Abstention in Elections with Asymmetric Information and Diverse Preferences.

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Abstract

We analyze a model of a two-candidate election with costless voting in which voters have asymmetric information and diverse preferences. We demonstrate that a strictly positive fraction of the electorate will abstain and, nevertheless, elections effectively aggregate voter's private information. Using examples we show how more informed voters are more likely to vote than their less informed counterparts. However, increasing the fraction of the electorate that is informed may lead to higher levels of abstention. We conclude by showing that a biased distribution of information can lead to a biased voting population but does not lead to biased outcomes.

1 Introduction

The following stylized facts occupy a central place in the empirical literature on voting and elections in the United States: (1) better educated and wealthy individuals participate in elections more frequently than others (Wolfinger and Rosenstone 1980); (2) over the last forty years participation in elections has declined significantly while education levels have increased (Brody 1978); In addition, recent research suggests that there is a correlation between participation and policy outcomes.[1] Facts (1) and (2) together with the likely correlation between participation and outcomes have led some to conclude that election outcomes are increasingly biased towards the wealthy and better educated (Lijphart 1997).

In this paper we analyze a game-theoretic model of participation in large elections. We derive necessary and sufficient conditions for abstention and show how the level of abstention depends upon the information environment. We then show that for a variety of model specifications our model gives results that are consistent with the stylized facts mentioned above without producing biased election outcomes.

The model presented here is an extension of a model developed by Feddersen and Pesendorfer (1996). They examine a setting with two alternatives and two states of the world. In their framework voters have identical state-dependent preferences preferring alternative 1 in state 1 and alternative 2 in state 2.[2] A subset of voters know the true state while others have no information. They show that equilibrium behavior in large elections has the following features:

1. uninformed voters may have a strict incentive to abstain even though they are not indifferent between the alternatives[3];

2. abstention levels may be high even in large electorates;

3. even with strategic abstention, elections satisfy "*full information equivalence*" i.e., the winning alternative is the alternative that would win an election in which all voters were fully informed and voted for their preferred alternative.

Our model generalizes their framework in three ways. First, we introduce a continuum of voter preference types and, hence, in a typical electorate no pair of voters has

[1] For example, Hill, Leighley and Hinton-Andersson (1995) conclude that welfare spending is correlated with participation of lower income groups. However, the connection between turnout and outcomes is disputed by Teixera (1992). See also Bartels (1996) and Lipjhart (1997).

[2] They also introduce voters who are "partisans" in the sense that their preferences do not depend upon the state.

[3] Suppose there are three voters who must choose between candidate 1 and 2 using majority rule. There are two states of the world: state 1 and state 2. State 1 is more likely than state 2. All of the voters prefer candidate 1 in state 1 and candidate 2 in state 2. All voters know that exactly one of them knows the true state. It is an equilibrium for those voters who do not know the state to abstain and to let the informed voter decide the outcome. To see why this is the case suppose that in the event the election is tied a coin is tossed to determine the winner. In the proposed equilibrium uninformed voters strictly prefer to abstain because their vote only changes the outcome when they vote for the wrong candidate–the candidate not supported by the perfectly informed voter–and the wrong candidate wins the coin toss.

exactly identical preferences. Preference diversity allows us to analyze participation when information and preferences are correlated. Second, we consider environments with more than two states. Third, we consider the effects of noisy information.

We assume that voter preferences depend on a preference type and on a state which represents the relative "quality" of the two alternatives. Quality refers to a characteristic of the alternatives that all voters enjoy. For example, suppose that an election is held to decide whether a bridge should be built. Each voter's preference type is determined by the frequency they would use the bridge and the cost of building it. The "state" in this example corresponds to the cost of the bridge. As the cost of the bridge increases all voters like the bridge less. In the case where two candidates are running we may view the candidates' "character" as a quality dimension. Candidates with "good character" stick closely to their announced position while those with "poor character" may not. Therefore, all risk averse voters prefer candidates with better character.

We assume that voters are asymmetrically informed about the quality of the alternatives. A variety of features of large elections make asymmetric information a realistic assumption. For example, not all voters follow elections in the media or pay attention to campaign advertisements. Moreover, campaigns target voter groups to receive specific messages. Finally, voters' ability to understand the messages they observe depends on their ability and expertise.

1.1 Theoretical Results

In the first part of the paper we present our model and analytical results. We show that the full-information equivalence result mentioned above is robust. Candidate 1 wins with high probability in states in which a majority of the electorate strictly prefers alternative 1 to alternative 2 and loses with high probability otherwise.

We also give necessary and sufficient conditions for abstention in a large electorate. We assume the set of states is finite and, therefore, there is a *critical state* in which the fraction of the electorate that prefers alternative 1 to alternative 2 is closest to 1/2. By the *critical margin* we denote the absolute value of the difference in the expected fraction of voters who prefer alternative 1 to alternative 2 in the critical state. Our main theoretical result is that in a large electorate the fraction of voters who abstain is proportional to the critical margin.

The fact that abstention can occur under some model specifications but not others may raise concerns about our model as an explanation for abstention. The concern is as follows: surely there is always some state of the world in which exactly half of any population would prefer one alternative to the other. Our results would then be taken as evidence that asymmetric information cannot provide an explanation for abstention.

Such an argument misunderstands what a state represents in our model. A state represents what a very large number of people may learn about the relative quality of the two alternatives. Thus, in the bridge example, it may be the case that the true cost of the bridge ranges continuously from \$50 to \$50,000,000 but the most any very large number of people could ever learn is that the true cost is either above or below \$25,000,000. In this case even though the true cost of the bridge is a continuous random variable it is appropriate to assume there are two states and hence the margin of victory in the critical state may be large.

1.2 Empirical Predictions

In the second part of the paper we provide comparative statics results for a series of examples that are consistent with the stylized facts mentioned above.

First, we analyze an example of a model in which some voters are perfectly informed while others have no information. Informed voters always participate while, depending on their preference type, uninformed voters abstain. In this setting we show that increasing the probability a voter is perfectly informed may result in higher levels of abstention. Thus, even though informed voters always participate, an increase in the fraction of informed voters changes the behavior of the uninformed so that the overall level of abstention rises.

Next, we provide examples with noisy signals. Our findings are as follows:

1. Individuals with better information are more likely to participate than individuals with worse information. At the same time, increasing the fraction of the electorate that receives a noisy signal can increase levels of abstention among *both* the informed and uninformed alike.

2. The voting electorate is biased towards the winning alternative, i.e., whenever the right alternative wins then voters with preference types to the left of the median are more likely to abstain and vice versa.

3. A biased distribution of information may result in an electorate that is systematically unrepresentative.

4. While the participating electorate may be systematically unrepresentative it does not follow that election outcomes are also unrepresentative.

2 Related Literature

There are a variety of papers that focus on the consequences of costs to vote on turnout. See Palfrey and Rosenthal (1983, 1985), Riker and Ordeshook (1968), Feddersen (1992), Morton (1991) and Lohmann (1993). For reviews of both the theoretical and empirical literature on turnout see Feddersen and Pesendorfer (1996), Aldrich (1993), Grofman (1993) or Matsusaka (1992). Katz and Ghirardato (1997) develop a decision-theoretic explanation of abstention that does not rely on costs to vote. Their explanation of abstention relies on voters having non-standard utility functions.

Traditionally, models of participation in elections have been decision-theoretic, and have focused on costs to participate as the key variable predicting participation (Riker and Ordeshook 1967, Matsusaka 1994). However, a decision theoretic model of participation runs into the following difficulty: empirically, education level is positively correlated with turnout.[4] To explain this cross-sectional variation, a decision-theoretic model must assume it is less costly for those with higher education to participate.[5] However, in the past forty years education levels in the population have increased and at the same time turnout has declined (Brody 1978). A decision theoretic model will have trouble simultaneously explaining the longitudinal phenomena of declining overall turnout while membership in high turnout categories grows.[6]

There is a substantial empirical literature that examines the relationship between participation and outcomes (Bartels 1996; Hill, Leighley and Hinton-Andersson 1995; Teixiera 1992; see Lijphart 1997 for a review of this literature). Some papers conclude that full participation would not change election results substantially (Teixeira 1992) while others argue that support for parties on the left would substantially increase (Lijphart 1997).

The empirical literature assumes that voter behavior will stay the same as the level of participation of other voters changes. This may be appropriate in a decision-theoretic model, however, in a game-theoretic voting model like ours it is not. In a game-theoretic model optimal behavior is predicated on the behavior of other agents. In our model the behavior of all agents changes if, e.g., voting is mandatory or the information levels of some voters change.[7] However, the election outcomes (as a function of the state) would remain unchanged.

Our model assumes that the number of voters is uncertain and distributed according to a Poisson distribution. The idea of using Poisson distributions to analyze large anonymous games is borrowed from Myerson (1997a,b). The advantage of the Poisson distribution is that it permits easy calculation of equilibrium profiles and examples even in very large electorates.

3 Model

We analyze a two alternative election. Alternatives are denoted by $j \in \{1, 2\}$. A voter's utility depends on a preference parameter $x \in [-1, 1] = X$, the chosen alternative j, and the state $s \in S \subset [0, 1]$. We assume the set of states S is finite and that $\{0, 1\} \subset S$, i.e., the smallest state is 0 and the largest state is 1.

We denote by $u(j, s, x)$ the utility function of voters. Let

$$v(s, x) \equiv u(2, s, x) - u(1, s, x) \tag{1}$$

denote the utility difference of a voter type x between alternative 2 and alternative 1 in state s.

At the beginning of the game nature selects a state s and an electorate. The electorate is chosen as follows. First, nature selects a number of players according

[4] Wolfinger and Rosenstone 1980.

[5] See Matsusaka (1992) for an example of such a theory.

[6] Of course, a decision theoretic model is consistent with the data if costs to vote may vary over time.

[7] See Bartels (1996) for an analysis in which it is assumed that the behavior of informed voters stays the same when the information levels of other voters changes.

to the Poisson distribution with parameter ν. Thus, the probability that there are n players is given by

$$\Pr(n) = \frac{e^{-\nu}\nu^n}{n!}$$

Second, every player is independently assigned a preference type according to the distribution function $F(x)$. Third, every player receives a signal $m \in \{\emptyset, 1, ..., M\} = \mathcal{M}$, where $\emptyset$ describes an uninformative signal. We assume that conditional on the state s the signal is independently distributed across agents. The probability that an agent receives the signal m in state s is $p(m|s)$.

We make the following assumptions:

Assumption 1 *$v(x, s)$ is defined for all $(x, s) \in [-1, 1] \times [0, 1]$; it is continuous in x and strictly increasing in (x, s). Furthermore, $v(-1, s) < 0$ for all s and $v(1, s) > 0$ for all $s \in [0, 1]$.*

Assumption 2 *The state s is chosen according to the probability distribution $g(s)$ where $g(s) > 0$ for all $s \in S$.*

Assumption 3 *The distribution function $F(x)$ has a continuous density $f(x)$ and $f(x)$ is bounded away from zero on $[-1, 1]$.*

Assumption 4 *$p(\emptyset|s) = 1 - q$ for some $0 < q \leq 1$ and all s.*

Assumption 5 *(SMLRP) Let (m, s) and (m', s') be signal-state pairs such that $m' > m$ and $s' > s$, where $m, m' \neq \emptyset$. Then $p(m|s)p(m'|s') \geq p(m|s')p(m'|s)$ and the inequality is strict if $p(m|s)p(m'|s') > 0$.*

Assumption 1 says that the payoff difference between the two alternatives is increasing both in the preference type and in the state. Thus, higher states make alternative 2 more attractive. Assumption 2 implies that every state occurs with positive probability and Assumption 3 implies that every open interval of preference types occurs with positive probability. Assumption 4 implies that signal $\emptyset$ does not provide any information about the state s and that the probability any voter type receives the uninformative signal is strictly less than 1 i.e., in a large electorate some voters will have private information with high probability. Assumption 5 says all signals other than $\emptyset$ satisfy the strict monotone likelihood ratio property. Specifically, when the ratio $\frac{p(m|s')}{p(m|s)}$ is defined, it is increasing in m if $s' > s$ and $m \in \{1, 2, ..., M\}$. Informally, SMLRP says that higher signals are more likely in higher states. For a careful discussion of the monotone likelihood ratio property see Milgrom (1981).

4 Equilibrium

Each player chooses an action $a \in A = \{0, 1, 2\}$ where $a = 0$ denotes abstention, $a = 1$ denotes a vote for alternative 1 and $a = 2$ denotes a vote for alternative 2. By n_a we denote the number of agents who take action a. We assume that the election outcome is determined by majority rule. Alternative 1 is the winner if and only if $n_2 \leq n_1$. This implies that in case of a tie, 1 will be the winner of the election.[8]

A symmetric strategy profile is a measurable function $\sigma : [-1,1] \times \mathcal{M} \to \Delta(A)$. The probability that a player with preference parameter x and signal m takes action a is $\sigma_a(x,m)$.

Given a symmetric strategy profile σ the ex-ante probability that a voter takes action a in state s is denoted by $t_a(s)$ where

$$t_a(s) = \sum_{m \in \mathcal{M}} p(m|s) \int_{-1}^{1} \sigma_a(x,m) dF(x) \tag{2}$$

For a given strategy profile σ we define the random variables $n_a(s)$ to be the number of agents who take action a in state s. Recall that the number of agents n is a Poisson random variable with parameter ν. Furthermore, signals are conditionally independent and preference types are chosen independently. It follows that $n_a(s), a \in \{\emptyset, 1, 2\}$ are independent Poisson random variables (see Myerson 1997a for an extensive discussion of the properties Poisson distribution in large voting games). The expected number of agents who take action a in state s is denoted $\nu_a(s)$ where

$$\nu_a(s) = \nu \cdot t_a(s).$$

It follows from the properties of the Poisson distribution and the symmetry of the strategy profile that from the perspective of any given voter, the number of *other* voters who take action a in state s is described by the same random variable $n_a(s)$.

The results in this paper depend on the fact that a voter only influences the outcome of an election when her vote is pivotal. In our model there are two ways in which a vote can be pivotal: a voter can influence the election outcome if the election is tied ($n_2 = n_1$) or if alternative 2 is ahead by one vote ($n_2 = n_1 + 1$). In the event that $n_2 = n_1 + 1$ voting for 2 and abstaining both lead to the outcome 2 and voting for 1 leads to the outcome 1. We label this event piv_1 and say that a vote is pivotal for alternative 1. In the event that $n_2 = n_1$ voting for 1 and abstaining both lead to the outcome 1 and voting for 2 leads to the outcome 2. We label this event piv_2. We label the event that a vote is pivotal (either piv_1 or piv_2) as piv. Note that the notation $n_1 = n_2$ or $n_1 + 1 = n_2$ refers to the outcome of the election from the perspective of a particular voter *without* his vote.

Since $n_1(s)$ and $n_2(s)$ are independent Poisson random variables, we can write the probabilities of the pivotal events for a given state s and a symmetric strategy profile σ as follows:

$$\begin{aligned} \Pr\{piv_1|s\} &= \sum_{x=0}^{\infty} \Pr(n_1(s) = x) \Pr(n_2(s) = x+1) \qquad (3) \\ &= \sum_{x=0}^{\infty} \frac{e^{-\nu(t_1(s)+t_2(s))}(\nu t_2(s))^{x+1}(\nu t_1(s))^x}{x!(x+1)!}. \end{aligned}$$

[8]This choice of tie breaking rule is made for technical convenience. All of our results would go through if in the case of a tie alternative 1 wins with probability $\pi \in (0,1)$.

The probability a vote is pivotal for alternative 2 as:

$$\begin{aligned}\Pr\{piv_2|s\} &= \sum_{x=0}^{\infty} \Pr(n_1(s)=x)\Pr(n_2(s)=x) \\ &= \sum_{x=0}^{\infty} \frac{e^{-\nu(t_1(s)+t_2(s))}(\nu t_2(s))^x(\nu t_1(s))^x}{x!x!};\end{aligned} \tag{4}$$

and the probability a vote is pivotal for either alternative as

$$\Pr\{piv|s\} = \Pr\{piv_1|s\} + \Pr\{piv_2|s\}. \tag{5}$$

Voters form beliefs about the distribution over states conditional on their private information and the event a vote is pivotal. If the probability an agent votes for alternative 2, $t_2(s)$, is greater than zero for all s then it follows from equations (3)-(4) that $\Pr(piv_j|s) > 0$ for all s and $j \in \{1,2\}$. Therefore, the probability distribution over states conditional on being pivotal for alternative $j \in \{1,2\}$ is well defined and given by

$$\beta(s|piv_j) = \frac{\Pr(piv_j|s)g(s)}{\sum_{w\in S}\Pr(piv_j|w)g(w)}. \tag{6}$$

The probability distribution over states conditional on being pivotal and observing signal m is then given by:

$$\beta(s|piv_j,m) = \frac{\Pr(piv_j|s)g(s)p(m|s)}{\sum_{w\in S}\Pr(piv_j|w)g(w)p(m|w)} = \frac{\beta(s|piv_j)p(m|s)}{\sum_{w\in S}\beta(w|piv_j)p(m|w)}. \tag{7}$$

Our first objective is to find a simple characterization of equilibrium behavior. Recall that we assumed that the payoff of alternative 2 is increasing in the preference type. Hence, a natural conjecture is for strategies to be monotone in the following sense: if type x votes for alternative 2 upon observing signal m then all types $x' > x$ vote for 2 upon observing the same signal. Similarly, if type x' votes for 1 after observing signal m then all types $x < x'$ vote for 1 after observing the same signal. Furthermore, (by SMLRP) agents place more weight on higher states when they observe a higher signal. Therefore, we might expect voting behavior to be monotone in the signal: if an agent with preference type x votes for 2 upon receiving signal m then he also votes for 2 upon receiving signal $m' > m$. The following definition of ordered cutpoints captures these two types of monotonicity.

Definition 1 *A strategy σ is characterized by* ordered cutpoints *if there are preference types $\{(x_m^1, x_m^2)\}_{m\in\mathcal{M}}$ with the following properties: for every $m \in \mathcal{M}$ $x_m^1 \leq x_m^2$; for every $j \in \{1,2\}$ $1 > x_1^j \geq .... \geq x_M^j > -1$; and $\sigma_1(x,m) = 1$ if $x < x_m^1$, $\sigma_2(x,m) = 1$ if $x > x_m^2$, and $\sigma_0(x,m) = 1$ if $x_m^1 < x < x_m^2$.*

Observe that if a strategy profile is characterized by ordered cutpoints then voters of type (x,m) with $x < x_M^1$ vote for alternative 1 regardless of their private signal whereas those for whom $x > x_1^2$ vote for alternative 2 regardless of their private signal. We say that voter types with $x \in (x_M^1, x_1^2)$ take *informative action*.

In the following we consider symmetric Nash equilibria. The first proposition demonstrates that every symmetric Nash equilibrium can be characterized by ordered cutpoints.

Proposition 1 *Suppose Assumptions 1-4 hold. Then a symmetric equilibrium exists and any such equilibrium can be characterized by ordered cutpoints.*

Figure 1 illustrates a symmetric strategy profile in a model with two informative signals that is consistent with ordered cutpoints.

<Figure 1 about here>

The fact that the cutpoints are strictly ordered together with SMLRP implies that in any symmetric equilibrium the expected vote share for alternative 1 is strictly decreasing in the state while the expected vote share for alternative 2 will be strictly increasing. This is demonstrated by the following proposition. We include the proof in the text because it is short and because we use the monotonicity of the vote shares repeatedly below.

Proposition 2 *In a symmetric equilibrium $t_1(s)$ is strictly decreasing in s and $t_2(s)$ is strictly increasing in s.*

Proof. Since a symmetric equilibrium can be characterized by ordered cutpoints we can rewrite Equation (2) as follows:

$$t_1(s) = \sum_{m \in \mathcal{M}} p(m|s) F(x_m^1) \tag{8}$$

and

$$t_2(s) = \sum_{m \in \mathcal{M}} p(m|s)(1 - F(x_m^2)) \tag{9}$$

Since x_m^j is strictly decreasing in m for every $j \in \{1, 2\}$ it follows from a standard property of SMLRP that $t_1(s)$ is strictly decreasing and $t_2(s)$ is strictly increasing in s. ■

The fact that every symmetric equilibrium is characterized by ordered cutpoints does not imply that there are always some voters who abstain. Consider the case in which there are two states $S = \{0, 1\}$and two messages $\mathcal{M} = \{1, 2\}$. Suppose each voter receives perfect information about the state s and message 1 implies the state is 0 while message 2 implies the state is 1. Thus, every voter knows the true state and, except for one type, has a strictly dominant strategy to vote for one of the two alternatives. In particular, voters get no additional information by conditioning on the event a vote is pivotal for either alternative i.e., $\beta(s|piv_1, m) = \beta(s|piv_2, m)$ for every s and m and hence it must be the case that $x_m^1 = x_m^2$ for all m.

We now show that if there are any messages that are not *perfectly informative* a strictly positive fraction of the electorate will abstain. We call a signal $m \in \mathcal{M}$ perfectly informative if there is a unique state s_m in which the signal can be received with strictly positive probability i.e., $p(m|s) > 0$ if and only if $s = s_m$.

Proposition 3 *Suppose Assumption 1 holds. Suppose σ is an equilibrium strategy and let (x_m^j), $j = 1, 2$ be the corresponding cutpoints. If m is not perfectly informative then $x_m^2 - x_m^1 > 0$.*

To gain an intuition for Proposition 3 consider a (not perfectly informed) voter who is indifferent between voting for alternative 2 and abstaining. This implies that conditional on the event that $n_1 = n_2$ the voter is indifferent between the two alternatives. Such a voter strictly prefers abstaining to voting for alternative 1. To see this note that voting for 1 instead of abstaining only makes a difference in the event that $n_2 = n_1 + 1$. Because the expected vote share of alternative 2 is strictly increasing in s the probability distribution over states conditional on $n_2 = n_1 + 1$ puts more weight on states that are favorable to alternative 2.[9] Thus, alternative 2 is even more desirable in the event that $n_2 = n_1 + 1$ than in the event $n_2 = n_1$ and the voter strictly prefers to abstain rather than vote for alternative 1. As a consequence there is an interval of preference types who prefer abstaining to voting for either alternative.[10]

5 Equilibrium Characterization for Large Populations

Proposition 3 does not exclude the possibility that the expected fraction of voters who choose to abstain in equilibrium converges to zero as the population goes to infinity. To conclude that our model predicts that a positive fraction of the electorate abstains we need a stronger result: we need to demonstrate that the fraction of voters who abstain stays bounded away from zero as the expected population size becomes very large.

In this section, we analyze equilibrium voting behavior for large electorates. We do so by considering a sequence of populations in which the expected size of the population grows to infinity. Recall that ν denotes the expected number of agents. In the following we consider a sequence of populations $(\nu_k)_{k=1}^{\infty}$ where $\nu_k \to \infty$. Along the sequence we fix the information structure defined by $g(s), p(m|s)$ and $F(x)$. We use the subscript k to indicate the dependence of the equilibrium on the parameter ν_k. Thus, for example, σ_k denotes a voting equilibrium for a population with expected size ν_k.

5.1 Preliminary Results

In this section we demonstrate three technical lemmas that will allow us to prove our main results on abstention and information aggregation. The main results of this section (Lemma 2 and Lemma 3) are concerned with the distribution over states conditional on a vote being pivotal. Since every voter behaves *as if* a vote is pivotal, characterizing this distribution is essential for our analysis. Basically, the results

[9]Technically, the distribution over states conditional on $n_2 = n_1 + 1$ first order stochastically dominates the probability distribution over states conditional on $n_2 = n_1$.

[10]Proposition 3 demonstrates that every symmetric equilibrium (under the stated assumptions) leads to abstention. Note that this relies on the assumption that the number of voters is uncertain. If, instead, the number of voters was common knowledge then there are also equilibria in which no abstention occurs. However, in large elections, it seems unreasonable to assume that every voter can *exactly* predict the number of other voters.

together imply that conditional on a vote being pivotal, the probability distribution over states must take the following simple form:

1. In the limit voters can put positive probability on at most two states s^1 and s^2 (Lemma 2).

2. The states s^1 and s^2 may be characterized as follows: Consider an election in which each agent knows the state and votes for her preferred alternative. Then, s^1 is the largest state in which alternative 1 wins and s^2 is the smallest state in which alternative 2 wins. (Lemma 2).

3. While uncertainty is limited to two states it does not disappear completely, i.e., voters assign non-negligible probability to both states in the limit (Lemma 3). This result requires the further assumption that not too many agents have perfectly informative signals.

The three items above should serve as a non-technical summary of the results of this subsection. Readers not interested in the technical details may skip ahead to subsection 5.2 below.

We define the ratio of expected vote shares for the two alternatives given state $s \in S$ and profile σ as:

$$\rho_k(s) = \frac{t_{1k}(s)}{t_{2k}(s)} \tag{10}$$

Lemma 1 says that in large elections the probability that a vote is pivotal for alternative 2 ($n_1 = n_2$) differs from the probability a vote is pivotal for alternative 1, ($n_1 = n_2 + 1$), by a factor that is approximately equal to $\sqrt{\rho_k(s)}$. Lemma 1 was first proven by Myerson (1997b).

Lemma 1 *1. There is an $\varepsilon > 0$ such that $t_{1k}(s) > \varepsilon$ and $t_{2k}(s) > \varepsilon$ for all k.*

2. As $k \to \infty$ $\left|\frac{\Pr_k\{piv_2|s\}}{\Pr_k\{piv_1|s\}} - \sqrt{\rho_k(s)}\right| \to 0$.

The first part of the Lemma follows from immediately from Assumption 1. The second part follows from a calculation of limits using the Poisson distribution. We give a proof in the Appendix. An alternative proof can be found in Myerson (1997b). Lemma 1 allows us to approximate the relative likelihood of the pivot probabilities by $\sqrt{\rho_k(s)}$. Recall that by Proposition 2 $t_{1k}(s)$ is decreasing in s and $t_{2k}(s)$ is increasing in s. Hence $\sqrt{\rho_k(s)}$ is decreasing in s. Therefore, a consequence of Lemma 1 is that the likelihood ratio of piv_1 and piv_2 is decreasing in s.

Let $x(s)$ denote the preference type who is indifferent between the two alternatives in state s. Thus $x(s)$ is defined by the equation

$$v(x(s), s) = 0 \tag{11}$$

Assumption 1 implies that $x(s)$ is well defined for every $s \in S$ and strictly decreasing in s. All voters with preference types $x < x(1)$ prefer alternative 1 in every state and hence have a strictly dominant strategy to vote for 1. Similarly, voters with preference types $x > x(0)$ always vote for alternative 2. If there are many voters and if $F(x(1)) > 1/2$ ($F(x(0)) < 1/2$) then with high probability the majority of voters prefers alternative 1 (alternative 2) irrespective of their information and hence alternative 1 (alternative 2) must win the election with high probability in every state. The following assumption rules out these cases.

Assumption 6 $F(x(0)) > 1/2, F(x(1)) < 1/2.$

We define s^1 to be the largest state in which 1 is elected if all voters know the state, i.e.,

$$s^1 = \max\{s : F(x(s)) \geq 1/2\}$$

Similarly, we define s^2 to be the smallest state in which 2 is elected if all voters know the state, i.e.,

$$s^2 = \min\{s : F(x(s)) < 1/2\}$$

Clearly $s^1 < s^2$ and (s^1, s^2) is a pair of consecutive states. Also note that s^1, s^2 are well defined if Assumption 6 holds.

The following Lemma says that in a voting equilibrium with a large electorate beliefs over states conditional on a voter being pivotal are concentrated on s^1 and s^2.

Lemma 2 *Suppose Assumptions 1-6 hold. Consider a sequence of voting equilibria. Then $\beta_k(s^1|piv) + \beta_k(s^2|piv) \to 1$ as $k \to \infty$. Also, for $j = 1, 2$, $\beta_k(s^1|piv_j) + \beta_k(s^2|piv_j) \to 1$ as $k \to \infty$.*

First, observe that in a large election, conditional on a vote being pivotal, voters must put almost all probability weight on those states in which the expected vote shares for each alternative are closest to being tied. Second, note that the set of states for which the election is closest to being tied consists either of a unique state or of a pair of consecutive states. Suppose the set is $\{s', s''\}$. We need to show that in fact $\{s', s''\} = \{s^1, s^2\}$. Suppose that $s^1 > s'$ and $s^1 > s''$. The fact that all beliefs are concentrated on the set $\{s', s''\}$ implies that $t_{1k}(s) > F(x(s^1)) > 1/2$ for $s \leq s^1$ since all voters with preference types $x > x(s^1)$ must vote for alternative 1 irrespective of their private information in states $s \leq s^1$. But now recall that the vote share of alternative 1 is decreasing in s and hence $1/2 < t_{1k}(s^1) < t_{1k}(s'')$ and therefore it is more likely that the election is tied in state s^1 than in either s' or s'' and we have a contradiction.

In the following Lemma we refine the characterization of the limit support of the distribution $\beta_k(s|piv)$: we give conditions under which both $s^1$and s^2 have positive probability conditional on a voter being pivotal. Let

$$\bar{M} = \{m : p(m, s^1)p(m, s^2) = 0 \text{ and } p(m, s^1) + p(m, s^2) > 0\}$$

denote the set of signals that allow an agent to distinguish states s^1 and s^2 with certainty. In other words, an agent who receives a signal $m \in \bar{M}$ and knows that $s \in \{s^1, s^2\}$ is perfectly informed. The expression $(F(x(s^1)) - F(x(s^2))\sum_{m\in\bar{M}} p(m|s)$

denotes the expected fraction of voters who receive signals in $\bar{M}$ in state s and who have state dependent preferences over the states in $\{s^1, s^2\}$. We call such voters perfectly informed swing voters.

Assumption 7 requires that $F(x(s^2)) < 1/2 < F(x(s^1))$ and puts a bound on the fraction of perfectly informed swing voters.

Assumption 7 $2|\frac{1}{2} - F(x(s^i))| > (F(x(s^1)) - F(x(s^2)) \sum_{m \in \bar{M}} p(m|s)$ *for all* s *and* $i = 1, 2$.

Assumption 7 is satisfied if there are no perfectly informative signals (i.e., $\bar{M}$ is empty) and $F(x(s^2)) < 1/2 < F(x(s^1))$. However, even if some fraction of agents receives perfectly informative signals, Assumption 7 is satisfied as long as $F(x^1)$ and $F(x^2)$ are approximately symmetric around $1/2$: if $F(x^1) = 1 - F(x^2)$ then Assumption 7 holds as long as there is a strictly positive probability that agents receive a signal that does not perfectly discriminate between s^1 and s^2.

The next lemma uses Assumption 7 to show that, conditional on a vote being pivotal, an agent believes both states s^1 and s^2 occur with probability bounded away from zero.

Lemma 3 *Suppose Assumptions 1-7 hold. Then there is an* $\epsilon > 0$ *and a* $k' < \infty$ *such that* $\beta_k(s^1|piv) \geq \epsilon, \beta_k(s^2|piv) \geq \epsilon$ *for all* $k \geq k'$.

To understand why Assumption 7 is needed in Lemma 3 consider the following example.

Example Suppose there are two states $s = 1, 2$. Further suppose that $F(x(1)) = 0.75$, i.e., the probability that a randomly drawn voter prefers 1 in state 1 is 0.75 and $F(x(2)) = 0.4$, i.e., the probability that a randomly drawn voter prefers 2 in state 2 is 0.6. Thus $2|.5 - F(x(2))| = 0.2$ and $F(x(0)) - F(x(1)) = .35$.

First consider the case where all voters know the state. Clearly this implies that in a large electorate alternative 1 gets close to 75% of the vote in state 1 and close to 40% of the vote in state 2. Conditional on a vote being pivotal the probability of state 2 converges to one in this case. Also note that Assumption 7 is violated in this case since $.35 > 2$.

Now assume that some voters do not know s, i.e., do not get a perfectly informative signal. In a large electorate, if there is a small fraction of uninformed voters, these voters must behave *as if* the state is $s = 2$. Thus an uninformed voter will vote for alternative 1 with probability 0.4 and for alternative 2 with probability 0.6. If q is the probability that a voter is informed then the expected vote share for alternative 1 in state 1 is now

$$t_1(s = 1) = q \cdot 0.75 + (1 - q) \cdot 0.4$$

and the expected vote share for alternative 2 in state 2 is

$$t_2(s = 2) = q \cdot 0.6 + (1 - q) \cdot 0.6 = 0.6.$$

Thus the described equilibrium strategies and the resulting limit distribution over states conditional on a vote being pivotal is valid as long as

$$q \cdot 0.75 + (1-q) \cdot 0.4 > 0.6$$

or

$$q > \frac{0.2}{0.35}$$

For $q < 0.2/.35$ Assumption 7 is satisfied and the Lemma shows that both states $s = 1$ and $s = 2$ must have strictly positive probability in the limit distribution over states conditional on a vote being pivotal.

5.2 Full Information Equivalence

We now give conditions under which elections satisfy *full information equivalence.* Full information equivalence is satisfied if the election outcome under asymmetric information converges in probability to the election outcome that would occur if all the voters knew the true state and voted for their preferred alternative. This result extends the results in Feddersen and Pesendorfer (1996, 1997). Recall that $x(s)$ denotes the preference type that is indifferent between the two alternatives in state s. Furthermore, s^1 denotes the largest state in which the expected fraction of voters who prefer alternative 1 is greater or equal to 1/2. Hence, $F(x(s^1)) \geq 1/2$. In the following proposition we assume that this inequality is strict.[11]

Proposition 4 *Suppose Assumptions 1-5 hold and $F(x(s^1)) > 1/2$. Then, as $k \to \infty$, the probability that alternative 1 is elected converges to 1 for $s \leq s^1$ and to 0 for $s > s^1$.*

To provide an intuition for the result we sketch the proof for the case where there are two states ($s \in \{0, 1\}$) and no agent is perfectly informed.

Sketch of Proof: Since no agent is perfectly informed we may apply Lemma 3 and conclude that, conditional on a vote being pivotal, the probability of state s must stay bounded away from zero for both states $s = 0$ and $s = 1$. Hence, a voter's private signal changes her beliefs about the state conditional on a vote being pivotal even as the expected size of the electorate grows to infinity. Consequently, the fraction of voters who change their vote as a function of their signal stays bounded away from zero. This implies that the expected vote share for alternative 1 is at least ε larger in state 0 than in state 1,i.e., $t_{1k}(0) > t_{1k}(1) + \varepsilon$; similarly, the expected vote share of alternative 2 is at least ε larger in state 1 than in state 0, i.e., $t_{2k}(1) > t_{2k}(0) + \varepsilon$. Thus, it follows that

$$t_{1k}(0) - t_{2k}(0) > t_{1k}(1) - t_{2k}(1) + 2\varepsilon$$

[11] If there cannot be a state in which the expected vote shares are exactly equal under full information the result would still hold. We excluded this case here because it would require a different proof. The reason is that in this case the fraction of voters who use their private information may converge to zero. See Feddersen and Pesendorfer (1997) for such an argument.

Observe that Lemma 3 implies that $t_{1k}(0) - t_{2k}(0) > 0 > t_{1k}(1) - t_{2k}(1)$. To see this assume to the contrary that $t_{1k}(0) - t_{2k}(0) \leq 0$. Then, we know that $t_{1k}(1) - t_{2k}(1) + 2\varepsilon < t_{1k}(0) - t_{2k}(0) \leq 0$. But this in turn implies that it is much more likely a vote is pivotal in state 0 than in state 1. Therefore, in a large electorate, a voter must put almost zero probability on being in state 1 conditional on a vote being pivotal - a contradiction of Lemma 3. Similarly, Lemma 3 implies that $|t_{1k}(s) - t_{2k}(s)|$ stays bounded away from zero for $s \in \{0, 1\}$.[12] Thus, there is an $\eta > 0$ such that

$$t_{1k}(0) - t_{2k}(0) > \eta > -\eta > t_{1k}(1) - t_{2k}(1)$$

and hence the law of large numbers implies that for large k, alternative 1 is elected with probability close to one in state 0 and alternative 2 is elected with probability close to one in state 1. ■

5.3 Abstention

In this section we give conditions under which the fraction of voters who abstain stays bounded away from zero as $k \to \infty$.

Proposition 5 *Suppose Assumptions 1-7 hold. Then there is an $\alpha > 0$ and a k' such that the expected fraction of voters who abstain in equilibrium is larger than α for all $k > k'$.*

To provide an intuition for Proposition 5 observe that Lemma 3 says that voters are uncertain about the state conditional on being pivotal even in the limit. This in turn allows us to conclude that the probability distribution over states conditional on a tie (the event piv_2) is bounded away from the probability distribution conditional on alternative 2 being up one vote (the event piv_1). More precisely, there is an $\epsilon' > 0$ such that

$$\beta_k(s^1|piv_2) < \beta_k(s^1|piv_1) - \epsilon',$$

for all k. But this is enough to allow us to make the argument given in Proposition 3 for an interval of preference type with length uniformly bounded away from zero and hence Proposition 5 follows.

The previous proposition demonstrated that a strictly positive fraction of the electorate will always abstain but it did not provide a sense of how large that fraction may be. The next proposition provides a bound on the fraction of agents who abstain in equilibrium. Define a *critical state* to be an $s^* \in \arg\min_s |F(x(s)) - 1/2|$, i.e., a state in which the fraction of the electorate that prefers alternative 1 to alternative 2 is closest to 1/2. In the example in section 4.1 above, the critical state is $s = 0$. It follows from the definition of s^1 and s^2 that $s^* \in \{s^1, s^2\}$.

We now show that the fraction of the electorate that abstains in a large election goes to zero as the fraction of the electorate that prefers alternative 1 in state s^* goes to 1/2.

[12] If there is a state for which $|t_{1k}(s) - t_{2k}(s)| \to 0$ then voters must believe that this state has occurred with probability converging to one conditional on being pivotal - again a contradiction to Lemma 3.

Proposition 6 *Suppose Assumptions 1-6 hold. For every $\varepsilon > 0$ there is an $\eta > 0$ such that $|F(x(s^*)) - 1/2| \leq \eta$ implies that $\limsup_k (1 - t_{1k}(s) - t_{2k}(s)) \leq \varepsilon$ for all $s \in S$, i.e., the expected fraction of voters who abstain is bounded above by ε for sufficiently large k.*

A key step in the proof is to show that if the fraction of the electorate that prefers alternative 1 to alternative 2 in the critical state is very close to $1/2$ then the equilibrium vote share of each alternative must be close to $1/2$ in both of the states s^1 *and* s^2 in which a vote is most likely to be pivotal. But then a single vote provides very little information about the true state since voters are expected to vote for either alternative with close to equal probability in both s^1 and s^2. Hence conditioning on the event piv_1 provides very similar information to conditioning on the event piv_2 and a small fraction of the electorate abstains.[13]

As a corollary this implies that if the state space is "fine", i.e., if the utility variation between any pair of consecutive states is small then the level of abstention is small.

Corollary 1 *Suppose Assumptions 1-6 hold. Then for every $\epsilon > 0$ there is an ν and a k' such that if $\max_x |v(x, s^2) - v(x, s^1)| < \nu$ then the expected fraction of voters who abstain is less than ϵ for all $k > k'$.*

We can use Propositions 1 and 5 to relate the level of abstention to the "aggregate" level of information.

Example Consider the bridge example from the introduction and suppose voters are uncertain about the true cost of the bridge. Specifically, let the true cost of the bridge be a continuous random variable $c \in [0, 1]$ that is drawn by nature according to some probability distribution. Suppose that there are K television stations who each do an independent investigation into the cost of the bridge and televises a news report only if they find that the bridge is likely to be very expensive. The probability that a television station reports the bridge is expensive is strictly increasing in the true cost. Each voter watches at most one television news report but some do not watch at all. The probability a voter watches a report is q. Let s_k denote the expected cost of the bridge conditional on k out of K television stations reporting that the bridge is expensive. Finally, suppose that the expected payoff of agents given k reports is given by

$$E\{v(x, t)|k\} = v(x, s_k)$$

where $v(x, s_k)$ satisfies Assumption 1. In words, the utility difference between building and not building the bridge is strictly decreasing in the number of reports the bridge is expensive.

[13] Assumption 7 implies that there cannot be a state in which the expected vote shares are exactly equal under full information. In this case the fraction of voters who abstain converges to zero. We excluded it here because it would require a different proof. The reason is that in this case the fraction of voters who use their private information may converge to zero. Therefore, a more delicate argument is needed to establish full information equivalence (see Feddersen and Pesendorfer (1997)) which in turn is needed to prove Proposition 5.

This example can be represented in our model as a finite state space model in which the state space is $S = \{s_0, ..., s_K\}$ and the probability a voter observes report $m \in \{\emptyset, l, h\}$ in state s_k is

$$p(m|s_k) = \begin{cases} \frac{qk}{K} & \text{if} \quad m = h \\ q(1 - \frac{k}{K}) & \text{if} \quad m = l \\ (1 - q) & \text{if} \quad m = \emptyset \end{cases}$$

When $K = 1$ there are two types of agents, those who are perfectly informed and those who are uninformed. When $K > 1$ there are no perfectly informed agents. When $K \to \infty$ the state space approximates a continuous state space $[0, 1]$.

Clearly, if the number of independent investigations by television stations is large then $s_{k+1} - s_k$ is small. Thus, many independent reports leads to better aggregate information and a smaller fraction of the population abstaining in equilibrium.

6 Information and Participation

In this section we use a simplified version of our model to demonstrate how private information combined with preference diversity can result in two seemingly contradictory comparative statics: (1) more informed voters participate with higher probability than less informed voters; and (2) increasing the fraction of the electorate that is better informed results in increased abstention.

Assume there are two states $s = 0, 1$ and

$$v(x, s) = \begin{cases} x - 1 \text{ if } s = 0 \\ x + 1 \text{ if } s = 1. \end{cases} \tag{12}$$

Thus, in state 0 all voter types strictly prefer alternative 1 to alternative 2 while in state 2 preferences are reversed. The agents' prior assigns probability 1/2 to each state. Agents are distributed uniformly on $[-1, 1]$. There are three signals $\mathcal{M} = \{\emptyset, 1, 2\}$. Signals 1 and 2 are informative while $\emptyset$ provides no information. Voters receive an informative signal with probability q. The conditional probability of observing signal $m \in \mathcal{M}$ given state s is $p(m|s)$ where:

$$p(m|s) = \begin{cases} pq & \text{if} & m = s + 1 \\ (1 - p)q & \text{if} & m \neq s + 1 \text{ and } m \neq \emptyset \\ (1 - q) & \text{if} & m = \emptyset \end{cases}$$

and $p > 1/2$.

An equilibrium in this model is characterized by the vector of cutpoints where x_m^j is the voter type who is indifferent between voting for alternative j and abstaining conditional upon observing signal m. Let $\beta(1|piv_j, m)$ be the probability of state 1 given piv_j and message m.

$$\beta(1|piv_j, m) = \frac{1}{1 + \frac{p(m|2)}{p(m|1)} \frac{\Pr\{piv_j|2\}}{\Pr\{piv_j|1\}}}$$

It follows from (12) that for any $j \in \{1,2\}$ and $m \in \mathcal{M}$ the cutpoint x_m^j is given by:

$$x_m^j = 2\beta(1|piv_j, m) - 1 \tag{13}$$

The expected vote share for alternative 1 in state s is $t_1(s)$ and, from equation (8), can be written

$$t_1(s) = .5\left(p(1|s)(1+x_1^1) + p(2|s)(1+x_2^1) + p(\emptyset|s)(1+x_\emptyset^1)\right) \tag{14}$$

Similarly the expected vote share for alternative 2 in state s is

$$t_2(s) = .5\left(p(1|s)(1-x_1^2) + p(2|s)(1-x_2^2) + p(\emptyset|s)(1-x_\emptyset^2)\right). \tag{15}$$

We analyze equilibrium behavior in large elections. Lemma 1 implies that, in the limit, as the expected population size goes to infinity, the following equation holds:

$$\frac{\Pr\{piv_1|1\}}{\Pr\{piv_1|0\}} = \frac{\sqrt{t_2(1)}\sqrt{t_1(0)}}{\sqrt{t_1(1)}\sqrt{t_2(0)}}\frac{\Pr\{piv_2|1\}}{\Pr\{piv_2|0\}} \tag{16}$$

Lemma 3 implies that the probability distribution over states conditional on a vote being pivotal must put strictly positive probability on both states in the limit. By Lemma 9 (in Appendix A) this requires that in the limit

$$\sqrt{t_1(0)} - \sqrt{t_2(0)} = \sqrt{t_2(1)} - \sqrt{t_1(1)} \tag{17}$$

The system of equations (13)-(17) may be solved to find a limit equilibrium strategy profile.

6.1 Perfectly informative signals

Suppose the signal $m \in \{1,2\}$ is perfectly informative i.e., $p = 1$. In this example the only variable is the fraction of the electorate that is perfectly informed (q). This assumption combined with the symmetry of the setting greatly simplifies the system of equations and permits an analytical solution. Observe that $x_1^j = -1$ and $x_2^j = 1$ for any $j \in \{1,2\}$ i.e., all voters who observe a perfectly informative signal vote for alternative 1 if they observe signal 1 and alternative 2 otherwise. By the assumption of a symmetric distribution of preference types it must be the case that $t_1(0) = t_2(1)$, $t_1(1) = t_2(0)$ and $x_\emptyset^1 = -x_\emptyset^2$. Some simple algebra can be used to show that the ratio of pivot probabilities reduces to

$$\begin{aligned}
\frac{\Pr\{piv_2|1\}}{\Pr\{piv_2|0\}} &= \sqrt{\frac{t_1(1)}{t_1(0)}} \\
\frac{\Pr\{piv_1|1\}}{\Pr\{piv_1|0\}} &= \sqrt{\frac{t_1(0)}{t_1(1)}}
\end{aligned}$$

We can now find the symmetric limit equilibrium by solving the following system of equations for the cutpoint $x_\emptyset^1$ and the expected vote shares $t_1(0)$ and $t_1(1)$:

$$\begin{array}{|l|}\hline x_\emptyset^1 = \frac{2}{1+\sqrt{\frac{t_1(1)}{t_1(2)}}} - 1 \\ \hline t_1(0) = q + .5(1-q)(1+x_\emptyset^1) \\ \hline t_1(1) = .5(1-q)(1+x_\emptyset^1) \\ \hline \end{array}$$

When we solve this system we get:

$$\begin{array}{|l|}\hline x_\emptyset^1 = \frac{q}{q-2} \\ \hline t_1(0) = \frac{1}{2-q} \\ \hline t_1(1) = \frac{(1-q)^2}{2-q} \\ \hline \end{array} \tag{18}$$

Figure 2 below illustrates the cutpoints for the case when $q = .65$. As noted above, informed agents have perfect information and always vote for alternative 1 when they observe signal 1 and alternative 2 when they observe signal 2. The uninformed agents are partitioned into three intervals of preference types: those who always vote for alternative 1 ($x \in (-1, -.48)$), those who abstain ($x \in (-.48, 48)$) and those who always vote for alternative 2 ($x \in (.48, 1)$).

<Figure 2 about here>

Abstention as a function of the fraction of the electorate that is perfectly informed is given by the equation:

$$1 - t_1(0) - t_1(1) = q\frac{1-q}{2-q}$$

Thus, in the example in Figure 2 the fraction of the electorate that abstains $.65\frac{1-.65}{2-.65} \simeq .17$. Figure 3 plots abstention as a function of the probability an agent is perfectly informed. Since perfectly informed voters never abstain one might imagine that increasing the fraction of such voters in the electorate would automatically reduce abstention. However, as the fraction of informed voters grows so does the informativeness of the election result. As a consequence the uninformed voters become more willing to abstain. When the fraction of the informed electorate is small this equilibrium effect dominates and abstention increases. When the fraction of the informed electorate is large the additional abstention of the uninformed is outweighed by the additional participation by the informed. Figure 3 illustrates that when the fraction of the electorate that is perfectly informed increases the result may be *more* abstention.

<Figure 3 about here>

6.2 Noisy Signals

Assume the same structure as in section 6 above except that those who are informed observe a noisy signal, i.e., $p \in (.5, 1)$. There are three cutpoints that must be calculated, one for each signal. The reason is that the inference of agents who receive signals $m \in \{1,2\}$ now also depends on the pivotal event. Therefore, $x_m^1 \neq x_m^2$ for $m \in \{\phi, 1, 2\}$. However, the symmetry of the example still implies that $x_\emptyset^1 = -x_\emptyset^2$, $x_1^1 = -x_2^2$ and $x_2^1 = -x_1^2$.

In this section we compute examples because we are unable to solve the system analytically. The system of equations used to compute the following examples along with a table giving the results for each of the examples computed may be found in appendix B.

Figure 4 below demonstrates the relationship between abstention and information when agents are not perfectly informed. Three consequences of increasing the fraction of informed agents stand out. First, abstention by informed voters is always strictly lower than abstention by the uninformed. Second, increasing the fraction of the electorate that is informed always results in higher levels of abstention by both the informed and uninformed. Third, increasing the fraction informed always results in a larger increase in abstention among the uninformed than among the informed.[14]

<Figure 4 about here>

Figure 5 shows the cutpoints and strategy profile for one particular example in which $p = .9$ and $q = .65$. To understand the effect of noise on the equilibrium strategy profile it is useful to compare the strategy profile in figure 5 with the profile in figure 2. The introduction of noise reduces the fraction of the uninformed who abstain but it creates two new categories of voter types: informed types who vote independent of their signal and informed abstainers. In the model with noise voters with types in the interval $(-1, -.95)$ always vote for alternative 1 while those in the interval $(.95, 1)$ always vote for alternative 2. Voter's whose type is in the interval $(-.95, -.80)$ vote for alternative 1 unless they are informed and observe signal 2 in which case they abstain. Similarly, those with types in the interval $(.80, .95)$ vote for alternative 2 unless they are informed and observe the signal 1 in which case they abstain.

The fact that some informed voters abstain as a function of their signal means that in any given election the population who actually turns out to vote is not representative of the electorate as a whole. Recall that the expected median preference type in this section is $x = 0$. We say that the voting electorate is *biased* in state s if the median preference type who votes is not equal to zero.

The strategy profile in Figure 5 implies that the voting electorate in state 0 will be biased in a negative direction while in state 1 the electorate will be biased in a positive direction. In this example the expected median participant has a preference type

[14] The reader may wonder if there is a theorem that might be proved to the effect that those with better information always participate more frequently and that abstention among the uniformed always increases faster as the electorate as a whole becomes more informed. While this pattern appears to be robust for a variety of examples it is possible to construct a counterexample in which those with better information abstain with higher probability than those with worse information. The example requires the introduction of considerable asymmetry in preferences. Details are available upon request from the authors.

$-.06$ in state 0 while in state 1 it is .06.[15] Since the election satisfies full information equivalence, alternative 1 always wins in state 0, while alternative 2 always wins in state 1.[16] Thus, in this example the voting electorate is always *biased towards the winner*.[17]

<Figure 5 about here>

While the voting electorate is always biased towards the winner the outcome is not biased. To define bias of the election outcome, we compare the election outcome in a model where abstention is permitted to the election outcome in a model where abstention is not permitted. Since the election without abstention satisfies full information equivalence it follows that the election outcome in the model with abstention is unbiased whenever it satisfies full information equivalence.[18]

The fact that turnout is biased in favor of the supporters of the winning alternative creates the possibility of the correlation between e.g., turnout of the lower classes and left wing policy outcomes. However, turnout has no independent effect on policy outcomes. Rather, it is the state s that determines both. In an empirical study, Hill, Leighley and Hinton-Anderson (1995) find a positive relationship between lower class voter turnout and welfare policy.[19] As we have argued, this correlation is consistent with our model. If, for example, economic conditions constitute the state variable, then both turnout and policy choice may be a function of economic conditions.[20] A regression of policy choice on turnout will discover a positive correlation. However, there is no direct relationship between those variables.

7 Bias and Abstention

Another stylized fact about participation in elections is that those on the left participate less frequently than those on the right (Lijphart 1997). The explanation for

[15]To compute the median *voter* type in state 0 we need to know the distribution of voters in state 0. Using the cutpoints the density of the preferences of voters who participate in state 0 is

$$f_0(x) = \begin{cases} \frac{1}{1.6575} = .60332 & \text{if } x \in [-1, -.95] \\ \frac{.65 \times .9 + .35}{1.6575} = .5641 & \text{if } x \in (-.95, -.8] \\ \frac{1}{1.6575} = .60332 & \text{if } x \in (-.8, -.35] \\ \frac{.65}{1.6575} = .39216 & \text{if } x \in (-.35, .35] \\ \frac{1}{1.6575} = .60332 & \text{if } x \in (.35, .8] \\ \frac{.65 \times .1 + .35}{1.6575} = .25038 & \text{if } x \in (.8, .95] \\ \frac{1}{1.6575} = .60332 & \text{if } x \in (.95, 1]. \end{cases}$$

[16]Feddersen and Pesendorfer (1997) demonstrate full information equivalence in a model without abstention. Their arguments can be adapted to yield full information equivalence here. It is also straightforward to adapt the arguments of Proposition 4 to the case without abstention.

[17]In general. since the states need not be equally likely it is also the case that the ex ante (prior to the realization of the state) expected median participant need not be equal to the population median type.

[18]Adapting the argument in the proof of Proposition 4 to the case in which abstention is not permitted is straightforward. For a proof in a related setting see Feddersen and Pesendorfer (1997).

[19]See Ringquist, Hill, Leighley and Hinton-Andersson (1997) as well.

[20]For example, during a recession everyone may be more disposed to support government spending.

this appears to be the correlation between education (a proxy for information) and political preferences. The correlation may work as follows. Those with higher levels of education enjoy higher incomes and this pushes their political preferences to the right. The following example demonstrates that a biased distribution of information in our model can lead to an electorate that is always biased towards the more informed end of the ideological spectrum. While the bias in the electorate may be dramatic this does not result in biased outcomes. Indeed, in this example the biased distribution of information results in strictly higher probabilities the election satisfies full information equivalence than an unbiased distribution of information.

For purposes of exposition call those with preference types below 0 leftists and those above 0 rightists. Assume the same structure as in example 6.1 above with the proviso that all rightists observe the perfectly informative signal while leftists receive the uninformative signal. Thus the distribution of information is maximally biased towards those on the right. It is clear that any abstention will occur only among leftists guaranteeing that, independent of the state, the voting electorate is biased to the right. Because the rightists all vote for 1 when they observe signal 1 and vote for 2 otherwise, it only remains to determine the behavior of the leftists. The cutpoints, expected vote shares and expected level of abstention (A) are given in column 1 of Table 1. The system of equations that was solved to generate this example may be found in Appendix B.

The first column of Table 1 below gives the cutpoints, vote shares and total level of abstention for this example. The total level of abstention is 30% however abstention only occurs among leftists. Indeed 60% of the leftists abstain. Nevertheless, by looking at the expected vote shares in each state it is clear that alternative 1 wins in a landslide in state 0 as does alternative 2 in state 1 and the expected margin of victory is the same in each state. Thus, the pattern of abstention may be dramatically skewed towards one end of the ideological spectrum without biasing the outcome of the election. It should be noted that in this example voters have different preferences but all prefer alternative 1 in state 1 and alternative 2 in state 2. It is a simple matter to generalize the example to include voters who always prefer one alternative or the other independent of the state.

To get a sense of the effect that a biased distribution of information has on equilibrium behavior suppose $q = .5$, $p = 1$ (those informed have perfect information) and there is no bias in the distribution of information. We can use the equations in (18) above to find the cutpoints and expected vote shares for each state (see column 2 of Table 1 below). In this case the fraction of the electorate that is informed is the same as in the above example. The difference is that a leftist is as likely to receive an informative signal as a rightist.

<Table 1 about here>

Abstention is lower in the unbiased example. While both the biased and the unbiased electorate choose alternative 1 in state 1 and alternative 2 in state 2 with high probability, in large finite elections the biased distribution of information actually results in *lower* probabilities of error than the unbiased example. This follows from the fact that all the informed voters vote correctly in each setting. The uninformed voters only introduce noise thus the fact that abstention is higher under the biased distribution of information marginally improves the performance of the electoral mechanism.

8 Conclusion

In the first section of the paper we demonstrated that abstention due to asymmetric information is robust to preference diversity and variable information environments.

In the second section of the paper we presented a set of examples that demonstrate the range of phenomena that are consistent with the model. In particular we demonstrated that a biased electorate may be caused by a biased distribution of information without creating a biased outcome. The fact that a large range of behavior may be supported as equilibrium in some information environment complicates the task of using this model as a predictive tool. However, the normative result that elections work well as information aggregation mechanisms even in the face of what would appear to be a systematically biased electorate and large scale abstention is robust.

It should be noted that we do not claim that outcomes of recent US elections were unbiased, i.e., would remain unchanged if all of those eligible to vote participated. Rather, we argue that correlations between outcomes and patterns of participation do not constitute evidence in favor of biased outcomes. Our examples show that such correlation may be generated in a model where outcomes are always unbiased.

One objection to our model is that computations of best responses are beyond the capabilities of actual voters. In our environment voting optimally requires correctly calculating two probability distributions over the state space, updating these distributions to include private information and then deciding how to vote or abstain. Objections are commonly raised against mathematical models of human behavior that the calculations are too difficult. The standard response is that mathematical models can not hope to capture the actual thought process of decision makers but aspire simply to predict behavior. The examples presented above suggest that our model is capable of predicting some of the stylized facts that appear puzzling in the context of decision-theoretic models. Whether our model is better than existing models is an open empirical question beyond the scope of this paper. However, recent papers by Ladha, Miller and Oppenhiemer (1996) and McKelvey and Palfrey (1998) show that the equilibrium predictions of strategic voting models in information environments like ours work remarkably well in laboratory experiments.

9 Appendix A

Proposition 1. *Suppose Assumptions 1-4 hold. Then a symmetric equilibrium exists and any such equilibrium can be characterized by ordered cutpoints.*
Proof. Existence is straightforward. For example, the existence proof in Myerson (1997) applies.

Since there is a strictly positive probability that any player is the only agent it follows that the probability a vote is pivotal for 2 is always strictly positive. Therefore, by Assumption 1, there is an $\varepsilon > 0$ such that all types $x \in [1-\varepsilon, 1]$ must vote for 2 independent of their signal m. By Assumption 3, the probability that a type is in the interval $[1-\varepsilon, 1]$ is strictly positive and hence $t_2(s)$ is strictly positive in any equilibrium. This in turn implies that there is a strictly positive probability that there are no votes for 1 and one vote for 2. Hence the probability that a voter is pivotal for 1 is strictly positive. Therefore, $\beta(s|piv_j, m), j = 1, 2$ and $\beta(s|piv, m)$ are well defined in equilibrium.

Fix a symmetric profile σ and let $E(v(x,s)|piv_j, m)$ denote the expectation of $v(x,s)$ with respect to $\beta(\cdot|piv_j, m)$. A voter of type (x,m) prefers to vote for 2 rather than abstain if

$$E\{v(x,s)|piv_2, m\} = \sum_s v(x,s)\beta(s|piv_2, m) > 0; \tag{19}$$

he prefers to vote for 1 rather than abstain if

$$E\{v(x,s)|piv_1, m\} = \sum_s v(x,s)\beta(s|piv_1, m) < 0, \tag{20}$$

and he prefers to vote for 2 rather than 1 if

$$E\{v(x,s)|piv, m\} = \sum_s v(x,s)\beta(s|piv, m) > 0 \tag{21}$$

By Assumption 1 $v(x,s)$ is strictly increasing in x. Thus, if it is a best response for type x to vote for 2 then for every type $x' > x$ the unique best response must be to vote for 2. Similarly, if it is a best response for type x to vote for 1 then the unique best response for every type $x' < x$ must be to vote for 1. It follows that any voting equilibrium can be characterized by cutpoints such that for any $m \in \mathcal{M}$ $x_m^1 \leq x_m^2$ and all types with $x < x_m^1$ vote for 1 when they receive signal m and all types with $x > x_m^2$ vote for 2 if they receive signal m. Types in the interval (x_m^1, x_m^2) abstain.

Observe that voters with types x_m^j are either indifferent between voting for j and abstaining (if (x_m^1, x_m^2) is non-empty) or indifferent between voting for 1 and 2 (if $x_m^1 = x_m^2$). As a consequence either

$$E\{v(x_m^j, s)|piv_j, m\} = 0$$

or

$$E\{v(x_m^j, s)|piv, m\} = 0$$

To show that cutpoints are ordered observe that for $m > m'$, such that $m, m' \neq \emptyset$, SMLRP implies that for all x

$$E\{v(x,s)|piv_j, m\} > E\{v(x,s)|piv_j, m'\}$$

and

$$E\{v(x,s)|piv, m\} > E\{v(x,s)|piv, m'\}$$

Therefore, it must be the case that $1 > x_1^j > .... > x_M^j > -1$ for $j = 1, 2$. ■

Proposition 3. *Suppose Assumption 1 holds. Suppose σ is an equilibrium strategy and let (x_m^j), $j = 1, 2$ be the corresponding cutpoints. If m is not perfectly informative then $x_m^2 - x_m^1 > 0$.*

Proof. Equations (3) and (4) imply that

$$\begin{aligned} \frac{\Pr\{piv_1|s\}}{\Pr\{piv_2|s\}} &= \frac{\sum_{x=0}^{\infty} \frac{t_1(s)^x t_2(s)^{x+1}}{x!(x+1)!}}{\sum_{x=0}^{\infty} \frac{t_1(s)^x t_2(s)^x}{x!x!}} \\ &\equiv L(t_1(s), t_2(s)). \end{aligned}$$

Below (Lemma A2) we demonstrate that

$$\frac{\partial}{\partial t_1} L(t_1, t_2) < 0, \frac{\partial}{\partial t_2} L(t_1, t_2) > 0$$

and therefore, since t_1 is strictly decreasing in s and t_2 is strictly increasing in s it follows that $L(t_1(s), t_2(s))$ is strictly increasing in s.

Let $\beta(s|Y)$ denote the probability of state s conditional on the event Y. We can interpret $\beta(s|piv_j, m)$ as the distribution that is achieved by updating $\beta(s|m) = \frac{g(s)p(m|s)}{\sum_m g(s)p(m|s)}$ with the signal $piv_j \in \{piv_1, piv_2\}$ which satisfies SMLRP since $L(t_1(s), t_2(s))$ is strictly increasing in s. If m is not perfectly informative then it is the case that $p(m|s)$ is non-zero for two or more states. Then by a standard property of SMLRP (see Milgrom (1979)) it follows that $\beta(s|piv_1, m)$ strictly first order stochastically dominates $\beta(s|piv_2, m)$. Since v is strictly increasing in s this allows us to conclude that

$$E\{v(x,s)|piv_1, m\} > E\{v(x,s)|piv_2, m\}. \tag{22}$$

Note that piv is the union of the events piv_1 and piv_2 which both occur with strictly positive probability and therefore

$$E\{v(x,s)|piv_1, m\} > E\{v(x,s)|piv, m\} > E\{v(x,s)|piv_2, m\}. \tag{23}$$

Consider a voter $(\hat{x}, m)$ who is indifferent between voting for 1 and voting for 2, that is,

$$E\{v(\hat{x}, s)|piv, m\} = 0.$$

Such a voter exists by Assumption 1. By inequality 23 this voter strictly prefers to abstain since conditional on piv_1, i.e., if a vote for 1 is decisive the voter strictly prefers alternative 2 and conditional on piv_2 (if a vote for 2 is decisive) the voter strictly prefers alternative 1. Since v is continuous it follows that there is an interval of voters who strictly prefer to abstain whenever m is not perfectly informative. ■

For the subsequent Lemmas we need to introduce the following notation. The modified Bessel function $I_v(x)$ is defined as

$$I_v(x) = \left(\frac{1}{2}x\right)^v \sum_{k=0}^{\infty} \frac{(\frac{1}{4}x^2)^k}{k!(v+k)!} \tag{24}$$

Note the following facts (see Abramowitz and Stegum (1970) p. 376):

$$\begin{aligned} I_0'(x) &= I_1(x) && (25) \\ I_1'(x) &= I_0(x) - \frac{1}{x}I_0(x) \\ I_2'(x) &= I_1(x) - \frac{2}{x}I_1(x) \\ I_0(0) &= 1, I_1(0) = 0, I_2(0) = 0 \\ I_0(x) - I_2(x) &= \frac{2}{x}I_1(x) \end{aligned}$$

Also note that from equation 9.7.1. in Abramowitz and Stegum (p. 377) it follows that for $\nu \in \{0,1,2\}$

$$\lim_{x\to\infty} \frac{\frac{e^x}{\sqrt{2\pi x}}}{I_\nu(x)} = 1 \tag{26}$$

Clearly, the first two equations of (25) (together with the initial conditions $I_0(0) = 1, I_1(0) = 0$) imply that for $x > 0$

$$I_0(x) > I_1(x) \tag{27}$$

since $I_1'(x) - I_0'(x) < I_0(x) - I_1(x)$ and hence $I_0(x) - I_1(x)$ cannot change sign (or be equal to zero) for $x > 0$.

Similarly, the second and the third equations imply that $I_2'(x) - I_1'(x) < \frac{1}{x}(I_1(x) - I_2(x))$ and together with the initial conditions this again implies that $I_1(x) > I_2(x)$ for $x > 0$. Also note that

$$I_0(x) - I_2(x) = \frac{2}{x}I_1(x)$$

This, together with $I_0(x) - I_2(x) > I_0(x) - I_1(x)$ implies that

$$0 < \frac{I_0(x)}{I_1(x)} - 1 < \frac{I_0(x) - I_2(x)}{I_1(x)} = \frac{2}{x}$$

Therefore it follows that

$$\frac{I_0(x)}{I_1(x)} \to 1 \tag{28}$$

Definition 2 *Define the function*

$$L(v,\omega)=\frac{\sum_{x=0}^{\infty}\frac{v^x\omega^{x+1}}{x!(x+1)!}}{\sum_{x=0}^{\infty}\frac{v^x\omega^x}{x!x!}}$$

for any $v>0$, $\omega>0$.

Lemma A1. *For* $I_v(x)$ *as defined in equation (24) we have that*

1. $I_0(2\sqrt{v\omega})=\sum_{x=0}^{\infty}\frac{v^x\omega^x}{x!x!}$
2. $I_1(2\sqrt{v\omega})=\sum_{x=0}^{\infty}\frac{v^x\omega^{x+1}}{x!(x+1)!}$
3. $L(v,\omega)=\frac{\sqrt{\omega}}{\sqrt{v}}\frac{I_1\left(2\sqrt{v\omega}\right)}{I_0\left(2\sqrt{v\omega}\right)}$.

Proof. To see part 1. note that

$$\begin{aligned}
I_0(2\sqrt{v\omega}) &= 1+\frac{\left(2\sqrt{v\omega}\right)^2}{2^2\left(1!\right)^2}+\frac{\left(2\sqrt{v\omega}\right)^4}{2^4(2!)^2}+\frac{\left(2\sqrt{v\omega}\right)^6}{2^6(3!)^2}+...\\
&= 1+\frac{\left(v\omega\right)^1}{\left(1!\right)^2}+\frac{\left(v\omega\right)^2}{(2!)^2}+\frac{\left(v\omega\right)^3}{(3!)^2}+...\\
&= \sum_{x=0}^{\infty}\frac{v^x\omega^x}{x!x!}
\end{aligned}$$

To see part 2. observe that

$$\begin{aligned}
\frac{\sqrt{\omega}}{\sqrt{v}}I_1(2\sqrt{v\omega}) &= \frac{\sqrt{\omega}}{\sqrt{v}}\left(\frac{(2\sqrt{v\omega})}{2}+\frac{(2\sqrt{v\omega})^3}{2^3 1!2!}+\frac{(2\sqrt{v\omega})^5}{2^5 2!3!}+...\right)\\
&= \frac{\sqrt{\omega}}{\sqrt{v}}\left(\frac{(\sqrt{v\omega})}{1}+\frac{(\sqrt{v\omega})^3}{1!2!}+\frac{(\sqrt{v\omega})^5}{2!3!}+...\right)\\
&= \left(\frac{\omega}{1}+\frac{v\omega^2}{1!2!}+\frac{v^2\omega^3}{2!3!}+...\right)=\sum_{x=0}^{\infty}\frac{v^x\omega^{x+1}}{x!(x+1)!}
\end{aligned}$$

Part 3. now follows by taking ratios. ■

Lemma A2. $\partial L(v,\omega)/\partial v<0, \partial L(v,\omega)/\partial\omega>0$.

Proof. ¿From Lemma A1 we know that

$$\begin{aligned}
L(v,\omega) &= \frac{\sqrt{\omega}}{\sqrt{v}}\frac{I_1\left(2\sqrt{v\omega}\right)}{I_0\left(2\sqrt{v\omega}\right)}\\
&= \frac{\sum_{x=0}^{\infty}\frac{v^x\omega^{x+1}}{x!(x+1)!}}{\sum_{x=0}^{\infty}\frac{v^x\omega^x}{x!x!}}
\end{aligned}$$

Therefore,

$$\begin{aligned} &\frac{\partial}{\partial v}L(v,\omega) \\ =\; &\frac{\left(\sum_{x=0}^{\infty}\frac{xv^{x-1}\omega^{x+1}}{x!(x+1)!}\right)\left(\sum_{x=0}^{\infty}\frac{v^x\omega^x}{x!x!}\right)-\left(\sum_{x=0}^{\infty}\frac{xv^{x-1}\omega^x}{x!x!}\right)\left(\sum_{x=0}^{\infty}\frac{v^x\omega^{x+1}}{x!(x+1)!}\right)}{\left(\sum_{x=0}^{\infty}\frac{v^x\omega^x}{x!x!}\right)^2}\end{aligned}$$

We need to show that the numerator is positive.

$$\begin{aligned} &\left(\sum_{x=0}^{\infty}\frac{v^{x-1}\omega^{x+1}}{(x-1)!(x+1)!}\right)\left(\sum_{x=0}^{\infty}\frac{v^x\omega^x}{x!x!}\right)-\left(\sum_{x=0}^{\infty}\frac{v^{x-1}\omega^x}{(x-1)!x!}\right)\left(\sum_{x=0}^{\infty}\frac{v^x\omega^{x+1}}{x!(x+1)!}\right) \\ =\; &\sum_{x=0}^{\infty}\sum_{y=0}^{\infty}\frac{v^{x-1}\omega^{x+1}}{(x-1)!(x+1)!}\frac{v^y\omega^y}{y!y!}-\frac{v^{x-1}\omega^x}{(x-1)!x!}\frac{v^y\omega^{y+1}}{y!(y+1)!} \\ =\; &\sum_{x=0}^{\infty}\sum_{y\le x}\frac{v^{x+y-1}\omega^{x+y+1}}{(x-1)!x!(y-1)!y!}\left[\frac{1}{(x+1)y}-\frac{1}{(y+1)y}+\frac{1}{(y+1)x}-\frac{1}{(x+1)x}\right]\end{aligned}$$

A straightforward calculation shows that

$$\left[\frac{1}{(x+1)y}-\frac{1}{(y+1)y}+\frac{1}{(y+1)x}-\frac{1}{(x+1)x}\right]=\frac{2xy-x^2-y^2}{(x+1)\,y(y+1)x}<0$$

Therefore we have established the first part of the Lemma.

For the second part we use the the following two properties of the Bessel functions (see Abramowitz and Stegum (p. 377)):

$$\begin{aligned} I_0'(x) &= I_1(x) \\ I_1'(x) &= I_0(x)-\frac{1}{x}I_0(x)\end{aligned}$$

Using these two equations, a straightforward calculation yields

$$\frac{\partial L}{\partial \omega}=\frac{I_0\left(2\sqrt{v\omega}\right)^2-I_1\left(2\sqrt{v\omega}\right)^2}{I_0\left(2\sqrt{v\omega}\right)^2}$$

Since $I_0(x)>I_1(x)>0$ for $x>0$ (Equation 27) the result follows. ■

Lemma 1.

1. *There is an* $\varepsilon>0$ such that $t_{1k}(s)>\varepsilon$ *and* $t_{2k}(s)>\varepsilon$ *for all* k.
2. *As* $k\to\infty$ $\left|\frac{\Pr_k\{piv_2|s\}}{\Pr_k\{piv_1|s\}}-\sqrt{\rho_k(s)}\right|\to 0.$

Proof. To see part (1) note that by Assumption 1 there is an $\epsilon>0$ such that for all $x\in[-1,-1+\epsilon)$ an agent strictly prefers alternative 1 for all states s and similarly for all $x\in(1-\epsilon,1]$ an agent strictly prefers alternative 2. This implies that $t_{jk}(s)$ is bounded away from zero for all k and all s.

To see part (2) note that by Equation 3 in Lemma A1

$$\begin{aligned}\frac{\Pr_k\{piv_2|s\}}{\Pr_k\{piv_1|s\}} &= \frac{\sqrt{\nu_k t_{1k}(s)}}{\sqrt{\nu_k t_{2k}(s)}}\frac{I_0\left(2\nu_k\sqrt{t_{1k}(s)t_2(s)}\right)}{I_1\left(2\nu_k\sqrt{t_{1k}(s)t_{2k}(s)}\right)} \\ &= \sqrt{\rho_k(s)}\frac{I_0\left(2\nu_k\sqrt{t_{1k}(s)t_{2k}(s)}\right)}{I_1\left(2\nu_k\sqrt{t_{1k}(s)t_{2k}(s)}\right)}\end{aligned}$$

Now the result follows from Equation 28 ■

Let

$$T_k = \left\{\arg\min_s\left|\sqrt{t_{1k}(s)} - \sqrt{t_{2k}(s)}\right|\right\}, \tag{29}$$

be the states that minimize the difference between the square roots of the expected vote shares for each alternative. The next lemma shows that in a large election conditional on a vote being pivotal almost all probability weight must be on states in T_k. The Lemma assumes that a positive fraction of agents takes informative action, i.e., $\left(x^j_{1k} - x^j_{Mk}\right)$ stays bounded away from zero.

Lemma A3. *Suppose that* $\left(x^j_{1k} - x^j_{Mk}\right) > \epsilon > 0$ *for* $j = 1$ *or* $j = 2$ *and all* k. *Then as* $k \to \infty$ $\sum_{s\in T_k}\beta_k(s|piv_j) \to 1$ *for* $j = 1, 2$ *and* $\sum_{s\in T_k}\beta_k(s|piv) \to 1$.

Proof. By the assumption of strict SMLRP and the fact that $\left(x^j_{1k} - x^j_{Mk}\right) > \epsilon > 0$ for $j = 1$ or $j = 2$ it follows that $t_1(s)$ is strictly decreasing while $t_{2k}(s)$ is strictly increasing in s with

$$\begin{aligned} t_{1k}(s) - t_{1k}(s') &> \epsilon' > 0 \\ t_{2k}(s') - t_{2k}(s) &> \epsilon' > 0 \end{aligned} \tag{30}$$

for $s' > s$ and for all k. Consider a convergent subsequence and let $t_j(s) = \lim_{k\to\infty} t_{jk}(s)$. Further, let

$$T = \left\{\arg\min\left|\sqrt{t_1(s)} - \sqrt{t_2(s)}\right|\right\} \tag{31}$$

It follows from 30 that T consists either of a singleton or a pair of consecutive states. We now show that agents must believe that conditional on a vote being pivotal the state is almost certainly in T.

We can write the ratio of pivot probabilities conditional on states s and s' as follows

$$\frac{\Pr_k\{piv_2|s'\}}{\Pr_k\{piv_2|s\}} = \frac{\sum_{x=0}^{\infty}\frac{e^{-\nu(t_{1k}(s')+t_{2k}(s'))}(\nu t_{2k}(s'))^x(\nu t_{1k}(s'))^x}{x!x!}}{\sum_{x=0}^{\infty}\frac{e^{-\nu(t_{1k}(s)+t_{2k}(s))}(\nu t_{2k}(s))^x(\nu t_{1k}(s))^x}{x!x!}}$$

Using equation (1) in Lemma A1 we rewrite the right hand side to yield

$$\begin{aligned}\frac{\Pr_k\{piv_2|s'\}}{\Pr_k\{piv_2|s\}} &= \frac{e^{-\nu(t_{1k}(s')+t_{2k}(s'))}I_0\left(2\nu\sqrt{t_{1k}(s')t_{2k}(s')}\right)}{e^{-\nu(t_{1k}(s)+t_{2k}(s))}I_0\left(2\nu\sqrt{t_{1k}(s)t_{2k}(s)}\right)} \\ &= e^{\nu(t_{1k}(s)-t_{1k}(s')+t_{2k}(s)-t_{2k}(s'))}\frac{I_0\left(2\nu\sqrt{t_{1k}(s')t_{2k}(s')}\right)}{I_0\left(2\nu\sqrt{t_{1k}(s)t_{2k}(s)}\right)}\end{aligned}$$

Note that from Equation (26) it follows that

$$\frac{\Pr_k\{piv_2|s'\}}{\Pr_k\{piv_2|s\}} \to e^{\nu_k(t_{1k}(s)-t_{1k}(s')+t_{2k}(s)-t_{2k}(s'))} \frac{\frac{e^{2\nu_k\sqrt{t_{1k}(s')t_{2k}(s')}}}{\sqrt{2\pi 2\nu_k\sqrt{t_{1k}(s')t_{2k}(s')}}}}{\frac{e^{2\nu_k\sqrt{t_{1k}(s)t_{2k}(s)}}}{\sqrt{2\pi 2\nu_k\sqrt{t_{1k}(s)t_{2k}(s)}}}}$$

$$= \frac{\sqrt{\sqrt{t_{1k}(s)t_{2k}(s)}}}{\sqrt{\sqrt{t_{1k}(s')t_{2k}(s')}}} e^{\nu_k\left(\left(\sqrt{t_{1k}(s)}-\sqrt{t_{2k}(s)}\right)^2-\left(\sqrt{t_{1k}(s')}-\sqrt{t_{2k}(s')}\right)^2\right)} \tag{32}$$

Expression 32 and the fact that $t_{1k}(s)$ and $t_{2k}(s')$ are bounded away from zero for all s (Lemma 1) allow us to conclude that

$$\frac{\Pr_k\{piv_2|s'\}}{\Pr_k\{piv_2|s\}} \to 0$$

if

$$\left(\sqrt{t_1(s')} - \sqrt{t_2(s')}\right) > \left(\sqrt{t_1(s)} - \sqrt{t_2(s)}\right). \tag{33}$$

Since 33 holds for any $s \in T$ and $s' \notin T$ it follows that

$$\frac{\sum_{s' \in T} \Pr_k\{piv_2|s'\}}{\sum_{s \in S} \Pr_k\{piv_2|s\}} \to 1. \tag{34}$$

Lemma 1 implies the ratio $\frac{\Pr_k\{piv_1|s'\}}{\Pr_k\{piv_2|s\}}$ is bounded away from zero and infinity since $t_{1k}(s)$ and $t_{2k}(s)$ are both bounded away from zero. Therefore, (34) implies that

$$\frac{\sum_{s' \in T} \Pr_k\{piv_1|s'\}}{\sum_{s \in S} \Pr_k\{piv_1|s\}} \to 1. \tag{35}$$

and, together, (34) and (35) imply

$$\frac{\sum_{s \in T} \Pr_k\{piv|s'\}}{\sum_{s \in S} \Pr_k\{piv|s\}} \to 1$$

Since

$$\beta_k(s|piv_j) = \frac{g(s)\Pr_k\{piv_j|s'\}}{\sum_{s \in S} g(s)\Pr_k\{piv_j|s'\}}$$

the Lemma follows. ■

Lemma 2. *Suppose Assumptions 1-6 hold. Consider a sequence of voting equilibria. Then* $\beta_k(s^1|piv) + \beta_k(s^2|piv) \to 1$ *as* $k \to \infty$*. Also, for* $j = 1, 2$, $\beta_k(s^1|piv_j) + \beta_k(s^2|piv_j) \to 1$ *as* $k \to \infty$.

Proof. Consider a sequence of voting equilibria (x^j_{mk}), $j = 1, 2$. Consider a convergent subsequence of (x^j_{mk}).

Step 1: Suppose $x^j_{1k} - x^j_{Mk} \to 0$ for $j = 1, 2$. Then $\beta_k(s^1|piv) \to 1$ as $k \to \infty$ and $F(x(s^1)) = 1/2$.

Proof. Since

$$\begin{aligned} E(v(x_{1k}^2,s)|piv_1,1) &= E(v(x_{Mk}^2,s)|piv_1,M) \\ E(v(x_{1k}^1,s)|piv_2,1) &= E(v(x_{Mk}^1,s)|piv_2,M) \end{aligned}$$

it follows from the continuity of v that for $x \in [x_{Mk}^2, x_{1k}^2]$ and $x' \in [x_{Mk}^1, x_{1k}^1]$

$$\begin{aligned} E(v(x,s)|piv_1,1) - E(v(x,s)|piv_1,M) &\to 0 \\ E(v(x',s)|piv_2,1) - E(v(x',s)|piv_2,M) &\to 0 \end{aligned} \tag{36}$$

Since the signal satisfies Assumption 2 it follows that there exists an s, s' such that $\Pr_k(s|piv_1) \to 1$ and $\Pr_k(s'|piv_2) \to 1$. Now recall that piv_1 is the event $n_1 = n_2 + 1$ and piv_2 is the event $n_1 = n_2$. Hence the difference is exactly one vote. Since $t_{jk}(s)$ is bounded away from zero it follows a vote is always a noisy signal of the state - hence contains limited information. Therefore, $\Pr_k(s'|piv_2) \to 1$ implies that also $\Pr_k(s'|piv_1) \to 1$ and hence $s' = s$. (A straightforward algebraic argument can be made using equations (3) and (4)).

We now show that it must be the case that $F(x(s')) = 1/2$. Suppose this were false and $F(x(s')) < 1/2$. Then the fraction of voters who prefer 2 in state s is less than $1/2 - \epsilon$ for some $\epsilon > 0$. Since $x_{1k}^j - x_{Mk}^j \to 0$ for $j = 1, 2$ it follows that the fraction of voters who vote for 2 is less than $1/2 - \epsilon/2$ for sufficiently large k for all s. This in turn implies that the probability that $n_2 = n_1$ is maximized at $s = 1$ and since every state has strictly positive prior it follows that $s' = 1$. But this contradicts the assumption that $F(x(1)) > 1/2$ (Assumption 6). An analogous argument can be made for the case where $F(x(s')) > 1/2$.

Step 2: Suppose $\lim \left(x_{1k}^j - x_{Mk}^j\right) > \epsilon > 0$ for $j = 1$ or $j = 2$. Then $\beta_k(s^1|piv) + \beta_k(s^2|piv) \to 1$ as $k \to \infty$.

Proof. By Lemma A3 we know that $\sum_T \beta_k(s|piv) \to 1$. Thus it is sufficient to show that $T \subseteq \{s^1, s^2\}$.

Consider a convergent subsequence and let $t_j(s) = \lim_{k\to\infty} t_{jk}(s)$. From the definition of T and the monotonicity of $t(s)$ it follows that if $t_1(s^1) \geq t_2(s^1)$ and $t_1(s^2) \leq t_2(s^2)$, then $T \subseteq \{s^1, s^2\}$.

Suppose, contrary to what we need to show, that

$$t_1(s^1) < t_2(s_2) \tag{37}$$

It follows that $\{s > s^1\} \cap T = \emptyset$ and hence

$$\beta_k(s > s^1|piv_1) \to 0. \tag{38}$$

Consider any m such that $p(m|s^1) > 0$. (38) implies that a voter who receives signal m and conditions on the event that a vote is pivotal believes the probability of state $s > s^1$ is arbitrarily close to zero for large k. Thus for all $\epsilon > 0$ there is a k' such that for $k > k'$

$$x_{mk}^1 > x(s^1) - \epsilon$$

and hence

$$\liminf_k x^1_{mk} \geq x(s^1).$$

But since $F(x(s^1)) > 1/2$, this in turn implies that $\liminf t_{1k}(s^1) - 1/2 \geq 0$ and hence $\sqrt{t_1(s^1)} - \sqrt{t_2(s^1)} > 0$ contradicting (37). An analogous argument can be used to establish $t_1(s^2) \leq t_2(s^2)$. ■

Lemma 3. *Suppose Assumptions 1-7 hold. Then there is an $\epsilon > 0$ and a $k' < \infty$ such that $\beta_k(s^1|piv) \geq \epsilon, \beta_k(s^2|piv) \geq \epsilon$ for all $k \geq k'$.*

Proof. Suppose contrary to the Lemma that there is a subsequence such that

$$\beta_k(s^1|piv) \to 0$$

This implies that

$$\beta_k(s^2|piv) \to 1$$

and hence $t_{1k}(s^2) \to F(x(s^2))$ and $t_{2k}(s^2) \to 1 - F(x(s^2))$. Moreover, since all voters who receive a noisy signal behave as if state 2 occurred,

$$t_{1k}(s^1) \to F(x(s^2)) + (F(x(s^1)) - F(x(s^2)) \sum_{\bar{M}} p(m|s^1).$$

and

$$t_{2k}(s^1) \to 1 - F(x(s^2)) - (F(x(s^1)) - F(x(s^2)) \sum_{\bar{M}} p(m|s^1).$$

Let $\Delta = (F(x(s^1)) - F(x(s^2)) \sum p(m|s^1)$ and observe that

$$\left(\sqrt{F(x(s^2)) + \Delta} - \sqrt{1 - F(x(s^2)) - \Delta}\right)^2 < \left(\sqrt{F(x(s^2))} - \sqrt{1 - F(x(s^2))}\right)^2 \quad (39)$$

whenever $0 < \Delta < 1 - 2F(x(s^2))$.

Assumption 7 says that $0 \leq \Delta < 1 - 2F(x(s^2))$. Hence 39 holds with a strict inequality whenever $\Delta > 0$ and therefore we have that for sufficiently large k

$$\left(\sqrt{t_{1k}(s^1))} - \sqrt{t_{2k}(s^1)}\right)^2 < \left(\sqrt{t_{1k}(s^2))} - \sqrt{t_{2k}(s^2))}\right)^2. \quad (40)$$

To see that (40) also holds (for large k) when $\Delta = 0$ observe that in this case $t_{1k}(s^j) \to F(x(s^2)) < 1/2$ and $t_{2k}(s^j) \to 1 - F(x(s^2)) > 1/2$ since every agent behaves as if state s^2 has occurred.. Thus we have that for large k

$$\begin{aligned} t_{1k}(s^2) &< t_{1k}(s^1) < 1/2 \\ t_{2k}(s^2) &> t_{2k}(s^1) > 1/2 \end{aligned}$$

and (40) follows for large k.

¿From the proof of Lemma 2 we know that

$$\frac{\Pr_k\{piv_j|s^1\}}{\Pr_k\{piv_j|s^2\}} \to \frac{\sqrt{\sqrt{t_{1k}(s^1)t_{2k}(s^1)}}}{\sqrt{\sqrt{t_{1k}(s^2)t_{2k}(s^2)}}} e^{\nu_k\left(\left(\sqrt{t_{1k}(s^2)} - \sqrt{t_{2k}(s^2)}\right)^2 - \left(\sqrt{t_{1k}(s^1)} - \sqrt{t_{2k}(s^1)}\right)^2\right)} \quad (41)$$

Inequality (40) then implies that for every $\epsilon > 0$ there is a k' such that for $k > k'$

$$\beta_k(s^1|piv) \geq \frac{\sqrt{\sqrt{t_{1k}(s^1)t_{2k}(s^1)}}}{\sqrt{\sqrt{t_{1k}(s^2)t_{2k}(s^2)}}}\frac{g(s^1)}{g(s^2)}\beta_k(s^2|piv) - \epsilon$$

Since $g(s)$ and $t_{jk}(s)$ are bounded away from zero for all s this in turn contradicts $\beta_k(s^1|piv) \to 0$. (The argument for the case where $\beta_k(s^2|piv) \to 0$ is analogous.) ■

Proposition 4. *Suppose Assumptions 1-5 hold and $F(x(s^1)) > 1/2$. Then, as $k \to \infty$, the probability that alternative 1 is elected converges to 1 for $s \leq s^1$ and to 0 for $s > s^1$.*

Proof. By Step 1 of the proof of Lemma 2 it follows from $F(x(s^1)) > 1/2$ that there is an $\varepsilon > 0$ such that $(x^j_{mk} - x^j_{m'k}) > \varepsilon$ for $m' > m$ and all k. As a consequence, there is an $\varepsilon' > 0$ such that

$$|t_j(s) - t_j(s')| > \varepsilon'. \tag{42}$$

Hence t_1 is strictly decreasing in s with slope bounded away from zero and t_2 are strictly increasing in s with slope bounded away from zero. By the proof of step 2 of Lemma 2 we know that for any convergent subsequence

$$\begin{aligned} &\lim\left(\sqrt{t_1(s^1)} - \sqrt{t_2(s^1)}\right) \\ \geq\ & 0 \geq \lim\left(\sqrt{t_1(s^2)} - \sqrt{t_2(s^2)}\right). \end{aligned}$$

If the above two inequalities are strict then (42) together with the strong law of large numbers implies the Theorem.

In the remainder of the proof we show that indeed both of these inequalities must be strict. To see this first note that (42) implies that at least one of the two inequalities is strict. Thus suppose that $\lim\left(\sqrt{t_1(s^1)} - \sqrt{t_2(s^1)}\right) = 0$. Then it follows that $T_k = \{s^1\}$ for large k and hence by Lemma A3 $\beta(s^1|piv_j) \to 1$. This implies that $t_1(s^1) = F(x(s^1)) > 1/2$ (by Assumption) which contradicts the hypothesis that $\lim\left(\sqrt{t_1(s^1)} - \sqrt{t_2(s^1)}\right) = 0$. An analogous argument shows that the second inequality is strict. ■

Proposition 5 *Suppose Assumptions 1-7 hold. Then there is an $\alpha > 0$ and a k' such that the expected fraction of voters who abstain in equilibrium is larger than α for all $k > k'$.*

Proof. Since Assumption 7 holds we can apply Lemma 3 to establish that $\beta_k(s^1|piv) > \epsilon > 0$ and $\beta_k(s^2|piv) > \epsilon > 0$ for large k. By Assumption 1 this in turn implies that $x^j_{1k} - x^j_{Mk} > \epsilon > 0$ for $j = 1$ or $j = 2$. for large k. Thus, there exists an $\epsilon' > 0$ such that $\rho_k(s) - \rho_k(s') \geq \epsilon'$ for all $s < s'$. From Lemma 1 we know that

$$\beta_k(s^1|piv_2) - \frac{g(s^1)\frac{\Pr_k(piv_1|s^1)}{\sqrt{\rho_k(s^1)}}}{g(s^1)\frac{\Pr_k(piv_1|s^1)}{\sqrt{\rho_k(s^1)}} + g(s^2)\frac{\Pr_k(piv_1|s^2))}{\sqrt{\rho_k(s^2)}}} \to 0$$

and hence there is an k' and an ϵ'' such that

$$\beta_k(s^1|piv_2) < \beta_k(s^1|piv_1) - \epsilon'' \tag{43}$$

for all $k > k'$. Consider an agent who receives a signal m with $p(m|s^1) > 0$ and $p(m|s^2) > 0$. Since (43) holds it follows that there is an $\epsilon''' > 0$ and a k' such that for $k > k'$

$$E\{v(x,s)|piv_1, m\} < E\{v(x,s)|piv_2, m\} - \epsilon'''$$

Since $x^1_{m,k}$ is given by $E\{v(x^1_{m,k}, s)|piv_1, m\} = 0$ and $x^2_{m,k}$ is given by $E\{v(x^2_{m,k}, s)|piv_2, m\} = 0$ the result now follows. ■

Proposition 6. *Suppose Assumptions 1-6 hold. For every $\varepsilon > 0$ there is an $\eta > 0$ such that $|F(x(s^*)) - 1/2| \leq \eta$ implies that $\limsup_k(1 - t_{1k}(s) - t_{2k}(s)) \leq \varepsilon$ for all $s \in S$, i.e., the expected fraction of voters who abstain is bounded above by ε for sufficiently large k.*

Proof. *Step 1:* First we demonstrate that the fraction of players who abstain and who receive a signal that allows them to exclude a state in the set $\{s^1, s^2\}$ converges to zero. I.e., we show that the fraction of voters who abstain and who receive a signal with the property $p(m|s^1)p(m|s^2) = 0$ converges to zero.

To prove Step 1 we distinguish two cases:

Case 1: $p(m|s^1) > 0$ and $p(m|s^2) = 0$ or $p(m|s^1) = 0$ and $p(m|s^2) > 0$. Suppose that $p(m|s^1) > 0$ and $p(m|s^2) = 0$. In this case receiving message m permits the agent to perfectly discriminate between states s^1 and s^2. Now it follows that

$$E(v(x,s)|piv_1, m) - E(v(x,s)|piv_2, m) \to 0$$

since

$$\begin{aligned} E(v(x,s)|piv_1, m) &\to v(x, s^1) \\ E(v(x,s)|piv_2, m) &\to v(x, s^1) \end{aligned}$$

The case $p(m|s^1) = 0$ and $p(m|s^2) > 0$ is analogous.

Case 2. $p(m|s^1) = p(m|s^2) = 0$. In this case receiving the message m tells the agent that the state is neither s^1 or s^2. It follows from SMLRP that either $p(m|s) = 0$ for any $s < s^1$ or for any $s > s^2$. Let $p(m|s) = 0$ for any $s < s^1$ (The other case is analogous). Let $\bar{s} < s^1$ be the largest state such that $p(\bar{s}|m) > 0$. Now it follows from $\rho(s)$ strictly decreasing in s that

$$\begin{aligned} E(v(x,s)|piv_1, m) &\to v(x, \bar{s}) \\ E(v(x,s)|piv_2, m) &\to v(x, \bar{s}) \end{aligned}$$

and the fraction of types receiving message m that abstain goes to zero.

Step 2. In Step 2 we demonstrate that the fraction of voters who receive a signal with the property $p(m|s^1)p(m|s^2) > 0$ and who abstain can be made arbitrarily small if η is sufficiently small. These are voters whose signal does not allow them to exclude a state in the set $\{s^1, s^2\}$. To prove this we demonstrate that for small η the beliefs conditional on the event piv_1 are very close to the beliefs conditional on the event piv_2.

Assume that the critical state is s^1 i.e., $s^1 \in \arg\min_s |F(x(s)) - 1/2|$. (The case where $s^2 = \arg\min_s |F(x(s)) - 1/2|$ is entirely analogous.)

Observe that $\liminf t_{2k}(s^1) \geq 1 - F(x(s^1))$ since every voter with type $x > x(s^1)$ prefers alternative 2 if the state is in the set $s \in \{s^1, s^2\}$ and by Lemma 2 we know that $\beta_k(s^1|piv_j) + \beta_k(s^2|piv_j) \to 1$ as $k \to \infty$. Since alternative 1 wins the election in state s^1 with probability close to one (Proposition 4) it follows that $t_{1k}(s^1) \geq t_{2k}(s^1)$. Since the expected vote shares must be less than or equal to 1 in each state it must be the case that $t_{1k}(s^1) \leq F(x(s^1))$ for sufficiently large k and we get the following inequality:

$$0 \geq \sqrt{t_{2k}(s^1)} - \sqrt{t_{1k}(s^1)} \geq \sqrt{1 - F(x(s^1))} - \sqrt{F(x(s^1))}.$$

Therefore,

$$1 \leq \liminf \sqrt{\rho_k(s^1)} \leq \limsup \sqrt{\rho_k(s^1)} \leq \frac{\sqrt{F(x(s^1))}}{\sqrt{1 - F(x(s^1))}}. \tag{44}$$

Further, observe that since vote shares are monotone it follows that

$$t_2(s^2) \geq 1 - F(x(s^1)).$$

By Lemma 3 $\beta_k(s^1|piv_j) \geq \varepsilon, \beta_k(s^2|piv_j) \geq \varepsilon > 0$ and by Lemma 2 this implies that

$$\left(\sqrt{t_{1k}(s^2)} - \sqrt{t_{2k}(s^2)}\right) - \left(\sqrt{t_{2k}(s^1)} - \sqrt{t_{1k}(s^1)}\right) \to 0$$

As a result we get that

$$\begin{aligned} \liminf \sqrt{t_{1k}(s^2)} &\geq \sqrt{1 - F(x(s^1))} + \sqrt{1 - F(x(s^1)} - \sqrt{F(x(s^1)} \\ &= 2\sqrt{1 - F(x(s^1))} - \sqrt{F(x(s^1)} \end{aligned}$$

and

$$1 \geq \limsup \sqrt{\rho_k(s^2)} \geq \liminf \sqrt{\rho_k(s^2)} \geq \frac{2\sqrt{1 - F(x(s^1))} - \sqrt{F(x(s^1)}}{\sqrt{1 - F(x(s^1))}}. \tag{45}$$

Equations 44 and 45 imply that for any $\varepsilon > 0$ there is an $\eta > 0$ such that for $F(x(s^1)) - 1/2 \leq \eta$ we have that

$$\begin{aligned} 1 &\leq \liminf \sqrt{\rho_k(s^1)} \leq \limsup \sqrt{\rho_k(s^1)} \leq 1 + \varepsilon \qquad (46) \\ 1 &\geq \limsup \sqrt{\rho_k(s^2)} \geq \liminf \sqrt{\rho_k(s^2)} \geq 1 - \varepsilon. \end{aligned}$$

Lemma 1 and the fact that $p(m|s^1)p(m|s^2) > 0$ then implies that for every ε' there is an η such that for $F(x(s^1)) - 1/2 \leq \eta$ implies that

$$\beta_k(s^1|piv_1, m) - \beta_k(s^1|piv_2, m) \leq \varepsilon'.$$

for sufficiently large k and therefore for every ε'' there is an η such that for $F(x(s^1)) - 1/2 \leq \eta$ and for m

$$E\{v(x,s)|piv_1, m\} - E\{v(x,s)|piv_2, m\} \leq \varepsilon'' \tag{47}$$

which yields the result. ■

10 Appendix B

The system of equations for the noisy signals example is given below:

$$\begin{aligned}
A &= 1 - t_1(1) - t_1(2) \\
A_i &= .5(x_1^2 - x_1^1) \\
A_u &= -x_\emptyset^1 \\
x_\emptyset^1 &= \frac{2}{1 + \sqrt{\frac{t_1(1)}{t_1(2)}}} - 1 \\
x_1^1 &= \frac{2}{1 + \frac{1-p}{p}\sqrt{\frac{t_1(1)}{t_1(2)}}} - 1 \\
x_1^2 &= \frac{2}{1 + \frac{1-p}{p}\sqrt{\frac{t_1(2)}{t_1(1)}}} - 1 \\
t_1(1) &= .5q\Big((1 + x_1^1)p + (1 - x_1^2)(1-p)\Big) + .5(1-q)(1 + x_\emptyset^1) \\
t_1(2) &= .5q\Big((1 + x_1^1)(1-p) + (1 - x_1^2)p\Big) + .5(1-q)(1 + x_\emptyset^1)
\end{aligned}$$

The system of equations for the biased distribution of information example is given below:

$$\begin{aligned}
A_L &= .5(x_\emptyset^2 - x_\emptyset^1) \\
x_\emptyset^1 &= \frac{2}{1 + \gamma_1} - 1 \\
x_\emptyset^2 &= \frac{2}{1 + \gamma_2} - 1 \\
\gamma_1 &= \sqrt{\frac{t_1(1)t_2(2)}{t_1(2)t_2(1)}}\gamma_2 \\
\sqrt{t_1(1)} - \sqrt{t_2(1)} &= \sqrt{t_2(2)} - \sqrt{t_1(2)} \\
t_1(1) &= .5 + .5(1 + x_\emptyset^1) \\
t_1(2) &= .5(1 + x_\emptyset^1) \\
t_2(1) &= .5(1 - x_\emptyset^2) \\
t_2(2) &= .5 + .5(1 - x_\emptyset^2)
\end{aligned}$$

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