

Project design with limited commitment and teams

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We study the interaction between a group of agents who exert effort to complete a project and a manager who chooses its objectives. The manager has limited commitment power so that she can commit to the objectives only when the project is sufficiently close to completion. We show that the manager has incentives to extend the project as it progresses. This result has two implications. First, the manager will choose a larger project if she has less commitment power. Second, the manager should delegate the decision rights over the project size to the agents unless she has sufficient commitment power.

1. Introduction

■ A key component of a project, such as the development of a new product, is choosing the features that must be included before the decision maker deems the product ready to market. Naturally, which features should be included must be communicated to the relevant stakeholders. When choosing these features, the decision maker must balance the added value derived from a bigger or a more complex project (i.e., one that contains more features) against the additional cost associated with designing and implementing the additional features. Such costs include not only engineering inputs but also the implicit cost associated with delayed cash flow.

An intrinsic challenge involved in choosing the requirements of a project is that the manager may not be able to commit to them in advance. This can be due to the fact that the requirements are difficult to describe; for example, if the project involves significant novelty in quality or design. What we have in mind about the incontractibility of the project requirements was eloquently posed by Tirole (1999):

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In practice, the parties are unlikely to be able to describe precisely the specifics of an innovation in an *ex ante* contract, given that the research process is precisely concerned with finding out these specifics, although they are able to describe it *ex post*.

In addition, committing to specific requirements may be difficult due to an asymmetry in the bargaining power of the parties involved. For example, if the project is undertaken in-house where the manager can significantly influence the team members' career paths and contracts are typically implicit, then the manager will tend to be less able to commit relative to the case in which the project is outsourced and contracts are explicit.

A fitting example stems from the development of Apple's first generation iPod. Anecdotal evidence indicates that Steve Jobs kept changing the requirements of the iPod as the project progressed. In particular, the development team would get orders such as, "Steve doesn't think it is loud enough," or "the sharps are not sharp enough," or "the menu is not coming up fast enough" (*Wired Magazine*, 2004). This suggests that committing to a set of features and requirements early on was not desirable in the development of an innovative new product such as the iPod back in 2001.

Similarly, consider a design project, such as the process of designing a new automobile. If the decision maker were able to provide a precise description of the design for it to be approved at the outset of the process, then there would be no need to undertake it in the first place. In practice, the decision maker must guide the design team as the final product takes shape to fulfill her objectives.

Finally, changing the requirements of a project (often referred to as *moving the goal post*) is common in project management (Brenner, 2001). One prominent example is Boeing's 787 Dreamliner project. The original set of project goals was to develop a lightweight, fuel-efficient aircraft that would meet the customers' needs for lower operating costs. On top of these original project goals, Boeing's senior management then decided to outsource parts of the design, engineering, and manufacturing processes to some 50 external "strategic partners" to reduce development costs and time. From the perspective of Boeing's engineers, a new goal was appended to the original set of project goals: restructure the design and manufacturing processes by overseeing and coordinating the work performed by internal engineers and those external strategic partners (Brenner, 2013). In addition, Boeing sought to maximize the usage of composite materials to reduce the weight of the 787 aircraft. However, due to unproven technology, Boeing's engineers were unable to provide accurate specifications at the outset of the project without the risk of compromising the intended performance and safety standards. Indeed, it was only in 2009, after examining the results of a series of fatigue and stress tests, that Boeing finally committed to the amount of composite materials to be used on the 787 aircraft (Tang and Zimmerman, 2009).

In this article, we propose a parsimonious model to analyze a dynamic contribution game in which a group of agents collaborate to complete a project. The project progresses at a rate that depends on the agents' costly effort, and it generates a payoff upon completion. Formally, the state of the project q_t starts at 0, and it progresses according to $dq_t = \sum_{i=1}^n a_{i,t} dt$, where $a_{i,t}$ denotes the effort level of agent i at time t . The project generates a payoff at the first stopping time τ such that $q_\tau = Q$, where Q is a one-dimensional parameter that captures the project requirements, or equivalently, the project size. The manager is the residual claimant of the project, and she possesses the decision rights over its size. It is noteworthy that the model is very tractable, and payoffs and strategies are derived in closed-form.

In Section 3 we analyze the agents' problem given a fixed project size. We characterize the (essentially) unique Markov Perfect Equilibrium (MPE), wherein at every moment, each agent's strategy depends solely on the current state of the project. In addition, we characterize a continuum of (non-Markovian) Public Perfect Equilibria (PPE), in which along the equilibrium path, the agents choose their effort by maximizing a convex combination of their individual and the team's discounted payoff. Motivated by the concepts of *insiders* and *outsiders* (who act in the best interest of the team and in their own best interest, respectively) introduced by Akerlof and

Kranton (2000), the weight that agents place on maximizing the team's payoff can be interpreted as a measure of the team's *cooperativeness*. A key result is that the agents exert greater effort the closer the project is to completion. Intuitively, this is because they discount time and are compensated upon completion, so that their incentives become stronger as the project progresses.

In Section 4 we analyze the manager's problem and determine her optimal project size. Her fundamental trade-off is that a larger project generates a bigger payoff upon completion but requires more effort (and hence takes longer) to be completed. To model the manager's limited ability to commit, we assume that given the current state of the project q_t , she can commit to any $Q \in [q_t, q_t + y]$, where $y \geq 0$ captures the commitment power. Therefore, the manager can commit to a project size $Q > y$ only after the agents have made sufficient progress such that $q_t \geq Q - y$.

For example, y will tend to be larger in a construction project, where the requirements are typically standardized and easy to define, than in a project that involves a significant innovation or quality component, where the requirements of the final output cannot be contracted on until the project is at a sufficiently advanced stage. Similarly, y will typically be small in design-related projects such as automotive design, as the requirements are difficult to describe. If the project is outsourced and contracts are explicit, then y will tend to be larger than the case in which it is undertaken in-house, where the manager can influence the team members' career paths and contracts are typically implicit.

The main result is that the manager's incentives propel her to extend the project as it progresses; for example, by introducing additional requirements. The manager chooses the project size by trading off the marginal benefit of a larger project against the marginal cost associated with a longer wait until the larger project is completed. However, as the project progresses, the agents increase their effort, so that this marginal cost decreases, whereas the respective marginal benefit does not change. As the project size will be chosen such that the two marginal values are equal, it follows that the manager's optimal project size increases as the project progresses.

An implication of this result is that the manager's optimal project size decreases in her commitment power. If the manager has sufficiently large commitment power, then she will commit to her optimal project size at time 0. Otherwise, she can commit to a smaller than ideal project at time 0, or she must wait until the project is at a sufficiently advanced stage so that she can commit to her optimal project size then. However, once such an advanced stage has been reached, her optimal project size is larger than it was originally, and the manager faces the same dilemma as at time 0. Nevertheless, we show that the manager always prefers to wait, and as a consequence, she will choose a bigger project the smaller her commitment power.

Anticipating that the manager will choose a larger project if she has less commitment power, the agents decrease their effort, which renders the manager worse off. To mitigate her commitment problem, assuming that the agents receive a share of the project's worth upon completion (i.e., an equity contract), the manager might delegate the decision rights over the project size to them. In this case, the agents will choose a smaller project than is optimal for the manager, but their preferences are time-consistent. Intuitively, because (unlike the manager) they also trade off the cost of effort when choosing the project size, their marginal cost associated with a larger project does not decrease as the project progresses. As a result, the manager's discounted profit is independent of the commitment power under delegation, whereas it increases in her commitment power when she retains the decision rights over the project size. We show that there exists an interior threshold such that delegation is optimal unless the manager has sufficient commitment power.

□ **Related literature.** First and foremost, this article is related to the literature on dynamic contribution games. The general theme of these games is that a group of agents interact repeatedly, and in every period (or moment), each agent chooses his contribution (or effort) to a joint project

at a private cost. Contributions accumulate until they reach a certain threshold, at which point the game ends. Agents receive flow payoffs while the game is in progress, a lump-sum payoff at the end, or a combination thereof. Admati and Perry (1991) and Marx and Matthews (2000) show that contributing little by little over multiple periods, each conditional on the previous contributions of the other agents, helps mitigate the free-rider problem. More recently, Yildirim (2006) and Kessing (2007) show that in contrast to the case in which the project generates flow payments while it is in progress, as studied by Fershtman and Nitzan (1991), efforts are strategic complements when the agents receive a payoff only upon completion. Georgiadis (2013) studies how the incentives to contribute to a public good project depend on the team composition and examines how a manager should choose the team composition and the agents' compensation scheme. A feature common to most articles in this literature is that the size of the project is given exogenously. However, in applications (e.g., new product development), the choice of the objectives of any given project is typically a central decision that must be made. Our contribution to this literature is to propose a tractable model to study this family of dynamic contribution games, to endogenize the size of the project, and to examine how the optimal project size depends on who has the decision rights and on the magnitude of the decision maker's commitment power.

A second strand of related literature is that on incomplete contracting. In particular, our article is closely related to the articles that study how *ex ante* contracting limitations generate incentives to renegotiate the initial contract *ex post* (Grossman and Hart, 1986; Hart and Moore, 1990; Aghion and Tirole, 1994; Tirole, 1999; Al-Najjar, Anderlini, and Felli, 2006; and others). A subset of this literature focuses on situations wherein the involved parties have asymmetric information. Here, ratchet effects have been shown to arise in principal-agent models in which the principal learns about the agent's ability over time, and the agent reduces his effort to manipulate the principal's beliefs about his ability (Freixas, Guesnerie, and Tirole, 1985; Laffont and Tirole, 1988). Another thread of this strand includes articles that consider the case in which the agent is better informed than the principal, or he has better access to valuable information. A common result is that delegating the decision rights to the agent is beneficial as long as he is sufficiently better informed and the incentive conflict is not too large (Aghion and Tirole, 1997; Dessein, 2002). In our model, however, all parties have full and symmetric information, so that ratchet effects and the incentives to delegate the decision rights to the agents arise purely out of moral hazard.

The article is organized as follows. We introduce the model in Section 2 and in Section 3, we analyze the agents' problem given a fixed project size. In Section 4, we study the manager's problem, her optimal choice of the project size, as well as her option to delegate the decision rights over the project size to the agents. Section 5 extends the model by incorporating deadlines, and by considering the case in which the agents' effort costs are linear. Finally, Section 6 concludes. In Appendix A, we extend our model to test the robustness of our results. All proofs are provided in Appendix 2.

2. The model

■ A group of n identical agents contracts with a manager to undertake a project. The agents exert (costly) effort over time to complete the project, they receive a lump sum compensation upon completing the project, and they are protected by limited liability.¹ The manager is the residual claimant of the project, and she possesses the decision rights over its size. A project of size $Q \geq 0$ generates a payoff equal to Q upon completion. This payoff is split between the parties

¹ We assume (for tractability) that the agents are compensated only by a lump sum upon completion of the project. In the third subsection of Appendix A, we consider the case in which the agents also receive a per unit of time compensation while the project is ongoing. The main results continue to hold.

as follows: each agent receives $\frac{\beta Q}{n}$, and the manager receives $(1 - \beta)Q$.^{2,3} Time $t \in [0, \infty)$ is continuous; all parties are risk neutral and they discount time at rate $r > 0$. The project starts at state $q_0 = 0$. At every moment t , each agent observes the state of the project denoted by q_t , and exerts costly effort to influence the process

$$dq_t = \left(\sum_{i=1}^n a_{i,t} \right) dt,$$

where $a_{i,t}$ denotes the (unverifiable) effort level of agent i at time t .⁴ Each agent's flow cost of exerting effort level a is $\frac{a^2}{2}$, and his outside option is equal to 0. The project is completed at the first stopping time τ such that $q_\tau = Q$.⁵

3. Agents' problem

■ In this section, we study the agents' problem, and we characterize the unique project-completing Markov Perfect Equilibrium (hereafter, MPE) wherein each agent conditions his strategy at t only on the current state of the project q_t , as well as a continuum of Public Perfect Equilibria (hereafter, PPE) wherein each agent's strategy at t is conditioned on the entire evolution path of the project $\{q_s\}_{s \leq t}$. Throughout this section, we take the project size Q as given, and we endogenize it in Section 4.

□ **Preliminaries.** Given a project of size Q , the current state of the project q_t , and others' strategies, agent i 's discounted payoff function is given by

$$\Pi_{i,t}(q; Q) = \max_{\{a_{i,s}\}_{s \geq t}} \left[e^{-r(\tau-t)} \frac{\beta Q}{n} - \int_t^\tau e^{-r(s-t)} \frac{a_{i,s}^2}{2} ds \mid \{a_{-i,s}\}_{s \geq t} \right] \quad (1)$$

where τ denotes the completion time of the project and it depends on the agents' strategies. The first term captures the agent's net discounted payoff upon completion of the project, whereas the second term captures his discounted cost of effort for the remaining duration of the project. Because payoffs depend solely on the state of the project (i.e., q) and not on the time t , this problem is stationary, and hence the subscript t can be dropped. Using standard arguments (Dixit, 1999), one can derive the Hamilton-Jacobi-Bellman (HJB) equation for the expected discounted payoff function for agent i ,

$$r \Pi_i(q; Q) = \max_{a_i} \left\{ -\frac{a_i^2}{2} + \left(\sum_{j=1}^n a_j \right) \Pi'_i(q; Q) \right\}, \quad (2)$$

subject to the boundary conditions

$$\Pi_i(q; Q) \geq 0 \quad \text{for all } q \quad \text{and} \quad \Pi_i(Q; Q) = \frac{\beta Q}{n}. \quad (3)$$

² This is essentially an equity contract. In the second subsection of Appendix A, we consider the case in which each agent receives a flat payment upon completion of the project that is independent of the project size Q . The main results continue to hold, and such contract is shown to aggravate the manager's commitment problem.

³ We assume that β is independent of Q ; otherwise, the assumption that the manager has limited ability to commit to a project size would be violated. However, we defer a detailed justification until after we have formalized the notion of limited commitment in Section 4.

⁴ The assumptions that efforts are perfect substitutes and the project progresses deterministically are made for tractability. However, we consider the cases in which efforts are complementary and the project progresses stochastically in the first subsection of Appendix A and the fifth subsection of Appendix A, respectively, and we show that all results continue to hold.

⁵ In the base model, the agents' reward upon completing the project does not depend on its completion time. In the first subsection of Section 6, we extend our model to incorporate deadlines.

The first boundary condition captures the fact that each agent’s discounted payoff must be non-negative as he can guarantee himself a payoff of 0 by exerting no effort, and hence, incurring no effort cost. The second boundary condition states that upon completing the project, each agent receives his reward and exerts no further effort.

□ **Markov Perfect Equilibrium (MPE).** In a MPE, at every moment, each agent i observes the state of the project q , and chooses his effort a_i to maximize his discounted payoff while accounting for the effort strategies of the other team members. It follows from (2) that the first-order condition for agent i ’s problem yields that $a_i(q; Q) = \Pi'_i(q; Q)$: at every moment, he chooses his effort such that the marginal cost of effort is equal to the marginal benefit associated with bringing the project closer to completion. By noting that the second-order condition is satisfied and that the first-order condition is necessary and sufficient, it follows that in any differentiable, project-completing MPE, the discounted payoff for agent i satisfies

$$r \Pi_i(q; Q) = -\frac{1}{2}[\Pi'_i(q; Q)]^2 + \left[\sum_{j=1}^n \Pi'_j(q; Q) \right] \Pi'_i(q; Q), \tag{4}$$

subject to the boundary conditions (3). The following proposition characterizes the MPE, and establishes conditions under which it is unique.

Proposition 1. For any given project size Q , there exists a MPE for the game defined by (1). This equilibrium is symmetric, each agent’s effort strategy satisfies

$$a(q; Q) = \frac{r}{2n-1} [q - C(Q)]^+, \quad \text{where } C(Q) = Q - \sqrt{\frac{2\beta Q}{r} \frac{2n-1}{n}},$$

and the project is completed at $\tau(Q) = \frac{2n-1}{rn} \ln[1 - \frac{Q}{C(Q)}]$.⁶ In equilibrium, each agent’s discounted payoff is given by

$$\Pi(q; Q) = \frac{r}{2} \frac{([q - C(Q)]^+)^2}{2n-1}.$$

If $Q < \frac{2\beta}{r}$, then this equilibrium is unique, and the project is completed in finite time. Otherwise, there also exists an equilibrium in which no agent ever exerts any effort and the project is never completed.

First note that if the project is too far from completion (i.e., $q < C(Q)$), then the discounted cost to complete it exceeds its discounted net payoff, and hence the agents are better off not exerting any effort, in which case the project is never completed. Because the project starts at $q_0 = 0$, this implies that the project is never completed if $C(Q) \geq 0$, or equivalently, if $Q \geq \frac{2\beta}{r} \frac{2n-1}{n}$. On the other hand, if $Q < \frac{2\beta}{r} \frac{2n-1}{n}$, then each agent’s effort level increases in the state of the project q . This is due to the facts that agents are impatient and they incur the cost of effort at the time the effort is exerted, whereas they are compensated only when the project is completed. As a result, their incentives are stronger, the closer the project is to completion.

Second, it is worth emphasizing that the MPE is always symmetric. This is due to the convexity of the agents’ effort costs. In contrast, if effort costs are linear, then the corresponding game has both symmetric and asymmetric MPE (see the second subsection of Section 5 for details). Finally, whereas the MPE need not be unique, it turns out that when the project size Q is endogenous, the manager will always choose it such that the equilibrium is unique (see Remark 1 in Section 4). As such, we shall restrict attention to the project-completing equilibrium characterized in Proposition 1.

⁶ To simplify notation, because the equilibrium is symmetric and unique, the subscript i is dropped throughout the remainder of this article. Moreover, $[\cdot]^+ = \max\{\cdot, 0\}$.

□ **Public Perfect Equilibria (PPE).** Although the restriction to MPE is reasonable when teams are large and members cannot monitor each other, there typically exist other PPE with history-dependent strategies; that is, strategies that at time t depend on the entire evolution path of the project $\{q_s\}_{s \leq t}$. In this section, we characterize a continuum of such equilibria in which at every moment, each agent chooses his effort to maximize a convex combination of his individual and the entire team's discounted payoff along the equilibrium path.

Building upon the concepts introduced in the seminal article on social identity by Tajfel and Turner (1979), Akerlof and Kranton (2000) argue that depending on the work environment, employees may behave as *insiders*, who act in the best interest of the organization or as *outsiders*, who act in their individual best interest. Therefore, the weight that an agent places on maximizing the team's discounted payoff can be interpreted as the degree to which he feels an insider, and we shall refer to an equilibrium as more cooperative the more weight each agent places on maximizing the team's discounted payoff.

We model this by assuming that given the current state of the project q , each agent chooses his effort to maximize the expected discounted payoff of $k \in [1, n]$ agents; that is, he solves

$$a(q; Q, k) \in \arg \max_a \left\{ a k \Pi(q; Q, k) - \frac{a^2}{2} \right\}. \tag{5}$$

Note that $k = 1$ ($k = n$) corresponds to the case in which each agent places all the weight on maximizing his individual (the team's) discounted payoff, whereas $k \in (1, n)$ corresponds to intermediate cooperation levels. The following proposition establishes that for all $k \in [1, n]$, there exists a PPE in which at every moment along the equilibrium path, each agent chooses his effort level by solving (5).⁷

Proposition 2. For any given $k \in [1, n]$ and project size Q , there exists a PPE in which each agent's effort strategy satisfies

$$a(q; Q, k) = \frac{r}{2n - k} [q - C(Q, k)]^+ \tag{6}$$

along the equilibrium path, where $C(Q, k) = Q - \sqrt{\frac{2\beta Q}{r} \frac{(2n-k)k}{n}}$, and the project is completed at $\tau(Q, k) = \frac{2n-k}{rn} \ln[1 - \frac{Q}{C(Q, k)}]$. After any deviation from the equilibrium path, all agents revert to the MPE (i.e., $k = 1$) for the remaining duration of the project. In equilibrium, each agent's discounted payoff is given by

$$\Pi(q; Q, k) = \frac{r}{2k} \frac{([q - C(Q, k)]^+)^2}{2n - k},$$

and it increases in k for all q and Q .

The intuition behind the existence of such PPE is as follows. First, if all agents choose their effort by solving (5) for some $k > 1$, then each agent is strictly better off relative to the case in which $k = 1$. Second, $k = 1$ corresponds to the MPE, so that the threat of punishment is credible. Third, by examining the progress made until time t , each agent can infer whether all agents followed the equilibrium strategy; that is, if q_t corresponds to the progress that should occur if all agents follow (6). Because a deviation from the equilibrium path is detectable (and punishable) arbitrarily quickly, the gain from a deviation is infinitesimally small. As a result, no agent has an incentive to deviate from the strategy dictated by (6), so that it constitutes a PPE.⁸

⁷ There also exist PPE in which each agent's cooperation level depends on q and varies across team members. However, we assume that it is constant throughout the duration of the project and identical across agents for tractability, and because we interpret k as part of the team's cohesiveness that is persistent over time.

⁸ There is a well-known difficulty associated with defining a trigger strategy in continuous-time games, which we resolve by using the concept of inertia strategies proposed by Bergin and MacLeod (1993). For details, the reader is referred to the proof of Proposition 2 in Appendix 2.

The following result establishes some comparative statics about how each agent’s effort level depends on the parameters of the problem.

Result 1. All other parameters held constant, each agent’s effort level $a(q; Q, k)$:

- (i) increases in k (and β);
- (ii) there exists a threshold Θ_r such that it increases in r if and only if $q \geq \Theta_r$; and
- (iii) there exists a threshold Θ_n such that it increases in n if and only if $q \geq \Theta_n$.

To see the intuition behind statement (i), note that the free-rider problem is mitigated as the agents’ cooperation level k increases (and it is eliminated when $k = n$). As a result, the agents work harder, the larger k is. The intuition behind the second part of statement (i) is straightforward: if the agents receive a larger reward upon completion, then their incentives are stronger. The threshold results in statements (ii) and (iii) are similar to Georgiadis (2013) who studies a stochastic version of this model with a fixed project size.⁹

Before we proceed to analyze the manager’s problem, it is instructive to characterize the first best outcome of this game.

Result 2. Consider a social planner whose objective is to maximize the total surplus of the team. For any given project size Q , the discounted payoff and the socially efficient strategy for each agent is given by

$$\Pi^{sp}(q; Q) = \frac{r}{2n} \left(\left[q - Q + \sqrt{\frac{2Qn}{r}} \right]^+ \right)^2 \quad \text{and} \quad a^{sp}(q; Q) = \frac{r}{n} \left[q - Q + \sqrt{\frac{2Qn}{r}} \right]^+,$$

respectively, and the project is completed at $\tau^{sp}(Q) = \frac{1}{r} \ln\left(\frac{\sqrt{2\beta n}}{\sqrt{2\beta n} - \sqrt{rQ}}\right)$.¹⁰

4. Project design and the commitment problem

■ In this section, we analyze the manager’s problem and we endogenize the project size Q . The manager has the decision rights over the choice of the project size, but she may not be able to commit to a specific Q until the project is sufficiently close to that Q . Formally, we assume that given the current state of the project q , the manager can only commit to a project size in the interval $[q, q + y]$, where $y \geq 0$ is common knowledge, and it can be interpreted as the manager’s commitment power.

□ **Manager’s problem.** Recall from Propositions 1 and 2 that the agents’ strategies depend on Q . Therefore, until the manager commits to a particular project size, the agents must form beliefs about the project size that she will choose, and condition their strategies on those beliefs. We assume (for tractability) that the manager uses pure strategies to determine her optimal project size, and we denote the agents’ beliefs about her choice of the project size by \tilde{Q} .¹¹ Finally, the solution concept of the game between the manager and the agents is Subgame Perfect Equilibrium (hereafter, SPE).

Given a project of size Q , the agents’ beliefs \tilde{Q} , and their cooperation level k , the manager’s discounted profit can be written as $W(q; Q, \tilde{Q}, k) = [e^{-r\tau}((1 - \beta)Q) | Q, \tilde{Q}]$, where the project’s completion time τ depends on the current state q and the agents’ strategies, which in turn depend

⁹ Note that when examining how each agent’s effort level depends on the team size, one must first consider how the agents’ cooperation level k depends on n . Statement (iii) holds for any fixed k as well as for $k = n$.

¹⁰ The socially efficient payoff and effort functions are obtained by substituting $\beta = 1$ and $k = n$ into the respective functions obtained in Proposition 2.

¹¹ Because the equilibrium is symmetric for any Q , all agents will share the same belief \tilde{Q} .

on \tilde{Q} and k . Of course, in equilibrium beliefs must be correct; that is, $Q = \tilde{Q}$. Using standard arguments, one can write the manager's discounted profit in differential form as

$$rW(q; Q, \tilde{Q}, k) = na(q; \tilde{Q}, k)W'(q; Q, \tilde{Q}, k),$$

subject to the boundary conditions

$$W(q; Q, \tilde{Q}, k) \geq 0 \quad \text{for all } q \quad \text{and} \quad W(Q; Q, \tilde{Q}, k) = (1 - \beta)Q,$$

where $a(q; \tilde{Q}, k)$ is given in (6). To interpret these conditions, note that the manager's discounted profit is non-negative at every state of the project, because she does not incur any cost or disburse any payments to the agents while the project is in progress. On the other hand, she receives her net profit $(1 - \beta)Q$, and the game ends as soon as the state of the project hits Q for the first time. After substituting (6) and solving the above ODE, it follows that

$$W(q; Q, \tilde{Q}, k) = (1 - \beta)Q \left(\frac{[q - C(\tilde{Q}, k)]^+}{Q - C(\tilde{Q}, k)} \right)^{\frac{2n-k}{n}}. \tag{7}$$

Note that $(1 - \beta)Q$ represents the manager's net profit upon completion of the project, whereas the next term can be interpreted as the present discounted value of the project, which depends on the current state q , the agents' beliefs about the project size and their cooperation level k , which in turn influence their strategies characterized in Proposition 2.

□ **Optimal project size.** To examine the manager's optimal project size, we first consider the case in which she has full commitment power (i.e., $y = \infty$), so that she can commit to any project size before the agents begin to work. Second, we consider the opposite extreme case in which she has no commitment power (i.e., $y = 0$), so that she knows only that the project is complete *when she sees it*. In this case, at every moment the manager observes the current state of the project q , and decides whether it is *good enough* (in which case, its size will be $Q = q$), or whether to let the agents continue to work and reevaluate the completion decision an instant later. Finally, we consider the case in which she has intermediate levels of commitment power (i.e., $0 < y < \infty$), and we examine how her optimal project size depends on y . Throughout the remainder of this section, we take the agents' cooperation level $k \in [1, n]$ as given. As such, we suppress k for notational convenience.

Full commitment power ($y = \infty$). If the manager has full commitment power, then she can commit to a project size before the agents begin to work. Therefore, at $q_0 = 0$, the manager leads a Stackelberg game in which she chooses the project size that maximizes her discounted profit and the agents follow by adopting the equilibrium strategy characterized in Proposition 2. As a result, her optimal project size with full commitment (FC) satisfies $Q_{FC}^M \in \arg \max_Q W(0; Q, Q)$. Noting from (7) that $W(0; Q, Q)$ is concave in Q , taking the first-order condition with respect to Q yields

$$Q_{FC}^M = \frac{\beta k(2n - k)}{r} \frac{4n}{2n} \left(\frac{4n}{4n - k} \right)^2.$$

Moreover, the concavity of her discounted profit function implies that she commits to Q_{FC}^M at $q = 0$ for any commitment power $y \geq Q_{FC}^M$.

No commitment power ($y = 0$). If the manager has no commitment power, then at every moment she observes the current state of the project q , and she decides whether to stop work and collect the net profit $(1 - \beta)q$ or to let the agents continue working and reevaluate her decision to complete the project a moment later. In this case, the manager and the agents engage in a simultaneous-action game, where the manager chooses Q to maximize her discounted profit given the agents' beliefs \tilde{Q} and the corresponding strategies, and the agents form their beliefs by anticipating the manager's choice Q . Therefore, her optimal project size with no commitment (NC) satisfies

$Q_{NC}^M \in \arg \max_Q \{W(q; Q, \tilde{Q})\}$, where in equilibrium beliefs must be correct; that is, $Q = \tilde{Q}$. By solving $\frac{\partial W(q; Q, \tilde{Q})}{\partial Q} \Big|_{q=\tilde{Q}=Q} = 0$, we have

$$Q_{NC}^M = \frac{\beta}{r} \frac{2kn}{2n - k}.$$

Observe that if $y = 0$, then the manager will choose a strictly larger project relative to the case in which she has full commitment power: $Q_{NC}^M > Q_{FC}^M$. We shall discuss the intuition behind this result in the third section of subsection 2 in Section 4 after we determine the manager's optimal project size for intermediate levels of commitment power.

This case raises the question of what happens to the agents' beliefs off the equilibrium path if the manager does not complete the project at Q_{NC}^M . Suppose that the manager did not complete the project at Q_{NC}^M so that $q > Q_{NC}^M$. Clearly, Q and $\tilde{Q} > Q_{NC}^M$, and it is straightforward to verify that $\frac{\partial W(q; Q, \tilde{Q})}{\partial Q} < 0$ for all q, Q and $\tilde{Q} > Q_{NC}^M$, which implies that the manager would be better off had she completed the project at Q_{NC}^M irrespective of the agents' beliefs.

Conceptually, this commitment problem could be resolved by allowing β to be contingent on the project size. In particular, suppose that the manager can fix β , and let $\hat{\beta}(Q)$ equal β if $Q = Q_{FC}^M$, and 1 otherwise. Then, her optimal project size is equal to Q_{FC}^M regardless of her commitment power because any other project size will yield her a net profit of 0. However, this implicitly assumes that Q_{FC}^M is contractible at $q = 0$, which is clearly not true for any $y < Q_{FC}^M$. Therefore, we rule out this possibility by assuming that β is independent of Q .

Partial commitment power ($0 < y < \infty$). Recall that for any given cooperation level k , the manager's optimal project size is equal to Q_{FC}^M for all $y \geq Q_{FC}^M$, and it is equal to Q_{NC}^M if $y = 0$. To determine her optimal project size when $y \in (0, Q_{FC}^M)$, we solve an auxiliary problem, and we show that there is a one-to-one correspondence between this (auxiliary) problem and the original problem.

Suppose that the manager can credibly commit to her optimal project size as soon as the state of the project hits (some exogenously given) x . In this case, the manager chooses Q_x^M to maximize her discounted profit at x , so that $Q_x^M \in \arg \max_{Q \geq x} \{W(x; Q, Q)\}$, and anticipating the manager's choice, the agents pick their strategies based on Q_x^M . We then show that for all $y \in (0, Q_{FC}^M)$, there exists a unique $x(y) \in (0, Q_{NC}^M)$, such that the manager will commit to the project size $Q_{x(y)}^M$ as soon as the project hits $x(y)$. The following Proposition characterizes this SPE.

Proposition 3. Suppose that given the current state q_t , the manager can commit to any project size $Q \in [q_t, q_t + y]$. There exists a Subgame Perfect Equilibrium in which at $q_t = x(y)$, the manager commits to project size $Q_{x(y)}^M$, where

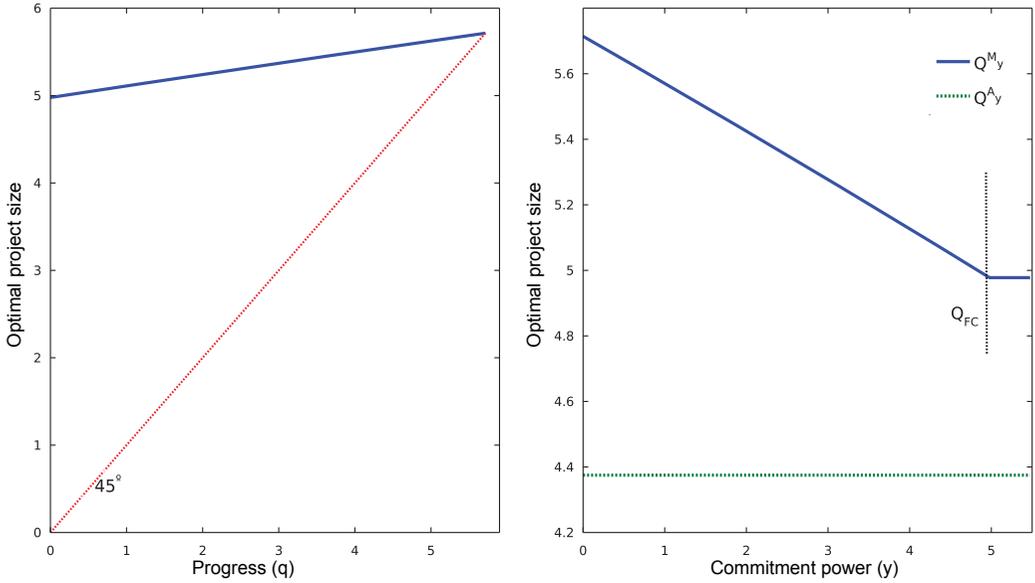
$$Q_{x(y)}^M = \left(\frac{2n}{4n - k} \right)^2 \left(\sqrt{\frac{\beta k(2n - k)}{r} \frac{1}{2n}} + \sqrt{\frac{\beta k(2n - k)}{r} \frac{1}{2n} + \frac{k(4n - k)}{4n^2} x(y)} \right)^2, \quad (8)$$

$x(y)$ is the unique solution to $\max\{Q_{x(y)}^M - y, 0\} = x(y)$, and each agent chooses his strategy according to (6). Moreover, $Q_{x(y)}^M$ and $x(y)$ decrease in y .

This proposition asserts that the manager has incentives to extend the project as it progresses: Q_x^M increases in x . The intuition is as follows: the manager trades off a larger project that yields a bigger net profit upon completion against having to wait longer until that profit is realized. However, she ignores the additional effort cost associated with a larger project. Moreover, recall that the agents raise their effort, and hence the manager's marginal cost associated with choosing a larger project decreases as the project progresses. On the other hand, her marginal benefit from choosing a larger project is independent of the progress made. Because the project size is chosen such that the two marginal values are equal, it follows that the manager's optimal project

FIGURE 1

OPTIMAL PROJECT SIZE WHEN $\beta = 0.5$, $r = 0.1$, $k = 1$, AND $n = 4$



Notes: The left panel illustrates the manager’s incentives to extend the project as it progresses: her optimal project size increases in the state of the project q , and there exists a state at which the manager is better off completing the project without further delay. The right panel illustrates that her optimal project size (solid line) decreases in her commitment power, whereas the agents’ optimal project size (dashed line) is independent of their commitment power.

size increases as the project progresses. By noting that the extreme cases in which the manager has full (no) commitment power correspond to $y = 0$ ($y = \infty$), this intuition also explains why $Q_{NC}^M > Q_{FC}^M$. This is illustrated in the left panel of Figure 1.

Noting that $\frac{\partial}{\partial x} Q_x^M < 1$ for all x , this proposition can be interpreted as follows: for any $y < Q_{FC}^M$, the manager finds it optimal to commit to her optimal project size when the project is at a sufficiently advanced stage such that the height of the wedge between Q_x^M and the 45° -line (as shown in the left panel of Figure 1) is equal to y .

After rearranging terms in (8), it is possible to write the manager’s optimal project size explicitly as a function of her commitment power y as

$$Q^M(y) = \frac{1}{2} \left(\sqrt{\frac{\beta}{r} \frac{kn}{2n-k}} + \sqrt{\frac{\beta}{r} \frac{kn}{2n-k} - \frac{k}{2n-k} \min\{y, Q_{FC}^M\}} \right)^2 .$$

Observe that $Q^M(y)$ decreases in y , which implies that if the manager has less commitment power, then she will commit at a later state and to a larger project. In addition, Proposition 3 together with the expression for the completion time of the project computed in Proposition 2 implies that the duration of the project will be larger if the manager has less commitment power.

Remark 1. Recall that (i) the MPE is unique if $Q < \frac{2\beta}{r}$, (ii) $Q_{NC}^M < \frac{2\beta}{r}$ for all $n \geq 2$, and (iii) $Q^M(y) \leq Q_{NC}^M$ for all y . Therefore, the game has a unique MPE for any level of commitment power when the project size is chosen by the manager.

It is important to emphasize that the agents internalize the manager’s limited ability to commit when choosing their effort strategy. In particular, one can verify from (6) that each agent’s effort increases in the manager’s commitment power (i.e., $a(q; Q^M(y))$ increases in y for

all q), which implies that the manager's inability to commit early on induces a ratchet effect: anticipating that she will choose a larger project, the agents scale down their effort. Although ratchet effects have been shown to arise in settings with asymmetric information (e.g., Freixas, Guesnerie, and Tirole, 1985; Laffont and Tirole, 1988), in our model they arise under moral hazard with full information.

The following result examines how the manager's optimal project size depends on the parameters of the problem.

Result 3. Other things equal, the manager's optimal project size $Q^M(y)$:

- (i) increases in k (and β) for all y ;
- (ii) decreases in r for all y ; and
- (iii) if $k = 1$, then there exists a threshold Φ_n such that it increases in n if and only if $y \geq \Phi_n$.
On the other hand, if $k = n$, then it increases in n .

Statements (i) and (ii) are not surprising. Because each agent's effort increases in k , the team can achieve more progress during any given time interval by playing a more cooperative equilibrium, and hence the manager has incentives to choose a larger project. If the agents receive a larger share of the project's value upon completion, then they will work harder along the equilibrium path, and as a result, the manager will (again) choose a bigger project. For the intuition behind (ii), recall that the manager trades off the higher payoff of a larger project against the longer delay until that payoff is collected. As the parties become more patient (i.e., as r decreases), the cost associated with the delay decreases, and hence the optimal project size increases.

To examine how the manager's optimal project size depends on the team size, one must first consider how the agents' cooperation level k depends on n . To obtain sharp results, we consider the cases $k = 1$ (which corresponds to the MPE) and $k = n$ (which corresponds to the efficient PPE). In the first case, observe from Proposition 1 that $\frac{\partial}{\partial q} a(q; Q) = \frac{r}{2n-1}$ decreases in n . Because the manager's incentive to extend the project is driven by the agents raising their effort as the project progresses, it follows that this incentive becomes weaker as the team size n increases. As a result, if the manager has sufficiently large commitment power so that she can commit at the early stages of the project, then her optimal project size increases in n , whereas otherwise it decreases in n . On the other hand, if $k = n$, then the result follows from the fact that the aggregate effort of the team increases in n at every state of the project.

This analysis also raises the question about the manager's optimal team size. Considering the cases $k = 1$ and $k = n$ as above, by substituting (8) into (7) and taking the first-order condition with respect to n , it is straightforward to show that with no commitment power (i.e., $y = 0$), the manager's optimal team size is $n^* = 2$ when $k = 1$, whereas the project is never completed and the manager's discounted profit equals 0 for any team size if $k = n$. On the other hand, with full commitment power (i.e., $y = \infty$), it is $n^* = 1$ and $n^* = \infty$ when $k = 1$ and $k = n$, respectively. With intermediate levels of commitment power, the expression for the manager's discounted profit is not sufficiently tractable to optimize with respect to n , but numerical analysis for the cases with $k = 1$ and $k = n$ indicates that the manager's optimal project size decreases in her commitment power.

□ **Optimal delegation.** The manager's limited ability to commit, in addition to disincentivizing the agents from exerting effort, is detrimental to her *ex ante* discounted profit; that is, $W(0; Q^M(y), Q^M(y))$ increases in y .¹² Thus, unable to commit sufficiently early, the manager might consider delegating the decision rights over the project size to the agents.

¹² This follows from the facts that $W(0; Q, Q)$ is concave in Q , the manager's *ex ante* discounted profit is maximized at Q_{FC}^M , $Q^M(y) \geq Q_{FC}^M$ for all y , and $Q^M(y)$ decreases in y .

We begin by examining how the agents would select the project size. Let $Q^A \in \arg \max_Q \{\Pi(x; Q)\}$ denote the agents' optimal project size given the current state x . Solving this maximization problem yields

$$Q^A = \frac{\beta k(2n - k)}{r - 2n}.$$

First, observe that the agents' optimal project size is independent of the current state x . This is also illustrated in the right panel of Figure 1. Intuitively, this is because the agents incur the cost of their effort, so that their effort cost increases together with their effort level as the project progresses. As a result, unlike the manager, their marginal cost associated with choosing a larger project does not decrease as the project evolves, and consequently they do not have incentives to extend the project as it progresses.

Second, observe that the agents always prefer a smaller project than the manager; i.e., $Q^A < Q^M(y)$ for all y . This is because they incur the cost of their effort, so that their marginal cost associated with a larger project is greater than that of the manager's.

The following proposition establishes conditions under which the manager finds it optimal to delegate the decision rights over the project size to the agents.

Proposition 4. Suppose that given the current state q_t , the manager can commit to any project size $Q \in [q_t, q_t + y]$. There exists an interior threshold Φ such that $W(0; Q^A, Q^A) > W(0; Q^M(y), Q^M(y))$ if and only if $y < \Phi$; that is, the manager should delegate the choice of the project size to the agents unless she has sufficient commitment power.

Recall that the agents' optimal project size is time-consistent, which implies that if the manager delegates the decision rights to the agents, then her *ex ante* discounted profit is independent of when the project size is chosen. The key part of this result is that if the manager has no commitment power (i.e., $y = 0$), then delegation is always optimal. By noting that the manager's *ex ante* discounted profit increases in her commitment power, the proposition follows.

Finally, one might also envision an intermediate decision rule, wherein all parties need to unanimously agree on a project size. If the *status quo* is to continue until all parties agree to complete the project, then, noting that the manager prefers a larger project than the agents regardless of her commitment power, the project will be completed at the manager's optimal project size. In contrast, if the *status quo* is to complete the project unless all parties agree to continue, then the project will be completed at the agents' optimal project size.

□ **Other equilibria.** In addition to the SPE characterized in the second subsection of Section 4 this game also admits other SPE in which the agents induce the manager to choose a smaller project that is closer to their optimal size (i.e., Q^A). The following proposition characterizes a family of SPE in which the agents revert to the MPE when the project reaches a certain threshold, thus inducing the manager to complete the project instantaneously. In establishing this result, it is convenient to assume that even if the manager has committed to a particular project size, she can renegotiate it to a smaller project size if such renegotiation is advantageous for both parties.

Proposition 5. Suppose that the agents play a PPE with cooperation level $k > 1$. For every $x \in [\max\{Q^A, Q^M(y)|_{k=1}\}, Q^M(y))$, there exists a Subgame Perfect Equilibrium (SPE) in which the agents revert to the MPE at $q_t = x$, and the manager best-responds by completing the project instantaneously.

To see the intuition, fix some $x \in [\max\{Q^A, Q^M(y)|_{k=1}\}, Q^M(y))$, and observe that the agents are better off if the project is completed at x instead of $Q^M(y)$. Recall that by Proposition 2, there exists a PPE in which the agents choose their effort by maximizing the discounted payoff of k agents while $q_t < x$, and they revert to the MPE at $q_t = x$, in which case the manager's best—response is to complete the project instantaneously. Moreover, it is useful to note that the agents'

discounted payoff is maximized if $x = \max\{Q^A, Q^M(y)|_{k=1}\}$, and if $Q^A \geq Q^M(y)|_{k=1}$ (which is true if the agents' cooperation level k is sufficiently large or the manager's commitment power y is sufficiently small), then the project size is effectively chosen by the agents.

Furthermore, there exist SPE in which all agents halt effort, thus inducing the manager to complete the project instantaneously. In particular, fix some $x \in [Q^A, Q^M(y))$, and consider the following strategy: each agent chooses his effort by maximizing the discounted payoff of k agents while $q_t < x$, and he exerts 0 effort for all $q_t \geq x$. After any deviation from this strategy, he reverts to the MPE. We shall argue that this strategy constitutes a SPE. First observe that absent any effort, the project will not progress, and hence the manager's best—response is to complete it instantaneously, so that $Q = x$. To see why no agent can profitably deviate from this strategy, observe that any deviation must involve the agent exerting positive effort and the manager completing the project at some $Q > x$. However, recall that each agent's discounted payoff decreases in the project size for all $Q > Q^A$, which together with the fact that $x \geq Q^A$ implies that no agent can increase his discounted payoff by deviating. Finally, reverting to the MPE after any deviation is subgame perfect. This leads us to the following result.

Result 4. For any $x \in [Q^A, Q^M(y))$, there exists a SPE in which all agents stop exerting effort when $q_t = x$, and the manager best-responds by completing the project instantaneously.

It is useful to highlight a refinement that eliminates this family of SPE. Suppose that given q_t , the manager is able to commit that $Q \geq q_t + \delta$, where $\delta > 0$ (e.g., by announcing that at least certain additional features will be included in the product, which is costly to retract). Then she effectively renders herself unable to best-respond to the agents halting effort, and hence such SPE do not exist.

□ **Socially optimal project size.** In this section, we characterize the optimal project size of a social planner who seeks to maximize the team's total discounted payoff.

Result 5. Consider a social planner who maximizes the sum of the agents' and the manager's discounted payoffs (but cannot control the agents' effort strategies). Moreover, suppose that given the current state q , he can commit to any project size $Q \in [q, q + y]$; and

- (i) His optimal project size $Q^{sp}(y)$ satisfies $Q^A < Q_{x(y)}^{sp} < Q^M(y)$ for all y .
- (ii) With 1 agent, his optimal project size decreases in his commitment power y .¹³

The social planner seeks to maximize the sum of the manager's and the agents' discounted payoff. As such, it is intuitive that his optimal project size will lie between the agents' and the manager's optimal project size for all y . In addition, because the agents' (manager's) optimal project size is independent of (decreases in) y , it is intuitive that the social planner's optimal project size decreases in his commitment power y . With $n \geq 2$ agents, this problem becomes intractable. However, numerical analysis indicates that his optimal project size continues to decrease in his commitment power y . Appealing to Result 4 in the fourth subsection of Section 4 it is useful to note that the socially optimal project size can be implemented as an outcome of a SPE.

Finally, if the social planner can also control the agents' strategies, then it is straightforward to verify from the social planner's discounted payoff function characterized in Result 2 that his optimal project size equals $\frac{n}{2r}$ and it is independent of his commitment power.

¹³ Given y , the social planner commits to $Q_{x(y)}^{sp} = \arg \max_Q \{n \Pi(q; Q) + W(q; Q, Q)\}$ at $q = x(y)$, where $x(y)$ satisfies $\max\{Q_{x(y)}^{sp} - y, 0\} = x(y)$. The result follows from the facts that this problem is strictly concave and $\frac{d}{dq} [n \Pi(q; Q) + W(q; Q, Q)]$ increases in q for all $Q \leq Q_{x(y)}^{sp}$, which in turn implies that $Q_{x(y)}^{sp}$ decreases in y .

5. Extensions

■ In this section, we present two extensions of our model. First, we allow the manager to impose a deadline by which the project must be completed, and second, we consider the case in which the agents' effort costs are linear. In the interest of brevity, we restrict attention to the MPE of the game in both extensions.

□ **Contracting on the duration of the project (deadlines).** Insofar, we have assumed that the manager can either contract on the size of the project (subject to her limited commitment constraint), or she can delegate the decision rights over the project size to the agents. It is plausible, however, that even if the manager has limited ability to commit to a particular project size at the outset of the project, she can commit to its duration. In this section, we consider the case in which she imposes a deadline (denoted by T), while delegating the choice of the project size (to be reached by T) to the agents.

Clearly, it is optimal for the manager to commit to the duration of the project (denoted by T) at time 0 rather than wait and commit at a later time. Therefore, the parties play a Stackelberg Equilibrium in which the manager announces T , and the agents begin to exert effort. At time T , the project is completed with a payoff equal to q_T , which is split as before; that is, the manager receives $(1 - \beta)q_T$, whereas each agent receives $\frac{\beta}{n}q_T$. The following proposition characterizes the MPE corresponding to this game with duration T .¹⁴ Unlike the earlier results where payoffs and strategies are characterized as a function of q , here it will be more convenient to characterize them as a function of t .

Proposition 6. Given a fixed deadline $T > 0$, there exists a MPE in which at every moment t , each agent's effort strategy satisfies

$$a_t = \frac{\beta}{n} e^{-\frac{rn(t-T)}{2n-1}}$$

and the project size at time T is equal to $q_T = \frac{\beta}{r} \frac{2n-1}{n} (1 - e^{-\frac{rnT}{2n-1}})$.

Given a fixed project duration T , the manager's *ex ante* discounted profit satisfies

$$W(0; T) = (1 - \beta) q_T e^{-rT}.$$

The following result characterizes the manager's optimal project duration T^* , and it shows that if she delegates the decision rights over the project size to the agents, then she can increase her *ex ante* discounted profit by fixing the duration of the project.

Result 6. The manager's optimal project duration and the corresponding project size in the MPE characterized in Proposition 6 is equal to

$$T^* = -\frac{2n-1}{rn} \ln\left(\frac{2n-1}{3n-1}\right) \quad \text{and} \quad q_{T^*} = \frac{\beta}{r} \frac{2n-1}{3n-1},$$

respectively, and her *ex ante* discounted profit $W(0; T^*) \geq W(0; Q^A, Q^A)|_{k=1}$, with strict inequality holding for all $n \geq 2$.

An implication of this result is that similar to Proposition 4 the manager finds it optimal to contract on the duration of the project instead of contracting on its size (while effectively delegating its duration) if and only if her commitment power is below a certain threshold. Moreover, one can show that contracting on the duration of the project is optimal even if the manager has full

¹⁴ The agents might be allowed to complete the project and collect their rewards at some $\tau < T$. However, it turns out that when the project duration is chosen optimally, the agents find it optimal to complete the project at $\tau = T$.

commitment power (i.e., $y = \infty$) as long as the group size n is sufficiently large. Intuitively, this is because larger groups have stronger incentives to procrastinate, and as we know from the existing literature, deadlines are an effective tool to mitigate procrastination (Bonatti and Hörner, 2011; Campbell, Ederer, and Spinnewijn, forthcoming).

Remark 2. Another possibility is for the manager to contract on both the size and the duration of the project. This is essentially a “forcing contract” in that the manager can choose a $\{Q, T\}$ pair to extract all surplus from the agents. An implicit assumption of such a contract is that the manager commits to destroy any surplus if the project size has not reached Q by time T . We believe that this case is neither interesting, nor realistic. Indeed, the manager would always prefer to renegotiate rather than destroy surplus, and anticipating this, the agents will behave as if there is no deadline or as if they have the decision rights over the project size.

□ **Linear effort costs.** In this section, we consider the case in which the agents have linear costs of effort. We assume that each agent’s marginal cost of effort is constant and equal to $\lambda > 0$, and each agent’s effort level $a_i \in [0, u]$, where $u \leq \infty$. For given Q , the discounted payoff function of each agent i satisfies the HJB equation

$$r \Pi_i(q; Q) = \max_{a_i \in [0, u]} \left\{ -\lambda a_i + \left(\sum_{j=1}^n a_j \right) \Pi'_i(q; Q) \right\}, \tag{9}$$

subject to the boundary conditions (3). We first characterize the symmetric MPE with bang-bang strategies.

Result 7. There exists a symmetric MPE in which each agent’s discounted payoff and effort strategy satisfies

$$\Pi(q; Q) = \left[-\frac{\lambda u}{r} + \left(\frac{\beta Q}{n} + \frac{\lambda u}{r} \right) e^{\frac{r(q-Q)}{nu}} \right] \mathbf{1}_{\{\psi \leq 0\}} \quad \text{and} \quad a(q; Q) = u \mathbf{1}_{\{\psi \leq 0\}},$$

respectively, where $\psi = Q + \frac{nu}{r} \ln\left(\frac{\lambda n^2 u}{r\beta Q + \lambda nu}\right)$.

The project is completed in this MPE only if $\psi \leq 0$, and this condition holds as long as $\beta \geq \lambda$ and the project size Q is in some medium range. Moreover, unless $\psi|_{n=1} \leq 0$ (i.e., a single agent is willing to undertake the entire project single-handedly), there exists another symmetric MPE in which no agent exerts any effort and the project is never completed. At the limit as $u \rightarrow \infty$ (i.e., without an upper bound on each agent’s maximal effort), $\psi \leq 0$ for all Q only if $\beta \geq \lambda$ and $n = 1$, in which case the project is completed instantaneously.

Turning to the manager’s problem, using a similar approach as in the third section of subsection two of Section 4, one can show that her optimal project size given the current state of the project x , is $Q_x^M = \min\{\frac{nu}{r}, \bar{Q}_x\}$, where \bar{Q}_x denotes the largest project size such that $x = Q + \frac{nu}{r} \ln\left(\frac{\lambda n^2 u}{r\beta Q + \lambda nu}\right)$. It is now necessary to distinguish between two cases:

- (i) If $\frac{nu}{r} \leq \bar{Q}_0$, which is true if and only if $\lambda \leq \frac{\beta}{ne-1}$, then the manager’s optimal project size is equal to $\frac{nu}{r}$ and it is independent of her commitment power y . An implication of this result is that she cannot benefit by delegating the decision rights over the project size to the agents.
- (ii) Otherwise, because \bar{Q}_x increases in x , anticipating that the manager will choose a project larger than \bar{Q}_0 , the agents will not exert effort until she commits to a project size. As such, her optimal project size is equal to $\min\{\bar{Q}_0, y\}$. In this case, the manager finds it optimal to delegate the decision rights over the project size to the agents if her commitment power is below a certain threshold.

A natural question is whether this game has other MPE. We answer this question affirmatively by characterizing, first, an asymmetric MPE with bang-bang strategies, and second, a symmetric MPE in which the agents are indifferent across any effort level along the equilibrium path.

Example 1. Let $r = 0.1$, $u = 10$, $\beta = 0.5$, $Q = 10$, $n = 4$, and $\lambda = 0.1$. This game has an asymmetric MPE in which only one agent exerts the maximal effort u at every moment, whereas the other agents never exert any effort.

More generally, one can construct MPE with bang-bang strategies in which initially, only few agents exert effort, and others join in as the project progresses. This is in contrast to the uniqueness result established in Proposition 1, indicating that the convexity of the effort costs plays a key role in the uniqueness of a (project-completing) MPE.

In addition to equilibria with bang-bang strategies, if the group comprises $n \geq 2$ agents, then one can construct equilibria with interior strategies as shown in the following result.

Result 8. Suppose that $\frac{\beta}{n} > \lambda$, $n \geq 2$, and u is sufficiently large. Then for any Q , there exists a symmetric project-completing MPE in which each agent's discounted payoff and effort strategy satisfies

$$\Pi(q; Q) = \left(\frac{\beta}{n} - \lambda\right) Q + \lambda q \quad \text{and} \quad a(q; Q) = \frac{r \left[\left(\frac{\beta}{n} - \lambda\right) Q + \lambda q\right]}{(n - 1)\lambda},$$

respectively. In contrast, if $\frac{\beta}{n} \leq \lambda$ or $n = 1$, then there exists no symmetric project-completing MPE with interior strategies.¹⁵

This MPE has a flavor of a mixed strategy equilibrium; observe that $\Pi'(q; Q) = \lambda$ for all q , so that at every moment, the agents are indifferent across any effort level. Observe that each agent's *ex ante* discounted payoff increases in the project size, which implies that the agents' (and it turns out also the manager's) optimal project size is equal to ∞ . This is intuitive as $\frac{\beta}{n} > \lambda$ implies that each agent's marginal benefit from a unit of progress exceeds its marginal cost, and the agents are risk neutral. Finally, it is useful to note that one can construct MPE in which some of the agents use bang-bang strategies, with the others using interior strategies, akin to those characterized in Results 7 and 8, respectively.

6. Concluding remarks

■ We propose a tractable model to study the interaction between a group of agents who collaborate over time to complete a project and a manager who chooses its size. A central feature of the model is that the manager has limited commitment power, in that she can only commit to the project size when the project is sufficiently close to completion. This is common in projects that involve a significant innovation or quality or design component that is difficult to contract on in advance.

In a setting in which both the manager and the agents are rational and they do not obtain new information about the difficulty or the value of the project, we show that the manager has incentives to extend the project as it progresses. As a result, if the manager has lower commitment power, then she will eventually commit to a bigger project. To mitigate her commitment problem, the manager might consider delegating the decision rights over the project size to the agents, who will choose a smaller project than is optimal for the manager, but their preferences are time-consistent. We show that delegation is optimal unless the manager has sufficient commitment power.

¹⁵ If $u < a(Q; Q) = \frac{r\beta Q}{n(n-1)\lambda}$, then there exists a symmetric MPE in which the agents' effort levels are interior up to some threshold Θ , while they are equal to u thereafter.

In our model, agents are compensated upon completion of the project and their compensation is independent of the completion time of the project. Although Georgiadis (2013) shows that backloading all rewards is optimal when the project size is given exogenously, it is unclear that this continues to be the case when the project size is endogenous and the manager has limited commitment power. It would be interesting to explore whether a more elaborate scheme in which the manager provides agents with flow payments while the project is in progress (e.g., Sannikov, 2008) or uses time-dependent contracts (e.g., milestones with deadlines) can improve her discounted profit. Second, Schelling (1960) made an important observation that free-riding is both a commitment problem and an information problem, and although significant advances have been made to analyze the former, little is known about the latter. Therefore, incorporating asymmetric or incomplete information to this class of games may provide valuable insights.

Appendix A

In this Appendix, we consider six extensions to our model to test the robustness of the main results.

□ **Production synergies and team coordination costs.** First, we consider the case in which at every moment, the total effort of the team is greater (due to production synergies) or smaller (due to coordination costs among the team members) than the sum of the agents' individual efforts. We show that all three main results continue to hold for any degree of complementarity.

To obtain tractable results, we consider the production function proposed by Bonatti and Hörner (2011), so that the project evolves according to $dq = (\sum_{i=1}^n a_i^{1/\gamma})^\gamma dt$, where $\gamma > 0$. Note that $\gamma \in (0, 1)$ ($\gamma > 1$) captures the case in which the total effort of the team is smaller (greater) than the sum of the agents' individual efforts, and a larger γ indicates smaller coordination costs or a stronger degree of complementarity. By assuming symmetric strategies, it follows that given the current state of the project q , cooperation level k , and the completion state Q , each agent's discounted payoff and effort strategy are given by

$$\Pi(q; Q, k) = \frac{rn^{2-2\gamma}}{2k} \frac{([q - C(Q, k)]^+)^2}{2n - k} \quad \text{and} \quad a(q; Q, k) = \frac{rn^{1-\gamma}}{2n - k} [q - C(Q, k)]^+,$$

respectively, where $C(Q, k) = Q - \sqrt{\frac{2\beta Q}{r} \frac{n^{2\gamma-2}(2n-k)k}{n}}$.¹⁶ Because (with other things equal) $\Pi(q; Q, k)$ increases in k for all γ , it follows that for all $k \in [1, n]$, there exists a PPE such that each agent follows the strategy dictated by $a(q; Q, k)$, and after any deviation from the equilibrium path, all agents revert to the MPE; that is, $k = 1$. Furthermore, each agent's discounted payoff, his equilibrium effort, as well as the aggregate effort of the entire team, increase in the degree of complementarity γ .

By using the agents' strategies, it follows that the manager's discounted profit satisfies

$$W(q; Q, \tilde{Q}, k) = (1 - \beta)Q \left(\frac{[q - C(\tilde{Q}, k)]^+}{Q - C(\tilde{Q}, k)} \right)^{\frac{2n-k}{n}}.$$

To streamline the exposition, we focus on the extreme cases in which the manager has either full or no commitment power. It follows that

$$Q_{FC}^M = \frac{\beta}{r} \frac{k(2n - k)}{2n} \left(\frac{4n}{4n - k} \right)^2 n^{2\gamma-2} \quad \text{and} \quad Q_{NC}^M = \frac{2\beta}{r} \frac{kn}{2n - k} n^{2\gamma-2}.$$

Observe that the manager's optimal project size increases in the degree of complementarity, and similar to the case analyzed in Section 4 $Q_{NC}^M > Q_{FC}^M$. Moreover, the counterpart of Proposition 3 continues to hold; that is, if the manager has less commitment power, then she will choose a bigger project.

We now examine the manager's option to delegate the choice of Q to the agents. To begin, note that the agents' optimal project size satisfies $Q^A = \frac{\beta}{2r} \frac{k(2n-k)}{n} n^{2\gamma-2}$. By following a similar approach as in the third subsection of Section 4, it follows that there exists a threshold Φ such that the manager is better off delegating the choice of the project size to the agents if and only if her commitment power $\gamma < \Phi$.

□ **Fixed compensation.** In the base model, we have assumed that the agents' net payoff upon completion of the project is proportional to its value. Whereas a more valuable project will typically yield a larger net payoff to the agents—for

¹⁶ As the algebra is straightforward and similar to that used to derive Propositions 1 and 2, it is omitted here in order to streamline the exposition.

example, a bigger bonus, a salary increase, greater job security, or a larger outside option, this assumption can be thought of as an extreme case, as any incentive scheme will likely consist of a fixed component that is independent of the project size, and a performance-based component. In this section, we consider the opposite extreme where each agent's net payoff is fixed and independent of the project size, while efforts are perfect substitutes; that is, $dq_t = (\sum_{i=1}^n a_{i,t}) dt$.

The main results continue to hold. In fact, the manager's commitment problem becomes so aggravated in this case, that the project may never be completed in equilibrium. Moreover, because the agents' net reward is independent of Q , their optimal project size is always 0. As such, the manager can no longer use delegation to mitigate her commitment problem.

To begin, suppose that each agent receives a lump sum $\frac{V}{n}$ as soon as the project is completed regardless of its size. Then given the current state of the project q , the cooperation level k , and the completion state Q , each agent's equilibrium effort is given by

$$\bar{a}(q; Q, k) = \frac{r}{2n-k} [q - \bar{C}(Q, k)]^+ \quad \text{where} \quad \bar{C}(Q, k) = Q - \sqrt{\frac{2V}{r} \frac{(2n-k)k}{n}},$$

whereas the manager's discounted profit satisfies

$$\bar{W}(q; Q, \bar{Q}, k) = (Q - V) \left(\frac{[q - \bar{C}(\bar{Q}, k)]^+}{Q - \bar{C}(\bar{Q}, k)} \right)^{\frac{2n-k}{n}}.$$

Using the same approach as in Section 3, one can show that for all $k \in (1, n]$, there exists a PPE such that each agent follows the strategy dictated by $\bar{a}(q; Q, k)$ contingent on all other agents following the same strategy, and reverts to the MPE (i.e., $k = 1$) after observing a deviation.

By examining the manager's optimal project size, it follows that with full and with no commitment power, we have

$$\bar{Q}_{FC}^M = \frac{2n-k}{3n-k} V + \frac{n}{3n-k} \sqrt{\frac{2V}{r} \frac{(2n-k)k}{n}} \quad \text{and} \quad \bar{Q}_{NC}^M = V + \sqrt{\frac{2V}{r} \frac{kn}{2n-k}},$$

respectively. Observe that $\bar{Q}_{NC}^M > \bar{Q}_{FC}^M$, and by solving for $\bar{Q}_x^M \in \arg \max_Q \bar{W}(q; Q, Q, k)$, it follows that \bar{Q}_x^M increases in x . Therefore, similar to the base model, the manager has incentives to extend the project as it progresses. In fact, these incentives can be so strong that there exists an equilibrium in which the project is not completed. To see why, note that the project is completed only if $\bar{C}(Q; k) < 0$, and this inequality is true at $Q = \bar{Q}_{NC}^M(k)$ if and only if $rV < \frac{2k(n-k)^2}{n(2n-k)}$.¹⁷ Moreover, if each agent's net payoff is independent of the project size, then delegating the choice of the project size to the agents is not beneficial, because they will choose a project of size 0.

□ **Flow payments while the project is in progress.** Throughout the analysis, we have maintained the assumption that the agents receive a lump-sum payment upon completion of the project, but they do not receive any flow payments while the project is in progress. Therefore, to extend the project, the manager must only incur the cost associated with having to wait longer until the project is completed. In this section, we consider the case in which the manager compensates each agent with a flow payment $\frac{w}{n} > 0$ per unit of time while the project is in progress, in addition to a lump-sum payment upon completing it, where w is not too large.

For a given project size Q , each agent's discounted payoff, and the manager's discounted profit satisfy

$$r\bar{\Pi}(q; Q) = \frac{w}{n} + \frac{k(2n-k)}{2} [\bar{\Pi}'(q; Q)]^2 \quad \text{s.t.} \quad \bar{\Pi}(Q; Q) = \beta Q$$

$$r\bar{W}(q; Q, \bar{Q}) = -w + [na(q; \bar{Q}, k)] \bar{W}'(q; Q, \bar{Q}) \quad \text{s.t.} \quad \bar{W}(Q; Q, \bar{Q}) = (1 - \beta)Q,$$

respectively, where $\bar{a}(q; \bar{Q}, k) = k\bar{\Pi}'(q; Q)$. As this model is analytically not tractable, to examine how the main results extend to this case, we present a numeric illustration (see Figure A1). The takeaway from this analysis is that the main results continue to hold when the agents receive flow payments while the project is in progress.

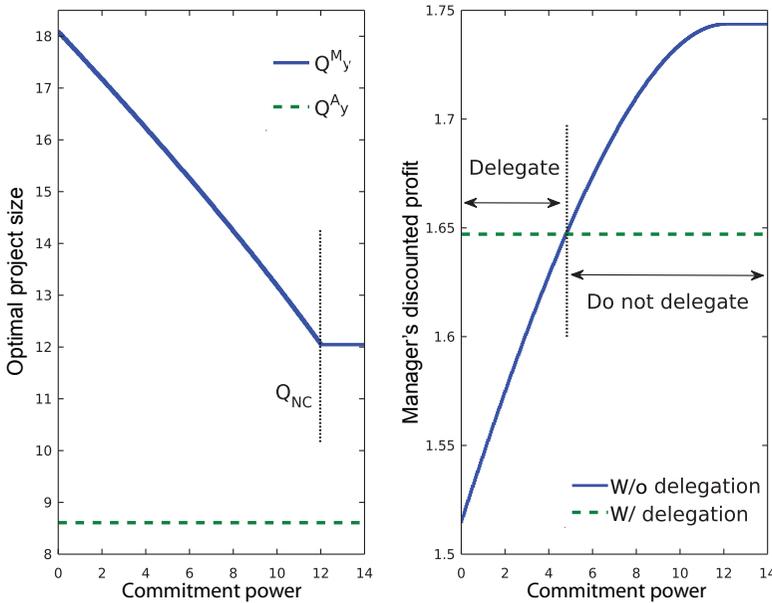
□ **Variable incentive power.** A key assumption of the model is that upon completion of the project, its value is split among the parties according to a fixed fraction: the manager receives $1 - \beta$ of its value, whereas each agent receives $\frac{\beta}{n}$. We have argued that this assumption is important to capture the manager's limited commitment power (see the last paragraph in the second section of subsection two of Section 4). Nevertheless, one might still wonder what is the impact on the overall dynamics of choosing some alternative class of reward functions. In this section, we consider the case in which $\beta(q) = \beta_0 + \beta_1 q$, where $\beta_0 \in (0, 1)$, and β_1 is either positive or negative, but sufficiently small.

For a given Q , the equilibrium characterization is identical to that developed in Section 3. Moreover, it is straightforward to show that similar to the base model, the agents' optimal project size is independent of when they commit to it. However, determining the manager's optimal project size is not tractable due to the second term in $\beta(q)$. As

¹⁷ In this case, the manager finds it optimal to commit to a project size equal to y at time 0.

FIGURE A1

AN EXAMPLE IN WHICH THE MANAGER COMPENSATES THE AGENTS PER UNIT OF TIME WHILE THE PROJECT IS IN PROGRESS WHEN $\beta = 0.5$, $r = 0.1$, $n = 4$, AND $w = 0.001$



Notes: The left panel illustrates that her optimal project size decreases in her commitment power, whereas the agents' optimal project size is independent of their commitment power. The right panel illustrates that delegating the decision rights over the project size to the agents is beneficial if and only if the manager doesn't have sufficient commitment power.

such, we examine how the main results of the article extend to this case numerically. Figure A2 illustrates an example, which indicates that the main results hold if the agents' share of the payoff is a (linear) function of the project size.

□ **Sequential projects.** Insofar, we have assumed that the manager interacts with the agents for the duration of a single project. However, because in practice, relationships between a manager and work teams are often persistent, it is important to verify that the main results of this article are robust to repeated interactions. In this section, we consider the case in which as soon as a project is completed, the manager and the agents interact for the duration of another project with probability $\alpha < 1$, whereas the relationship is terminated with probability $1 - \alpha$ and each party receives its outside option, which is normalized to 0.¹⁸

Indeed, we find that when the manager and the agents engage in sequential projects, all the main results continue to hold. Moreover, we observe that if the relationship is more persistent (i.e., α is larger), then the manager has stronger incentives to delegate the choice of the project size to the agents.

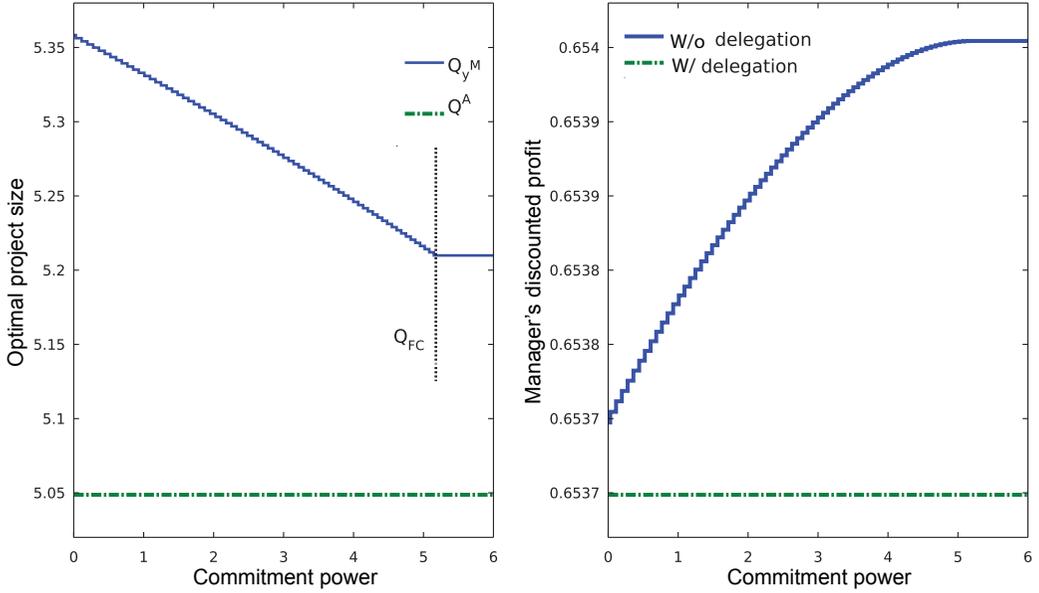
Because the problem is stationary, the manager will choose the same project size every time. Both the agents' and the manager's problem remain unchanged, except for the boundary conditions, which become $\bar{\Pi}(Q; Q) = \frac{\beta Q}{n} + \alpha \bar{\Pi}(0; Q)$ and $\bar{W}(Q; Q, \tilde{Q}) = (1 - \beta)Q + \alpha \bar{W}(0; Q, \tilde{Q})$, respectively. The interpretation of these conditions is that upon completion of each project, each party receives its net payoff from this project, plus the expected continuation value from future projects.

Unfortunately, it is not possible to derive the desired results analytically. As such, we use a numerical example to illustrate how the main results of the article extend to this case. Figure A3 illustrates that the main results continue to hold. In particular, (i) the manager's optimal project size decreases in her commitment power, whereas the agents' optimal project size is independent of their commitment power, and (ii) the manager should delegate the decision rights over Q unless she has sufficient commitment power.

¹⁸ Because the value of the project has been assumed to be linear in its size and it generates a payoff only upon completion, as $\alpha \rightarrow 1$, both the manager and the agents choose an arbitrarily small project, which is completed arbitrarily quickly. Therefore, we restrict attention to the cases in which $\alpha < 1$.

FIGURE A2

AN EXAMPLE IN WHICH THE AGENTS' SHARE OF THE PROJECT'S PAYOFF INCREASES WITH THE PROJECT SIZE WHEN $r = 0.2$, $n = 4$, $k = 1$, AND $\beta(q) = 0.5 + 0.005q$



Notes: The left panel illustrates that the manager's optimal project size decreases in her commitment power. From the right panel, observe the manager cannot benefit by delegating the decision rights over the project size to the agents. More generally, numerical examples indicate that this is true for sufficiently large values of β_1 .

□ **Stochastic evolution of the project.** A key assumption that provides tractability to our model is that the project progresses deterministically. To obtain some insights as to how the results in this article depend on this assumption, consider the case in which the project progresses stochastically according to

$$dq_t = \left(\sum_{i=1}^n a_{i,t} \right) dt + \sigma dW_t,$$

where $\sigma > 0$ captures the degree of uncertainty associated with the evolution of the project, and W_t is a standard Brownian motion. It is straightforward to show that for a given project size Q , in any MPE, each agent's expected discounted payoff and the manager's expected discounted profit satisfy

$$r \bar{\Pi}(q; Q) = \frac{(2n-1)}{2} [\bar{\Pi}'(q; Q)]^2 + \frac{\sigma^2}{2} \bar{\Pi}''(q; Q) \quad \text{and}$$

$$r \bar{W}(q; Q, \tilde{Q}) = [n\bar{a}(q; \tilde{Q})] \bar{W}'(q; Q, \tilde{Q}) + \frac{\sigma^2}{2} \bar{W}''(q; Q, \tilde{Q}),$$

subject to $\bar{\Pi}(Q; Q) = \frac{\beta Q}{n}$, $\bar{W}(Q; Q, \tilde{Q}) = (1 - \beta)Q$, $\lim_{q \rightarrow -\infty} \bar{\Pi}(q; Q) = 0$, and $\lim_{q \rightarrow -\infty} \bar{W}(q; Q) = 0$, respectively, where $\bar{a}(q; \tilde{Q}) = \bar{\Pi}'(q; Q)$. This problem is studied by Georgiadis (2013) for a fixed project size, who shows that similar to Proposition 1, a unique solution to this system of ODE exists, and $\bar{\Pi}'(q; Q) > 0$ and increasing in q , which in turn implies that the MPE is unique. It is worth noting that the non-Markovian strategies characterized in this article no longer constitute a PPE if the project progresses according to the above stochastic process. Intuitively, this is because a deviation from the equilibrium path cannot be detected instantaneously with probability 1.

Georgiadis (2011) shows that the agents are time-consistent with respect to their optimal project size, which implies that similar to the deterministic case, their optimal project size is independent of the commitment power y . Unfortunately however, the analysis of the manager's choice of Q is not tractable. Therefore, in Figure A4, we use a numerical example to illustrate that the main results continue to hold when the project progresses stochastically: the manager's optimal project size decreases in her commitment power, whereas the agents' optimal project size is independent of their commitment power, and delegation is optimal unless the manager has sufficient commitment power.

FIGURE A3

AN EXAMPLE IN WHICH THE MANAGER INTERACTS WITH THE AGENTS REPEATEDLY WHEN $\beta = 0.5$, $r = 0.1$, $n = 4$, and $\alpha = 0.25$. SIMILAR TO FIGURE A1, IT ILLUSTRATES THAT THE MAIN RESULTS CONTINUE TO HOLD IN THIS CASE

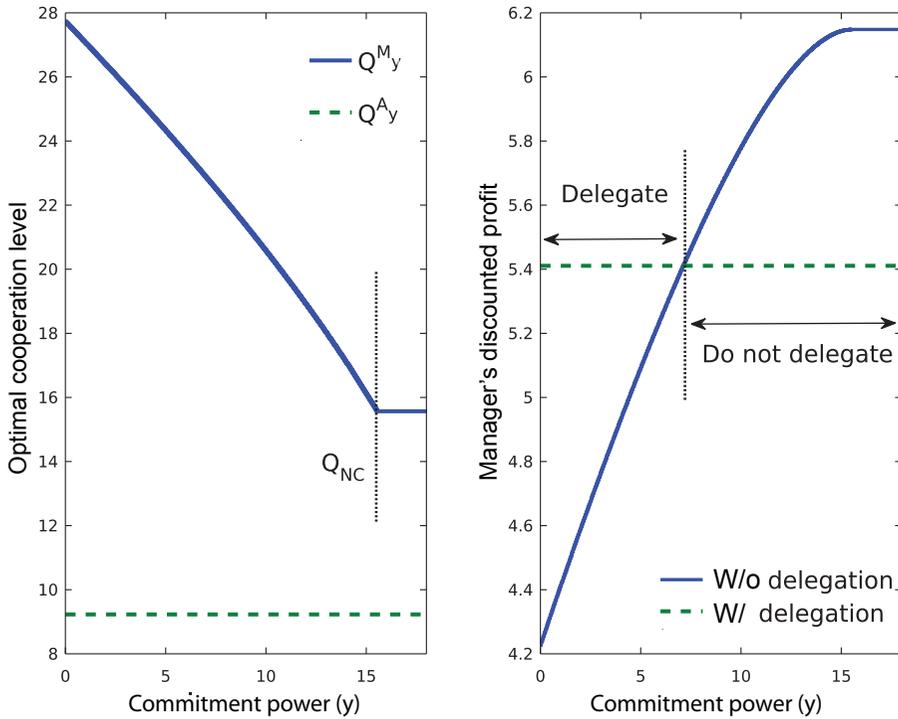
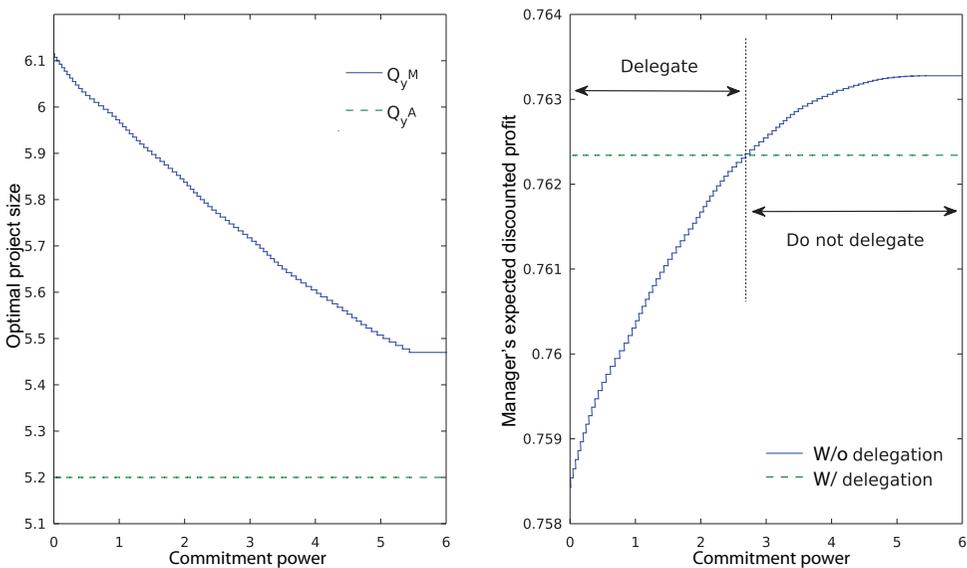


FIGURE A4

AN EXAMPLE IN WHICH THE PROJECT PROGRESSES STOCHASTICALLY WHEN $\beta = 0.5$, $r = 0.1$, $n = 3$, AND $\sigma = 1$. SIMILAR TO FIGURE A1, IT ILLUSTRATES THAT THE MAIN RESULTS CONTINUE TO HOLD IN THIS CASE



Appendix B

This Appendix contains all the proofs.

Proof of Proposition 1. To show that a MPE with differentiable strategies exists for this game, it suffices to show that a solution to (4) exists. To show this, we derive a symmetric solution analytically. In particular, for symmetric strategies (i.e., $\Pi_i(q; Q) = \Pi_j(q; Q)$ for all i and j), (4) can be rewritten as

$$r\Pi(q; Q) = \frac{2n-1}{2} [\Pi'(q; Q)]^2, \quad (\text{B1})$$

and the solution to this ODE satisfies

$$\Pi(q; Q) = \frac{r}{2} \frac{([q - C(Q)]^+)^2}{2n-1}, \quad \text{where } C(Q) = Q - \sqrt{\frac{2\beta Q}{r} \frac{2n-1}{n}}$$

is determined by the value matching condition. By using the first-order condition, it follows that each agent's effort strategy is given by

$$a(q; Q) = \frac{r}{2n-1} [q - C(Q)] \mathbf{1}_{\{q \geq C(Q)\}}.$$

To show that there do not exist any asymmetric solutions to (4), we proceed by contradiction. Fix $Q > 0$, and suppose there exists at least two agents a and b whose discounted payoff functions $\Pi_a(q; Q)$ and $\Pi_b(q; Q)$ satisfy (4), but $\Pi_a(q; Q) \neq \Pi_b(q; Q)$ for at least some $q < Q$. Then let $D(q) = \Pi_a(q; Q) - \Pi_b(q; Q)$, and note that $D(Q) = 0$ and $D(\cdot)$ is differentiable. Then using (4), we can write $2rD(q) = [2\sum_i \Pi_i(q; Q) - \Pi_a(q; Q) - \Pi_b(q; Q)]D'(q)$. Moreover, because agents are impatient ($r > 0$) and the amount of effort that needs to be exerted until the project is completed diverges to infinity as $q \rightarrow -\infty$, it must be true that $\Pi_i(q; Q) \rightarrow 0$ as $q \rightarrow -\infty$. Therefore, $\lim_{q \rightarrow -\infty} D(q) = 0$, so if $D(q) \neq 0$ for at least some $q < Q$, then it must be the case that there exists some interior $z < Q$ such that $D(z) \neq 0$ and $D'(z) = 0$, which yields a contradiction. Hence, we conclude that (4) cannot admit an asymmetric solution.

To show that (B1) has a unique symmetric solution, we use a similar approach. Fix $Q > 0$, and suppose that there exists $\Pi_A(q; Q)$ and $\Pi_B(q; Q)$ that both satisfy (B1). Then let $\Delta(q) = \Pi_A(q; Q) - \Pi_B(q; Q)$, and note that $\Delta(Q) = 0$ and $\Delta(\cdot)$ is differentiable. Therefore, (B1) can be rewritten as $2r\Delta(q) = (2n-1)[\Pi'_A(q; Q) + \Pi'_B(q; Q)]\Delta'(q)$. Moreover, $\lim_{q \rightarrow -\infty} \Delta(q) = 0$ by the same argument as above, so if $\Delta(q) \neq 0$ for at least some $q < Q$, then it must be the case that there exists some interior $z < Q$ such that $\Delta(z) \neq 0$ and $\Delta'(z) = 0$, which yields a contradiction. Therefore, there exists a unique symmetric solution to (4).

We have insofar shown that there exists a unique solution to (4), and that this solution is symmetric. Moreover, note that if $C(Q) \geq 0$ (or equivalently, $Q \geq \frac{2\beta}{r} \frac{2n-1}{n}$), then the equilibrium strategy dictates that no agent ever exerts any effort, in which case the project is never completed. On the other hand, as long as $C(Q) < 0$, the strategy $a(q; Q)$ constitutes the unique *project-completing* MPE. Next, suppose that $C(Q) < 0 \leq C(Q)|_{n=1}$ (or equivalently, $\frac{2\beta}{r} \leq Q < \frac{2\beta}{r} \frac{2n-1}{n}$), and fix all effort strategies except that of agent i to 0. Then agent i 's best-response is to also exert 0 effort, as $C(Q)|_{n=1} \geq 0$; that is, he is not willing to undertake the entire project by himself. As a result, if $Q \geq \frac{2\beta}{r}$, then in addition to the *project-completing* MPE, there also exists an equilibrium in which no agent exerts any effort, and the project is never completed.

Finally, to compute the completion time of the project, we substitute the agents' effort function $a(q; Q)$ into $dq_t = na_t dt$, we solve the resulting ODE $q'(t) = \frac{r}{2n-1} [q(t) - C(Q)]^+$ subject to $q(0) = 0$, and we obtain the completion time of the project τ by solving for $q(\tau) = Q$. Q.E.D.

Proof of Proposition 2. This proof is organized as follows. First, we show that each agent's discounted payoff function is given by $\Pi(q; Q, k)$ if at every moment, all agents choose their effort level according to (5), and that (6) is the corresponding effort level. Then we show that there exists a PPE in which each agent's effort level and discounted payoff satisfies (6) and $\Pi(q; Q, k)$ along the equilibrium path, respectively.

From (5), given the current state of the project q , the first-order condition yields $a(q; Q, k) = k\Pi'(q; Q, k)$, and the second-order condition is always satisfied. Substituting the first-order condition into each agent's HJB equation yields

$$\Pi(q; Q, k) = \frac{(2n-k)k}{2r} [\Pi'(q; Q, k)]^2,$$

subject to $\Pi(Q; Q, k) \geq 0$ for all q and $\Pi(Q; Q, k) = \frac{\beta Q}{n}$.

A solution to the above HJB equation is given by $\Pi(q; Q, k) = \frac{r}{2k} \frac{([q - C(Q, k)]^+)^2}{2n-k}$, and by using the first-order condition, it follows that each agent's effort strategy satisfies (6). If $k = 1$, then (6) corresponds to the Markov equilibrium. To compute the completion time of the project, we substitute (6) into $dq_t = na_t dt$, solving the resulting differential equation $q'(t) = \frac{r}{2n-k} [q(t) - C(Q, k)]^+$ subject to $q(0) = 0$ yields that $q(t) = -C(Q, k)(e^{-\frac{r}{2n-k}t} - 1)$. By solving for $q(\tau) = Q$, we obtain the completion time of the project $\tau(Q, k) = \frac{2n-k}{rn} \ln[1 - \frac{Q}{C(Q, k)}]$.

We now show that for any $k \in (1, n]$, there exists a PPE in which along the equilibrium path, each agent's effort level is given by (6). It follows that in equilibrium, each agent's discounted payoff satisfies $\Pi(q; Q, k)$. Because the project progresses deterministically, there is a one-to-one mapping between t and q_t , and it will be convenient to rewrite (6) as a function of time, so that (abusing notation) $a(t; Q, k) = -\frac{rC(Q, k)}{2n-k} e^{\frac{rnt}{2n-k}}$. Fix some Q and $k \in (1, n]$, and consider the following strategy:

$$a_t = \begin{cases} a(t; Q, k) & \text{if } q_t = -C(Q, k) [e^{\frac{rnt}{2n-k}} - 1] \\ a(q_t; Q, 1) & \text{otherwise,} \end{cases}$$

where $a(q; Q, k)$ is given in (6) and $q_t = \int_0^t n a(s; Q, k) ds$. This strategy designates that at every moment t , each agent chooses his effort level by maximizing the discounted payoff of k agents as long as every other agent has insofar followed this strategy, and he reverts to the MPE otherwise. There is a well-known problem associated with defining such trigger strategy in continuous-time games. To see why, suppose that a deviation occurs at some t' , and agents revert to the MPE at some $t'' > t$. Because there is no first moment after t' , there always exists some $t \in (t', t'')$ such that the agents are better off reverting to the MPE at that t ; that is, subgame perfection fails. To resolve this problem, following Bergin and MacLeod (1993), we define the following inertia strategies:

$$a_t^\epsilon = \begin{cases} a(t; Q, k) & \text{if } q_{t-\epsilon} = -C(Q, k) [e^{\frac{r n(t-\epsilon)}{2n-k}} - 1] \\ a(q_t; Q, 1) & \text{otherwise,} \end{cases} \tag{B2}$$

where $\epsilon > 0$ and sufficiently small, and $q_t = 0$ for all $t \leq 0$. First, observe that at the limit as $\epsilon \rightarrow 0$, a_t^ϵ converges to a_t . Second, because $a_t^\epsilon < \infty$ for all ϵ, t, Q , and $k \in (1, n]$, for any decreasing sequence $\{\epsilon^m\}_{m=1}^\infty$ converging to 0, the strategies $a_t^{\epsilon^m}$ form a Cauchy sequence. Third, whereas the strategy in (B2) cannot form a PPE, we shall show that it forms an ϵ -PPE. To see why, first note that if at every moment all agents choose their effort level according to (5), then given the current state of the project q , each agent's discounted payoff is equal to $\Pi(q; Q, k)$. Moreover, $\Pi(q; Q, k) > \Pi(q; Q, 1)$ for all $q \geq C(Q, k)$ and $k > 1$. Second, observe that for sufficiently small ϵ , the optimal strategy against (B2) is for an agent to choose $a(t; Q, k)$ at every $t < \tilde{t}$, and deviate to the MPE (so that he maximizes his discounted payoff given the current state of the project) at $t = \tilde{t}$, where \tilde{t} is chosen such that the project is completed at $\tilde{t} + \epsilon$ when the agents follow (B2). In this case, project completion is delayed by at most ϵ units of time, and each agent's discounted payoff when the state of the project is equal to q is at least $\Pi(q; Q, k) - \epsilon r e^{-rC(Q, k)} - \frac{\epsilon}{2} [a(Q; Q, k)]^2$. Therefore, the strategy in (B2) forms an ϵ -PPE, and by applying Theorem 3 of Bergin and MacLeod (1993), it follows that the limit strategy as $\epsilon \rightarrow 0$ forms a PPE. Consequently, there exists a PPE such that along the equilibrium path, each agent's effort level is given by (6), and his discounted payoff satisfies $\Pi(q; Q, k)$. Q.E.D.

Proof of Proposition 3. To begin, fix $k \in [1, n]$, and note that for any x , $W(x; Q, Q, k)$ is strictly concave in Q . Applying the first-order condition yields (8). It is straightforward to verify that $\frac{\partial}{\partial x} Q_x^M \in (0, 1)$ and $\frac{\partial^2}{\partial x^2} Q_x^M < 0$ for all $q > 0$. Finally, solving the fixed point $Q_Q^M = Q$ yields $Q_Q^M = \frac{\beta}{r} \frac{2kn}{2n-k}$.

Next, let $g(x) = Q_x^M - x$, and observe that $g(0) = Q_0^M > 0$ and $g(Q_Q^M) = 0$. Moreover, it is easy to check that $g'(x) < 0$ on $[0, Q_Q^M]$, which implies that given any $y \leq Q_0^M$, there exists a unique $x(y)$ such that $g(x(y)) = y$.

Clearly, if $y \geq Q_0^M$, then the manager finds it optimal to commit to Q_0^M at $x = 0$. Therefore, for all $y \geq 0$, there exists a unique $x(y)$ that solves $\max\{Q_{x(y)}^M - y, 0\} = x(y)$. Furthermore, noting that $\frac{\partial}{\partial x} Q_x^M \in (0, 1)$, it follows that $x(y)$ and $Q_{x(y)}^M$ both decrease in y .

To proceed, suppose that $y < Q_0^M$, and note that $W(q; Q, Q)$ is strictly concave in Q for all $Q \geq q$. Given the current state of the project q , the manager can either commit to a completion state in the interval $[q, q + y]$, in which case her discounted payoff is equal to $\max_{q \leq Q \leq q+y} W(q, Q, Q)$, or she can delay committing, anticipating that she will be able to commit to some completion state $q' > q + y$ later, which will yield her a discounted payoff $W(q, q', q')$. Therefore, the manager will choose to commit to a completion state at q if and only if $\max_{q \leq Q \leq q+y} W(q, Q, Q) \geq W(q, q', q')$ for all $q' > q + y$, or equivalently if and only if $\arg \max_{Q \geq q} W(q, Q, Q) \leq q + y$. By noting that $Q_q^M \in \arg \max_{Q \geq q} \{W(q; Q, Q)\}$, it follows that the manager finds it optimal to commit to project size $Q_{x(y)}^M$ at $q = x(y)$, where $x(y)$ is the unique solution to the equation $\max\{Q_{x(y)}^M - y, 0\} = x(y)$, and $Q_{x(y)}^M$ is given by (8). Q.E.D.

Proof of Proposition 4. To begin, fix $n \geq 1$ and $k \in [1, n]$. If the project size is chosen by the agents, then they will choose $Q^A = \frac{\beta}{r} \frac{k(2n-k)}{2n}$, and by substituting this into the manager's expected discounted profit, yields $W(0; Q^A, Q^A) = \frac{(1-\beta)\beta}{r} \frac{k(2n-k)}{2n} (\frac{1}{2})^{\frac{2n-k}{n}}$.

Next, consider the case in which the completion state is chosen by the manager, and she has no commitment power (i.e., $y = 0$) so that she eventually completes the project at $Q_{NC}^M = \frac{\beta}{r} \frac{2kn}{2n-k}$. By substituting this, the manager's expected discounted profit we have that $W(0; Q_{NC}^M, Q_{NC}^M) = \frac{(1-\beta)\beta}{r} \frac{2kn}{2n-k} (\frac{n-k}{2n-k})^{\frac{2n-k}{n}}$.

Now consider the ratio $\frac{W(0; Q_{NC}^M, Q_{NC}^M)}{W(0; Q^A, Q^A)} = (\frac{2n}{2n-k})^2 (\frac{2n-2k}{2n-k})^{\frac{2n-k}{n}}$, and for the purpose of this proof, let $h(n, k) = (\frac{2n}{2n-k})^2 (\frac{2n-2k}{2n-k})^{\frac{2n-k}{n}}$. Observe that $h(k, k) = 0$ and $\lim_{n \rightarrow \infty} h(n, k) = 1$. Differentiating with respect to n yields $\frac{d}{dn} h(n, k) = \frac{4[(2n-k)(n-k) \ln(\frac{2n-2k}{2n-k}) + nk]}{(2n-k)^3 (n-k)} (\frac{2n-2k}{2n-k})^{\frac{2n-1}{n}} > 0$ if and only if $(2n-k)(n-k) \ln(\frac{2n-2k}{2n-k}) + nk > 0$ or equivalently if $\ln(\frac{2n-2k}{2n-k}) + \frac{nk}{(2n-k)(n-k)} > 0$. Now observe that $\lim_{n \rightarrow \infty} [\ln(\frac{2n-2k}{2n-k}) + \frac{nk}{(2n-k)(n-k)}] = 0$, and $\frac{\partial}{\partial n} [\ln(\frac{2n-2k}{2n-k}) + \frac{nk}{(2n-k)(n-k)}] < 0$ for all $n \geq k$.

This implies that $\ln\left(\frac{2n-2k}{2n-k}\right) + \frac{nk}{(2n-k)(n-k)} > 0$, and hence $\frac{d}{dn}h(n, k) > 0$. By noting that $h(k, k) = 0$ and $\lim_{n \rightarrow \infty} h(n, k) = 1$, it follows that $h(n, k) < 1$ for all $n \geq k$, which implies that $W(0; Q^A, Q^A) > W(0; Q_{NC}^M, Q_{NC}^M)$ for all $n \geq k$.

We have thus far established that $W(0; Q^A, Q^A) > W(0; Q_{NC}^M, Q_{NC}^M)$. Moreover, it is straightforward to verify that $W(0; Q_{FC}^M, Q_{FC}^M) > W(0; Q^A, Q^A)$; that is, the manager should not delegate the choice of Q to the agents if she has full commitment power. Because $Q^M(y)$ is strictly decreasing in y for all $y < Q_{FC}^M$, $W(0; Q, Q)$ is strictly concave in Q , and $Q_{FC}^M < Q_{NC}^M$, it follows that $W(0; Q^M(y), Q^M(y))$ is strictly increasing in y on $[0, Q_{FC}^M]$. By noting that $W(0; Q^A, Q^A)$ is independent of y , it follows that there exists some threshold $\Phi < Q_{FC}^M$ such that $W(0; Q^A, Q^A) > W(0; Q^M(y), Q^M(y))$ if and only if $y < \Phi$. Q.E.D.

Proof of Proposition 5. Fix some $x \in [\max\{Q^A, Q^M(y)|_{k=1}\}, Q^M(y)]$. By Proposition 2, there exists a PPE in which the agents choose their effort to maximize the discounted payoff of k agents at every t such that $q_t < x$, and they revert to the MPE at $q_t = x$. Now observe that by reverting to the MPE, the agents cannot induce the manager to choose a project size smaller than $Q^M(y)|_{k=1}$. Moreover, if the state of the project is q_t and the agents are playing the MPE, then the manager's best-response is to complete the project at $Q = \max\{q_t, Q^M(y)|_{k=1}\}$. Therefore, there exists a SPE in which the agents switch to the MPE at $q_t = x$, and the manager best-responds by completing the project instantaneously. Q.E.D.

Proof of Proposition 6. We use Pontryagin's maximum principle to characterize a MPE. The Hamiltonian associated with each agent i 's objective function can be written as

$$H_{i,t} = -e^{-rt} \frac{a_{i,t}^2}{2} + \lambda_{i,t} \left(\sum_{j=1}^n a_{j,t} \right),$$

where $\lambda_{i,t} \geq 0$ is the costate variable associated with agent i 's Hamiltonian, and his bequest function is $\phi_{i,T} = \frac{\beta Q}{n} e^{-rT}$. By Pontryagin's principle, for each agent i , there exists a continuous function $\lambda_{i,t}$ that satisfies the following conditions for all $t \geq 0$: (i) the optimality equation $\frac{dH_{i,t}}{da_{i,t}} = 0$; (ii) the adjoint equation $\dot{\lambda}_{i,t} = -\frac{dH_{i,t}}{dq}$; and (iii) the transversality condition $\lambda_{i,T} = \frac{d\phi_{i,T}}{dQ}$. The optimality equation implies that $a_{i,t} = \lambda_{i,t} e^{rt}$, and the adjoint equation can be rewritten as

$$\dot{\lambda}_{i,t} = - \sum_{j \neq i} \frac{dH_{i,t}}{da_{j,t}} \frac{da_{j,t}}{dt} \frac{dt}{dq} = - \sum_{j \neq i} \frac{\lambda_{i,t} (r\lambda_{j,t} + \dot{\lambda}_{j,t})}{\sum_{l=1}^n \lambda_{l,t}} \Rightarrow \dot{\lambda}_{i,t} = -r \frac{n-1}{2n-1} \lambda_{i,t},$$

where the last equality follows from the restriction to symmetric equilibria (and hence, we drop the subscript i). A solution to the adjoint equation is $\lambda_t = c e^{-rt} \frac{n-1}{2n-1}$, and by using the transversality condition to determine the constant c , it follows that $c = \frac{\beta}{n} e^{\frac{r\beta T}{2n-1}}$. Therefore, each agent's effort function satisfies the desired expression, and the state of the project at the deadline is equal to $q_T = \int_0^T na_t dt = \frac{\beta}{r} \frac{2n-1}{n} (1 - e^{-\frac{r\beta T}{2n-1}})$. Finally, it is straightforward to verify that the maximized Hamiltonian in concave is q , which implies that this solution does indeed constitute a MPE. Q.E.D.

Proof of Result 6. The manager's optimal project duration T^* follows by taking the first-order condition of $W(0; T)$ with respect to T , and noting that the second-order condition is satisfied. The project size at $t = T^*$ follows by substituting T^* into the expression for q_T derived in Proposition 6. By substituting T^* and q_{T^*} into $W(0; T)$, it follows that

$$W(0; T^*) = \frac{(1-\beta)\beta}{r} \left(\frac{2n-1}{3n-1} \right)^{\frac{3n-1}{n}},$$

and it is straightforward to verify that $W(0; T^*) \geq W(0; Q^A, Q^A) = \frac{(1-\beta)\beta}{r} \frac{2n-1}{2n} \left(\frac{1}{2} \right)^{\frac{2n-1}{n}}$ for all n , with strict inequality holding for all $n \geq 2$. Q.E.D.

Proof of Result 7. Taking the first-order condition in (9) with respect to a_i yields that agent i finds it optimal to exert effort u if $\Pi_i(q; Q) \geq \lambda$, whereas he exerts no effort otherwise. By substituting this into (9), assuming symmetry and solving the resulting ODE yields the desired discounted payoff function. Differentiating this with respect to q yields that $\Pi_i'(q; Q) \geq \lambda$ if and only if $q \geq Q + \frac{nu}{r} \ln\left(\frac{\lambda n^2 u}{r\beta Q + \lambda nu}\right) = \psi$, and by noting that $q_0 = 0$ and $\Pi_i'(q; Q)$ increases in q , it follows that each agent always exerts the maximal effort u if and only if $\psi \leq 0$. Q.E.D.

Proof of Example 1. Suppose that one agent (say, $i = 1$) exerts the maximal effort u at every moment, whereas the other agents (i.e., $i = 2, \dots, n$) never exert any effort. Then it follows from (9) that

$$\Pi_1(q; Q) = -\frac{\lambda u}{r} + \left(\frac{\beta Q}{n} + \frac{\lambda u}{r} \right) e^{\frac{r(q-Q)}{n}}, \text{ and}$$

$$\text{for all } i = 2, \dots, n, \quad \Pi_i(q; Q) = \frac{\beta Q}{n} e^{\frac{r(q-Q)}{n}}.$$

Noting that $\Pi'_i(q; Q)$ increases in q , for this set of strategies to constitute a MPE, it must be the case that $\Pi'_i(0; Q) \geq \lambda > \Pi'_i(Q; Q)$ for all $i = 2, \dots, n$, or equivalently, that

$$\frac{r}{u} \left(\frac{\beta Q}{n} + \frac{\lambda u}{r} \right) e^{-\frac{rQ}{u}} \geq \lambda > \frac{r}{u} \frac{\beta Q}{n}.$$

It is straightforward to verify that these inequalities are satisfied when $r = 0.1$, $u = 10$, $\beta = 0.5$, $Q = 10$, $n = 4$, and $\lambda = 0.1$. Q.E.D.

Proof of Result 8. Let us guess that there exists a symmetric MPE in which for given Q , $\Pi'(q; Q) = \lambda$ for all q . Integrating with respect to q and solving for the boundary condition yields that $\Pi(q; Q) = \left(\frac{\beta}{n} - \lambda\right)Q + \lambda q$. Substituting this into (9) and solving for each agent's effort level yields that $a(q; Q) = \frac{r\left(\frac{\beta}{n} - \lambda\right)Q + \lambda q}{(n-1)\lambda}$. By noting that the second-order condition is satisfied, it follows that this strategy does indeed constitute a MPE. Q.E.D.

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