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Deadlines and infrequent monitoring in the dynamic provision of public goods☆

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ABSTRACT

We consider a dynamic game of private provision of a discrete public good. In our model, a group of agents contributes to a project over time, which is completed once the cumulative contributions reach a threshold. Provided that this occurs prior to a prespecified deadline, each agent receives a lump-sum payoff. We show that a shorter deadline can induce the agents to raise their efforts, but no matter the length of the deadline, effort provision is inefficient due to the agents' frontloading incentives. Only if the agents do not monitor progress until the deadline are their frontloading incentives eliminated, so by committing to a deadline equal to the first-best completion time, it is possible to restore efficiency. Recognizing that deadlines are not renegotiation proof, we show that by committing to monitor progress to date at the first-best completion time, and then again at a sufficiently later date, efficiency can be attained. In this case, that monitoring date acts as a self-enforcing deadline.

1. Introduction

Collaboration is pervasive in markets and organizations, and often takes place over extended periods of time. Entrepreneurs collaborate on their ventures. Corporations and divisions collaborate on research and development and even the production of new products. Citizens collaborate on the provision of public goods, such as building and supporting churches, library collections, public parks, and protecting the environment. In all these cases, collaboration is not a once-off effort, but one that takes place repeatedly and sometimes continuously over weeks, months, and even years.

It is well known that free riding is a key impediment to collaboration (Olson, 1965). Since some of the benefits of my collaborative efforts accrue to you, I have a natural tendency to underprovide such effort, and so do you (Holmström, 1982). In a dynamic setting, however, there is a second, less well understood inefficiency: collaborative efforts are front-loaded relative to the social optimum (Kessing, 2007). I provide too much collaborative effort early on to bring forward the completion date, and thereby motivate you to provide more effort in the future.

The goal of this paper is to explore instruments agents can use to mitigate both of these inefficiencies. Perhaps the most commonly used instrument to combat free riding is the imposition of a deadline by which the project has to be completed (Lindkvist et al., 1998). We first show that while deadlines do mitigate free riding as collaborators are forced to work harder, they do not alleviate frontloading. The source of these frontloading incentives is that the total progress made to date is publicly observable: if I provide too much effort today, then my partners will observe this tomorrow and will be motivated to raise their efforts to my benefit. We thus explore complementing deadlines with infrequent monitoring of progress to date. We show that together deadlines and infrequent monitoring can alleviate both inefficiencies, and may even restore efficiency.

To fix ideas, imagine a group of innovators collaborating towards obtaining a patent. Each agent can exert effort to progress the project, and she receives a prize once the patent is awarded. Echoing this example, our model has three key ingredients. First, progress towards a particular goal is made gradually over time at a rate that depends on the agents' costly efforts. Second, the agents discount time. Finally, the project generates a payoff once the cumulative efforts reach a pre-specified threshold, provided that this occurs by a given deadline.

We begin our analysis by formulating the optimal control problem, and considering the benchmark case in which a social planner chooses the agents' effort levels to maximize the team's total

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discounted payoff. Because effort costs are convex and the agents discount time, efficiency requires that the agents perfectly smooth their effort over time; *i.e.*, choose it such that their discounted marginal cost of effort is constant over time.

We proceed to characterize Markov Perfect equilibria (hereafter MPE), wherein at every moment, each agent observes the cumulative progress and the time remaining until the deadline, and chooses her effort level to maximize her discounted payoff. We characterize the unique symmetric MPE with differentiable strategies in closed form, and we show that for any given deadline, equilibrium efforts are inefficient. When the deadline is long or non-binding, efforts are inefficiently low and frontloaded, and the project is completed at a later date compared to the first-best outcome. Intuitively, each agent has incentives to shirk, because she does not internalize the positive externality associated with completing the project sooner. Moreover, because effort is increasing over time, each agent has an incentive to frontload her effort to induce others to raise their future efforts. Shortening the deadline forces the agents to work harder, but effort continues to be frontloaded. As a result, if the deadline is sufficiently short, then equilibrium efforts are inefficiently high during the early stages of the project, and inefficiently low during its later stages.

A source of the agents' frontloading incentives is that they observe the state of the project continuously, so each agent can influence the future efforts of her peers by raising or lowering her present effort. This observation, together with the fact that in many settings, the members of a project team observe how close they are to their goal periodically (*e.g.*, during group meetings where progress is tallied), motivated us to explore the case in which the agents monitor how close the project is to completion only at discrete, prespecified dates. Between those dates, they must choose their strategies based on their beliefs about their peers' strategies. Intuitively, the key difference relative to the continuous-monitoring case is that each agent's actions can affect the future actions of the other team members only following the next monitoring date, and as we show, the agents' incentives to frontload effort are eliminated.

If the agents observe the project state at some prespecified deadline for the first time, then they perfectly smooth their effort along any equilibrium path. As a result, if they can commit to any deadline at the outset of the game and they do not observe the project state at any intermediate date, then it is possible to implement the first-best outcome.

Noting that deadlines are not renegotiation-proof, we consider the case of an exogenous deadline, and we show that the agents can maximize their ex ante discounted payoff by choosing a single intermediate monitoring date such that the project is completed at that date in equilibrium. Moreover, if the deadline is sufficiently long, then it is possible to implement the first-best outcome. Intuitively, by monitoring the project state at some date and then not monitoring it again for a sufficiently long period of time, makes it in every agent's interest to complete the project by that first monitoring date. In this sense, the first monitoring date acts as a self-enforcing deadline.

First and foremost, our work contributes to the literature on dynamic public good provision. Our model is based on the differential game framework of Kessing (2007), to which we incorporate deadlines and we consider the case in which progress is observed infrequently. Early contributions to this literature were made by Admati and Perry (1991), who characterizes the MPE in a two-player game and shows that contributions are inefficiently low due to the free rider problem. Marx and Matthews (2000) generalizes this result to games with n players and characterizes equilibria with Markovian, as well as non-Markovian strategies. Yildirim (2006) and Kessing (2007) show that if the project generates a payoff only upon completion, then in an MPE, efforts are strategic complements across time, which implies that the agents have incentives to frontload effort. This is in contrast to the case in which the project only generates flow payoffs while in progress as in Fershtman and Nitzan (1991).

This paper is related to a literature that explores how deadlines influence incentives in dynamic public good provision problems. Bonatti and Hörner (2011) incorporates deadlines into a strategic experimentation in teams problem, and shows that the first-best outcome can be implemented in equilibrium if the deadline is chosen appropriately. Campbell et al. (2013) analyzes a similar model in which each player is privately informed about the outcome of her past efforts, and she can verifiably disclose said outcome. They show that short deadlines induce high effort but may disincentivize the agents from disclosing their private information. While a number of authors have considered finite-horizon dynamic contribution models (*e.g.*, Marx and Matthews, 2000 and Rahmani et al., 2015), to the best of our knowledge, this is the first paper to study how the length of the deadline affects the agents' incentives in such a setting.

This paper joins a recent literature that studies how the informational environment affects incentives in dynamic games with externalities. For example, Campbell et al. (2013) shows that agents may optimally not disclose a successful breakthrough to maintain their partners' motivation. Bimpikis and Drakopoulos (2014) shows that in the context of a strategic experimentation problem, efficiency can be restored if the agents commit to sharing no information until a certain date, and disclosing all available information at that date. Heidhues et al. (2015) considers a similar setting, and allowing for cheap-talk communication, characterizes an equilibrium that implements the first-best outcome. Bonatti and Hörner (2011) shows that in a "good news" strategic experimentation problem, public monitoring of experimentation levels leads to weaker incentives, whereas Bonatti and Hörner (2017) shows that in the "bad news" counterpart of this problem, monitoring leads to stronger experimentation incentives. In contrast, the present paper focuses on the effect of deadlines and the monitoring frequency of progress to date on the agents' incentives in a dynamic public good provision framework.

Finally, authors have considered other instruments to mitigate the inefficiencies that arise in this framework. For instance, Cvitanic and Georgiadis, G.´ (2016) constructs a mechanism that uses state-dependent flow payments to induce the agents to always exert the first-best effort level as the outcome of an MPE. Georgiadis (2015) examines how a profit-maximizing principal would choose the team size and the agents' incentive contracts. Bowen et al. (2017) considers a similar model with heterogeneous agents in which the scope of the project is endogenous, and studies how different collective choice institutions (*e.g.*, dictatorship and unanimity) affect the project scope that is implemented in equilibrium.

2. Model

A group of $n \geq 2$ agents collaborate on a project. Time $t \in [0, \infty)$ is continuous. Agents are risk neutral and credit constrained, they discount time at rate $r > 0$, and the value of their outside option is normalized to 0. The project starts at state $q_0 = 0$, and at every moment, each agent privately exerts costly effort to influence the process

$$dq_t = \left(\sum_{i=1}^{n} a_{i,t}\right) dt,$$

where q_t denotes the state of the project and $a_{i,t} \geq 0$ denotes the effort level of agent i at time t.[1] Each agent's flow cost of exerting

[1] Consistent with the existing literature (*e.g.*, Marx and Matthews, 2000, Compte and Jehiel., 2004), the assumption that the project progresses deterministically is made for tractability; that is because along an equilibrium path, there is a one-to-one mapping between time t and the state q_t.

effort level a is $\frac{a^2}{2}$.[2] The project is completed at the first τ such that the agents observe $q_\tau \geq Q$. If $\tau \leq T$, where $T \leq \infty$ is a given deadline, then each agent receives a prespecified reward V upon completion, and receives no reward otherwise.[3] Finally, we shall restrict attention to symmetric equilibria.

3. Building blocks

In this section, we set up the building blocks for the analysis, and we evaluate the first-best outcome of the game.

3.1. Foundations

For a given set of strategies $\{a_i(t,q)\}_{i=1}^n$, at time t and given the project state $q_t = q$, the discounted payoff function of agent i satisfies

$$J_i(t,q) = \underbrace{e^{-r(\tau-t)}V\,\mathbf{1}_{\{\tau\leq T\}}}_{\text{discounted net payoff}} - \underbrace{\int_t^\tau e^{-r(s-t)}\,\frac{[a_i(t,q)]^2}{2}ds}_{\text{discounted effort costs}},$$

where τ denotes the completion time of the project. The first term captures agent i's discounted net payoff from completing the project, while the second term captures her discounted effort costs along the evolution path of the project.

To analyze this game, it is convenient to modify the model by assuming that the agents *must* complete the project by the deadline T along the equilibrium path.[4] Conditional on completing the project at some $\tau \leq T$, each agent minimizes her discounted effort costs, while anticipating that the other agents behave in the same cost-minimizing manner. The solution of this modified game will be an equilibrium for the original game if each agent's discounted payoff at time 0 is weakly larger than her outside option (which has been normalized to 0), and otherwise, there exists no project-completing equilibrium.

We use the maximum principle of optimal control and write strategies and payoffs as a function of time t.[5] The Hamiltonian corresponding to each agent i's objective function is

$$H_{i,t} = -e^{-rt}\,\frac{a_{i,t}^2}{2} + \lambda_{i,t}\left(\sum_{j=1}^n a_{j,t}\right)\ ,$$

where $\lambda_{i,t} \geq 0$ is the co-state variable associated with agent i's payoff function, and it can be interpreted as agent i's marginal value of an additional unit of progress at time t.[6] Her terminal value function is $\phi_{i,\tau} = e^{-r\tau}V$, and the requirement that the project be completed by the deadline imposes the constraint

$$\int_0^\tau \sum_{i=1}^n a_{i,t}dt = Q, \text{where } \tau \leq T\ . \tag{1}$$

Using Pontryagin's maximum principle (Kamien and Schwartz, 2012), the optimality and adjoint equation for each agent is

$$\frac{dH_{i,t}}{da_{i,t}} = 0,\ \text{and} \tag{2}$$

$$\dot{\lambda}_{i,t} = -\frac{dH_{i,t}}{dq_t} = -\sum_{j=1}^n \frac{dH_{i,t}}{da_{j,t}}\,\frac{da_{j,t}}{dt}\,\frac{dt}{dq_t}, \tag{3}$$

respectively. Eq. (2) specifies that at every moment, each agent chooses effort to maximize her Hamiltonian, while Eq. (3) follows from a variational argument that characterizes the law of motion of $\lambda_{i,t}$ along an optimal effort path. Noting that the agents' strategies are a function of both time t and the project state q_t, the second equality in Eq. (3) follows by totally differentiating each agent's Hamiltonian with respect to q_t. The transversality condition for each agent is

$$H_{i,\tau} + \frac{d\phi_\tau}{d\tau} \geq 0\,(= 0 \text{ if} \tau < T)\ . \tag{4}$$

Conditions (1)–(4) are necessary for an optimal solution. Sufficiency follows by noting that $H_{i,t}$ is strictly concave in $a_{i,t}$, and applying the Mangasarian theorem (see Seierstad and Sydsaeter, 1987).

3.2. First-best outcome

As a benchmark, we characterize the first-best effort paths, where at every moment t, each agent chooses her effort to maximize the team's (as opposed to her individual) discounted payoff. Because the agents have identical and convex effort costs, their strategies will be symmetric, so we can drop the subscript i. We denote each agent's first-best effort and discounted payoff function by a_t^{fb} and J_t^{fb}, respectively. The team Hamiltonian is $\mathcal{H}_t = \sum_{i=1}^n H_{i,t}$, and the optimality and adjoint equation can be rewritten as

$$e^{-rt}a_t^{fb} = \lambda_t n \text{ and } \dot{\lambda}_t = 0\ , \tag{5}$$

respectively, where the second equality follows by noting that in the first-best outcome, $dH_{i,t}/da_{j,t} = 0$ for all i and j. Note that the optimality equation requires that at every moment, each agent's discounted marginal cost of effort is equal to the total marginal benefit associated with moving the project closer to completion. Therefore, the co-state variable $\lambda_t = c$ is a constant to be determined, and from the optimality equation, we obtain $a_t^{fb} = cne^{rt}$. We can then determine the constant c as a function of the completion time τ by solving Eq. (1), and the first-best completion time is pinned down by the transversality condition $\sum_{i=1}^n \left(H_{i,\tau} + \frac{d\phi_\tau}{d\tau}\right) \geq 0\,(= 0 \quad \text{if}\tau < T)$.

The above analysis pins down the strategies that minimize the agents' joint discounted payoffs conditional on the project being completed by the deadline. Therefore, efficiency stipulates that the project is completed if and only if each agent's corresponding ex ante discounted payoff is nonnegative. The following proposition characterizes the first-best outcome.

[2] All qualitative results continue to hold if the agents face a more general convex effort cost function, and the production function is super- or sub-additive to capture complementarities or coordination costs, respectively. Similarly, one might allow the agents to be asymmetric, and while it is no longer possible to obtain closed-form characterizations, the main results continue to be valid.

[3] Our results continue to hold if, in addition to a lump-sum reward upon completion, each agent receives a constant flow payoff while the project is in progress. If the flow payoff is increasing in the state q_t, then the effort initially increases and then decreases with progress (see Appendix A.1 in Georgiadis, 2015 for details). As a result, in an MPE, agents frontload their effort in the early stages of the project, and backload their effort in the later stages.

[4] This is equivalent to assuming that if the project is not completed by the deadline, then each agent incurs an arbitrarily large penalty.

[5] Because the project progresses deterministically, in any equilibrium, there will be a one-to-one correspondence between time t and the project state q_t. As such, one can equivalently use the Hamilton-Jacobi-Bellman approach to solve this problem. However, it turns out that the optimal control approach is more tractable in this case.

[6] Note that each agent i's Hamiltonian is a function of t, q, $\{a_{j,t}\}_{j=1}^n$, and $\lambda_{i,t}$. For notational simplicity, we suppress the latter three arguments and simply write $H_{i,t}$.

Proposition 1. *Suppose that the agents choose their strategies to maximize the team's total discounted payoff. If* $\frac{rQ^2}{2Vn^2} < 1$ *and* $T \geq -\frac{1}{r}\ln\left(1-\frac{rQ^2}{2Vn^2}\right)$, *then each agent's effort and ex ante discounted payoff satisfies*

$$a_t^{fb} = \frac{rQ}{n}\,\frac{e^{rt}}{e^{r\tau^{fb}}-1} \text{ and } J_0^{fb} = e^{-r\tau^{fb}}V - \frac{rQ^2}{2n^2}\,\frac{1}{e^{r\tau^{fb}}-1}\,, \tag{6}$$

respectively, and the project is completed at $\tau^{fb} = \min\left\{T,\ \bar{\tau}^{fb}\right\}$, *where* $\bar{\tau}^{fb} = -\frac{1}{r}\ln\left(1-\sqrt{\frac{rQ^2}{2Vn^2}}\right)$. *Otherwise, each agent exerts no effort, and receives zero payoff.*

All proofs are provided in Appendix B.

Observe that in the first-best outcome, each agent's effort level increases exponentially in time at rate r (*i.e.*, $a_t^{fb} \propto e^{rt}$). Given a completion time τ, each agent minimizes her effort costs subject to completing $1/n$ of the project by $t=\tau$. As effort costs are convex, this is accomplished by the agent smoothing her effort along the evolution path of the project; that is by ensuring that her discounted marginal cost of effort $e^{-rt}a_t^{fb}$ is constant for all t.

If the deadline $T > \bar{\tau}^{fb}$, then the project is completed at the first-best completion time $\bar{\tau}^{fb}$, and each agent's ex-ante discounted payoff J_0^{fb} is maximized. On the other hand, if the deadline is shorter than $\bar{\tau}^{fb}$ (but not too short that it is efficient to abandon the project), then the project is completed at T.

4. Continuous monitoring of progress

In this section, we consider the case in which each agent observes the state of the project q_t continuously. In Section 4.1, we characterize the MPE of this game, wherein at every moment, each agent forms a conjecture about the strategies of the other agents, and chooses her effort conditional on time t and the project state q to maximize her discounted payoff. In Section 4.2, we examine how the agents' strategies and payoffs depend on the parameters of the problem.

Introducing a deadline implies that both the project state q and time t are payoff-relevant state variables. Therefore, we can define Markov strategies that punish the other agents following a deviation from the equilibrium path, as in Abreu et al. (1986).[7] In combination with the perfect-monitoring and continuous-time assumptions, this implies that there exists an MPE in which each agent exerts the first-best effort level along the equilibrium path. A feature of this equilibrium is that it requires non-differentiable strategies at the switching point from a cooperation to a punishment regime. Therefore, we shall distinguish between MPE with differentiable strategies, which are akin to MPE in the game without a deadline, and MPE with non-differentiable strategies, which allow for punishments following a deviation from the equilibrium path. The former is characterized in Section 4.1. Noting that the latter is a knife-edge equilibrium, its analysis is deferred to Appendix A.

4.1. Markov perfect equilibrium

For each agent, the optimality equation yields $a_{i,t} = \lambda_{i,t}e^{rt}$.[8] Following Starr and Ho (1969), the adjoint equation can be rewritten as

$$\dot{\lambda}_{i,t} = -\sum_{j\neq i}\frac{dH_{i,t}}{da_{j,t}}\,\frac{da_{j,t}}{dt}\,\frac{dt}{dq_t} = -\sum_{j\neq i}\frac{\lambda_{i,t}\left(r\lambda_{j,t}+\dot{\lambda}_{j,t}\right)}{\sum\limits_{l=1}^{n}\lambda_{l,t}} \Rightarrow \dot{\lambda}_t = -\frac{n-1}{2n-1}r\lambda_t\,, \tag{7}$$

where the second equality follows by using that $\frac{dH_{i,t}}{da_{j,t}} = \lambda_{i,t}$ for all $j\neq i$, $\frac{da_{j,t}}{dt} = \left(r\lambda_{j,t}+\dot{\lambda}_{j,t}\right)e^{rt}$, and $\frac{dt}{dq_t} = \frac{e^{-rt}}{\sum_{l=1}^{n}\lambda_{l,t}}$, and the last equality follows by rearranging terms and the restriction to symmetric equilibria, after dropping the subscript i. A candidate MPE is characterized by a trajectory of the co-state variable λ_t subject to Eqs. (1) and (4), which pin down the completion time of the project. This candidate is an MPE if and only if each agent obtains a nonnegative ex ante discounted payoff. The following proposition provides a characterization.

Proposition 2. *There exists a unique candidate for a symmetric, project-completing MPE with differentiable strategies, wherein each agent's effort and ex ante discounted payoff satisfies*

$$a_t^{mpe} = \frac{rQ}{2n-1}\,\frac{e^{\frac{rnt}{2n-1}}}{e^{\frac{rn\tau^{mpe}}{2n-1}}-1} \text{ and } J_0^{mpe} = e^{-r\tau^{mpe}}V - \frac{rQ^2}{2\,(2n-1)}\,\frac{e^{\frac{r\tau^{mpe}}{2n-1}}-1}{\left(e^{\frac{rn\tau^{mpe}}{2n-1}}-1\right)^2}\,, \tag{8}$$

respectively, $\tau^{mpe} = \min\left\{T,\ \bar{\tau}^{mpe}\right\}$ *and* $\bar{\tau}^{mpe} = -\frac{2n-1}{rn}\ln\left(1-\sqrt{\frac{rQ^2}{2V}\frac{1}{2n-1}}\right)$. *This candidate is an MPE if and only if* $J_0^{mpe} \geq 0$.

Proposition 2 shows that the agents increase their effort over time (since $a_t^{mpe} \propto e^{\frac{rnt}{2n-1}}$), but they do so at a slower rate than in the first-best outcome (where $a_t^{fb} \propto e^{rt}$ and $\frac{n}{2n-1} < 1$ for all $n \geq 2$).[9] In particular, in the MPE, each agent's discounted marginal cost of effort (*i.e.*, $e^{-rt}a_t^{mpe}$) decreases over time, which implies that equilibrium efforts are frontloaded. Notice that effort increases with progress and this is a game with positive externalities. Therefore, each agent has an incentive to distort her effort path so as to induce the other agents to raise their future effort levels, which is accomplished by raising her effort at the early stages of the project; *i.e.*, by frontloading effort.[10]

This frontloading effect can be interpreted as peer pressure, in the sense that each agent works hard *today* to incentivize her peers to work harder in the future. However, in contrast to Kandel and Lazear (1992) where the incentives to exert peer pressure are ingrained

[7] Such strategies are typically referred to as non-Markov in the literature, and the corresponding equilibria, Public or Subgame Perfect (Marx and Matthews, 2000).

[8] Recall from Section 3.2 that $e^{-rt}a_t^{fb} = \lambda_t n$, and notice that in contrast to the first-best outcome, in the MPE, each agent chooses her effort such that her marginal cost of effort is equal to her own (as opposed to the group's total) marginal benefit of progress.

[9] Propositions 1 and 2 parallels Propositions 1 and 2 in Kessing (2007), which characterizes the first-best outcome and MPE, respectively, in the game without a deadline.

[10] This is a consequence of this game exhibiting positive externalities (as can be seen from $\frac{dH_{i,t}}{da_{j,t}} = \lambda_{i,t} \geq 0$), which together with $\frac{da_{j,t}}{dq_t} = \frac{r\lambda_t+\dot{\lambda}_t}{n\lambda_t} = \frac{r}{2n-1} > 0$ implies that efforts are strategic complements across time. This strategic complementarity was first established by Bolton and Harris (1999) in the context of strategic experimentation in teams, and Kessing (2007) in the context of dynamic public good provision.

into the agents' utility functions, in this model, peer pressure arises endogenously.

The project will be completed (if at all) either at the deadline, or at $\bar{\tau}^{mpe}$, whichever comes first. Note that the unconstrained equilibrium completion time $\bar{\tau}^{mpe}$ does not maximize the agents' ex-ante discounted payoff J_0^{mpe}, which, while intuitive, raises the question of what deadline maximizes J_0^{mpe}. While an analytic characterization is not possible, numerical analysis indicates that for a wide range of parameters, the optimal deadline is *approximately* the same as the first-best completion time $\bar{\tau}^{fb}$. Note however that a problematic feature of deadlines is that they are not renegotiation proof. That is, if the project is not completed by the deadline set ex ante, then the agents will have an incentive to postpone it ex post, undermining its intended purpose.

4.2. Comparative statics

In this section, we conduct comparative statics to understand how the agents' incentives depend on the parameters of the problem.

4.2.1. Effect of deadline (T)

To examine how the deadline T influences the agents' strategies, we begin with the first-best outcome characterized in Proposition 1. Noting that $\bar{\tau}^{fb}$ denotes the efficient completion time of the project, if $T \geq \bar{\tau}^{fb}$, then each agent's effort path coincides with that of the infinite horizon problem, the project is completed at $\bar{\tau}^{fb}$, and the deadline has no effect on the agents' strategies. If instead $T < \bar{\tau}^{fb}$, then by shortening the deadline, as one can see from Eq. (6), each agent's effort path is scaled up by a constant factor.

Let us now turn to the MPE characterized in Proposition 2. If the deadline $T \geq \bar{\tau}^{mpe}$, then in the MPE, the agents exert inefficiently low effort at every state, and the project is completed inefficiently late; *i.e.*, $a_t^{fb} > a_t^{mpe}$ for all t, and $\bar{\tau}^{fb} < \bar{\tau}^{mpe}$. This result is consistent with the earlier literature on dynamic public good provision (*e.g.*, Admati and Perry, 1991; Yildirim, 2006 and Georgiadis, 2015), and is illustrated by the cyan dash-dot line in Fig. 1. As the deadline becomes shorter, each agent's equilibrium effort a_t^{mpe} is scaled up by a constant factor as can be seen from Eq. (8) and the green dotted line in Fig. 1. If the deadline is sufficiently short, then due to the agents' frontloading incentives, equilibrium effort is inefficiently high during the early stages of the project. This statement is formalized in the following remark, and it can be visualized by comparing the blue solid line to the red dashed line in Fig. 1.

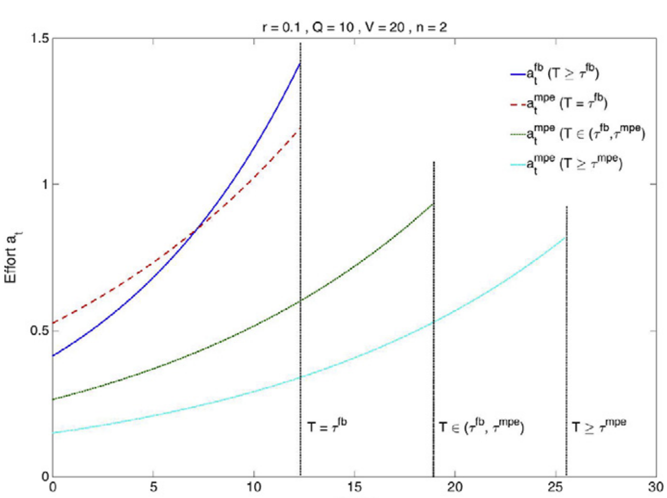

Fig. 1. First-best effort vs. equilibrium effort paths.

Remark 1. For any deadline T, there exists a threshold $\theta > 0$ such that $a_t^{mpe} \geq a_t^{fb}$ if and only if $t \leq \theta$. This threshold is interior (*i.e.*, $\theta > 0$) if the deadline is sufficiently short.[11]

Finally, if the deadline is too short, then the agents must exert too high effort to meet the deadline, so they are better off abandoning the project altogether.

4.2.2. Other comparative statics

Observe that as a function of time, each agent's effort function is defined on $[0, \tau]$ where the completion time τ depends on the chosen parameters, whereas as a function of the state q, it is defined on $[0, Q]$, and the completion state Q is exogenous. Moreover, as there is a one-to-one correspondence between t and q along the equilibrium path (and the first-best outcome), it will be more convenient to characterize the following comparative statics in terms of q instead of t.

Using Propositions 1 and 2, each agent's effort level can be rewritten as a function of the state q_t as

$$a^{fb}(q_t) = \frac{r}{n}\left(q_t + \frac{Q}{e^{r\bar{\tau}^{fb}} - 1}\right) \text{ and } a^{mpe}(q_t) = \frac{r}{2n-1}\left(q_t + \frac{Q}{e^{\frac{rn\bar{\tau}^{mpe}}{2n-1}} - 1}\right), \tag{9}$$

for the first-best outcome and the MPE, respectively. The following remark characterizes how the agents' incentives depend on the completion state Q and payoff V.

Remark 2. Suppose that the game has a project-completing MPE with differentiable strategies as characterized in Proposition 2.

(i) If the deadline is binding (*i.e.*, $T < \bar{\tau}^{mpe}$), then $\frac{\partial}{\partial Q} a^{mpe}(q) > 0$, whereas otherwise, $\frac{\partial}{\partial Q} a^{mpe}(q) < 0$ for all q.
(ii) If the deadline is binding, then $a^{mpe}(q)$ is independent of V, whereas otherwise, $\frac{\partial}{\partial V} a^{mpe}(q) > 0$ for all q.

These comparative statics also apply to the first-best outcome.

If the deadline is binding, then all else equal, an increase in the completion state Q induces the agents to raise their efforts, for otherwise they would miss the deadline. On the other hand, if the deadline is not binding, then as can be seen from Eq. (9), increasing Q has two opposing effects: a direct effect, which incentivizes the agents to raise their effort, and an indirect effect, which induces the agents to complete the project later, thus weakening their incentives. It turns out that the second effect always dominates, so increasing the completion state Q induces the agents to decrease their efforts.

Turning to the second statement, note from Eq. (9) that the reward V affects the unconstrained completion time $\bar{\tau}^{mpe}$, but does not directly affect the agents' equilibrium effort function. As a result, if the deadline is binding, then increasing V does not influence the agents' strategies. If, however, the deadline is not binding, then unsurprisingly, if the agents expect a larger reward upon completion, then they work harder to complete the project sooner.

The comparative statics with respect to the discount rate r and the group size n are similar to the case without a deadline, and they are characterized in Proposition 1 and Theorem 2 of Georgiadis (2015), respectively. For completeness, we state them below. First, if the agents are less patient, then their effort path is steeper and

[11] Formally, there exists a critical deadline $T^{crit} \in \left(\bar{\tau}^{fb}, \bar{\tau}^{mpe}\right)$ such that the threshold is interior (*i.e.*, $\theta > 0$ if $T < T^{crit}$).

they frontload their efforts less. Formally, there exists an interior threshold ϑ such that $\frac{\partial}{\partial r} a^{mpe}(q) < 0$ if and only if $q \leq \vartheta$.

To examine how the group size n influences the agents' incentives, assume that the project generates a fixed lump-sum B upon completion, and each agent receives $V_n = \frac{B}{n}$; *i.e.*, each agent's reward is inversely proportional to the size of her group.

Remark 3. Suppose that the group size increases from n to $m > n$, and a project-completing MPE exists in both cases.

(i) The unconstrained completion time may be $\bar{\tau}_n^{mpe} \lesseqgtr \bar{\tau}_m^{mpe}$ depending on the parameters.
(ii) There exists a threshold θ such that individual effort $a_m^{mpe}(q) \geq a_n^{mpe}(q)$ if and only if $q \leq \theta$.
(iii) There exists an interior threshold ϕ such that aggregate effort $m\, a_m^{mpe}(q) \geq n\, a_n^{mpe}(q)$ if and only if $q \leq \phi$.

In the first-best outcome, the unconstrained completion time decreases (*i.e.*, $\bar{\tau}_m^{fb} < \bar{\tau}_n^{fb}$), statement (ii) is the same as above, and the total effort increases; *i.e.*, $m\, a_m^{fb}(q) \geq n\, a_n^{fb}(q)$ for all q.

5. Infrequent monitoring of progress

In this section, we consider the case in which the agents learn the state of the project q_t at discrete dates $t \in \{t_1, \ldots, T\}$ (which are predetermined and common knowledge), but they do not obtain any information about q_t between those dates. For example, in corporate team projects, the members of the team may learn how close they are to their goal during group meetings where progress is accounted for. As holding meetings is costly and time-consuming, they occur infrequently.

In this case, the agents must choose their strategies based on the project state observed at the discrete dates, and their expectations about their peers' strategies. Because the project progresses deterministically, along any equilibrium path, the agents have correct beliefs about the current state q_t. The key difference relative to the continuous-monitoring case analyzed in the previous section is that now, an agent's effort level at $s \in (t_i, t_{i+1})$ cannot influence the effort levels of the other agents until t_{i+1}, and therefore the agents no longer have an incentive to frontload effort in between monitoring dates. Note that even if the cumulative effort reaches the completion threshold between two monitoring dates, the agents cannot verify this and collect their payoffs before the next monitoring date. As a result, the project can only be completed at a monitoring date.

5.1. No monitoring

We begin by characterizing the equilibrium for the case in which the agents obtain no information about the state q_t until the deadline T.[12] Together with the facts that effort costs are convex and the agents cannot collect their reward before T, this implies that they have no incentive to reach the completion threshold Q strictly before the deadline. Therefore, in a project-completing equilibrium, the state q_t will reach Q exactly at $t = T$.

Proposition 3. *(No monitoring) Suppose that $T < \infty$, and the agents observe the state of the project q_t only a $t = T$. There exists a unique candidate for a symmetric project-completing equilibrium in which each agent's effort function satisfies*

$$a_t^{\{T;T\}} = \frac{rQ}{n} \frac{e^{rt}}{e^{rT} - 1}, \tag{10}$$

$q_T = Q$, and each agent's ex ante discounted payoff is equal to

$$J_0^{\{T;T\}} = e^{-rT} V - \frac{rQ^2}{2n^2} \frac{1}{e^{rT} - 1}.$$

This candidate is an equilibrium if and only if $J_0^{\{T;T\}} \geq 0$.[13]

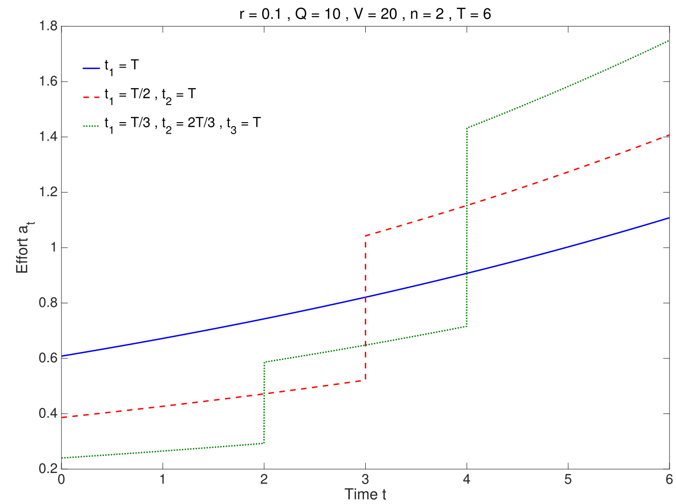

Fig. 2. Effort paths when the project state is monitored at 0, 1, and 2 intermediate dates.

Because the agents do not monitor the state of the project while in progress, they cannot use their current action to influence the future actions of their peers, and hence they have no incentive to frontload their effort as in the MPE. Therefore, similar to what a social planner would do, each agent optimally chooses her effort path at $t = 0$ such that her discounted marginal cost of effort $e^{-rt} a_t^{\{T;T\}}$ is constant. An example is illustrated by the blue solid line in Fig. 2.

This equilibrium need not be unique. If $J_0^{\{T;T\}}\big|_{n=1} \leq 0$ (*i.e.*, if an agent working alone does not find it optimal to complete the project), then there also exists an equilibrium in which no agent ever exerts any effort and the project is not completed. In addition, this game has a continuum of asymmetric equilibria, in which the agents exert different portions of the total effort necessary to complete the project. In all these equilibria, each agent smooths her efforts perfectly along the equilibrium path, and because effort costs are convex, the agents' total discounted payoff is maximized when the symmetric equilibrium is played.

5.2. One intermediate monitoring date

Next, we consider the case in which the agents observe the state q_t at an intermediate date t_1, in addition to the deadline T. There exist two project-completing equilibrium candidates: a candidate in which the project is completed at t_1 and a candidate in which it is

[12] This is often referred to as an "open-loop equilibrium" in the literature; see for instance Fershtman and Nitzan (1991). See also Kessing (2003) and Van Long (2010) for a characterization of the open loop equilibria of this model without a deadline. However, these authors did not relate their theoretical analyses to monitoring.

[13] The superscript $\{T; T\}$ denotes the dates at which the agents monitor the state of the project and its completion time, respectively.

completed at T. A candidate is an equilibrium if (a) it yields each agent a nonnegative ex ante discounted payoff, and (b) no agent has an incentive to unilaterally deviate, thus causing the project to be completed at a different date, or to not be completed at all. The following proposition provides a characterization.

Proposition 4. *(Infrequent monitoring) Suppose that the agents observe the state q_t only at $t = \{t_1, T\}$. There exist two symmetric project-completing equilibrium candidates:*

(i) *A candidate in which each agent's effort level satisfies*

$$a_t^{\{t_1,T;t_1\}} = \begin{cases} \frac{rQ}{n}\frac{e^{rt}}{e^{r\,t_1}-1} & \text{if} \quad t \le t_1 \\ \frac{r(Q-q_{t_1})}{n}\frac{e^{rt}}{e^{rT}-e^{r\,t_1}} & \text{if} \quad t > t_1\ , \end{cases} \tag{11}$$

the project is completed at $\tau = t_1$, and her ex ante discounted payoff is equal to

$$J_0^{\{t_1,T;t_1\}} = e^{-rt_1}V - \frac{rQ^2}{2n^2}\frac{1}{e^{rt_1}-1}\ . \tag{12}$$

This is an equilibrium if and only if $J_0^{\{t_1,T;t_1\}} \ge 0$ and

$$J_0^{\{t_1,T;t_1\}} \ge e^{-rT}V - \frac{rQ^2}{2n^2}\frac{1}{n^2\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)}\ . \tag{13}$$

(ii) *A candidate in which each agent's effort level satisfies*

$$a_t^{\{t_1,T;T\}} = \begin{cases} \frac{rQ}{n}\frac{e^{rt}}{n\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)} & \text{if} \quad t \le t_1 \\ \frac{r(Q-q_{t_1})}{n}\frac{e^{rt}}{e^{rT}-e^{r\,t_1}} & \text{if} \quad t > t_1\ , \end{cases} \tag{14}$$

the project is completed at $\tau = T$, and her ex ante discounted payoff is equal to

$$J_0^{\{t_1,T;T\}} = e^{-rT}V - \frac{rQ^2}{2n^2}\frac{n^2\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)}{\left[n\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)\right]^2}\ .$$

This is an equilibrium if and only if $J_0^{\{t_1,T;T\}} \ge 0$ and

$$J_0^{\{t_1,T;T\}} \ge e^{-rt_1}V - \frac{rQ^2}{2n^2}\frac{1}{e^{r\,t_1}-1}\frac{\left[n^2\left(e^{rT}-e^{rt_1}\right)+\left(e^{r\,t_1}-1\right)\right]^2}{\left[n\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)\right]^2}\ . \tag{15}$$

In the first candidate, similar to the case analyzed in Section 5.1, each agent perfectly smooths her effort over $[0, t_1]$.[14] The only potentially profitable deviation is one in which, expecting all other agents to choose the strategy specified by Eq. (11), an agent chooses the strategy with the smallest discounted cost such that the project is completed at T instead of t_1.[15] This strategy gives the agent an ex ante discounted payoff equal to the right-hand-side of Eq. (13), which for an agent to not have an incentive to deviate this way, must be no greater than $J_0^{\{t_1,T;t_1\}}$; *i.e.*, the payoff corresponding to completing the project at t_1.

Turning to the second candidate, each agent smooths her efforts perfectly during the intervals $[0, t_1]$ and $(t_1, T]$. This is intuitive, since she cannot influence the strategies of the other agents except at t_1. However, observe that if all agents choose their efforts according to Eq. (14) on $[0, t_1]$, then immediately after monitoring the state at t_1, each agent's effort level jumps upwards by a factor of n. Intuitively, this occurs because by shirking prior to t_1, an agent saves the cost of effort that she "skipped", but in equilibrium, she will only exert $1/n$ of the skipped effort after the state is observed at t_1. She cannot shirk during $[t_1, T]$, for otherwise, the project will miss the deadline and she will receive no reward. An example of each agent's effort path is illustrated by the red dashed line in Fig. 2.

Note that the only potentially profitable deviation is one in which, anticipating that all other agents choose their effort levels according to Eq. (14), an agent chooses her strategy to minimize her discounted cost of effort while completing the project at t_1. In this case, this agent's ex ante discounted payoff is given by the right-hand side of Eq. (15), which for this candidate to be an equilibrium, must be no greater than $J_0^{\{t_1,T;T\}}$; *i.e.*, the payoff corresponding to completing the project at T.

An implication of this result is that if the state is observed at an intermediate date along the equilibrium path, then effort is backloaded (*i.e.*, each agent's marginal cost of effort $e^{-rt}a_t$ is increasing in t), which is in contrast to the frontloading theme of Section 4. Recall that frontloading stems from each agent's incentive to motivate her peers to raise their future efforts, and in turn collect the reward from completing the project sooner. If the completion date is fixed, as is the case in this candidate equilibrium, then there is no incentive to frontload effort, and the agents backload effort instead.

In general, there may exist no, one, or two project-completing equilibria.[16] If both candidates constitute an equilibrium, then notice that the right hand side of Eq. (13) is greater than $J_0^{\{t_1,T;T\}}$, which implies that $J_0^{\{t_1,T;t_1\}} > J_0^{\{t_1,T;T\}}$. Therefore, in this case, the equilibrium in which the project is completed at t_1 provides the agents with a strictly higher ex ante discounted payoff.

5.3. Multiple intermediate monitoring dates

Using a similar approach, one can characterize equilibria in which the agents observe the project state at potentially multiple intermediate dates.

Remark 4. Suppose that the agents observe the state q_t only at $t \in \{t_1, \ldots, t_{m-1}, t_m = T\}$, where $m \ge 2$. For every $k \in \{1, \ldots, m\}$, there exists a candidate symmetric equilibrium in which the project is completed at t_k, and along the equilibrium path, each agent's effort level satisfies

$$a_t = \begin{cases} \lambda e^{rt} & \text{if} \quad t \in [0,\ t_1) \\ \lambda n e^{rt} & \text{if} \quad t \in [t_1,\ t_2) \\ \ldots \\ \lambda n^{k-1} e^{rt} & \text{if} \quad t \in [t_{k-1},\ t_k]\ , \end{cases}$$

where $\lambda = \frac{rQ}{\sum_{j=1}^{k} n^j\left(e^{rt_j}-e^{rt_{j-1}}\right)}$.[17]

[14] Note that strategies must specify effort levels for all $t \in [0, T]$ even if in equilibrium, the project is completed at $t_1 < T$. If, off the equilibrium path, the project is not completed at t_1, then the agent's effort path for $t > t_t$ will depend on q_{t_1}.

[15] Note that a deviation that causes the project to not be completed cannot be profitable.

[16] Simulations over a wide range of parameters suggest that an equilibrium in which the project is completed at T (t_1) is more likely to exist if $\frac{rQ^2}{2n^2V}$ is larger (smaller), or t_1 and $T - t_1$ are smaller (larger).

[17] To ensure that this candidate is an equilibrium, one needs to verify that no profitable deviation exists. In the interest of brevity, this step, which involves tedious algebra, is omitted.

Observe that following every monitoring date, each agent's effort level jumps upwards by a factor of n. The intuition is the same as for the first candidate characterized in Proposition 4. An example for the case in which the state is monitored at two intermediate dates is illustrated by the green dotted line in Fig. 2.

5.4. Endogenous monitoring dates

In this section, we are concerned with endogenizing the set of monitoring dates. For example, consider the case in which monitoring is costly (*e.g.*, group meetings are time-consuming and may require members to travel), so the agents can monitor the project state at most once before the deadline. When should they choose to monitor their progress?[18]

Assuming that $\frac{rQ^2}{2Vn^2} < 1$ so that the project is socially desirable, if the agents can commit to any deadline ex ante, then they find it optimal to set $T = \bar{\tau}^{fb}$ and to not monitor the project state at any intermediate date. In this case, the agents' payoffs and strategies coincide with the first-best outcome characterized in Proposition 1. Recall however that deadlines are not renegotiation-proof, and if the project is not completed by $\bar{\tau}^{fb}$, then it will be in their interest to renegotiate the deadline. In the remainder of this section, we consider the case in which the agents face some deadline $T > \bar{\tau}^{fb}$, and they cannot commit to a shorter deadline. The following proposition shows that the agents can maximize their ex ante discounted payoffs by monitoring the state at some appropriately chosen intermediate date t_1 such that the project is completed at that date. Moreover, if T is sufficiently long, then it is possible to implement the first-best outcome.

Proposition 5. *(Endogenous monitoring dates) Suppose that the agents face some deadline $T > \bar{\tau}^{fb}$, they cannot commit to not renegotiating a shorter deadline, but they can choose (ex ante) to observe the project state at some intermediate date t_1.*

(i) *The agents will optimally choose the smallest $t_1 \geq \bar{\tau}^{fb}$ such that Eq. (13) is satisfied; i.e., such that there exists an equilibrium in which the project is completed at t_1.*
(ii) *If T is sufficiently large, then $t_1 = \bar{\tau}^{fb}$, and there exists a unique symmetric project-completing equilibrium, which implements the first-best outcome.*

This result follows from two observations. First, conditional on the project being completed at some date τ, the agents are better off not observing the project state at any $t < \tau$; *i.e.*, $J_0^{\{\tau;\tau\}} > J_0^{\{t,\tau;\tau\}}$ for all $t \in (0,\tau)$. This is because without intermediate monitoring the agents will perfectly smooth their effort over $[0,\tau]$, which is payoff-maximizing. Second, observe from Eq. (12) that if the project is completed at t_1, then each agent's ex ante discounted payoff decreases in t_1 for all $t_1 \geq \bar{\tau}^{fb}$. Therefore, the agents will choose the smallest $t_1 \geq \bar{\tau}^{fb}$ such that Eq. (13) is satisfied; *i.e.*, such that there exists an equilibrium in which the project is completed at t_1. The second part of the proposition follows by noting that Eq. (13) is trivially satisfied when T is sufficiently large.[19]

This result has two implications. First, even if the agents cannot commit to not renegotiate a deadline, they can provide themselves with stronger, possibly first-best incentives by choosing the intermediate monitoring date t_1 appropriately. As a result, t_1 acts as a self-enforcing deadline. Second, absent an exogenous deadline (*i.e.*, if $T = \infty$), then the agents can implement the first-best outcome by committing to monitoring the project state at $t_1 = \bar{\tau}^{fb}$, and then to not monitoring it again until a much later date t_2. In this case, no agent has an incentive to deviate, as that would delay completion until at least t_2, and as a result, completing the project at $\bar{\tau}^{fb}$ is the unique symmetric project-completing equilibrium outcome.[20]

Last, we discuss the case in which the agents face an exogenous deadline T and can choose multiple intermediate dates at which to monitor the state of the project. Because $J_0^{\{\tau;\tau\}} > J_0^{\{t_1,\tau;\tau\}}$ for all $\tau > 0$ and $t_1 \in (0,\tau)$, the agents will choose these dates such that in equilibrium, the project is completed at the first monitoring date t_1. We know from Proposition 5 that if T is sufficiently large such that Eq. (13) is satisfied when $t_1 = \bar{\tau}^{fb}$, then the agents will optimally choose to monitor the project state only at $\bar{\tau}^{fb}$ and at the deadline T, and they will attain the first-best payoffs as a result. In general, the right-hand-side of Eq. (13) is not monotone decreasing in T, so if T is sufficiently small (but larger than $\bar{\tau}^{fb}$), then the agents may find it optimal to monitor the project state at some t_1, and at a second intermediate state $t_2 < T$.

6. Discussion

We use a tractable dynamic model to study the provision of discrete public goods. At every moment, each member of a team chooses her costly effort to make progress on a project, which is completed once the cumulative efforts reach a certain threshold. Provided that this occurs prior to a prespecified deadline, each agent receives a lump-sum reward. We contribute to the extant literature by studying how deadlines and infrequent monitoring of progress affect equilibrium behavior.

We establish three key insights. First, a shorter deadline induces the agents to raise their efforts (provided that a project-completing MPE exists), but no matter the length of the deadline, effort provision is inefficient due to the agents' frontloading incentives. Second, not monitoring the project state until the deadline eliminates the agents' frontloading incentives, so by committing to a deadline equal to the first-best completion time, it is possible to restore efficiency. Recognizing that deadlines are not renegotiation proof, our third main result shows that by committing to monitor the state at the first-best completion time, and then again at a sufficiently later date, efficiency can be attained. In this case, that monitoring date acts as a self-enforcing deadline.

Finally, it is valuable to discuss two assumptions that are important for our results. In particular, that each agent has complete information about the preferences and productivity of the other team members, and the project progresses deterministically. As a result, along the equilibrium path, there is a one-to-one correspondence between time and the project state, so even if the agents cannot monitor the state continuously, their beliefs are correct. While these assumptions provide tractability, they imply that there is no positive role for monitoring. For example, absent either assumption, monitoring the state of the project would give each agent the opportunity to learn about the preferences and productivity of her peers or shocks to progress, and adapt her strategy to the new information. In such a setting, to determine whether monitoring is desirable, one would need to trade off the value of adaptation to the efficiency cost associated with the agents' frontloading incentives. See Campbell et al. (2013) and Cetemen et al. (2016) for steps in that direction.

[18] We discuss the case in which the project state can be monitored at multiple intermediate dates following Proposition 5.

[19] If the deadline $T \leq \bar{\tau}^{fb}$, then the agents find it optimal to not monitor the project at any intermediate state and complete it at T.

[20] Intuitively, committing to not monitoring the state for a sufficiently large duration of time is analogous to introducing a provision threshold in a static voluntary contribution mechanism. If the provision threshold is chosen appropriately, then it may be possible to induce efficient contributions in equilibrium; *e.g.*, see Bagnoli and Lipman (1989) and Andreoni (1998) for details.

Disclosure statement

I, George Georgiadis, declare that I have no relevant or material financial interests that relate to the research described in this paper.

Appendix A. MPE with non-differentiable strategies

In this appendix, we expand the strategy space by also considering non-differentiable strategies that allow agents to punish their peers following a deviation from the equilibrium path. We characterize an MPE in which, along the equilibrium path, all agents exert the first-best effort levels. In particular, consider the following strategy. Given the current state of the project q_t and time t, each agent chooses the first-best effort level characterized in Proposition 1 as long as no deviation from this strategy by any agent has been detected; *i.e.*, as long as the state of the project at time t is consistent with all agents having exerted first-best effort thus far. If a deviation is detected, then all agents *immediately* revert to the MPE characterized in Proposition 2 for the remaining duration of the project.[21]

Proposition 6. *There exists an MPE with non-differentiable strategies in which at every moment along the equilibrium path, each agent exerts the first-best effort level a_t^{fb} and her ex ante discounted payoff is equal to J_0^{fb} , both characterized in Proposition 1. After any deviation from the equilibrium path, all agents revert to the MPE characterized in Proposition 2 for the remaining duration of the project.*

The result that efficiency can be sustained with non-differentiable strategies should not come as a surprise. First, each agent is strictly better off relative to the MPE characterized in Proposition 2 if all agents exert first-best effort levels. Second, the threat to revert to that MPE is credible, because such play constitutes a subgame-perfect equilibrium in the continuation game. Lastly, because agents can detect (and punish) a deviation arbitrarily quickly, the gain from a deviation is arbitrarily small, and consequently, no agent ever has an incentive to deviate.

Appendix B. Proofs

Proof of Proposition 1. The team Hamiltonian can be written as

$$\mathcal{H}_t = -e^{-rt}\sum_{i=1}^{n}\frac{a_{i,t}^2}{2} + \lambda_t n\sum_{i=1}^{n} a_{i,t} \ ,$$

where $\lambda_t \geq 0$ is the co-state variable. Letting $\tau \leq T$ denote the completion time of the project, we have the project-completion condition $\int_0^\tau \sum_{i=1}^n a_{i,t}dt = Q$, and the terminal value function is $\varphi_\tau = e^{-r\tau}nV$. Applying Pontryagin's maximum principle yields the optimality and adjoint equations

$$\frac{d\mathcal{H}_t}{da_{i,t}} = 0 \quad \text{for all} \quad i \text{ and } \dot{\lambda}_t = -\frac{d\mathcal{H}_t}{dq} \ ,$$

respectively. It follows from the adjoint equation that λ_t is constant (say $\lambda_t = c$), and from the optimality equation that $a_{i,t} = cne^{rt}$. Note that the agents' strategies are symmetric, so we can drop the subscript i. Substituting $a_t = cne^{rt}$ into the project-completion condition $\int_0^\tau na_t dt = Q$ yields $c = \frac{rQ}{n^2}(e^{r\tau}-1)^{-1}$, and therefore, for a given completion time τ, each agent's effort and discounted payoff function satisfies the desired expression. By substituting c and a_t into Eq. (4) it follows that

$$1 - \sqrt{\frac{rQ^2}{2Vn^2}} \leq e^{-r\tau} \quad (= \quad \text{if} \quad \tau < T)$$

By noting that the right-hand-side in the last inequality decreases in τ and is ¡1, it follows that if the project is completed in the first-best outcome, then its completion time $\tau = \min\left\{T,\ \bar{\tau}^{fb}\right\}$, where $\bar{\tau}^{fb} = -\frac{1}{r}\ln\left(1-\sqrt{\frac{rQ^2}{2Vn^2}}\right)$. By noting that $\mathcal{H}_t$ is strictly concave in $a_{i,t}$ for all i and it is independent of q_t, it follows from the Mangasarian Theorem (see Seierstad and Sydsaeter, 1987) that the above conditions are sufficient for an optimal solution. Each agent's discounted payoff is equal to $J_t^{fb} = e^{-r(\tau-t)}V - \frac{rQ^2}{2n^2}\frac{e^{rt}(e^{r\tau}-e^{rt})}{(e^{r\tau}-1)^2}$. Finally, the project is completed in the first-best outcome (*i.e.*, it is socially desirable) if and only if $J_0^{fb} \geq 0$, or equivalently $1-e^{-r\tau} \geq \frac{rQ^2}{2Vn^2}$. □

Proof of Proposition 2. The unique (non-trivial) solution to the adjoint equation $\dot{\lambda}_t = -r\frac{n-1}{2n-1}\lambda_t$ is $\lambda_t = c\,e^{-rt\,\frac{n-1}{2n-1}}$, where c is a constant to be determined.[22] Substituting λ_t into Eq. (1) yields

$$\begin{aligned} cn\int_0^\tau e^{rt\left(1-\frac{n-1}{2n-1}\right)}dt = Q &\Rightarrow \frac{(2n-1)\,c}{r}\left(e^{\frac{rn\tau}{2n-1}}-1\right) \\ &= Q \Rightarrow c = \frac{rQ}{2n-1}\left(e^{\frac{rn\tau}{2n-1}}-1\right)^{-1} \ , \end{aligned}$$

and Eq. (4) can be rewritten as

$$1-\sqrt{\frac{rQ^2}{2V}\frac{1}{2n-1}} \leq e^{-\frac{rn\tau}{2n-1}} \quad (=0 \quad \text{if} \quad \tau < T) \ .$$

By noting that the right-hand-side in the last inequality decreases in τ and is ¡1, it follows that the project is completed at $\tau = \min\{T,\ \bar{\tau}^{mpe}\}$, where $\bar{\tau}^{mpe} = -\frac{2n-1}{rn}\ln\left(1-\sqrt{\frac{rQ^2}{2V}\frac{1}{2n-1}}\right)$. Noting that $H_{i,t}$ is strictly concave in $a_{i,t}$ for all i and it is independent of q_t, it follows from the Mangasarian Theorem (see Seierstad and Sydsaeter, 1987) that the above conditions are sufficient for an optimal solution.

For a given completion time τ, at time t along the candidate equilibrium path, each agent's effort strategy is equal to $a_t^{mpe} = \frac{rQ}{2n-1}\frac{e^{\frac{rnt}{2n-1}}}{e^{\frac{rn\tau}{2n-1}}-1}$, and substituting this into each agent's discounted payoff function, it follows that $J_t^{mpe} = e^{-r(\tau-t)}V - \frac{rQ^2}{2(2n-1)}\frac{e^{rt}\left(e^{\frac{r\tau}{2n-1}}-e^{\frac{rt}{2n-1}}\right)}{\left(e^{\frac{rn\tau}{2n-1}}-1\right)^2}$. For a project-completing MPE to exist, it suffices to show that $J_t^{mpe} \geq 0$ at every t. By noting that J_t^{mpe} increases in t, it follows that a project-completing MPE exists if $J_0^{mpe} \geq 0$, or equivalently if $1-e^{-\frac{rn\tau}{2n-1}} \geq \frac{rQ^2}{2V}\frac{1}{2n-1}$. Noting that the adjoint equation has a unique non-trivial solution and that the necessary conditions are also sufficient, it follows that the candidate equilibrium described above is the unique project-completing MPE if $J_0^{mpe} \geq 0$. □

[21] There is a well-known challenge associated with defining trigger strategies in continuous time games. To see why, suppose that a deviation occurs at some t', and agents revert to the MPE at t''. Because there is no first time after t', there always exists some $t \in (t', t'')$ such that the agents are better off reverting to the MPE at that t; *i.e.*, subgame perfection fails. To resolve this problem, we use the concept of inertia strategies proposed by Bergin and MacLeod (1993). Also see Georgiadis et al. (2014), who consider the infinite-horizon counterpart of this problem.

[22] The adjoint equation has a trivial solution $\lambda_t = 0$ for all t, which implies that $a_t = 0$ for all t. However this solution cannot satisfy the project-completion condition (1).

Proof of Remark 1. Let

$$\rho(t) = \frac{a_t^{fb}}{a_t^{mpe}} = \frac{2n-1}{n}\left(\frac{e^{\frac{rn\tau^{mpe}}{2n-1}}-1}{e^{r\tau^{fb}}-1}\right)e^{r\left(\frac{n-1}{2n-1}\right)t},$$

where $\tau^{fb} = \min\left\{T, \bar{\tau}^{fb}\right\}$ and $\tau^{mpe} = \min\left\{T, \bar{\tau}^{mpe}\right\}$ denotes the completion time of the project in the first-best outcome and the MPE, respectively. Observe that

$$\rho(0)|_{\tau^{mpe}=\tau^{fb}=T} = \frac{2n-}{n}\left(\frac{e^{\frac{rnT}{2n-}}-}{e^{rT}-}\right) < \qquad 1$$

for all $T > 0$. Therefore, if $T \le \bar{\tau}^{fb}$ so that $\tau^{fb} = \tau^{mpe} = T$, then $a_0^{mpe} > a_0^{fb}$. Moreover, observe that $\rho(0)$ is increasing in T and $\frac{2n-1}{n}\frac{e^{\frac{rn\bar{\tau}^{mpe}}{2n-1}}-1}{e^{r\bar{\tau}^{fb}}-1} > 1$, so there exists some threshold $T_{crit} \in \left(\bar{\tau}^{fb}, \bar{\tau}^{mpe}\right)$ such that $a_0^{mpe} \ge a_0^{fb}$ if and only if $T \le T_{crit}$. Finally, note that for any deadline T, $\rho(t)$ is strictly increasing in t, which implies the desired result. □

Proof of Remark 2. If the deadline is binding (*i.e.*, if the project is completed at $\tau = T$), then $a^{mpe}(q) = \frac{r}{2n-1}\left(q + \frac{Q}{e^{\frac{rnT}{2n-1}}-1}\right)$, whereas if it is not (and so the project is completed at $\tau = \bar{\tau}^{mpe}$), then $a^{mpe}(q) = \frac{r}{2n-1}\left(q + Q\frac{1-\sqrt{\frac{rQ^2}{2V}\frac{1}{2n-1}}}{\sqrt{\frac{rQ^2}{2V}\frac{1}{2n-1}}}\right)$. The comparative statics follow by inspecting how $a^{mpe}(q)$ depends on Q and V in each case. □

Proof of Remark 3. For part (i), one can construct numerical examples such that by increasing the group size, the unconstrained completion time of the project $\bar{\tau}^{mpe} = -\frac{2n-1}{rn}\ln\left(1-\sqrt{\frac{rQ^2}{2B}\frac{n}{2n-1}}\right)$ increases and others in which it decreases. To establish parts (ii) and (iii), fix some n and $m > n$. Denote the completion time of the project when the agents play the MPE by $\tau(n) = \min\left\{T, \bar{\tau}^{mpe}(n)\right\}$, and recall that along the path of the MPE, each agent's effort level satisfies $a^{mpe}(q;n) = \frac{r}{2n-1}\left(q + \frac{Q}{e^{\frac{r\,n\tau(n)}{2n-1}}-1}\right)$. Consider the ratio $\frac{a^{mpe}(q;m)}{a^{mpe}(q;n)} = \frac{2n-1}{2m-1}\frac{q+\frac{Q}{e^{\frac{r\,m\tau(m)}{2m-1}}-1}}{q+\frac{Q}{e^{\frac{r\,n\tau(n)}{2n-1}}-1}}$, and notice that it decreases in q if and only if $\frac{m\,\tau(m)}{2m-1} < \frac{n\,\tau(n)}{2n-1}$. Observe that $e^{\frac{r\,n\bar{\tau}^{mpe}(n)}{2n-1}} = -\ln\left(1-\frac{rQ^2}{2B}\frac{n}{2n-1}\right)$ decreases in n, and so $\frac{m\,\bar{\tau}^{mpe}(m)}{2m-1} < \frac{n\,\bar{\tau}^{mpe}(n)}{2n-1}$. Noting that $\frac{m}{2m-1} < \frac{n}{2n-1}$, the desired inequality holds if $\bar{\tau}^{mpe}(m) \le \bar{\tau}^{mpe}(n)$. Suppose that $\bar{\tau}^{mpe}(m) > \bar{\tau}^{mpe}(n)$. If $T \le \bar{\tau}^{mpe}(n) < \bar{\tau}^{mpe}(m)$, then $\tau(n) = \tau(m) = T$, and so the desired inequality holds. If $\bar{\tau}^{mpe}(n) < T < \bar{\tau}^{mpe}(m)$, then $\tau(n) = \bar{\tau}^{mpe}(n)$ and $\tau(m) = T$, and because $\frac{m\,\bar{\tau}^{mpe}(m)}{2m-1} < \frac{n\,\bar{\tau}^{mpe}(n)}{2n-1}$, we have $\frac{m\,\tau(m)}{2m-1} < \frac{m\,\bar{\tau}^{mpe}(m)}{2m-1} < \frac{n\,\tau(n)}{2n-1}$. Lastly, if $\bar{\tau}^{mpe}(n) < \bar{\tau}^{mpe}(m) \le T$, then $\tau(n) = \bar{\tau}^{mpe}(n)$ and $\tau(m) = \bar{\tau}^{mpe}(m)$, and the desired inequality again holds. Therefore, there exists some threshold θ such that $a^{mpe}(q;m) \ge a^{mpe}(q;n)$ if and only if $q \le \theta$. Finally, by noting that the ratio $\frac{m\,a^{mpe}(q;m)}{n\,a^{mpe}(q;n)}$ also decreases in q, it follows that there exists another threshold ϕ such that $m\,a^{mpe}(q;m) \ge n\,a^{mpe}(q;n)$ if and only if $q \le \phi$.

To establish the counterpart of the above results for the case in which the MPE with non-differentiable strategies is played, first notice that $\bar{\tau}^{fb}(n) = -\frac{1}{r}\ln\left(\left[1-\sqrt{\frac{rQ^2}{2Bn}}\right]^+\right)$ decreases in n. Turning to part (ii), fix some n and $m > n$. Denoting the first-best completion time of the project when the group size is n by $\tau(n) = \min\left\{T, \bar{\tau}^{fb}(n)\right\}$, along the efficient path, each agent's effort level satisfies $a^{fb}(q;n) = \frac{r}{n}\left(q + \frac{Q}{e^{r\tau(n)}-1}\right)$. Noting that $\tau(m) \le \tau(n)$ for any deadline, observe that the ratio $\frac{a^{fb}(q;m)}{a^{fb}(q;n)} = \frac{n}{m}\frac{q+\frac{Q}{e^{r\tau(m)}-1}}{q+\frac{Q}{e^{r\tau(n)}-1}}$ decreases in q. Therefore, there exists some threshold θ such that $a^{fb}(q;m) \ge a^{fb}(q;n)$ if and only if $q \le \theta$. Finally, consider the ratio of aggregate efforts $\frac{m\,a^{fb}(q;m)}{n\,a^{fb}(q;n)} = \frac{q+\frac{Q}{e^{r\tau(m)}-1}}{q+\frac{Q}{e^{r\tau(n)}-1}}$. It decreases in q, and using that $\tau(m) \le \tau(n)$, it is straightforward to verify that $\frac{m\,a^{fb}(Q;m)}{n\,a^{fb}(Q;n)} > 1$. Therefore, $m\,a^{fb}(q;m) > n\,a^{fb}(q;n)$ for all q. □

Proof of Proposition 3. The optimality and the adjoint equation can be written as $a_t = \lambda_t e^{rt}$ and $\dot{\lambda}_t = -\frac{dH_t}{dq} = 0$, respectively. The second condition implies that λ_t is a constant (say equal to c), and using Eq. (1) to determine this constant yields that $c = \frac{rQ}{n(e^{r\tau}-1)}$. Because effort costs are convex, the agents discount time, and they do not receive a reward until $t = T$, if the project is completed in equilibrium, then it will be completed at $t = T$. As in the continuous monitoring case, sufficiency follows from the Mangasarian Theorem by noting that $H_{i,t}$ is strictly concave in $a_{i,t}$ and independent of q_t for all i. Therefore, in any project-completing equilibrium, $\tau = T$, each agent's effort function satisfies

$$a_t^{\{T;T\}} = \frac{rQ}{n}\frac{e^{rt}}{e^{rT}-1},$$

and her discounted payoff function

$$J_t^{\{T;T\}} = e^{-r(\tau-t)}V - \frac{rQ^2}{2n^2}\frac{e^{rt}\left(e^{rT}-e^{rt}\right)}{\left(e^{rT}-1\right)^2}.$$

Finally, a project-completing equilibrium exists as long as $J_0^{\{T;T\}} \ge 0$. □

Proof of Proposition 4. Part (i). For candidate (i), it follows from Proposition 4 that in an equilibrium in which the project is completed at t_1, each agent's strategy and ex ante discounted payoff must be as given in Eq. (11) and $J_0^{\{t_1,T;t_1\}}$. Similar to the previous case, for this to be an equilibrium, it is necessary that no agent has an incentive to unilaterally deviate to a different strategy, and $J_0^{\{t_1,T;t_1\}} \ge 0$. Moreover, the only potentially optimal deviation is for an agent to choose her effort path such that the project is not completed at t_1, and so it is completed at T instead. To analyze this case, suppose that all agents but i choose their effort according to Eq. (11), and so they exert total effort $\frac{n-1}{n}Q$ by t_1.

Suppose that the state of the project at t_1 is $q_1 \ge \frac{n-1}{n}Q$. It follows from the analysis in part (i) of the proof for the game during $[t_1, T]$ that agent i's discounted payoff at t_1 is $J_{i,t_1}^{dev}(q_1) = e^{-r(T-t_1)}V - \frac{re^{rt_1}}{2n^2}\frac{(Q-q_1)^2}{e^{rT}-e^{rt_1}}$. Turning to the game during $[0, t_1]$, agent i's effort path has the form $a_{i,t} = \nu e^{rt}$, where ν satisfies the transversality condition $\nu = \frac{q}{dq_1}e^{-rt_1}J_{t_1}^{dev}(q_1) = \frac{r(Q-q_1)}{n^2\left(e^{rT}-e^{rt_1}\right)}$, and feasibility requires that $\int_0^{t_1}\nu e^{rt}dt + \frac{n-1}{n}Q = q_1$. Together, these conditions pin down

$$q_1 = Q\frac{n(n-1)\left(e^{rT}-e^{rt_1}\right)+\left(e^{rt_1}-1\right)}{n^2\left(e^{rT}-e^{rt_1}\right)+\left(e^{rt_1}-1\right)}.$$

Therefore, $\nu = \frac{rQ}{n}\frac{1}{n^2\left(e^{rT}-e^{rt_1}\right)+\left(e^{rt_1}-1\right)}$, and agent i's ex ante discounted payoff is equal to

$$J_{i,0}^{dev} = e^{-rt_1}J_{i,t_1}^{dev}(q_1) - \int_0^{t_1} e^{-rt}\frac{1}{2}\nu^2 e^{2rt}dt = e^{-rT}V - \frac{rQ^2}{2n^2}\frac{n^2\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)}{\left[n^2\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)\right]^2}$$

For candidate (i) to be an equilibrium, it must be the case that $J_0^{\{t_1,T;t_1\}} \geq \max\left\{0, J_{i,0}^{dev}\right\}$.

Part (ii). We begin by characterizing the second candidate (in which the project is completed at T), and establish the conditions under which it is an equilibrium. Fix the state q_1 that the agents observe at t_1, and consider the game for $t \in [t_1, T]$. The optimality and the adjoint equation are $a_t = \lambda_t e^{rt}$ and $\dot{\lambda}_t = -\frac{dH_t}{dq} = 0$, respectively. For the project to be completed at T, it must be the case that $\int_{t_1}^T n\lambda(q_1)e^{rt}dt = Q - q_1$, or equivalently $\lambda(q_1) = \frac{r(Q-q_1)}{n\left(e^{rT}-e^{rt_1}\right)}$, where q_1 denotes the project state at t_1. Therefore, for $t \geq t_1$, each agent's effort level $a_t = \frac{r(Q-q_1)e^{rt}}{n\left(e^{rT}-e^{rt_1}\right)}$. Next, compute each agent's discounted payoff at t_1:

$$J_{t_1}(q_1) = e^{-r(T-t_1)}V - \int_{t_1}^T e^{-r(t-t_1)}\frac{1}{2}[\lambda(q_1)]^2 e^{2rt}dt = e^{-r(T-t_1)}V - \frac{re^{rt_1}}{2n^2}\frac{(Q-q_1)^2}{e^{rT}-e^{rt_1}}.$$

Next, consider the game for $t \in [0, t_1)$. Similar to the previous case, the optimality and the adjoint equation can be written as $a_t = \mu_t e^{rt}$ and $\dot{\mu}_t = -\frac{dH_t}{dq} = 0$, respectively. Feasibility requires that $\int_0^{t_1} n\mu e^{rt}dt = q_1$, or equivalently $\mu = \frac{rq_1}{n\left(e^{rt_1}-1\right)}$. The transversality condition is $\mu = \frac{q}{dq_1}e^{-rt_1}J_{t_1}(q_1) = \frac{r(Q-q_1)}{n^2\left(e^{rT}-e^{rt_1}\right)}$, which pins down

$$q_1 = Q\frac{e^{rt_1}-1}{n\left(e^{rT}-e^{rt_1}\right)+\left(e^{rt_1}-1\right)},$$

and subsequently, the parameters μ and $\lambda(q_1)$, as well as the effort function given in Eq. (14). Each agent's ex ante discounted payoff is equal to

$$\begin{aligned}
J_0^{\{t_1,T;T\}} &= e^{-rt_1}J_{t_1}(q_1) - \int_0^{t_1} e^{-rt}\frac{1}{2}\mu^2 e^{2rt}dt \\
&= e^{-rT}V - \frac{rQ^2}{2n^2}\frac{(Q-q_1)^2}{e^{rT}-e^{rt_1}} - \frac{rq_1^2}{2n\left(e^{r\,t_1}-1\right)} \\
&= e^{-rT}V - \frac{rQ^2}{2n^2}\frac{n^2\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)}{\left[n\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)\right]^2}.
\end{aligned}$$

For the strategies given in Eq. (14) to constitute an equilibrium, it is necessary that no agent has an incentive to unilaterally deviate from said strategies, and $J_0^{\{t_1,T;T\}} \geq 0$. Notice that the only potentially optimal deviation is for an agent to choose her effort path such that the project is completed at t_1 instead of T. To analyze this case, suppose that all agents but i choose their effort according to Eq. (14), and so they exert total effort $\frac{n-1}{n}q_1$ by t_1. To complete the project by t_1, agent i must exert total effort $Q - \frac{n-1}{n}q_1$ during $[0, t_1]$. Her effort path has the form $a_{i,t} = \nu e^{rt}$, where ν is pinned down by the feasibility condition $\int_0^{t_1}\nu e^{rt}dt = Q - \frac{n-1}{n}q_1$. It follows that $\nu = \frac{rQ}{e^{rt_1}-1}\frac{n^2\left(e^{rT}-e^{rt_1}\right)+\left(e^{rt_1}-1\right)}{n^2\left(e^{rT}-e^{rt_1}\right)+n\left(e^{rt_1}-1\right)}$. In this case, agent i's ex ante discounted payoff is equal to

$$J_{i,0}^{dev} = e^{-rt_1}V - \int_0^{t_1} e^{-rt}\frac{1}{2}\nu^2 e^{2rt}dt = e^{-rt_1}V - \frac{rQ^2}{2n^2}\frac{1}{e^{r\,t_1}-1}\frac{\left[n^2\left(e^{rT}-e^{rt_1}\right)+\left(e^{r\,t_1}-1\right)\right]^2}{\left[n\left(e^{rT}-e^{r\,t_1}\right)+\left(e^{r\,t_1}-1\right)\right]^2}.$$

For candidate (ii) to be an equilibrium, it must be the case that $J_0^{\{t_1,T;T\}} \geq \max\left\{0, J_{i,0}^{dev}\right\}$. □

Proof of Proposition 5. First observe that $J_0^{\{t_1,T;T\}} < J_0^{\{T;T\}}$ for all $t_1 \in (0, T)$; that is, if the project is completed T, then the agents are strictly better off not observing the project state at any intermediate date $t_1 \in (0, T)$. This implies that if the agents choose to observe the project state at some intermediate date t_1, then there must exist an equilibrium in which the project is completed at t_1.

Next, note from Eq. (12) that $J_0^{\{t_1,T;t_1\}}$ decreases in t_1; *i.e.*, conditional on the project being completed at t_1 in equilibrium, each agent's ex ante discounted payoff decreases in t_1. Therefore, the agents' ex ante discounted payoff is maximized by choosing the smallest t_1 such that Eq. (13) is satisfied.

Finally, note that the right-hand-side of Eq. (13) diminishes to 0 as $T \to \infty$, which implies that for any t_1, Eq. (13) is trivially satisfied if T is sufficiently large. Therefore, in that case, the agents will optimally choose $t_1 = \bar{\tau}^{fb}$. □

Proof of Remark 4. Consider a candidate equilibrium in which the project is completed at t_k. For $t \in [t_{j-1}, t_j]$, where $j \in \{2, \ldots, m\}$, the adjoint equation implies that the co-state variable λ_j is constant, and the optimality and the feasibility condition is

$$a_t = \lambda_j e^{rt} \text{ and } \int_{t_{j-1}}^{t_j} n\lambda_j e^{rt}dt = q_{t_j} - q_{t_{j-1}},$$

respectively, where the latter can be rewritten as $\lambda_j = \frac{r\left(q_{t_j}-q_{t_{j-1}}\right)}{n\left(e^{rt_j}-e^{rt_{j-1}}\right)}$. Each agent's payoff at t_{j-1} can be written as a function of q_{j-1} as follows:

$$J_{t_{j-1}} = e^{-r\left(t_j-t_{j-1}\right)}J_{t_j} - \int_{t_{j-1}}^{t_j} e^{-r\left(t-t_{j-1}\right)}\frac{1}{2}\lambda_j^2 e^{2rt}dt = e^{-r\left(t_j-t_{j-1}\right)}J_{t_j} - \frac{re^{rt_{j-1}}}{2n^2}\frac{\left(q_{t_j}-q_{t_{j-1}}\right)^2}{e^{rt_j}-e^{rt_{j-1}}}.$$

Now let us turn our attention to the interval $[t_{j-2}, t_{j-1}]$, where $t_0 = 0$. As in the previous case, the co-state variable is constant, and the optimality and the feasibility condition is $a_t = \lambda_{j-1}e^{rt}$ and $\int_{t_{j-2}}^{t_{j-1}} n\lambda_{j-1}e^{rt}dt = q_{t_{j-1}} - q_{t_{j-2}}$, respectively, where the latter can be rewritten as $\lambda_{j-1} = \frac{r\left(q_{t_{j-1}}-q_{t_{j-2}}\right)}{n\left(e^{rt_{j-1}}-e^{rt_{j-2}}\right)}$. Note that the transversality condition $\lambda_{j-1} = e^{-rt_{j-1}}\frac{dJ_{t_{j-1}}}{dq_{t_{j-1}}}$ must be satisfied, which implies that $\lambda_{j-1} = \frac{r}{n^2}\frac{q_{t_j}-q_{t_{j-1}}}{e^{rt_j}-e^{rt_{j-1}}}$.

Observe that $\lambda_j = n\lambda_{j-1}$, and note that this relationship must hold for all $j \in \{2, \ldots, m\}$. Therefore, in a symmetric equilibrium candidate in which the project is completed at t_k, each agent's effort level satisfies

$$a_t = \begin{cases} \lambda e^{rt} & \text{if} \quad t \in [0,\ t_1) \\ \lambda n e^{rt} & \text{if} \quad t \in [t_1,\ t_2) \\ \dots \\ \lambda n^{k-1} e^{rt} & \text{if} \quad t \in [t_{k-1},\ t_k] \end{cases},$$

where $\lambda = \frac{rQ}{\sum_{j=1}^{k} n^j \left(e^{rt_j} - e^{rt_{j-1}}\right)}$ follows from the feasibility condition; *i.e.*, $\int_0^{t_k} n a_t dt = Q$. □

Proof of Proposition 6. To begin, suppose that the agents revert to the MPE at some time $\bar{t}$, and let $\bar{q} = q_{\bar{t}}$ denote the corresponding state. Then observe that they face the same problem as if they were starting a project of size $Q - \bar{q}$ with deadline $T - \bar{t}$, and they were playing the corresponding MPE. Assuming that a project-completing MPE exists, each agent's discounted effort function on $[\bar{t}, T]$ and her discounted payoff at $\bar{t}$ satisfies

$$a_t^{mpe}(\bar{t}, \bar{q}) = \frac{r(Q - \bar{q})}{2n - 1} \frac{e^{\frac{rn(t - \bar{t})}{2n-1}}}{e^{\frac{rn\tau}{2n-1}} - 1} \text{ and } J_{\bar{t}}^{mpe}(\bar{t}, \bar{q}) = e^{-r(\tau - \bar{t})} V - \frac{r(Q - \bar{q})^2}{2(2n - 1)} \frac{e^{\frac{r\tau}{2n-1}} - 1}{\left(e^{\frac{rn\tau}{2n-1}} - 1\right)^2},$$

respectively, where the completion time $\tau = \min\{T,\ \bar{\tau} + \bar{t}\}$, and $\bar{\tau} = -\frac{2n-1}{rn} \ln\left(\left[1 - \sqrt{\frac{r(Q-\bar{q})^2}{2V} \frac{1}{2n-1}}\right]^+\right)$. If a project-completing MPE does not exist, then each agent exerts no effort for all $t \geq \bar{t}$ and her discounted payoff is equal to 0. Letting $J_{\bar{t}}^{fb}(\bar{t}, \bar{q})$ denote each agent's discounted payoff at $\bar{t}$ if all agents exert the first-best effort levels for all $t \geq \bar{t}$, it is straightforward to verify that $J_{\bar{t}}^{fb}(\bar{t}, \bar{q}) > J_{\bar{t}}^{mpe}(\bar{t}, \bar{q})$ for all $\bar{t}$ and $\bar{q} < Q$.

The remainder of this proof follows the proof of Proposition 2 in Georgiadis et al. (2014). Consider the following strategy:

$$a_t = \begin{cases} a_t^{fb} & \text{if} \quad q_t = \int_0^t n a_s^{fb} ds \\ a_t^{mpe}(\bar{t}, \bar{q}) & \text{otherwise,} \end{cases}$$

where a_t^{fb} is given in Eq. (6), $\bar{t} = \inf_t \left\{q_t \neq \int_0^t n a_s^{fb} ds\right\}$, and $\bar{q} = q_{\bar{t}}$. This strategy specifies that at every moment t, each agent exerts the first-best effort level as long as every other agent has insofar followed this strategy, and she reverts to the MPE otherwise.

There is a well known problem associated with defining such trigger strategy in continuous-time games. To see why, suppose that a deviation occurs at some t', and agents revert to the MPE at some $t'' > t$. Because there is no first moment after t', there always exists some $t \in (t', t'')$ such that the agents are better off reverting to the MPE at that t; *i.e.*, subgame perfection fails. To resolve this problem, following Bergin and MacLeod (1993), consider the following inertia strategy:

$$a_t^{\epsilon} = \begin{cases} a_t^{fb} & \text{if} q_{t-\epsilon} = \int_0^{t-\epsilon} n a_s^{fb} ds \\ a_t^{mpe}(\bar{t}, \bar{q}) & \text{otherwise,} \end{cases}$$

where $q_t = 0$ for all $t \leq 0$. First observe that at the limit as $\epsilon \to 0$, a_t^{ϵ} converges to a_t. Second, because $a_t^{\epsilon} < \infty$ for all ϵ, t, and Q, for any decreasing sequence $\{\epsilon^m\}_{m=1}^{\infty}$ converging to 0, the strategies a_t^{ϵ} form a Cauchy sequence. Third, whereas the strategy a_t^{ϵ} cannot form a PPE, it does form an ϵ-PPE. To see why, first observe that if at every moment t, all agents choose effort level a_t^{ϵ}, then each agent's discounted payoff is strictly greater than if they play the MPE for all $s \geq t$. Second, observe that for sufficiently small ϵ, the optimal strategy against a_t^{ϵ} is for an agent to choose a_t^{fb} for all $t < \tilde{\tau}$, and deviate to some $a_t^{\epsilon,dev}$ at $t = \tilde{\tau}$, where $\tilde{\tau}$ is chosen such that the project is completed at $\min\{\tilde{\tau} + \epsilon, T\}$. In this case, project completion is delayed by at most ϵ units of time, and each agent's discounted payoff at time t is at least $J_t^{fb} - \epsilon r\, e^{-r\tau} - \frac{\epsilon}{2}\left[a_{\tau^{fb}}^{fb}\right]^2$. Therefore, the strategy a_t^{ϵ} forms an ϵ-PPE, and by applying Theorem 3 of Bergin and MacLeod (1993), it follows that the limit strategy as $\epsilon \to 0$ forms a PPE. This competes the proof. □

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