{HH}), (IC_{LH}) and (IR_{LH}) as $(1-\psi_H)d_1 + \psi_H r_1 \le (1-\psi_H)d_2 + \psi_H r_2$, $d_1 \le \psi_H(V_H - r_1)/(1-\psi_H)$, $(1-\psi_L)d_2 + \psi_L r_2 \le (1-\psi_L)d_1 + \psi_L r_1$, and $d_2 \le \psi_L(V_H - r_2)/(1-\psi_L)$, respectively. As a result, (IC_{HH}) and (IR_{LH}) can be written as $(1-\psi_H)d_1 = (1-\psi_H)d_2 + \psi_H(r_2 - r_1) - \tau_1$ and $d_2 = \psi_L(V_H - r_2)/(1-\psi_L) - \tau_2$, where $\tau_1, \tau_2 \ge 0$ are decision variables. Let us first suppose that the firm prioritizes customers with option-2 reservations over customers with option-1 reservations. Then by using (5.1), the optimal two-option policy can be determined by solving the following linear program:

$$\max_{r_1, r_2, \tau_1, \tau_2} \sum_{k=0}^{n} \sum_{i=0}^{k} p_i q_{k-i} \left\{ i(1-\psi_H) \left[\frac{\psi_L(V_H - r_2)}{1-\psi_L} - \tau_2 + \frac{\psi_H(r_2 - r_1) - \tau_1}{1-\psi_H} \right] + (k-i)(1-\psi_L) \left[\frac{\psi_L(V_H - r_2)}{1-\psi_L} - \tau_2 \right] + \sum_{j_1=0}^{i} \sum_{j_2=0}^{k-i} \Pr(j_1|i) \cdot \Pr(j_2|k-i) \cdot \right.$$
$$\left. [\min\{j_1, (m-j_2)^+\} r_1 + \min\{j_2, m\} r_2 - \max\{j_1 + j_2 - m, 0\} c] \right\}$$

subject to (IR_{HH}), (IC_{LH}), (IR_{HL}), $\tau_1 \ge 0$, and $\tau_2 \ge 0$.

Also observe that the above linear program is separable in (τ_1, τ_2) and (r_1, r_2), and after omitting the constant terms the problem with respect to τ_1 and τ_2 can be written as

$$\max_{\tau_1, \tau_2} \left\{ \sum_{k=0}^{n} \sum_{i=0}^{k} p_i q_{k-i} \{ -i\tau_1 - [k(1-\psi_L) - i(\psi_H - \psi_L)]\tau_2 \} \text{ s.t. } \tau_1 \ge 0 \text{ and } \tau_2 \ge 0 \right\}$$

By noting that $k \ge i \ge 0$ and $(1-\psi_L) \ge (\psi_H - \psi_L)$, it follows that $\tau_1^* = \tau_2^* = 0$. As a result, (IC_{HH}) and (IR_{LH}) will bind. Because (IR_{LH}) binds and because $\psi_L/\psi_H \ge V_L/V_H$, ($IR_{HL}$) is rendered redundant.

By considering the case in which customers with option-1 reservations are prioritized over customers with option-2 reservations, it follows that $\tau_1^* = \tau_2^* = 0$. This proves (i) and (ii) for either prioritization rule that the firm chooses to use.

By using (IC_{LH}), (IR_{LH}) and $d_2 \le r_2$, we have that $r_1 \le r_2 \le V_H$. By using (IC_{HH}), (IR_{LH}) and $0 \le d_2 \le r_2$, we have that

$$\frac{\psi_L(1-\psi_H)V_H + (\psi_H - \psi_L)r_2}{(1-\psi_L)} \le r_1 \le \frac{\psi_L(1-\psi_H)V_H + (\psi_H - \psi_L)r_2}{(1-\psi_L)\psi_H}$$

This proves (iii).

By using (5.1) and $r_2 \ge r_1$ from (iii), observe that for any given booking capacity n, the firm can increase its expected revenue by prioritizing customers who show up with a reservation under option 2 over customers who show up with an option 1 reservation. This completes the proof. □

Proof of Proposition 4. For any booking capacity n, by using (5.1) and by substituting

$$d_1 = \frac{\psi_L(V_H - r_2)}{1-\psi_L} + \frac{\psi_H(r_2 - r_1)}{1-\psi_H} \quad \text{and} \quad d_2 = \frac{\psi_L(V_H - r_2)}{1-\psi_L},$$

the optimal two-option policy (r_1, d_1, r_2, d_2) can be determined by solving the following linear program:

$$\max \sum_{k=0}^{n} \sum_{i=0}^{k} p_i q_{k-i} \left\{ i\left[\left(\frac{1-\psi_H}{1-\psi_L} \right) \psi_L(V_H - r_2) + \psi_H(r_2 - r_1) \right] + (k-i)[\psi_L(V_H - r_2)] + \sum_{j_1=0}^{i} \sum_{j_2=0}^{k-i} \Pr(j_1|i) \cdot \Pr(j_2|k-i)[\min\{j_1, (m-j_2)^+\} r_1 + \min\{j_2, m\} r_2] \right\} \tag{A.3}$$

subject to

$$r_1 \le r_2 \le V_H \text{ and } \frac{\psi_L(1-\psi_H)V_H + (\psi_H - \psi_L)r_2}{(1-\psi_L)} \le r_1 \le \frac{\psi_L(1-\psi_H)V_H + (\psi_H - \psi_L)r_2}{(1-\psi_L)\psi_H}$$

By re-arranging terms in the objective function one can re-write (A.3) as

$$\max \sum_{k=0}^{n} \sum_{i=0}^{k} p_i q_{k-i} \left\{ \left[\sum_{j_1=0}^{i} \sum_{j_2=0}^{k-i} \Pr(j_1|i) \cdot \Pr(j_2|k-i) \min\{j_1, (m-j_2)^+\} - i\psi_H \right] r_1 + \left[\sum_{j_2=0}^{k-i} \Pr(j_2|k-i) \min\{j_2, m\} - (k-i)\psi_L + i\left(\frac{\psi_H - \psi_L}{1-\psi_L} \right) \right] r_2 \right\}$$

By using that $\min\{j_1, m\}$ increases in m, it follows that $\forall j_1$, $\sum_{j_2=0}^{k-i} \Pr(j_2|k-i) \min\{j_1, (m-j_2)^+\} \le \min\{j_1, m\}$. Hence by applying Lemma 2 it follows that $\sum_{j_1=0}^{i} \sum_{j_2=0}^{k-i} \Pr(j_1|i)\Pr(j_2|k-i) \min\{j_1, (m-j_2)^+\} - i\psi_H < 0 \quad \forall 0 \le i \le k$. Consequently $r_1^*(r_2) = (\psi_L(1-\psi_H)V_H + (\psi_H - \psi_L)r_2)/(1-\psi_L)$ and (A.3) can be re-written as a single-variable linear program subject to $r_1^*(r_2) \le r_2 \le V_H$. By noting that the solution r_2^* is on the boundary of the feasibility set, it follows that $r_2^* \in \{r_1^*, V_H\}$.

By using statement (ii) in Lemma 4 and $r_1^* = (\psi_L(1-\psi_H)V_H + (\psi_H - \psi_L)r_2^*)/(1-\psi_L)$, we have that

$$\text{if } r_2^* = \begin{cases} r_1^* & \text{then } (r_1^*, d_1^*) = (r_2^*, d_2^*) = (\psi_L V_H, \psi_L V_H) \\ V_H & \text{then } (r_1^*, d_1^*) = (\psi_H V_H, \psi_H V_H) \text{ and } (r_2^*, d_2^*) = (V_H, 0) \end{cases}$$

This completes the proof. □

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