The Review of Economic Studies Advance Access published October 6, 2014

Review of Economic Studies (2014) **0**, 1–32 doi:10.1093/restud/rdu031

Projects and Team Dynamics

GEORGE GEORGIADIS

Boston University and California Institute of Technology

First version received November 2011*; final version accepted June* 2014 (*Eds.*)

I study a dynamic problem in which a group of agents collaborate over time to complete a project. The project progresses at a rate that depends on the agents' efforts, and it generates a pay-off upon completion. I show that agents work harder the closer the project is to completion, and members of a larger team work harder than members of a smaller team—both individually and on aggregate—if and only if the project is sufficiently far from completion. I apply these results to determine the optimal size of a self-organized partnership, and to study the manager's problem who recruits agents to carry out a project, and must determine the team size and its members' incentive contracts. The main results are: (i) that the optimal symmetric contract compensates the agents only upon completing the project; and (ii) the optimal team size decreases in the expected length of the project.

Key words: Projects, Moral hazard in teams, Team formation, Partnerships, Differential games

JEL Codes: D7, H4, L22, M5

1. INTRODUCTION

Teamwork and projects are central in the organization of firms and partnerships. Most large corporations engage a substantial proportion of their workforce in teamwork (Lawler *et al.*, 2001), and organizing workers into teams has been shown to increase productivity in both manufacturing and service firms (Ichniowski and Shaw, 2003). Moreover, the use of teams is especially common in situations in which the task at hand will result in a defined deliverable, and it will not be ongoing, but will terminate (Harvard Business School Press, 2004). Motivated by these observations, I analyse a dynamic problem in which a group of agents collaborate over time to complete a project, and I address a number of questions that naturally arise in this environment. In particular, what is the effect of the group size to the agents' incentives? How should a manager determine the team size and the agents' incentive contracts? For example, should they be rewarded for reaching intermediate milestones, and should rewards be equal across the agents?

I propose a continuous-time model, in which at every moment, each of n agents exerts costly effort to bring the project closer to completion. The project progresses stochastically at a rate that is equal to the sum of the agents' effort levels (*i.e.* efforts are substitutes), and it is completed when its state hits a pre-specified threshold, at which point each agent receives a lump sum pay-off and the game ends.

This model can be applied both within firms, for instance, to research teams in new product development or consulting projects, and across firms, for instance, to R&D joint ventures. More broadly, the model is applicable to settings in which a group of agents collaborate to complete a project, which progresses gradually, its expected duration is sufficiently large such that the agents discounting time matters, and it generates a pay-off upon completion. A natural example is the Myerlin Repair Foundation (MRF): a collaborative effort among a group of leading scientists in quest of a treatment for multiple sclerosis (Lakhani and Carlile, 2012). This is a long-term venture, progress is gradual, each principal investigator incurs an opportunity cost by allocating resources to MRF activities (which gives rise to incentives to free-ride), and it will pay off predominantly when an acceptable treatment is discovered.

In Section 3, I characterize the Markov perfect equilibrium (MPE) of this game, wherein at every moment, each agent observes the state of the project (*i.e.* how close it is to completion), and chooses his effort level to maximize his expected discounted pay-off, while anticipating the strategies of the other agents. A key result is that each agent increases his effort as the project progresses. Intuitively, because he discounts time and is compensated upon completion, his incentives are stronger the closer the project is to completion. An implication of this result is that efforts are strategic complements across time, in that a higher effort level by one agent at time t brings the project (on expectation) closer to completion, which in turn incentivizes himself, as well as the other agents to raise their future efforts.

In Section 4, I examine the effect of the team size to the agents' incentives. I show that members of a larger team work harder than members of a smaller team—both individually and on aggregate—if and only if the project is sufficiently far from completion.[1] Intuitively, by increasing the size of the team, two forces influence the agents' incentives. First, they obtain stronger incentives to free-ride. However, because the total progress that needs to be carried out is fixed, the agents benefit from the ability to complete the project quicker, which increases the present discounted value of their reward, and consequently strengthens their incentives. I refer to these forces as the *free-riding* and the *encouragement effect*, respectively. Because the marginal cost of effort is increasing and agents work harder the closer the project is to completion, the free-riding effect becomes stronger as the project progresses. On the other hand, the benefit of being able to complete the project faster in a bigger team is smaller the less progress remains, and hence the encouragement effect becomes weaker with progress. As a result, the encouragement effect dominates the free-riding effect, and consequently members of a larger team work harder than those of a smaller team if and only if the project is sufficiently far from completion.

I first apply this result to the problem faced by a group of agents organizing into a partnership. If the project is a public good so that each agent's reward is independent of the team size, then each agent is better off expanding the partnership *ad infinitum*. However, if the project generates a fixed pay-off upon completion that is shared among the team members, then the optimal partnership size increases in the length of the project.[2]

Motivated by the fact that projects are often run by corporations (rather than self-organized partnerships), in Section 5, I introduce a manager who is the residual claimant of the project, and he/she recruits a group of agents to undertake it on his/her behalf. His/Her objective is to determine the size of the team and each agent's incentive contract to maximize his/her expected discounted profit.

First, I show that the optimal symmetric contract compensates the agents only upon completion of the project. The intuition is that by backloading payments (compared to rewarding the agents for reaching intermediate milestones), the manager can provide the same incentives at the early stages of the project (via continuation utility), while providing stronger incentives when the project is close to completion. This result simplifies the manager's problem to determining the team size and his/her budget for compensating the agents. Given a fixed team size, I show that the manager's optimal budget increases in the length of the project. This is intuitive: to incentivize

1. This result holds both if the project is a public good so that each agent's reward is independent of the team size, and if the project generates a fixed pay-off that is shared among the team members so that doubling the team size halves each agent's reward.

2. The length of the project refers to the expected amount of progress necessary to complete it (given a fixed pay-off).

the agents, the manager should compensate them more, the longer the project. Moreover, the optimal team size increases in the length of the project. Recall that a larger team works harder than a smaller one if the project is sufficiently far from completion. Therefore, the benefit from a larger team working harder while the project is far from completion outweighs the loss from working less when it is close to completion only if the project is sufficiently long. Lastly, I show that the manager can benefit from dynamically decreasing the size of the team as the project nears completion. The intuition is that he/she prefers a larger team while the project is far from completion since it works harder than a smaller one, while a smaller team becomes preferable near completion.

The restriction to symmetric contracts in not without loss of generality. In particular, the scheme wherein the size of the team decreases dynamically as the project progresses can be implemented with an asymmetric contract that rewards the agents upon reaching different milestones. Finally, with two (identical) agents, I show that the manager is better off compensating them asymmetrically if the project is sufficiently short. Intuitively, the agent who receives the larger reward will carry out the larger share of the work in equilibrium, and hence she/he cannot free-ride on the other agent as much.

First and foremost, this article is related to the moral hazard in teams literature (Holmström, 1982; Ma *et al*., 1988; Bagnoli and Lipman, 1989; Legros and Matthews, 1993; Strausz, 1999, and others). These papers focus on the free-rider problem that arises when each agent must share the output of his/her effort with the other members of the team, and they explore ways to restore efficiency. My article ties in with this literature in that it analyzes a dynamic game of moral hazard in teams with stochastic output.

Closely related to this article is the literature on dynamic contribution games, and in particular, the papers that study *threshold* or *discrete* public good games. Formalizing the intuition of Schelling (1960), Admati and Perry (1991), and Marx and Matthews (2000) show that contributing little by little over multiple periods, each conditional on the previous contributions of the other agents, mitigates the free-rider problem. Lockwood and Thomas (2002) and Compte and Jehiel (2004) show how gradualism can arise in dynamic contribution games, while Battaglini, Nunnari and Palfrey (2013) compare the set of equilibrium outcomes when contributions are reversible to the case in which they are not. Whereas these papers focus on characterizing the equilibria of dynamic contribution games, my primary focus is on the organizational questions that arise in the context of such games.

Yildirim (2006) studies a game in which the project comprises of multiple discrete stages, and in every period, the current stage is completed if at least one agent exerts effort. Effort is binary, and each agent's effort cost is private information, and re-drawn from a common distribution in each period. In contrast, in my model, following Kessing (2007), the project progresses at a rate that depends smoothly on the team's aggregate effort. Yildirim (2006) and Kessing (2007) show that if the project generates a pay-off only upon completion, then contributions are strategic complements across time even if there are no complementarities in the agents' production function. This is in contrast to models in which the agents receive flow pay-offs while the project is in progress (Fershtman and Nitzan, 1991), and models in which the project can be completed instantaneously (Bonatti and Hörner, 2011), where contributions are strategic substitutes. Yildirim also examines how the team size influences the agents' incentives in a dynamic environment, and he shows that members of a larger team work harder than those of a smaller team at the early stages of the project, while the opposite is true at its later stages.[3] This result is similar to Theorem 2(i) in this article. However, leveraging the tractability of my

3. It is worth pointing out, however, that in Yildirim's model, this result hinges on the assumption that in every period, each agent's effort cost is re-drawn from a non-degenerate distribution. In contrast, if effort costs are deterministic,

model, I also characterize the relationship between aggregate effort and the team size, which is the crucial metric for determining the manager's optimal team size.

In summary, my contributions to this literature are 2-fold. First, I propose a natural framework to analyse the dynamic problem faced by a group of agents who collaborate over time to complete a project. The model provides several testable implications, and the framework proposed in this article can be useful for studying other dynamic moral hazard problems with multiple agents; for example, the joint extraction of an exhaustible common resource, or a tug of war between two teams (in the spirit of Cao, 2014), or a game of oligopolistic competition with demand that is correlated across time (as in Section IV of Sannikov and Skrzypacz, 2007). Moreover, in an earlier version of this article, I also analyse the cases in which the agents are asymmetric and the project size is endogenous (Georgiadis, 2011). Secondly, I derive insights for the organization of partnerships, and for team design where a manager must determine the size of his/her team and the agents' incentive contracts. To the best of my knowledge, this is one of the first papers to study this problem; one notable exception being Rahmani *et al.* (2013), who study the contractual relationship between the members of a two-person team.

This paper is also related to the literature on free-riding in groups. To explain why teamwork often leads to increased productivity in organizations in spite of the theoretical predictions that effort and group size should be inversely related (Olson, 1965; Andreoni, 1988), scholars have argued that teams benefit from mutual monitoring (Alchian and Demsetz, 1972), peer pressure to achieve a group norm (Kandel and Lazear, 1992), complementary skills (Lazear, 1998), warm-glow (Andreoni, 1990), and non-pecuniary benefits such as more engaging work and social interaction. While these forces are helpful for explaining the benefits of teamwork, this paper shows that they are actually not necessary in settings in which the team's efforts are geared towards completing a project.

Lastly, the existence proofs of Theorems 1 and 3 are based on Hartman (1960), while the proof techniques for the comparative statics draw from Cao (2014), who studies a continuous-time version of the patent race of Harris and Vickers (1985).

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the MPE of the game, and establishes some basic results. Section 4 examines how the size of the team influences the agents' incentives, and characterizes the optimal partnership size. Section 5 studies the manager's problem, and Section 6 concludes. Appendix A contains a discussion of non-Markovian strategies and four extensions of the base model. The major proofs are provided in Appendix B, while the omitted proofs are available in the online Appendix.

2. THE MODEL

A team of n agents collaborate to complete a project. Time $t \in [0,\infty)$ is continuous. The project starts at some initial state $q_0 < 0$, its state q_t evolves according to a stochastic process, and it is completed at the first time τ such that q_t hits the completion state which is normalized to 0. Agent $i \in \{1,\dots,n\}$ is risk neutral, discounts time at rate $r > 0$, and receives a pre-specified reward $V_i > 0$ upon completing the project.[4] An incomplete project has zero value. At every moment t, each

then this comparative static is reversed: the game becomes a dynamic version of the "reporting a crime" game (ch. 4.8 in Osborne, 2003), and one can show that in the unique symmetric, mixed-strategy MPE, both the probability that each agent exerts effort, and the probability that at least one agent exerts effort at any given stage of the project (which is the metric for individual and aggregate effort, respectively) decreases in the team size.

4. In the base model, the project generates a pay-off only upon completion. The case in which the project also generates a flow pay-off while it is in progress is examined in Appendix A.1, and it is shown that the main results continue to hold.

agent observes the state of the project q_t, and privately chooses his/her effort level to influence the drift of the stochastic process

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t}\right) dt + \sigma dW_t,$$

where $a_{i,t} \geq 0$ denotes the effort level of agent i at time t, $\sigma > 0$ captures the degree of uncertainty associated with the evolution of the project, and W_t is a standard Brownian motion.[5,6] As such, $|q_0|$ can be interpreted as the expected length of the project.[7] Finally, each agent is credit constrained, his effort choices are not observable to the other agents, and his flow cost of exerting effort a is given by $c(a) = \frac{a^{p+1}}{p+1}$, where $p \geq 1$.[8]

At every moment t, each agent i observes the state of the project q_t, and chooses his/her effort level $a_{i,t}$ to maximize his/her expected discounted pay-off while taking into account the effort choices $a_{-i,s}$ of the other team members. As such, for a given set of strategies, his/her expected discounted pay-off is given by

$$J_i(q_t) = \mathbb{E}_\tau \left[e^{-r(\tau - t)} V_i - \int_t^\tau e^{-r(s-t)} c\left(a_{i,s}\right) ds \right], \tag{1}$$

where the expectation is taken with respect to τ: the random variable that denotes the completion time of the project.

Assuming that $J_i(\cdot)$ is twice differentiable for all i, and using standard arguments (Dixit, 1999), one can derive the Hamilton–Jacobi–Bellman (hereafter HJB) equation for the expected discounted pay-off function of agent i:

$$rJ_i(q) = -c\left(a_{i,t}\right) + \left(\sum_{j=1}^{n} a_{j,t}\right) J_i'(q) + \frac{\sigma^2}{2} J_i''(q) \tag{2}$$

defined on $(-\infty, 0]$ subject to the boundary conditions

$$\lim_{q \to -\infty} J_i(q) = 0 \quad \text{and} \quad J_i(0) = V_i. \tag{3}$$

Equation (2) asserts that agent i's flow pay-off is equal to his/her flow cost of effort, plus his marginal benefit from bringing the project closer to completion times the aggregate effort of the team, plus a term that captures the sensitivity of his/her pay-off to the volatility of the project.

5. For simplicity, I assume that the variance of the stochastic process (*i.e.* σ) does not depend on the agents' effort levels. While the case in which effort influences both the drift and the diffusion of the stochastic process is intractable, numerical examples with $dq_t = \left(\sum_{i=1}^{n} a_{i,t}\right) dt + \sigma \left(\sum_{i=1}^{n} a_{i,t}\right)^{1/2} dW_t$ suggest that the main results continue to hold. See Appendix A.3 for details.

6. I assume that efforts are perfect substitutes. To capture the notion that when working in teams, agents may be more (less) productive due to complementary skills (coordination costs), one can consider a super- (sub-) additive production function such as $dq_t = \left(\sum_{i=1}^{n} a_{i,t}^{1/\gamma}\right)^{\gamma} dt + \sigma dW_t$, where $\gamma > 1$ $(0 < \gamma < 1)$. The main results continue to hold.

7. Because the project progresses stochastically, the total amount of effort to complete it may be greater or smaller than $|q_0|$.

8. The case in which $c(\cdot)$ is an arbitrary, strictly increasing, and convex function is discussed in Remark 1, while the case in which effort costs are linear is analysed in Appendix A.5 The restriction that $p \geq 1$ is necessary only for establishing that a MPE exists. If the conditions in Remark 1 are satisfied, then all results continue to hold for any $p > 0$.

To interpret equation (3), observe that as $q \to -\infty$, the expected time until the project is completed so that agent i collects his/her reward diverges to ∞, and because $r > 0$, his/her expected discounted pay-off asymptotes to 0. However, because he/she receives his/her reward and exerts no further effort after the project is completed, $J_i(0) = V_i$.

3. MARKOV PERFECT EQUILIBRIUM

I assume that strategies are Markovian, so that at every moment, each agent chooses his/her effort level as a function of the current state of the project.[9] Therefore, given q, agent i chooses his/her effort level $a_i(q)$ such that

$$a_i(q) \in \arg\max_{a_i \geq 0} \left\{ a_i J_i'(q) - c(a_i) \right\}.$$

Each agent chooses his/her effort level by trading off marginal benefit of bringing the project closer to completion and the marginal cost of effort. The former comprises of the direct benefit associated with the project being completed sooner, and the indirect benefit associated with influencing the other agents' future effort choices.[10] By noting that $c'(0) = 0$ and $c(\cdot)$ is strictly convex, it follows that for any given q, agent i's optimal effort level $a_i(q) = f\left(J_i'(q)\right)$, where $f(\cdot) = c'^{-1}(\max\{0, \cdot\})$. By substituting this into equation (2), the expected discounted pay-off for agent i satisfies

$$rJ_i(q) = -c\left(f\left(J_i'(q)\right)\right) + \left[\sum_{j=1}^{n} f\left(J_j'(q)\right)\right] J_i'(q) + \frac{\sigma^2}{2} J_i''(q) \tag{4}$$

subject to the boundary conditions (3).

An MPE is characterized by the system of ordinary differential equations (ODE) defined by equation (4) subject to the boundary conditions (3) for all $i \in \{1, \ldots, n\}$. To establish existence of a MPE, it suffices to show that a solution to this system exists. I then show that this system has a unique solution if the agents are symmetric (*i.e.*, $V_i = V_j$ for all $i \neq j$). Together with the facts that every MPE must satisfy this system and the first-order condition is both necessary and sufficient, it follows that the MPE is unique in this case.

Theorem 1. *An MPE for the game defined by equation (1) exists. For each agent i, the expected discounted pay-off function $J_i(q)$ satisfies:*

(i) $0 < J_i(q) \leq V_i$ *for all q.*
(ii) $J_i'(q) > 0$ *for all q, and hence the equilibrium effort* $a_i(q) > 0$ *for all q.*
(iii) $J_i''(q) > 0$ *for all q, and hence* $a_i'(q) > 0$ *for all q.*
(iv) If agents are symmetric (i.e. $V_i = V_j$ *for all* $i \neq j$*), then the MPE is symmetric and unique.*[11]

9. The possibility that the agents play non-Markovian strategies is discussed in Remark 5, in Section 3.2.

10. Because each agent's effort level is a function of q, his/her current effort level will impact his/her and the other agents' future effort levels.

11. To simplify notation, if the agents are symmetric, then the subscript i is interchanged with the subscript n to denote the team size throughout the remainder of this article.

$J_i'(q)>0$ implies that each agent is strictly better off, the closer the project is to completion. Because $c'(0)=0$ (*i.e.* the marginal cost of *little* effort is negligible), each agent exerts a strictly positive amount of effort at every state of the project: $a_i(q)>0$ for all q.[12]

Because the agents incur the cost of effort at the time effort is exerted but are only compensated upon completing the project, their incentives are stronger, the closer the project is to completion: $a_i'(q)>0$ for all q. An implication of this result is that efforts are strategic complements across time. That is because a higher effort by an agent at time t brings the project (on expectation) closer to completion, which in turn incentivizes himself/herself, as well as the other agents to raise their effort at times $t'>t$.

Note that Theorem 1 hinges on the assumption that $r>0$. If the agents are patient (*i.e.* $r=0$), then in equilibrium, each agent will always exert effort 0.[13] Therefore, this model is applicable to projects whose expected duration is sufficiently large such that the agents discounting time matters.

Remark 1. *For an MPE to exist, it suffices that $c(\cdot)$ is strictly increasing and convex with $c(0)=0$, it satisfies the INADA condition $\lim_{a\to\infty}c'(a)=\infty$, and $\frac{\sigma^2}{4}\int_0^\infty \frac{s ds}{r\sum_{i=1}^n V_i+nsf(s)}>\sum_{i=1}^n V_i$. If $c(a)=\frac{a^{p+1}}{p+1}$ and $p\geq 1$, then the LHS equals ∞, so that the inequality is always satisfied. However, if $p\in(0,1)$, then the inequality is satisfied only if $\sum_{i=1}^n V_i$, r and n are sufficiently small, or if σ is sufficiently large. More generally, this inequality is satisfied if $c(\cdot)$ is sufficiently convex.*

The existence proof requires that $J_i(\cdot)$ and $J_i'(\cdot)$ are always bounded. It is easy to show that $J_i(q)\in[0,V_i]$ and $J_i'(q)\geq 0$ for all i and q. The inequality in Remark 1 ensures that the marginal cost of effort $c'(a)$ is sufficiently large for large values of a that no agent ever has an incentive to exert an arbitrarily high effort, which by the first-order condition implies that $J_i'(\cdot)$ is bounded from above.

Remark 2. *An important assumption of the model is that the agents are compensated only upon completion of the project. In Appendix A.1, I consider the case in which during any interval $(t,t+dt)$ while the project is in progress, each agent receives a flow pay-off $h(q_t)dt$, in addition to the lump sum reward V upon completion. Assuming that $h(\cdot)$ is increasing and satisfies certain regularity conditions, there exists a threshold ω (not necessarily interior) such that $a_n'(q)\geq 0$ if and only if $q\leq\omega$;* i.e. *effort is hump-shaped in progress.*

The intuition why effort can decrease in q follows by noting that as the project nears completion, each agent's flow pay-off becomes larger, which in turn decreases his/her marginal benefit from bringing the project closer to completion. Numerical analysis indicates that this threshold is interior as long as the magnitude of the flow pay-offs is sufficiently large relative to V.

Remark 3. *The model assumes that the project is never "cancelled". If there is an exogenous* cancellation *state $Q_C<q_0<0$ such that the project is cancelled (and the agents receive pay-off 0) at the first time that q_t hits Q_C, then statements (i) and (ii) of Theorem 1 continue to hold,*

12. If $c'(0)>0$, then there exists a quitting threshold Q_q, such that each agent exerts 0 effort on $(-\infty,Q_q]$, while he/she exerts strictly positive effort on $(Q_q,0]$, and his/her effort increases in q.

13. If $\sigma=0$, because effort costs are convex and the agents do not discount time, in any equilibrium in which the project is completed, each agent finds it optimal to exert an arbitrarily small amount of effort over an arbitrarily large time horizon, and complete the project asymptotically. (A project-completing equilibrium exists only if $c'(0)$ is sufficiently close to 0.)

but effort needs no longer be increasing in q. Instead, there exists a threshold ω (not necessarily interior) such that $a_n'(q) \le 0$ if and only if $q \le \omega$; i.e. *effort is U-shaped in progress. See Appendix A.2 for details.*

Intuitively, the agents have incentives to exert effort (i) to complete the project, and (ii) to avoid hitting the cancellation state Q_C. Because the incentives due to the former (latter) are stronger the closer the project is to completion (to Q_C), depending on the choice of Q_C, the agent's incentives may be stronger near Q_C and near the completion state relative to the midpoint. Numerical analysis indicates that $\omega = 0$ so that effort increases monotonically in q if Q_C is sufficiently small; it is interior if Q_C is in some intermediate range, and $\omega = -\infty$ so that effort always decreases in q if Q_C is sufficiently close to 0.

Remark 4. *Agents have been assumed to have outside option* 0. *In a symmetric team, if each agent has a positive outside option $u > 0$, then there exists an optimal* abandonment *state $Q_A > -\infty$ satisfying the smooth-pasting condition $\frac{\partial}{\partial q} J_n(q, Q_A)\Big|_{q=Q_A} = 0$ such that the agents find it optimal to abandon the project at the first moment q hits Q_A, where $J_n(\cdot, Q_A)$ satisfies equation (4) subject to $J_n(Q_A, Q_A) = u$ and $J_i(0, Q_A) = V_i$. In this case, each agent's effort increases monotonically with progress.*

3.1. *Comparative statics*

This section establishes some comparative statics, which are helpful to understand how the agents' incentives depend on the parameters of the problem. To examine the effect of each parameter to the agents' incentives, I consider two symmetric teams that differ in exactly one attribute: their members' rewards V, patience levels r, or the volatility of the project σ.[14]

Proposition 1. *Consider two teams comprising symmetric agents.*

(i) If $V_1 < V_2$, then all other parameters held constant, $a_1(q) < a_2(q)$ for all q.
(ii) If $r_1 > r_2$, then all other parameters held constant, there exists an interior threshold Θ_r such that $a_1(q) \le a_2(q)$ if and only if $q \le \Theta_r$.
(iii) If $\sigma_1 > \sigma_2$, then all other parameters held constant, there exist interior thresholds $\Theta_{\sigma,1} \le \Theta_{\sigma,2}$ such that $a_1(q) \ge a_2(q)$ if $q \le \Theta_{\sigma,1}$ and $a_1(q) \le a_2(q)$ if $q \ge \Theta_{\sigma,2}$.[15]

The intuition behind statement (i) is straightforward. If the agents receive a bigger reward, then they always work harder in equilibrium.

Statement (ii) asserts that less patient agents work harder than more patient agents if and only if the project is sufficiently close to completion. Intuitively, less patient agents have more to gain from an earlier completion (provided that the project is sufficiently close to completion). However, bringing the completion time forward requires that they exert more effort, the cost of which is incurred at the time that effort is exerted, whereas the reward is only collected upon completion of the project. Therefore, the benefit from bringing the completion time forward (by exerting more effort) outweighs its cost only when the project is sufficiently close to completion.

14. Since the teams are symmetric and differ in a single parameter (*e.g.* their reward V_i in statement (i)), abusing notation, I let $a_i(\cdot)$ denote each agent's effort strategy corresponding to the parameter with subscript i.

15. Unable to show that $J_i'''(q)$ is unimodal in q, this result does not guarantee that $\Theta_{\sigma,1} = \Theta_{\sigma,2}$, which implies that it does not provide any prediction about how the agents' effort depends on σ when $q \in [\Theta_{\sigma,1}, \Theta_{\sigma,2}]$. However, numerical analysis indicates that in fact $\Theta_{\sigma,1} = \Theta_{\sigma,2}$.

Finally, statement (iii) asserts that incentives become stronger in the volatility of the project σ when it is far from completion, while the opposite is true when it gets close to completion. As the volatility increases, it becomes more likely that the project will be completed either earlier than expected (*upside*), or later than expected (*downside*). If the project is sufficiently far from completion, then $J_i(q)$ is close to 0 so that the downside is negligible, while $J_i''(q)>0$ implies that the upside is not (negligible), and consequently $a_1(q)\geq a_2(q)$. However, because the completion time of the project is non-negative, the upside diminishes as it approaches completion, which implies that the downside is bigger than the upside, and consequently $a_1(q)\leq a_2(q)$.

3.2. *Comparison with first-best outcome*

To obtain a benchmark for the agents' equilibrium effort levels, I compare them to the first-best outcome, where at every moment, each agent chooses his effort level to maximize the team's, as opposed to his individual expected discounted pay-off. I focus on the symmetric case, and denote by $\hat{J}_n(q)$ and $\hat{a}_n(q)$ the first-best expected discounted pay-off and effort level of each member of an n-person team, respectively. The first-best effort level satisfies $\hat{a}_n(q)\in\arg\max_a\left\{an\hat{J}_n'(q)-c(a)\right\}$, and the first-order condition implies that $\hat{a}_n(q)=f\left(n\hat{J}_n'(q)\right)$. Substituting this into equation (2) yields

$$r\hat{J}_n(q)=-c\left(f\left(n\hat{J}_n'(q)\right)\right)+nf\left(n\hat{J}_n'(q)\right)\hat{J}_n'(q)+\frac{\sigma^2}{2}\hat{J}_n''(q)$$

subject to the boundary conditions (3). It is straightforward to show that the properties established in Theorem 1 apply for $\hat{J}_n(q)$ and $\hat{a}_n(q)$. In particular, the first-best ODE subject to equation (3) has a unique solution, and $\hat{a}_n'(q)>0$ for all q; *i.e.* similar to the MPE, the first-best effort level increases with progress.

Proposition 2 compares each agent's effort and his/her expected discounted pay-off in the MPE to the first-best outcome.

Proposition 2. *In a team of $n\geq 2$ agents, $a_n(q)<\hat{a}_n(q)$ and $J_n(q)<\hat{J}_n(q)$ for all q.*

This result is intuitive: because each agent's reward is independent of his/her contribution to the project, he/she has incentives to free-ride. As a result, in equilibrium, each agent exerts strictly less effort and he/she is strictly worse off at every state of the project relative to the case in which agents behave collectively by choosing their effort level at every moment to maximize the team's expected discounted pay-off.

Remark 5. *A natural question is whether the agents can increase their expected discounted pay-off by adopting non-Markovian strategies, so that their effort at t depends on the entire evolution path of the project $\{q_s\}_{s\leq t}$. While a formal analysis is beyond the scope of this article, the analysis of Sannikov and Skrzypacz (2007), who study a related model, suggests that no, there does not exist a symmetric public perfect equilibrium (PPE) in which agents can achieve a higher expected discounted pay-off than the MPE at any state of the project. See Appendix A.4 for details.*

It is important to emphasize, however, that this conjecture hinges upon the assumption that the agents cannot observe each other's effort choices. For example, if efforts are publicly observable, then in addition to the MPE characterized in Theorem 1, using a similar approach as in Georgiadis *et al*. (2014), who study a deterministic version of this model (*i.e.* with $\sigma=0$), one can show that there exists a PPE in which the agents exert the first-best effort level along the equilibrium path. Such equilibrium is supported by trigger strategies, wherein at every moment t, each agent exerts the first-best effort level if all agents have exerted the first-best effort level for all $s < t$, while he/she reverts to the MPE otherwise.[16]

4. THE EFFECT OF TEAM SIZE

When examining the relationship between the agents' incentives and the size of the team, it is important to consider how each agent's reward depends on the team size. I consider the following (natural) cases: the *public good allocation* scheme, wherein each agent receives a reward V upon completing the project irrespective of the team size, and the *budget allocation* scheme, wherein each agent receives a reward V/n upon completing the project.

With n symmetric agents, each agent's expected discounted pay-off function satisfies

$$rJ_n(q) = -c\left(f\left(J_n'(q)\right)\right) + nf\left(J_n'(q)\right)J_n'(q) + \frac{\sigma^2}{2}J_n''(q)$$

subject to $\lim_{q\to-\infty} J_n(q) = 0$ and $J_n(0) = V_n$, where $V_n = V$ or $V_n = V/n$ under the public good or the budget allocation scheme, respectively.

Theorem 2 below shows that under both allocation schemes, members of a larger team work harder than members of a smaller team—both individually and on aggregate—if and only if the project is sufficiently far from completion. Figure 1 illustrates an example.

Theorem 2. *Consider two teams comprising n and m > n identical agents. Under both allocation schemes, all other parameters held constant, there exist thresholds $\Theta_{n,m}$ and $\Phi_{n,m}$ such that*

(i) $a_m(q) \geq a_n(q)$ if and only if $q \leq \Theta_{n,m}$; and
ii) $ma_m(q) \geq na_n(q)$ if and only if $q \leq \Phi_{n,m}$.

By increasing the size of the team, two opposing forces influence the agents' incentives: First, agents obtain stronger incentives to free-ride. To see why, consider an agent's dilemma at time t to (unilaterally) reduce his/her effort by a *small* amount ε for a *short* interval Δ. By doing so, he/she saves approximately $\varepsilon c'(a(q_t))\Delta$ in effort costs, but at $t+\Delta$, the project is $\varepsilon\Delta$ farther from completion. In equilibrium, this agent will carry out only $1/n$ of that lost progress, which implies that the benefit from shirking increases in the team size. Secondly, recall that each agent's incentives are proportional to the marginal benefit of bringing the completion time τ forward: $-d/d\tau V_n \mathbb{E}\left[e^{-r\tau}\right] = rV_n\mathbb{E}\left[e^{-r\tau}\right]$, which implies that holding strategies fixed, an increase in the team size decreases the completion time of the project, and hence strengthens the agents' incentives. Following the terminology of Bolton and Harris (1999), who study an experimentation in teams problem, I refer to these forces as the *free-riding* and the *encouragement effect*, respectively, and the intuition will follow from examining how the magnitude of these effects changes as the project progresses.

It is convenient to consider the deterministic case in which $\sigma = 0$. Because $c'(0) = 0$ and effort vanishes as $q \to -\infty$, and noting that each agent's gain from free-riding is proportional to $c'(a(q))$, it follows that the free-riding effect is negligible when the project is sufficiently far from completion. As the project progresses, the agents raise their effort, and because effort costs are convex, the free-riding effect becomes stronger. The magnitude of the encouragement effect

16. There is a well-known difficulty associated with defining trigger strategies in continuous-time games, which Georgiadis *et al*. (2014) resolve using the concept of inertia strategies proposed by Bergin and MacLeod (1993).

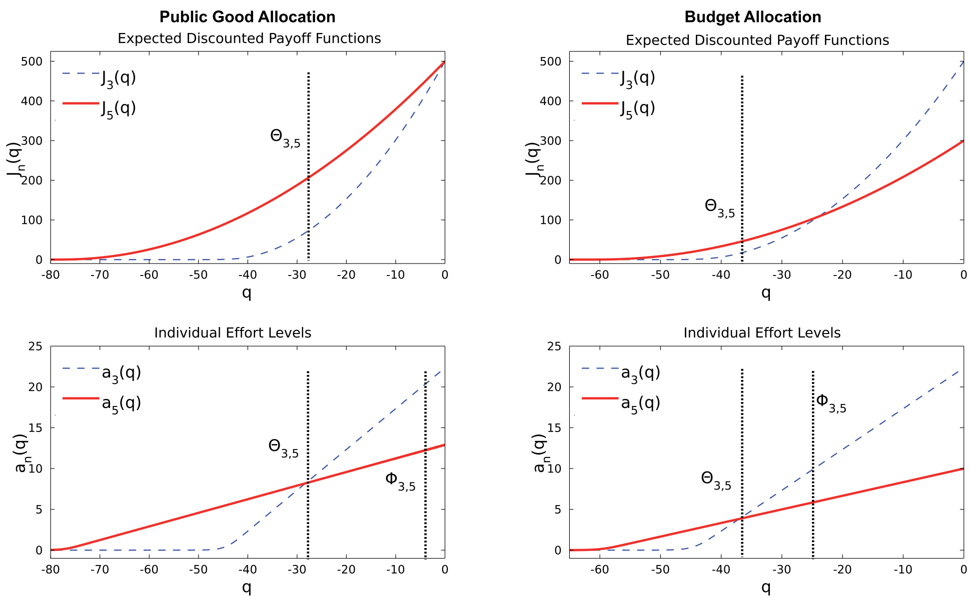

FIGURE 1

Illustration of Theorem 2.

The upper panels illustrate each agent's expected discounted pay-off under public good (left) and budget (right) allocation for two different team sizes: $n=3$ and 5. The lower panels illustrate each agent's equilibrium effort.

can be measured by the ratio of the marginal benefits of bringing the completion time forward: $\frac{rV_{2n}e^{-\frac{r\tau}{2}}}{rV_n e^{-r\tau}} = \frac{V_{2n}}{V_n} e^{\frac{r\tau}{2}}$. Observe that this ratio increases in τ, which implies that the encouragement effect becomes weaker as the project progresses (*i.e.* as τ becomes smaller), and it diminishes under public good allocation (since $\frac{V_{2n}}{V_n}=1$) while it becomes negative under budget allocation (since $\frac{V_{2n}}{V_n}<1$).

In summary, under both allocation schemes, the encouragement effect dominates the free-riding effect if and only if the project is sufficiently far from completion. This implies that by increasing the team size, the agents obtain stronger incentives when the project is far from completion, while their incentives become weaker near completion.

Turning attention to the second statement, it follows from statement (i) that aggregate effort in the larger team exceeds that in the smaller team if the project is far from completion. Perhaps surprisingly, however, when the project is near completion, not only the individual effort, but also the aggregate effort in the larger team is less than that in the smaller team. The intuition follows by noting that when the project is *very* close to completion (*e.g.* $q_t=-\epsilon$), this game resembles the (static) "reporting a crime" game (ch. 4.8 in Osborne, 2003), and it is well known that in the unique symmetric mixed-strategy Nash equilibrium of this game, the probability that at least one agent exerts effort (which is analogous to aggregate effort) decreases in the group size.

The same proof technique can be used to show that under both allocation schemes, the first-best aggregate effort increases in the team size at every q. This difference is a consequence of the free-riding effect being absent in this case, so that the encouragement effect alone leads a larger team to always work on aggregate harder than a smaller team.

It is noteworthy that the thresholds of Theorem 2 need not always be interior. Under budget allocation, it is possible that $\Theta_{n,m}=-\infty$, which would imply that each member of the smaller

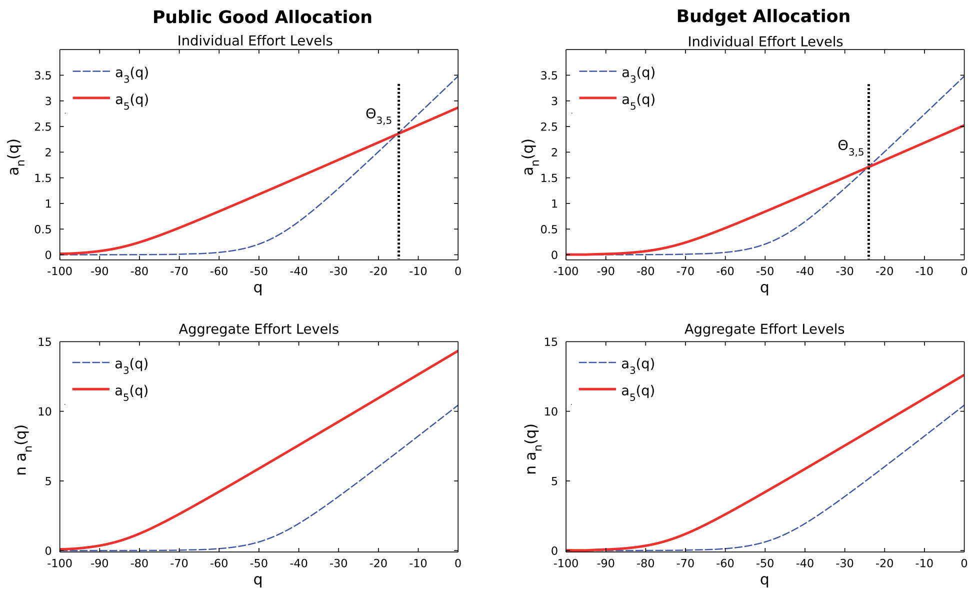

FIGURE 2

An example with quartic effort costs ($p=3$).

The upper panels illustrate that under both allocation schemes, $\Theta_{n,m}$ is interior, whereas the lower panels illustrate that $\Phi_{n,m}=0$, in which case the aggregate effort in the larger team always exceeds that of the smaller team.

team always works harder than each member of the larger team. However, numerical analysis indicates that $\Theta_{n,m}$ is always interior under both allocation schemes. Turning to $\Phi_{n,m}$, the proof of Theorem 2 ensures that it is interior only under budget allocation if effort costs are quadratic, while one can find examples in which $\Phi_{n,m}$ is interior as well as examples in which $\Phi_{n,m}=0$ otherwise. Numerical analysis indicates that the most important parameter that determines whether $\Phi_{n,m}$ is interior is the convexity of the effort cost function, and it is interior as long as $c(\cdot)$ is not too convex (*i.e.* p is sufficiently small). This is intuitive, as more convex effort costs favour the larger team more.[17] In addition, under public good allocation, for $\Phi_{n,m}$ to be interior, it is also necessary that n and m are sufficiently small. Intuitively, this is because the size of the pie increases in the team size under this scheme, which (again) favours the larger team. Figure 2 illustrates an example with quartic effort costs (*i.e.* $p=3$) in which case $\Theta_{n,m}$ is interior but $\Phi_{n,m}=0$ under both allocation schemes.

4.1. *Partnership formation*

In this section, I examine the problem faced by a group of agents who seek to organize into a partnership. Proposition 3 characterizes the optimal partnership size.

17. This finding is consistent with the results of Esteban and Ray (2001), who show that in a static setting, the aggregate effort increases in the team size if effort costs are sufficiently convex. In their setting, however, individual effort always decreases in the team size irrespective of the convexity of the effort costs. To further examine the impact of the convexity of the agents' effort costs, in Appendix A.5, I consider the case in which effort costs are linear, and I establish an analogous result to Theorem 2: members an $(n+1)$-person team have stronger incentives relative to those of an n-person team as long as n is sufficiently small.

Proposition 3. *Suppose that the partnership composition is finalized before the agents begin to work, so that the optimal partnership size satisfies* $\arg\max_n \{J_n(q_0)\}$.

(i) Under public good allocation, the optimal partnership size $n = \infty$ *independent of the project length* $|q_0|$.
(ii) Under budget allocation, the optimal partnership size n *increases in the project length* $|q_0|$.

Increasing the size of the partnership has two effects. First, the expected completion time of the project changes; from Theorem 2 it follows that it decreases, thus increasing each agent's expected discounted reward, if the project is sufficiently long. Secondly, in equilibrium, each agent will exert less effort to complete the project, which implies that his total expected discounted cost of effort decreases. Proposition 3 shows that if each agent's reward does not depend on the partnership size (*i.e.* under public good allocation), then the latter effect always dominates the former, and hence agents are better off the bigger the partnership. Under budget allocation, however, these effects outweigh the decrease in each agent's reward caused by the increase in the partnership size only if the project is sufficiently long, and consequently, the optimal partnership size increases in the length of the project.

An important assumption underlying Proposition 3 is that the partnership composition is finalized before the agents begin to work. Under public good allocation, this assumption is without loss of generality, because the optimal partnership size is equal to ∞ irrespective of the length of the project. However, it may not be innocuous under budget allocation, where the optimal partnership size does depend on the project length. If the partnership size is allowed to vary with progress, an important modelling assumption is how the rewards of new and exiting members will be determined. While a formal analysis is beyond the scope of this article, abstracting from the above modelling issue and based on Theorem 2, it is reasonable to conjecture that the agents will have incentives to expand the partnership after setbacks, and to decrease its size as the project nears completion.

5. MANAGER'S PROBLEM

Most projects require substantial capital to cover infrastructure and operating costs. For example, the design of a new pharmaceutical drug, in addition to the scientists responsible for the drug design (*i.e.* the project team), necessitates a laboratory, expensive and maintenance-intensive machinery, as well as support staff. Because individuals are often unable to cover these costs, projects are often run by corporations instead of the project team, which raises the questions of: (i) how to determine the optimal team size; and (ii) how to best incentivize the agents. These questions are addressed in this section, wherein I consider the case in which a third party (to be referred to as a manager) is the residual claimant of the project, and he/she hires a group of agents to undertake it on his/her behalf. Section 5.1 describes the model, Section 5.2 establishes some of the properties of the manager's problem, and Section 5.3 studies his/her contracting problem.

5.1. *The model with a manager*

The manager is the residual claimant of the project, he/she is risk neutral, and he/she discounts time at the same rate $r > 0$ as the agents. The project has (expected) length $|q_0|$, and it generates a pay-off $U > 0$ upon completion. To incentivize the agents, at time 0, the manager commits to an incentive contract that specifies the size of the team, denoted by n, a set of milestones $q_0 < Q_1 < \ldots < Q_K = 0$

(where $K \in \mathbb{N}$), and for every $k \in \{1,\ldots,K\}$, allocates non-negative payments $\left\{V_{i,k}\right\}_{i=1}^{n}$ that are due upon reaching milestone Q_k for the first time.[18]

5.2. *The manager's profit function*

I begin by considering the case in which the manager compensates the agents only upon completing the project, and I show in Theorem 3 that his/her problem is well-defined and it satisfies some desirable properties. Then I explain how this result extends to the case in which the manager also rewards the agents for reaching intermediate milestones.

Given the team size n and the agents' rewards $\{V_i\}_{i=1}^{n}$ that are due upon completion of the project (where I can assume without loss of generality that $\sum_{i=1}^{n} V_i \leq U$), the manager's expected discounted profit function can be written as

$$F(q) = \left(U - \sum_{i=1}^{n} V_i\right) \mathbb{E}_\tau \left[e^{-r\tau} \,|\, q\right],$$

where the expectation is taken with respect to the project's completion time τ, which depends on the agents' strategies and the stochastic evolution of the project.[19] By using the first-order condition for each agent's equilibrium effort as determined in Section 3, the manager's expected discounted profit at any given state of the project satisfies

$$rF(q) = \left[\sum_{i=1}^{n} f\left(J_i'(q)\right)\right] F'(q) + \frac{\sigma^2}{2} F''(q) \tag{5}$$

defined on $(-\infty, 0]$ subject to the boundary conditions

$$\lim_{q \to -\infty} F(q) = 0 \text{ and } F(0) = U - \sum_{i=1}^{n} V_i, \tag{6}$$

where $J_i(q)$ satisfies equation (2) subject to equation (3). The interpretation of these conditions is similar to equation (3). As the state of the project diverges to $-\infty$, its expected completion time diverges to ∞, and because $r > 0$, the manager's expected discounted profit diminishes to 0. The second condition asserts that the manager's profit is realized when the project is completed, and it equals her pay-off U less the payments $\sum_{i=1}^{n} V_i$ disbursed to the agents.

Theorem 3. *Given $\left(n, \{V_i\}_{i=1}^{n}\right)$, a solution to the manager's problem defined by equation (5) subject to the boundary conditions (6) and the agents' problem as defined in Theorem 1 exists, and it has the following properties:*

(i) $F(q) > 0$ and $F'(q) > 0$ for all q.
(ii) $F(\cdot)$ is unique if the agents' rewards are symmetric (i.e. if $V_i = V_j$ for $i \neq j$).

18. The manager's contracting space is restricted. In principle, the optimal contract should condition each agent's pay-off on the path of q_t (and hence on the completion time of the project). Unfortunately, however, this problem is not tractable; for example, the contracting approach developed in Sannikov (2008) boils down a partial differential equation with $n+1$ variables (*i.e.* the state of the project q and the continuation value of each agent), which is intractable even for the case with a single agent. As such, this analysis is left for future research.

19. The subscript k is dropped when $K = 1$ (in which case $Q_1 = 0$).

Now let us discuss how Theorems 1 and 3 extend to the case in which the manager rewards the agents upon reaching intermediate milestones. Recall that he/she can designate a set of milestones, and attach rewards to each milestone that are due as soon as the project reaches the respective milestone for the first time. Let $J_{i,k}(\cdot)$ denote agent i's expected discounted pay-off given that the project has reached $k-1$ milestones, which is defined on $(-\infty, Q_k]$, and note that it satisfies equation (4) subject to $\lim_{q\to-\infty} J_{i,k}(q)=0$ and $J_{i,k}(Q_k)=V_{i,k}+J_{i,k+1}(Q_k)$, where $J_{i,K+1}(0)=0$. The second boundary condition states that upon reaching milestone k, agent i receives the reward attached to that milestone, plus the continuation value from future rewards. Starting with $J_{i,K}(\cdot)$, it is straightforward that it satisfies the properties of Theorem 1, and in particular, that $J_{i,K}\left(Q_{k-1}\right)$ is unique (as long as rewards are symmetric) so that the boundary condition of $J_{i,K-1}(\cdot)$ at Q_{K-1} is well defined. Proceeding backwards, it follows that for every k, $J_{i,k}(\cdot)$ satisfies the properties of Theorem 1.

To examine the manager's problem, let $F_k(\cdot)$ denote his/her expected discounted profit given that the project has reached $k-1$ milestones, which is defined on $(-\infty, Q_k]$, and note that it satisfies equation (5) subject to $\lim_{q\to-\infty} F_k(q)=0$ and $F_k(Q_k)=F_{k+1}(Q_k)-\sum_{i=1}^{n} V_{i,k}$, where $F_{K+1}(Q_k)=U$. The second boundary condition states that upon reaching milestone k, the manager receives the continuation value of the project, less the payments that he/she disburses to the agents for reaching this milestone. Again starting with $k=K$ and proceeding backwards, it is straightforward that for all k, $F_k(\cdot)$ satisfies the properties established in Theorem 3.

5.3. *Contracting problem*

The manager's problem entails choosing the team size and the agents' incentive contracts to maximize his/her *ex ante* expected discounted profit subject to the agents' incentive compatibility constraints.[20] I begin by analysing symmetric contracts. Then I examine how the manager can increase his/her expected discounted profit with asymmetric contracts.

5.3.1. Symmetric contracts. Theorem 4 shows that within the class of symmetric contracts, one can without loss of generality restrict attention to those that compensate the agents only upon completion of the project.

Theorem 4. *The optimal symmetric contract compensates the agents only upon completion of the project.*

To prove this result, I consider an arbitrary set of milestones and arbitrary rewards attached to each milestone, and I construct an alternative contract that rewards the agents only upon completing the project and renders the manager better off. Intuitively, because rewards are sunk in terms of incentivizing the agents after they are disbursed, and all parties are risk-neutral and they discount time at the same rate, by backloading payments, the manager can provide the same incentives at the early stages of the project, while providing stronger incentives when it is close to completion.[21]

20. While it is possible to choose the team size directly via the incentive contract (*e.g.* by setting the reward of $\bar{n}<n$ agents to 0, the manager can effectively decrease the team size to $n-\bar{n}$), it is analytically more convenient to analyse the two "levers" (for controlling incentives) separately.

21. As shown in part II of the proof of Theorem 4, the agents are also better off if their rewards are backloaded. In other words, each agent could strengthen his/her incentives and increase his/her expected discounted pay-off by depositing any rewards from reaching intermediate milestones in an account with interest rate r, and closing the account upon completion of the project.

This result is consistent with practice, as evidenced for example by Lewis and Bajari (2014), who study incentive contracts in highway construction projects. Moreover, it is valuable from an analytical perspective, because it reduces the infinite-dimensional problem of determining the team size, the number of milestones, the set of milestones, and the rewards attached to each milestone into a two-dimensional problem, in which the manager only needs to determine his/her budget $B=\sum_{i=1}^{n} V_i$ for compensating the agents and the team size. Propositions 4–6 characterize the manager's optimal budget and his/her optimal team size.

Proposition 4. *Suppose that the manager employs n agents whom she compensates symmetrically. Then her optimal budget B increases in the length of the project* $|q_0|$.

Contemplating an increase in his/her budget, the manager trades off a decrease in her net profit $U-B$ and an increase in the project's expected present discounted value $\mathbb{E}_\tau\left[e^{-r\tau}\,|q_0\right]$. Because a longer project takes (on average) a larger amount of time to be completed, a decrease in his/her net profit has a smaller effect on his/her *ex ante* expected discounted profit the longer the project. Therefore, the benefit from raising the agents' rewards outweighs the decrease in his/her net profit if and only if the project is sufficiently long, which in turn implies that the manager's optimal budget increases in the length of the project.

Lemma 1. *Suppose that the manager has a fixed budget B and he/she compensates the agents symmetrically. For any* $m>n$*, there exists a threshold* $T_{n,m}$ *such that he/she prefers employing an m-member team instead of an n-member team if and only if* $|q_0|\geq T_{n,m}$.

Given a fixed budget, the manager's objective is to choose the team size to minimize the expected completion time of the project. This is equivalent to maximizing the aggregate effort of the team along the evolution path of the project. Hence, the intuition behind this result follows from statement (B) of Theorem 2. If the project is short, then on expectation, the aggregate effort of the smaller team will be greater than that of the larger team due to the free-riding effect (on average) dominating the encouragement effect. The opposite is true if the project is long. Figure 3 illustrates an example.

Applying the Monotonicity Theorem of Milgrom and Shannon (1994) leads one to the following Proposition.

Proposition 5. *Given a fixed budget to (symmetrically) compensate a group of agents, the manager's optimal team size n increases in the length of the project* $|q_0|$.

Proposition 5 suggests that a larger team is more desirable while the project is far from completion, whereas a smaller team becomes preferable when the project gets close to completion. Therefore, it seems desirable to construct a scheme that dynamically decreases the team size as the project progresses. Suppose that the manager employs two identical agents on a fixed budget, and he/she designates a *retirement state* R, such that one of the agents is permanently retired (*i.e.* he/she stops exerting effort) at the first time that the state of the project hits R. From that point onwards, the other agent continues to work alone. Both agents are compensated only upon completion of the project, and the payments (say V_1 and V_2) are chosen such that the agents are indifferent with respect to who will retire at R; *i.e.* their expected discounted pay-offs are equal at $q_t=R$.[22]

22. Note that this is one of many possible *retirement schemes*. A complete characterization of the optimal dynamic team size management scheme is beyond the scope of this article, and is left for future research.

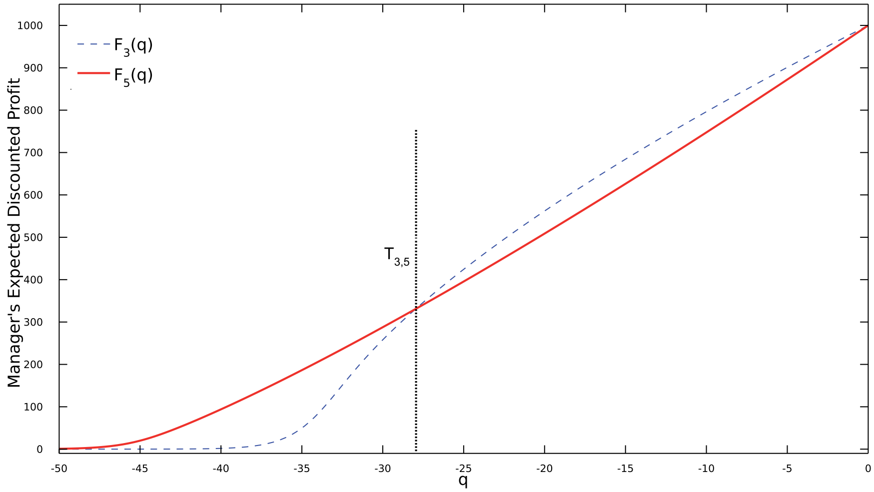

FIGURE 3

Illustration of Lemma 1.

Given a fixed budget, the manager's expected discounted profit is higher if she recruits a 5-member team relative to a 3-member team if and only if the initial state of the project q_0 is to the left of the threshold $-T_{3,5}$; or equivalently, if and only if $|q_0| \geq T_{3,5}$.

Proposition 6. *Suppose the manager employs two agents with quadratic effort costs. Consider the retirement scheme described above, where the retirement state* $R > \max\{q_0, -T_{1,2}\}$ *and* $T_{1,2}$ *is taken from Lemma 1. There exists a threshold* $\Theta_R > |R|$ *such that the manager is better off implementing this retirement scheme relative to allowing both agents to work together until the project is completed if and only if its length* $|q_0| < \Theta_R$.

First, note that after one agent retires, the other will exert first-best effort until the project is completed. Because the manager's budget is fixed, this retirement scheme is preferable only if it increases the aggregate effort of the team along the evolution path of the project. A key part of the proof involves showing that agents have weaker incentives before one of them is retired as compared to the case in which they always work together (*i.e.* when a retirement scheme is not used). Therefore, the benefit from having one agent exert first-best effort *after* one of them retires outweighs the loss from the two agents exerting less effort before one of them retires (relative to the case in which they always work together) only if the project is sufficiently short. Hence, this retirement scheme is preferable if and only if $|q_0| < \Theta_R$.

From an applied perspective, this result should be approached with caution. In this environment, the agents are (effectively) restricted to playing the MPE, whereas in practice, groups are often able to coordinate to a more efficient equilibrium, for example, by monitoring each other's efforts, thus mitigating the free-rider problem (and hence weakening this result). Moreover, Weber (2006) shows that while efficient coordination does not occur in groups that start off large, it is possible to create efficiently coordinated large groups by starting with small groups that find it easier to coordinate, and adding new members gradually who are aware of the group's history. Therefore, one should be aware of the tension between the free-riding effect becoming stronger with progress, and the force identified by Weber.

5.3.2. Asymmetric contracts. Insofar, I have restricted attention to contracts that compensate the agents symmetrically. However, Proposition 6 suggests that an asymmetric contract that rewards the agents upon reaching intermediate milestones can do better than the best symmetric one if the project is sufficiently short. Indeed, the retirement scheme proposed above can be implemented using the following asymmetric rewards-for-milestones contract.

Remark 6. *Let $Q_1 = R$, and suppose that agent 1 receives V as soon as the project is completed, while he/she receives no intermediate rewards. However, agent 2 receives the equilibrium present discounted value of $B-V$ upon hitting R for the first time* (i.e. $(B-V)\mathbb{E}_\tau\left[e^{-r\tau}|R\right]$)*, and he/she receives no further compensation, so that he/she effectively retires at that point. From Proposition 6 we know that there exists a $V \in (0,B)$ and a threshold Θ_R such that this asymmetric contract is preferable to a symmetric one if and only if $|q_0| < \Theta_R$.*

It is important to note that while the expected cost of compensating the agents in the above asymmetric contract is equal to B, the actual cost is stochastic, and in fact, it can exceed the project's pay-off U. As a result, unless the manager is sufficiently solvent, there is a positive probability that he/she will not be able to honour the contract, which will negatively impact the agents' incentives.

The following result shows that an asymmetric contract may be preferable even if the manager compensates the (identical) agents upon reaching the same milestone; namely, upon completing the project.

Proposition 7. *Suppose that the manager has a fixed budget $B > 0$, and he/she employs two agents with quadratic effort costs whom he/she compensates upon completion of the project. Then for all $\epsilon \in \left(0, \frac{B}{2}\right]$, there exists a threshold T_ϵ such that the manager is better off compensating the two agents asymmetrically such that $V_1 = \frac{B}{2} + \epsilon$ and $V_2 = \frac{B}{2} - \epsilon$ instead of symmetrically, if and only if the length of the project $|q_0| \leq T_\epsilon$.*[23]

Intuitively, asymmetric compensation has two effects: first, it causes an efficiency gain in that the agent who receives the smaller share of the payment has weak incentives to exert effort, and hence the other agent cannot free-ride as much. At the same time however, because effort costs are convex, it causes an efficiency loss, as the total costs to complete the project are minimized when the agents work symmetrically; which occurs in equilibrium only when they are compensated symmetrically. By noting that the efficiency loss is increasing in the length of the project, and that the manager's objective is to allocate his/her budget so as to maximize the agents' expected aggregate effort along the evolution path of the project, it follows that the manager prefers to compensate the agents asymmetrically if the project is sufficiently short.

6. CONCLUDING REMARKS

To recap, I study a dynamic problem in which a group of agents collaborate over time to complete a project, which progresses at a rate that depends on the agents' efforts, and it generates a pay-off upon completion. The analysis provides several testable implications. In the context of the MRF, for example, one should expect that principal investigators will allocate more resources to MRF activities as the goal comes closer into sight. Secondly, in a drug discovery venture for

23. Note that the solution to the agents' problem need not be unique if the contract is asymmetric. However, this comparative static holds for every solution to equation (5) subject to equations (6), (4), and (3) (if more than one exists).

instance, the model predicts that the amount of time and resources (both individually and on aggregate) that the scientists allocate to the project will be positively related to the group size at the early stages of the project, and negatively related near completion. Moreover, this prediction is consistent with empirical studies of voluntary contributions by programmers to open-source software projects (Yildirim, 2006). These studies report an increase in the average contributions with the number of programmers, especially in the early stages of the projects, and a decline in the mature stages. Thirdly, the model prescribes that the members of a project team should be compensated asymmetrically if the project is sufficiently short.

In a related paper, Georgiadis *et al*. (2014) consider the case in which the project size is endogenous. Motivated by projects involving *design* or *quality* objectives that are often difficult to define in advance, they examine how the manager's optimal project size depends on his/her ability to commit to a given project size in advance. In another related paper, Ederer *et al*. (2014) examine how the team size affects incentives in a discrete public good contribution game using laboratory experiments. Preliminary results support the predictions of Theorem 2.

This article opens several opportunities for future research. First, the optimal contracting problem is an issue that deserves further exploration. As discussed in Section 5, I have considered a restricted contracting space. Intuitively, the optimal contract will be asymmetric, and it will backload payments (*i.e.* each agent will be compensated only at the end of his/her involvement in the project). However, each agent's reward should depend on the path of q_t, and hence on the completion time of the project. Secondly, the model assumes that efforts are unobservable, and that at every moment, each agent chooses his/her effort level after observing the current state of the project. An interesting extension might consider the case in which the agents can obtain a noisy signal of each other's effort (by incurring some cost) and the state of the project is observed imperfectly. The former should allow the agents to coordinate to a more efficient equilibrium, while the latter will force the agents to form beliefs about how close the project is to completion, and to choose their strategies based on those beliefs. Finally, from an applied perspective, it may be interesting to examine how a project can be split into subprojects that can be undertaken by separate teams.

APPENDIX A

A. ADDITIONAL RESULTS

A.1. *Flow payoffs while the project is in progress*

An important assumption of the base model is that the agents are compensated only upon completion of the project. In this section, I extend the model by considering the case in which during any *small* $[t,t+dt)$ interval while the project is in progress, each agent receives $h(q_t)dt$, in addition to the lump sum reward V upon completion. To make the problem tractable, I shall make the following assumptions about $h(\cdot)$:

Assumption 1. *$h(\cdot)$ is thrice continuously differentiable on $(-\infty,0]$, it has positive first, second, and third derivatives, and it satisfies $\lim_{q\to-\infty}h(q)=0$ and $h(0)\le rV$.*

Using a similar approach as in Section 3, it follows that in an MPE, the expected discounted pay-off function of agent i satisfies

$$rJ_i(q)=\max_{a_i}\left\{h(q)-c(a_i)+\left(\sum_{j=1}^{n}a_j\right)J_i'(q)+\frac{\sigma^2}{2}J_i''(q)\right\}$$

subject to equation (3), and his optimal effort level satisfies $a_i(q)=f\left(J_i'(q)\right)$, where $f(\cdot)=c'^{-1}(\max\{0,\cdot\})$.

Proposition 8 below characterizes the unique MPE of this game, and it shows: (i) that each agent's effort level is either increasing, or hump-shaped in q; and (ii) the team size comparative static established in Theorem 2 continues to hold.

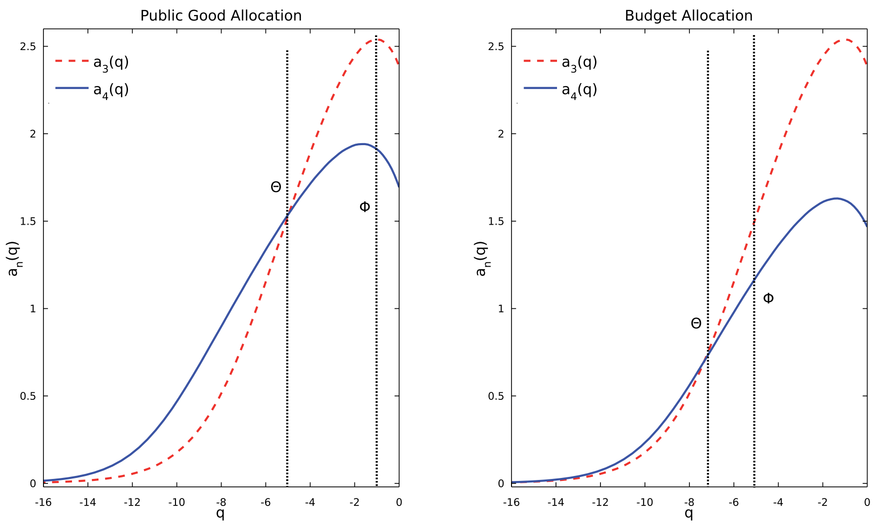

FIGURE A1

An example in which agents receive flow pay-offs while the project is in progress with $h(q) = 10e^{q/2}$. Observe that effort strategies are hump shaped in q, and the predictions of Theorem 2 continue to hold under both allocation schemes.

Proposition 8. *Suppose that each agent receives a flow pay-off $h(q)$ while the project is in progress, , and $h(\cdot)$ satisfies Assumption 1.*

(i) A symmetric MPE for this game exists, it is unique, and it satisfies $0 \le J_n(q) \le V$ and $J_n'(q) \ge 0$ for all q.

(ii) There exists a threshold ω (not necessarily interior) such that each agent's effort $a_n'(q) \ge 0$ if and only if $q \le \omega$.

(iii) Under both allocation schemes and for any $m > n$, there exists a threshold $\Theta_{n,m}$ ($\Phi_{n,m}$) such that $a_m(q) \ge a_n(q)$ ($ma_m(q) \ge na_n(q)$) if and only if $q \le \Theta_{n,m}$ ($q \le \Phi_{n,m}$).

The intuition why effort can be decreasing in q when the project is close to completion can be explained as follows: far from completion, the agents are incentivized by the future flow pay-offs and the lump sum V upon completion. As the project nears completion, the current flow pay-offs become larger, and hence the agents have less to gain by bringing the project closer to completion, and consequently, they decrease their effort. While establishing conditions under which ω is interior does not seem possible, numerical analysis indicates that this is the case if $h(0)/r$ is sufficiently close to V.

Finally, statement (iii) follows by noting that $J_n'(q)$ being unimodal in q is sufficient for the proof of Theorem 2. Figure A1 illustrates an example.

A.2. *Cancellation states*

In this section, I consider the case in which the project is cancelled at the first moment that q_t hits some (exogenous) cancellation state $Q_C > -\infty$ and the game ends with the agents receiving 0 pay-off. The expected discounted pay-off for each agent i satisfies equation (4) subject to the boundary conditions

$$J_i(Q_C) = 0 \quad \text{and} \quad J_i(0) = V.$$

In contrast to the model analysed in Section 3, with a finite cancellation state, it need not be the case that $J_i'(Q_C) = 0$. It follows that all statements of Theorem 1 hold except for (iii) (which asserts that effort increases with progresses).[24] Instead, there exists some threshold ω (not necessarily interior), such that $a_n'(q) \ge 0$ if and only if $q \ge \omega$.

Similarly, by noting that $J_n'(q)$ being unimodal in q is sufficient for the proof of Theorem 2, it follows that even with cancellation states, members of a larger team work harder than members of a smaller team, both individually and on aggregate, if and only if the project is sufficiently far from completion. These results are summarized in the Proposition 9 below.

24. This result requires that $\lim_{q \to -\infty} J_i'(q) = 0$.

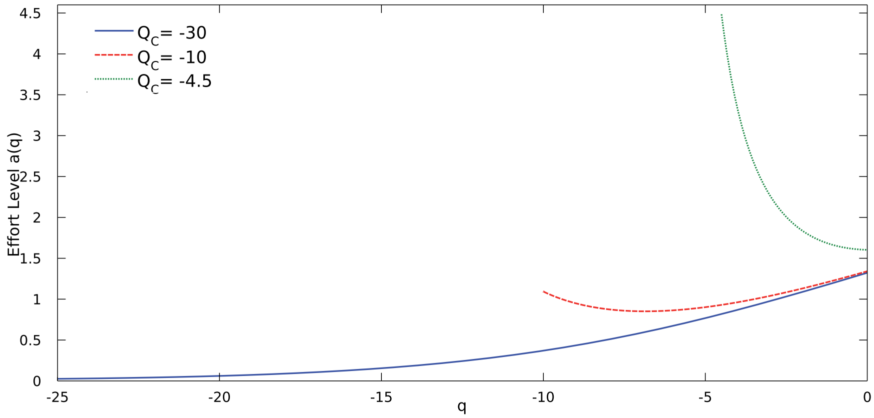

FIGURE A2

Illustration of the agents' effort functions given three different cancellation states. Observe that when Q_C is small ($e.g.$ $Q_C = -30$), effort increases in q. When Q_C is in an intermediate range ($e.g.$ $Q_C = -10$), then effort is U-shaped in q, while it decreases in q if Q_C is sufficiently large ($e.g.$ $Q_C = -4.5$).

Proposition 9. *Suppose that the project is cancelled at the first moment such that q_t hits a given cancellation state $Q_C > -\infty$ and the game ends with the agents receiving 0 pay-off.*

(i) A symmetric MPE for this game exists, it is unique, and it satisfies $0 \leq J_n(q) \leq V$ and $J_n'(q) \geq 0$ for all q.

(ii) There exists a threshold ω (not necessarily interior) such that each agent's effort $a_n'(q) \geq 0$ if and only if $q \geq \omega$.

(iii) Under both allocation schemes and for any $m > n$, there exists a threshold $\Theta_{n,m}$ ($\Phi_{n,m}$) such that $a_m(q) \geq a_n(q)$ ($ma_m(q) \geq na_n(q)$) if and only if $q \leq \Theta_{n,m}$ ($q \leq \Phi_{n,m}$).

While a sharper characterization of the MPE is not possible, numerical analysis indicates that effort increases in q if Q_C is sufficiently small ($i.e.$ $\omega = -\infty$), it is U-shaped in q if Q_C is in some intermediate range ($i.e.$ ω is interior), while it decreases in q ($i.e.$ $\omega = 0$) if Q_C is close to 0. An example is illustrated in Figure A2.

Intuitively, the agents have incentives to exert effort to: (i) complete the project; and (ii) avoid hitting the cancellation state Q_C. Moreover, observe that the incentives due to the former (latter) are stronger the closer the project is to completion (to Q_C). Therefore, if Q_C is small, then the latter incentive is weak, so that the agents' incentives are driven primarily by (i), and effort increases with progress. As Q_C increases, (ii) becomes stronger, so that effort becomes U-shaped in q, and if Q_C is sufficiently close to 0, then the incentives from (ii) dominate those from (i), and consequently, effort decreases in q.

A.3. *Effort affects drift and variance of stochastic process*

A simplifying assumption in the base model is that the variance of the process that governs the evolution of the project ($i.e.$ σ) does not depend on the agents' effort levels. As a result, even if no agent ever exerts any effort, the project is completed in finite time with probability 1. To understand the impact of this assumption, in this section, I consider the case in which the project progresses according to

$$dq_t = \sum_{i=1}^{n} a_{i,t} dt + \sqrt{\sum_{i=1}^{n} a_{i,t}}\, \sigma dW_t .$$

[25] The expected discounted pay-off function of agent i satisfies the HJB equation

$$rJ_i(q) = -c\left(a_{i,t}\right) + \left(\sum_{j=1}^{n} a_{j,t}\right)\left(J_i'(q) + \frac{\sigma^2}{2} J_i''(q)\right)$$

25. Note that the total effort of the team is instantly observable here. Therefore, there typically exist non-Markovian equilibria that are sustained via trigger strategies that revert to the MPE after observing a deviation. Moreover, provided that the state q_t is verifiable, the team's total effort becomes contractible.

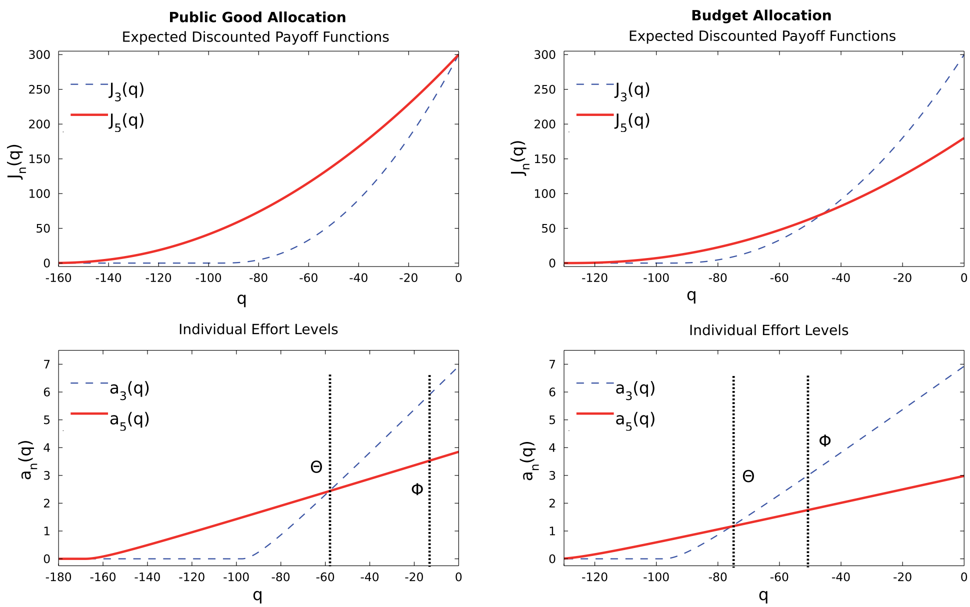

FIGURE A3

An example in which the agents' effort influences both the drift and the variance of the stochastic process. Observe that effort increases in q, and that the predictions of Theorem 2 continue to hold under both allocation schemes.

subject to equation (3). Restricting attention to symmetric MPE and guessing that each agent's first order condition always binds, it follows that his effort level satisfies $a(q)=f\left(J'(q)+\frac{\sigma^2}{2}J''(q)\right)$. Using a similar approach to that used to prove Theorem 1, one can show that a non-trivial solution to this ODE exists. However, the MPE need not be unique in this case: unless a single agent is willing to undertake the project single handedly, then there exists another equilibrium in which no agent ever exerts any effort, and the project is never completed.

Unfortunately, analysing how the agents' effort levels change with progress and how individual and aggregate effort depends on the team size is analytically intractable. However, as illustrated in Figure A3, numerical examples indicate that the main results of the base model continue to hold: effort increases with progress (*i.e.* $a'(q)\geq 0$ for all q) and the predictions of Theorem 2 continues to hold: under both allocation schemes and for any $m>n$, there exists a threshold $\Theta_{n,m}$ ($\Phi_{n,m}$) such that $a_m(q)\geq a_n(q)$ ($ma_m(q)\geq na_n(q)$) if and only if $q\leq\Theta_{n,m}$ ($q\leq\Phi_{n,m}$).

A.4. *Equilibria with non-markovian strategies*

Insofar, I have restricted attention to Markovian strategies, so that at every moment, each agent's effort is a function of only the current state of the project q_t. This raises the question whether agents can increase their expected discounted pay-off by adopting non-Markovian strategies that at time t depend on the entire evolution path of the project $\{q_s\}_{s\leq t}$. Sannikov and Skrzypacz (2007) study a related model in which the agents can change their actions only at times $t=0,\Delta,2\Delta,\ldots$, where $\Delta>0$ (but *small*), and the information structure is similar; *i.e.* the state variable evolves according to a diffusion process whose drift is influenced by the agents' actions. They show that the pay-offs from the best symmetric PPE converge to the pay-offs corresponding to the MPE as $\Delta\to 0$ (see their Proposition 5).

A natural, discrete-time analogue of the model considered in this article is one in which at $t\in\{0,\Delta,2\Delta,\ldots\}$ each agent chooses his effort level $a_{i,t}$ at cost $c\left(a_{i,t}\right)\Delta$, and at $t+\Delta$ the state of the project is equal to $q_{t+\Delta}=q_t+\left(\sum_{i=1}^n a_{i,t}\right)\Delta+\epsilon_{t+\Delta}$, where $\epsilon_{t+\Delta}\sim N\left(0,\sigma^2\Delta\right)$. In light of the similarities between this model and the model in Section VI of Sannikov and Skrzypacz (2007), it is reasonable to conjecture that in the continuous-time limit (*i.e.* as $\Delta\to 0$), there does not exist a PPE in which agents can achieve a higher expected discounted pay-off than the MPE at any state of the project. However, because a rigorous proof is difficult for the continuous-time game and the focus of this article is on team formation and contracting, a formal analysis of non-Markovian PPE of this game is left for future work.

Nevertheless, it is useful to present some intuition. Following Abreu *et al*. (1986), an optimal PPE involves a collusive regime and a punishment regime, and in every period, the decision whether to remain in the collusive regime or to switch is guided by the outcome in that period alone. In the context of this model, at $t+\Delta$, each agent will base his decision on $\frac{q_{t+\Delta}-q_t}{\Delta}$. As Δ decreases, two forces influence the scope of cooperation. First, the gain from a deviation in a single period decreases, which helps cooperation. On the other hand, because $\mathbb{V}\left(\frac{q_{t+\Delta}-q_t}{\Delta}\right)=\frac{\sigma^2}{\Delta}$, the agents must decide whether to switch to the punishment regime by observing noisier information, which increases the probability of type I errors (*i.e.* triggering a punishment when no deviation has occurred), thus hurting cooperation. As Sannikov and Skrzypacz (2007) show, the latter force becomes overwhelmingly stronger than the former as $\Delta \to 0$, thus eradicating any gains from cooperation.

A.5. *Linear effort costs*

The assumption that effort costs are convex affords tractability as it allows for comparative statics despite the fact that the underlying system of HJB equations does not admit a closed-form solution. However, convex effort costs also favour larger teams. Therefore, it is useful to examine how the comparative statics with respect to the team size extend to the case in which effort costs are linear; *i.e.* $c(a)=a$. In this case, the marginal value of effort is equal to $J_i'(q)-1$, so agent i finds it optimal to exert the largest possible effort level if $J_i'(q)>1$, he is indifferent across any effort level if $J_i'(q)=1$, and he exerts no effort if $J_i'(q)<1$. As a result, I shall impose a bound on the maximum effort that each agent can exert: $a\in[0,u]$. Moreover, suppose that agents are symmetric, and $\sigma=0$ so that the project evolves deterministically.[26] This game has multiple MPE: (i) a symmetric MPE with bang-bang strategies; (ii) a symmetric MPE with interior strategies; and (iii) asymmetric MPE. The reader is referred to Section 5.2 of Georgiadis *et al*. (2014) for details. Because (ii) is sensitive to the assumption that $\sigma=0$, I shall focus on the symmetric MPE with bang-bang strategies.[27]

By using equation (2) subject to equation (3) and the corresponding first-order condition, it follows that there exists a symmetric MPE in which each agent's discounted pay-off and effort strategy satisfies

$$J_n(q)=\left[-\frac{u}{r}+\left(V_n+\frac{u}{r}\right)e^{\frac{rq}{nu}}\right]\mathbf{1}_{\{q\geq\psi_n\}} \text{ and } a_n(q)=u\mathbf{1}_{\{q\geq\psi_n\}},$$

where $\psi_n=\frac{nu}{r}\ln\left(\frac{nu}{rV_n+u}\right)$. In this equilibrium, the project is completed only if $q_0\geq\psi_n$.[28] Observe that agents have stronger incentives the closer the project is to completion, as evidenced by the facts that $J_n''(q)\geq 0$ for all q, and $a_n(q)=1$ if and only if $q\geq\psi_n$. To investigate how the agents' incentives depend on the team size, one needs to examine how ψ_n depends on n. This threshold decreases in the team size n under both allocation schemes (*i.e.* both if $V_n=V$ and $V_n=V/n$ for some $V>0$) if and only if n is sufficiently small. This implies that members of an $(n+1)$-member team have stronger incentives relative to those of an n-member team as long as n is sufficiently small.

If agents maximize the team's rather than their individual discounted pay-off, then the first-best threshold $\hat{\psi}_n=\frac{nu}{r}\ln\left(\frac{u}{rV_n+u}\right)$, and it is straightforward to show that it decreases in n under both allocation schemes. Therefore, similar to the case in which effort costs are convex, members of a larger team always have stronger incentives than those of a smaller one.

B. PROOFS

This proof is organized in seven parts. I first show that an MPE for the game defined by equation (1) exists. Next I show that properties (i) through (iii) hold, and that the value functions are infinitely differentiable. Finally, I show that with symmetric agents, the equilibrium is symmetric and unique.

Part I: *Existence of an MPE.*

To show that an MPE exists, it suffices to show that a solution satisfying the system of ordinary nonlinear differential equations defined by equation (4) subject to the boundary conditions (3) for all $i=1,\ldots,n$ exists.

26. While the corresponding HJB equation can be solved analytically if effort costs are linear, the solution is too complex to obtain the desired comparative statics if $\sigma>0$.

27. In the MPE with interior strategies, $J_n'(q)=1$ for all q, and the equilibrium effort is chosen so as to satisfy this indifference condition. Together with the boundary condition $J_n(0)=V_n$, this implies that $J_n(q)=0$ and $a_n(q)=0$ for all $q\leq -V_n$. However, such an equilibrium cannot exist if $\sigma>0$, because in this case, $J_n(q)>0$ for all q even if $a_n(q)=0$.

28. If $q_0\in[\psi_n,\psi_1)$ so that each agent is not willing to undertake the project single handedly, then there exists another equilibrium in which no agent exerts any effort and the project is never completed.

To begin, fix some arbitrary $N\in\mathbb{N}$ and rewrite equations (4) and (3) as

$$J''_{i,N}(q) = \frac{2}{\sigma^2}\left[rJ_{i,N}(q)+c\left(f\left(J'_{i,N}(q)\right)\right)-\left(\sum_{j=1}^{n} f\left(J'_{j,N}(q)\right)\right)J'_{i,N}(q)\right] \tag{B.1}$$

$$\text{subject to } J_{i,N}(-N)=0 \text{ and } J_{i,N}(0)=V_i$$

for all i. Let $g_i\left(J_N,J'_N\right)$ denote the the RHS of equation (B.1), where J_N and J'_N are vectors whose i-th row corresponds to $J_{i,N}(q)$ and $J'_{i,N}(q)$, respectively , and note that $g_i(\cdot,\cdot)$ is continuous. Now fix some arbitrary $K>0$, and define a new function

$$g_{i,K}\left(J_N,J'_N\right)=\max\left\{\min\left\{g_i\left(J_N,J'_N\right),K\right\},-K\right\}.$$

Note that $g_{i,K}(\cdot,\cdot)$ is continuous and bounded. Therefore, by Lemma 4 in Hartman (1960), there exists a solution to $J''_{i,N,K}=g_{i,K}\left(J_{N,K},J'_{N,K}\right)$ on $[-N,0]$ subject to $J_{i,N,K}(-N)=0$ and $J_{i,N,K}(0)=V_i$ for all i. This Lemma, which is due to Scorza-Dragoni (1935), states:

> Let $g\left(q,J,J'\right)$ be a continuous and bounded (vector-valued) function for $\alpha\le q\le\beta$ and arbitrary $\left(J,J'\right)$. Then, for arbitrary q_α and q_β, the system of differential equations $J''=g\left(q,J,J'\right)$ has at least one solution $J=J(q)$ satisfying $J(\alpha)=q_\alpha$ and $J(\beta)=q_\beta$.

The next part of the proof involves showing that there exists a $\bar{K}$ such that $g_{i,K}\left(J_{i,N,K}(q),J'_{i,N,K}(q)\right)\in\left(-\bar{K},\bar{K}\right)$ for all i, K, and q, which will imply that the solution $J_{i,N,\bar{K}}(\cdot)$ satisfies equation (B.1) for all i. The final step involves showing that a solution exists when $N\to\infty$, so that the first boundary condition in equation (B.1) is replaced by $\lim_{q\to-\infty}J_i(q)=0$.

First, I show that $0\le J_{i,N,K}(q)\le V_i$ and $J'_{i,N,K}(q)\ge 0$ for all i and q. Because $J_{i,N,K}(0)>J_{i,N,K}(-N)=0$, either $J_{i,N,K}(q)\in[0,V_i]$ for all q, or it has an interior extreme point z^* such that $J_{i,N,K}(z^*)\notin[0,V_i]$. If the former is true, then the desired inequality holds. Suppose the latter is true. By noting that $J_{i,N,K}(\cdot)$ is at least twice differentiable, $J'_{i,N,K}(z^*)=0$, and hence $J''_{i,N,K}(z^*)=\max\left\{\min\left\{\frac{2r}{\sigma^2}J_{i,N,K}(z^*),K\right\},-K\right\}$. Suppose z^* is a global maximum. Then $J''_{i,N,K}(z^*)\le 0\Longrightarrow J_{i,N,K}(z^*)\le 0$, which contradicts the fact that $J_{i,N,K}(0)>0$. Now suppose that z^* is a global minimum. Then, $J''_{i,N,K}(z^*)\ge 0\Longrightarrow J_{i,N,K}(z^*)\ge 0$. Therefore, $0\le J_{i,N,K}(q)\le V_i$ for all i and q.

Next, let us focus on $J'_{i,N,K}(\cdot)$. Suppose that there exists a z^{**} such that $J'_{i,N,K}(z^{**})<0$. Because $J_{i,N,K}(-N)=0$, either $J_{i,N,K}(\cdot)$ is decreasing on $[-N,z^{**}]$, or it has a local maximum $\bar{z}\in(-N,z^{**})$. If the former is true, then $J'_{i,N,K}(z^{**})<0$ implies that $J_{i,N,K}(q)<0$ for some $q\in(-N,z^{**}]$, which is a contradiction because $J_{i,N,K}(q)\ge 0$ for all q. So the latter must be true. Then, $J'_{i,N,K}(\bar{z})=0$ implies that $J''_{i,N,K}(\bar{z})=\max\left\{\min\left\{\frac{2r}{\sigma^2}J_{i,N,K}(\bar{z}),K\right\},-K\right\}$. However, because $\bar{z}$ is a maximum, $J''_{i,N,K}(\bar{z})\le 0$, and together with the fact that $J_{i,N,K}(q)\ge 0$ for all q, this implies that $J_{i,N,K}(q)=0$ for all $q\in[-N,z^{**})$. But since $J'_{i,N,K}(z^{**})<0$, it follows that $J_{i,N,K}(q)<0$ for some q in the neighbourhood of z^{**}, which is a contradiction. Therefore, it must be the case that $J'_{i,N,K}(q)\ge 0$ for all i and q.

The next step involves establishing that there exists an $\bar{A}$, independent of N and K, such that $J'_{i,N,K}(q)<\bar{A}$ for all i and q. First, let $S_{N,K}(q)=\sum_{i=1}^{n}\left|J_{i,N,K}(q)\right|$. By summing $J''_{i,N,K}=g_{i,K}\left(J_{i,N,K},J'_{i,N,K}\right)$ over i, using that (i) $0\le J_{i,N,K}(q)\le V_i$ and $0\le J'_{i,N,K}(q)\le S'_{N,K}(q)$ for all i and q, (ii) $f(x)=x^{1/p}$, and (iii) $c(x)\le xc'(x)$ for all $x\ge 0$, and letting $\Gamma=r\sum_{i=1}^{n}V_i$, we have that for all q

$$\begin{aligned}
\left|S''_{N,K}(q)\right| &\le \frac{2}{\sigma^2}\sum_{i=1}^{n}\left[rJ_{i,N,K}(q)+c\left(f\left(J'_{i,N,K}(q)\right)\right)+\left[\sum_{j=1}^{n}f\left(J'_{j,N,K}(q)\right)\right]J'_{i,N,K}(q)\right]\\
&\le \frac{2}{\sigma^2}\left[\Gamma+\sum_{i=1}^{n}c'\left(c'^{-1}\left(J'_{i,N,K}(q)\right)\right)c'^{-1}\left(J'_{i,N,K}(q)\right)+S'_{N,K}(q)\sum_{j=1}^{n}f\left(J'_{j,N,K}(q)\right)\right]\\
&\le \frac{4}{\sigma^2}\left[\Gamma+nS'_{N,K}(q)f\left(S'_{N,K}(q)\right)\right]=\frac{4}{\sigma^2}\left[\Gamma+n\left(S'_{N,K}(q)\right)^{\frac{p+1}{p}}\right].
\end{aligned}$$

By noting that $S_{N,K}(0)=\sum_{i=1}^{n}V_i$, $S_{N,K}(-N)=0$, and applying the mean value theorem, it follows that there exists a $z^*\in[-N,0]$ such that $S'_{N,K}(z^*)=\frac{\sum_{i=1}^{n}V_i}{N}$. It follows that for all $z\in[-N,0]$

$$\sum_{i=1}^{n}V_i>\int_{z^*}^{z}S'_{N,K}(q)\,dq\ge\frac{\sigma^2}{4}\int_{z^*}^{z}S'_{N,K}(q)\frac{S''_{N,K}(q)}{\Gamma+n\left(S'_{N,K}(q)\right)^{\frac{p+1}{p}}}dq\ge\frac{\sigma^2}{4}\int_{0}^{S'_N(z)}\frac{s}{\Gamma+ns^{\frac{p+1}{p}}}ds,$$

where I let $s=S'_{N,K}(q)$ and used that $S'_{N,K}(q)S''_{N,K}(q)dq=S'_{N,K}(q)dS'_{N,K}(q)$. It suffices to show that there exists a $\bar{A}<\infty$ such that $\frac{\sigma^2}{4}\int_0^{\bar{A}}\frac{s}{\Gamma+ns^{\frac{p+1}{p}}}ds=\sum_{i=1}^{n}V_i$. This will imply that $S'_{N,K}(q)<\bar{A}$, and consequently $J'_{i,N,K}(q)\le\bar{A}$ for all

$q\in[-N,0]$. To show that such $\bar{A}$ exists, it suffices to show that $\int_0^\infty \frac{s}{\Gamma+ns^{\frac{p+1}{p}}}ds=\infty$. First, observe that if $p=1$, then $\int_0^\infty \frac{s}{\Gamma+ns^2}ds=\frac{1}{2n}\ln\left(\Gamma+ns^2\right)\big|_0^\infty=\infty$. By noting that $\frac{s}{\Gamma+ns^2}$ is bounded for all $s\in[0,1]$, $\frac{s}{\Gamma+ns^{\frac{p+1}{p}}}>\frac{s}{\Gamma+ns^2}$ for all $s>1$ and $p>1$, and $\int_0^\infty \frac{s}{\Gamma+ns^2}ds=\infty$, integrating both sides over $[0,\infty]$ yields the desired inequality.

Because $\bar{A}$ is independent of both N and K, this implies that $J'_{i,N,K}(q)\in\left[0,\bar{A}\right]$ for all $q\in[-N,0]$, $N\in\mathbb{N}$ and $K>0$. In addition, we know that $J_{i,N,K}(q)\in[0,V_i]$ for all $q\in[-N,0]$, $N\in\mathbb{N}$ and $K>0$. Now let $\bar{K}=\max_i\left\{\frac{2}{\sigma^2}\left[rV_i+c\left(f\left(\bar{A}\right)\right)\right]\right\}$, and observe that a solution to $J''_{i,N,\bar{K}}=g_{i,\bar{K}}\left(J_{N,\bar{K}},J'_{N,\bar{K}}\right)$ subject to $J_{i,N,\bar{K}}(-N)=0$ and $J_{i,N,\bar{K}}(0)=V_i$ for all i exists, and $g_{i,\bar{K}}\left(J_{N,\bar{K}}(q),J'_{N,\bar{K}}(q)\right)=g_i\left(J_{N,\bar{K}}(q),J'_{N,\bar{K}}(q)\right)$ for all i and $q\in[-N,0]$. Therefore, $J_{i,N,\bar{K}}(\cdot)$ solves equation (B.1) for all i.

To show that a solution for equation (B.1) exists at the limit as $N\to\infty$, I use the Arzela–Ascoli theorem, which states that:

> Consider a sequence of real-valued continuous functions $(f_n)_{n\in\mathbb{N}}$ defined on a closed and bounded interval $[a,b]$ of the real line. If this sequence is uniformly bounded and equicontinuous, then there exists a subsequence $\left(f_{n_k}\right)$ that converges uniformly.

Recall that $0\le J_{i,N}(q)\le V_i$ and that there exists a constant $\bar{A}$ such that $0\le J'_{i,N}(q)\le\bar{A}$ on $[-N,0]$ for all i and $N>0$. Hence the sequences $\left\{J_{i,N}(\cdot)\right\}$ and $\left\{J'_{i,N}(\cdot)\right\}$ are uniformly bounded and equicontinuous on $[-N,0]$. By applying the Arzela–Ascoli theorem to a sequence of intervals $[-N,0]$ and letting $N\to\infty$, it follows that the system of ODE defined by equation (4) has at least one solution satisfying the boundary conditions (3) for all i.

Finally, note that (i) the RHS of (2) is strictly concave in a_i so that the first-order condition is necessary and sufficient for a maximum and (ii) $J_i(q)\in[0,V_i]$ for all q and i so that the transversality condition $\lim_{t\to\infty}\mathbb{E}\left[e^{-rt}J_i(q_t)\right]=0$ is satisfied. Therefore, the verification theorem is satisfied (p. 123 in Chang, 2004), thus ensuring that a solution to the system given by equation (4) subject to equation (3) is indeed optimal for equation (1).

Part II: $J_i(q)>0$ *for all* q *and* i.

By the boundary conditions we have that $\lim_{q\to-\infty}J_i(q)=0$ and $J_i(0)=V_i>0$. Suppose that there exists an interior z^* that minimizes $J_i(\cdot)$ on $(-\infty,0]$. Clearly $z^*<0$. Then $J'_i(z^*)=0$ and $J''_i(z^*)\ge0$, which by applying equation (4) imply that

$$rJ_i\left(z^*\right)=\frac{\sigma^2}{2}J''_i\left(z^*\right)\ge0.$$

Because $\lim_{q\to-\infty}J_i(q)=0$, it follows that $J_i(z^*)=0$. Next, let $z^{**}=\arg\max_{q\le z^*}\{J_i(q)\}$. If z^{**} is on the boundary of the desired domain, then $J_i(q)=0$ for all $q\le z^*$. Suppose that z^{**} is interior. Then $J'_i(z^{**})=0$ and $J''_i(z^{**})\le0$ imply that $J_i(z^{**})\le0$, so that $J_i(q)=J'_i(q)=0$ for all $q<z^*$. Using equation (4) we have that

$$\left|J''_i(q)\right|\ \le\ \frac{2r}{\sigma^2}\left|J_i(q)\right|+\frac{2}{\sigma^2}(n+1)f\left(\bar{A}\right)\left|J'_i(q)\right|,$$

where this bound follows from part I of the proof. Now let $h_i(q)=|J_i(q)|+\left|J'_i(q)\right|$, and observe that $h_i(q)=0$ for all $q<z^*$, $h_i(q)\ge0$ for all q, and

$$h'_i(q)\le\left|J'_i(q)\right|+\left|J''_i(q)\right|\le\frac{2r}{\sigma^2}\left|J_i(q)\right|+\frac{2}{\sigma^2}\left[(n+1)f\left(\bar{A}\right)+\frac{\sigma^2}{2}\right]\left|J'_i(q)\right|\le Ch_i(q),$$

where $C=\frac{2}{\sigma^2}\max\left\{r,(n+1)f\left(\bar{A}\right)+\frac{\sigma^2}{2}\right\}$. Fix some $\hat{z}<z^*$, and applying the differential form of Grönwall's inequality yields $h_i(q)\le h_i\left(\hat{z}\right)\exp\left(\int_{\hat{z}}^q Cdx\right)$ for all q. Because (i) $h_i\left(\hat{z}\right)=0$, (ii) $\exp\left(\int_{z^*}^q Cdx\right)<\infty$ for all q, and (iii) $h_i(q)\ge0$ for all q, this inequality implies that $J_i(q)=0$ for all q. However this contradicts the fact that $J_i(0)=V_i>0$. As a result, $J_i(\cdot)$ cannot have an interior minimum, and there cannot exist a $z^*>-\infty$ such that $J_i(q)=0$ for all $q\le z^*$. Hence $J_i(q)>0$ for all q.

Part III: $J'_i(q)>0$ *for all* q *and* i.

Pick a K such that $J_i(0)<J_i(K)<V_i$. Such K is guaranteed to exist, because $J_i(\cdot)$ is continuous and $J_i(0)>0=\lim_{q\to-\infty}J_i(q)$. Then by the mean-value theorem there exists a $z^*\in(K,0)$ such that $J'_i(z^*)=\frac{J_i(0)-J_i(K)}{-K}=\frac{V_i-J_i(K)}{-K}>0$. Suppose that there exists a $z^{**}\le0$ such that $J'_i(z^{**})\le0$. Then by the intermediate value theorem, there exists a $\bar{z}$ between z^* and z^{**} such that $J'_i(\bar{z})=0$, which using equation (4) and part II implies that $rJ_i(\bar{z})=\frac{\sigma^2}{2}J''_i(\bar{z})>0$ (*i.e.* $\bar{z}$ is a local minimum). Consider the interval $(-\infty,\bar{z}]$. Because $\lim_{q\to-\infty}J_i(q)=0$, $J_i(\bar{z})>0$ and $J''_i(\bar{z})>0$, there exists an interior local maximum $\hat{z}<\bar{z}$. Since $\hat{z}$ is interior, it must be the case that $J'_i\left(\hat{z}\right)=0$ and $J''_i\left(\hat{z}\right)\le0$, which using equation (4) implies that $J_i\left(\hat{z}\right)\le0$. However, this contradicts the fact that $J_i(q)>0$ for all q. As a result, there cannot exist a $\bar{z}\le0$ such that $J'_i(\bar{z})\le0$. Together with part II, this proves properties (i) and (ii).

Part IV: $J_i(q)$ *is infinitely differentiable on* $(-\infty,0]$ for all i.

By noting that $\lim_{q\to-\infty} J_i(q) = \lim_{q\to-\infty} J_i'(q) = 0$ for all i, and by twice integrating both sides of equation (B.1) over the interval $(-\infty, q]$, we have that

$$J_i(q) = \int_{-\infty}^{q}\int_{-\infty}^{y} \frac{2r}{\sigma^2} J_i(z) + \frac{2}{\sigma^2}\left[c\left(f\left(J_i'(z)\right)\right) - \left(\sum_{j=1}^{n} f\left(J_j'(z)\right)\right) J_i'(z)\right] dz dy.$$

Recall that $c(a) = \frac{a^{p+1}}{p+1}$, $f(x) = x^{1/p}$, and $J_i'(q) > 0$ for all q. Since $J_i(q)$ and $J_i'(q)$ satisfy equation (4) subject to the boundary conditions (3) for all i, $J_i(q)$ and $J_i'(q)$ are continuous for all i. As a result, the function under the integral is continuous and infinitely differentiable in $J_i(z)$ and $J_i'(z)$ for all i. Because $J_i(q)$ is differentiable twice more than the function under the integral, the desired result follows by induction.

Part V: $J_i''(q) > 0$ *and* $a_i'(q) > 0$ *for all* q *and* i.

I have thus far established that for all q, $J_i(q) > 0$ and $J_i'(q) > 0$. By applying the envelope theorem to equation (4) we have that

$$rJ_i'(q) = \left[f\left(J_i'(q)\right) + A_{-i}(q)\right] J_i''(q) + \frac{\sigma^2}{2} J_i'''(q), \tag{B.2}$$

where $A_{-i}(q) = \sum_{j\neq i}^{n} f\left(J_j'(q)\right)$. Choose some finite $z \le 0$, and let $z^{**} = \arg\max\left\{J_i'(q) : q \le z\right\}$. By part III, $J_i'(z^{**}) > 0$ and because $\lim_{q\to-\infty} J_i'(q) = 0$, either $z^{**} = z$, or z^{**} is interior. Suppose z^{**} is interior. Then $J_i''(z^{**}) = 0$ and $J_i'''(z^{**}) \le 0$, which using equation (B.2) implies that $J_i'(z^{**}) \le 0$. However, this contradicts the fact that $J_i'(z^{**}) > 0$, and therefore $J_i'(\cdot)$ does not have an interior maximum on $(-\infty, z]$ for any $z \le 0$. Therefore, $z^{**} = z$, and since z was chosen arbitrarily, $J_i'(\cdot)$ is strictly increasing; *i.e.* $J_i''(q) > 0$ for all q. By differentiating $a_i(q)$ and using that $J_i'(q) > 0$ for all q, we have that

$$\frac{d}{dq} a_i(q) = \frac{d}{dq} c'^{-1}\left(J_i'(q)\right) = \frac{J_i''(q)}{c''\left(c'^{-1}\left(J_i'(q)\right)\right)} > 0.$$

Part VI: *When the agents are symmetric, the MPE is also symmetric.*

Suppose agents are symmetric; *i.e.* $V_i = V_j$ for all $i \neq j$. In any MPE, $\{J_i(\cdot)\}_{i=1}^n$ must satisfy equation (4) subject to equation (3). Pick two arbitrary agents i and j, and let $\Delta(q) = J_i(q) - J_j(q)$. Observe that $\Delta(\cdot)$ is smooth, and $\lim_{q\to-\infty}\Delta(q) = \Delta(0) = 0$. Therefore, either $\Delta(\cdot) \equiv 0$ on $(-\infty,0]$, which implies that $J_i(\cdot) \equiv J_j(\cdot)$ on $(-\infty, 0]$ and hence the equilibrium is symmetric, or $\Delta(\cdot)$ has at least one interior global extreme point. Suppose the latter is true, and denote this extreme point by z^*. By using equation (4) and the fact that $\Delta'(z^*) = 0$, we have $r\Delta(z^*) = \frac{\sigma^2}{2}\Delta''(z^*)$. Suppose that z^* is a global maximum. Then $\Delta''(z^*) \le 0$, which implies that $\Delta(z^*) \le 0$. However, because $\Delta(0) = 0$ and z^* is assumed to be a maximum, $\Delta(z^*) = 0$. Next, suppose that z^* is a global minimum. Then $\Delta''(z^*) \ge 0$, which implies that $\Delta(z^*) \ge 0$. However, because $\Delta(0) = 0$ and z^* is assumed to be a minimum, $\Delta(z^*) = 0$. Therefore, it must be the case that $\Delta(\cdot) \equiv 0$ on $(-\infty, 0]$. Since i and j were chosen arbitrarily, $J_i(\cdot) \equiv J_j(\cdot)$ on $(-\infty,0]$ for all $i \neq j$, which implies that the equilibrium is symmetric.

Part VII: *Suppose that* $V_i = V_j$ *for all* $i \neq j$. *Then the system of ordinary nonlinear differential equations defined by equation (4)* subject *to equation (3) has at most one solution.*

From part VI of the proof, we know that if agents are symmetric, then the MPE is symmetric. Therefore to facilitate exposition, I drop the notation for the i-th agent. Any solution $J(\cdot)$ must satisfy

$$rJ(q) = -c\left(f\left(J'(q)\right)\right) + nf\left(J'(q)\right)J'(q) + \frac{\sigma^2}{2} J''(q) \text{ subject to } \lim_{q\to-\infty} J(q) = 0 \text{ and } J(0) = V.$$

Suppose that there exist two functions $J_A(q), J_B(q)$ that satisfy the above boundary value problem. Then define $D(q) = J_A(q) - J_B(q)$, and note that $D(\cdot)$ is smooth and $\lim_{q\to-\infty} D(q) = D(0) = 0$. Hence, either $D(\cdot) \equiv 0$ in which case the proof is complete, or $D(\cdot)$ has an interior global extreme point z^*. Suppose the latter is true. Then $D'(z^*) = 0$, which implies that $rD(z^*) = \frac{\sigma^2}{2} D''(z^*)$. Suppose that z^* is a global maximum. Then $D''(z^*) \le 0 \Rightarrow D(z^*) \le 0$, and $D(0) = 0$ implies that $D(z^*) = 0$. Next, suppose that z^* is a global minimum. Then $D''(z^*) \ge 0 \Rightarrow D(z^*) \ge 0$, and $D(0) = 0$ implies that $D(z^*) = 0$. Therefore, it must be the case that $D(\cdot) \equiv 0$ and the proof is complete.

In light of the fact that $J_i'(q) > 0$ for all q, it follows that the first-order condition for each agent's best response always binds. As a result, any MPE must satisfy the system of ODE defined by equation (4) subject to equation (3). Since this system of ODE has a unique solution with n symmetric, it follows that in this case, the dynamic game defined by equation (1) has a unique MPE. ‖

Proof of Proposition 1. See online Appendix. ‖

Proof of Proposition 2. See online Appendix. ‖

Proof of Theorem 2. This proof is organized in four parts.

Proof for (i) under public good allocation

To begin, let us define $D_{n,m}(q)=J_m(q)-J_n(q)$, and note that $D_{n,m}(q)$ is smooth, and $D_{n,m}(0)=\lim_{q\to-\infty}D_{n,m}(q)=0$. Therefore, either $D_{n,m}(\cdot)\equiv 0$, or it has an interior extreme point. Suppose the former is true. Then $D_{n,m}(\cdot)\equiv D'_{n,m}(\cdot)\equiv D''_{n,m}(\cdot)\equiv 0$ together with equation (4) implies that $f\left(J'_n(q)\right)J'_n(q)=0$ for all q. However, this contradicts Theorem 1 (ii), so that $D_{n,m}(\cdot)$ must have an interior extreme point, which I denote by z^*. Then $D'_{n,m}(z^*)=0\Rightarrow J'_m(z^*)=J'_n(z^*)$, and using equation (4) yields

$$rD_{n,m}(z^*) = \frac{\sigma^2}{2}D''_{n,m}(z^*)+(m-n)f\left(J'_n(z^*)\right)J'_n(z^*).$$

By noting that any local interior minimum must satisfy $D''_{n,m}(z^*)\geq 0$ and hence $D_{n,m}(z^*)>0$, it follows that z^* must satisfy $D_{n,m}(z^*)\geq 0$. Therefore, $J_m(q)\geq J_n(q)$ (*i.e.* $D_{n,m}(q)\geq 0$) for all q.

I now show that $D_{n,m}(q)$ is single peaked. Suppose it is not. Then there must exist a local maximum z^* followed by a local minimum $\bar{z}>z^*$. Clearly, $D_{n,m}(\bar{z})<D_{n,m}(z^*)$, $D'_{n,m}(\bar{z})=D'_{n,m}(z^*)=0$, $D''_{n,m}(\bar{z})\geq 0\geq D''_{n,m}(z^*)$, and by Theorem 1 (iii), $J'_n(\bar{z})>J'_n(z^*)$. By using equation (4), at $\bar{z}$ we have

$$\begin{aligned} rD_{n,m}(\bar{z}) &= \frac{\sigma^2}{2}D''_{n,m}(\bar{z})+(m-n)f\left(J'_m(\bar{z})\right)J'_m(\bar{z}) \\ &> \frac{\sigma^2}{2}D''_{n,m}(z^*)+(m-n)f\left(J'_m(z^*)\right)J'_m(z^*)=rD_{n,m}(z^*), \end{aligned}$$

which contradicts the assumption that z^* is a local maximum and $\bar{z}$ is a local minimum. Therefore, there exists a $\Theta_{n,m}\leq 0$ such that $J'_m(q)\geq J'_n(q)$ (because $D'_{n,m}(q)\geq 0$), and consequently $a_m(q)>a_n(q)$, if and only if $q\leq\Theta_{n,m}$.

Proof for (i) under budget allocation

Recall that under the public good allocation scheme, we had the boundary condition $D_{n,m}(0)=0$. This condition is now replaced by $D_{n,m}(0)=\frac{V}{m}-\frac{V}{n}<0$. Therefore, $D_{n,m}(\cdot)$ is either decreasing, or it has at least one extreme point. Using similar arguments as above, it follows that any extreme point z^* is a global maximum and $D_{n,m}(\cdot)$ may be at most single peaked. Hence either $D_{n,m}(\cdot)$ is decreasing in which case $\Theta_{n,m}=-\infty$, or there exists an interior $\Theta_{n,m}$ such that $a_m(q)\geq a_n(q)$ if and only if $q\leq\Theta_{n,m}$. The details are omitted.

Proof for (ii) under public good allocation

Note that $c(a)=\frac{a^{p+1}}{p+1}$ implies that $f(x)=x^{1/p}$ and $c(f(x))=\frac{x^{\frac{p+1}{p}}}{p+1}$. As a result, equation (4) can be written for an n-member team as

$$rJ_n(q)=\left(n-\frac{1}{p+1}\right)\left(J'_n(q)\right)^{\frac{p+1}{p}}+\frac{\sigma^2}{2}J''_n(q). \tag{B.3}$$

To compare the total effort of the teams at every state of the project, we need to compare $mf\left(J'_m(q)\right)=\left(m^pJ'_m(q)\right)^{1/p}$ and $nf\left(J'_n(q)\right)=\left(n^pJ'_n(q)\right)^{1/p}$. Define $\bar{D}_{n,m}(q)=m^pJ_m(q)-n^pJ_n(q)$, and observe that $\bar{D}'_{n,m}(q)\geq 0\Longleftrightarrow ma_m(q)\geq na_n(q)$. Note that $\bar{D}_{n,m}(0)=(m^p-n^p)V>0$ and $\lim_{q\to-\infty}\bar{D}_{n,m}(q)=0$. As a result, either $\bar{D}_{n,m}(q)$ is increasing for all q, which implies that $ma_m(q)\geq na_n(q)$ for all q and hence $\Phi_{n,m}=0$, or $\bar{D}_{n,m}(q)$ has an interior extreme point z^*. Suppose the latter is true. Then $\bar{D}'_{n,m}(z^*)=0$ implies that $J'_m(z^*)=\left(\frac{n}{m}\right)^pJ'_n(z^*)$. Multiplying both sides of equation (B.3) by m^p and n^p for $J_m(\cdot)$ and $J_n(\cdot)$, respectively, and subtracting the two quantities yields

$$r\bar{D}_{n,m}(z^*) = \frac{n^p}{p+1}\left(\frac{m-n}{m}\right)\left(J'_n(z^*)\right)^{\frac{p+1}{p}}+\frac{\sigma^2}{2}\bar{D}''_{n,m}(z^*),$$

and observe that the first term in the RHS is strictly positive. Now suppose z^* is a global minimum. Then $\bar{D}''_{n,m}(z^*)\geq 0$, which implies that $\bar{D}_{n,m}(z^*)>0$, but this contradicts the facts that $\lim_{q\to-\infty}\bar{D}_{n,m}(q)=0$ and z^* is interior. Hence, z^* must be a global maximum or a local extreme point satisfying $\bar{D}_{n,m}(z^*)\geq 0$.

To complete the proof for this case, I now show that $\bar{D}_{n,m}(\cdot)$ can be at most single peaked. Suppose that the contrary is true. Then there exists a local maximum z^* followed by a local minimum $\bar{z}>z^*$. Because $\bar{D}'_{n,m}(z^*)=\bar{D}'_{n,m}(\bar{z})=0$, $\bar{D}''_{n,m}(\bar{z})\geq 0\geq\bar{D}''_{n,m}(z^*)$, and by Theorem 1 (iii) $J'_n(z^*)<J'_n(\bar{z})$, it follows that $\bar{D}_{n,m}(z^*)<\bar{D}_{n,m}(\bar{z})$. However, this contradicts the facts that z^* is a local maximum and $\bar{z}$ is a local minimum, which implies that $\bar{D}_{n,m}(\cdot)$ is either strictly increasing in which case $\Phi_{n,m}=0$, or it has a global interior maximum and no other local extreme points, in which case there exists an interior $\Phi_{n,m}$ such that $ma_m(q)\geq na_n(q)$ if and only if $q\leq\Phi_{n,m}$.

Proof for (ii) under budget allocation

The only difference compared to the proof under public good allocation is the boundary condition at 0; *i.e.* $\bar{D}_{n,m}(0)=m^pJ_m(0)-n^pJ_n(0)=\left(m^{p-1}-n^{p-1}\right)V>0$ (recall $p\geq 1$). As a result, the same proof applies. Note that if $p=1$ (*i.e.* effort costs are quadratic), then $\bar{D}_{n,m}(0)=0$ and hence $\Phi_{n,m}$ must be interior (whereas otherwise $\bar{D}_{n,m}(0)>0$ and hence $\Phi_{n,m}\leq 0$). ‖

Proof of Proposition 3. Let us first consider the statement under public good allocation. In the proof of Theorem 2 (i), I showed that $D_{n,n+1}(q)=J_{n+1}(q)-J_n(q)\geq 0$ for all q. This implies that $J_{n+1}(q_0)\geq J_n(q_0)$ for all n and q_0, and hence the optimal partnership size $n=\infty$ for any project length $|q_0|$.

Now consider the statement under budget allocation. In the proof of Theorem 2 (i), I showed that $D_{n,m}(\cdot)=J_m(\cdot)-J_n(\cdot)$ is either decreasing, or it has exactly one extreme point which must be a maximum. Because $\lim_{q\to-\infty}D_{n,n+1}(q)=0$ and $D_{n,m}(0)<0$, there exists a threshold $T_{n,m}$ (may be $-\infty$) such that $J_m(q_0)\geq J_n(q_0)$ if and only if $q_0\leq -T_{n,m}$, or equivalently if and only if $|q_0|\geq T_{n,m}$. By noting that the necessary conditions for the Monotonicity Theorem (*i.e.* Theorem 4) of Milgrom and Shannon (1994) to hold are satisfied, it follows that the optimal partnership size increases in the length of the project $|q_0|$. ‖

Proof of Theorem 3. See online Appendix. ‖

Proof of Theorem 4. To prove this result, first fix a set of arbitrary milestones $Q_1<\ldots<Q_K=0$ where K is arbitrary but finite, and assume that the manager allocates budget $w_k>0$ for compensating the agents upon reaching milestone k for the first time. Now consider the following two compensation schemes. Let $B=\sum_{k=1}^{K}w_k$. Under *scheme (a)*, each agent is paid B/n upon completion of the project and receives no intermediate compensation while the project is in progress. Under *scheme (b)*, each agent is paid $w_k/n\mathbb{E}_{\tau_k}[e^{r\tau_k}|Q_k]$ when q_t hits Q_k for the first time, where τ_k denotes the random time to completion given that the current state of the project is Q_k. I shall show that the manager is always better off using scheme (a) relative to scheme (b). Note that scheme (b) ensures that the expected total cost for compensating each agent equals B/n to facilitate comparison between the two schemes.

This proof is organized in three parts. In part I, I introduce the necessary functions (*i.e.* ODEs) that will be necessary for the proof. In part II, I show that each agent exerts higher effort under scheme (a) relative to scheme (b). Finally, in part III, I show that the manager's expected discounted profit is higher under scheme (a) relative to scheme (b) for any choice of Q_k's and w_k's.

Part I: To begin, I introduce the expected discounted pay-off and discount rate functions that will be necessary for the proof. Under *scheme (a)*, given the current state q, each agent's expected discounted pay-off satisfies

$$rJ(q)=-c\left(f\left(J'(q)\right)\right)+nf\left(J'(q)\right)J'(q)+\frac{\sigma^2}{2}J''(q) \text{ subject to } \lim_{q\to-\infty}J(q)=0 \text{ and } J(0)=\frac{B}{n}.$$

However, under *scheme (b)*, given the current state q and that $k-1$ milestones have been reached, each agent's expected discounted pay-off, which is denoted by $J_k(q)$, satisfies

$$rJ_k(q)=-c\left(f\left(J_k'(q)\right)\right)+nf\left(J_k'(q)\right)J_k'(q)+\frac{\sigma^2}{2}J_k''(q) \text{ on } (-\infty,Q_k]$$

subject to

$$\lim_{q\to-\infty}J_k(q)=0 \text{ and } J_k(Q_k)=\frac{w_k}{n\mathbb{E}_{\tau_k}[e^{r\tau_k}|Q_k]}+J_{k+1}(Q_k),$$

where $J_{K+1}(Q_K)=0$.[29] The second boundary condition states that upon reaching milestone Q_k for the first time, each agent is paid $w_k/n\mathbb{E}_{\tau_k}[e^{r\tau_k}|Q_k]$, and he receives the continuation value $J_{k+1}(Q_k)$ from future progress. Eventually upon reaching the K-th milestone, the project is completed so that each agent is paid w_K/n, and receives no continuation value. Note that due to the stochastic evolution of the project, even after the k-th milestone has been reached for the fist time, the state of the project may drift below Q_k. Therefore, the first boundary condition ensures that as $q\to-\infty$, the expected time until the project is completed so that each agent collects his/her reward diverges to ∞, which together with the fact that $r>0$, implies that his/her expected discounted pay-off asymptotes to 0. It follows from Theorem 1 that for each k, $J_k(\cdot)$ exists, it is unique, smooth, strictly positive, strictly increasing, and strictly convex on its domain.

Next, let us denote the expected *present discounted value* function under scheme (a), given the current state q, by $T(q)=\mathbb{E}_\tau\left[e^{-r\tau}|q\right]$. Using the same approach as used to derive the manager's HJB equation, it follows that

$$rT(q)=nf\left(J'(q)\right)T'(q)+\frac{\sigma^2}{2}T''(q) \text{ subject to } \lim_{q\to-\infty}T(q)=0 \text{ and } T(0)=1.$$

The first boundary condition states that as $q\to-\infty$, the expected time until the project is completed diverges to ∞, so that $\lim_{q\to-\infty}T(q)=0$. However, when the project is completed so that $q=0$, then $\tau=0$ with probability 1, which implies that $T(0)=1$.

29. Since this proof considers a fixed team size n, we use to subscript k to denote that $k-1$ milestones have been reached.

Next, let us consider scheme (b). Similarly, we denote the expected *present discounted value* function, given the current state q and that $k-1$ milestones have been reached, by $T_k(q)=\mathbb{E}_{\tau_k}\left[e^{-r\tau_k}\,|q\right]$. Then, it follows that

$$rT_k(q)=nf\left(J_k'(q)\right)T_k'(q)+\frac{\sigma^2}{2}T_k''(q) \text{ on } (-\infty,Q_k]$$

subject to

$$\lim_{q\to-\infty}T_k(q)=0\ ,\ T_k(Q_k)=T_{k+1}(Q_k) \text{ for all } k\leq n,$$

where $T_{K+1}(Q_K)=1$. The first boundary condition has the same interpretation as above. The second boundary condition ensures value matching; *i.e.* that upon reaching milestone k for the first time, $T_k(Q_k)=T_{k+1}(Q_k)$. Using the same approach as used in Theorem 3, it is straightforward to show that $T(\cdot)$ and for each k, $T_k(\cdot)$ exists, it is unique, smooth, strictly positive, and strictly increasing on its domain.

Note that by Jensen's inequality, $\frac{1}{\mathbb{E}_{\tau_k}\left[e^{r\tau_k}\right]}\leq\mathbb{E}_{\tau_k}\left[e^{-r\tau_k}\right]$. Therefore, using this inequality, and the second boundary condition for $J_k(\cdot)$, it follows that $J_k(Q_k)\leq\frac{w_k}{n}T_k(Q_k)+J_{k+1}(Q_k)$.

Part II: The next step of the proof is to show that for any k, $J(Q_k)\geq J_k(Q_k)$, and as a consequence of Proposition 1 (i), $J'(q)\geq J_k'(q)$ for all $q\leq Q_k$. This will imply that agents exert higher effort under scheme (a) at every state of the project. To proceed, let us define $\Delta_k(q)=J(q)-J_k(q)-\frac{1}{n}\left(\sum_{i=1}^{k-1}w_i\right)T_k(q)$ on $(-\infty,Q_k]$ for all k, and note that $\lim_{q\to-\infty}\Delta_k(q)=0$ and $\Delta_k(\cdot)$ is smooth.

First, I consider the case in which $k=K$, and then I proceed by backward induction. Noting that $\Delta_K(Q_K)=0$ (where $Q_K=0$), either $\Delta_K(\cdot)\equiv0$ on $(-\infty,Q_K]$, or $\Delta_K(\cdot)$ has some interior global extreme point z. If the former is true, then $\Delta_K(q)=0$ for all $q\leq Q_K$, so that $J(Q_K)\geq J_K(Q_K)$. Now suppose that the latter is true. Then $\Delta_K'(z)=0$ so that

$$\begin{aligned}r\Delta_K(z) &= -c\big(f\left(J'(z)\right)\big)+nf\left(J'(z)\right)J'(z)+c\big(f\left(J_K'(z)\right)\big)-nf\left(J_K'(z)\right)J_K'(z)\\ &\quad-\left(\sum_{i=1}^{m-1}w_i\right)f\left(J_K'(z)\right)T_K'(z)+\frac{\sigma^2}{2}\Delta_K''(z).\end{aligned}$$

Because $\Delta_K'(z)=0$ implies that $\left(\sum_{i=1}^{k-1}w_i\right)T_K'(z)=n\big[J'(z)-J_K'(z)\big]$, the above equation can be re-written as

$$\begin{aligned}r\Delta_K(z) &= c\big(f\left(J_K'(z)\right)\big)-c\big(f\left(J'(z)\right)\big)+nf\left(J'(z)\right)J'(z)-nf\left(J_K'(z)\right)J'(z)+\frac{\sigma^2}{2}\Delta_K''(z)\\ &= \left\{\frac{\big[J_K'(z)\big]^{\frac{p+1}{p}}-\big[J'(z)\big]^{\frac{p+1}{p}}}{p+1}+n\big[J'(z)\big]^{\frac{p+1}{p}}-n\big[J_K'(z)\big]^{\frac{1}{p}}J'(z)\right\}+\frac{\sigma^2}{2}\Delta_K''(z).\end{aligned}$$

To show that the term in brackets is strictly positive, note that $J(Q_K)>J_K(Q_K)$ so that $J'(z)>J_K'(z)$ by Proposition 1 (i), and $J_K'(z)>0$. Therefore, let $x=\frac{J_K'(z)}{J'(z)}$, where $x<1$, and observe that the term in brackets is non-negative if and only if

$$\begin{aligned}n(p+1)\big[J'(z)\big]^{\frac{p+1}{p}}-\big[J'(z)\big]^{\frac{p+1}{p}} &\geq n(p+1)\big[J_K'(z)\big]^{\frac{1}{p}}J'(z)-\big[J_K'(z)\big]^{\frac{p+1}{p}}\\ \Longrightarrow n(p+1)-1 &\geq n(p+1)x^{\frac{1}{p}}-x^{\frac{p+1}{p}}.\end{aligned}$$

Because the RHS is strictly increasing in x, and it converges to the LHS as $x\to1$, it follows that the above inequality holds.

Suppose that z is a global minimum. Then, $\Delta_K''(z)\geq0$ together with the fact that the term in brackets is strictly positive implies that $\Delta_K(z)>0$. Therefore, any interior global minimum must satisfy $\Delta_K(z)\geq0$, which in turn implies that $\Delta_K(q)\geq0$ for all q. As a result, $\Delta_K(Q_{K-1})\geq0$ or equivalently $J(Q_{K-1})\geq J_K(Q_{K-1})+\frac{1}{n}\left(\sum_{i=1}^{K-1}w_i\right)T_K(Q_{K-1})$.

Now consider $\Delta_{K-1}(\cdot)$, and note that $\lim_{q\to-\infty}\Delta_{K-1}(q)=0$. By using the last inequality, that $J_{K-1}(Q_{K-1})\leq\frac{w_{K-1}}{n}T_{K-1}(Q_{K-1})+J_K(Q_{K-1})$, and $T_{K-1}(Q_{K-1})=T_K(Q_{K-1})$, it follows that

$$\Delta_{K-1}(Q_{K-1})=J(Q_{K-1})-J_{K-1}(Q_{K-1})-\frac{1}{n}\left(\sum_{i=1}^{K-2}w_i\right)T_{K-1}(Q_{K-1})\geq0.$$

Therefore, either $\Delta_{K-1}(\cdot)$ is increasing on $(-\infty,Q_{K-1}]$, or it has some interior global extreme point $z<Q_{K-1}$ such that $\Delta_{K-1}'(z)=0$. If the former is true, then $\Delta_{K-1}(Q_{K-2})\geq0$. If the latter is true, then by applying the same technique as above we can again conclude that $\Delta_{K-1}(Q_{K-2})\geq0$.

Proceeding inductively, it follows that for all $k\in\{2,\ldots,K\}$, $\Delta_k(Q_{k-1})\geq0$ or equivalently $J(Q_{k-1})\geq J_k(Q_{k-1})+\frac{1}{n}\left(\sum_{i=1}^{k-1}w_i\right)T_k(Q_{k-1})$ and using that $J_{k-1}(Q_{k-1})\leq\frac{w_{k-1}}{n}T_k(Q_{k-1})+J_k(Q_{k-1})$, it follows that $J(Q_{k-1})\geq J_{k-1}(Q_{k-1})$.

Finally, by using Proposition 1 (i), it follows that for all k, $J'(q) \geq J_k'(q)$ for all $q \leq Q_k$. In addition, it follows that for all k, $J(q) \geq J_k(q)$ for all $q \leq Q_k$, which implies that given a fixed expected budget, the agents are better off if their rewards are backloaded.

Part III: Given a fixed expected budget B, the manager's objective is to maximize $\mathbb{E}_\tau\left[e^{-r\tau}\,|q_0\right]$ or equivalently $T(q_0)$, where τ denotes the completion time of the project, and it depends on the agents' strategies, which themselves depend on the set of milestones $\{Q_k\}_{k=1}^K$ and payments $\{w_k\}_{k=1}^K$. Since $q_0 < Q_1 < \ldots < Q_K$, it suffices to show that $T(q_0) \geq T_1(q_0)$ to conclude that given any arbitrary choice of $\{Q_k, w_k\}_{k=1}^K$, the manager is better off compensating the agents only upon completing the project relative to also rewarding them for reaching intermediate milestones.

Define $D_k(q) = T(q) - T_k(q)$ on $(-\infty, Q_k]$ for all $k \in \{1,\ldots,K\}$, and note that $D_k(\cdot)$ is smooth and $\lim_{q\to-\infty} D_k(q) = 0$. Let us begin with the case in which $k = K$. Note that $D_K(Q_K) = 0$ (where $Q_K = 0$). So either $D_K(\cdot) \equiv 0$ on $(-\infty, Q_K]$, or $D_K(\cdot)$ has an interior global extreme point $\bar{z} < Q_K$. Suppose that $\bar{z}$ is a global minimum. Then $D_K'(\bar{z}) = 0$ so that

$$rD_K(\bar{z}) = n\left[J'(\bar{z}) - J_K'(\bar{z})\right]T'(\bar{z}) + \frac{\sigma^2}{2} D_K''(\bar{z}).$$

Recall that $J'(q) \geq J_k'(q)$ for all $q \leq Q_k$ from part II. Since $\bar{z}$ is assumed to be a minimum, it must be true that $D_K''(\bar{z}) \geq 0$, which implies that $D_K(\bar{z}) \geq 0$. Therefore, any interior global minimum must satisfy $D_K(\bar{z}) \geq 0$, which implies that $D_K(q) \geq 0$ for all $q \leq Q_K$. As a result, $T(Q_{K-1}) \geq T_K(Q_{K-1}) = T_{K-1}(Q_{K-1})$.

Next, consider $D_{K-1}(\cdot)$, recall that $\lim_{q\to-\infty} D_{K-1}(q) = 0$, and note that the above inequality implies that $D_{K-1}(Q_{K-1}) \geq 0$. By using the same technique as above, it follows that $T(Q_{K-2}) \geq T_{K-1}(Q_{K-2}) = T_{K-2}(Q_{K-2})$, and proceeding inductively we obtain that $D_1(q) \geq 0$ for all $q \leq Q_1$ so that $T(q_0) \geq T_1(q_0)$. ‖

Proof of Proposition 4. See online Appendix. ‖

Proof of Lemma 1. Let us denote the manager's expected discounted profit when he/she employs n (symmetric) agents by $F_n(\cdot)$, and note that $\lim_{q\to-\infty} F_n(q) = 0$ and $F_n(0) = U - V > 0$ for all n. Now let us define $\Delta_{n,m}(\cdot) = F_m(\cdot) - F_n(\cdot)$ and note that $\Delta_{n,m}(\cdot)$ is smooth and $\lim_{q\to-\infty} \Delta_{n,m}(q) = \Delta_{n,m}(0) = 0$. Note that either $\Delta_{n,m}(\cdot) \equiv 0$, or $\Delta_{n,m}(\cdot)$ has at least one global extreme point. Suppose that the former is true. Then, $\Delta_{n,m}(q) = \Delta_{n,m}'(q) = \Delta_{n,m}''(q) = 0$ for all q, which together with equation (5) implies that $[A_m(q) - A_n(q)]F_n'(q) = 0$ for all q, where $A_n(\cdot) \equiv na_n(\cdot)$. However, this is a contradiction, because $A_m(q) > A_n(q)$ for at least some q by Theorem 2 (ii), and $F_n'(q) > 0$ for all q by Theorem 3 (i). Therefore, $\Delta_{n,m}(\cdot)$ has at least one global extreme point, which I denote by $\bar{z}$. By using that $\Delta_{n,m}'(\bar{z}) = 0$ and (5), we have that

$$r\Delta_{n,m}(\bar{z}) = [A_m(\bar{z}) - A_n(\bar{z})]F_n'(\bar{z}) + \frac{\sigma^2}{2}\Delta_{n,m}''(\bar{z}).$$

Recall that $F_n'(\bar{z}) > 0$, and from Theorem 2 (ii) that for each n and m there exists an (interior) threshold $\Phi_{n,m}$ such that $A_m(q) \geq A_n(q)$ if and only if $q \leq \Phi_{n,m}$. It follows that $\bar{z}$ is a global maximum if $\bar{z} \leq \Phi_{n,m}$, while it is a global minimum if $\bar{z} \geq \Phi_{n,m}$. Next observe that if $\bar{z} \leq \Phi_{n,m}$ then any local minimum must satisfy $\Delta_{n,m}(\bar{z}) \geq 0$, while if $\bar{z} \geq \Phi_{n,m}$ then any local maximum must satisfy $\Delta_{n,m}(\bar{z}) \leq 0$. Therefore, either one of the following three cases must be true: (i) $\Delta_{n,m}(\cdot) \geq 0$ on $(-\infty, 0]$, or (ii) $\Delta_{n,m}(\cdot) \leq 0$ on $(-\infty, 0]$, or (iii) $\Delta_{n,m}(\cdot)$ crosses 0 exactly once from above. Therefore, there exists a $T_{n,m}$ such that $\Delta_{n,m}(q_0) \geq 0$ if and only if $q_0 \leq -T_{n,m}$, or equivalently the manager is better off employing $m > n$ rather than n agents if and only if $|q_0| \geq T_{n,m}$. By noting that $T_{n,m} = 0$ under case (i), and $T_{n,m} = \infty$ under case (ii), the proof is complete. ‖

Proof of Proposition 5. All other parameters held constant, the manager chooses the team size $n \in \mathbb{N}$ to maximize his/her expected discounted profit at q_0; *i.e.* he/she chooses $n(|q_0|) = \arg\max_{n\in\mathbb{N}}\{F_n(q_0)\}$. By noting that the necessary conditions for the Monotonicity Theorem (*i.e.* Theorem 4) of Milgrom and Shannon (1994) to hold are satisfied, it follows that the optimal team size $n(|q_0|)$ is (weakly) increasing in the project length $|q_0|$. ‖

Proof of Propositions 6–9. See online Appendix. ‖

Acknowledgments. I am grateful to the co-editor, Marco Ottaviani, and to three anonymous referees whose comments have immeasurably improved this paper. I am indebted to Simon Board and Chris Tang for their guidance, suggestions and criticisms. I also thank Andy Atkeson, Sushil Bikhchandani, Andrea Bertozzi, Miaomiao Dong, Florian Ederer, Hugo Hopenhayn, Johannes Hörner, Moritz Meyer-Ter-Vehn, Kenny Mirkin, James Mirrlees, Salvatore Nunnari, Ichiro Obara, Tom Palfrey, Gabriela Rubio, Tomasz Sadzik, Yuliy Sannikov, Pierre-Olivier Weill, Bill Zame, Joe Zipkin, as well as seminar participants at Bocconi, BU, Caltech, Northwestern University, NYU, TSE, UCLA, UCSD, the University of Chicago, the University of Michigan, USC, UT Austin, UT Dallas, the Washington University in St. Louis, the 2012 Southwest Economic Theory conference, the 2012 North American Summer Meetings of the Econometric Society, GAMES 2012, and the SITE 2013 Summer Workshop for many insightful comments and suggestions.

Supplementary Data

Supplementary materials are available at *Review of Economic Studies* online.

REFERENCES

ABREU, D., PEARCE, D. and STACCHETTI, E. (1986), "Optimal Cartel Equilibria with Imperfect Monitoring", *Journal of Economic Theory*, **39**, 251–269.

ADMATI, A. R. and PERRY, M. (1991), "Joint Projects without Commitment", *Review of Economic Studies*, **58**, 259–276.

ALCHIAN, A. A. and DEMSETZ, H. (1972), "Production, Information Costs, and Economic Organization", *American Economic Review*, **62**, 777–795.

ANDREONI, J. (1988), "Privately Provided Public Goods in a Large Economy: The Limits of Altruism", *Journal of Public Economics*, **35**, 57–73.

ANDREONI, J. (1990), "Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving", *Economic Journal*, **100**, 464–477.

BAGNOLI, M. and LIPMAN, B. L. (1989), "Provision of Public Goods: Fully Implementing the Core through Private Contributions", *Review of Economic Studies*, **56**, 583–601.

BARON, J. and KREPS, D. (1999), *Strategic Human Resources: Frameworks for General Managers* (New York: John Wiley).

BATTAGLINI, M., NUNNARI, S. and PALFREY, T. (2013), "Dynamic Free Riding with Irreversible Investments", *American Economic Review*, forthcoming.

BERGIN, J. and MACLEOD, W. B. (1993), "Continuous Time Repeated Games", *International Economic Review*, **43**, 21–37.

BOLTON, P. and HARRIS, C. (1999), "Strategic Experimentation", *Econometrica*, **67**, 349–374.

BONATTI, A. and HÖRNER, J. (2011), "Collaborating", *American Economic Review*, **101**, 632–663.

CAMPBELL, A., EDERER, F. P. and SPINNEWIJN, J. (2013), "Delay and Deadlines: Freeriding and Information Revelation in Partnerships", *American Economic Journal: Microeconomics*, forthcoming.

CAO, D. (2014), "Racing Under Uncertainty: Boundary Value Problem Approach", *Journal of Economic Theory*, **151**, 508–527.

CHANG, F.R. (2004), *Stochastic Optimization in Continuous Time* (Cambridge: Cambridge University Press).

COMPTE, O. and JEHIEL, P. (2004), "Gradualism in Bargaining and Contribution Games", *Review of Economic Studies*, **71**, 975–1000.

DIXIT, A. (1999), "The Art of Smooth Pasting", Taylor & Francis.

EDERER, F. P., GEORGIADIS, G. and NUNNARI, S. (2014), "Team Size Effects in Dynamic Contribution Games: Experimental Evidence", Work in Progress.

ESTEBAN, J. and RAY, D. (2001), "Collective Action and the Group Size Paradox", *American Political Science Review*, **95**, 663–972.

FERSHTMAN, C. and NITZAN, S. (1991), "Dynamic Voluntary Provision of Public Goods", *European Economic Review*, **35**, 1057–1067.

GEORGIADIS, G. (2011), "Projects and Team Dynamics", Unpublished Manuscript.

GEORGIADIS, G., LIPPMAN, S. A. and TANG, C. S. (2014), "Project Design with Limited Commitment and Teams", *RAND Journal of Economics*, forthcoming.

GRADSTEIN, M. (1992), "Time Dynamics and Incomplete Information in the Private Provision of Public Goods", *Journal of Political Economy*, **100**, 581–597.

HARTMAN, P. (1960), "On Boundary Value Problems for Systems of Ordinary, Nonlinear, Second Order Differential Equations", *Transactions of the American Mathematical Society*, **96**, 493–509.

HAMILTON, B. H., NICKERSON, J. A. and OWAN, H. (2003), "Team Incentives and Worker Heterogeneity: An Empirical Analysis of the Impact of Teams on Productivity and Participation", *Journal of Political Economy*, **111**, 465–497.

HARRIS, C. and VICKERS, J. (1985), "Perfect Equilibrium in a Model of a Race", *Review of Economic Studies*, **52**, 193–209.

HARVARD BUSINESS SCHOOL PRESS. (2004), *Managing Teams: Forming a Team that Makes a Difference* (Harvard Business School Press).

HOLMSTRÖM, B. (1982), "Moral Hazard in Teams", *Bell Journal of Economics*, **13**, 324–340.

ICHNIOWSKI, C. and SHAW, K. (2003), "Beyond Incentive Pay: Insiders' Estimates of the Value of Complementary Human Resource Management Practices", *Journal of Economic Perspectives*, **17**, 155–180.

KANDEL, E. and LAZEAR, E. P. (1992), "Peer Pressure and Partnerships", *Journal of Political Economy*, **100**, 801–817.

KESSING, S.G. (2007), "Strategic Complementarity in the Dynamic Private Provision of a Discrete Public Good", *Journal of Public Economic Theory*, **9**, 699–710.

LAKHANI, K. R. and CARLILE, P. R. (2012), "Myelin Repair Foundation: Accelerating Drug Discovery Through Collaboration", HBS Case No. 9-610-074 (Boston: Harvard Business School).

LAWLER, E. E., MOHRMAN, S. A. and BENSON, G. (2001), *Organizing for High Performance: Employee Involvement, TQM, Reengineering, and Knowledge Management in the Fortune 1000* (San Fransisco: Jossey-Bass).

LAZEAR, E. P. (1989), "Pay Equality and Industrial Politics", *Journal of Political Economy*, **97**, 561–580.

LAZEAR, E. P. (1998), *Personnel Economics for Managers* (New York: Wiley).
LAZEAR, E. P. and SHAW, K. L. (2007), "Personnel Economics: The Economist's View of Human Resources", *Journal of Economic Perspectives*, **21**, 91–114.
LEGROS, P. and MATTHEWS, S. (1993), "Efficient and Nearly-Efficient Partnerships", *Review of Economic Studies*, **60**, 599–611.
LEWIS, G. and BAJARI, P. (2014), "Moral Hazard, Incentive Contracts, and Risk: Evidence from Procurement", *Review of Economic Studies*, forthcoming.
LOCKWOOD, B. and THOMAS, J. P. (2002), "Gradualism and Irreversibility", *Review of Economic Studies*, **69**, 339–356.
MA, C., MOORE, J. and TURNBULL, S. (1988), "Stopping agents from Cheating", *Journal of Economic Theory*, **46**, 355–372.
MARX, L. M. and MATTHEWS, S. A. (2000), "Dynamic Voluntary Contribution to a Public Project", *Review of Economic Studies*, **67**, 327–358.
MATTHEWS, S. A. (2013), "Achievable Outcomes of Dynamic Contribution Games", *Theoretical Economics*, **8**, 365–403.
MILGROM, P. and SHANNON, C. (1994), "Monotone Comparative Statics", *Econometrica*, **62**, 157–180.
OLSON, M. (1965), *The Logic of Collective Action: Public Goods and the Theory of Groups* (Cambridge, MA: Harvard University Press).
OSBORNE, M. J. (2003), *An Introduction to Game Theory* (New York: Oxford University Press).
RAHMANI, M., ROELS, G. and KARMARKAR, U. (2013), "Contracting and Work Dynamics in Collaborative Projects" (UCLA Anderson School of Management, Working Paper).
SANNIKOV, Y. and SKRZYPACZ, A. (2007), "Impossibility of Collusion under Imperfect Monitoring with Flexible Production", *American Economic Review*, **97**, 1794–1823.
SANNIKOV, Y. (2008), "A Continuous-Time Version of the Principal-Agent Problem", *Review of Economic Studies*, **75**, 957–984.
SCHELLING, T. C. (1960), *The Strategy of Conflict* (Cambridge, Massachusetts: Harvard University Press).
STRAUSZ, R. (1999), "Efficiency in Sequential Partnerships", *Journal of Economic Theory*, **85**, 140–156.
WEBER, R. (2006), "Managing Growth to Achieve Efficient Coordination in Large Groups", *American Economic Review*, **96**, 114–126.
YILDIRIM, H. (2006), "Getting the Ball Rolling: Voluntary Contributions to a Large-Scale Public Project", *Journal of Public Economic Theory*, **8**, 503–528.