

# Political model of social evolution

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Edited by Paul Robert Milgrom, Stanford University, Stanford, CA, and approved September 22, 2011 (received for review January 19, 2011)

**Almost all democratic societies evolved socially and politically out of authoritarian and nondemocratic regimes. These changes not only altered the allocation of economic resources in society but also the structure of political power. In this paper, we develop a framework for studying the dynamics of political and social change. The society consists of agents that care about current and future social arrangements and economic allocations; allocation of political power determines who has the capacity to implement changes in economic allocations and future allocations of power. The set of available social rules and allocations at any point in time is stochastic. We show that political and social change may happen without any stochastic shocks or as a result of a shock destabilizing an otherwise stable social arrangement. Crucially, the process of social change is contingent (and history-dependent): the timing and sequence of stochastic events determine the long-run equilibrium social arrangements. For example, the extent of democratization may depend on how early uncertainty about the set of feasible reforms in the future is resolved.**

history dependence | political equilibrium

**A**lmost all today's social, political, and economic institutions have evolved over decades and sometimes, hundreds of years. For example, many scholars trace the roots of British political institutions to 1215, when the barons forced King John to sign the Magna Carta, introducing the first formal restrictions on the authority of the monarchs and paving the way to the Glorious Revolution of 1688 and the Bill of Rights. The more recent wave of democratization in Britain started with the first Reform Act of 1832, which modestly extended the franchise. This act was followed by a series of reforms gradually extending voting rights over the next three-quarters of a century, first to a broader franchise of adult men and then to women. Both the protesters demanding change in 1832 and the political elites that agreed to address these demands by extending the franchise modestly likely understood many of the consequences of these political changes (1). However, they were as unlikely to perfectly foresee—or care about—the sequence of events, including the repeal of the Corn Laws in 1846 and the arrival of universal mass suffrage, unleashed by this reform as the barons that forced King John to sign the Magna Carta were unlikely to understand the importance that this document would have centuries later.

This brief description suggests that a satisfactory framework of social evolution should incorporate three key ingredients.

- i) Strategic agents: any institutional change is implemented by strategic individuals (even if their preferences reflect the preferences of larger social groups). These individuals seek to maximize their future benefits in the context of a well-defined game.
- ii) Discounting: agents discount future payoffs and thus put limited weight on events that will take place in the very far future. For example, the barons that pushed for the Magna Carta cared greatly about its consequences in the near future, but its consequences in 800 y would have meant little to them.
- iii) Stochastic future: the environment is subject to stochastic shocks. This finding is important for modeling the contingent nature of social and political change, and it also allows

us to better capture the notion that agents are not able to perfectly foresee the future path of events (although they do understand what set of events are feasible in the future).

In this paper, we provide a general but tractable framework of social, economic, and political evolution incorporating these ingredients. The society consists of agents that care about current and future social rules and allocations that are comprised of economic as well as social elements. A change in social rules not only affects current allocations, but it also alters the balance of power in society and potentially, impacts future social rules and allocations.

We assume that the set of social rules and allocations can be represented by an ordered set (e.g., by a set of rules that can be thought of as less democratic or more democratic). Although restrictive, this assumption, together with stage payoffs that satisfy a single-crossing condition (*Example 1*), enables us to provide a sharp analysis of the process of social evolution. In particular, under these assumptions and some additional, mostly technical assumptions, we prove that the dynamic game representing the process of social evolution always has a pure strategy (Nash) equilibrium, and we provide a fairly tight characterization of the structure of equilibria. We view this framework as a political model of social evolution, because the extent of social change is determined and restricted by political equilibria, particularly, by concerns about how political power will shift as a result of possible social changes.

Social rules and allocations change in this framework when those people who are currently (socially or politically) powerful agree to a change. Social change may happen without any stochastic shocks; it may happen as a result of a stochastic shock destabilizing an otherwise stable social arrangement, or because a sequence of changes is desirable for those currently holding power. However, social change may fail to occur at times, even if it is Pareto improving, precisely because it may reduce the future power of the currently powerful.

We also show, using a simple example, how the set of possible equilibrium configurations is both history-dependent and contingent on the nature of stochastic events. For example, the extent of democratization may depend on how early uncertainty about the set of feasible reforms in the future is resolved. If this uncertainty is resolved early and implies that other reforms are likely to follow the current one, this uncertainty might discourage current reforms (for instance, the fact that early political concessions led to greater democratic reforms in Tunisia may have discouraged Syrian elites from making any concessions). Intuitively, current

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This paper results from the Arthur M. Sackler Colloquium of the National Academy of Sciences, "Dynamics of Social, Political, and Economic Institutions," held December 3–4, 2010, at the Arnold and Mabel Beckman Center of the National Academies of Sciences and Engineering in Irvine, CA. The complete program and audio files of most presentations are available on the NAS Web site at [www.nasonline.org/Sackler\\_Dynamics](http://www.nasonline.org/Sackler_Dynamics).

Author contributions: D.A., G.E., and K.S. designed research, performed research, and wrote the paper.

The authors declare no conflict of interest.

This article is a PNAS Direct Submission.

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This article contains supporting information online at [www.pnas.org/lookup/suppl/doi:10.1073/pnas.1019454108/-DCSupplemental](http://www.pnas.org/lookup/suppl/doi:10.1073/pnas.1019454108/-DCSupplemental).

reform may correspond to a gamble for the political elites, with the downside being a process of additional reforms reducing their political power and social status. If they think that such reforms are not likely, they will be willing to agree to the current reform; however, if feasibility of such reforms is revealed early, this revelation may increase their resolve against reform.

The next example illustrates some of the situations that can be studied with this framework.

**Example 1.** Consider a society consisting of  $i = 1, \dots, n$  agents. Suppose that lower-indexed agents are richer or more aristocratic than higher-indexed agents. Suppose that the society chooses one of  $s = 1, \dots, m$  social rules, with higher-indexed social rules being more democratic or allocating greater social status to poorer and less aristocratic agents than lower-indexed ones. Social rules lead to different types of economic, social, and political relationships among the agents. Rather than modeling these relationships in detail, we simply assign a stage utility  $u_i(s)$  that agent  $i$  will obtain from social rule  $s$ . Each agent maximizes the sum of stage payoffs discounted with factor  $\beta \in (0, 1)$ . We impose the single-crossing assumption that  $u_i(s) - u_j(s)$  is increasing in  $s$  whenever  $i > j$ . This assumption implies that less aristocratic individuals are less averse to (or more keen on) more democratic social rules than more aristocratic ones. We impose some reasonable conditions on how political and social power is distributed under different social rules, loosely capturing the notion that higher-indexed social rules are more democratic. This finding may result from an explicit political process in which agents vote or bargain or from social interactions giving more decision-making capacity to agents with greater social status. Finally, we model the stochastic nature of the environment by assuming that, at time  $t$ , only a subset of the states,  $l_t, \dots, r_t$ , are feasible, and additional states become feasible stochastically. Therefore, when elites agreed to the modest extension of the franchise in 1832, they may not have known for sure whether and when future reform possibilities would present themselves (although in our model, they understand that there is some likelihood of this event happening).

Our paper is most closely related to the work in ref. 2 and our own previous work (3, 4). In the work in ref. 3, we study a related model but with two crucial differences. First, the discount factor is taken to be large, and therefore, only the stable long-run outcomes matter. Second, there are no stochastic transitions (refs. 2 and 5 also study related models; the work in ref. 2 considers an environment with side payments, and the work in ref. 5 focuses on submajority voting rules). In the work in ref. 4, we study political selection and government formation in a population with heterogeneous and stochastically changing competencies, but these stochastic shocks are assumed to be very infrequent, and the discount factor is again taken to be large (close to one). Thus, the key issues related to gradual and stochastic social evolution, which is our main focus in this paper, do not arise in these papers. Finally, the analysis is also related to models of voting in clubs, franchise extension, political reform, and coalition formation. The related literature on these topics is discussed in ref. 3, and we do not repeat this discussion here.

The rest of the paper is organized as follows. *Model* introduces our baseline model. *Analysis* analyzes the structure of equilibria and provides additional characterization results on the structure of equilibrium transitions. *Conclusion* concludes. *Appendix* contains some proofs and sketches of other proofs, whereas the rest and some additional examples are in the *SI Appendix*.

## Model

Time is discrete and infinite. The society consists of  $n$  agents  $N = \{1, \dots, n\}$ , which may be interpreted as groups or individuals, and there are  $m$  social rules, which we will refer to as states for short, represented by the set  $S = \{1, \dots, m\}$ . These social rules/

states determine both current payoffs and the distribution of social and political power in society. Each state is characterized by the (stage) payoffs that individuals receive in a period when society is in that state and also, by the set of winning coalitions (i.e., coalitions of players that may decide to change the present state in favor of a new one). Individual  $i$  obtains stage payoff  $u_i(s)$  in state  $s \in S$ . Denoting the state at time  $t$  by  $s_t$  and the expectation at time  $t$  by  $\mathbb{E}_t$ , the expected discounted payoff of individual  $i$  at time  $t$  can be written as (Eq. 1)

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k u_i(s_{t+k}), \quad [1]$$

where  $\beta \in (0, 1)$  is the common discount factor. Throughout, we impose assumption 1.

**Assumption 1 (Single Crossing).** For any agents  $i, j \in N$  such that  $i > j$  (Eq. 2),

$$u_i(s) - u_j(s) \quad [2]$$

is increasing in  $s$  (on  $S$ ).

In addition, we sometimes also assume that preferences are single peaked, meaning that, for each agent  $i$ , there exists some state  $s_i$  (his most preferred state) such that for any  $s'' > s' \geq s_i$  or  $s'' < s' \leq s_i$ ,  $u_i(s'') < u_i(s')$ . In other words, different states are ranked according to how close they are to this most preferred state.

We refer to nonempty subsets  $X$  of  $N$  as coalitions. Each state  $s$  is associated with a subset  $\mathcal{W}_s$  of the set of coalitions, which designates the set of winning coalitions in state  $s$  and summarizes (in an economical fashion) the agents that have political and social power to alter existing social rules. For example, if, in state  $s$ , a majority is required for decision-making,  $\mathcal{W}_s$  includes all subsets of  $N$  that form a majority; if, in state  $s$ , individual  $i$  is a dictator,  $\mathcal{W}_s$  includes all coalitions that include  $i$ . We impose the following relatively mild and natural assumptions on the set of winning coalitions.

**Assumption 2 (Winning Coalitions).** For any given  $s$ , the set of winning coalitions  $\mathcal{W}_s$  satisfies

- i) if  $X \subset Y \subset N$  and  $X \in \mathcal{W}_s$ , then  $Y \in \mathcal{W}_s$ ,
- ii) if  $X \in \mathcal{W}_s$ , then  $N \setminus X \notin \mathcal{W}_s$ , and
- iii)  $\mathcal{W}_s \neq \emptyset$ .

The first part imposes the natural condition that, if a coalition has the capacity to implement change, then a larger coalition also does. The second part ensures that, if some coalition has the capacity to implement change, then the remaining players (its complement) do not; this rules out submajority-type rules. The third part states that a winning coalition always exists.

We next introduce a generalization of the notion of median voter.

**Definition 1 (Quasi-Median Voter).** Player  $i \in N$  is a quasi-median voter in state  $s$  if, for any winning coalition,  $X \in \mathcal{W}_s$ ,  $\min X \leq i \leq \max X$ .

Definition 1 is adopted from ref. 3. In light of assumption 2, part i, it is equivalent to assuming that player  $i$  is a quasi-median voter if he is a member of any connected winning coalition in  $s$  (i.e., any coalition  $X$  for which there exists some  $a, b \in N$  such that  $j \in X \Leftrightarrow a \leq j \leq b$ ). In the case where the set of winning coalitions corresponds to majority voting, the quasi-median voter coincides with the median voter (i.e., the voter whose most preferred allocation is the median of the most preferred allocations of all voters).

Let us denote the set of quasi-median voters in  $s$  by  $M_s$ . In the work in ref. 3, we prove that, under assumption 2,  $M_s$  is nonempty. Throughout this paper, we impose assumption 3.

**Assumption 3 (Monotonic Quasi-Median Voters).** Sequences  $\{\min M_s\}_{s \in S}$  and  $\{\max M_s\}_{s \in S}$  are nondecreasing in  $s$ .

Assumption 3 ensures that, if a certain number of rich people is sufficient to implement some social change decisions in one state, then it is also sufficient to do so in lower states; similarly, if a certain number of poor people is sufficient to implement such change in some state, then it can also do so in higher states. In terms of example 1 in the Introduction, it captures the idea that higher-indexed states, corresponding to more democratic states, give less power to more aristocratic individuals. In that example, assumptions 1–3 are satisfied provided that, as stated there,  $u_i(s) - u_i(s')$  is increasing in  $s$  and that we adopt some weighted (super) majority rule for changing the composition of the club or social rules.

We next introduce stochastic shocks. We assume that, at each date  $t$ , the set of available states is  $L_t = \{l_t, \dots, r_t\}$ , (i.e., the set of feasible states is always assumed to be connected). New states,  $l_t - 1$  and  $r_t + 1$ , are assumed to become available stochastically at the beginning of period  $t$  with probabilities  $p_{L_t}^l$  and  $p_{L_t}^r$ , respectively.

The last aspect of the dynamic game left to be described is the process by which proposals are made. We model this aspect using a set of protocols. A protocol  $\pi^s$  for state  $s$  is a finite sequence of players  $\pi^s = (\pi_1^s, \dots, \pi_k^s)$  such that, for any player  $i \in N$ ,  $\pi_k^s = i$  for some  $k \in \{1, \dots, K_s\}$ . This last condition assures that each player has a chance to be the agenda setter in each state.

Finally, in what follows, we assume that only one-step transitions are allowed, meaning that, if the current state is  $s_t$ , then  $s_{t+1} \in \{s_t - 1, s_t, s_t + 1\}$ . This assumption significantly simplifies the analysis. The general case where any  $s_{t+1} \in L_t$  is feasible is studied in a companion paper (3).

The timing of each period is as follows.

- i) Period  $t$  begins with state  $s_{t-1}$  inherited from the previous period ( $s_0$  is exogenously given).
- ii) With probability  $(1 - p_{L_{t-1}, s_{t-1}}^l)(1 - p_{L_{t-1}, s_{t-1}}^r)$ , the set of available states stays the same ( $L_t = L_{t-1}$ ); otherwise, it expands to the left, right, or both sides.
- iii) Players become agenda setters, one at a time, according to the protocol  $\pi^{s_{t-1}}$ . Agenda setter  $i$  proposes an alternative,  $a_{t,i} \in L_t$ , such that  $s_{t-1} - 1 \leq a_{t,i} \leq s_{t-1} + 1$ .
- iv) All players vote sequentially over the proposal  $a_{t,i}$  (e.g., we can assume that the voting sequence is  $1, 2, \dots, n$ , although any deterministic or stochastic sequence would also give identical results). If the set of players that supports the transition,  $Y$ , is a winning coalition (i.e.,  $Y \in \mathcal{W}_{s_{t-1}}$ ), then  $s_t = a_{t,i}$  (i.e., a transition to  $a_{t,i}$  takes place). Otherwise, the next person makes a proposal, and if the last agent in the protocol has already made a proposal and no proposal has been accepted, then  $s_t = s_{t-1}$ .
- v) Each player  $i$  gets instantaneous utility  $u_i(s_t)$ .

We are interested in Markov perfect equilibria (MPE) of the above game, which essentially means (subgame perfect) equilibria of this dynamic game are Markovian in the sense that they are conditioned on the payoffs-relevant variables (such as the current state  $s_{t-1}$ ).

## Analysis

**Definition 2 (Transition Mapping).** Consider the game above with protocol  $\{\pi^s\}_{s \in S}$  and MPE  $\sigma$  in pure strategies. Mapping  $\phi_L: S \rightarrow S$  is referred to as the transition mapping corresponding to equilibrium  $\sigma$  and the set of available states  $L$  if, whenever  $s_{t-1} = s$  and  $L_t = L$ ,  $s_t = \phi_L(s)$  along the equilibrium path of  $\sigma$ .

The transition mapping is an economical way of summarizing the structure of equilibria, because it specifies how the state  $s_t$  will evolve.

**Definition 3 (Monotone Mappings).** Transition mapping  $\phi_L: S \rightarrow S$  is monotone if, for any  $s_1, s_2 \in S$  with  $s_1 \leq s_2$ ,  $\phi_L(s_1) \leq \phi_L(s_2)$ .

Monotone transition mappings rule out cycles, and we will see that our equilibria will induce such monotone mappings.

Some of our results hold not for all but for almost all possible configurations of stage payoffs. To formalize this notion, we say that some statement is held generically if it is true for all combinations of payoffs  $\{u_i(s)\}_{i \in N, s \in S}$ , except, perhaps, for a set of Lebesgue measure zero. We now formulate our first result.

## Proposition 1.

- i) For any set of protocols  $\{\pi^s\}_{s \in S}$  and any initial state  $s_0$ , there exists a Markov perfect equilibrium of the game in pure strategies that induces a monotone transition mapping  $\phi_L$  for every set of available states  $L \subset S$ .
- ii) Generically, any pure strategy Markov perfect equilibrium induces monotone transition mappings  $\phi_L$  for every set of available states  $L \subset S$ .
- iii) Generically, the transition mapping  $\phi_L$  is unique whenever either (i) preferences are single peaked or (ii) every set of quasi-median voters is a singleton. This unique mapping does not depend on  $\{\pi^s\}_{s \in S}$ .

Proposition 1 proves the existence of a pure strategy Markov perfect equilibrium. For this proof, we first construct an equilibrium for the environment where all states have become available (i.e.,  $L_t = S$  for any  $t$ ) and show that the implied transition mapping is monotone. The monotonicity of the mapping implies that single crossing (assumption 1) holds not only for stage payoffs but also for continuation values. This finding allows us to use the following inductive argument: iterate by one step to the situation where only one state is not available yet, and assign payoffs to different states that are linear combinations of stage payoffs in the current state and continuation values after a possible shock (which we already know by induction). The linear combinations are such that the single-crossing condition is preserved, and thus, we can apply the same argument in a situation in which two states are not available yet and so on. Moreover, this construction ensures that the transition mapping is monotone for all  $L$ .

The second part shows that all equilibria have a simple and intuitive structure: generically, all equilibria can be represented by monotone transition mappings. By definition, this structure ensures that there are no equilibrium cycles (where the equilibrium path periodically visits some states  $s_1, \dots, s_k$ ). An immediate implication is that each nonmonotone switch (e.g., a transition from  $s$  to  $s' < s$  followed by a transition to  $s'' > s'$ ) in the observed evolution of states must be a result of a shock.

Finally, the third part establishes the uniqueness of the transition mapping provided that one of two (or both) additional assumptions hold. Although theoretically restrictive, these assumptions hold in a number of applications, and the uniqueness result is, therefore, applicable to these applications.

**Proposition 2.** Generically, the evolution of states has a limit state  $s^\infty$ : as  $t \rightarrow \infty$ ,  $s_t$  converges almost surely to some  $s^\infty$ . The limit state  $s^\infty$  might depend on the timing and sequence of shocks.

This proposition implies that, despite stochastic shocks, the process of social evolution will lead to a (long-run stable) limit state. Intuitively, without any shocks, the monotonicity of transition paths ensures convergence to such a limit state. When there are shocks, they will ultimately lead to the entire set of feasible states that are available ( $L = S$ ), and then monotonicity again ensures convergence.

**Proposition 3.** Suppose that either (i) preferences are single-peaked or (ii) all sets of quasi-median voters are singletons.

Suppose also that, given the currently available set of states  $L = \{l, \dots, r\}$ , transition paths have stabilized by period  $t$  (i.e.,  $s_t = s_{t-1} = s_{t-2}$ ). Then, generically after a shock that adds a higher-indexed state at time  $t$  so that  $L_t = \{l, \dots, r+1\}$ , there cannot be a transition to a lower state (i.e.,  $s_\tau \geq s_t$  for all  $\tau$  such that  $L_\tau = L_t$ ), whereas a transition to a higher state is possible. Similarly, generically after a shock that adds a lower-indexed state at time  $t$  so that  $L_t = \{l-1, \dots, r\}$ , there cannot be a transition to a higher state (i.e.,  $s_\tau \leq s_t$  for all  $\tau$  such that  $L_\tau = L_t$ ), whereas a transition to a lower state is possible.

This proposition shows that the addition of a new high state does not lead to a transition to a lower state. Roughly speaking, this finding is because such a new state makes higher states relatively more attractive (given the single crossing). Similarly, the addition of a low state does not lead to the transition to a higher state. This result also implies that, if shocks are sufficiently rare, the direction of response to a shock is predictable.

We next provide an example showing how the path of social change is contingent and history-dependent [i.e., depends on the timing (and exact sequence) of realization of uncertainty].

**Example 2 (Early Shocks May Make Reforms More Difficult).** Suppose that  $S = \{1, 2, 3\}$ , then the initial state is  $s_0 = 1$ . Suppose also that state 2 corresponds to limited democracy, and a change to this state will shift political power away from player 1. Let player 2 be the quasi-median voter in state 2 (i.e.,  $M_2 = \{2\}$ ). Finally, suppose that  $u_2(3) > u_2(2)$  and  $u_1(2) > u_1(1) > u_1(3)$ , and therefore, player 2 would prefer transition to a more democratic state 3 if such transitions were feasible. However, this transition is disliked by player 1, who prefers state 2. For instance, we can think of player 1 as corresponding to the king or aristocracy and being in favor of limited democracy but disliking full democracy (ref. 3 has a more detailed discussion of this example). Suppose that  $\beta$  is sufficiently high so that, when  $L = \{1, 2, 3\}$ , player 1 prefers to maintain state 1 (because a change to state 2 will immediately induce a change to state 3, which he dislikes).

Suppose we start with  $L_0 = \{1, 2\}$  and let the probability that the set of feasible states will expand to  $L = \{1, 2, 3\}$  be  $p$ . For  $p$  sufficiently low, player 1 would be in favor of a switch to state 2, because he would expect that society will spend a long time in this state. Suppose, however, that there is an early shock, revealing that state 3 is feasible at time  $t = 0$ . This early shock makes the path of reform more difficult, because it discourages player 1 from accepting the change to state 2. In terms of the discussion of democracy in the British context in the Introduction, an early shock would correspond to the elite believing in 1832 that there would be a very rapid reform to a much more inclusive franchise immediately. If many members of the elite supported the reforms of 1832 with the understanding that these reforms would be relatively stable, such a shock might have made them less willing to accept the more modest reforms of 1832 in the first place.

## Conclusion

Almost all democratic societies have evolved socially and politically out of authoritarian and nondemocratic regimes. This evolution has often been a result of intense political and social conflict and even revolutionary changes. These changes not only altered the allocation of economic resources in society but also the structure of political power. In this paper, we developed a framework for studying dynamics of political and social change that alters the balance of power in society, thus paving the way for future changes.

## Appendix: Proofs

**Proof of Proposition 1.** Here, we provide a short version of the proof, which contains the main ideas and main steps but omits some technical details. A not for publication appendix contains the complete proof.

We prove a stronger result obtained by weakening one condition in Proposition 1, part iii. We replace single peakedness with, for each state  $s$ , there is a player  $i(s) \in M_s$  such that there does not exist two states  $x < s$  and  $y > s$  such that  $u_i(x) > u_i(s)$  and  $u_i(y) > u_i(s)$  (single peakedness trivially implies this condition). We start by proving this stronger proposition in the nonstochastic case (i.e., where  $L_0 = S$ ). We do so by induction by the number of states  $m = |S|$ . The base  $m = 1$  is trivial. We now assume that the proposition has been proved for all configurations with  $|S| < m$  and now prove the induction step for each part of the proposition.

**Proof for the Nonstochastic Case. Part i.** Consider two possibilities. Suppose first that  $u_i(1) \leq u_i(2)$  for at least one  $i \in M_2$ . We consider a new game with the same set of players  $N$ , the same set of states  $S' = [2, m]$ , the same set of winning coalitions on  $S'$ , and the same protocols on  $S'$ , and payoffs are given by  $\tilde{u}_i(x) = u_i(x)$  for each  $x \in S'$ . For this new game, assumption 1 holds, and by induction, it has MPE  $\sigma'$  with transition mapping  $\phi'$ . Let  $\{V'_i(x)\}$  be the continuation values in this MPE. Let us define  $\phi: S \rightarrow S'$  by setting  $\phi(x) = \phi'(x)$  if  $x \in S'$  and  $\phi(1) = 1$  if  $V'_i(2) \leq u_i(1)/(1-\beta)$  for some  $i \in M_1$  and  $\phi(1) = 2$  otherwise. It is then easy to construct MPE  $\sigma$  that implements  $\phi$ , which completes the induction step in this case.

The second possibility is that  $u_i(1) > u_i(2)$  for all  $i \in M_2$ . We take a new game with the set of states  $S' = [2, m]$ , the same sets of winning coalitions, and the same protocols on  $S'$  but with payoffs given by  $\tilde{u}_i(x) = u_i(x)$  for each  $x \geq 3$  and  $\tilde{u}_i(x) = (1-\beta)u_i(2) + \beta u_i(1)$  if  $x = 2$ . Again, assumption 1 holds, and therefore, we can take MPE  $\sigma'$  with transition mapping  $\phi'$ ; denote the continuation values by  $\{V'_i(x)\}$ . We then define  $\phi(x) = \phi'(x)$  for all  $x \geq 3$  and consider the following cases separately. First, if at least one  $i \in M_2$  has  $u_i(2)/(1-\beta) \geq V'_i(3)$ , then we let  $\phi(2) = \phi(1) = 1$ .

Second, suppose that all players  $i \in M_2$  have  $V'_i(3) > u_i(2)/(1-\beta)$ . Take the player (not necessarily in  $M_2$ ) for whom these two inequalities are satisfied and who is the last to propose when the state is two; denote this player by  $j$ . If either  $u_i(1)/(1-\beta) \geq V'_i(3)$  for all  $i \in M_2$  or this expression is true for at least one player in  $M_2$  and player  $j$ , then let  $\phi(2) = \phi(1) = 1$ .

The remaining case is where all players  $i \in M_2$  have  $V'_i(3) > u_i(2)/(1-\beta)$  but  $u_i(1)/(1-\beta) < V'_i(3)$  for at least one  $i \in M_2$ , and moreover, this finding holds for either all players in  $M_2$  or player  $j$ ; in this case, let  $\phi(2) = 3$ . Now, let  $\phi(1) = 2$  if  $(1-\beta)u_i(2) + \beta V'_i(3) > u_i(1)$  for all  $i \in M_1$ , and  $\phi(1) = 1$  otherwise. In all cases, it is straightforward to construct MPE  $\sigma$  that implements  $\phi$ . This finding completes the proof of existence.

**Part ii.** Generically,  $u_i(x) \neq u_i(y)$  for any  $x, y \in S$  and  $i \in N$ . Suppose, to obtain a contradiction, that mapping  $\phi$  supported by MPE  $\sigma$  is nonmonotonic. Then, there are  $x, y \in S$  such  $x < y$  and  $\phi(x) > \phi(y)$ . Because transitions are one step, we have  $y = x + 1$ .

Let us prove that  $u_i(y) > u_i(x)$  for all  $i \in M_x$ . For every  $i \in S$ ,  $V_i(x) = u_i(x) + \beta V_i(y)$  and  $V_i(y) = u_i(y) + \beta V_i(x)$ , which implies  $V_i(x) - V_i(y) = (1 + \beta)(u_i(x) - u_i(y))$ . In the not for publication appendix, we examine agenda setting and voting strategies to prove that  $V_i(y) \geq V_i(x)$  for all  $i \in M_x$ . Given genericity,  $u_i(y) > u_i(x)$  for all  $i \in M_x$ .

We can similarly prove that  $u_i(x) > u_i(y)$  for all  $i \in M_y$ . However, by assumptions 1 and 3, the same must hold for every  $i \in M_x$ , which contradicts the opposite inequality established in the previous paragraph. This contradiction completes the proof.

**Part 3.** Suppose that there are two MPEs,  $\sigma_1$  and  $\sigma_2$ , and two different transition mappings,  $\phi_1$  and  $\phi_2$ , corresponding to these MPEs, respectively. Without loss of generality, assume that  $m$  is the minimal number of states for which this is possible (i.e., if  $|S| < m$ , then transition mapping is unique). Obviously,  $m \geq 2$ .

Let us first prove that if  $\phi_1(x) = x$ , then  $x = 1$  or  $x = m$ . Indeed, suppose the opposite and consider  $\phi_2(x)$ . If  $\phi_2(x) < x$ , then

$\phi_1|_{[1,x]}$  and  $\phi_2|_{[1,x]}$  are two different mappings, both of which may, which is easy to show, be transition mappings for MPE in the game with the same players but with the set of states  $S' = [1, x]$ . This finding would contradict the assertion that  $m$  is the minimal number of players for which this possibility occurs. If  $\phi_2(x) > x$ , we get a similar contradiction by considering the subset of states  $[x, m]$ , and if  $\phi_2(x) = x$ , we get a contradiction by considering  $[1, x]$  or  $[x, m]$  depending on where  $\phi_1$  and  $\phi_2$  differ. We similarly prove that, if  $\phi_2(x) = x$ , then either  $x = 1$  or  $x = m$ .

We now consider the two cases of the proposition separately.

(i) Generically, no player gets the same utilities in two different states, and both mappings are monotone. If  $\phi_1(x) < x < \phi_2(x)$  or vice versa, then for all  $i \in M_x$ , there must be both a state  $x_1 < x$  and a state  $x_2 > x$  such that  $u_i(x_1) > u_i(x)$  and  $u_i(x_2) > u_i(x)$ , which contradict the assumption in this case. Because for  $1 < x < m$ ,  $\phi(x) \neq x$ , we get that  $\phi_1(x) = \phi_2(x)$  for such  $x$ . Let us prove that  $\phi_1(1) = \phi_2(1)$ . If this equation is not the case, then  $\phi_1(1) = 1$  and  $\phi_2(1) = 2$  (or vice versa). If  $m = 2$ , then monotonicity implies  $\phi_2(2) = 2$ , and if  $m > 2$ , then as proved earlier, we have  $\phi_2(x) = x + 1$  for  $1 < x < m$  and  $\phi_2(m) = m$ . In both cases, we have  $\phi_1(x) = \phi_2(x) > 1$  for  $1 < x \leq m$ . Hence,  $V_i^1(2) = V_i^2(2)$  for all  $i \in N$  (where  $V^1$  and  $V^2$  are continuation payoffs under  $\phi_1$  and  $\phi_2$ , respectively). Because  $\sigma_1$  is MPE, we must have  $u_i(1)/(1-\beta) \geq V_i^1(2)$  for  $i \in M_1$ , and because  $\sigma_2$  is MPE, we must have  $u_i(1)/(1-\beta) \leq V_i^2(2)$ . Generically, these expressions cannot hold together, and this finding proves that  $\phi_1(1) = \phi_2(1)$ . We can similarly prove that  $\phi_1(m) = \phi_2(m)$ , which implies that  $\phi_1 = \phi_2$ . This equation contradicts the hypothesis of nonuniqueness.

(ii) In this case, let  $M_x$  denote the unique quasi-median voter in state  $x \in S$ , and let  $b(x)$  be the state that maximizes  $u_{M_x}(y)$  on  $S$  (generically, it is unique). By assumption 1, the sequence  $\{b(x)\}_{x=1}^m$  is nondecreasing. Let us prove that  $b(2) \geq 2$ . Indeed, if  $b(2) = 1$ , then  $b(1) = 1$  by monotonicity; hence, we must have  $\phi_1(1) = \phi_2(1) = 1$ , and therefore,  $\phi_1(2) = \phi_2(2) = 1$ . Now consider a game with the same set of players, set of states  $S' = [2, m]$ , same sets of winning coalitions, and payoffs given by  $\tilde{u}_i(x) = u_i(x)$  for  $x > 2$ ,  $\tilde{u}_i(2) = (1-\beta)u_i(2) + \beta u_i(1)$ . Now, notice that perpetual state 2 in the new game delivers exactly the continuation utility  $V_i^1(2) = V_i^2(2)$  of the original game. It is now easy to see that the two mappings  $\phi_1$  and  $\phi_2$  given by  $\tilde{\phi}_j(x) = \phi_j(x)$  if  $x > 2$ ,  $\tilde{\phi}_j(2) = 2$  may be supported by MPE in the new game. However, these mappings are different, which contradicts that  $m$  is the minimal number of players for which this expression is possible. Hence,  $b(2) \geq 2$ . We can similarly prove that  $b(m-1) \leq m-1$ . Because  $\{b(x)\}_{x=2}^{m-1}$  is nondecreasing,  $b(x) = x$  for some  $x \in [2, m-1]$ . However, this expression would imply that  $\phi_1(x) = x$ , which we earlier proved to be impossible. This contradiction completes the proof in the nonstochastic case.

**Proof for the Stochastic Case. Part 1.** The proof proceeds by induction by the number of states that are in  $S$  but not  $L$ . The base was proved earlier. We prove the step for the case where only states on the right may be added, leaving the general case for the not for publication appendix. Let  $L = [l, r]$  and  $L' = [l, r+1]$ ; for  $L'$ , we know, by induction, that there is an MPE  $\sigma'$  with a transition mapping  $\phi'$ , which generates continuation utilities  $\{V'_i(s)\}_{i \in N}^{s \in L'}$ .

Let us define (Eq. 3)

$$\tilde{u}_i(s) = u_i(s) + \beta p_L^r V'_i(\phi'(s)) \text{ and (Eq. 4)} \quad [3]$$

$$\tilde{\beta} = \beta(1 - p_L^r), \quad [4]$$

and we consider a game without shocks, with a set of states  $L$ , and with the same winning coalitions and protocols but stage payoffs given by  $\tilde{u}_i(s)$  and discount factor  $\tilde{\beta}$ . Because assumption 1 holds, this game has an MPE  $\tilde{\sigma}$  with monotone transition mapping  $\tilde{\phi}$ . In this game, continuation utilities satisfy (Eq. 5)

$$\tilde{V}_i^L(s) = \tilde{u}_i(s) + \tilde{\beta} \tilde{V}_i^L(\tilde{\phi}(s)) \quad [5]$$

and therefore, are continuation utilities in the original game when the current set of states is  $L$ ,  $\tilde{\phi}$  is implemented before the shock, and  $\phi'$  is implemented afterward. Consequently, for  $\phi = \tilde{\phi}$ , we can construct MPE  $\sigma$  in the original game, and therefore,  $\phi$  is implemented before the shock. This finding proves the induction step and completes the proof of part 1.

**Part 2.** This part follows directly from part 2 above (mapping  $\tilde{\phi}$  is constructed as a transition mapping of some game without shocks).

**Part 3.** It suffices to prove that respective conditions hold in a game with the set of states  $L$ , utilities given by  $\tilde{u}_i(s)$ , and a discount factor  $\beta$ . In case *ii*, it follows from the hypothesis for the game with a set of states  $S$ . In case *i*, take any state  $s$  and player  $i(s)$  for whom the condition holds for utilities  $u_i(s)$ , and without loss of generality, suppose  $u_i(s) \geq u_i(x)$  for all  $x > s$ . If this supposition holds, in the game with the set of states  $L'$ ,  $\phi'(s) > s$  holds only if  $V'_i(s) = u_i(s)/(1-\beta)$ , in which case  $V'_i(x) \leq V'_i(s)$ , because the trajectory may take  $x$  only to states to the right of  $s$ . If  $\phi'(s) > s$ , then  $V'_i(s) \geq u_i(s)/(1-\beta)$  [this expression holds for all  $i \in M_s$ , including  $i(s)$ ]. The trajectory starting from  $x > s$  will either involve states that yield at most  $u_i(s)$  per period or lead to state  $s$ , thus delivering a continuation value of  $V'_i(s)$ . This finding implies that  $V'_i(x) \leq V'_i(s)$  for all  $x$  in this case as well. However, this finding implies that  $\tilde{u}_i(s) \geq \tilde{u}_i(x)$  for all  $x > s$ . This argument proves that the property in case *i* is satisfied, and part 3 of the proposition in the nonstochastic case is applicable.

**Proof of Proposition 2.** Suppose not. Then, there is an infinite number of transitions from one state to another. Because the total number of shocks is finite, this proposition means that an infinite number of transitions happen between shocks. However, in an equilibrium with monotone transition mapping, this is impossible.

**Proof of Proposition 3.** The assumptions of the proposition imply that a transition mapping is uniquely determined and monotone both for  $L$  and  $L' = L_r$  as the set of available states. Without loss of generality, suppose that  $L'$  is the entire set of states. Suppose, to obtain a contradiction, that  $s \in L$  is such that  $\phi(s) = s$ , but  $\phi'(s) < s$ , where  $\phi$  and  $\phi'$  are the unique transition mappings for  $L$  and  $L'$ , respectively. Notice, however, that  $\phi'|_{[1,s]}$  and  $\phi|_{[1,s]}$  may be sustained as MPE in a game with the set of states  $[1, s]$ , same sets of winning coalitions, same protocols, and same stage payoffs as in the initial game where the entire set of states  $L'$  is available. But  $\phi'$  and  $\phi$  are different mappings, which contradicts the uniqueness that should hold in this case.

**ACKNOWLEDGMENTS.** We thank the editor, two anonymous referees, and seminar participants at Stanford University for useful comments and suggestions.

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