American Economic Journal: Microeconomics 2019, 11(1): 243–298
https://doi.org/10.1257/mic.20170217

Collective Choice in Dynamic Public Good Provision†

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Two heterogeneous agents contribute over time to a joint project and collectively decide its scale. A larger scale requires greater cumulative effort and delivers higher benefits upon completion. We show that the efficient agent prefers a smaller scale, and preferences are time-inconsistent: as the project progresses, the efficient (inefficient) agent's preferred scale shrinks (expands). We characterize the equilibrium outcomes under dictatorship and unanimity, with and without commitment. We find that an agent's degree of efficiency is a key determinant of control over the project scale. From a welfare perspective, it may be desirable to allocate decision rights to the inefficient agent. (*JEL* C73, D71, H43)

In many economic settings agents must collectively decide the *scale* of a joint project. A greater scale yields a larger reward upon completion but requires more cumulative effort. For example, the General Agreement on Tariffs and Trade (GATT), the largest trade agreement, is periodically extended by way of negotiating rounds. These rounds are formally launched with objectives agreed to by member countries. A broader scale of negotiations (such as a greater number of sectors or tariff lines to be included in negotiations) yields a higher reward when the agreement enters into force, but requires greater effort from all parties. Similarly, entrepreneurs collaborating on a joint business venture must choose whether to seek a blockbuster product or one that may have a quicker, if smaller, payoff. Academics working on a joint research project face a similar trade-off when deciding the scale of a data collection exercise, for example. A critical concern in such joint decisions is the disproportionate control of large contributors to the project. Of the GATT's Uruguay round of trade negotiations, one Nigerian newspaper commented that "It is

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† Go to https://doi.org/10.1257/mic.20170217 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

not the GATT of the whole world but that of the rich and powerful" (Preeg 1995). This paper investigates the source of control in joint projects and asks how it is affected by the formal collective choice institution.

We focus on projects with three key features, which are shared by the previous examples. First, progress on the project is gradual, and hence the problem is dynamic in nature. Second, each participant's payoff is realized predominantly upon completion of the project, and it depends on the scale that is implemented, which is endogenous.[1] Finally, the participants are heterogeneous with respect to their opportunity cost of contributing and their stake in the project.

We take the dynamic public good provision framework of Marx and Matthews (2000) as the starting point for our analysis. It is well-known that free-riding occurs in this setting, and basic comparative statics are well understood when agents are symmetric (Admati and Perry 1991, Compte and Jehiel 2004, and Bonatti and Hörner 2011). However, little is known about this problem when agents are heterogeneous. We begin by studying a simple two-agent model. The agent with the lower effort cost per unit of benefit is referred to as the *efficient* agent, and the agent with the higher effort cost per unit of benefit is referred to as the *inefficient* agent. The solution concept we use is Markov perfect equilibrium (hereafter MPE), as is standard in this literature. When multiple equilibria exist, we refine the set of equilibria to the Pareto-dominant ones.

To lay the foundations for the collective choice analysis, we first consider the setting in which the project scale is exogenously fixed. We show that at every stage of the project, the efficient agent not only exerts more effort than the inefficient agent, but he also obtains a lower discounted payoff (normalized by his project stake). Each agent's effort increases as the project nears completion, and furthermore, we show that the efficient agent's effort increases at a faster rate than that of the inefficient agent. Intuitively, both agents' incentives grow as the project gets closer to completion, but the agent with the lower effort cost per unit of benefit has stronger incentives to raise his effort.

We use these results to derive the agents' preferences over the project scale. A lower normalized payoff for the efficient agent implies that at every stage of the project, he prefers a smaller project scale than the inefficient agent. Moreover, the project scale that maximizes the efficient agent's discounted payoff *decreases* as the project progresses, while the opposite is true for the inefficient agent. This is because the efficient agent increases his effort at a faster rate than the inefficient agent, so the efficient agent's share of the remaining project cost *increases* as the project gets closer to completion. The opposite is true for the inefficient agent. The agents' preferences over the project scale are thus time-inconsistent and divergent. This is illustrated in Figure 1.

[1] For example, negotiating parties in the GATT could consider the benefits of adding sectors or tariff lines as a number of studies calculated these. Harrison, Rutherford, and Tarr (1997) calculated increases in global GDP of \$58.3, \$18.8, and \$16.0 billion on agriculture, manufactures, and textiles, respectively, as a consequence of the Uruguay round. Francois, McDonald, and Nordström (1994); Goldin, Knudsen, and van der Mensbrugghe (1993); and Page, Davenport, and Hewitt (1991), among others, also provided estimates of the impact of the Uruguay round for developing and developed countries at various stages of the negotiation.

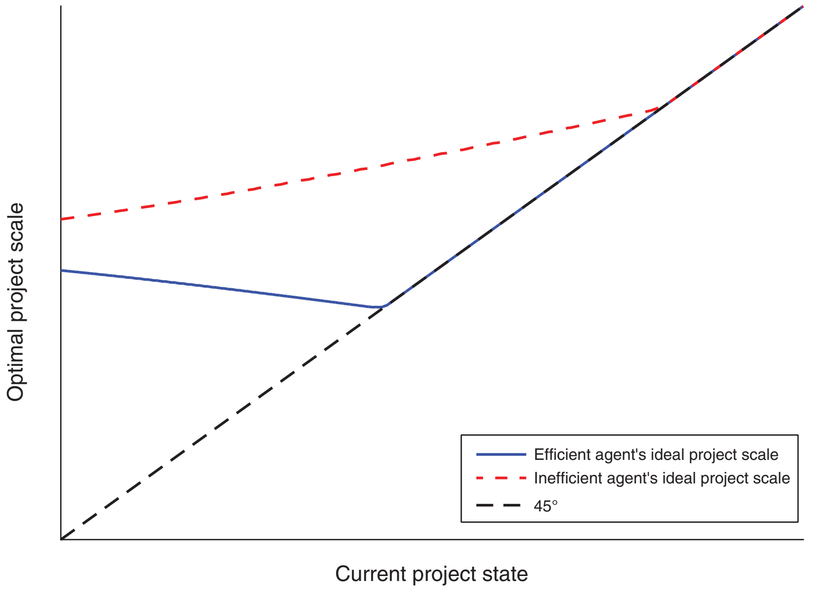

Figure 1. Agent Preferences over Project Scale

Next, we endogenize the project scale and analyze the equilibrium outcomes under two commonly studied collectiv.e choice institutions: dictatorship and unanimity. We consider that the parties may or may not be able to commit to an ex ante decision to implement a particular project scale.[2] For example, the GATT negotiating rounds often miss deadlines, and the final scale differs from the original agreement. The Uruguay round of negotiations was scheduled to conclude in 1990 but was not finalized until 1994. In reference to this delay, the World Trade Organization (WTO) states "The delay had some merits. It allowed some negotiations to progress further than would have been possible in 1990." It is also common for the scale of public infrastructure projects to change throughout their development, a phenomenon often referred to as "scope creep." In such cases, the parties cannot commit to the chosen scale. In other cases, the parties can commit to a binding decision about the project scale at any time, preventing subsequent renegotiation. We consider the ability to commit part of the economic environment and not a choice of the agents.

With commitment, we show that the project scale is decided at the start of the project in equilibrium under any institution. When either agent is dictator, he chooses his ex ante payoff-maximizing project scale, whereas under unanimity, the project scale lies between the agents' ex ante optimal scales.

Without commitment, if the efficient agent is dictator, then there exists a unique MPE in which he completes the project at his preferred scale. However, if the inefficient agent is dictator, then there exists a continuum of equilibrium project scales. All these scales are smaller than the inefficient agent's ideal, but more preferred by

[2] Commitment refers to the case in which the agents can commit to a decision about the project scale at any time. In the case without commitment, the agents cannot commit to an ex ante decision, so, at every moment they decide to either complete the project immediately or continue.

the inefficient agent than the project scale that the efficient agent would choose if he were dictator. That is because the project scale that is implemented in equilibrium depends on when the inefficient agent expects the efficient agent to stop working. Last, because the inefficient agent prefers a larger project scale than the efficient agent, the set of equilibria under unanimity are the same as when the inefficient agent is dictator.

These findings are consistent with stylized facts from the GATT negotiations. For example, the Trade Facilitation Agreement negotiations formally concluded in 2013, but countries still had to ratify the agreement through their domestic legislative process. This ratification was, in general, completed earlier by larger countries, and later by smaller countries indicating that larger countries preferred to complete the agreement sooner.[3]

While formal collective choice institutions exist, the project scale that is implemented remains an equilibrium outcome. That is, even if an agent has dictatorship rights, he has to account for the other agent's actions when deciding the project scale. We say that an agent has effective control if his preferences are implemented in equilibrium. With commitment, whichever agent has formal control (i.e., the dictator), also has effective control. In contrast, without commitment, regardless of which agent is dictator, at completion, it is the efficient agent who has effective control. As indicated, the final scale of the Uruguay round was narrower than some participants had hoped for and left many developing countries with the impression that they had little control. Our findings help explain why items are left off multilateral agreements. This is because larger contributing countries prefer narrower agreements that can be concluded faster, and they have a credible threat to end negotiations.

The socially optimal project scale lies between the two agents' ex ante payoff-maximizing project scales. Therefore, when the efficient agent is dictator, the equilibrium project scale is too small relative to the social optimum. The reason is that he retains full control of the scale and his ideal project scale does not internalize the inefficient agent's higher dynamic payoff. In contrast, if the inefficient agent is dictator or under unanimity, the socially optimal project scale belongs to the set of equilibrium project scales. Therefore, it may be desirable to confer some formal control to the inefficient agent (via dictatorship or unanimity) as a means to counter the effective control that the efficient agent obtains in equilibrium. This provides a rationale for unanimity as the collective choice institution in many international agreements.

To test the robustness of our results, we consider four extensions of the model. If transfers are allowed, then the social planner's project scale can be implemented in equilibrium under all institutions. When the agents can choose the stakes (or shares) of the project ex ante, simulations show that the efficient agent is always allocated a higher share than the inefficient agent. With the efficient agent as dictator, the share awarded to him is naturally the largest. Second, we consider the possibility that the agents play non-Markov equilibria, and using simulations, we examine how

[3] See http://www.tfafacility.org/ratications. Note that the agreement would not go into effect until ratification was complete by a sufficient number of countries, hence, payoffs could not be realized.

the equilibrium project scale depends on the collective choice institution. We also consider the case in which the project progresses stochastically, and we illustrate that the main results continue hold.[4] Finally, we discuss the case in which the group comprises of more than two agents. We find that agents' preferences over the project scale are ordered by their level of efficiency. This can provide the basis for richer collective choice analysis in future work.

The remainder of the paper is organized as follows. We review the related literature in the following subsection. In Section I, we present the model. To lay the foundation for the collective choice analysis, in Section II, we characterize the MPE given a fixed project scale, as well as the agents' preferences over project scales. In Section III, we endogenize the project scale and examine the outcome under two collective choice institutions—dictatorship and unanimity. Section IV discusses extensions. In Section V, we conclude. All proofs are relegated to Appendix A, and we provide some supplemental results in Appendix B.

Related Literature.—Our model draws from the literature on the dynamic provision of public goods, including classic contributions by Levhari and Mirman (1980) and Fershtman and Nitzan (1991). Similar to our approach, Admati and Perry (1991), Marx and Matthews (2000), Compte and Jehiel (2004), Kessing (2007), Yildirim (2006), Georgiadis and Tang (2017), and Georgiadis (2017) consider the case of public good provision when the benefit is received predominantly upon completion. Bonatti and Rantakari (2016) consider collective choice in a public good game, where each agent exerts effort on an independent project, and the collective choice is made to adopt one of the projects at completion. Battaglini, Nunnari, and Palfrey (2014) study a public good provision game without a terminal date, in which each agent receives a flow benefit that depends on the stock of the public good, in contrast to our setting. We contribute to this literature by endogenizing the provision point of the public good, and studying how different collective choice institutions influence the project scale that is implemented in equilibrium.

This paper also joins a large political economy literature studying collective decision-making when the agents' preferences are heterogeneous, including the seminal work of Romer and Rosenthal (1979). More recently, this literature has turned its attention to the dynamics of collective decision making, including papers by Baron (1996); Dixit, Grossman, and Gul (2000); Battaglini and Coate (2008); Strulovici (2010); Diermeier and Fong (2011); Besley and Persson (2011); and Bowen, Chen, and Eraslan (2014). Other papers, for example, Lizzeri and Persico (2001), have looked at alternative collective choice institutions. To the best of our knowledge, this is the first paper to study collective decision-making in the context of a group of agents collaborating to complete a project.

The application to public projects without the ability to commit relates to a large number of articles studying international agreements. Several of these study environmental agreements (for example, Nordhaus 2015, Battaglini and Harstad

[4] The models with uncertainty and endogenous choice of project shares in the voluntary contribution game with heterogeneous agents is analytically intractable, so we examine them numerically. All other results are obtained analytically.

2016) and trade agreements (see Maggi 2014).[5] To our knowledge, this literature has not examined the dynamic selection of project scale (or goals) in these agreements with asymmetric agents or identified the source of control. Our theory sheds light on the dominance of large countries in many trade and environmental agreements in spite of unanimity being the formal institution.

Finally, our interest in effective control relates to a literature studying the source of authority and power, including the influential work of Aghion and Tirole (1997) and more recent contributions by Callander (2008), Levy (2014), Callander and Harstad (2015), Hirsch and Shotts (2015), and Akerlof (2012). Unlike this paper, these authors focus on the role of information in determining real authority. Bester and Krähmer (2008) and Georgiadis, Lippman, and Tang (2014) consider a principal-agent setting in which the principal has formal control to choose which project to implement, but that choice is restricted by the agent's effort incentives; or she can delegate the project choice decision to the agent. Acemoglu and Robinson (2008) consider the distinction between de jure and de facto political power, which are the analogs of formal and effective control, but the source of the latter is attributed to various forces outside the model. In contrast, we are able to endogenously attribute the source of effective control under different collective choice institutions to the agents' effort costs and stake in our simpler setting of a public project.

I. Model

We present a stylized model of two heterogeneous agents $\{i \in 1,2\}$ deciding the scale of a public project $Q \geq 0$. Time is continuous and indexed by $t \in [0,\infty)$. A project of scale Q requires voluntary effort from the agents over time to be completed. Let $a_{it} \geq 0$ be agent i's instantaneous effort level at time t, which induces flow cost $c_i(a_{it}) = \gamma_i a_{it}^2/2$ for some $\gamma_i > 0$. Agents are risk-neutral and discount time at common rate $r > 0$.

We denote the cumulative effort (or progress on the project) up to time t by q_t, which we call the project state. The project starts at initial state $q_0 = 0$ and progresses according to

$$dq_t = (a_{1t} + a_{2t})dt.$$

It is completed at the moment that the state reaches the chosen scale Q.[6] The project yields no payoff while it is in progress, but upon completion, it yields a payoff $\alpha_i Q$ to agent i, where $\alpha_i \in \mathbb{R}_+$ is agent i's stake in the project.[7] Agent i's project stake

[5] Bagwell and Staiger (2002) discusses the economics of trade agreements in depth. Others look at various aspects of specific trade agreements, such as flexibility or forbearance in a nonbinding agreement (see, for example, Beshkar, Bond, and Rho 2015; Bowen 2013; and Beshkar and Bond 2017).

[6] Note that the project scale Q is simply the aggregate effort exerted on the project. This can be interpreted as scale in the case of a quantitative variable, or scope in the case of a qualitative variable. We maintain the interpretation of scale throughout the paper. We make the simplifying assumption that the project state progresses deterministically. See Section IVC for an extension in which the state progresses stochastically.

[7] Without loss of generality, one can assume that upon completion, the project yields a stochastic payoff to agent i that has expected value $\alpha_i Q$.

therefore captures all of the expected benefit from the project.[8] All information is common knowledge.

Given an arbitrary set of effort paths $\{a_{1s}, a_{2s}\}_{s \geq t}$ and project scale Q, agent i's discounted payoff at time t satisfies

$$J_{it} = e^{-r(\tau - t)} \alpha_i Q - \int_t^{\tau} e^{-r(s-t)} \frac{\gamma_i}{2} a_{is}^2 \, ds, \tag{1}$$

where τ denotes the equilibrium completion time of the project (and $\tau = \infty$ if the project is never completed).

By convention, we assume that the agents are ordered such that $\gamma_1/\alpha_1 \leq \gamma_2/\alpha_2$. Intuitively, this means that agent 1 is *relatively more efficient* than agent 2, in that his marginal cost of effort relative to his stake in the project is smaller than that of agent 2. In sequel, we say that agent 1 is *efficient* and agent 2 is *inefficient*.

The project scale Q is decided by collective choice at any time $t \geq 0$, i.e., at the start of the project, or after some progress has been made. The set of decisions available to each agent depends on the collective choice institution, which is either dictatorship or unanimity. To lay the foundations for the collective choice analysis, we shall assume that the project scale Q is fixed in the next section. When we consider the collective choice problem in Section III, we will enrich the model by introducing additional notation as necessary.

II. Analysis with Fixed Project Scale Q

In this section, we lay the foundations for the collective choice analysis. We begin by considering the case in which the project scale Q is specified exogenously at the outset of the game, and characterize the stationary Markov Perfect equilibrium (MPE) of this game.[9] We then derive each agent's preferences over the project scale Q given the MPE payoffs induced by a choice of Q. Finally, we characterize the social planner's benchmark. In Section III, we consider the case in which the agents decide the project scale via collective choice.

A. *Markov Perfect Equilibrium with Exogenous Project Scale*

In a MPE, at every moment, each agent chooses his effort level as a function of the current project state q to maximize his discounted payoff while anticipating the other agents' effort choices. Let us denote each agent i's discounted continuation payoff and effort level when the project state is q by $J_i(q)$ and $a_i(q)$, respectively. Using standard arguments (for example, Kamien and Schwartz 2012) and

[8] The sum $\alpha_1 + \alpha_2$ reflects the *publicness* of the project. If $\alpha_1 + \alpha_2 = 1$, then the project stake can be interpreted as the project share. We assume that these stakes are exogenously fixed. In Section IVA, we extend our model to allow the agents to use transfers to reallocate shares.

[9] We focus on MPE, as is standard in the literature. These equilibria require minimal information and coordination between the agents, and appear natural in our model. Moreover, simulations suggest that the MPE is robust to uncertainty in the progress of the project. For completeness, we discuss non-Markov equilibria in Section VB. In particular, we illustrate that under certain conditions, a Public Perfect equilibrium may exist, in which the agents exert the first-best effort along the equilibrium path. However, this equilibrium is sensitive to the assumption that the project progresses deterministically, and its analysis is not is not as tractable as the MPE.

assuming that $\{J_1(\cdot), J_2(\cdot)\}$ are continuously differentiable, it follows that agent i best-responds to $a_j(\cdot)$ by solving the Hamilton-Jacobi-Bellman (hereafter HJB) equation

$$(2) \qquad rJ_i(q) = \max_{\hat{a}_i \geq 0}\left\{-\frac{\gamma_i}{2}\hat{a}_i^2 + \left(\hat{a}_i + a_j(q)\right)J_i'(q)\right\},$$

subject to the boundary condition

$$(3) \qquad J_i(Q) = \alpha_i Q.$$

We refer to MPE, where $\{J_1(\cdot), J_2(\cdot)\}$ are continuously differentiable, as *well-behaved*.

The right side of (2) is maximized when $\hat{a}_i = \max\{0, J_i'(q)/\gamma_i\}$. Intuitively, at every moment, each agent either does not put in any effort, or he chooses his effort level such that the marginal cost of effort is equal to the marginal benefit associated with bringing the project closer to completion. In any equilibrium we have $J_i'(q) \geq 0$ for all i and q, that is, each agent is better off the closer the project is to completion.[10] Naturally, in a MPE, $a_1(\cdot)$ and $a_2(\cdot)$ must be a best-response to each other. By substituting each agent's first-order condition into (2), it follows that in a MPE, each agent i's discounted payoff function satisfies

$$(4) \qquad rJ_i(q) = \frac{\left[J_i'(q)\right]^2}{2\gamma_i} + \frac{1}{\gamma_j}J_i'(q)\,J_j'(q),$$

subject to the boundary condition (3), where j denotes the agent other than i. By noting that each agent's problem is concave, and thus the first-order condition is necessary and sufficient for a maximum, it follows that every well-defined MPE is characterized by the system of ordinary differential equations (ODEs) defined by (4) subject to (3).[11] The following Proposition characterizes the MPE.

PROPOSITION 1: *For any project scale Q, there exists a unique well-behaved MPE. Moreover, for any project scale Q, exactly one of two cases can occur.*

(*i*) *The MPE is project-completing: both agents exert effort at all states and the project is completed. Then, $J_i(q) > 0$, $J_i'(q) > 0$, and $a_i'(q) > 0$ for all i and $q \geq 0$.*

(*ii*) *The MPE is not project-completing: agents do not ever exert any effort, and the project is not completed.*

[10] See the proof of Proposition 1.

[11] This system of ODEs can be normalized by letting $\tilde{J}_i(q) = J_i(q)/\gamma_i$. This becomes strategically equivalent to a game in which $\gamma_1 = \gamma_2 = 1$, and agent i receives $\frac{\alpha_i}{\gamma_i}Q$ upon completion of the project.

If Q is sufficiently small, then case (1) *applies, while, otherwise, case* (2) *applies.*

All proofs are provided in Appendix A.

Proposition 1 characterizes the unique MPE for any given project scale Q. In any project-completing MPE, payoffs and efforts are strictly positive, and each agent increases his effort as the project progresses toward completion, i.e., $a_i'(q) > 0$ for all i and q. Because the agents discount time and they are rewarded only upon completion, their incentives are stronger the closer the project is to completion.

If the agents are symmetric (i.e., if $\gamma_1/\alpha_1 = \gamma_2/\alpha_2$), which is the case studied by Kessing (2007) (with exogenous project scale), then in the unique project-completing MPE, each agent i's discounted payoff and effort function can be characterized analytically as follows:

$$J_i(q) = \frac{r\gamma_i (q - C)^2}{6} \quad \text{and} \quad a_i(q) = \frac{r(q - C)}{3}, \tag{5}$$

where $C = Q - \sqrt{\frac{6\alpha_i Q}{r\gamma_i}}$. A project-completing MPE exists if $C < 0$. While the solution to the system of ODEs given by (4) subject to (3) can be found with relative ease in the case of symmetric agents, no closed-form solution can be obtained for the case of asymmetric agents. Nonetheless, we are able to derive properties of the solution, which will be useful for understanding the intuition behind the results in Section B. The following proposition compares the equilibrium effort levels and payoffs of the two agents.

PROPOSITION 2: *Suppose that $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$. In any project-completing MPE:*

(*i*) *Agent* 1 *exerts higher effort than agent* 2 *in every state, and agent* 1*'s effort increases at a greater rate than agent* 2*'s. That is, $a_1(q) \geq a_2(q)$ and $a_1'(q) \geq a_2'(q)$ for all $q \geq 0$.*

(*ii*) *Agent* 1 *obtains a lower discounted payoff normalized by project stake than agent* 2*. That is, $J_1(q)/\alpha_1 \leq J_2(q)/\alpha_2$ for all $q \geq 0$.*

Suppose instead that $\gamma_1/\alpha_1 = \gamma_2/\alpha_2$. In any project-completing MPE, $a_1(q) = a_2(q)$ and $J_1(q)/\alpha_1 = J_2(q)/\alpha_2$ for all $q \geq 0$.

The intuition behind this result is as follows. First, because each agent's marginal cost of effort is linear in his effort level, agent i's effort incentives are proportional to his marginal benefit of bringing the project closer to completion. This marginal benefit is the marginal increase of his normalized gross payoff $e^{-r(\tau-t)}\frac{\alpha_i}{\gamma_i}Q$ due to a marginal decrease of the time to completion, $\tau - t$. Note that this marginal benefit is always larger for the efficient agent (i.e., agent 1). As a result, the efficient agent always exerts higher effort than the inefficient agent. Then, as the project progresses, marginal benefits increase for both agents, but it increases faster for the efficient agent. As a result, both agents raise their effort level over time, but the efficient agent raises his effort at a faster rate than the inefficient agent.

What is perhaps surprising is that the efficient agent obtains a lower discounted payoff (normalized by his stake) than the other agent. This is because the efficient agent not only works harder than the other agent, but he also incurs a higher total discounted cost of effort (normalized by his stake). To examine the robustness of this result, in Appendix B, we consider a larger class of effort cost functions, and we show that this result holds as long as each agent's effort cost function is weakly log-concave in the effort level.

B. *Preferences over Project Scale*

In this section, we characterize each agent's optimal project scale without institutional restrictions. That is, we determine the project scale Q that maximizes each agent's discounted payoff given the current state q and assuming that both agents follow the MPE characterized in Proposition 1 for that particular Q. Note that the agents will choose a project scale such that the project is completed in equilibrium.

Agents Working Jointly.—To make the dependence on the project scale explicit, we let $J_i(q;Q)$ denote agent i's payoff at project state q when the project scale is Q. Let $Q_i(q)$ denote agent i's ideal project scale when the state of the project is q. That is,

$$Q_i(q) = \arg\max_{Q \geq q} \left\{ J_i(q;Q) \right\}. \tag{6}$$

For each agent i there exists a unique state q, denoted by $\bar{Q}_i$, such that he is indifferent between terminating the project immediately or an instant later, and $\bar{Q}_2 \geq \bar{Q}_1$.[12] Throughout the remainder of this paper, we shall assume that the parameters of the problem are such that $Q \mapsto J_i(q;Q)$ is strictly concave on $\left[q, \bar{Q}_2\right]$.[13] Observe that the strict concavity assumption implies that $J_i(0,Q) > 0$ for all i and $Q \in \left(0, \bar{Q}_2\right)$, so the corresponding MPE is project-completing.

The following proposition establishes properties of each agent's ideal project scale.

PROPOSITION 3: *Consider agent i's optimal project scale*, $Q_i(q)$, *defined in* (6):

(*i*) *If the agents are symmetric* (*i.e.*, $\gamma_1/\alpha_1 = \gamma_2/\alpha_2$), *then for all states up to* $\frac{3\alpha_i}{2\gamma_i r}$, *their ideal project scales are equal and given by* $Q_1(q) = Q_2(q) = \frac{3\alpha_i}{2\gamma_i r}$.

[12] The value of $\bar{Q}_i$ is provided in Lemma 7 in the proof of Proposition 3.

[13] This condition is satisfied in the symmetric case $\gamma_1/\alpha_1 = \gamma_2/\alpha_2$ (see Georgiadis, Lippman, and Tang 2014 for details) and, by a continuity argument, it is also satisfied for neighboring, asymmetric parameter values. While we do not make a formal claim regarding the set of parameters values for which the condition is satisfied, numerical simulations suggest that this condition holds generically. We provide examples of numerical simulations with various parameter values in Section A of Appendix B.

If $q > \frac{3\alpha_i}{2\gamma_i r}$, *then* $Q_i(q) = q$; *i.e., both agents prefer to complete the project immediately.*

(ii) *If the agents are asymmetric* $(i.e., \gamma_1/\alpha_1 < \gamma_2/\alpha_2)$, *then:*

(a) *The efficient agent prefers a strictly smaller project scale than the inefficient agent at all states up to* $\bar{Q}_2$, *i.e.,* $Q_1(q) < Q_2(q)$ *for all* $q < \bar{Q}_2$.

(b) *The efficient agent's ideal scale is strictly decreasing in the project state up to* $\bar{Q}_1$, *while the inefficient agent's scale is strictly increasing for all* q, *i.e.,* $Q_1'(q) < 0$ *for all* $q < \bar{Q}_1$ *and* $Q_2'(q) > 0$ *for all* q.

(c) *Agent i's ideal is to complete the project immediately at all states greater than* $\bar{Q}_i$, *i.e.,* $Q_i(q) = q$ *for all* $q \geq \bar{Q}_i$.

Proposition 3(i) asserts that when the agents are symmetric, they have identical preferences over project scale, and these preferences are time-consistent.

Proposition 3(ii) characterizes each agent's ideal project scale when the agents are asymmetric, and is illustrated in Figure 2. Proposition 3(ii), part (a) asserts that the more efficient agent always prefers a strictly smaller project scale than the less efficient agent for $q < \bar{Q}_2$.[14] Note that each agent trades off the bigger gross payoff from a project with a larger scale and the cost associated with having to exert more effort and wait longer until the project is completed. Moreover, agent 1 not only always works harder than agent 2, but at every moment, his discounted total cost remaining to complete the project normalized by his stake (along the equilibrium path) is larger than that of agent 2. Therefore, it is intuitive that agent 1 prefers a smaller project scale than agent 2.

Proposition 3(ii), part (b) shows that both agents are time-inconsistent with respect to their preferred project scale: as the project progresses, agent 1's optimal project scale becomes smaller, whereas agent 2 would like to choose an ever larger project scale. To see the intuition behind this result, recall that $a_1'(q) \geq a_2'(q) > 0$ for all q; that is, both agents increase their effort with progress, but the rate of increase is greater for agent 1 than it is for agent 2. This implies that for a given project scale, the closer the project is to completion, the larger is the share of the remaining effort carried out by agent 1, so his optimal project scale decreases. The converse holds for agent 2, and as a result, his preferred project scale grows as the project progresses.

Recall that $\bar{Q}_i$ is the project scale such that agent i is indifferent between stopping immediately (when $q = \bar{Q}_i$) and stopping at a marginally larger scale. This is the value of the state at which $Q_i(q)$ hits the 45° line. Proposition 3(ii), part (c) shows that at every state $q \geq \bar{Q}_i$, agent i prefers to stop immediately.

[14] The agents' ideal project scales are equal for $q \geq \bar{Q}_2$ by Proposition 3(ii), part (c).

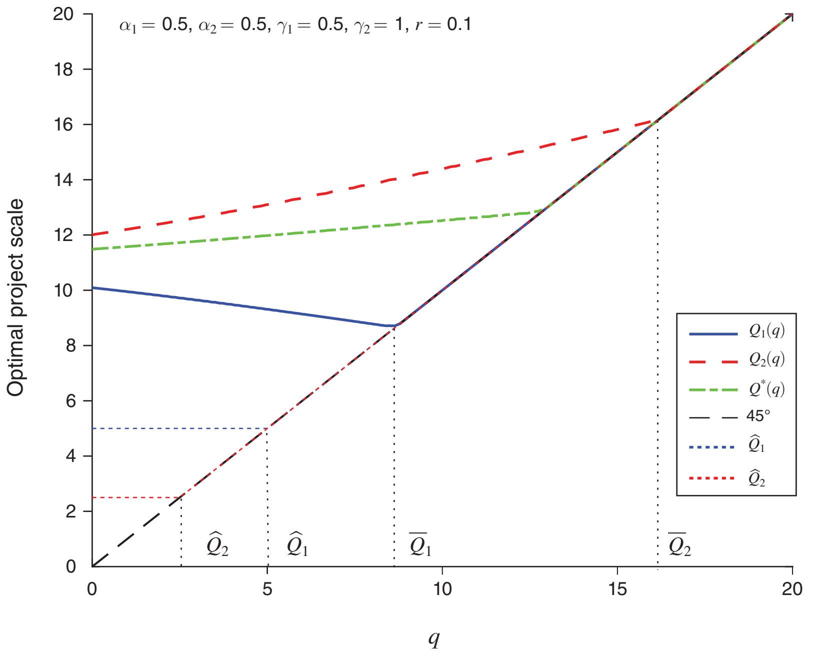

Figure 2. Agents' and Social Planner's Ideal Project Scale

Agents Working Independently.—This section characterizes each agent's optimal project scale when he works alone. We use this to characterize the equilibrium with endogenous project scale in Section IV. Let $\hat{J}_i(q;Q)$ denote agent i's discounted payoff function when he works alone, the project scale is Q, and he receives $\alpha_i Q$ upon completion.[15] We define agent i's optimal project scale as

$$\hat{Q}_i(q) = \arg\max_{Q \geq q} \left\{ \hat{J}_i(q;Q) \right\}.$$

The following lemma characterizes $\hat{Q}_i(q)$.

LEMMA 1: *Suppose that agent i works alone and he receives $\alpha_i Q$ upon completion of a project with scale Q. Then his optimal project scale satisfies*

$$\hat{Q}_i(q) = \frac{\alpha_i}{2r\gamma_i},$$

for all $q \leq \frac{\alpha_i}{2r\gamma_i}$, *and otherwise,* $\hat{Q}_i(q) = q$. *Moreover, for all q,*

$$\hat{Q}_2(q) \leq \hat{Q}_1(q) \leq Q_1(q) \leq Q_2(q).$$

[15] The value of $\hat{J}_i(q;Q)$ is given in the proof of Lemma 1 in the Appendix.

The first part of the lemma is a direct consequence of Bellman's Principle of Optimality: for single-agent decision problems, optimal policies are time consistent. Thus, if an agent works alone, then his preferences over project scales are time consistent (as long as he does not want to stop immediately). As such, we write $\hat{Q}_i = \alpha_i/(2r\gamma_i)$.

Intuitively, when the agent works alone, he bears the entire cost to complete the project, in contrast to the case in which the two agents work jointly. The second part of this lemma rank-orders the agents' ideal project scales. If an agent works in isolation, then he cannot rely on the other to carry out any part of the project, and therefore the less efficient agent prefers a smaller project scale than the more efficient one. Last, it is intuitive that the more efficient agent's ideal project scale is larger when he works with the other agent relative to when he works alone.

C. *Social Optimum*

To conclude this section, we consider a social planner choosing the project scale that maximizes the sum of the agents' discounted payoffs, conditional on the agents choosing effort strategically. For this analysis, we assume that the social planner cannot coerce the agents to exert effort, but she can dictate the state at which the project is completed.[16] Let

$$Q^*(q) = \arg\max_{Q \geq q} \left\{ J_1(q;Q) + J_2(q;Q) \right\}$$

denote the project scale that maximizes the agents' total discounted payoff.

LEMMA 2: *The project scale that maximizes the agents' total discounted payoff satisfies* $Q^*(q) \in \left(Q_1(q), Q_2(q)\right)$.

Lemma 2 shows that the social planner's optimal project scale $Q^*(q)$ lies between the agents' optimal project scales for every state of the project. This is intuitive, since she maximizes the sum of the agents' payoffs. Note that in general, $Q^*(q)$ is dependent on q; i.e., the social planner's optimal project scale is also time-inconsistent. We illustrate Proposition 3, and Lemmas 1 and 2 in Figure 2.

III. Endogenous Project Scale

In this section, we allow agents to choose the project scale via a collective choice institution. The project scale in this section is thus *endogenous*, in contrast to the analysis in Section II. In Section IVA and IVB, we characterize the MPE under dictatorship and unanimity, respectively, while in Section IVC we discuss the implications for effective control and welfare. Finally, in Section IVD we consider an

[16] This implies that the social planner is unable to completely overcome the free-rider problem. We consider the benchmark in which the social planner chooses both the agents' effort levels, and the project scale in Appendix B, part C. However, as it is unlikely that a social planner can coerce agents to exert a specific amount of effort, we use the result in the following lemma as the appropriate benchmark.

equilibrium refinement by imposing a restriction on the agents' off-path strategies. To maintain tractability, we restrict attention to equilibria in pure strategies. Throughout most of this section, we focus on equilibria on the Pareto frontier (i.e., equilibria whose outcomes are such that in no other equilibrium outcome can a party get a strictly larger ex ante payoff without a reduction of the other party's payoff). To avoid ambiguity, we write Pareto-efficient MPE when we refer to an MPE on the Pareto frontier.

A. *Dictatorship*

In this section, one of the two agents, denoted agent i, has dictatorship rights. The other agent, agent j, can contribute to the project, but has no formal control to end it. We consider that the dictator can either commit to the project scale or not.

We enrich the baseline model of Section I by defining a strategy for agent i (the dictator) to be a pair of maps $\{a_i(q,Q),\theta_i(q)\}$, where $q \in \mathbb{R}_+$, $Q \in \mathbb{R}_+ \cup \{-1\}$, and $Q = -1$ denotes the case in which the project scale has not yet been decided.[17] The function $a_i(q,Q)$ gives the dictator's effort level in state q when project scale Q has been decided, where $Q = -1$ represents the case in which a decision about the project scale is yet to be made. The value $\theta_i(q)$ gives the dictator's choice of project scale in state q, which applies under the assumption that no project scale has been committed to before state q. We set by convention $\theta_i(q) = -1$ if the dictator does not yet wish to commit to a project scale at state q, and $\theta_i(q) \geq q$ otherwise. Similarly, a strategy for agent $j \neq i$ is a map $a_j(q,Q)$ associated with his effort level in state q and the project scale decided by the dictator Q (or $Q = -1$ if a decision has not yet been made). Notice that each agent's strategy conditions only on the payoff-relevant variables q and Q, and hence they are Markov in the sense of Maskin and Tirole (2001).

Dictatorship with Commitment.—We first consider dictatorship with commitment. In this case, the dictator can announce a particular project scale at any time, and, following this announcement, the project scale is set once and for all. Therefore, at every state q before some project scale Q has been committed to, the dictator chooses $\theta_i(q) \in \{-1\} \cup [q,\infty)$. After a project scale has been set, it is definitive, so $\theta_i(\cdot)$ becomes obsolete.

After a project scale Q has been committed to, it is completed, and each agent obtains his reward as soon as the cumulative contributions reach Q. If the agents do not make sufficient contributions, then the project is never completed: both agents incur the cost of their effort, but neither collects any reward. The project cannot be completed before the dictator announces a project scale.

The following proposition characterizes the equilibrium. Under commitment, each agent finds it optimal to impose his ideal project scale. The time inconsistency of the dictator's preferences implies that the scale is always chosen at the beginning of the project.

[17] Before the project scale has been decided, in equilibrium, the agents correctly anticipate the project scale that will be implemented, and choose their effort levels optimally.

PROPOSITION 4: *Under dictatorship with commitment, there exists a unique MPE. In this equilibrium, agent i commits to his ex ante ideal project scale* $Q_i(0)$, *and the project is completed.*

Dictatorship without Commitment.—We now consider dictatorship without commitment. In this case, the dictator does not have the ability to credibly commit to a particular project scale, so at every instant he must decide whether to complete the project immediately or continue one more instant. Formally, at every state q while the project is in progress, the dictator chooses $\theta_i(q) \in \{-1, q\}$.[18] Note that in contrast to the commitment case, the strategies no longer condition on any agreed upon project scale Q, as no agreement on the project scale is reached before the project is completed. As soon as the project is completed, both agents collect their payoffs. The following Proposition characterizes the outcomes of Pareto-efficient MPE.

PROPOSITION 5: *Under dictatorship without commitment, if agent* 1 (*i.e., the efficient agent*) *is the dictator, then there exists a unique Pareto-efficient MPE, in which the project is completed at* $\bar{Q}_1$. *If agent* 2 *is the dictator, then a Pareto-efficient MPE in which the project is completed at* Q *exists if and only if* $Q \in [Q_1(0), Q_2(0)]$.

We provide a heuristic proof, which is useful for understanding the intuition for this result. First, recall from Lemma 1 that $\hat{Q}_2 < \hat{Q}_1 < \bar{Q}_1 < \bar{Q}_2$. Assume that agent i is dictator, fix some $Q \in [\hat{Q}_i, \bar{Q}_i]$ such that a project-completing MPE exists, and consider the following strategies. For all $q < Q$, both agents exert effort according to the MPE with fixed project scale Q characterized in Proposition 1, and exert no effort thereafter. Agent i stops the project immediately when $q \geq Q$. We shall argue that neither agent has an incentive to deviate, and hence these strategies constitute an MPE. Notice that the agents' efforts constitute an MPE for a fixed project scale Q, so they have no incentive to exert more or less effort at any $q < Q$. Because $Q \leq \bar{Q}_i$, agent i has no incentive to stop the project at any $q < Q$. Moreover, anticipating that he will contribute alone to the project at any $q \geq Q$, and noting that $Q \geq \hat{Q}_i$, agent i cannot benefit by completing the project at any state greater than Q. Finally, observe that both agents' ex ante payoffs increase (decrease) in the project scale for all $Q < Q_1(0)$ ($Q > Q_2(0)$). Therefore, if agent 1 is the dictator, then there exists a unique Pareto-efficient MPE in which $Q = \bar{Q}_1$. If agent 2 is the dictator, then any $Q \in [Q_1(0), Q_2(0)]$ can be a Pareto-efficient MPE outcome.[19]

[18] Any announcement of project scale other than the current state cannot be committed to. Thus, any announcement by agent i other than the current state is ignored by agent j in equilibrium. Thus, agent i's strategy collapses to an announcement to complete the project immediately, or keep working.

[19] Note that inefficient MPE typically exist. For example, the arguments used to prove Proposition 5 lead to the conclusion that, absent the restriction to Pareto-efficient MPE, if the efficient agent is dictator, then for every $Q \in [\hat{Q}_1, \bar{Q}_1]$ there exists an MPE in which the project is completed at Q. And conversely, for any MPE—Pareto efficient or not—the equilibrium scale Q is in the range $[\hat{Q}_1, \bar{Q}_1]$. If instead the inefficient agent is dictator, then for every $Q \in [\hat{Q}_2, \min\{\bar{Q}_2, \tilde{Q}\}]$ there exists an MPE in which the project is completed at Q, where $\tilde{Q}$ denotes the largest scale such that a project-completing MPE exists in a project with given exogenous scale $\tilde{Q}$. And conversely, for any MPE in which the inefficient agent is dictator, the equilibrium scale Q is in the range $[\hat{Q}_2, \min\{\bar{Q}_2, \tilde{Q}\}]$.

B. *Unanimity*

In this section, we consider the case in which both agents must agree on the project scale. One of the agents, whom we denote by i, is (exogenously) chosen to be the agenda setter, and he has the right to make proposals for the project scale. The other agent (agent j) must respond to the agenda setter's proposals by either accepting or rejecting each proposal.[20] If a proposal is rejected, then no decision is made about the project scale at that time. The project cannot be completed until a project scale has been agreed to.

A strategy for agent i (the agenda setter) is a pair of maps $\{a_i(q,Q),\theta_i(q)\}$ defined for $q \in \mathbb{R}_+$ and $Q \in \mathbb{R}_+ \cup \{-1\}$. Here, $a_i(q,Q)$ denotes the effort level of the agenda setter when the project state is q and the project scale agreed upon is Q; by convention, $a_i(q,-1)$ denotes his effort level when no agreement has been reached yet. The value of $\theta_i(q)$ is the project scale proposed by the agenda setter in project state q; by convention, $\theta_i(q) = -1$ if the agent does not make a proposal at state q. Similarly, the map $a_j(q,Q)$ denotes the effort level in state q when project scale Q has been agreed upon; by convention, $Q = -1$ if no agreement has been reached yet. The map $Y_j(q,Q)$ is the acceptance strategy of agent j if agent i proposes project scale Q at state q, where $Y_j(q,Q) = 1$ if agent j accepts, and $Y_j(q,Q) = 0$ if he rejects.

Unanimity with Commitment.—We first consider the case in which the agents can commit to a decision about the project scale. At any instant, the agenda setter can propose a project scale. Upon proposal, the other agent must decide to either accept or reject the offer. If he accepts, then the project scale agreed upon is set once and for all, and cannot be changed. From that instant onward, the agenda setter stops making proposals, so $\{\theta_i(\cdot),Y_j(\cdot)\}$ become obsolete. The agents may continue to work on the project, and the project is completed and the agents collect their payoffs if and only if the state reaches the agreed upon project scale. If agent j rejects the proposal, then no project scale is decided upon, and the agenda setter may continue to make further proposals.

The following proposition characterizes the set of Pareto-efficient MPE for the game in which both agents must agree to a particular project scale, and they can commit ex ante.

PROPOSITION 6: *Under unanimity with commitment, there exists a Pareto-efficient MPE in which the agents agree to complete the project at Q at the outset of the game if and only if $Q \in [Q_1(0),Q_2(0)]$.*

In other words, the equilibrium project scale lies between the agents' ideal project scales.

[20] The set of equilibrium project scales is independent of who is the agenda-setter.

Unanimity without Commitment.—Now suppose that the agenda setter cannot commit to a future project scale. Given the current state q, the agenda setter either proposes to complete the project immediately, or he does not make any proposal; i.e., $\theta_i(q) \in \{-1, q\}$. The following proposition shows that without commitment, unanimity generates the same set of Pareto-efficient equilibrium outcomes as the game when the inefficient agent is the dictator.

PROPOSITION 7: *Without commitment, under unanimity, the set of Pareto-efficient MPE outcomes are the same as when agent* 2 (*i.e., the inefficient agent*) *is the dictator. That is, a Pareto-efficient MPE in which the project is completed at Q exists if and only if* $Q \in [Q_1(0), Q_2(0)]$.

Recall from Proposition 3 that agent 2 always prefers a larger project scale than agent 1 (i.e., $Q_2(q) \geq Q_1(q)$ for all q). Therefore, at any state q such that agent 2 would like to complete the project immediately, agent 1 wants to do so as well, but the opposite is not true. Because both agents must agree to complete the project, effectively, it is agent 2 who has the decision rights over the project scale.

Note that there is another institution wherein at every moment, the agents must both agree to continue the project. By a symmetric argument, the set of Pareto-efficient MPE outcomes are the same as when agent 1 is the dictator; i.e., there exists a unique Pareto-efficient MPE in which $Q = \bar{Q}_1$ is implemented. However, to remain consistent with the previous cases analyzed, we focus on the institution in which both agents must agree to stop the project.[21]

C. *Implications*

In this section, we elaborate on two implications of our results. First, we seek to understand how closely the equilibrium project scale is aligned with each agent's preferences. Second, we examine the welfare implications associated with each collective choice institution.

Control.—While institutions can influence the extent of an agent's control, the scale that is eventually implemented remains an equilibrium outcome. The agent with decision power has to account for the other agent's actions, and the equilibrium scale may be better aligned with the preferences of the agent who does not have decision power.

We define formal control as the right to determine the state at which the project ends and rewards are collected. It is determined by the collective choice institution. Under dictatorship, the dictator has formal control, whereas in the unanimity setting, the agents share formal control. In contrast, we say that the agent whose preferences are implemented in equilibrium has effective control over the project scale.

[21] The protocol in which participants must "agree to stop" the project is consistent with many international agreements that must have all participants' consent to be implemented. Formally, GATT agreements require the ratification of member countries to enter into force, and thus negotiations do not end until each member ratifies.

DEFINITION 1: *Suppose the state is q, and a project scale has not been decided at any $\tilde{q} < q$. Agent i has effective control if either:*

(*i*) *The project scale Q is decided at q and $Q = Q_i(q)$; or,*

(*ii*) *The project scale Q is not decided at q and $Q_i(q) > q$.*

Note that this definition applies only until a project scale is committed to. After the project scale has been decided, the game becomes one of dynamic contributions with a fixed, exogenous scale, and the concept of effective control is no longer relevant. For example, consider a developed country assisting a developing country to construct a large infrastructure project. The project, being carried out on the developing country's soil, is subject to its laws and jurisdiction. The developing country thus has formal control over the project and can specify the termination state, but it is not clear that the developing country does so at a state that is *its* ideal scale, due to the incentives of the donor developed country.[22]

With commitment, the project scale is decided at the beginning of the project, and whichever agent has formal control (i.e., dictatorship rights), also has effective control. Under unanimity, recall that any $Q \in \left[Q_1(0), Q_2(0)\right]$ is part of a Pareto-efficient equilibrium, so depending on which scale is implemented, either agent can have effective control, or neither.

Without commitment, because the agents' preferences over project scale are time-inconsistent, effective control has a temporal component, and therefore richer implications. The following remark elaborates.

REMARK 1: *Consider the case without commitment. For all $q < \bar{Q}_1$, the agents share effective control. For $q \geq \bar{Q}_1$:*

(*i*) *If agent 1 is dictator, then he has effective control at the completion state $q = \bar{Q}_1$.*

(*ii*) *If agent 2 is dictator (or under unanimity) and $Q \in \left[Q_1(0), Q_2(0)\right]$ is implemented, then he has effective control for all $q \in \left[\bar{Q}_1, Q\right)$. However, agent 1 has effective control at the completion state Q.*

Note that the domain in which the agents have conflicting preferences is $\left[\bar{Q}_1, \bar{Q}_2\right]$. If the efficient agent is dictator, then he completes the project at his ideal project scale, so he has effective control at the completion state $\bar{Q}_1$. In contrast, if the

[22] Our notions of effective and formal control are different from the real and formal authority described in Aghion and Tirole (1997). As with the agents endowed with real authority of Aghion and Tirole, the agent endowed with effective control in our setting may end up deciding, indirectly, when to stop the project. However, for Aghion and Tirole, the key to real versus formal authority is the asymmetric information between the two agents: the agent with less information may decide to follow the agent with more information. In contrast, in our setting, there is no private information, and the key to effective versus formal control is that the agent who has formal control lacks the ability to decide directly on the effort level of the other agent, because this effort level is an equilibrium object. As a result, the optimal stopping decision of the agent who has formal control may end up being better aligned with the preferences of the other agent.

inefficient agent is the dictator (or under unanimity), the inefficient agent has effective control while the project is ongoing (since he prefers to continue, whereas the efficient agent would like to complete the project immediately), but his effective control eventually "runs out," and upon completion, it is the efficient agent who has effective control.

This mechanism is reflected in the Uruguay round of GATT negotiations. Near the end of the Uruguay round "[t]he frustration was [. . .] directed at the two principal participants in the world trading system, the United States and the [European Community]. The Uruguay Round had been launched at strong US initiative, with a far broader sweep of issues and country participation than any previous negotiation. But now, more than six years later, and after others had done their part, the two principals proved incapable of bridging the final gaps for a comprehensive agreement, ostensibly over relatively modest tariff reductions in a few sectors." (Preeg 1995). Thus, our results may help explain why it is often the case that international organizations formally governed by unanimity (such as the WTO) appear to be heavily influenced by large contributors. These large contributors are the more efficient agents, who contribute more to the project and hence prefer to conclude the project earlier than the less efficient agents.[23]

Welfare.—Finally, we discuss the welfare implications associated with each collective choice institution. In particular, we are interested in which institutions can maximize total welfare. The following remark summarizes.

REMARK 2: *With commitment, the social planner's ex ante ideal project scale can be implemented only with unanimity. Without commitment, the social planner's project scale can be implemented if the inefficient agent is dictator or with unanimity.*

The main takeaway is that from a welfare perspective, it may be desirable to give the weaker party (i.e., the inefficient agent) formal control, because the stronger party obtains effective control in equilibrium. If instead the efficient agent is conferred formal control, then because he does not internalize the positive externality associated with a larger project scale, total welfare will be lower. This provides a rationale for unanimity as the collective choice institution in international agreements, and it resonates with Galbraith (1952), who argues that when one party is strong and the other weak, it is preferable to give formal authority to the latter.

D. *An Equilibrium Refinement*

In some cases of our analysis, multiple (Pareto-efficient) MPE exist. This multiplicity owes to the threat an agent can pose on another by halting effort if the state of the project exceeds the equilibrium scale. Therefore, it is natural to ask if one can

[23] This disproportionate influence can be explained by appealing to "bargaining power." In this paper, we demonstrate one potential source of this bargaining power—the credible threat by more efficient agents to stop contributing to the project.

refine the set of equilibria by imposing constraints on the agents' strategies off the equilibrium path.

Suppose that each agent's Markovian effort strategy with respect to the project state must be left continuous and must satisfy $a_i(q_+) \geq \phi a_i(q)$ for some fixed $\phi \in [0,1]$; i.e., an agent's effort cannot jump downward by a fraction bigger than $1-\phi$ while the project is in progress. Intuitively, this restriction bounds the threat that an agent can pose on another by reducing effort if the latter does not complete the project at a particular state.[24]

To illustrate how such refinement can impact equilibrium outcomes, consider the case without commitment in which the inefficient agent (i.e., agent 2) is dictator. Let us look for an equilibrium in which the dictator completes the project at (some) $Q \leq \tilde{Q}$ (and, off equilibrium path, completes the project at all states greater than Q) where $\tilde{Q}$ denotes the largest scale such that a project-completing equilibrium exists, and the agents' effort strategies are as follows. At every $q \leq Q$, each agent's effort level $a_i(q)$ is as characterized in Proposition 1 given project scale Q. Then, for some arbitrarily small $\epsilon > 0$, if the dictator does not complete the project at $q_\tau = Q$, then on $(Q, Q+\epsilon]$ the agent drops effort to $\phi a_1(Q)$, on $(Q+\epsilon, Q+2\epsilon]$ the agent drops effort to $\phi^2 a_1(Q)$, and so on until the dictator terminates the project. Following an argument analogous to the proof of Proposition 5, to show that this is an equilibrium profile, it suffices to show that the dictator finds it optimal to complete the project at Q. Informally, it will be the case if

$$\alpha_2 Q \geq \max_{a_2 \geq 0}\left\{-\frac{\gamma_2}{2} a_2^2\, dt + (1 - r dt)\,\alpha_2\left[Q + \phi a_1(Q) dt + a_2\, dt\right]\right\};$$

i.e., if he is better off completing the project at Q instead of an instant later. Using equation (13) in Appendix A.3, it follows that the dictator will optimally complete the project at Q if

$$Q \geq \left[\frac{\phi}{2}\frac{\sqrt{\mu}+\sqrt{3\nu}}{\sqrt{6r}} + \frac{1}{2}\sqrt{\left(\phi\frac{\sqrt{\mu}+\sqrt{3\nu}}{\sqrt{6r}}\right)^2 + \frac{2\alpha_2}{r\gamma_2}}\,\right]^2 \equiv \underline{Q}_2(\phi),$$

where μ and ν are constants defined in Lemma 7. Conversely, the left continuity of the effort strategies together with the bounded discontinuities imply by the same argument that in any MPE that satisfies the refinement, $Q \geq \underline{Q}_2(\phi)$.

Thus, the scale Q is a Pareto-efficient MPE outcome if and only if

$$Q \in \left[\max\left\{Q_1(0), \underline{Q}_2(\phi)\right\}, Q_2(0)\right].$$

Note, first, that $\underline{Q}_2(0) = \hat{Q}_1 < Q_1(0)$; second, that $\underline{Q}_2(\cdot)$ is increasing, and; third, that $\underline{Q}_2(1) = \bar{Q}_2 > Q_2(0)$. Recall from Proposition 5 that the Pareto-efficient

[24] Note that the analysis in the previous sections is included in the case $\phi = 0$, and recall that in every equilibrium, each agent's effort $a_i(q)$ is monotonically increasing in q.

MPE scales span the range $\left[Q_1(0), Q_2(0)\right]$. Therefore, if ϕ is small enough, then the set of Pareto-efficient MPE coincides with that characterized in Proposition 5, and the set of equilibrium outcomes remains unchanged if the agents' efforts can only drop gradually. As ϕ increases, the set of MPE shrinks. There exists an interval of values of ϕ for which $\underline{Q}_2(\phi) \in \left[Q_2(0), \tilde{Q}\right]$. In this interval, there exists a unique Pareto-efficient MPE in which the project is completed at $\underline{Q}_2(\phi)$.[25] Finally, if ϕ is such that $\underline{Q}_2(\phi) > \tilde{Q}$, then no project-completing MPE exists. If instead the efficient agent is dictator, then a similar argument applies and $\bar{Q}_1$ continues to be the unique Pareto-efficient MPE scale, as in Proposition 5.

IV. Extensions

In this section, we extend our model in three directions. First, we allow the agents to use monetary transfers in exchange for implementing a particular project scale, or re-allocating the shares $\{\alpha_1, \alpha_2\}$. Second, we consider the possibility that the agents play non-Markov equilibria. Third, we consider the case in which the project progresses stochastically.

A. *Transfers*

So far we have assumed that each agent's project stake α_i is exogenous and transfers are not permitted. These are reasonable assumptions if agents are liquidity constrained. However, if transfers are available, then there are various ways to mitigate the inefficiencies associated with the collective choice problem. Our objective in this section is to shed light on how transfers can be useful for improving the efficiency properties of the collective choice institutions. We consider that agents choose effort levels strategically, so free-riding still occurs. We look at two types of transfers. First, we discuss the possibility that the agents can make lump-sum transfers at the beginning of the game to directly influence the project scale that is implemented. Second, we consider the case in which the agents can bargain over the allocation of shares in the project in exchange for transfers. In both cases, we assume that the agents commit to the project scale, transfers, and reallocation of shares at the outset of the game.

Transfers Contingent on Project Scale.—Let us consider the case in which one of the agents is dictator, and he can commit to a particular project scale.[26] Assume that agent 1 is dictator and makes a take-it-or-leave-it offer to agent 2, which specifies a transfer (from agent 2 to agent 1) in exchange for committing to some project scale Q. Then agent 1 solves the following problem:

$$\max_{Q \geq 0, T \in \mathbb{R}} J_1(0; Q) + T$$

[25] Recall from footnote 19 that without commitment, if agent 2 is dictator, then any $Q \in \left[\hat{Q}_2, \tilde{Q}\right]$ can be part of an MPE (but not necessarily on the Pareto frontier).

[26] The analysis for the other cases is similar, and yields the same insights.

subject to

$$J_2\big(0;Q\big) - T \;\geq\; J_2\big(0;Q_1\big(0\big)\big).$$

Put in words, agent 1 chooses the project scale and the corresponding transfer to maximize his ex ante discounted payoff, subject to agent 2 obtaining a payoff that is at least as great as his payoff if he were to reject agent 1's offer, in which case agent 1 would commit to the status quo project scale $Q_1(0)$, and no transfer would be made. Because transfers are unlimited, the constraint binds in the optimal solution, and the problem reduces to

$$\max_{Q\geq 0}\Big\{J_1\big(0;Q\big) + J_2\big(0;Q\big) - J_2\big(0;Q_1\big(0\big)\big)\Big\}.$$

Note that the optimal choice of project scale, $Q^*(0)$, maximizes total surplus. This is intuitive: because the agents are cash-unconstrained and they have complete information, bargaining is efficient. Moreover, it is straightforward to verify that the same result holds under any one-shot bargaining protocol irrespective of which agent has dictatorship rights, and for any initial status quo.[27]

If agent 2 faces a cash constraint, say $\bar{T}$, then agent 1 solves

$$\max_{Q\geq 0}\Big\{J_1\big(0;Q\big) + \min\Big\{\bar{T}, J_2\big(0;Q\big) - J_2\big(0;Q_1\big(0\big)\big)\Big\}\Big\}.$$

Because both total surplus and $J_2(0;Q)$ is increasing in Q for all $Q \leq Q^*(0)$, the agents will agree to the total surplus maximizing project scale $Q^*(0)$ if $\bar{T} \geq T^* \equiv J_2\big(0;Q^*(0)\big) - J_2\big(0;Q_1(0)\big)$. Otherwise, the equilibrium project scale solves $\bar{T} = J_2(0;Q) - J_2\big(0;Q_1(0)\big)$, and in exchange, agent 2 transfers $\bar{T}$ to agent 1. Note that because $J_2(0;Q)$ increases in Q (as long as $Q \leq Q_2(0)$), it follows that the equilibrium project scale is (weakly) increasing in $\bar{T}$.

Transfers Contingent on Reallocation of Shares.—We now consider $\alpha_1 + \alpha_2 = 1$, so the project stakes can be interpreted as project shares. We consider an extension of the model in which, at the outset, the agents start with an exogenous allocation of shares and then engage in a bargaining game in which shares can be reallocated in exchange for a transfer. After the reallocation of shares, the collective choice institution determines the choice of scale as given in Section III. Note that the allocation of shares influences the agents' incentives and consequently the equilibrium project scale. Because this is a game with complete information, the agents reallocate the shares so as to maximize the ex ante total discounted surplus, taking the collective choice institution as given. For the cases in which the Pareto-efficient

[27] One might also consider the case in which commitment is not possible. Because $Q_1(q) \leq Q_2(q)$ for all q, to influence the project scale at some state, agent 1 might offer a lump-sum transfer to agent 2 in exchange for completing the project immediately, whereas agent 2 might offer flow transfers to agent 1 to extend the scale of the project. This model is intractable, so we do not pursue it in the current paper.

MPE is not unique, we further refine the MPE to the one in which total surplus is maximal.[28]

Based on the analysis of Section III, there are three cases to consider:

(i) Agent i is dictator, for $i \in \{1,2\}$, and he has the ability to commit. As such, he commits to $Q = Q_i(0)$ at the outset, by Proposition 4.

(ii) Agent 1 is dictator, but he is unable to commit. In this case, the project is completed at state $\bar{Q}_1$, by Proposition 5.

(iii) Agent 2 is dictator, but he is unable to commit, or decisions must be made unanimously, with or without commitment. In these cases, the equilibrium project scale is $Q^*(0)$ by Propositions 5, 6, and 7, and the refinement to the total surplus-maximizing MPE.

We focus the analysis on the case in which agent 1 is dictator and can commit to a particular project scale at the outset; the other cases lead to similar insights. To begin, let $Q_1(0;\alpha)$ denote the (unique) equilibrium project scale when agent 1 is dictator and has the ability to commit, conditional on the shares $\{\alpha_1, 1-\alpha_1\}$. Assume that agent 1 makes a take-it-or-leave-it offer to agent 2, which specifies a transfer in exchange for reallocating the parties' shares from the status quo shares $\{\bar{\alpha}_1, 1-\bar{\alpha}_1\}$ to $\{\alpha_1, 1-\alpha_1\}$. Let $J_i(q;Q,\alpha)$ denote the continuation value for agent i when the state is q, the chosen project scale is Q, and the chosen share to agent 1 is α. Then, agent 1 solves the following problem:

$$\max_{\alpha_1 \in [0,1], T \in \mathbb{R}} J_1\big(0; Q_1(0;\alpha_1), \alpha_1\big) - T$$

subject to

$$J_2\big(0; Q_1(0;\alpha_1), \alpha_1\big) + T \geq J_2\big(0; Q_1(0;\bar{\alpha}_1), \bar{\alpha}_1\big).$$

Because transfers are unlimited and each agent's discounted payoff increases in his share, the incentive compatibility constraint binds in the optimal solution, and so the problem reduces to

$$\max_{\alpha_1 \in [0,1]} \Big\{J_1\big(0; Q_1(0;\alpha_1), \alpha_1\big) + J_2\big(0; Q_1(0;\alpha_1), \alpha_1\big) - J_2\big(0; Q_1(0;\bar{\alpha}_1), \bar{\alpha}_1\big)\Big\}.$$

The optimal choice of α_1 maximizes total surplus, conditional on the scale subsequently selected by the collective choice institution. In all other cases, and under any one-shot bargaining protocol, the agents will agree to re-allocate their shares to maximize total surplus.

[28] This is the case under dictatorship without commitment, and unanimity with or without commitment. Simulations indicate that the findings are robust to the equilibrium selection rule.

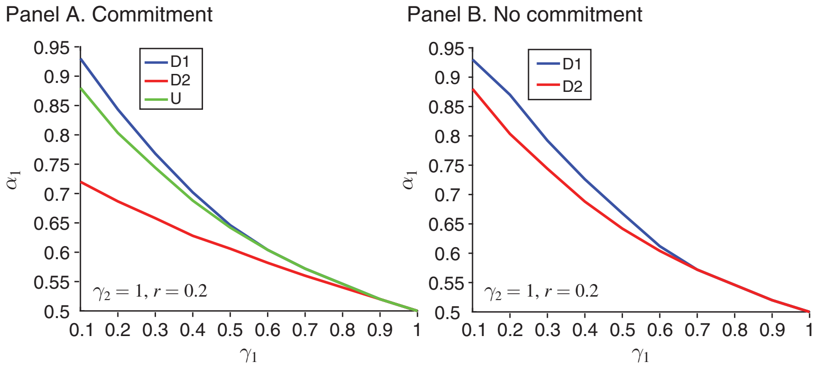

Figure 3. Agent 1's Optimal Project Share

The problem of optimally reallocating shares is analytically intractable, therefore we find the solution numerically. Figure 3 below illustrates the share allocated to agent 1, as a function of his effort cost. Note that without commitment, both the case of unanimity and the case in which agent 2 is dictator deliver the same result, and hence the result for unanimity is omitted.

In all cases, it is optimal for agent 1, who is more productive (i.e., $\gamma_1 < \gamma_2$), to possess the majority of the shares. Moreover, his optimal allocation decreases as his effort costs increase; i.e., as he becomes less productive. In other words, if one agent is substantially more productive than the other, then the former should possess the vast majority of the shares. Indeed, it is efficient to provide the stronger incentives to the more productive agent, and the smaller the disparity in productivity between the agents, the smaller should be the difference in the shares that they possess.

B. *Public Perfect Equilibria*

One may ask if other outcomes can be obtained when relaxing the restriction to Markovian strategies. In particular, one may ask if the first-best effort levels can be achieved. In this section, we answer the question positively. We construct a non-Markov, public perfect equilibrium (hereafter PPE) in which agents exert the first-best effort levels along the equilibrium path. This equilibrium is supported by the threat of reverting to the MPE characterized in Proposition 1 following any deviation, which is detected arbitrarily quickly since the project progresses deterministically.[29]

Let us consider the baseline model of exogenous scale of Section I, fix a scale Q, and suppose that at every instant, each agent chooses his effort to maximize the

[29] Such a PPE is characterized for the case of symmetric agents in Georgiadis, Lippman, and Tang (2014).

agents' joint payoffs. From the analysis of the first-best outcome in Appendix B2, it follows that each agent i's discounted payoff is given by

$$J_i^{eff}(q;Q) = \alpha_i\left[q - Q + \beta\sqrt{Q}\right]\left[\frac{\sqrt{Q}}{\beta} - z_i(Q-q)\right], \tag{7}$$

where[30]

$$\beta = \sqrt{\frac{2(\alpha_1+\alpha_2)(\gamma_1+\gamma_2)}{r\gamma_1\gamma_2}} \quad \text{and} \quad z_i = \frac{r\gamma_i}{2\alpha_i}\left(\frac{\gamma_{-i}}{\gamma_1+\gamma_2}\right)^2.$$

Incentive compatibility implies that a PPE in which each agent chooses the first-best effort level along the equilibrium path exists if $J_i^{eff}(q;Q) > J_i(q;Q)$ for all $i \in \{1,2\}$ and $q < Q$, where $J_i(q;Q)$ is characterized by Proposition 1. If the agents are symmetric, then this condition is satisfied as long as $Q < \beta^2$. (Otherwise, it is inefficient for any agent to exert any effort.) However, when the agents are asymmetric, this need not be the case. To see why, suppose that the agents have identical marginal costs of effort, but agent i has no stake in the project (i.e., $\alpha_i = 0$). In such a PPE, both agents must exert the same effort, but agent i receives none of the benefit, so he prefers to deviate.

Let us assume that such a PPE exists, and consider agent i's ideal project scale when the current state is q, assuming that both agents exert the first-best effort throughout the duration of the project. This agent solves

$$Q_i^{eff}(q) = \arg\max_{Q \geq q}\left\{J_i^{eff}(q;Q)\right\}.$$

It follows from (7) that if $q < \bar{Q}_i^{eff} := \left(1/\beta + \beta z_i\right)^{-2}$, then agent i's ideal project scale satisfies the first-order condition

$$1 + 2z_i(Q-q) = \frac{3Q-q}{2\sqrt{Q}}\left(\frac{1}{\beta} + \beta z_i\right).$$

Otherwise, agent i prefers that the project is completed immediately. Simulations indicate that both agents would like to extend the project as it progresses (i.e., $Q_i^{eff}(\cdot)$ is increasing), and agent 1 prefers a smaller project than agent 2 (i.e.,

[30] From Appendix B, part B, we have that each agent i's effort level satisfies $a_i^{eff}(q,Q) = \frac{r\gamma_{-i}}{\gamma_1+\gamma_2}\left[q - Q + \beta\sqrt{Q}\right]^+$. One obtains the desired expression by substituting the effort path into (1) and using that the completion time of a project of scale Q is equal to $\tau = \frac{1}{r}\ln\left(\frac{\beta}{\beta - \sqrt{Q}}\right)$.

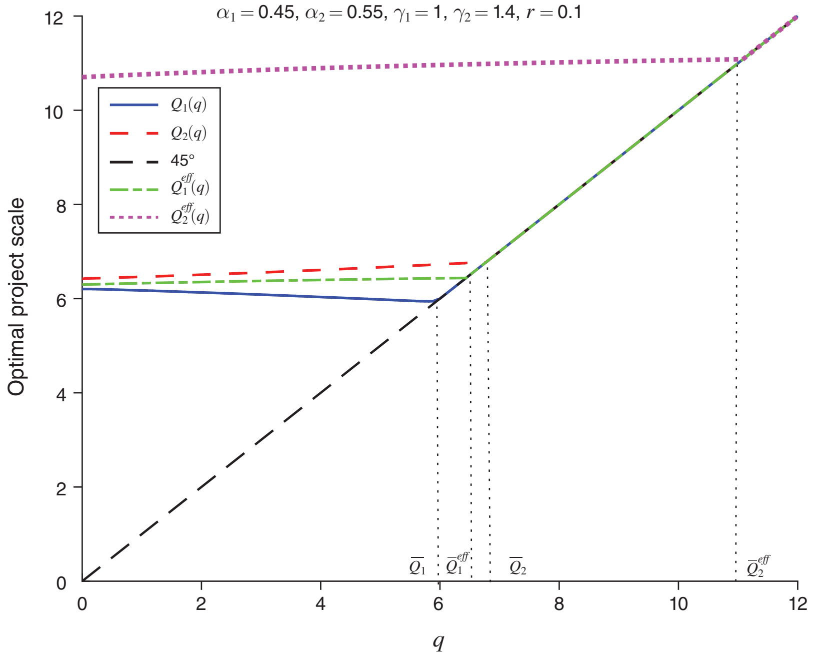

Figure 4. Agents' Ideal Project Scale in a PPE in Which They Exert First-Best Effort along the Equilibrium Path

$Q_1^{eff}(q) < Q_2^{eff}(q)$ for all q) if and only if $\gamma_1 \alpha_1 < \gamma_2 \alpha_2$.[31] Figure 4 illustrates an example.

Of course, an important question is whether such a PPE exists. Absent an analytical expression for $J_i(q;Q)$, we are unable to establish necessary or sufficient conditions such that this is the case. However, simulations indicate that if $Q_i^{eff}(q) \geq Q_i(q)$ for all i and q, where $Q_i(q)$ is characterized in Proposition 3, then for every $Q \leq \max\left\{\bar{Q}_1^{eff}, \bar{Q}_2^{eff}\right\}$, a PPE in which both agents exert the first-best effort along the equilibrium path exists.

Finally, one may ask how the collective choice institution influences the equilibrium project scale, when such a PPE exists and it is played in the settings of Section III. First, suppose that the agents can commit to a particular project scale ex ante. The set of equilibria is then similar to the case analyzed in Section III: if agent i is dictator, then he will choose his ideal project scale $Q_i^{eff}(0)$, whereas under unanimity, any project scale between $Q_1^{eff}(0)$ and $Q_2^{eff}(0)$ can be part of an equilibrium. Second, consider the case in which the agents are unable to commit. Then, irrespective of the collective choice institution, there exists a unique equilibrium in which the project is completed at $Q = \min\left\{\bar{Q}_1^{eff}, \bar{Q}_2^{eff}\right\}$ (in the class of PPE with first-best efforts along the equilibrium path). The reason is that at any $q > Q$, one

[31] Recall that in the MPE characterized in Proposition 1, agent 1 prefers a smaller project if and only if $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$.

of the agents will have an incentive to deviate, triggering a reversion to the MPE, in which case both agents will prefer to complete the project immediately. Noting that neither agent finds it optimal to complete the project at any $q < Q$, it follows that only Q can be part of an equilibrium.

C. *Collective Choice under Uncertainty*

To examine the robustness of our results, in this section, we consider the case in which the project progresses stochastically according to

$$dq_t = (a_{1t} + a_{2t})dt + \sigma dZ_t,$$

where Z_t is a standard Brownian motion, and $\sigma > 0$ captures the degree of uncertainty associated with the evolution of the project. We discuss the results for collective choice under this form of uncertainty.

As in the deterministic case studied in Section II, we begin by establishing the existence of an MPE with an exogenous project scale Q. In an MPE, each agent's discounted payoff function satisfies

$$rJ_i(q) = \frac{\left[J_i'(q)\right]^2}{2\gamma_i} + \frac{1}{\gamma_j}J_i'(q)\,J_j'(q) + \frac{\sigma^2}{2}J_i''(q)$$

subject to the boundary conditions $\lim_{q\to-\infty} J_i(q) = 0$ and $J_i(Q) = \alpha_i Q$ for each i. It follows from Georgiadis (2015) that for any project scale Q, an MPE exists and satisfies $J_i(q) > 0$, $J_i'(q) > 0$, $a_i(q) > 0$, and $a_i'(q) > 0$ for all i and q. This is the analog of Proposition 1 in the case of uncertainty.

We next establish the key properties of the MPE with exogenous project scale for asymmetric agents.

PROPOSITION 8: *Consider the model with uncertainty, and suppose that* $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$:

- (i) *Agent* 1 *exerts higher effort than agent* 2 *in every state, and agent* 1*'s effort increases at a greater rate than agent* 2*'s. That is,* $a_1(q) \geq a_2(q)$ *and* $a_i'(q) \geq a_2'(q)$ *for all* q.

- (ii) *Agent* 1 *obtains a lower discounted payoff normalized by project stake than agent* 2. *That is,* $J_1(q)/\alpha_1 \leq J_2(q)/\alpha_2$ *for all* q.

If instead $\gamma_1/\alpha_1 = \gamma_2/\alpha_2$, *then* $a_1(q) = a_2(q)$ *and* $J_1(q)/\alpha_1 = J_2(q)/\alpha_2$ *for all* q.

Proposition 8 is the analog of Proposition 2 in the case of uncertainty. It asserts that, under uncertainty, if agents are asymmetric, then the efficient agent exerts higher effort at every state of the project, and the efficient agent's effort increases at a higher rate than that of the inefficient agent. Furthermore, the efficient agent

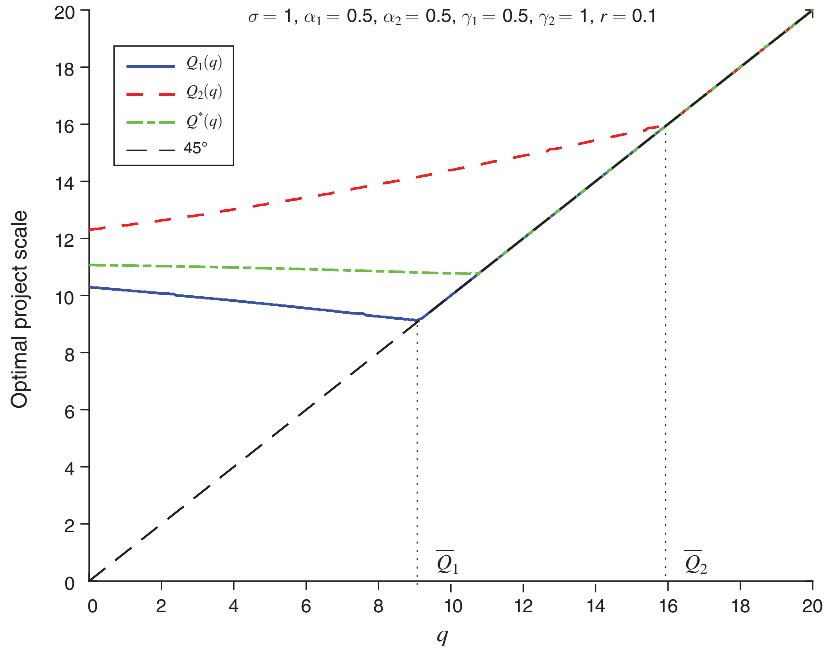

Figure 5. Agent i's Ideal Project Scale $Q_i(q)$ with Uncertainty

achieves a lower discounted payoff (normalized by the stake α_i) at every state of the project.

As for the agents' preferences over project scales, while we are unable to prove the counterpart of the results in Section B, numerical computations suggest that they continue to hold. This is not surprising given the result in Proposition 8 and because the intuition for the ordering and divergence of preferences is identical to that for the case without uncertainty. An example is illustrated in Figure 5.

As Figure 5 illustrates, the inefficient agent prefers a larger scale than the efficient agent at every state, and furthermore, his ideal project scale increases over the course of the project, whereas the efficient agent's ideal project scale decreases. Moreover, for each agent, there exists a threshold such that he prefers to complete the project immediately at every state larger than that threshold.

Notice that the results of Section III rely on the key properties of the preferences illustrated in Figure 5. Conditional on these preferences, all results of Propositions 4–7 will hold.

V. Concluding Remarks

We study a dynamic game in which two heterogeneous agents make costly contributions toward the completion of a public project. The scale (i.e., the size) of the project is endogenous, and it can be decided by a predetermined collective choice institution at any time.

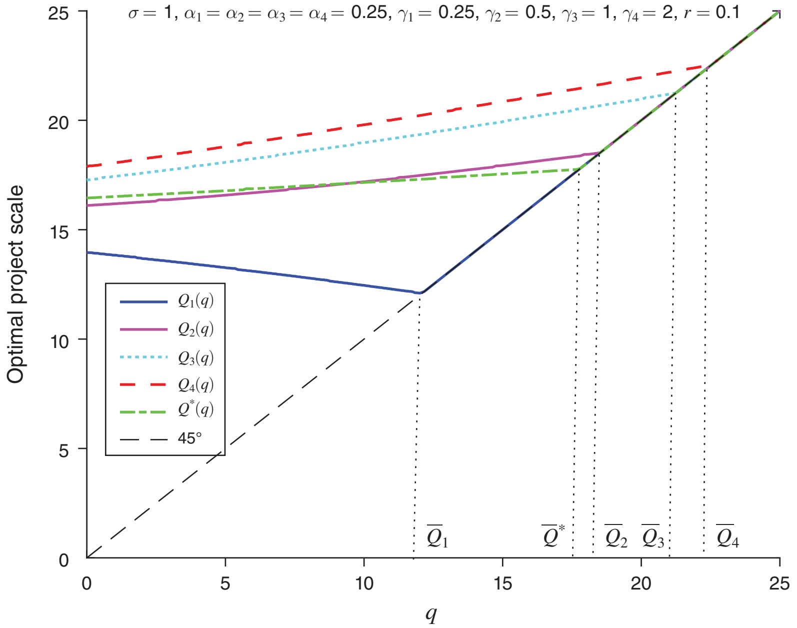

Figure 6. Agents' and Social Planner's Ideal Project Scales with Four Agents

Three main takeaways arise from our analysis. First, due to free-riding incentives, the agents' preferences with respect to their ideal project scale are time-inconsistent, and the more efficient agent prefers to implement a smaller project relative to the less efficient agent. Second, absent the ability to commit to a decision about the project scale, if the efficient agent has dictatorship rights, then he also has effective control of the project scale that is implemented. In contrast, if the inefficient agent is the dictator or under unanimity, then effective control has a temporal component: for a duration of time, the dictator has effective control, but it eventually runs out, and upon completion of the project, it is the efficient agent who has effective control. Third, from a welfare perspective, it may be desirable to assign formal control to the inefficient agent (via dictatorship rights or unanimity). These insights are applicable to international agreements, joint ventures, and other dynamic public projects with heterogeneous agents.

Our paper also leaves a number of open questions and directions for future research. First, one could allow for an arbitrary number of players. As an example, Figure 6 illustrates each agent's ideal project scale, as well as the socially optimal project scale for a group of 4 agents. Note that, similar to the two-player case, the agents' preferences over project scale are time inconsistent, and rank-ordered from most to least efficient. Second, our work suggests interesting institutional design questions. One could take a step back and model an institution as the specification of a default behavior of whether to continue or stop the project, together with a voting rule to decide to overrule the default behavior (noting that, with more than two agents, other voting rules are relevant, and unanimity and dictatorship become the extreme ends of a spectrum), and possibly constraints on the final project scale or completion time. One could also introduce an institution designer and an objective, such as maximization of welfare or of the total quantity of work. Third, a richer contracting space may be considered, in which, for example, the payoff of an agent is conditioned both on the project scale, and the completion time. Fourth, and finally, one may also depart from the complete information assumption and study, for example, the case of private costs of effort. In this case, the efficient agent may have an incentive to mimic the inefficient agent, thus contributing a smaller amount of effort. This may lead to a greater ideal project scale for the efficient agent, which will be welfare enhancing if the efficient agent is the dictator, but the welfare implications are not immediate because the distribution of work will likely be further away from that of the social planner.

Appendix A: Proofs

A. *Proof of Proposition* 1

We first establish two lemmas that will be used throughout the proof of this Proposition, as well as in the proof of Proposition 3. We consider the benchmark game of Section II, with exogenous project scale Q.

LEMMA 3: *Let* $\left(J_1, J_2\right)$ *be a pair of well-behaved value functions associated with an MPE. Then* $J_i\left(q\right) \in \left[0, \alpha_i Q\right]$ *and* $J_i'(q) \geq 0$ *for all* i *and* q.

PROOF:

Because each agent i can guarantee himself a payoff of 0 by not exerting any effort, in any equilibrium, it must be the case that $J_i(q) \geq 0$ for all q. Moreover, because he receives reward $\alpha_i Q$ upon completion of the project, he discounts time, and the cost of effort is nonnegative, his payoff satisfies $J_i(q) \leq \alpha_i Q$ for all q. Next, suppose that $J_i'(q^*) < 0$ for some i and q^*. Then agent i exerts 0 effort at q^*, and it must be the case that agent $j \neq i$ also exerts 0 effort, because otherwise it implies $J_i(q^*) < 0$, which cannot occur in equilibrium. Since both agents exert 0 effort at q^*, the project is never completed, and so $J_1(q^*) = J_2(q^*) = 0$. Therefore, for sufficiently small $\epsilon > 0$, we have $J_i(q^* + \epsilon) < 0$, which is a contradiction, implying $J_i'(q) \geq 0$ for all i and q. ∎

Observe that dividing both sides of equation (4) by γ_i the system of ODEs defined by (4) subject to (3) can be rewritten as

$$
(8) \qquad r\tilde{J}_i(q) = \frac{1}{2}\left[\tilde{J}_i'(q)\right]^2 + \tilde{J}_i'(q)\,\tilde{J}_j'(q)
$$

subject to $\tilde{J}_i(Q) = \frac{\alpha_i}{\gamma_i} Q$ for all $i \in \{1,2\}$ and $j \neq i$. The following lemma derives an explicit system of ODEs that is equivalent to the implicit form given in (8) of Section II.

LEMMA 4: *Let* (J_1, J_2) *be a pair of well-behaved value functions associated with an MPE, and let* $\tilde{J}_i(q) = J_i(q)/\gamma_i$. *If at state q the project is completing at* $Q > q$, *then the following explicit ODEs are satisfied on the range* (q, Q):[32]

$$\tilde{J}_1' = \sqrt{\frac{r}{6}}\sqrt{2\sqrt{\tilde{J}_1^2+\tilde{J}_2^2-\tilde{J}_1\tilde{J}_2}+\left(\tilde{J}_1+\tilde{J}_2\right)}+\sqrt{\frac{r}{2}}\sqrt{2\sqrt{\tilde{J}_1^2+\tilde{J}_2^2-\tilde{J}_1\tilde{J}_2}-\left(\tilde{J}_1+\tilde{J}_2\right)},$$

$$\tilde{J}_2' = \sqrt{\frac{r}{6}}\sqrt{2\sqrt{\tilde{J}_1^2+\tilde{J}_2^2-\tilde{J}_1\tilde{J}_2}+\left(\tilde{J}_1+\tilde{J}_2\right)}-\sqrt{\frac{r}{2}}\sqrt{2\sqrt{\tilde{J}_1^2+\tilde{J}_2^2-\tilde{J}_1\tilde{J}_2}-\left(\tilde{J}_1+\tilde{J}_2\right)}.$$

PROOF:

In an MPE in which the project is completing at state q, $\tilde{J}_1' + \tilde{J}_2' > 0$ on $[q, Q)$ as otherwise both agents put 0 effort at some intermediary state and the project is not completed.

Using (8), subtracting $\tilde{J}_2$ from $\tilde{J}_1$ and adding $\tilde{J}_2$ to $\tilde{J}_1$ yields

$$r\left(\tilde{J}_1-\tilde{J}_2\right)-\frac{1}{2}\left(\tilde{J}_1'+\tilde{J}_2'\right)\left(\tilde{J}_1'-\tilde{J}_2'\right) = 0, \quad \text{and}$$
$$r\left(\tilde{J}_1+\tilde{J}_2\right)-\frac{1}{2}\left(\tilde{J}_1'+\tilde{J}_2'\right)^2 = \tilde{J}_1'\tilde{J}_2',$$

respectively, where for notational simplicity we drop the argument q. Letting $G = \tilde{J}_1 + \tilde{J}_2$ and $F = \tilde{J}_1 - \tilde{J}_2$, these equations can be rewritten as

$$rF - \frac{1}{2}F'G' = 0,$$

$$rG - \frac{1}{2}(G')^2 = \frac{1}{4}(G')^2 - \frac{1}{4}(F')^2.$$

From the first equation we have $F' = 2rF/G'$ (and recall that we have assumed $G' > 0$), while the second equation, after plugging in the value of F', becomes

$$rG - \frac{1}{2}(G')^2 = \frac{1}{4}(G')^2 - r^2\frac{F^2}{(G')^2}.$$

This equation is quadratic in $(G')^2$, and noting by Lemma 3 that in any project-completing MPE we have $G' > 0$ on $[0, Q]$, the unique, strictly positive root is

$$(G')^2 = \frac{2r}{3}\left(\sqrt{G^2+3F^2}+G\right) \Rightarrow G' = \sqrt{\frac{2r}{3}}\sqrt{\sqrt{G^2+3F^2}+G}.$$

[32] We say that the *project is completing at state q* to indicate that if the state is q, then the project will be completed. In contrast, we say that the *project is completed at state Q* to indicate that state Q is the termination state.

Since $G' > 0$ on the interval of interest, we have

$$F' = \frac{2rF}{G'} = \frac{\sqrt{6r}F}{\sqrt{\sqrt{G^2+3F^2}+G}} \Rightarrow F' = \sqrt{2r}\sqrt{\sqrt{G^2+3F^2}-G}.$$

By using that $\tilde{J}_1 = \frac{1}{2}(G+F)$ and $\tilde{J}_2 = \frac{1}{2}(G-F)$, we obtain the desired expressions. ∎

Existence.—Fix some $Q > 0$, and let $\tilde{J}_i(q) = J_i(q)/\gamma_i$. As in Lemma 4, we note that the system of ODEs of Section II defined by (4) subject to (3) can be rewritten as

$$r\tilde{J}_i(q) = \frac{1}{2}\left[\tilde{J}_i'(q)\right]^2 + \tilde{J}_i'(q)\tilde{J}_j'(q) \tag{9}$$

subject to $\tilde{J}_i(Q) = \frac{\alpha_i}{\gamma_i}Q$ for all $i \in \{1,2\}$ and $j \neq i$. If a solution to this system of ODEs exists and $\tilde{J}_i'(q) \geq 0$ for all i and q, then it constitutes an MPE, and each agent i's effort level satisfies $a_i(q) = \tilde{J}_i'(q)$.

LEMMA 5: *For every $\epsilon \in \left(0, \min_i\left\{\frac{\alpha_i}{\gamma_i}Q\right\}\right)$, there exists some $q_\epsilon < Q$ such that there exists a unique solution $\left(\tilde{J}_1, \tilde{J}_2\right)$ to the system of ODEs on $\left[q_\epsilon, Q\right]$ that satisfies $\tilde{J}_i \geq \epsilon$ on that interval for all i.*

PROOF:

This proof follows the proof of Lemma 4 in Cvitanić and Georgiadis (2016) closely. It follows from Lemma 4 above that we can write (4) as

$$\tilde{J}_i'(q) = H_i\left(\tilde{J}_1(q), \tilde{J}_2(q)\right), \tag{10}$$

with

$$H_1(x,y) = \sqrt{\frac{r}{6}}\sqrt{2\sqrt{x^2+y^2-xy}+(x+y)} + \sqrt{\frac{r}{2}}\sqrt{2\sqrt{x^2+y^2-xy}-(x+y)},$$

$$H_2(x,y) = \sqrt{\frac{r}{6}}\sqrt{2\sqrt{x^2+y^2-xy}+(x+y)} - \sqrt{\frac{r}{2}}\sqrt{2\sqrt{x^2+y^2-xy}-(x+y)}.$$

For given $\epsilon > 0$, let

$$M_H = \max_i \max_{\epsilon \leq x_i \leq \frac{\alpha_i}{\gamma_i}Q} H_i(x_1, x_2).$$

Let us choose $q_\epsilon < Q$ sufficiently large such that, for all i,

$$\frac{\alpha_i}{\gamma_i} Q - (Q - q_\epsilon) M_H \geq \epsilon.$$

Then, define $\Delta q = \frac{Q - q_\epsilon}{N}$ and functions $\tilde{J}_i^N$ by Euler iterations (see, for example, Atkinson, Han, and Stewart 2011). Going backwards from Q,

$$\begin{aligned}
\tilde{J}_i^N(Q) &= \frac{\alpha_i}{\gamma_i} Q, \\
\tilde{J}_i^N(Q - \Delta q) &= \frac{\alpha_i}{\gamma_i} Q - \Delta q H_i\Big(\frac{\alpha_1}{\gamma_1} Q, \frac{\alpha_2}{\gamma_2} Q\Big), \\
\tilde{J}_i^N(Q - 2\Delta q) &= J_i^N(Q - \Delta q) - \Delta q H_i\big(J_1^N(Q - \Delta q), \ldots, J_n^N(Q - \Delta q)\big) \\
&= \frac{\alpha_i}{\gamma_i} Q - \Delta q H_i\Big(\frac{\alpha_1}{\gamma_1} Q, \frac{\alpha_2}{\gamma_2} Q\Big) \\
&\quad - \Delta q H_i\big(J_1^N(Q - \Delta q), \ldots, J_n^N(Q - \Delta q)\big),
\end{aligned}$$

and so on, until $\tilde{J}_i^N(Q - N\Delta q) = \tilde{J}_i(q_\epsilon)$. We then complete the definition of function $\tilde{J}_i^N$ by making it piecewise linear between the points $Q - k\Delta q$, $k = 1, \ldots, N$. Note from the assumption on $Q - q_\epsilon$ that $\tilde{J}_i^N(Q - k\Delta q) \geq \epsilon$, for all $k = 1, \ldots, N$. Since the H_is are continuously differentiable, they are Lipschitz continuous on the 2-dimensional bounded domain $\big[\epsilon, \frac{\alpha_1}{\gamma_1} Q\big] \times \big[\epsilon, \frac{\alpha_2}{\gamma_2} Q\big]$. Therefore, following standard arguments, the sequence $\big\{\tilde{J}_i^n\big\}_{n=1}^N$ converges to a unique solution $\tilde{J}_i$ of the system of ODEs, and we have $\tilde{J}_i(q) > \epsilon$ for all $q \in [q_\epsilon, Q]$. ∎

Let

$$\underline{q} = \inf_{\epsilon > 0} q_\epsilon. \tag{11}$$

Lemma 5 shows that the system of ODEs has a unique solution on $[q_\epsilon, Q]$ for every $\epsilon > 0$. Thus, there exists a unique solution on $(\underline{q}, Q]$. Then, by standard optimal control arguments, it follows that $\tilde{J}_i(q)$ is the value function of agent i for every initial project value $q > \underline{q}$.

To establish convexity, we differentiate (8) with respect to q to obtain

$$r\tilde{J}_i'(q) = \big[\tilde{J}_1'(q) + \tilde{J}_2'(q)\big] \tilde{J}_i''(q) + \tilde{J}_i'(q) \tilde{J}_j''(q),$$

or equivalently, in matrix form,

$$r\begin{bmatrix} \tilde{J}_1' \\ \tilde{J}_2' \end{bmatrix} = \begin{bmatrix} \tilde{J}_1' + \tilde{J}_2' & \tilde{J}_1' \\ \tilde{J}_2' & \tilde{J}_1' + \tilde{J}_2' \end{bmatrix} \begin{bmatrix} \tilde{J}_1'' \\ \tilde{J}_2'' \end{bmatrix} \Rightarrow \begin{bmatrix} \tilde{J}_1'' \\ \tilde{J}_2'' \end{bmatrix} = \frac{r}{\big(\tilde{J}_1'\big)^2 + \big(\tilde{J}_2'\big)^2 + \tilde{J}_1' \tilde{J}_2'} \begin{bmatrix} \big(\tilde{J}_1'\big)^2 \\ \big(\tilde{J}_2'\big)^2 \end{bmatrix}. \tag{12}$$

Note that $a_i'(q) = \tilde{J}_i''(q) > 0$ if and only if $\tilde{J}_i'(q) > 0$ for all i, or equivalently, if and only if $q > \underline{q}$.

So far, we have shown that for any given Q, there exists some $\underline{q} < Q$ (which depends on the choice of Q) such that the system of ODEs defined by (4) subject to (3) has a project-completing solution on $(\underline{q}, Q]$. In this solution, $J_i(q) > 0$, $J_i'(q) > 0$, and $a_i'(q) > 0$ for all i and $q > \underline{q}$. On the other hand, Lemma 6 implies that $J_i(q) = J_i'(q) = 0$ for all $q \leq \underline{q}$. Therefore, the game starting at $q_0 = 0$ has a project-completing MPE if and only if $\underline{q} < 0$.

As shown in Lemma 1 regarding the single agent case, for small enough Q, each agent would be exerting effort and completing the project by himself even if the other agent were to exert no effort. A fortiori, the project will complete in an equilibrium where both agents can exert effort. Hence, for Q small enough, the MPE is project-completing.

As is shown in Section B regarding the socially optimal effort levels, for large enough Q, agents are better off not starting the project. A fortiori, for such project scales, the project will not complete in an equilibrium where both agents can exert effort. Hence, for Q large enough, the MPE is not project-completing. Instead, neither agent puts any effort on the project and the project is never started.

Uniqueness.—We show that if (J_1^a, J_2^a) and (J_1^b, J_2^b) are 2 well-behaved solutions to (4) subject to the boundary constraint (3) and subject to the constraint that each of the 4 functions is nondecreasing, then $(J_1^a, J_2^a) = (J_1^b, J_2^b)$ on the entire range $[0, Q]$. If the value functions associated with some MPE are well-behaved, then they must satisfy (4) subject to (3), and by Lemma 3 they must be nondecreasing. As the value functions uniquely pin down the equilibrium actions, it implies that for any project scale Q there exists a unique MPE with well-behaved solutions to the HJB equations.

The following lemma shows that at every state q, $J_1(q) > 0$ if and only if $J_2(q) > 0$.

LEMMA 6: *Let (J_1, J_2) be a pair of well-behaved value functions associated with an MPE. Then for every state q, $J_1(q) > 0$ if and only if $J_2(q) > 0$. Furthermore, if the project is completing at state q, then both J_1' and J_2' are strictly positive on (q, Q).*

PROOF:

Fix agent i and let j denote the other agent. If $J_i(q) > 0$, then the project is completing at state q. By Lemma 4, $\tilde{J}_1'$ is bounded strictly above 0 on (q, Q), thus J_1' is also bounded strictly above 0 on that range, and as an agent's action is proportional to the slope of the value function, agent 1's effort is also bounded strictly above 0 on the range (q, Q). This implies that, if agent 2 chooses to exert no effort on (q, Q), potentially deviating from his equilibrium strategy, the project is still completed by agent 1—and thus agent 2 makes a strictly positive discounted payoff at state q without exerting any effort from state q onwards. Agent 2's equilibrium strategy provides at least as much payoff as in the case of agent 2 exerting no effort past state q, thus agent 2's equilibrium discounted payoff at state q, $J_2(q)$ should be strictly positive. To summarize, $J_1(q) > 0$ and $J_2(q) > 0$. Thus, if the project is completing at state q, then $J_1(q)$ and $J_2(q)$ are both strictly positive. By Lemma 3, $J_1'(q) \geq 0$ and $J_2'(q) \geq 0$ and therefore J_1 and J_2 are strictly positive on (q, Q). Equation (8) then implies that J_1' and J_2' are strictly positive on (q, Q). Hence, if in some MPE the project is completing at state q, both agents exert strictly positive effort at all states beyond q (and up to completion of the project). ∎

First, consider the case $J_1^a(0) > 0$. Then, $J_2^a(0) > 0$ by Lemma 6. As J_1^a and J_2^a are nondecreasing, it follows from Lemma 5 that $(J_1^a, J_2^a) = (J_1^b, J_2^b)$ on the entire range $[0, Q]$. If instead $J_1^b(0) > 0$, the symmetric argument applies.

Next consider the case $J_1^a(0) = J_1^b(0) = 0$, and let $q^a = \sup\{q \geq 0 | J_1^a(q) = 0\}$. As $J_1^a(0) = 0$ we have $q^a \geq 0$. The boundary condition (3) and the continuity of J_1 implies that $q^a < Q$. Moreover, on the non-empty interval $(q^a, Q]$ we have $J_1^a > 0$, and thus by Lemma 6, $J_1^b > 0$ on that same interval. Lemma 5 then implies that $(J_1^a, J_2^a) = (J_1^b, J_2^b)$ on every $[q^a + \epsilon, Q]$ for $\epsilon > 0$, and thus that $(J_1^a, J_2^a) = (J_1^b, J_2^b)$ on $(q^a, Q]$. Now let us consider the range $[0, q^a]$. By continuity of J_1^a we have $J_1^a(q^a) = 0$. As J_1^a is nondecreasing and nonnegative, then $J_1^a(q^a) = 0$ implies that $J_1^a = 0$ on the interval $[0, q^a]$. As $J_1^a(q) = 0$ if and only if $J_2^a(q) = 0$, we get that $J_2^a = 0$ on the interval $[0, q_0]$. Thus, $(J_1^a, J_2^a) = 0$ on $[0, q^a]$.

Similarly let $q^b = \sup\{q | J_1^b(q) = 0\}$. We have $q^b \in [0, Q)$, and by a symmetric argument $(J_1^b, J_2^b) = 0$ on $[0, q^b]$. If $q^b < q^a$, then we get by Lemma 5 that $(J_1^a, J_2^a) = (J_1^b, J_2^b) > 0$ on $(q^b, Q]$, which contradicts $(J_1^a, J_2^a) = 0$ on $[0, q^a]$. If instead $q^b > q^a$, then we get that $(J_1^a, J_2^a) = (J_1^b, J_2^b) > 0$ on $(q^a, Q]$, which contradicts that $(J_1^b, J_2^b) = 0$ on $[0, q^b]$. Hence, $q^a = q^b$.

Altogether this implies that on the interval $[0, q^a]$, $(J_1^a, J_2^a) = (J_1^b, J_2^b) = 0$, and on the interval $(q^a, Q]$, $(J_1^a, J_2^a) = (J_1^b, J_2^b) > 0$. Hence, the HJB equations define a unique value function and thus a unique MPE. ∎

B. *Proof of Proposition* 2

First, we fix some $Q > 0$, and we use the normalization $\tilde{J}_i(q) = J_i(q)/\gamma_i$ as in the proof of Proposition 1.

To prove part 1, assume that $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$, let $\tilde{D}(q) = \tilde{J}_1(q) - \tilde{J}_2(q)$, and note that $\tilde{D}(\cdot)$ is smooth, $\tilde{D}(q) = 0$ for $q \leq \underline{q}$, and $\tilde{D}(Q) = \left(\frac{\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}\right)Q > 0$, where $\underline{q}$ is given by (11), in the proof of Proposition 1. Observe that either $\tilde{D}'(q) > 0$ for all $q \geq 0$, or there exists some $\bar{q} \in [0, Q]$ such that $\tilde{D}'(\bar{q}) = 0$. Suppose that the latter is the case. Then it follows from (8) that $\tilde{D}(\bar{q}) = 0$, which implies that $\tilde{D}(q) \geq 0$ for all q, and $\tilde{D}'(q) >(=)\ 0$ if and only if $\tilde{D}(q) >(=)\ 0$. Therefore, $\tilde{D}'(q) \geq 0$, which implies that $a_1(q) \geq a_2(q)$ for all $q \geq 0$. Observe from equation (12) in the proof of Proposition 1, that $J_i''(q) = \beta \cdot (J_i'(q))^2$, where

$\beta = r/\left[(\tilde{J}_1')^2 + (\tilde{J}_2')^2 + \tilde{J}_1'\tilde{J}_2'\right]$, and note that $a_i(q) = \tilde{J}_i'(q)$. Moreover, we know from part 1 of Proposition 2 that $a_1(q) \geq a_2(q)$, which implies that $J_1''(q) \geq J_2''(q)$, or equivalently, $a_1'(q) \geq a_2'(q)$ for all $q \geq 0$.

To prove part 2, note first the result for actions follows from the previous paragraph with all weak inequalities replaced with strict inequalities. Let $D(q) = \frac{J_1(q)}{\alpha_1} - \frac{J_2(q)}{\alpha_2}$, and note that $D(\cdot)$ is smooth, $D(q) = 0$ for q sufficiently small, and $D(Q) = 0$. Therefore, either $D(q) = 0$ for all q, or $D(\cdot)$ has an interior extreme point. Suppose that the former is true. Then for all q, we have $D(q) = D'(q) = 0$, which using (4) implies that

$$rD(q) = \frac{\left[J_1'(q)\right]^2}{2\alpha_1^2}\left(\frac{\alpha_2}{\gamma_2} - \frac{\alpha_1}{\gamma_1}\right) = 0 \Rightarrow J_1'(q) = 0.$$

However, this is a contradiction, and so the latter must be true. Then there exists some $\bar{q}$ such that $D'(\bar{q}) = 0$. Using (4) and the fact that $J_i'(q) \geq 0$ for all q and $J_i'(q) > 0$ for some q, this implies that $D(\bar{q}) \leq 0$. Therefore, $D(q) \leq 0$ for all q, which completes the proof.

Finally, if $\alpha_1/\gamma_1 = \alpha_2/\gamma_2$, then it follows from the analysis above that $\tilde{D}'(q) = 0$ and $D(q) = 0$, which implies that $a_1(q) = a_2(q)$ and $J_1(q)/\alpha_1 = J_2(q)/\alpha_2$ for all $q \geq 0$. ∎

C. *Proof of Proposition* 3

To prove part 1, first suppose that $\gamma_1/\alpha_1 = \gamma_2/\alpha_2$. In this case, we know from equation (5) that each agent's discounted payoff function satisfies

$$J_i(q;Q) = \frac{r\gamma_i}{6}\left[q - Q + \sqrt{\frac{6\alpha_i Q}{r\gamma_i}}\right],$$

and by maximizing $J_i(q;Q)$ with respect to Q, we obtain that $Q_1(q) = Q_2(q) = \frac{3\alpha_i}{2r\gamma_i}$ for all q.

To prove part 2, consider the case in which $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$. This part of the proof comprises 3 steps. To begin, in the following lemma, we characterize the values $\bar{Q}_i$ for $i = 1,2$ that are defined to be the project state that makes each agent i indifferent between terminating the project at this state, and continuing the project one more instant.

LEMMA 7: *Assume the agents are asymmetric, i.e.,* $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$. *The values of* $\bar{Q}_1$ *and* $\bar{Q}_2$ *are unique and given by*

$$\sqrt{\bar{Q}_1} = \frac{\sqrt{2/3}\,\sqrt{\mu}\,\alpha_1/\gamma_1}{\sqrt{r}\,\alpha_1/\gamma_1 + \frac{\sqrt{r}}{12}\left[\sqrt{\mu} + \sqrt{3\nu}\right]^2}$$

and

$$\sqrt{\bar{Q}_2} = \frac{\sqrt{2/3}\,\sqrt{\mu}\,\alpha_2/\gamma_2}{\sqrt{r}\,\alpha_2/\gamma_2 + \frac{\sqrt{r}}{12}\left[\sqrt{\mu} - \sqrt{3\nu}\right]^2},$$

where

$$\mu = 2\sqrt{\left(\frac{\alpha_1}{\gamma_1}\right)^2 + \left(\frac{\alpha_2}{\gamma_2}\right)^2 - \frac{\alpha_1}{\gamma_1}\frac{\alpha_2}{\gamma_2}} + \left(\frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2}\right)$$

and

$$\nu = 2\sqrt{\left(\frac{\alpha_1}{\gamma_1}\right)^2 + \left(\frac{\alpha_2}{\gamma_2}\right)^2 - \frac{\alpha_1}{\gamma_1}\frac{\alpha_2}{\gamma_2}} - \left(\frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2}\right).$$

Furthermore, $\bar{Q}_1 < \bar{Q}_2$.

PROOF:

Throughout this proof, we consider a project of a given scale Q. Let $\bar{a}_i(Q)$ denote the equilibrium effort agent i exerts at the very end of the project, when the terminal state is Q. Recall that, in equilibrium, the action of agent i at state q is given by

$$a_i(q) = J_i'(q)/\gamma_i,$$

and thus $\bar{a}_i(Q) = J_i'(Q)/\gamma_i = \tilde{J}_i'(Q)$. From Lemma 4 and noting that $\tilde{J}_i(Q) = (\alpha_i/\gamma_i)Q$, we get

$$\bar{a}_1(Q) = \sqrt{\frac{rQ}{6}}\left(\sqrt{\mu} + \sqrt{3\nu}\right), \tag{13}$$

$$\bar{a}_2(Q) = \sqrt{\frac{rQ}{6}}\left(\sqrt{\mu} - \sqrt{3\nu}\right), \tag{14}$$

with μ and ν defined as in the statement of the current lemma.

For a project of scale Q, agent i gets value $\alpha_i Q$ at the completion of the project, when $q = Q$. If the project is instead of scale $Q + \Delta Q$ (for small enough ΔQ), and if the current state is $q = Q$, there is a delay ϵ before the project is completed. To the first order in ϵ, the relationship $\Delta Q = \left(\bar{a}_1(Q) + \bar{a}_2(Q)\right)\epsilon$ holds. Thus, to the first order in ϵ, the net discounted value of the project to agent i at state $q = Q$ is

$$\alpha_i\left[Q + \left(\bar{a}_1(Q) + \bar{a}_2(Q)\right)\epsilon\right]e^{-r\epsilon} - \frac{\gamma_i}{2}\left(\bar{a}_i(Q)\right)^2\epsilon.$$

At project scale $Q = \bar{Q}_i$, the agent is indifferent between stopping the project now (corresponding to a project scale $\bar{Q}_i$) and waiting an instant later (corresponding to a project scale $\bar{Q}_i + \Delta Q$ for an infinitesimal ΔQ). So to the first order,

$$\alpha_i \bar{Q}_i = \alpha_i \Big(\bar{Q}_i + \big(\bar{a}_1(\bar{Q}_i) + \bar{a}_2(\bar{Q}_i) \big) \epsilon \Big) e^{-r\epsilon} - \frac{\gamma_i}{2} \big(\bar{a}_i(\bar{Q}_i) \big)^2 \epsilon.$$

So:

$$\alpha_i \big(\bar{a}_1(\bar{Q}_i) + \bar{a}_2(\bar{Q}_i) \big) - r \alpha_i \bar{Q}_i - \frac{\gamma_i}{2} \big(\bar{a}_i(\bar{Q}_i) \big)^2 = 0.$$

Solving this equation for $i = 1, 2$ yields

$$\sqrt{\bar{Q}_1} = \frac{\sqrt{2/3}\,\sqrt{\mu}\,\alpha_1/\gamma_1}{\sqrt{r}\,\alpha_1/\gamma_1 + \frac{\sqrt{r}}{12}\big[\sqrt{\mu} + \sqrt{3\nu}\big]^2}$$

and

$$\sqrt{\bar{Q}_2} = \frac{\sqrt{2/3}\,\sqrt{\mu}\,\alpha_2/\gamma_2}{\sqrt{r}\,\alpha_2/\gamma_2 + \frac{\sqrt{r}}{12}\big[\sqrt{\mu} - \sqrt{3\nu}\big]^2}.$$

Note that

$$\frac{\sqrt{\bar{Q}_1}}{\sqrt{\bar{Q}_2}} = \frac{12 + \left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\big[\sqrt{\mu} - \sqrt{3\nu}\big]^2}{12 + \left(\frac{\alpha_1}{\gamma_1}\right)^{-1}\big[\sqrt{\mu} + \sqrt{3\nu}\big]^2} = \frac{12\nu^{-1} + \left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\big[\sqrt{\mu/\nu} - 3\big]^2}{12\nu^{-1} + \left(\frac{\alpha_1}{\gamma_1}\right)^{-1}\big[\sqrt{\mu/\nu} + 3\big]^2}.$$

In particular, $\bar{Q}_1 < \bar{Q}_2$ if and only if the inequality

$$\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\big[\sqrt{\mu} + \sqrt{3\nu}\big] - \left(\frac{\alpha_1}{\gamma_1}\right)^{1/2}\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\big[\sqrt{\mu} - \sqrt{3\nu}\big] > 0 \tag{15}$$

holds. Let

$$f(x) = \sqrt{2\sqrt{1+x^2} - x + 1 + x} \quad \text{and} \quad g(x) = \sqrt{2\sqrt{1+x^2} - x - 1 - x}.$$

Note that

$$\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\big[\sqrt{\mu} + \sqrt{3\nu}\big] = f\left(\left(\frac{\alpha_1}{\gamma_1}\right)\left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\right) + \sqrt{3}\,g\left(\left(\frac{\alpha_1}{\gamma_1}\right)\left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\right)$$

and

$$\left(\frac{\alpha_2}{\gamma_2}\right)^{-1/2}\big[\sqrt{\mu} - \sqrt{3\nu}\big] = f\left(\left(\frac{\alpha_1}{\gamma_1}\right)\left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\right) - \sqrt{3}\,g\left(\left(\frac{\alpha_1}{\gamma_1}\right)\left(\frac{\alpha_2}{\gamma_2}\right)^{-1}\right).$$

Since, by assumption, $\alpha_1/\gamma_1 < \alpha_2/\gamma_2$, (15) is satisfied if $\left[f(x) + \sqrt{3}\,g(x)\right] - x\left[f(x) - \sqrt{3}\,g(x)\right] > 0$ for every $x \in (0,1)$. Note that, as $f, g > 0$ on $(0,1)$, so

$$\begin{aligned}\left[f(x) + \sqrt{3}g(x)\right] - x\left[f(x) - \sqrt{3}g(x)\right] &\geq x\left[f(x) + \sqrt{3}g(x)\right] - x\left[f(x) - \sqrt{3}g(x)\right]\\ &= 2\sqrt{3}xg(x) > 0.\end{aligned}$$

This establishes the inequality (15), and thus $\bar{Q}_1 < \bar{Q}_2$. ∎

Equations (13) and (14) show that the agent's action at time of termination is strictly increasing with the project scale.

LEMMA 8: *The value $J_i'(Q;Q)$ is strictly increasing in Q. Furthermore $\bar{Q}_i$ is the unique solution to the equation in Q, $J_i'\big(Q_i(Q);Q_i(Q)\big) = \alpha_i$.*

PROOF8:

Consider agent i's optimization problem given state q. We seek to find the unique q such that $q = \operatorname{argmax}_{Q \geq q}\{J_i(q;Q)\}$. For such q, we have $\frac{\partial}{\partial Q} J_i(q;Q)\Big|_{q=Q} = 0$. Note that $J_i(Q;Q) = \alpha_i Q$, and totally differentiating this with respect to Q yields

$$\frac{dJ_i(Q;Q)}{dQ} = J_i'(Q;Q) + \frac{\partial J_i(q;Q)}{\partial Q}\bigg|_{q=Q}$$

thus

$$J_i'(Q;Q) = \alpha_i. \tag{16}$$

By our assumption that $J_i(q;Q)$ is strictly concave in Q for all $q \leq Q \leq \bar{Q}_2$, it follows that (16) is necessary and sufficient for a maximum.

Noting that the explicit form of the HJB equations of Lemma 4 implies that $J_i'(Q;Q) = J_i'(1;1)\sqrt{Q}$, it follows that $J_i'(Q;Q)$ is strictly increasing in Q. Therefore, the solution to (16) is unique. ∎

Step 1: We show that $Q_2'(q) \geq 0$ for all $q \geq \bar{Q}_1$.

To begin, we differentiate $\tilde{J}_i(q;Q)$ in (8) with respect to Q to obtain

$$r\partial_Q \tilde{J}_1(q;Q) = \partial_Q a_1(q;Q)\left[a_1(q;Q) + a_2(q;Q)\right] + a_1(q;Q)\partial_Q a_2(q;Q),$$

$$r\partial_Q \tilde{J}_2(q;Q) = \partial_Q a_2(q;Q)\left[a_1(q;Q) + a_2(q;Q)\right] + a_2(q;Q)\partial_Q a_1(q;Q),$$

where we note $\partial_Q \tilde{J}_i(q;Q) = \frac{\partial}{\partial Q}\tilde{J}_i(q;Q)$, and where $\partial_Q a_i(q;Q) = \partial_Q \tilde{J}_i'(q;Q) = \frac{\partial^2}{\partial Q \partial q}\tilde{J}_i(q;Q)$, and $a_i(q;Q) = \tilde{J}_i'(q;Q) = \frac{\partial}{\partial q}\tilde{J}_i(q;Q)$.[33] Rearranging terms yields

$$\frac{(a_1 + a_2)^2 - a_1 a_2}{r}(\partial_Q a_1) = (a_1 + a_2)(\partial_Q \tilde{J}_1) - a_1(\partial_Q \tilde{J}_2), \tag{17}$$

$$\frac{(a_1 - a_2)^2 + a_1 a_2}{r}(\partial_Q a_2) = (a_1 + a_2)(\partial_Q \tilde{J}_2) - a_2(\partial_Q \tilde{J}_1), \tag{18}$$

where we drop the arguments q and Q for notational simplicity. Because $a_i, a_j > 0$, note that $(a_1 + a_2)^2 - a_1 a_2 > 0$ and $(a_1 - a_2)^2 + a_1 a_2 > 0$. Recall $Q_i(q)$ is agent i's ideal project scale given the current state q. Then for all $q < Q_i(q)$ and for the smallest q such that $q = Q_i(q)$, we have $\frac{\partial}{\partial Q}\tilde{J}_i(q;Q_i(q)) = 0$. Differentiating this with respect to q yields

$$\frac{\partial^2}{\partial Q \partial q}\tilde{J}_i(q;Q_i(q)) + \frac{\partial^2}{\partial Q^2}\tilde{J}_i(q;Q_i(q))\,Q_i'(q) = 0 \quad \Rightarrow$$

$$Q_i'(q) = -\frac{\partial_Q a_i(q;Q_i(q))}{\partial_Q^2 \tilde{J}_i(q;Q_i(q))}.$$

Since $\partial_Q^2 \tilde{J}_i(q;Q) < 0$ (by our strict concavity assumption), it follows that $Q_i'(q) \leq 0$ if and only if $\partial_Q a_i(q;Q) \geq 0$.

Next, fix some $\hat{q} \in (\bar{Q}_1, \bar{Q}_2)$. By the strict concavity of $\tilde{J}_i(q;Q)$ in Q, it follows that $\partial_Q \tilde{J}_1(\hat{q}, Q_2(\hat{q})) < 0$ and $\partial_Q \tilde{J}_2(\hat{q}, Q_2(\hat{q})) = 0$; i.e., agent 1 would prefer to have completed the project at a smaller project scale than $Q_2(\hat{q})$, whereas agent 2 finds it optimal to complete the project at $Q_2(\hat{q})$ (the latter statement being true by definition of $Q_2(\hat{q})$). Using (18) it follows that $\partial_Q a_2(\hat{q}, Q_2(\hat{q})) > 0$, which implies that $Q_2'(\hat{q}) > 0$. Therefore, $Q_2'(q) > 0$ for all $q \in (\bar{Q}_1, \bar{Q}_2)$ and $Q_2(\bar{Q}_1) > \bar{Q}_1$, where the last inequality follows from the facts that by assumption $\tilde{J}_2(q;Q)$ is strictly concave in Q for $q \leq Q \leq \bar{Q}_2$ and so it admits a unique maximum, and that $\tilde{J}_2'(\bar{Q}_1;\bar{Q}_1) < \alpha_2/\gamma_2$, which implies that he prefers to continue work on the project rather than complete it at $\bar{Q}_1$.

[33] Note $a_i(q;Q)$ is distinct from agent strategies in the case of commitment $a_i(q,Q)$. Here, $a_i(q;Q)$ denotes agents' actions in the MPE with exogenous project scale Q.

Step 2: We show that $Q_1'(q) \leq 0 \leq Q_2'(q)$ for all $q \leq \bar{Q}_1$. Moreover, $Q_1'(q) < 0 < Q_2'(q)$ for all q such that $Q_1(q) < Q_2(q)$.

Because $Q_2(\bar{Q}_1) > \bar{Q}_1$ and $Q_i(\cdot)$ is smooth, there exists some $\bar{q} \geq 0$ such that $Q_2(q) > Q_1(q)$ for all $q \in (\bar{q}, \bar{Q}_1)$. Pick some q in this interval, and note that $\partial_Q \tilde{J}_1(q, Q_2(q)) < 0$ and $\partial_Q \tilde{J}_2(q, Q_2(q)) = 0$, which together with (18) implies that $\partial_Q a_2(q, Q_2(q)) > 0$. Similarly, we have $\partial_Q \tilde{J}_1(q, Q_1(q)) = 0$ and $\partial_Q \tilde{J}_2(q, Q_1(q)) > 0$, which together with (17) implies that $\partial_Q a_1(q, Q_1(q)) < 0$. Therefore, $Q_1'(q) < 0 < Q_2'(q)$ for all $q \in (\bar{q}, \bar{Q}_1)$.

Next, by way of contradiction, assume that there exists some q such that $Q_1(q) > Q_2(q)$ for some $q < \bar{q}$. Because $Q_i(q)$ is smooth, by the intermediate value theorem, there exists some $\tilde{q}$ such that $Q_1(\tilde{q}) > Q_2(\tilde{q})$ and at least one of the following statements is true: $Q_1'(\tilde{q}) < 0$ or $Q_2'(\tilde{q}) > 0$. This implies that for such $\tilde{q}$, we must have $\partial_Q \tilde{J}_1(\tilde{q}, Q_2(\tilde{q})) > 0$, $\partial_Q \tilde{J}_2(\tilde{q}, Q_2(\tilde{q})) = 0$, $\partial_Q \tilde{J}_1(\tilde{q}, Q_1(\tilde{q})) = 0$, and $\partial_Q \tilde{J}_2(\tilde{q}, Q_1(\tilde{q})) < 0$. Then it follows from (17) and (18) that $\partial_Q a_1(\tilde{q}, Q_2(\tilde{q})) > 0$ and $\partial_Q a_2(\tilde{q}, Q_1(\tilde{q})) < 0$. This in turn implies that $Q_1'(\tilde{q}) > 0 > Q_2'(\tilde{q})$, which is a contradiction. Therefore, it must be the case that $Q_2(q) \geq Q_1(q)$ for all q, and therefore $Q_1'(q) \leq 0$ *for all* $q \leq \bar{Q}_1$ *and* $Q_2'(q) \geq 0$ *for all* $q \leq \bar{Q}_2$.

Step 3: We show that there does not exist any q such that $Q_1(q) = Q_2(q)$.

First, we show that if there exists some $\bar{q}$ such that $Q_1(\bar{q}) = Q_2(\bar{q})$, then it must be the case that $Q_1(q) = Q_2(q)$ for all $q \leq \bar{q}$. Suppose that the converse is true. Then by the intermediate value theorem, there exists some $\tilde{q}$ such that $Q_1(\tilde{q}) < Q_2(\tilde{q})$ and at least one of the following statements is true: either $Q_1'(\tilde{q}) > 0$ or $Q_2'(\tilde{q}) < 0$. This implies that for such $\tilde{q}$, we must have $\partial_Q \tilde{J}_1(\tilde{q}, Q_2(\tilde{q})) < 0$, $\partial_Q \tilde{J}_2(\tilde{q}, Q_2(\tilde{q})) = 0$, $\partial_Q \tilde{J}_1(\tilde{q}, Q_1(\tilde{q})) = 0$, and $\partial_Q \tilde{J}_2(\tilde{q}, Q_1(\tilde{q})) > 0$. Then it follows from (17) and (18) that $\partial_Q a_1(\tilde{q}, Q_2(\tilde{q})) < 0$ and $\partial_Q a_2(\tilde{q}, Q_1(\tilde{q})) > 0$. This in turn implies that $Q_1'(\tilde{q}) < 0 < Q_2'(\tilde{q})$, which is a contradiction. Therefore, if there exists some $\bar{q}$ such that $Q_1(\bar{q}) = Q_2(\bar{q})$, then $Q_1(q) = Q_2(q)$ and $\partial_Q a_1(q; Q) = \partial_Q a_2(q; Q) = 0$ for all $q \leq \bar{q}$ and $Q = Q_1(q)$.

Next, note that each agent's normalized discounted payoff function can be written in integral form as

$$\tilde{J}_i(q_t; Q) = e^{-r[\tau(Q)-t]} \frac{\alpha_i}{\gamma_i} Q - \int_t^{\tau(Q)} e^{-r(s-t)} \frac{\left(a_i(q_s; Q)\right)^2}{2} ds.$$

Differentiating this with respect to Q yields the first-order condition

$$(19) \qquad e^{-r[\tau(Q)-t]} \frac{\alpha_i}{\gamma_i}\left[1 - rQ\tau'(Q)\right] - e^{-r[\tau(Q)-t]} \tau'(Q) \frac{\left(a_i(Q; Q)\right)^2}{2} - \int_t^{\tau(Q)} e^{-r(s-t)} a_i(q_s; Q) \partial_Q a_i(q_s; Q) ds = 0.$$

Now, by way of contradiction, suppose there exists some $\bar{q}$ and some Q^* such that $Q_1(\bar{q}) = Q_2(\bar{q}) = Q^*$.[34] Then we have $Q_1(q) = Q_2(q)$ and $\partial_Q a_1(q;Q^*) = \partial_Q a_2(q;Q^*) = 0$ for all $q \leq \bar{q}$. Therefore, fixing some $q \leq \bar{q}$ and $Q^* = Q_1(\bar{q})$, it follows from (19) that

$$2\left[1 - rQ^*\tau'(Q^*)\right] = \tau'(Q^*)\frac{\gamma_1}{\alpha_1}\left(a_1(Q^*;Q^*)\right)^2 = \tau'(Q^*)\frac{\gamma_2}{\alpha_2}\left(a_2(Q^*;Q^*)\right)^2.$$

Observe that $\partial_Q a_1(q;Q^*) = \partial_Q a_2(q;Q^*) = 0$, which implies that $\partial_Q\left[a_1(q;Q^*) + a_2(q;Q^*)\right] = 0$, and hence $\tau'(Q^*) > 0$. By assumption, $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$, and we shall now show that $\frac{\gamma_1}{\alpha_1}\left(a_1(Q^*;Q^*)\right)^2 > \frac{\gamma_2}{\alpha_2}\left(a_2(Q^*;Q^*)\right)^2$. Let $D(q;Q^*) = \sqrt{\frac{\gamma_1}{\alpha_1}}\tilde{J}_1(q;Q^*) - \sqrt{\frac{\gamma_2}{\alpha_2}}\tilde{J}_2(q;Q^*)$, and note that $D(q;Q^*) = 0$ for q sufficiently small, $D(Q^*;Q^*) = \left(\sqrt{\frac{\alpha_1}{\gamma_1}} - \sqrt{\frac{\alpha_2}{\gamma_2}}\right)Q^* > 0$, and $D(\cdot;\ Q^*)$ is smooth. Therefore, either $D'(q;Q^*) > 0$ for all q, or there exists some extreme point z such that $D'(z;Q^*) = 0$. If the former is true, then $D'(Q^*;Q^*) > 0$, and we obtain the desired result. Now suppose that the latter is true. It follows from (8) that

$$rD(z;Q^*) = \frac{\left[\tilde{J}_1'(z;Q^*)\right]^2}{2}\left(\sqrt{\frac{\gamma_1}{\alpha_1}\frac{\alpha_2}{\gamma_2}} - 1\right) < 0,$$

which implies that any extreme point z must satisfy $D(z;Q^*) < 0 < D(Q^*;Q^*)$, and hence $D'(Q^*;Q^*) > 0$. Therefore, $\frac{\gamma_1}{\alpha_1}\left(a_1(Q^*;Q^*)\right)^2 > \frac{\gamma_2}{\alpha_2}\left(a_2(Q^*;Q^*)\right)^2$, which contradicts the assumption that there exists some q such that $Q_1(q) = Q_2(q)$.

We complete the proof of Proposition 3. From Lemma 7, we know that $\bar{Q}_1 < \bar{Q}_2$. Steps 1 and 2 show that $Q_1'(q) \leq 0$ for all $q \leq \bar{Q}_1$ and $Q_2'(q) \geq 0$ for all $q \leq \bar{Q}_2$, respectively, while step 3 shows that there exists no $q < \bar{Q}_2$ such that $Q_1(q) = Q_2(q)$. This proves part 2(a). To see part 2(b), Step 3 shows that $Q_2(q) > Q_1(q)$ for all q (i.e., $\bar{q} = 0$), which together with Step 2, implies that $Q_2'(q) > 0 > Q_1'(q)$ for all $q > 0$. Finally, it follows from the strict concavity of $J_i(q;Q)$ in Q that $Q_i(q) = q$ for all $q \geq \bar{Q}_i$, which completes the proof of part 2(c). ∎

D. *Proof of Lemma* 1

First, we characterize each agent i's effort and payoff function when he works alone on the project (and receives $\alpha_i Q$ upon completion).

Let $\hat{J}_i(q;Q)$ be agent i's discounted payoff at state q for a project of scale Q. By standard arguments, under regularity conditions, the function $\hat{J}_i(\cdot;\ Q)$ satisfies the HJB equation

$$\text{(20)} \qquad r\hat{J}_i(q;Q) = \max_{\check{a}_i}\left\{-\frac{\gamma}{2}\check{a}_i^2 + \check{a}_i\hat{J}_i'(q;Q)\right\}$$

[34] Note that the notation Q^* as used in this proof is distinct from the function $Q^*(q)$ which denotes the social planner's ideal project scale.

subject to the boundary condition

$$\hat{J}_i(Q;Q) = \alpha_i Q. \tag{21}$$

The game defined by (20) subject to the boundary condition (21) has a unique solution on $(\underline{q}, Q]$ in which the project is completed, where $\underline{q} = Q - \sqrt{\frac{2\alpha_i Q}{r\gamma_i}}$. Then agent i's effort strategy and discounted payoff satisfies

$$\hat{a}_i(q;Q) = r\left(q - Q + \sqrt{\frac{2\alpha_i Q}{r\gamma_i}}\right)$$

and

$$\hat{J}_i(q;Q) = \frac{r\gamma_i}{2}\left(q - Q + \sqrt{\frac{2\alpha_i Q}{r\gamma_i}}\right)^2,$$

respectively. Define

$$\hat{Q}_i(q) = \arg\max_{Q \geq q}\left\{\hat{J}_i(q;Q)\right\}.$$

It is straightforward to verify that $\hat{Q}_i(q) = \frac{\alpha_i}{2r\gamma_i}$. The inequality $\hat{Q}_2(q) < \hat{Q}_1(q)$ follows from the fact that by assumption $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$.

Next, we show that $\hat{Q}_1(q) < \bar{Q}_1$. Define $\hat{\Delta}(q) = J_1(q;\bar{Q}_1) - \hat{J}_1(q;\bar{Q}_1)$. Note that $J_1'(\bar{Q}_1;\bar{Q}_1) = \alpha_1$, $\hat{\Delta}(\bar{Q}_1) = 0$, $\hat{\Delta}(q) = 0$ for sufficiently small q, and $\hat{\Delta}(\cdot)$ is smooth. Therefore, either $\hat{\Delta}(q) = 0$ for all q, or it has an interior local extreme point. In either case, there exists some z such that $\hat{\Delta}'(z) = 0$. Using (4) and the fact that, from the single agent HJB equation, $r\hat{J}_1(q;Q) = \left[\hat{J}_1'(q;Q)\right]^2/(2\gamma_1)$, it follows that

$$r\hat{\Delta}(z) = \frac{J_1'(z;\bar{Q}_1)\,J_2'(z;\bar{Q}_1)}{\gamma_2}.$$

Because $J_1'(q;\bar{Q}_1)\,J_2'(q;\bar{Q}_1) > 0$ for at least some q, it follows that it cannot be the case that $\hat{\Delta}(q) = 0$ for all q. Because $J_1'(q;\bar{Q}_1)\,J_2'(q;\bar{Q}_1) \geq 0$, it follows that any extreme point z must satisfy $\hat{\Delta}(z) \geq 0$, which together with the boundary conditions implies that $\hat{\Delta}(q) \geq 0$ for all q. Therefore, $\hat{\Delta}'(\bar{Q}_1) < 0$, which in turn implies that $\hat{J}_1'(\bar{Q}_1;\bar{Q}_1) > J_1'(\bar{Q}_1;\bar{Q}_1) = \alpha_1$. By noting that $\hat{J}_1'(\hat{Q}_1(q);\hat{Q}_1(q)) = \alpha_1$ and $\hat{J}_1'(Q;Q)$ is strictly increasing in Q, it follows that $\hat{Q}_1(q) < \bar{Q}_1$.

Since $Q_1'(q) < 0$ for all q, it follows that $\hat{Q}_1(q) < Q_1(q)$ for all q, and we know from Proposition 3 that $Q_1(q) < Q_2(q)$ for all q. ∎

E. *Proof of Lemma* 2

Let $S(q;Q) = J_1(q;Q) + J_2(q;Q)$. Because, by assumption, $J_i(q;Q)$ is strictly concave in Q for all i and $q \leq Q \leq \bar{Q}_2$, it follows that $S(q;Q)$ is also strictly concave in Q for all $q \leq Q \leq \bar{Q}_2$. Therefore, $Q^*(q)$ will satisfy $\frac{\partial}{\partial Q}S(q;Q) = 0$ at $Q = Q^*(q)$ and $\frac{\partial}{\partial Q}S(q;Q)$ is strictly decreasing in Q for all q. We know from Proposition 3 that $Q_1(q) < Q_2(q)$ for all $q \leq \bar{Q}_2$. Moreover, we know that (i) $\frac{\partial}{\partial Q}J_1(q;Q) \geq 0$ and $\frac{\partial}{\partial Q}J_2(q;Q) > 0$ and so $\frac{\partial}{\partial Q}S(q;Q) > 0$ for all $q \leq Q_1(q)$, and (ii) $\frac{\partial}{\partial Q}J_1(q;Q) < 0$ and $\frac{\partial}{\partial Q}J_2(q;Q) \leq 0$ and so $\frac{\partial}{\partial Q}S(q;Q) < 0$ for all $q \geq Q_2(q)$. Because $\frac{\partial}{\partial Q}S(q;Q)$ is strictly decreasing in Q, it follows that $\frac{\partial}{\partial Q}S(q;Q) = 0$ for some $Q \in \big(Q_1(q), Q_2(q)\big)$. ∎

F. *Proof of Proposition* 4

We first construct a project-completing MPE with project scale $Q_i(0)$, and then argue the uniqueness of the equilibrium project scale.

Consider the following strategy profile:

- *Effort levels:* let both agents exert no effort at all states before the project scale has been decided. Once a project scale Q has been decided, let both agents choose their respective effort level as in the benchmark setting of Section II for a project of exogenous scale Q at all states $q \leq Q$, and let them exert no effort for all states $q > Q$.
- *Dictator's decision:* at any state q, where no scale has yet been decided, let the dictator set the project scale $Q_i(q)$.

We verify that such strategy profile is an MPE.

First, let us fix the strategy of the dictator. Then at any state q, if the dictator's decision is yet to be made, agent j anticipates the scale to be set immediately, and exerting no effort is a best response. At any state q, if a decision of scale Q has been made by the dictator, agent j's effort levels are, by definition, a best response to the dictator's effort strategy.

Second, let us fix the effort strategy of agent j. If, at state q, the project scale has not been decided, the dictator never profits by delaying the decision to commit because agent j exerts no effort before the project scale is decided. Therefore, it is a best response to commit at state q. Furthermore, if he commits to project scale $Q \neq Q_i(q)$, the dictator's discounted payoff is $J_i(q;Q) \leq J_i\big(q;Q_i(q)\big)$. Hence, committing at state q to project scale $Q_i(q)$ is a best response. The effort levels of the dictator are, by definition, a best response to agent j's strategy.

Finally, we note that in any MPE, the dictator commits at the beginning of the project. Suppose he were to commit after the project started, say when the project reaches state $\check{q} > 0$. Since $J_i(\check{q};Q)$ has a unique maximum in

Q, he commits to $Q_i(\check{q})$ and obtains payoff $J_i(\cdot; Q_i(\check{q}))$. Then at state $q = 0$ there is a profitable deviation to commit immediately to $Q_i(0)$ and obtain payoff $J_i(0; Q_i(0)) > J_i(0; Q_i(\check{q}))$. Hence, there is no MPE in which the dictator delays the announcement of the project scale. ■

G. *Proof of Proposition* 5

We begin by showing that if a project-completing equilibrium exists with scale Q, and if agent i is dictator, then $Q \leq \bar{Q}_i$. This helps identify the set of Pareto-efficient equilibrium outcomes.

In an equilibrium of project scale Q, both agents anticipate that the project will be completed at state Q. Therefore, they will both work as they would in the benchmark game of fixed project scale Q described in Section III. In particular, at any state $q \in [0, Q]$, each agent $k \in \{1, 2\}$ gets continuation payoff $J_k(q; Q)$.

If $Q > \bar{Q}_i$, then at any state $q \in (\bar{Q}_i, Q)$, Proposition 3 implies that $J_i(q; q) > J_i(q; Q)$, i.e., the dictator is strictly better off stopping the project when at state q, instead of stopping at state Q. Thus, $Q \leq \bar{Q}_i$ in equilibrium.

Next, we show that, if agent 1 is the dictator, then $Q = \bar{Q}_1$ can be sustained in an MPE, whereas if agent 2 is the dictator, then any $Q \in [Q_1(0), Q_2(0)]$ can be sustained in an MPE. Observe that these project scales are the Pareto-efficient ones, subject to the constraint that $Q \leq \bar{Q}_i$ when agent i is dictator.

Let $Q^\dagger = \bar{Q}_1$ if agent 1 is the dictator and let $Q^\dagger \in [Q_1(0), Q_2(0)]$ if agent 2 is the dictator. Recall that, as explained in Section II, for any *fixed*, *exogenous* scale $Q \in [0, \bar{Q}_2)$, the resulting MPE is completing, owing to the assumed strict concavity of $Q \mapsto J_2(0; Q)$ over that range. We verify that there exists an MPE with project scale $Q^\dagger$.

Consider the following strategy profile:

- *Effort levels:* for any state $q \leq Q^\dagger$, let both agents choose their effort optimally in a game of fixed project scale $Q^\dagger$, and for all $q > Q^\dagger$, let them exert no effort. Note that, because the unique MPE of a project of fixed scale $Q^\dagger$ is completing, both agents put positive effort at every state up to $Q^\dagger$.
- *Dictator's decision:* let the dictator stop the project immediately whenever $q \geq Q^\dagger$.

To show such strategy profile is an MPE, we must show that agents play a best response to each other at every state.

First, fix the dictator's strategy. Then agent j anticipates to be working on a project of scale $Q^\dagger$, and it follows directly from agent j's effort strategy that agent j plays a best response at every state $q \leq Q^\dagger$. At any state $q > Q^\dagger$, agent j anticipates that the dictator completes the project immediately, and so putting no effort is a weakly best response.

Now, let us fix agent j's strategy. If the dictator completes the project at state $Q^\dagger$, then his effort level is optimal given j's effort level, by definition of agent i's effort strategy.

Let us check that terminating the project at every state $q \geq Q^{\dagger}$ is optimal for the dictator. Consider state $q \geq Q^{\dagger}$. As agent j exerts no effort for all states greater that $Q^{\dagger}$, and as $Q^{\dagger} \geq \bar{Q}_1 > \hat{Q}_i$, the dictator has no incentive to continue the project by himself: he is always better off stopping the project immediately.

Now consider state $q < Q^{\dagger}$:

- If agent 1 is the dictator, then as $q < \bar{Q}_1 < Q_1(q)$, by our assumption that $Q \mapsto J_1(q;Q)$ is strictly concave on $\left[q, \bar{Q}_2\right)$ and is maximized for $Q = Q_1(q)$, it is also strictly increasing $\left[q, Q_1(q)\right]$. This implies that $J_1\left(q; Q_1(q)\right) > J_1\left(q; \bar{Q}_1\right) > J_1(q;q)$, and so the agent has no incentive to collect the termination payoff before reaching state $\bar{Q}_1$.
- If agent 2 is the dictator, then by Lemma 8 (see the proof of Proposition 3), $Q \mapsto J_2'(Q;Q)$ increases on $\left[\bar{Q}_1, \bar{Q}_2\right]$, and $J_2'\left(\bar{Q}_2; \bar{Q}_2\right) = J_2'\left(Q_2\left(\bar{Q}_2\right); Q_2\left(\bar{Q}_2\right)\right) = \alpha_2$. Additionally, $J_2(Q;Q) = \alpha_2 Q$ and Proposition 1shows that $J_2(q;Q)$ is strictly convex in q for $q \leq Q \leq \bar{Q}_2$. Hence $J_2'(q;Q) < \alpha_2$ for $q < Q < \bar{Q}_2$, which in turn implies that $J_2(q;Q) > \alpha_2 q$ for all $q < Q$ with $Q < \bar{Q}_2$. So, if $q < Q^{\dagger}$, then $J_2(q;q) = \alpha_2 q < J_2\left(q; Q^{\dagger}\right)$, and hence agent 2 has no incentive to complete the project before reaching state $Q^{\dagger}$.

In conclusion, the strategies defined above form a project-completing MPE with project scale $Q^{\dagger}$. ∎

H. *Proof of Proposition* 6

Fix some $Q^{\dagger} \in \left[Q_1(0), Q_2(0)\right]$. We construct a project-completing MPE with project scale $Q^{\dagger}$. Observe that any project scale $Q' \notin \left[Q_1(0), Q_2(0)\right]$ is Pareto-dominated; that is, there exists some $Q^* \in \left[Q_1(0), Q_2(0)\right]$ such that $J_i(0;Q^*) \geq J_i(0;Q')$ for all i. Consider the following strategy profile:

- *Effort levels:* before a project scale has been committed to, each agent i exerts effort $a_i(q;-1) = a_i\left(q; Q^{\dagger}\right)\mathbf{1}_{\{q<Q^{\dagger}\}}$. After a project scale Q has been committed to, each agent exerts effort $a_i(q;Q)\,\mathbf{1}_{\{q<Q\}}$, where $a_i(q;Q)$ is characterized in the benchmark setting of Section II for a project of exogenous scale Q.
- *Agenda setter proposals:* let the agenda setter propose project scale $Q^{\dagger}$ at every state $q \leq Q^{\dagger}$, and propose to stop the project immediately at every state $q > Q^{\dagger}$.
- *Agent j's decisions:* in a project state $q > Q^{\dagger}$, agent j accepts the agenda setter's proposal to stop at Q for all Q with $J_j(q;Q) \geq J_j(q;q)$, and rejects the proposal otherwise. In a state $q \leq Q^{\dagger}$, let agent j accept the agenda setter's proposal to stop at Q whenever $J_j(q;Q) \geq J_j\left(q; Q^{\dagger}\right)$ and reject the proposal otherwise.

We now show that such strategy profile is an MPE. First, fix the agenda setter's strategy. It follows directly from agent j's strategy that agent j plays a best response at every state—both in terms of effort and response to proposals of the agenda setter.

Now take the strategy of agent j as given. If at state q a project scale Q has already been agreed upon, the agenda setter, who can no longer change the project scale, plays a best response (in terms of effort level) to the strategy of agent j. It remains to show that the agenda setter plays a best response at every q when no project state has been agreed on yet. If he anticipates the project scale to be $Q^\dagger$, then his effort levels are optimal in every state. Let us check that the proposal strategy is indeed optimal, and yield project scale $Q^\dagger$:

- If $q \geq Q^\dagger$, and agent 1 is the agenda setter, then agent 1 is better off if the project stops immediately: since $Q_1(q) = q$ as $Q^\dagger \geq \bar{Q}_1$, $J_1(q;q) > J_1(q;Q)$ for every $Q > q$. If agent 1 proposes to stop the project at state q, then agent 2 accepts, by definition of agent 2's strategy. Hence, it is optimal for agent 1 to propose to stop the project at state q, and the conjectured equilibrium strategy of agent 1 is a best response to agent 2's strategy.
- If $q \geq Q^\dagger$, and agent 2 is the agenda setter, then agent 2 would prefer in some cases to pursue the project with agent 1, but never wants to pursue the project by himself, because $Q^\dagger > \hat{Q}_2$. As agent 1 only accepts proposals to stop right away, and as he exerts no effort past state $Q^\dagger$ until a scale proposed is accepted, agent 2 is better off proposing to stop the project at the current state q—proposition accepted by agent 1. Hence, the conjectured equilibrium strategy of agent 2 is a best response to agent 1's strategy.
- If $q < Q^\dagger$, and agent 1 is the agenda setter, then the agenda setter can guarantee himself a continuation payoff $J_i\left(q;Q^\dagger\right)$ by following the strategy defined in the above conjectured equilibrium profile. Assume by contradiction that there is an alternative strategy for the agenda setter that yields a strictly higher payoff. Such strategy must generate a different project scale, Q. In addition, that project scale must be less than $Q^\dagger$ for agent 1 to be better off, and so an agreement must be reached before state $Q^\dagger$. But then $J_2(q;Q) < J_2\left(q;Q^\dagger\right)$, and by definition of agent 2's strategy, agent 2 would not accept agent 1's proposal to set scale Q at any state $q < Q^\dagger$. Hence, the conjectured equilibrium strategy of agent 1 is a best response to agent 2's strategy.
- If $q < Q^\dagger$, and agent 2 is the agenda setter, then as before the agenda setter can guarantee himself a continuation payoff $J_2\left(q;Q^\dagger\right)$ by following the strategy defined in the above conjectured equilibrium profile. Assume by contradiction that there is an alternative strategy for the agenda setter that yields strictly higher payoff with a different project scale Q. Then, as agent 2 is strictly better off, it must be that $Q > Q^\dagger$, as $J_2(q;Q)$ is strictly increasing in Q when $Q < Q^\dagger$. However, agent 1 would not accept such a proposal of project Q before reaching state $Q^\dagger$. He may accept such a proposal in state $q = Q$, however, between state $Q^\dagger$ and Q exerts no effort. As $Q^\dagger > \hat{Q}_2$, agent 2 is never better off pursuing and completing the project by himself past state $Q^\dagger$, and thus a project scale $Q = Q^\dagger$ is optimal. Hence, the conjectured equilibrium strategy of agent 2 is a best response to agent 1's strategy.

Therefore the conjectured strategy profile constitutes a project-completing MPE with project scale $Q^\dagger$. ∎

I. *Proof of Proposition* 7

Fix some $Q^{\dagger} \in \left[Q_1(0), Q_2(0)\right]$. As in the Proof of Proposition 6, we show that $Q^{\dagger}$can be sustained in some MPE. Let us consider the following strategy profile.

(i) *Effort levels:* let both agents choose an effort level optimal for a project of fixed scale $Q^{\dagger}$, and put zero effort for any state $q > Q^{\dagger}$.

(ii) *Agenda setter proposals:* let the agenda setter propose to stop the project for any state $q \geq Q^{\dagger}$, and continue to project for all $q < Q^{\dagger}$.

(iii) *Agent j's decisions:* let agent *j* accept the agenda setter's proposal to stop for all states $q \geq Q^{\dagger}$, and otherwise accept to stop whenever $J(q;q) \geq J\left(q;Q^{\dagger}\right)$.

Let us show that such strategy profile is an MPE.

Let us fix the strategy of the agenda setter and check that agent *j*'s strategy is a best response at every state:

- First, suppose agent 1 is the agenda setter. If he proposes to stop the project at a state $q \geq Q^{\dagger}$, agent 2 should accept: agent 1 puts no effort past state $Q^{\dagger}$, and agent 2 would rather not work alone on the project as $\hat{Q}_2 < Q^{\dagger}$. If agent 1 proposes to stop at a state $q < Q^{\dagger}$, then agent 2 should accept only if the payoff he makes from immediate project termination, $J_2(q;q)$ is no less than the payoff he makes by rejecting—which then pushes back the next anticipated proposal at state $Q^{\dagger}$, $J_2\left(q;Q^{\dagger}\right)$. Given the agenda setter's strategy, agent 2 expects to complete the project in state $Q^{\dagger}$, and by definition of agent 2's effort strategy, the effort levels of agent 2 are optimal at all states.
- Second, suppose agent 2 is the agenda setter. If agent 1 is offered to stop the project at $q \geq Q^{\dagger}$, then agent 1 finds it optimal to accept because $Q_1(q) = q$ for all $q \geq \bar{Q}_1$. If agent 1 is offered to stop the project at $q < Q^{\dagger}$, then he should accept only if the payoff from immediate project termination $J_1(q;q)$ is no less than the payoff he expects to make from rejecting, which as before is $J_1\left(q;Q^{\dagger}\right)$. Given the agenda setter's strategy, agent 1 expects to complete the project in state $Q^{\dagger}$, and by definition of agent 1's effort strategy, the effort levels of agent 1 are optimal at all states.

Next let us fix the strategy of agent *j* and check that the agenda setter's strategy is a best response at every state:

- First, suppose agent 1 is the agenda setter. Then agent 1 expects to make payoff $J_1\left(q;Q^{\dagger}\right)$ by following the conjectured equilibrium strategy. To make a better payoff, he would have to complete the project at a state $Q < Q^{\dagger}$. However, such a proposal to stop the project early would not be accepted by agent 2, who is better off working toward a project of scale $Q^{\dagger}$ because $J_2(q;Q)$is increasing in Q for all $Q \leq Q^{\dagger} \leq Q_2(q)$. Hence, not proposing to stop before state $Q^{\dagger}$ is a (weak) best response. As agent 2 accepts to stop at all states $q \geq Q^{\dagger}$, agent 1 is better off

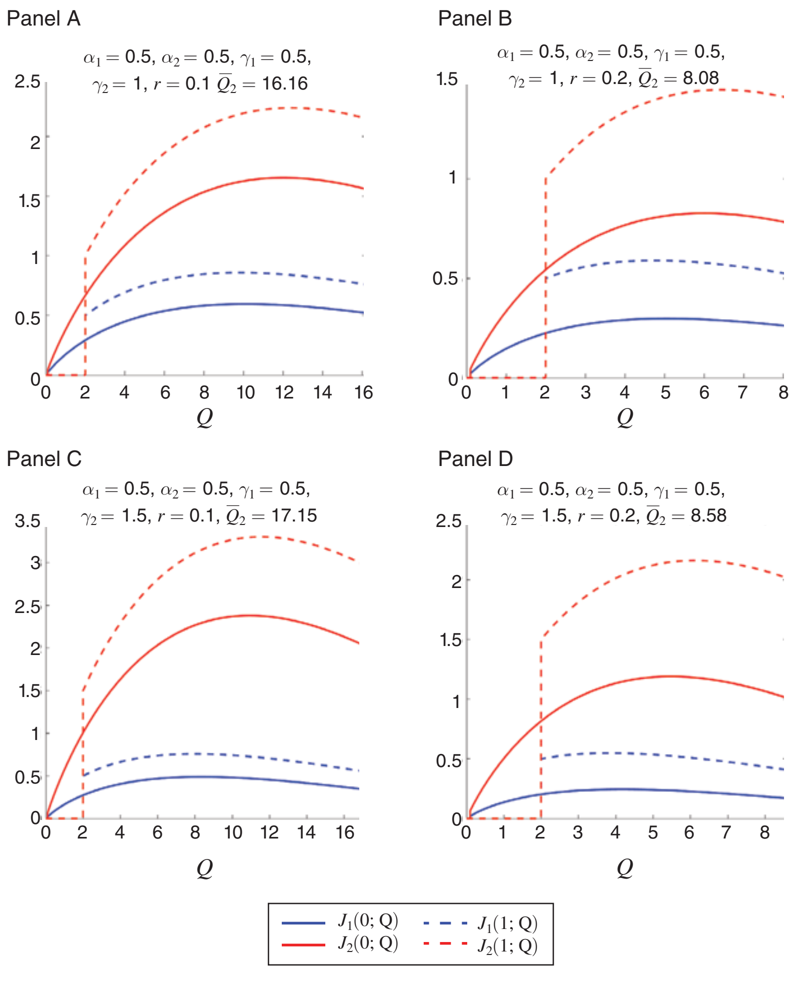

FIGURE 7. AGENTS' DYNAMIC PAYOFFS AS A FUNCTION OF Q, FOR $Q \in [q, \bar{Q}_2]$.

proposing to stop at all states $q \geq Q^\dagger$ because $Q_1(q) = q$ for all $q \geq Q^\dagger \geq \bar{Q}_1$. Therefore, agent 1 anticipates the project scale to be $Q^\dagger$ and his effort levels are optimal for such a project scale.

- Second, suppose agent 2 is the agenda setter. Then agent 2 expects to make payoff $J_2(q; Q^\dagger)$ by following the conjectured equilibrium strategy, and to make a larger payoff would require completing the project at a state $Q > Q^\dagger$. Therefore it is never optimal for agent 2 to stop at any $Q < Q^\dagger$. However, it is always optimal to stop at every $Q \geq Q^\dagger$, as agent 1 plans to put in no effort after Q, and agent 2 prefers not to work alone on the project since $\hat{Q}_2 < Q^\dagger$.

Hence, the conjectured strategy profile constitutes a project-completing MPE with project scale $Q^\dagger$. ∎

J. *Proof of Proposition* 8

Fix some $Q > 0$. We use the normalization $\tilde{J}_i(q) = J_i(q)/\gamma_i$ as in the proof of Proposition 1.

To prove part 1, assume that $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$, let $\tilde{D}(q) = \tilde{J}_1(q) - \tilde{J}_2(q)$, and note that $\tilde{D}(\cdot)$ is smooth, $\lim_{q\to-\infty}\tilde{D}(q) = 0$ and $\tilde{D}(Q) = \left(\frac{\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}\right)Q > 0$. Suppose that $\tilde{D}(\cdot)$ has an interior global extreme point, and denote such extreme point by $\bar{q}$. Because $\tilde{D}(\cdot)$ is smooth, it must be the case that $\tilde{D}'(\bar{q}) = 0$. Then it follows from (8) that $r\tilde{D}(\bar{q}) = \frac{\sigma^2}{2}\tilde{D}''(\bar{q})$. If $\bar{q}$ is a maximum, then $\tilde{D}''(\bar{q}) \leq 0$, so $\tilde{D}(\bar{q}) \leq 0$, which contradicts the fact that $\lim_{q\to-\infty}\tilde{D}(q) = 0$ and the assumption that $\bar{q}$ is a maximum. On the other hand, if $\bar{q}$ is a minimum, then $\tilde{D}''(\bar{q}) \geq 0$, so $\tilde{D}(\bar{q}) \geq 0$, which contradicts the fact that $\lim_{q\to-\infty}\tilde{D}(q) = 0$ and the assumption that $\bar{q}$ is a minimum. Therefore, $\tilde{D}'(q) > 0$ for all q, which implies that $a_1(q) > a_2(q)$ for all q.

To prove part 2, let $D(q) = \frac{J_1(q)}{\alpha_1} - \frac{J_2(q)}{\alpha_2}$, and note that $D(\cdot)$ is smooth, $\lim_{q\to-\infty}D(q) = 0$, and $D(Q) = 0$. Therefore, either $D(q) = 0$ for all q, or $D(\cdot)$ has an interior global extreme point. Suppose that the former is true. Then for all q, we have $D(q) = D'(q) = D''(q) = 0$, which using (4) implies that

$$rD(q) = \frac{\left[J_1'(q)\right]^2}{2\alpha_1^2}\left(\frac{\alpha_2}{\gamma_2} - \frac{\alpha_1}{\gamma_1}\right) = 0 \Rightarrow J_1'(q) = 0.$$

By Proposition 1, we have $J_i' > 0$ in any project-completing MPE, so this is a contradiction. Thus, the latter must be true. Then there exists some $\bar{q}$ such that $D'(\bar{q}) = 0$. Using (4), this implies that

$$rD(\bar{q}) = \frac{\left[J_1'(\bar{q})\right]^2}{2\alpha_1^2}\left(\frac{\alpha_2}{\gamma_2} - \frac{\alpha_1}{\gamma_1}\right) + \frac{\sigma^2}{2}D''(\bar{q}),$$

and note that $J_1'(\bar{q}) > 0$. Suppose that $\bar{q}$ is a maximum. Then $D''(\bar{q}) \leq 0$, which together with the fact that $\alpha_2/\gamma_2 < \alpha_1/\gamma_1$ implies that $D(\bar{q}) < 0$. Therefore, $D(q) \leq 0$ for all q, which completes the proof of part 2.

Finally, if $\alpha_1/\gamma_1 = \alpha_2/\gamma_2$, then it follows from the analysis above that $\tilde{D}(q) = \tilde{D}'(q) = 0$ and $D(q) = 0$, which implies that $a_1(q) = a_2(q)$ and $J_1(q)/\alpha_1 = J_2(q)/\alpha_2$ for all $q \geq 0$. ∎

Appendix B: Additional Results

A. *About the Concavity of Players Payoffs*

We illustrate on Figure 7 the concavity of the players' payoffs with respect to the project scale, for different parameter values. The plots are obtained by solving numerically the differential equations that the value functions must satisfy in equilibrium.

B. *Propositions* 1 *and* 2 *Hold under Broader Assumptions*

In this section, we show that Propositions 1 and 2 hold under a broader class of effort cost functions. In particular, suppose that effort level a induces flow cost equal to $c_i(a) = \gamma_i c(a)$ to agent i, where $\gamma_i > 0$, and $c(\cdot)$ is some arbitrary function that satisfies $c', c'' > 0$, $c''' \geq 0$, $c(0) = 0$, and $\lim_{a\to\infty} c(a) = \infty$. Using similar arguments as in Section III, it follows that for any fixed $Q > 0$, each agent i's payoff function satisfies the HJB equation

$$rJ_i(q) = \max_{\hat{a}_i}\left\{-\gamma_i c(\hat{a}_i) + \left(\hat{a}_i + a_j(q)\right) J_i'(q)\right\}$$

subject to the boundary condition $J_i(Q) = \alpha_i Q$. By using the normalization $\tilde{J}_i(q) = J_i(q)/\gamma_i$, it follows that in a well-behaved MPE, each agent's discounted payoff satisfies the following system of ODE:

$$r\tilde{J}_i(q) = -c\left(f\left(\tilde{J}_i'(q)\right)\right) + \left[f\left(\tilde{J}_i'(q)\right) + f\left(\tilde{J}_j'(q)\right)\right]\tilde{J}_i'(q) \tag{22}$$

subject to $\tilde{J}_i(Q) = \frac{\alpha_i}{\gamma_i}Q$, where $f(\cdot) = c'^{-1}(\cdot)$, and each agent's effort level is given by $a_i(q) = f\left(\tilde{J}_i'(q)\right)$. Cvitanić and Georgiadis (2016) show that if a project-completing MPE exists, then an analogous result to Proposition 1(i) holds; i.e., $J_i(q) > 0$, $J_i'(q) > 0$, and $a_i'(q) > 0$ for all i and $q \geq 0$.

The following result establishes conditions such that Proposition 2 holds under a broader class of effort cost functions.

PROPOSITION 9: *Suppose that* $\gamma_1/\alpha_1 < \gamma_2/\alpha_2$. *In any project-completing MPE:*

(*i*) *Agent* 1 *exerts higher effort than agent* 2 *in every state; i.e.*, $a_1(q) \geq a_2(q)$ *for all* $q \geq 0$.

(*ii*) *Agent* 1*'s effort increases at a greater rate than agent* 2 (*i.e.*, $a_1'(q) \geq a_2'(q)$ *for all* $q \geq 0$) *if* $c'(\cdot)$ *is weakly log-concave; i.e.*, $\log c'(\cdot)$ *is weakly concave.*

(*iii*) *Agent* 1 *obtains a lower discounted payoff normalized by project state than agent* 2 (*i.e.*, $J_1(q)/\alpha_1 \leq J_2(q)/\alpha_2$ *for all* $q \geq 0$) *if* $c(\cdot)$ *is weakly log-concave.*

PROOF:

Statement 1: Define $\tilde{D}(\cdot) = \tilde{J}_1(\cdot) - \tilde{J}_2(\cdot)$, and note that $\tilde{D}(\cdot)$ is smooth, $\tilde{D}(q) = 0$ for q sufficiently small (possibly $q < 0$), and $\tilde{D}(Q) = \left(\frac{\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}\right)Q > 0$. Therefore, either $\tilde{D}'(q) \geq 0$ for all q, or $\tilde{D}(\cdot)$ has at least one interior extreme point. Suppose that the latter is true. Then there exists some z such that $\tilde{D}'(z) = 0$ and substituting into (22) yields

$$r\tilde{D}(z) = 0.$$

Because any interior extreme point z must satisfy $\tilde{D}(z) = 0$ and $\tilde{D}(\cdot)$ is continuous, it must be the case that $\tilde{D}(q) \geq 0$ and $\tilde{D}'(q) \geq 0$ for all q. Therefore, $\tilde{J}_1'(q) \geq \tilde{J}_2'(q)$ for all q, and because $f(\cdot)$ is monotone, it follows that $a_1(q) \geq a_2(q)$ for all q.

Statement 2: To prove the second part, we differentiate (22) with respect to q, which yields in matrix form

$$r\begin{bmatrix}\tilde{J}_1' \\ \tilde{J}_2'\end{bmatrix} = \begin{bmatrix} f(\tilde{J}_1') + f(\tilde{J}_2') & \tilde{J}_1' f'(\tilde{J}_2') \\ \tilde{J}_2' f'(\tilde{J}_1') & f(\tilde{J}_1') + f(\tilde{J}_2')\end{bmatrix}\begin{bmatrix}\tilde{J}_1'' \\ \tilde{J}_2''\end{bmatrix},$$

where we used that $c'(f(x)) = x$, and we omitted the dependence of $\{\tilde{J}_1, \tilde{J}_2\}$ on q for notational convenience. If the determinant of the above matrix is positive; i.e., if

$$\det \equiv \left[f(\tilde{J}_1') + f(\tilde{J}_2')\right] - \tilde{J}_1'\tilde{J}_2' f'(\tilde{J}_1') f'(\tilde{J}_2') > 0,$$

then it is invertible. A sufficient condition for this to be true is that $c''' \geq 0$.[35] Then we have that

$$\begin{bmatrix}\tilde{J}_1'' \\ \tilde{J}_2''\end{bmatrix} = \frac{r}{\det}\begin{bmatrix} f(\tilde{J}_1') + f(\tilde{J}_2') & -\tilde{J}_1' f'(\tilde{J}_2') \\ -\tilde{J}_2' f'(\tilde{J}_1') & f(\tilde{J}_1') + f(\tilde{J}_2')\end{bmatrix}\begin{bmatrix}\tilde{J}_1' \\ \tilde{J}_2'\end{bmatrix}.$$

Note that $a_1'(q) \geq a_2'(q)$ if and only if $\tilde{J}_1''(q) \geq \tilde{J}_2''(q)$, which is true if and only if

$$\text{(23)} \quad \left[f(\tilde{J}_1') + f(\tilde{J}_2')\right]\tilde{J}_1' - \tilde{J}_1'\tilde{J}_2' f'(\tilde{J}_2') \geq -\tilde{J}_1'\tilde{J}_2' f'(\tilde{J}_1') + \left[f(\tilde{J}_1') + f(\tilde{J}_2')\right]\tilde{J}_2'$$
$$\Leftrightarrow \left[f(\tilde{J}_1') + f(\tilde{J}_2')\right](\tilde{J}_1' - \tilde{J}_2') + \tilde{J}_1'\tilde{J}_2'\left[f'(\tilde{J}_1') - f'(\tilde{J}_2')\right] \geq 0.$$

Recall that $\left[f(\tilde{J}_1') + f(\tilde{J}_2')\right]^2 > \tilde{J}_1'\tilde{J}_2' f'(\tilde{J}_1') f'(\tilde{J}_2')$ and $\tilde{J}_1' \geq \tilde{J}_2'$. Therefore, (23) is satisfied if

$$\tilde{J}_2' f'(\tilde{J}_2')(\tilde{J}_1' - \tilde{J}_2') + \tilde{J}_1'\tilde{J}_2'\left[f'(\tilde{J}_1') - f'(\tilde{J}_2')\right] \geq 0$$
$$\Leftrightarrow \tilde{J}_2'\left[\tilde{J}_1' f'(\tilde{J}_1') - \tilde{J}_2' f'(\tilde{J}_2')\right] \geq 0.$$

Noting that $f(\tilde{J}_i') = a_i$, $\tilde{J}_i' = c'(a_i)$, $f = c'^{-1}$, and $f'(\tilde{J}_i') = 1/c''(a_i) > 0$, it follows that the above inequality holds if and only if $c'(a)/c''(a)$ is increasing in a. This is true if and only if

$$\left[c''(a)\right]^2 \geq c'(a)c'''(a) \quad \text{for all} \quad a,$$

or equivalently if $c'(\cdot)$ is weakly log-concave.

[35] For details, see Cvitanić and Georgiadis (2016, 333, footnote 20).

Statement 3: Recall that in any well-defined MPE, each agent's payoff satisfies the system of ODE

$$rJ_i(q) = -\gamma_i c\left(f\left(\frac{J_i'(q)}{\gamma_i}\right)\right) + \left[f\left(\frac{J_i'(q)}{\gamma_i}\right) + f\left(\frac{J_j'(q)}{\gamma_j}\right)\right] J_i'(q) \tag{24}$$

such that

$$J_i(Q) = \alpha_i Q.$$

Define $D(\cdot) = \frac{J_1(\cdot)}{\alpha_1} - \frac{J_2(\cdot)}{\alpha_2}$, and note that $D(\cdot)$ is smooth, $D(q) = 0$ for q sufficiently small, and $D(Q) = 0$. Therefore, there must exist an interior point z such that $D'(z) = 0$, and substituting into (24) yields

$$rD(z) = -\frac{\gamma_1}{\alpha_1} c\left(f\left(\frac{J_1'(z)}{\gamma_1}\right)\right) + \frac{\gamma_2}{\alpha_2} c\left(f\left(\frac{J_2'(z)}{\gamma_2}\right)\right)$$

$$\Rightarrow r\alpha_1 D(z) = -\gamma_1 c\left(f\left(\frac{J_1'(z)}{\gamma_1}\right)\right) + \underbrace{\frac{\alpha_1\gamma_2}{\alpha_2}}_{>\gamma_1} c\left(f\left(\frac{\alpha_2}{\alpha_1}\frac{J_1'(z)}{\gamma_2}\right)\right).$$

Notice that if $D(z) \leq 0$, then this will imply that $D(q) \leq 0$ for all q, which will complete the proof. To establish $D(z) \leq 0$, notice that it suffices to show that $c(f(\lambda x))/\lambda$ is increasing in λ for all $x > 0$ and $\lambda > 0$. That is because letting $x = J_1'(z)$, $\lambda_1 = 1/\gamma_1$, and $\lambda_2 = \frac{\alpha_2}{\alpha_1\gamma_2}$, where $\lambda_1 > \lambda_2$, we will have $-\frac{c(f(\lambda_1 x))}{\lambda_1} + \frac{c(f(\lambda_2 x))}{\lambda_2} \leq 0$.

Fix x, and let $g(\lambda) = c(f(\lambda x))/\lambda$. Then

$$g'(\lambda) = \frac{x}{\lambda} c'(f(\lambda x)) f'(\lambda x) - \frac{c(f(\lambda x))}{\lambda^2} = x^2 \frac{1}{c''(f(\lambda x))} - \frac{c(f(\lambda x))}{\lambda^2} \geq 0$$

$$\Leftrightarrow (\lambda x)^2 \geq c(f(\lambda x)) c''(f(\lambda x)).$$

Letting $a = f(\lambda x) = c'^{-1}(\lambda x)$, observe that $\lambda x = c'(a)$, and substituting this into the above inequality yields

$$[c'(a)]^2 \geq c(a) c''(a),$$

which holds for all a if and only if $c(\cdot)$ is weakly log-concave. ∎

C. *Social Planner's Project Scale and Effort Level*

A classic benchmark of the literature is the cooperative environment in which agents follow the social planner's recommendations for effort. Here, we present, for completeness, the solution when the social planner chooses both the agents' level of effort and the project scale.

For a fixed project scale Q, the social planner's relevant HJB equation is

$$rS(q) = \max_{a_1,a_2}\left\{-\frac{\gamma_1}{2}a_1^2 - \frac{\gamma_2}{2}a_2^2 + (a_1+a_2)S'(q)\right\},$$

subject to $S(Q) = Q$. Each agent's first-order condition is $a_i = S'(q)/\gamma_i$, and substituting this into the HJB equation, we obtain the ordinary differential equation $rS(q) = \frac{\gamma_1+\gamma_2}{2\gamma_1\gamma_2}\left[S'(q)\right]^2$. This admits the closed-form solution for the social planner's value function $S(q) = \frac{r\gamma_1\gamma_2}{2(\gamma_1+\gamma_2)}(q-C)^2$, where $C = Q - \sqrt{\frac{2Q(\gamma_1+\gamma_2)(\alpha_1+\alpha_2)}{r\gamma_1\gamma_2}}$. Agent i's effort level is thus $a_i(q) = \frac{r\gamma_{-i}}{\gamma_1+\gamma_2}(q-C)$. Note that $a_1(q) > a_2(q)$ for all q if and only if $\gamma_1 < \gamma_2$. That is, the social planner would have the efficient agent do the majority of the work, and incur the majority of the effort cost. It is straightforward to show that the social planner's discounted payoff function is maximized at

$$Q^{**} = \frac{(\gamma_1+\gamma_2)(\alpha_1+\alpha_2)}{2r\gamma_1\gamma_2}$$

at every state of the project, and thus, the planner's preferences are time-consistent. This is intuitive, as the time-inconsistency problem is due to the agents not internalizing the externality of their actions and choices.

REFERENCES

Acemoglu, Daron, and James A. Robinson. 2008. "The Persistence of Power, Elites, and Institutions." *American Economic Review* 98 (1): 267–93.

Admati, Anat R., and Motty Perry. 1991. "Joint Projects without Commitment." *Review of Economic Studies* 58 (2): 259–76.

Aghion, Philippe, and Jean Tirole. 1997. "Formal and Real Authority in Organizations." *Journal of Political Economy* 105 (1): 1–29.

Akerlof, Robert J. 2012. "A Theory of Authority." https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2140472.

Atkinson, Kendall E., Weimin Han, and David Stewart. 2011. *Numerical Solution of Ordinary Differential Equations*. Hoboken: Wiley.

Bagwell, Kyle, and Robert W. Staiger. 2002. *The Economics of the World Trading System*. Cambridge, MA: MIT Press.

Baron, David P. 1996. "A Dynamic Theory of Collective Goods Programs." *American Political Science Review* 90 (2): 316–30.

Battaglini, Marco, and Stephen Coate. 2008. "A Dynamic Theory of Public Spending, Taxation, and Debt." *American Economic Review* 98 (1): 201–36.

Battaglini, Marco, and Bård Harstad. 2016. "Participation and Duration of Environmental Agreements." *Journal of Political Economy* 124 (1): 160–204.

Battaglini, Marco, Salvatore Nunnari, and Thomas R. Palfrey. 2014. "Dynamic Free Riding with Irreversible Investments." *American Economic Review* 104 (9): 2858–71.

Beshkar, Mostafa, and Eric W. Bond. 2017. "Cap and Escape in Trade Agreements." *American Economic Journal: Microeconomics* 9 (4): 171–202.

Beshkar, Mostafa, Eric W. Bond, and Youngwoo Rho. 2015. "Tariff binding and overhang: Theory and evidence." *Journal of International Economics* 97 (1): 1–13.

Besley, Timothy, and Torsten Persson. 2011. *Pillars of Prosperity: The Political Economics of Development Clusters*. Princeton: Princeton University Press.

Bester, Helmut, and Daniel Krähmer. 2008. "Delegation and incentives." *RAND Journal of Economics* 39 (3): 664–82.

Bonatti, Alessandro, and Johannes Hörner. 2011. "Collaborating." *American Economic Review* 101 (2): 632–63.

Bonatti, Alessandro, and Heikki Rantakari. 2016. "The Politics of Compromise." *American Economic Review* 106 (2): 229–59.

Bowen, T. Renee. 2013. "Forbearance in Optimal Multilateral Trade Agreement." https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1958291.

Bowen, T. Renee, Ying Chen, and Hülya Eraslan. 2014. "Mandatory versus Discretionary Spending: The Status Quo Effect." *American Economic Review* 104 (10): 2941–74.

Callander, Steven. 2008. "A Theory of Policy Expertise." *Quarterly Journal of Political Science* 3 (2): 123–40.

Callander, Steven, and Bård Harstad. 2015. "Experimentation in Federal Systems." *Quarterly Journal of Economics* 130 (2): 951–1002.

Compte, Olivier, and Philippe Jehiel. 2004. "Gradualism in Bargaining and Contribution Games." *Review of Economic Studies* 71 (4): 975–1000.

Cvitanić, Jakša, and George Georgiadis. 2016. "Achieving Efficiency in Dynamic Contribution Games." *American Economic Journal: Microeconomics* 8 (4): 309–42.

Diermeier, Daniel, and Pohan Fong. 2011. "Legislative Bargaining with Reconsideration." *Quarterly Journal of Economics* 126 (2): 947–85.

Dixit, Avinash, Gene M. Grossman, and Faruk Gul. 2000. "The Dynamics of Political Compromise." *Journal of Political Economy* 108 (3): 531–68.

Fershtman, Chaim, and Shmuel Nitzan. 1991. "Dynamic voluntary provision of public goods." *European Economic Review* 35 (5): 1057–67.

Francois, Joseph F., Brad McDonald, and Håkan Nordström. 1994. "The Uruguay Round: A Global General Equilibrium Assessment." Centre for Economic Policy Research (CEPR) Discussion Paper DP1067.

Galbraith, John Kenneth. 1952. *American Capitalism: The Concept of Countervailing Power*. Abingdon, UK: Routledge.

Georgiadis, George. 2015. "Project and Team Dynamics." *Review of Economic Studies* 82 (1): 187–218.

Georgiadis, George. 2017. "Deadlines and infrequent monitoring in the dynamic provision of public goods." *Journal of Public Economics* 152: 1–12.

Georgiadis, George, Steven A. Lippman, and Christopher S. Tang. 2014. "Project design with limited commitment and teams." *RAND Journal of Economics* 45 (3): 598–623.

Georgiadis, George, and Christopher S. Tang. 2017. "Project Contracting Strategies for Managing Team Dynamics." In *Handbook of Information Exchange in Supply Chain Management*, edited by Albert Y. Ha and Christopher S. Tang, 89–105. Berlin: Springer.

Goldin, Ian, Odin Knudsen, and Dominique van der Mensbrugghe. 1993. *Trade Liberalisation: Global Economic Implications*. Washington, DC: World Bank.

Harrison, Glenn W., Thomas F. Rutherford, and David G. Tarr. 1997. "Quantifying the Uruguay Round." *Economic Journal* 107 (444): 1405–30.

Hirsch, Alexander V., and Kenneth W. Shotts. 2015. "Competitive Policy Development." *American Economic Review* 105 (4): 1646–64.

Kamien, Morton I., and Nancy L. Schwartz. 2012. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. 2nd ed. Mineola, NY: Dover Publications.

Kessing, Sebastian G. 2007. "Strategic Complementarity in the Dynamic Private Provision of a Discrete Public Good." *Journal of Public Economic Theory* 9 (4): 699–710.

Levhari, David, and Leonard J. Mirman. 1980. "The Great Fish War: An Example Using a Dynamic Cournot-Nash Solution." *Bell Journal of Economics* 11 (1): 322–34.

Levy, Nadav. 2014. "Domain knowledge, ability, and the principal's authority relations." *RAND Journal of Economics* 45 (2): 370–94.

Lizzeri, Alessandro, and Nicola Persico. 2001. "The Provision of Public Goods under Alternative Electoral Incentives." *American Economic Review* 91 (1): 225–39.

Maggi, Giovanni. 2014. "International Trade Agreements." In *Handbook of International Economics*, Vol. 4, edited by Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, 317–90. Amsterdam: North-Holland.

Marx, Leslie M., and Steven A. Matthews. 2000. "Dynamic Voluntary Contribution to a Public Project." *Review of Economic Studies* 67 (2): 327–58.

Maskin, Eric, and Jean Tirole. 2001. "Markov Perfect Equilibrium: I. Observable Actions." *Journal of Economic Theory* 100 (2): 191–219.

Nordhaus, William. 2015. "Climate Clubs: Overcoming Free-Riding in International Climate Policy." *American Economic Review* 105 (4): 1339–70.

Page, Sheila, Michael Davenport, and Adrian Hewitt. 1991. *The GATT Uruguay Round: Effects on Developing Countries*. London: Overseas Development Institute.

Preeg, Ernest H. 1995. *Traders in a Brave New World: The Uruguay Round and the Future of the International Trading System*. Chicago: University of Chicago Press.

Romer, Thomas, and Howard Rosenthal. 1979. "Bureaucrats Versus Voters: On the Political Economy of Resource Allocation by Direct Democracy." *Quarterly Journal of Economics* 93 (4): 563–87.

Strulovici, Bruno. 2010. "Learning While Voting: Determinants of Collective Experimentation." *Econometrica* 78 (3): 933–71.

Yildirim, Huseyin. 2006. "Getting the Ball Rolling: Voluntary Contributions to a Large-Scale Public Project." *Journal of Public Economic Theory* 8 (4): 503–28.

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