Project Contracting Strategies for Managing Team Dynamics

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Abstract

In this chapter we study a team dynamic problem in which a group of agents collaborate over time to complete a project. The project progresses at a rate that depends on the agents' efforts, and it generates a payoff upon completion. First, we show that agents work harder the closer the project is to completion, and members of a larger team work harder than members of a smaller team - both individually and on aggregate - if and only if the project is sufficiently far from completion. Second, we analyze the problem faced by a manager who is the residual claimant of the project and she chooses the size of the team and the agents' incentive contracts to maximize her discounted payoff. We show that the optimal symmetric contract compensates the agents only upon completion of the project. Finally, we endogenize the size of the project, where a bigger project is one that requires greater cumulative effort and generates a larger upon upon completion. We show that if the manager can commit to her optimal project size at the outset of the game, then she will choose a smaller project relative to the case without commitment. An implication of this result is that without commitment, the manager is better off delegating the decision rights over the project size to the agents.

1 Introduction

Teamwork and projects are omnipresent. Lawler, Mohrman and Benson (2001) reported that most large corporations engage a substantial proportion of their workforce in teams. This is because teamwork has been shown to increase productivity in both manufacturing and service firms (Ichniowski and Shaw (2003)). Moreover, the use of teams is especially

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common in situations when the task at hand will result in a defined deliverable (Harvard Business School Press (2004)). A key component of most projects is choosing the features that must be included before the decision maker deems the product ready to market. When choosing these features, the decision maker must trade off the added value derived from a bigger or a more complex project (*i.e.*, one that contains more features) against the additional cost associated with designing and implementing the additional features.

Motivated by these observations, we analyze a team dynamic problem in which a group of agents collaborate over time to complete a project, and we address a number of questions that naturally arise in this environment. In particular, what is the impact of group size on the agents' incentives? How should a manager determine the agents' incentive contracts to maximize her profit; for example, should they be rewarded for reaching intermediate milestones? How and by whom should the optimal project size be chosen; for example, can the manager benefit by delegating the decision rights over the project size to the agents?

Our model can be applied to both within firms (e.g., research teams in new product development or consulting projects) and across firms (e.g., R&D joint ventures). More broadly, the model is applicable to settings in which a group of agents collaborate to complete a project, which generates a payoff upon completion. The expected project completion time is sufficiently long such that the agents discounting time matters. A natural example is the Myerlin Repair Foundation (MRF): a collaborative effort among a group of leading scientists in quest of a treatment for multiple sclerosis (Lakhani and Carlile (2012)). This is a long-term venture, progress is gradual, each principal investigator incurs an opportunity cost by allocating resources to MRF activities (which gives rise to incentives to free-ride), and it will pay off predominantly when an acceptable treatment is discovered.

We use a parsimonious model to analyze a *dynamic contribution game* in which a group of agents collaborate to complete a project. The project progresses at a rate that depends on the agents' costly effort, and it generates a payoff upon completion. Formally, the state of the project q_t starts at 0, and it progresses according to $dq_t = \sum_{i=1}^{n} a_{i,t}\, dt$, where $a_{i,t}$ denotes the effort level of agent i at time t. The project generates a payoff at the first stopping time τ such that $q_\tau = Q$, where Q is a one-dimensional parameter that captures the project requirements, or equivalently, the project size. The manager is the residual claimant of the project, and she possesses the decision rights over its size, as well as the agents' contracts. The model is very tractable, and payoffs and strategies are derived in closed-form.

In Section 3, we analyze the agents' problem given a fixed project size. We characterize the (essentially) unique Markov Perfect equilibrium, wherein at every moment, each agent's strategy depends solely on the current state of the project. A key result is that the agents exert greater effort the closer the project is to completion. Intuitively, this is because they discount time and they are compensated upon completion; hence, their incentives become stronger as the project progresses. An implication of this result is that efforts are strategic complements (across time). This is because by raising his effort, an agent brings the project closer to completion, thus incentivizing the other agents to raise their future efforts.

We also examine the impact of team size on the agents' incentives. In contrast to static moral hazard in teams models where an increase in the group size leads to lower effort levels (Olson (1965)), we obtain a *partially opposite result* in our dynamic model. In particular, members of a larger team work harder than members of a smaller team - both individually and on aggregate - if and only if the project is sufficiently far from completion. Intuitively, when the project is close to completion, then the game resembles the static one, and the standard free-riding effect which asserts that smaller teams work harder than larger ones holds. However, in contrast to the static game and because at every moment each agent observes the state of the project before choosing his effort, the strategic complementarity is stronger in a larger team. That is because by raising his effort, each agent induces a greater number of other agents to raise their future efforts, which in turn renders him better off. Noting that this effect is stronger at the early stages of the project, because the benefits from greater effort have a longer lasting effect, it follows that this encouragement effect dominates the free-riding effect when the project is far from completion.

In Section 4, we analyze the manager's problem, and we show that the optimal symmetric contract compensates the agents only upon completion. The intuition is that by backloading payments (compared to rewarding the agents for reaching intermediate milestones), the manager can provide the same incentives at the early stages of the project (via continuation utility), while providing stronger incentives when the project is close to completion. This result simplifies the manager's problem to determining the team size and her budget for compensating the agents.

In Section 5, we endogenize the size of the project, where a larger project requires greater cumulative effort, and delivers a bigger payoff upon completion. We consider the case in which the manager can commit to her optimal project size at the outset of the game, and the case in which she cannot and at every moment, she observes the state of the project and decides whether to complete it immediately, or to let the agents continue working and re-evaluate her decision to complete the project a moment later. To motivate this case, note that an intrinsic challenge involved in choosing the requirements of a project is that the manager may not be able to commit to them in advance. This can be due to the fact that the requirements are difficult to describe; for example, if the project involves significant novelty in quality or design. What we have in mind about the incontractibility of the project requirements was eloquently posed by Tirole (1999):

> *In practice, the parties are unlikely to be able to describe precisely the specifics of an innovation in an ex ante contract, given that the research process is precisely concerned with finding out these specifics, although they are able to describe it ex post.*

For example, anecdotal evidence from the development of Apple's first generation iPod indicates that Steve Jobs kept changing the requirements of the iPod as the project progressed. In particular, the development team would get orders such as "Steve doesn't think it is loud enough", or "the sharps are not sharp enough", or "the menu is not coming up fast enough" (Wired Magazine (2004)). This suggests that committing to a set of features and requirements early on was not desirable in the development of an innovative new product such as the iPod back in 2001.

We show that without commitment, the manager chooses a larger project relative to the case with commitment. Practically, this result asserts that the manager extends the project; for example, by introducing additional requirements. The manager chooses the project size by trading off the marginal benefit of a larger project against the marginal cost associated with a longer wait until the larger project is completed. However, as the project progresses, the agents increase their effort, so that this marginal cost decreases, while the respective marginal benefit does not change. As the project size will be chosen such that the two marginal values are equal, it follows that if the manager cannot commit to her optimal project size at the outset, she will end up choosing a bigger project.

Anticipating that if the manager cannot commit, then she will choose a larger project, the agents decrease their effort, which renders the manager worse off. We show that without commitment and assuming that the agents receive a share of the project's worth upon completion (*i.e.*, an equity contract), the manager finds it optimal to delegate the decision rights over the project size to the agents. In this case, the agents will choose a smaller project than is optimal for the manager, but their preferences are time-consistent. Intuitively, because (unlike the manager) they also trade off the cost of effort when choosing the project size, their marginal cost associated with a larger project does not decrease as the project progresses.

Related Literature

First and foremost, this chapter is related to the literature on dynamic contribution games. The general theme of these games is that a group of agents interact repeatedly, and in every period (or moment), each agent chooses his contribution (or effort) to a joint project at a private cost. Contributions accumulate until they reach a certain threshold, at which point the game ends. Agents receive flow payoffs while the game is in progress, a lump-sum payoff at the end, or a combination thereof. Admati and Perry (1991) and Marx and Matthews (2000) show that contributing little by little over multiple periods, each conditional on the previous contributions of the other agents helps mitigate the free-rider problem. More recently, Yildirim (2006) and Kessing (2007) show that in contrast to the case in which the project generates flow payments while it is in progress as studied by Fershtman and Nitzan (1991), efforts are strategic complements when the agents receive a payoff only upon completion.

A second strand of related literature is that on incomplete contracting. In particular, our article is closely related to the articles that study how ex-ante contracting limitations generate incentives to renegotiate the initial contract ex-post (Grossman and Hart (1986), Aghion and Tirole (1994), Tirole (1999), and others). A subset of this literature focuses on situations wherein the involved parties have asymmetric information. Here, ratchet effects have been shown to arise in principal-agent models in which the principal learns about the agent's ability over time, and the agent reduces his effort to manipulate the principal's beliefs about his ability (Freixas, Guesnerie and Tirole (1985) and Laffont and Tirole (1988)). Another thread of this strand includes articles that consider the case in which the agent is better informed than the principal, or he has better access to valuable information. A common result is that delegating the decision rights to the agent is beneficial as long as the he is sufficiently better informed and the incentive conflict is not too large (Aghion and Tirole (1997) and Dessein (2002)). In our model however, all parties have full and symmetric information, so that ratchet effects and the incentives to delegate the decision rights to the agents arise purely out of moral hazard.

The chapter is organized as follows. We introduce the model in Section 2, and in Section 3 we analyze the agents' problem given a fixed project size. In Section 4, we study the manager's problem, and we characterize the optimal contract. In Section 5 we endogenize the project size, and we conclude in Section 6. This paper unifies results from Georgiadis et al. (2014) and Georgiadis (2015) and the proofs are provided therein.

2 The Model

A group of n identical agents contracts with a manager to undertake a project. The agents exert (costly) effort over time to complete the project, they receive a lump-sum compensation upon completing the project, and they are protected by limited liability.[1] The manager is the residual claimant of the project, and she possesses the decision rights over its size, as well as the agents' compensation. A project of size $Q \geq 0$ generates a payoff equal to Q upon completion. Time $t \in [0, \infty)$ is continuous; all parties are risk neutral, they discount time at rate $r > 0$, and have outside option 0. The project starts at state $q_0 = 0$. At every moment t, each agent observes the state of the project denoted by q_t, and exerts costly effort to influence the progress of the project according to:

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t} \right) dt \, ,$$

where $a_{i,t}$ denotes the (unverifiable) effort level of agent i at time t.[2] The project is completed at the first stopping time τ such that $q_\tau = Q$.[3] Each agent is credit constrained, his effort choices are not observable to the other agents, and his flow cost of exerting effort level a is $\frac{a^2}{2}$. Finally, we assume that (i) strategies are Markovian, so that at every moment, each agent chooses his effort level as a function of the current state of the project q_t, and (ii) incentive contracts are symmetric.[4]

[1]In practice, the relevant employees are rewarded by a combination of flow payments (*i.e.*, periodic salary) and compensation after completion of the project. The latter can take the form of bonus lump-sum payments, stock options (that are correlated to the profit generated by the project), and reputational benefits. In the base model, we assume (for tractability) that the agents are compensated only by a lump-sum upon completion of the project. Georgiadis (2015a) also considers the case in which, in addition to a lump-sum payment upon completion, they also receive a per unit of time compensation while the project is ongoing.

[2]The assumptions that efforts are perfect substitutes and the project progresses deterministically are made for tractability. Georgiadis et. al. (2014) and Georgiadis (2015a) also examine the case in which efforts are complementary and the project progresses stochastically, and they show that all results continue to hold.

[3]We implicitly assume that the agents do not face a deadline to complete the project. This assumption is made (i) for simplicity, and (ii) because deadlines are generally not renegotiation proof. As a result, if the project has not been completed by the deadline, the agents find it mutually beneficial to extend the deadline. For a treatment of deadlines, see Georgiadis (2015b).

[4]When progress is deterministic, as Georgiadis et. al. (2014) show, the game also admits non-Markovian equilibria where at every moment t, each agent's strategy is a function of the entire path of the project

3 Agents' Problem

In this Section, we study the agents' problem, and we characterize the unique project-completing Markov Perfect equilibrium (hereafter MPE) wherein each agent conditions his strategy at t only on the current state of the project q_t. Throughout Sections 3 and 4 we take the project size Q as given, and we endogenize Q in Section 5.

3.1 Preliminaries

We assume (for now) that each agent receives a lump sum reward $\frac{V}{n}$ upon completion of the project and no intermediate compensation. We then show in Section 4 that the optimal symmetric contract rewards the agents only upon completion of the project. Moreover, we will carry out the analysis in this Section assuming that the project size Q is given, and we will endogenize it in Section 5.

Given the current state of the project q_t, and others' strategies, agent i's discounted payoff function is given by

$$\Pi_{i,t}(q) = \max_{\{a_{i,s}\}_{s \geq t}} \left[e^{-r(\tau - t)} V - \int_t^{\tau} e^{-r(s-t)} \frac{a_{i,s}^2}{2} ds \mid \{a_{-i,s}\}_{s \geq t} \right] , \tag{1}$$

where τ denotes the completion time of the project and it depends on the agents' strategies. The first term captures the agent's net discounted payoff upon completion of the project, while the second term captures his discounted cost of effort for the remaining duration of the project. Because payoffs depend solely on the state of the project (*i.e.*, q) and not on the time t, this problem is stationary, and hence the subscript t can be dropped. Using standard arguments (Dixit (1999)), one can derive the Hamilton-Jacobi-Bellman equation for the expected discounted payoff function for agent i

$$r\Pi_i(q) = \max_{a_i} \left\{ -\frac{a_i^2}{2} + \left(\sum_{j=1}^{n} a_j \right) \Pi_i'(q) \right\} \tag{2}$$

subject to the boundary conditions

$$\Pi_i(q) \geq 0 \text{ for all } q \quad \text{and} \quad \Pi_i(Q) = V \, . \tag{3}$$

$\{q_s\}_{s \leq t}$. We use the deterministic specification as a reduced form for a stochastic process, in which case as Georgiadis (2015a) conjectures, the agents' payoffs from the best symmetric Public Perfect equilibrium are equal to the payoffs corresponding to the MPE.

The first boundary condition captures the fact that each agent's discounted payoff must be non-negative as he can guarantee himself a payoff of 0 by exerting no effort and hence incurring no effort cost. The second boundary condition states that upon completing the project, each agent receives his reward and exerts no further effort.[5]

3.2 Markov Perfect Equilibrium (MPE)

In a MPE, at every moment, each agent i observes the state of the project q, and chooses his effort a_i to maximize his discounted payoff while accounting for the effort strategies of the other team members. It follows from (2) that the first order condition for agent i's problem yields that $a_i(q) = \Pi_i'(q)$: at every moment, he chooses his effort such that the marginal cost of effort is equal to the marginal benefit associated with bringing the project closer to completion. By noting that the second order condition is satisfied and that the first order condition is necessary and sufficient, it follows that in any differentiable, project-completing MPE, the discounted payoff for agent i satisfies

$$r\Pi_i(q) = -\frac{1}{2}\left[\Pi_i'(q)\right]^2 + \left[\sum_{j=1}^{n}\Pi_j'(q)\right]\Pi_i'(q) \tag{4}$$

subject to the boundary conditions (3). The following Proposition characterizes the MPE, and establishes conditions under which it is unique.

Proposition 1. *For any given project size Q, there exists a Markov Perfect equilibrium for the game defined by (1). This equilibrium is symmetric, each agent's effort strategy satisfies*

$$a(q) = \frac{r}{2n-1}\left[q - C\right]^+ \text{ , where } C = Q - \sqrt{\frac{2V}{r}\frac{2n-1}{n}}\,, \tag{5}$$

and the project is completed at $\tau(Q) = \frac{2n-1}{rn}\ln\left[1 - \frac{Q}{C}\right]$.[6] In equilibrium, each agent's discounted payoff is given by

$$\Pi(q) = \frac{r}{2}\frac{\left(\left[q-C\right]^+\right)^2}{2n-1}\,. \tag{6}$$

If $Q^2 < \frac{2V}{r}\frac{2n-1}{n}$, then this equilibrium is unique, and the project is completed in finite time. Otherwise, there also exists an equilibrium in which no agent ever exerts any effort and the project is never completed.

[5]Because the agents' rewards are independent of the completion time, the game is stationary, and so we can drop the subscript t.

[6]To simplify notation, because the equilibrium is symmetric and unique, the subscript i is dropped throughout the remainder of this article. Moreover, $[\cdot]^+ = \max\{\cdot, 0\}$.

Because the project starts at $q_0 = 0$, notice that if $C \geq 0$, or equivalently if $Q^2 \geq \frac{2V}{r}\frac{2n-1}{n}$, then there exists no MPE in which the project is completed. This is because the discounted cost to complete the project exceeds its discounted net payoff, and hence the agents are better off not exerting any effort, in which case the project is never completed. On the other hand, if $C < 0$, then the project is completed, and each agent's effort level increases in the state of the project q. This is due to the facts that agents are impatient and they incur the cost of effort at the time the effort is exerted, while they are compensated only when the project is completed. As a result, their incentives are stronger, the closer the project is to completion. An implication of this observation is that efforts are strategic complements (across time) in this game. That is because by raising his effort, an agent brings the project closer to completion, which induces the other agents to raise their future efforts.

It is worth emphasizing that the MPE is always symmetric, but it need not be unique. In particular, if $C|_{n=1} \geq 0$, or equivalently if $Q^2 \geq \frac{2V}{r}$, then there exists an equilibrium in which no agent ever exerts any effort and the project is never completed.[7] However, it turns out that when the project size Q is endogenous, the manager will always choose it such that the equilibrium is unique. As such, we shall restrict attention to the project-completing MPE characterized in Proposition 1.

Lastly, one can show that in the MPE, each agent exerts less effort and is worse off relative to the first best outcome; *i.e.*, the case in which at every moment, a social planner chooses each agent's effort to maximize their total discounted payoff. This in a direct implication of the free-rider problem: in equilibrium, each agent chooses his effort by trading off the marginal cost of effort and his marginal benefit from bringing the project closer to completion, but he ignores the positive externality of his effort on the other agents.

3.3 Comparative Statics

In this Section, we examine how each agent's payoff and effort level depends on the parameters of the problem. The following result establishes some comparative statics about how each agent's effort level depends on the parameters of the problem.

Proposition 2. *All other parameters held constant, each agent's effort level $a(q)$ possesses the following properties:*
(i) $a(q)$ is increasing in V ; and

[7] So see why such an equilibrium exists, suppose that all agents except for i exert no effort at $q_0 = 0$. Then agent i finds it optimal to also exert no effort, because he is not willing to undertake the entire project single-handedly (since $C|_{n=1} \geq 0$).

(ii) there exists a threshold Θ_r such that $a(q)$ is increasing in r if and only if $q \geq \Theta_r$.

Statement (i) is intuitive: if the agents receive a larger reward upon completion, then they have stronger incentives. Statement (ii) asserts that as agents become less patient, they tend to work harder when the project is close to completion, and less hard when it is far from completion. To see the intuition, notice that the marginal benefit of bringing the completion time forward (which occurs if the agents raise their effort) is equal to $-\frac{d}{d\tau}\left(e^{-r\tau}\frac{V}{n}\right) = \frac{rV}{n}e^{-r\tau}$, and it increases in r if τ is sufficiently small; *i.e.*, if the project is sufficiently close to completion.

We next consider the effect of the team size on the agents' incentives. For this analysis, it is necessary to consider how each agent's reward depends on the team size. We assume that upon completion of the project, each agent receives reward $V_n = \frac{V}{n}$, so that the total rewards disbursed to the agents is independent of n.

Proposition 3. *Consider two teams comprising of n and $m > n$ identical agents. Other things equal, there exist thresholds $\Theta_{n,m}$ and $\Phi_{n,m}$ such that*
(i) $a_m(q) \geq a_n(q)$ if and only if $q \leq \Theta_{n,m}$; and
(ii) $ma_m(q) \geq na_n(q)$ if and only if $q \leq \Phi_{n,m}$.[8]

By increasing the size of the team, two opposing forces influence the agents' incentives: First, as is well known in the static moral hazard in teams literature starting with Olson (1965), the agents obtain stronger incentives to free-ride. Moreover, because the agents' incentives to free-ride are proportional to the cost of effort they are incurring, effort increases with progress, and effort costs are convex, the free-riding effect becomes more intense as the project progresses. Second, because efforts are strategic complements in this game, when agents are part of a larger team, they have incentives to raise their effort, because doing so will induce a greater number of other agents to raise their future efforts, which in turn makes them better off. This encouragement effect is strong at the outset of the game where a lot of progress remains to complete the project, and it becomes weaker as the project nears completion.

In summary, the encouragement effect dominates the free-riding effect if and only if the project is sufficiently far from completion. This implies that by increasing the team size, the agents obtain stronger incentives when the project is far from completion, while their incentives become weaker near completion.

[8]This result follows by noting that $C = Q - \sqrt{\frac{2V}{r}\frac{2n-1}{n}}$ decreases in n, while both $a(Q)$ and $na(Q)$ decrease in n.

4 Manager's Problem

In this section, we analyze the manager's problem who is the residual claimant of the project, and his objective is to choose the team size and the agents' (symmetric) incentive contracts to maximize her ex-ante discounted profit. As in the previous section, we will keep the project size fixed. Moreover, we will restrict attention to incentive contracts that specify a set of milestones $q_0 < Q_1 < .. < Q_K = 0$ (where $K \in \mathbb{N}$), and for every $k \in \{1, .., K\}$, allocates non-negative payments $\{V_k\}_{i=1}^{n}$ that are payable to the agents upon reaching milestone Q_k for the first time.[9]

We set out by considering the case in which the manager compensates the agents only upon completing the project, and we characterize the manager's discounted profit function. Then we explain how this result extends to the case in which the manager also rewards the agents for reaching intermediate milestones.

Given the team size and the agents' rewards that are due upon completion of the project (where we can assume without loss of generality that $V \leq Q$), the manager's expected discounted profit function can be written as

$$W(q_t) = e^{-r(\tau - t)}(Q - V) \ ,$$

where τ denotes the completion time of the project and it depends on the agents' strategies as defined in Proposition 1. By using the first order condition for each agent's equilibrium effort as determined in Section 3, the manager's expected discounted profit at any given state of the project satisfies

$$rW(q) = na(q)\, W'(q) \tag{7}$$

subject to the boundary conditions

$$W(q) \geq 0 \text{ for all } q \quad \text{and} \quad W(Q) = (Q - V) \ . \tag{8}$$

To interpret these conditions, note that manager's discounted profit is non-negative at every state of the project, because she does not incur any cost or disburse any payments to the

[9] The manager's contracting space is restricted. In principle, the optimal contract should condition each agent's payoff on the path of q_t (and hence on the completion time of the project). However, when the project progresses deterministically, the problem becomes trivial as efforts effectively become contractible, and in the stochastic case the problem is not tractable. For example, the contracting approach developed in Sannikov (2008) boils down a partial differential equation with at least variables (*i.e.*, the state of the project q and the continuation value of each agent), which is intractable.

agents while the project is in progress. On the other hand, she receives her net profit $Q-V$, and the game ends as soon as the state of the project hits Q for the first time. After substituting (5) and solving the above ODE, it follows that

$$W(q) = (Q-V)\left(\frac{[q-C]^{+}}{Q-C}\right)^{\frac{2n-1}{n}}, \text{ where } C = Q - \sqrt{\frac{2V}{r}\frac{2n-1}{n}} \tag{9}$$

We now discuss how each agent's payoff function and the principal's profit function extend to the case in which the manager rewards the agents upon reaching intermediate milestones. Recall that she can designate a set of milestones, and attach rewards to each milestone that are due as soon as the project reaches the respective milestone for the first time. Let $\Pi_k(\cdot)$ denote each agent's discounted payoff given that the project has reached $k-1$ milestones, which is defined for $q \leq Q_k$, and note that it satisfies (4) subject to $\Pi_k(q) \geq 0$ for all q and $\Pi_k(Q_k) = \frac{V_k}{n} + \Pi_{k+1}(Q_k)$, where $\Pi_{K+1}(0) = 0$. The second boundary condition states that upon reaching milestone k, each agent receives the reward attached to that milestone, plus the continuation value from future rewards. Starting with $\Pi_K(\cdot)$ and proceeding by backward induction, it is straightforward to derive each agent's discounted payoff by solving (4) subject to the corresponding boundary conditions.

To examine the manager's problem, let $W_k(\cdot)$ denote her expected discounted profit given that the project has reached $k-1$ milestones, which is defined for $q \leq Q_k$, and note that it satisfies (7) subject to $W_k(q) \geq 0$ for all q and $W_k(Q_k) = W_{k+1}(Q_k) - V_k$, where $F_{K+1}(Q_k) = Q$. The second boundary condition states that upon reaching milestone k, the manager receives the continuation value of the project, less the payments that she disburses to the agents for reaching this milestone. Again starting with $k = K$ and proceeding backwards, it is straightforward to derive the principal's discounted profit.

The following Proposition shows that one can without loss of generality restrict attention to those that compensate the agents only upon completion of the project.

Proposition 4. *The optimal symmetric contract compensates the agents only upon completion of the project.*

To prove this result, Georgiadis (2015a) considers an arbitrary set of milestones and arbitrary rewards attached to each milestone, and constructs an alternative contract that rewards the agents only upon completing the project and renders the manager better off. Intuitively, because rewards are sunk in terms of incentivizing the agents after they are disbursed, and all parties are risk neutral and they discount time at the same rate, by backloading payments, the manager can provide the same incentives at the early stages of the project, while providing stronger incentives when it is close to completion.

The value of this result lies in that it reduces the infinite-dimensional problem of determining the team size, the number of milestones, the set of milestones, and the rewards attached to each milestone into a two-dimensional problem, in which the manager only needs to determine her budget V for compensating the agents and the team size n to maximize her ex-ante discounted profit. In other words, the manager solves

$$\max_{V,n}\left\{(Q-V)\left(\frac{-C}{Q-C}\right)^{\frac{2n-1}{n}} \quad \text{s.t. } C<0\right\}.$$

Solving this optimization problem analytically is not tractable. Figure 1 illustrates how the optimal contract depends on the parameters of the problem: the discount rate r and the project size Q.

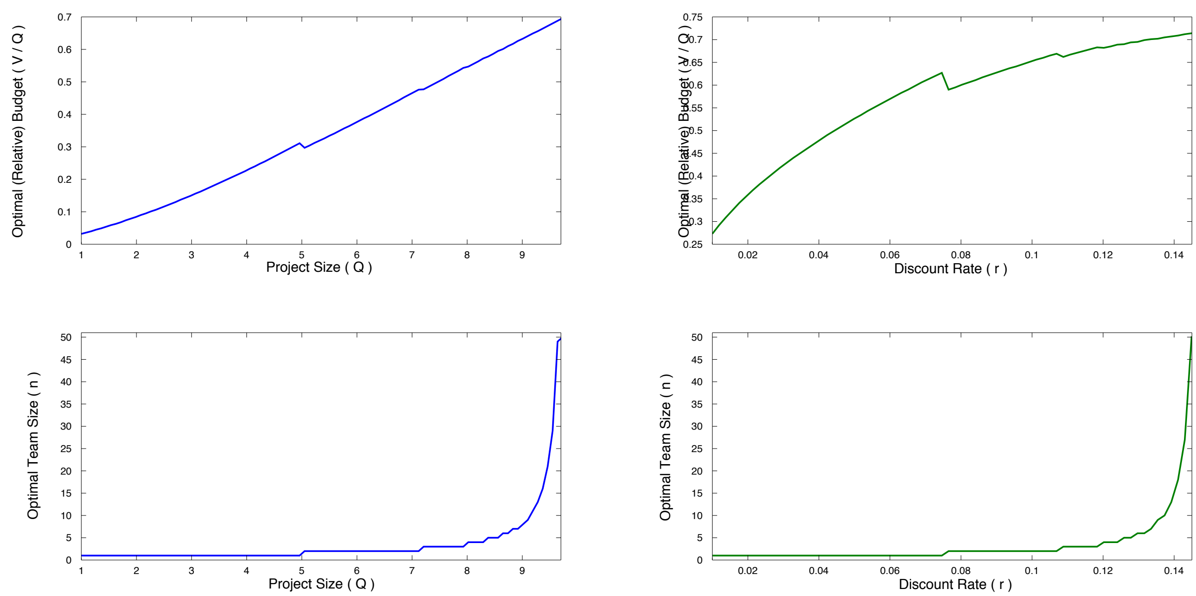

Figure 1: **Optimal contract**. The **left panel** illustrates how the optimal (relative) budget (*i.e.*, $\frac{V^*}{Q}$) and the optimal team size n^* depend on the project size, given $r = 0.15$. The **right panel** illustrates how optimal (relative) budget and the optimal team size depend on the discount rate, given $Q = 10$.

By examining the left panel in figure 1, one observes that if the project is larger, then the manager optimally hires a larger team, and she allocates a larger proportion of her budget to compensate the agents. From the right panel, notice that if the discount rate r increases, then the manager show employ a bigger team and allocate more of her budget to compensate the agents.

5 Project Design with and without Commitment

In this Section, we endogenize the project size Q. To obtain tractable results, we assume that the manager's budget $V = \beta Q$; *i.e.*, each agent receives a fixed proportion of the payoff that the project generates, and this proportion does not depend on Q. We consider the case in which the manager is able to commit to a particular project size at the outset of the game, as well as the case in which she cannot, in which case at every moment she observes the current state of the project q, and she decides whether to stop work and collect the net profit $(1-\beta)\, q$ or to let the agents continue working and re-evaluate her decision to complete the project a moment later.

To analyze the case in which the project size Q is endogenous and the manager cannot commit to a particular Q at the outset of the game, the agents must form beliefs about the project size that the manager will choose at a later date and choose their efforts accordingly. Following rational expectations, we assume that in equilibrium, the agents can correctly anticipate the manager's choice. Therefore, we write the manager's discounted profit function as

$$W\left(q;\, Q, \tilde{Q}\right) = (1-\beta)\, Q \left(\frac{\left[q - C\left(\tilde{Q}\right)\right]^{+}}{Q - C\left(\tilde{Q}\right)}\right)^{\frac{2n-1}{n}}, \text{ where } C\left(\tilde{Q}\right) = \tilde{Q} - \sqrt{\frac{2\beta\tilde{Q}}{r}\frac{2n-1}{n}} \quad (10)$$

and $\tilde{Q}$ denotes the agents' (common) belief about the project size that the manager will choose.

Full Commitment

If the manager can commit to a project size at the outset of the game, then at $q_0 = 0$, she leads a Stackelberg game in which she chooses the project size that maximizes her discounted profit and the agents follow by adopting the equilibrium strategy characterized in Proposition 1. As a result, her optimal project size with full commitment (FC) satisfies $Q_{FC}^{M} \in \arg\max_{Q} W\left(0\,; Q, Q\right)$. Noting from (10) that $W\left(0\,; Q, Q\right)$ is concave in Q, taking the first order condition with respect to Q yields

$$Q_{FC}^{M}=\frac{\beta}{r}\frac{2n-1}{2n}\left(\frac{4n}{4n-1}\right)^{2}.$$

No Commitment

If the manager has no commitment power, then at every moment she observes the current state of the project q, and she decides whether to stop work and collect the net profit $(1-\beta)\,q$ or to let the agents continue working and re-evaluate her decision to complete the project a moment later. In this case, the manager and the agents engage in a simultaneous-action game, where the manager chooses Q to maximize her discounted profit given the agents' belief $\tilde{Q}$ and the corresponding strategies, and the agents form their beliefs by anticipating the manager's choice Q. Therefore, her optimal project size with no commitment (NC) satisfies $Q_{NC}^{M}\in\arg\max_{Q}\left\{W\left(q\,;Q,\tilde{Q}\right)\right\}$, where in equilibrium beliefs must be correct; *i.e.*, $Q=\tilde{Q}$. By solving $\left.\frac{\partial W\left(q\,;Q,\tilde{Q}\right)}{\partial Q}\right|_{q=\tilde{Q}=Q}=0$, we have

$$Q_{NC}^{M}=\frac{\beta}{r}\frac{2n}{2n-1}.$$

Observe that with no commitment, the manager will choose a strictly larger project relative to the case in which she commitment power: $Q_{NC}^{M}>Q_{FC}^{M}$.[10,11]

The following Proposition summarizes:

Proposition 5. *If the manager can commit to her optimal project size at the outset, then she finds it optimal to choose* $Q_{FC}^{M}=\frac{\beta}{r}\frac{2n-1}{2n}\left(\frac{4n}{4n-1}\right)^{2}$*. In contrast, without commitment, she completes the project at* $Q_{NC}^{M}=\frac{\beta}{r}\frac{2n}{2n-1}>Q_{FC}^{M}$*. Moreover, the manager's ex-ante discounted profit is higher with commitment than without; i.e.,* $W\left(0;Q_{FC}^{M},Q_{FC}^{M}\right)>W\left(0\,;Q_{NC}^{M},Q_{NC}^{M}\right)$.

[10] This case raises the question of what happens to the agents' beliefs off the equilibrium path if the manager does not complete the project at Q_{NC}^{M}. Suppose that the manager did not complete the project at Q_{NC}^{M} so that $q>Q_{NC}^{M}$. Clearly, Q and $\tilde{Q}>Q_{NC}^{M}$, and it is straightforward to verify that $\frac{\partial W\left(q\,;Q,\tilde{Q}\right)}{\partial Q}<0$ for all q, Q and $\tilde{Q}>Q_{NC}^{M}$, which implies that the manager would be better off had she completed the project at Q_{NC}^{M} irrespective of the agents' beliefs.

[11] Conceptually, this commitment problem could be resolved by allowing β to be contingent on the project size. In particular, suppose that the manager can fix β, and let $\hat{\beta}\,(Q)$ equal β if $Q=Q_{FC}^{M}$, and 1 otherwise. Then, her optimal project size is equal to Q_{FC}^{M} regardless of her commitment power because any other project size will yield her a net profit of 0. However, this implicitly assumes that Q_{FC}^{M} is contractible at $q=0$, which is clearly not true without commitment. Therefore, we rule out this possibility by assuming that β is independent of Q.

Intuitively, the manager chooses the project size by trading off the marginal benefit of a larger project against the marginal cost associated with a longer wait until the larger project is completed. However, as the project progresses, the agents increase their effort, so that this marginal cost decreases, while the respective marginal benefit does not change. As the project size will be chosen such that the two marginal values are equal, it follows that the manager's optimal project size increases as the project progresses.[12] As a result, without commitment, the manager chooses a bigger project relative to the case with commitment.

5.1 Optimal Delegation

The manager's limited ability to commit, in addition to disincentivizing the agents from exerting effort, is detrimental to her ex-ante discounted profit; *i.e.*, $W\left(0\,;Q_{NC}^{M},Q_{NC}^{M}\right) < W\left(0;Q_{FC}^{M},Q_{FC}^{M}\right)$. Thus, unable to commit, the manager might consider delegating the decision rights over the project size to the agents.

We begin by examining how the agents would select the project size. Let $Q^{A} \in \arg\max_{Q}\left\{\Pi\left(q\,;Q\right)\right\}$ denote the agents' optimal project size given the current state x. Solving this maximization problem yields

$$Q^{A} = \frac{\beta}{r}\frac{2n-1}{2n}\,.$$

Observe that the agents' optimal project size is independent of the current state q. Intuitively, this is because the agents incur the cost of their effort, so that their effort cost increases together with their effort level as the project progresses. As a result, unlike the manager, their marginal cost associated with choosing a larger project does not decrease as the project evolves, and consequently they do not have incentives to extend the project as it progresses.

Second, observe that the agents always prefer a smaller project than the manager; *i.e.*, $Q^{A} < Q_{FC}^{M} < Q_{NC}^{M}$. This is because they incur the cost of their effort, so that their marginal cost associated with a larger project is greater than that of the manager's.

The following Proposition shows that without commitment, the manager finds it optimal to delegate the decision rights over the project size to the agents.

Proposition 6. *If the manager can commit to her optimal project size at the outset, then she find it optimal to retain the decision rights over the project size. In contrast, without commitment, she should delegate the decision rights over Q to the agents; i.e.,* $W\left(0;Q_{NC}^{M},Q_{NC}^{M}\right) < W\left(0;Q^{A},Q^{A}\right) < W\left(0;Q_{FC}^{M},Q_{FC}^{M}\right)$.

[12] Letting $Q^{M}\left(q\right) = \arg\max_{Q\geq q}\left\{W\left(q\,;Q,Q\right)\right\}$, one can show that $Q^{M}\left(q\right)$ increases in q.

Since Q^M_{NC} maximizes the manager's ex-ante discounted profit, it is straightforward that she cannot benefit by delegating the decision rights over the project size to the agents. Without commitment however, because the agents' preferences over Q are time-consistent whereas the manager's are not, delegation turns out to be always optimal.

6 Concluding Remarks

We use a tractable model to study the interaction between a group of agents who collaborate over time to complete a project and a manager who chooses its size. First, we analytically characterize the Markov Perfect equilibrium, and we show that in contrast to the static moral hazard in teams, larger teams may be more effective in completing the project than smaller ones. We then study the manager's problem who chooses the size of the team and the contracts of the team members to maximize her ex-ante discounted profit. We show that the optimal symmetric contract compensates the agents only upon completion of the project. This result reduces the infinite-dimensional contracting problem to a two-dimensional one where the manager needs to only choose the team size and her budget for compensating the agents. Lastly, we endogenize the size of the project, and we show that without the ability to commit to a particular project size at the outset of the game, the manager will end up choosing a larger project relative to the case in which she can commit. An implication of this result is that without commitment, she finds it optimal to delegate the decision rights over the project size to the agents, who will choose a smaller project but their preferences are time-consistent.

In subsequent work, Cvitanic and Georgiadis (2015) characterize a mechanism that induces each agent to at every moment exert the efficient effort level in a MPE. Bowen, Georgiadis and Lambert (2015) study a dynamic contribution game with two heterogeneous agents and endogenous project size, and they analyze how different collective choice institutions influence the size of the project that is implemented in equilibrium. Georgiadis (2015b) considers the effect of deadlines and the agents' ability to monitor the state of the project on the agents' incentives and their payoffs. Finally, Ederer, Georgiadis and Nunnari (2015) examine how the team size and the monitoring structure affects incentives in a discrete public good contribution game using laboratory experiments. Preliminary results support the theoretical predictions.

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