International evidence on investment and fundamentals

Janice C. Eberly a,b, *

a Department of Finance, Wharton School, University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104-6367, USA
b National Bureau of Economic Research, Cambridge, MA 02138, USA

Received 1 June 1996; revised 1 November 1996

Abstract

If a firm's costs of installing capital are not quadratic, then its optimal investment is not a linear function of fundamentals, such as the returns and costs of capital. This study specifies a model in which a firm may face fixed, linear, and convex costs of investing, and estimates the resulting investment function using firm-level data from 11 countries. The evidence suggests important nonlinearities, consistent with the presence of fixed or other non-quadratic costs, in the relationship between investment and fundamentals for most countries. These findings are statistically significant at the level of the firm, and economically significant when aggregated by country. © 1997 Elsevier Science B.V.

JEL classification: E22

Keywords: Capital; Investment; Adjustment costs; Irreversibility

1. Introduction

If firms face fixed costs, linear installation costs, or other non-quadratic forms of adjustment costs, then optimal investment will not be a simple linear function of its fundamentals, e.g. the returns to and price of capital. A large theoretical
literature suggests that such costs can have important effects on the dynamics of capital accumulation. Moreover, a growing body of work on U.S. data suggests that these effects may be substantive empirically. This study explores whether international data support the presence of nonlinearities in the relationship between investment and its fundamental determinants.

Theoretically, the finding that non-quadratic adjustment costs induce a nonlinear relationship between investment and fundamentals developed simultaneously with the use of quadratic adjustment costs. Arrow (1968) studied irreversibility of investment as an extreme case of kinked, linear adjustment costs. More recently, work by Bernanke (1983), Bertola (1987), Pindyck (1988), Dixit (1989), Bertola and Caballero (1994), and many others (summarized by Dixit and Pindyck (1994)) emphasize the importance of linear adjustment costs and the special case of irreversibility in the presence of uncertainty. Bertola and Caballero (1990) also allow a role for fixed costs in a general specification of capital accumulation.

The development of quadratic adjustment costs is historically linked to the literature on Tobin's \( q \), sparked by the Tobin (1969) suggestion that a firm's investment should be related to the eponymous ratio — of the market value of the firm's capital to its replacement cost. Tobin's suggestion was then linked to the assumption of quadratic adjustment costs by the findings of Mussa (1977) and Abel (1983), who showed (under certainty and uncertainty, respectively) that such costs ensure that investment is indeed a linear function of fundamentals. These findings were empirically linked to Tobin's \( q \) by Hayashi (1982), who showed the conditions under which investment fundamentals would be accurately measured by Tobin's, or average, \( q \).

The linear specification implied by quadratic adjustment costs produced a large, but in many ways unsatisfying, empirical literature relating investment to fundamentals. The more complex relationships between investment and fundamentals implied by the non-quadratic models have only recently been explored empirically. Using a balanced panel of U.S. plant level data, Cooper et al. (1995) estimate the investment hazard functions implied by a model of machine replacement. They examine only large values of the investment rate and find that after an initial decline, the hazard function is increasing in the time since last adjustment. Using a balanced panel of the same data, Caballero et al. (1995) show that plants invest disproportionately more when they are far from an estimated value of their desired capital stock.

This study follows Abel and Eberly (1995) in using Tobin's insight as the basis for measuring the returns to capital, but allows the more general costs of adjustment emphasized in recent literature and the empirical studies noted above. The work of Abel and Eberly (1994) shows how fixed, linear, and convex (not necessarily quadratic) adjustment costs can be incorporated into an optimizing model of the firm. Optimal investment can then be characterized by a threshold rule: above an upper threshold value of \( q \), investment is positive and increasing in \( q \); below a lower threshold value of \( q \), investment is negative and increasing in \( q \);
between the two thresholds, investment is zero and unresponsive to \( q \). Moreover, in the regions where investment responds to \( q \), the relationship between investment and fundamentals need not be linear.

This general model is developed in more detail in the next section of the paper. Section 3 describes the firm-level data from 11 countries used to estimate the model. Section 4 estimates the linear model for comparison to the existing literature and as a benchmark against which to compare the nonlinear models estimated in Sections 5 and 6. The first nonlinear model assumes capital is homogeneous and relaxes the assumption of quadratic convex adjustment costs. The second nonlinear model allows for capital heterogeneity and the possibility of fixed costs; this combination implies a more general nonlinear investment function than the isoelastic specification resulting with homogeneous capital. The aggregate importance of these nonlinearities is explored in Section 7. Allowing a nonlinear relationship between investment and fundamentals substantially improves the predictive ability of aggregated investment equations in many of the countries considered. Explicit aggregation, accounting for the nonlinearities in the firm-level specification, increases \( R^2 \) of an aggregated linear investment equation from approximately zero in national accounts data to 0.38 in pooled aggregated data. The final section of the paper offers interpretations of these findings and suggestions for further work.

2. A model of optimal investment

Optimal investment is analyzed using a discrete-time, parametric case of the model in Abel and Eberly (1994), allowing for the possibility of fixed, linear, and convex costs of investing, and extended to allow for stochastic costs of investing. Consider a firm maximizing the present value of expected operating profits, \( \pi(X,K) \), less its cost of investing, \( c(I,K,u) \). Operating profits represent the revenues of the firm less variable costs. Uncertainty about demand and/or productivity enters through the stochastic term \( X \), and uncertainty about the costs of investing enters through the stochastic vector, \( u \). Operating profits and costs of investment are assumed to be linearly homogeneous in the capital stock, \( K \), and the level of gross investment, \( I \). Together, these assumptions allow the use of average, or Tobin's, \( q \) as an empirical measure of the shadow value of capital. They also insure that the marginal revenue product of capital does not “jump” when the capital stock is moved discretely; thus optimal investment does not follow an S–s policy (even when the firm must pay a fixed cost to invest), but instead follows a threshold policy governed by the value of \( q \). The optimal

---

1 A linearly homogeneous operating profit function can be derived for a competitive firm with a constant-returns-to-scale production function.
investment policy in this case is derived by writing the total returns to investing in period \( t \) as:

\[
\psi(q_t, K_t) = \max_{I_t, \eta_t} \left[ q_t I_t - \eta_t c(I_t, K_t) \right],
\]

where \( q_t \) is the shadow value of an additional unit of capital, \( I_t \) is the level of gross investment, and \( \eta_t \) is a dummy variable equal to one if investment is positive and zero otherwise. Conditional on \( \eta_t = 1 \), the optimal amount of investment, \( I_t^* \), satisfies the first-order condition, \( q_t = c(I_t^*, K_t, \eta_t) \). The dummy variable, \( \eta_t \), is then set equal to one if and only if \( [q_t I_t^* - c(I_t^*, K_t, \eta_t)] \geq 0 \), so that the total returns to the (conditionally) optimal level of investment exceed their total costs. This second condition defines two threshold levels of \( q_t \): between these two thresholds, returns are neither high enough to justify investing, nor low enough to justify disinvesting. Above the upper threshold, \( q_2 \), total returns are high enough to at least pay total investment costs. Below the lower threshold, \( q_1 \), the foregone returns from selling capital are so low as to be exceeded by the resale value of capital, so the firm disinvests. Since the data used in this study report only positive investment, this section focuses on the non-negative component of the optimal investment policy. Specifically, the optimal level of non-negative investment satisfies:

\[
\hat{I}_t = \begin{cases} 
I_t^* & \text{for } q_t \geq q_{2,t}, \text{ and} \\
0 & \text{for } q_t < q_{2,t}.
\end{cases}
\]

A parametric form for the \( c(I, K, \nu) \) function is chosen in order to empirically specify the form of the relationship between the observables, \( \hat{I}_t \) and \( q_t \). The cost function is allowed to have three types of terms:

\[
\text{fixed costs}: \quad a_t K_t, \\
\text{linear costs}: \quad (b_t + \nu_{1,t}) I_t, \text{ and} \\
\text{convex costs}: \quad \frac{v_{2,t} \beta}{1 + \beta} \left( \frac{I_t}{K_t} \right)^{(1+\beta)/\beta} K_t.
\]

The fixed costs are proportional to the size of the firm's capital stock and are independent of the amount of the investment. Linear investment costs may include both an acquisition price of capital, \( b_t \), as well as a linear and potentially stochastic installation cost, \( \nu_{1,t} \). Convex costs are positive and increasing in the rate of investment, \( I/K \), for \( I > 0 \). Thus, it is costlier to install or disinstall a given amount of capital immediately rather than over time. Using this cost

\footnote{If \( I < 0 \) were considered, its absolute value would enter the convex portion of Eq. (3) and convex costs would be positive and decreasing in the rate of disinvestment, \( I/K \) for \( I < 0 \).}
function and the definition of $I^*$, the expression for optimal investment in Eq. (2) becomes:

$$\frac{I_t}{K_t} = \begin{cases} 
(q_t - b_t - v_{1,t})^\beta 
& \text{for } q_t \geq q_2(a_t, v_t), \\
0 
& \text{for } q_t < q_2(a_t, v_t).
\end{cases}$$

(4)

Choosing a parametric form for the costs of investing restricts the form of the relationship between $I$ and $q$, but allows interpretation of the empirical findings in terms of the environment facing the firm. Moreover, as demonstrated in Fig. 1, the above specification nests a large family of investment functions. In the absence of fixed costs, the linear costs determine the intercept with the horizontal axis. The curvature of the investment function is then determined by the parameter $\beta$, and the slope of the investment function is also affected by $v_2$. In addition to the above considerations, with fixed costs the investment function is discontinuous.

3. The data

The data are from the Global Vantage industrial data set. The data set includes annual balance sheet, flow of funds, and income statement information from publicly traded companies. In this study, data from 11 countries are used: Belgium, Canada, France, Germany, Japan, the Netherlands, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Each firm typically reports 10 years of data beginning as early as 1981; the data extend through 1994. Thus, there is little entry and exit from the data set, so the panel is nearly balanced. Each firm’s entries are temporally consistent, and accounting standards
Table 1
Descriptive statistics by country a

<table>
<thead>
<tr>
<th></th>
<th>BEL</th>
<th>CAN</th>
<th>FRA</th>
<th>GER</th>
<th>JPN</th>
<th>NLD</th>
<th>SPN</th>
<th>SWE</th>
<th>CH</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I/K)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.30</td>
<td>0.23</td>
<td>0.30</td>
<td>0.31</td>
<td>0.24</td>
<td>0.22</td>
<td>0.18</td>
<td>0.27</td>
<td>0.26</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>medium</td>
<td>0.21</td>
<td>0.16</td>
<td>0.25</td>
<td>0.26</td>
<td>0.22</td>
<td>0.18</td>
<td>0.12</td>
<td>0.20</td>
<td>0.19</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>(q)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>1.14</td>
<td>1.31</td>
<td>1.24</td>
<td>0.73</td>
<td>1.50</td>
<td>0.92</td>
<td>1.08</td>
<td>0.85</td>
<td>1.31</td>
<td>1.09</td>
<td>1.56</td>
</tr>
<tr>
<td>median</td>
<td>0.93</td>
<td>1.00</td>
<td>0.97</td>
<td>0.62</td>
<td>1.30</td>
<td>0.81</td>
<td>0.96</td>
<td>0.78</td>
<td>1.09</td>
<td>0.92</td>
<td>1.18</td>
</tr>
<tr>
<td>(p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>1.72</td>
<td>0.98</td>
<td>1.79</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
<td>1.19</td>
<td>0.93</td>
<td>1.01</td>
<td>0.92</td>
<td>1.06</td>
</tr>
<tr>
<td>median</td>
<td>3.89</td>
<td>1.99</td>
<td>5.84</td>
<td>4.64</td>
<td>3.77</td>
<td>4.24</td>
<td>2.09</td>
<td>3.71</td>
<td>3.06</td>
<td>4.16</td>
<td>4.19</td>
</tr>
<tr>
<td>(R/K)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>5.87</td>
<td>4.33</td>
<td>7.82</td>
<td>10.23</td>
<td>7.82</td>
<td>6.57</td>
<td>4.14</td>
<td>4.85</td>
<td>4.06</td>
<td>6.74</td>
<td>6.73</td>
</tr>
<tr>
<td>median</td>
<td>3.89</td>
<td>1.99</td>
<td>5.84</td>
<td>4.64</td>
<td>3.77</td>
<td>4.24</td>
<td>2.09</td>
<td>3.71</td>
<td>3.06</td>
<td>4.16</td>
<td>4.19</td>
</tr>
<tr>
<td>obs</td>
<td>369</td>
<td>2140</td>
<td>2095</td>
<td>2324</td>
<td>1461</td>
<td>775</td>
<td>458</td>
<td>624</td>
<td>566</td>
<td>6260</td>
<td>14741</td>
</tr>
</tbody>
</table>

\(a\) Source: Author's calculation from Global Vantage data. The table reports summary statistics, by country, of the investment rate, \(I/K\), Tobin's \(q\), the tax-adjusted relative price of capital, \(p\), and the revenue rate, \(R/K\).
Table 2
Characteristics of investment rates by country

<table>
<thead>
<tr>
<th></th>
<th>BEL</th>
<th>CAN</th>
<th>FRA</th>
<th>GER</th>
<th>JPN</th>
<th>NLD</th>
<th>SPN</th>
<th>SWE</th>
<th>CH</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness</td>
<td>5.92</td>
<td>4.78</td>
<td>6.26</td>
<td>5.86</td>
<td>2.87</td>
<td>3.52</td>
<td>2.36</td>
<td>3.45</td>
<td>2.75</td>
<td>6.81</td>
<td>5.00</td>
</tr>
<tr>
<td>$\rho$ serial</td>
<td>0.24</td>
<td>0.60</td>
<td>0.45</td>
<td>0.46</td>
<td>0.48</td>
<td>0.50</td>
<td>0.50</td>
<td>0.41</td>
<td>0.58</td>
<td>0.48</td>
<td>0.54</td>
</tr>
<tr>
<td>$\rho$ maxima</td>
<td>0.64</td>
<td>0.51</td>
<td>0.51</td>
<td>0.49</td>
<td>0.61</td>
<td>0.51</td>
<td>0.49</td>
<td>0.45</td>
<td>0.59</td>
<td>0.54</td>
<td>0.45</td>
</tr>
</tbody>
</table>

a Source: Author’s calculations from Global Vantage data. Line 2, serial correlation is calculated by firm. Line 3, “$\rho$, maxima”, calculates the share of firms in which the largest and second largest years’ investment rates occur in consecutive years.
are consistent across firms within a country. Reporting conventions are consistent across countries within the database, however, accounting standards may differ across countries. Where such differences exist, they are noted in the footnotes to the database.

The means and medians of the primary variables of interest are reported by country in Table 1 (the median of $p$ is not reported since there is little cross-section variation). Notice that in virtually all cases, the median exceeds the mean, indicating positive skewness in the data. Investment rates are examined more closely in Table 2, where the first row explicitly calculates skewness by country. The second row reports the serial correlation coefficient for firm-level investment rates, which is between 0.4 and 0.6 for all countries except Belgium. The third row reports the share of the firms for which the two highest annual investment rates occur in consecutive years. These data indicate that while in some years investment is very high (positive skewness), these years tend to occur consecutively (serial correlation).

4. Linear specification

The first specification estimated is the standard linear investment equation; this serves as a benchmark for comparison to earlier studies and to the nonlinear estimates in the next section. The investment Eq. (4) nests the standard linear investment equation when adjustment costs are quadratic ($\beta = 1$). The investment equation is therefore:

$$\frac{I_{i,t}}{K_{i,t}} = \frac{1}{v_2} q_{i,t} - \frac{1}{v_2} b_{i,t} - \frac{1}{v_2} \nu_1.$$  

Now allow the linear component of adjustment costs, $\nu_1$, to have a firm specific component, $\nu_{1,i}$, as well as a normally-distributed stochastic component, $\nu_{1,i,t}$. Together with the assumption that $v_2$ is constant, this yields the estimation equation:

$$\frac{I_{i,t}}{K_{i,t}} = \frac{1}{v_2} q_{i,t} + \frac{\alpha}{v_2} p_{i,t} - \frac{v_{1,i}}{v_2} + \epsilon_{i,t},$$

where estimating the coefficient $\alpha$ allows the actual acquisition price of capital to be proportional to the measured price, $p_{i,t}$, and its sign gives the sign of the effect.

---

3 Only positive values of investment are considered in this study. Appendix A provides more details on the sample selection.
4 Since investment is a flow and the independent variables below are measured at a point in time, the timing convention is to measure $q_{i,t}$ using end-of-previous-period values and similarly for the price of capital. Further details on measurement are reported in Appendix A.
<table>
<thead>
<tr>
<th></th>
<th>BEL</th>
<th>CAN</th>
<th>FRA</th>
<th>GER</th>
<th>JPN</th>
<th>NLD</th>
<th>SFN</th>
<th>SWE</th>
<th>CH</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OLS:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tobin’s $q$</td>
<td>0.14</td>
<td>0.12</td>
<td>0.10</td>
<td>0.19</td>
<td>0.21</td>
<td>0.08</td>
<td>0.15</td>
<td>0.12</td>
<td>0.16</td>
<td>0.19</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$p$</td>
<td>0.19</td>
<td>-0.78</td>
<td>-0.37</td>
<td>-1.16</td>
<td>-0.22</td>
<td>-0.10</td>
<td>0.36</td>
<td>0.07</td>
<td>-1.76</td>
<td>-1.20</td>
<td>-0.11</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.18)</td>
<td>(0.17)</td>
<td>(0.56)</td>
<td>(0.23)</td>
<td>(0.18)</td>
<td>(0.40)</td>
<td>(0.51)</td>
<td>(0.85)</td>
<td>(0.14)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.02</td>
<td>0.13</td>
<td>0.03</td>
<td>0.04</td>
<td>0.17</td>
<td>0.01</td>
<td>0.11</td>
<td>0.01</td>
<td>0.14</td>
<td>0.09</td>
<td>0.06</td>
</tr>
</tbody>
</table>

|                |     |     |     |     |     |     |     |     |     |     |     |
| **QUANT:**     |     |     |     |     |     |     |     |     |     |     |     |
| Tobin’s $q$    | 0.11| 0.08| 0.10| 0.17| 0.16| 0.08| 0.11| 0.06| 0.09| 0.09| 0.06|
|                | (0.01)| (0.002)| (0.01)| (0.01)| (0.01)| (0.01)| (0.01)| (0.02)| (0.01)| (0.002)| (0.001)|
| $p$            | -0.04| -0.32| -0.24| -0.97| -0.14| 0.02| 0.37| -0.04| -1.10| -0.54| -0.01|
|                | (0.05)| (0.07)| (0.06)| (0.22)| (0.14)| (0.38)| (0.15)| (0.20)| (0.29)| (0.04)| (0.02)|
| $\bar{R}^2$   | 0.01| 0.04| 0.02| 0.03| 0.07| 0.01| 0.04| 0.01| 0.05| 0.03| 0.03|

|                |     |     |     |     |     |     |     |     |     |     |     |
| **IV:**        |     |     |     |     |     |     |     |     |     |     |     |
| Tobin’s $q$    | 0.57| 0.14| 0.26| 0.13| 0.33| 0.11| 0.43| 0.47| 0.23| 0.24| 0.17|
|                | (0.12)| (0.02)| (0.03)| (0.07)| (0.02)| (0.04)| (0.10)| (0.07)| (0.03)| (0.02)| (0.01)|
| $p$            | -0.26| -0.69| -0.51| -0.83| -0.86| 0.23| 0.07| -0.20| -1.33| -0.88| -0.31|
|                | (0.55)| (0.18)| (0.24)| (0.71)| (0.28)| (0.31)| (0.59)| (0.47)| (10.11)| (0.15)| (0.10)|
| $\bar{R}^2$   | 0.11| 0.08| 0.01| 0.02| 0.15| 0.01| 0.01| 0.01| 0.08| 0.07| 0.01|

---

1 Source: Author’s calculations from *Global Vantage* data. The dependent variable is the investment rate, and the first column reports the estimation method and regressors. The instrument set includes two lags of revenue and profits (scaled by capital stock), the corporate tax rate and the investment tax credit, and the second lag of the change in $q$ and $I/K$. 
of the tax-adjusted price of capital on investment. The residual is defined so that 
\( e_{i,t} = -\frac{v_{1,i,t}}{v_2} \). The firm-specific intercept is removed by first differencing. The results of this estimation are reported in Table 3 using ordinary-least-squares (OLS), quantile regression (QUANT), and instrumental variables (IV). The coefficient on Tobin's \( q \) is positive and significant for all countries and estimation methods. Generally, the lowest values (0.06) of the coefficient are found using quantile estimation, while the highest values (over 0.25) are found using instrumental variables. The sign and significance of the coefficient on the tax-adjusted relative price of capital are mixed. When statistically significant, the coefficient is generally negative (as expected), but it is often insignificant. The \( R^2 \) varies from 0.01 to 0.17. These results are consistent with those reported elsewhere in the literature for specific countries, though the coefficients on Tobin's \( q \) and the \( R^2 \) statistics are higher than those found in some other studies.

5. Non-linear specification: homogeneous capital

From Fig. 1, it is evident that the simplest form of nonlinearity in the investment function may arise from non-quadratic adjustment costs (\( \beta \neq 1 \)). Relaxing this assumption allows the relationship between investment and fundamentals (returns and costs) to be convex or concave; in fact, \( \beta \) can be explicitly estimated. The estimation equation for positive investment is derived from the general formulation in Eq. (4) when the linear adjustment cost, \( v_1 \), constant. This produces an isoelastic specification which, taking natural logs, yields:

\[
\ln\left(\frac{I_t}{K_t}\right) = \beta \ln(q_t - b_t - v_1) - \beta \ln(v_2).
\]

(7)

Assume that the cost shock, \( v_2 \), has a firm-specific component, \( v_{2,i,t} \), and a stochastic, log-normally distributed component, \( v_{2,i,t} \). Again, allow the measured price of capital, \( p_{1,i,t} \), to be proportional to the actual acquisition price of capital, with factor of proportionality \( -\alpha \) so that the sign of \( \alpha \) gives the sign of the effect of the price of capital on investment. This produces the estimation equation:

\[
\ln\left(\frac{I_t}{K_t}\right) = \beta \ln(q_t + \alpha p_{1,i,t} - v_1) - \beta \ln(v_{2,i,t}) + e_{i,t},
\]

(8)

where \( e_{i,t} = -\beta \ln(v_{2,i,t}) \). This equation is first-differenced to eliminate the firm-specific component and then estimated by nonlinear least squares.

5 Alternatively, using fixed effects estimation yields similar results to those reported in Table 3.

6 If observations on zero investment were available, a censored regression could be estimated at this point. As explained in the data appendix, zero observations in the Global Vantage investment data can arise because of missing data, so these observations are unreliable.
The estimation results are reported in Table 4. In some cases a linear specification cannot be rejected compared to a general isoelastic model; specifically, in Belgium, the Netherlands, Spain, and Switzerland, the constrained ($\beta = 1$) model cannot be rejected with at least 90 percent confidence. For the other seven countries, however, the constrained model is rejected at confidence levels from 90 to over 99 percent. With the exception of Sweden, in each of these cases the estimated value of $\beta$ significantly exceeds 1, with point estimates ranging from 1.22 (U.S.) to 1.95 (Japan). In this isoelastic framework, these results indicate adjustment costs less convex than quadratic, so that investment is more responsive
to $q$ at higher values of $q$ than at lower values of $q$. Using this specification, however, note that the elasticity of investment with respect to fundamentals is restricted to be a constant, an assumption that is relaxed in the next section. Given this restriction, however, the results are consistent with the analysis of a balanced panel of U.S. manufacturing plants by Caballero et al. (1995), who emphasize the result that observed positive adjustments of the capital stock are larger when the current capital stock is significantly less than an estimate of the plant's desired capital stock.

6. Non-linear specification: heterogeneous capital

The value of capital expenditures reported by the firms in the Global Vantage data set aggregates over purchases of many types of capital. This is particularly clear when firms are observed to purchase and sell capital in the same period. This section follows Abel and Eberly (1995) in explicitly modeling the firm's optimal choice of investment in many types of capital, which may differ both in their returns and their costs.

Consider a particular type of capital, indexed by $n$. The return to investing in type $n$ is $q_n$ and cost of investing in type $n$ includes a fixed cost $a$, a linear cost $(b_n + v_{1,n})$ and an adjustment cost $\frac{1}{2} v_2 (I_n/K)^2 K$. The investment function for type $n$ is then derived from the general specification in Eq. (4), yielding:

$$\frac{I_n}{K} = \frac{1}{v_2} (q_n - b_n - v_{1,n}) \text{ if } q_n \geq q_{z,n} = b_n + v_{1,n} + \sqrt{2av_2}. \quad (9)$$

Investment in type $n$ is linear in fundamentals if returns to type $n$ are sufficiently high to justify paying the linear and fixed costs. In order to calculate total investment by the firm, now specify the characteristics of a type relative to an average for the firm. The returns to type $n$ differ from the average $q$ for the firm.

---

7. This result is in contrast to those of Abel and Eberly (1995) and Burnett and Sakellaris (1995), who find a declining response to investment to $q$ using U.S. Compustat data from the 1970s through the early 1990s. The difference may have several sources, since the two Compustat studies use unbalanced panels over a longer and different time period. This point is discussed further in the conclusions.

8. This form of aggregation may explain why there are few observations of zero investment in this and other data sets. Another possible explanation is aggregation over time. This section emphasizes aggregation in types of capital because, as we see below, it can lead nonlinearities in the investment equation of the sort estimated in the previous section.

9. Since operating profits and investment costs are assumed to be linearly homogeneous in the stock and investment in all types of capital, write $q$ as $q(X,v) = q(X,v)K/K = \int_0^s q_n(K_n/K) dF(n)$, the weighted average of the shadow values associated with each individual type of capital.
by an amount $\omega_{1,n}$, and the linear investment costs for type $n$ differ from the
average, $b + v_1$, for the firm by an amount $\omega_{2,n}$. Substituting into Eq. (9) yields:
\[
\frac{I_n}{K} = \frac{1}{\nu_2} \left[ q + \omega_{1,n} - (b + v_1 + \omega_{2,n}) \right] = \frac{1}{\nu_2} \left( q - b - v_1 + \omega_n \right)
\]
if $q_n \geq q_{2,n} \equiv b + v_1 + \omega_{2,n} + \sqrt{2av_2}$,
\[0 \text{ otherwise.}
\]
where $\omega_n \equiv \omega_{1,n} - \omega_{2,n}$. Integrating this expression over all types of capital, $n$, with the cumulative distribution function $F_n(n)$ defined over the interval $[0, \bar{n}]$, we obtain:
\[
\frac{I}{K} = \int_0^\bar{n} \frac{I_n}{K} dF_n(n) = \frac{1}{\nu_2} \int_0^q (q - b - v_1 + \omega_n) dF(q_{2,n} - \omega_{1,n}),
\]
where $F(q_{2,n} - \omega_{1,n})$ is the cumulative distribution function of the investment
thresholds, $q_{2,n}$, less idiosyncratic returns, $\omega_{1,n}$, across types. Total investment of
the firm is given by the amount invested in each type, $\nu_2^{-1}(q - b - v_1 + \omega_n)$, integrating over types above the threshold, $q_n \geq q_{2,n}$. Subtracting $\omega_{1,n}$ from both
sides of this inequality, we obtain $q_{2,n} - \omega_{1,n} < q_n - \omega_{1,n} = q$, or the range of
integration in Eq. (11) (noting that $q$ is bounded below by zero in this parametric
case). Carrying out the integration in Eq. (11) yields:
\[
\frac{I}{K} = \frac{1}{\nu_2} \left[ F(q)(q - b - v_1) + \int_0^q \omega_n dF(q_{2,n} - \omega_{1,n}) \right]
\]
\[
= \frac{1}{\nu_2} \{ F(q)(q - b - v_1 + \phi(q)) \},
\]
where
\[
\phi(q) \equiv \frac{1}{F(q)} \int_0^q \omega_n dF(q_{2,n} - \omega_{1,n})
\]

The total investment rate of the firm is the product of the extensive margin, $F(q)$, and the intensive margin, $[q - b - v_1 + \phi(q)]$; the former summarizes how
many types of capital in which the firm invests, while the latter describes how
much on average the firm invests in each type. The firm-level investment function
implied by this specification is sigmoidal: first convex and then concave, though
monotonically increasing. The initial convexity arises from the product of the
intensive and extensive margins: as fundamentals improve the firm invests in more
types, as well as in more of each type. When fundamentals become very high,
however, the extensive margin becomes exhausted and the response of investment to fundamentals is damped\textsuperscript{10}.

When the extensive margin is degenerate [$F(q) = 1$] and $\phi(q)$ is therefore a constant, Eq. (12) reverts to the linear investment equation; this result obtains as $\sigma \to \infty$. Intuitively, this indicates that the firm is investing in all types continuously, so heterogeneity is not meaningful for investment. When the intensive margin is degenerate, a fixed amount is invested in each type of capital conditional on investing in that type; the amount invested in each type is independent of fundamentals\textsuperscript{11}. Heterogeneity completely determines the rate of investment in this case, since the investment decision of the firm is solely a choice of how many types in which to invest.

Taking logs of Eq. (12) and allowing $v_2$ to have a firm-specific and a log-normally distributed stochastic component, produces:

$$\ln\left(\frac{I_{i,t}}{K_{i,t}}\right) = \ln[F(q_{i,t})] + \ln[q_{i,t} + \alpha P_{i,t} - v_1 + \phi(q_{i,t})] - \ln(v_{2,i,t}) + \epsilon_{i,t},$$

where $\epsilon_{i,t} = - \ln(v_{2,i,t})$ and the coefficient $\alpha$ permits the measured price of capital to be proportional to the true acquisition cost\textsuperscript{12}.

After first-differencing to eliminate the firm-specific effect, this equation is estimated using nonlinear least squares. In the empirical implementation, the distribution function $F(\cdot)$ is assumed to be the cumulative distribution function (CDF) of a normal distribution, with mean $\mu$ and standard deviation $\sigma$. This distribution is consistent with a normal distribution of $\omega_n$, the measure of underlying heterogeneity in returns and costs across types of capital. This functional form was chosen primarily for parsimony, since it can be characterized with a minimum of parameters. Other functional forms are feasible; however, note that the function of interest is the CDF, not the probability density. For all density functions, the CDF will be strictly increasing, asymptote below at zero and above at one, and for a broad family of density functions, the CDF will be (at least weakly) convex over some range and subsequently (at least weakly) concave. These are the properties reflected in the estimated investment equation. Three

\textsuperscript{10} These properties rely on the fact that the cumulative distribution function $F(q)$ is monotonically increasing and bounded above by 1.

\textsuperscript{11} This might be thought of as investment in a project of exogenous, though not necessarily fixed, size. Empirically, this result is obtained as $- v_1$ becomes large, so that the elasticity of the investment rate with respect to fundamentals through the intensive margin, $1/[q - b - v_1 + \phi(q)]$ becomes small.

\textsuperscript{12} We adopt the timing assumption that the firm first chooses the types of capital in which it will invest, then observes the realization of $v_{2,i,t}$, and finally chooses the amount to invest in each of the types. This guarantees that the distribution function $F(q_{2,n} - \omega_{1,n})$ is time-invariant. Otherwise, notice from Eq. (10) that the value of $v_2$ affects the investment threshold, so the distribution of the thresholds would depend on the (unobserved) value of $v_{2,i,t}$.
Table 5
Estimation of the nonlinear model with heterogeneous capital

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}$</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\nu}_1$</th>
<th>$R^2$</th>
<th>$p (\sigma \to \infty)$</th>
<th>$p (I_n / K = i_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEL</td>
<td>2.10</td>
<td>1.22</td>
<td>0.07</td>
<td>-</td>
<td>0.10</td>
<td>0.003</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td>(0.31)</td>
<td>(0.33)</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>-1.82</td>
<td>2.81</td>
<td>-6.24</td>
<td>-8.58</td>
<td>0.10</td>
<td>0.015</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(0.68)</td>
<td>(1.21)</td>
<td>(1.22)</td>
<td>(3.63)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>-0.13</td>
<td>4.30</td>
<td>-2.36</td>
<td>-5.40</td>
<td>0.06</td>
<td>0.001</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.69)</td>
<td>(2.71)</td>
<td>(0.71)</td>
<td>(1.94)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>-3.16</td>
<td>1.18</td>
<td>-5.62</td>
<td>-</td>
<td>0.10</td>
<td>1x10^-7</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>(0.75)</td>
<td>(0.13)</td>
<td>(0.92)</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>37.65</td>
<td>435</td>
<td>-0.09</td>
<td>0.08</td>
<td>0.11</td>
<td>0.029</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>(1.5x10^5)</td>
<td>(1.7x10^6)</td>
<td>(0.67)</td>
<td>(0.68)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NLD</td>
<td>1.04</td>
<td>1.03</td>
<td>0.34</td>
<td>-6.60</td>
<td>0.04</td>
<td>0.161</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(0.90)</td>
<td>(0.43)</td>
<td>(0.72)</td>
<td>(20.37)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPN</td>
<td>1.68</td>
<td>1.14</td>
<td>0.42</td>
<td>-5.68</td>
<td>0.07</td>
<td>0.205</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(1.69)</td>
<td>(0.40)</td>
<td>(1.47)</td>
<td>(23.59)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SWE</td>
<td>4.70</td>
<td>0.63</td>
<td>2.81</td>
<td>-2.34</td>
<td>0.05</td>
<td>0.089</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(1.69)</td>
<td>(0.27)</td>
<td>(1.18)</td>
<td>(10.31)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CH</td>
<td>-2.87</td>
<td>1.18</td>
<td>-4.33</td>
<td>-5.71</td>
<td>0.14</td>
<td>0.045</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(1.69)</td>
<td>(0.43)</td>
<td>(2.02)</td>
<td>(3.84)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>-1.44</td>
<td>1.83</td>
<td>-3.95</td>
<td>5.54</td>
<td>0.11</td>
<td>1.8x10^-7</td>
<td>1.6x10^-4</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
<td>(0.39)</td>
<td>(0.45)</td>
<td>(1.59)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.37</td>
<td>2.02</td>
<td>-0.41</td>
<td>-2.84</td>
<td>0.10</td>
<td>1.2x10^-10</td>
<td>1.3x10^-8</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.17)</td>
<td>(0.24)</td>
<td>(1.21)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Author's calculations from *Global Vantage* data. The first column of coefficients reports the estimated value of $\hat{\mu} = (\mu - \nu_1)/\hat{\sigma}$. The last two columns of coefficients report the probability that the $F$-statistic exceeds the value associated with tests of the unrestricted model against a model with a degenerate extensive margin ($\sigma \to \infty$) and against a model with a degenerate intensive margin, ($I_n / K = i_0$), respectively.
versions of the above equation are estimated: (i) the unrestricted Eq. (13); (ii) a restricted version with a degenerate extensive margin; and (iii) a restricted version with a degenerate intensive margin. Parameter estimates are reported in Table 5 for the restricted model if it cannot be rejected, and for the unrestricted model if the restricted model can be rejected. The first column of Table 5 reports the value of \( \hat{\mu} = (\mu - v_1)/\sigma \), since under the restricted model with a degenerate intensive margin, the parameter \( v_1 \) is not separately identified. The last two columns of Table 5 report the \( p \)-values obtained by comparing the unrestricted model to the restricted models.

These results suggest a significant role for nonlinearities in virtually all countries. With the exception of the Netherlands and Spain, linearity (e.g., a degenerate extensive margin) can be rejected with greater than 90 percent confidence; in 8 of these 9 countries this exceeds 95 percent confidence. For the Netherlands and Spain, the evidence is generally mixed; linearity can be rejected with only 84 and 79 percent confidence, respectively. This is consistent with the results for the isoelastic model in Table 4, where these two countries also showed the weakest rejection of the linear model. In addition to the Netherlands and Spain, in five more countries a degenerate intensive margin cannot be rejected with at least 90 percent confidence; this indicates that for these countries, the extensive margin sufficiently captures the response of investment to fundamentals. This finding rejects the linear relationship between investment and fundamentals in favor of a nonlinear relationship. For the remaining countries, both margins play statistically significant roles. The importance of the specific coefficient estimates is discussed below.

7. Aggregate implications

Estimation of the firm-level model suggests statistically significant nonlinearities in the relationship between investment and its fundamental determinants in some countries. This section addresses the economic significance of this finding. Two metrics are employed to assess the importance of the nonlinearities. First, the aggregate rate of investment observed in the data is compared to that predicted by each of the three models estimated above. This gauges how well the firm-level estimates fare in predicting aggregate values. Second, an aggregated linear investment equation is estimated, relating aggregated values of investment to aggregated fundamentals. When the true relationship between investment and fundamentals is nonlinear, higher moments of the fundamentals (in addition to the first) should affect investment.

\[ \text{where the restricted model with a degenerate intensive margin cannot be rejected (Belgium and Germany), a parameter estimate for } v_1 \text{ is not reported since it is not identified in this case.} \]
Table 6
Predictive ability of the firm-level models for aggregate investment rates

<table>
<thead>
<tr>
<th>Country</th>
<th>Linear model</th>
<th>Isoelastic</th>
<th>Heterogeneous K</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEL</td>
<td>0.49</td>
<td>0.50</td>
<td>0.48</td>
</tr>
<tr>
<td>CAN</td>
<td>0.11</td>
<td>0.62</td>
<td>0.60</td>
</tr>
<tr>
<td>FRA</td>
<td>0.74</td>
<td>0.73</td>
<td>0.72</td>
</tr>
<tr>
<td>GER</td>
<td>0.67</td>
<td>0.80</td>
<td>0.81</td>
</tr>
<tr>
<td>JPN</td>
<td>0.79</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>NLD</td>
<td>0.33</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>SPA</td>
<td>0.64</td>
<td>0.64</td>
<td>0.60</td>
</tr>
<tr>
<td>SWE</td>
<td>0.91</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>CH</td>
<td>0.78</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
<td>UK</td>
<td>0.53</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>US</td>
<td>0.25</td>
<td>0.29</td>
<td>0.28</td>
</tr>
</tbody>
</table>

* Source: Author's calculations from Global Vantage data. The $\bar{R}^2$ values reflect the $\bar{R}^2$ of a regression of the average value of log investment rates across firms on that predicted by each of the three models: the linear model estimated in Section 4 (using the OLS results), the isoelastic model estimated in Section 5, and the model with heterogeneous capital estimated in Section 6.

The estimation results of Sections 4–6 are first used to obtain predicted values of (log) investment rates, by firm. The actual values and the estimated values are each averaged across firms for a given year. This produces an implied time series of (log) investment rates in the data, as well as estimated by each of the three models. Table 6 reports the $\bar{R}^2$ associated with each of the three models compared to the actual aggregated data.

The results in Table 6 indicate that the nonlinear models typically do at least as well as the linear model in predicting aggregate investment rates, and in some cases substantially better. Generally, in the countries (Canada, Germany, Switzerland, the U.K., and the U.S.), where significant nonlinearities were indicated from the earlier tables of results, we find that the nonlinear models are substantially more successful at predicting aggregate investment.

The most dramatic results are in Canada, where allowing for the nonlinearity increases the $\bar{R}^2$ by 50 percentage points. More representative, however, are Germany, where the $\bar{R}^2$ rises by 13 points and Switzerland, where it rises by 6–7 points. The countries where the linear model performs approximately as well as the nonlinear models are those for which linearity could not be most confidently rejected in the isoelastic model (Belgium, the Netherlands, Spain and Sweden from Table 4), or for whom the variance estimate in the heterogeneous capital

---

14. The OLS results are used from the linear model so that the effect of nonlinearities is not confounded with the effect of different estimation methods. However, the models are still non-nested because the linear model is estimated in first-differences of levels, while the nonlinear models are estimated in first-differences of logs. Care should be taken in interpreting comparisons of these models.
Table 7
Pooled aggregate investment equations

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Regressor</th>
<th>$\frac{\Sigma_i V_i}{\Sigma_i K_i}$</th>
<th>$\frac{\Sigma_i \left( \frac{V_i}{K_i} \right)}{\Sigma_i}$</th>
<th>$\Sigma_i w_i \left( \frac{V_i}{K_i} \right)$</th>
<th>$P$</th>
<th>$sd(q)$</th>
<th>$sk(q)$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log\left( \frac{\Sigma_i I_i}{\Sigma_i K_i} \right)$</td>
<td>0.40</td>
<td>0.06</td>
<td>$-0.01$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($0.20$)</td>
<td>($0.40$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log\left( \frac{\Sigma_i I_i}{\Sigma_i K_i} \right)$</td>
<td>0.32</td>
<td>0.01</td>
<td>0.11</td>
<td>$-0.01$</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($0.20$)</td>
<td>($0.40$)</td>
<td>($0.16$)</td>
<td>($0.009$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log\left( \frac{\Sigma_i I_i}{\Sigma_i K_i} \right)$</td>
<td>0.40</td>
<td>$-0.05$</td>
<td>$-0.03$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($0.27$)</td>
<td>($0.40$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log\left( \frac{\Sigma_i I_i}{\Sigma_i K_i} \right)$</td>
<td>0.62</td>
<td>0.01</td>
<td>0.10</td>
<td>$-0.02$</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($0.27$)</td>
<td>($0.39$)</td>
<td>($0.16$)</td>
<td>($0.009$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma_i w_i \log\left( \frac{I_i}{K_i} \right)$</td>
<td>0.56</td>
<td>$-0.005$</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($0.11$)</td>
<td>($0.005$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Sigma_i w_i \log\left( \frac{I_i}{K_i} \right)$</td>
<td>0.72</td>
<td>$-0.006$</td>
<td>0.005</td>
<td>$-4.4 \times 10^{-4}$</td>
<td>0.38</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($0.11$)</td>
<td>($0.005$)</td>
<td>($0.002$)</td>
<td>($1.3 \times 10^{-4}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Source: Author's calculations from Global Vantage data. The independent variables $sd(q)$ and $sk(q)$ are the cross-section standard deviation and skewness, respectively, of Tobin's $q$, by country and year. All equations are estimated in first-differences. Country-specific dummy variables are included, but not reported; there are 81 country-level observations. The firm weights, $w_i$, are based on 1991 levels of the capital stock.

model (Table 5) is high enough that the model approaches linearity in practice (France and, especially, Japan)\(^{15}\). The two nonlinear models perform approximately as well as each other; this suggests that the isoelastic restriction may not be

\(^{15}\) While the $F$-tests in Table 5 strongly reject excluding the extensive margin from the model for France and Japan, the estimated variances are sufficiently large that the CDF term in the estimation equation becomes nearly linear.
overly strong or alternatively, that most of the data lie in the convex region of the sigmoidal investment function allowed by the heterogeneous capital model.

Finally, the potential importance of a nonlinear relationship between investment and fundamentals is assessed using an aggregated linear investment equation, but allowing for higher moments of fundamentals (Tobin's $q$) to affect investment. In order to evaluate the importance of aggregation bias in aggregate investment equations, the firm-level data are aggregated in two different ways. First, aggregates are calculated as if they were National Accounts data. That is, the investment rate is calculated as aggregate investment divided by the aggregate capital stock, or $\frac{\sum_i I_i}{\sum_i K_i}$, and similarly, aggregated Tobin's $q$ is calculated as $\frac{\sum_i V_i}{\sum_i K_i}$. Since only 7 or 8 years of aggregated data are available per country by aggregating the Global Vantage data, a pooled regression with country-specific dummy variables is estimated. Since the estimation is in first differences, country-specific level effects have already been removed and the dummy variables control for differences in growth rates across countries. A linear regression of investment on fundamentals calculated in this manner is reported in the first row of Table 7. While the coefficient on aggregated Tobin's $q$ is positive and significant, the effect of prices is of the wrong sign (though statistically insignificant), and the $R^2$ is slightly negative. Adding higher moments of $q$ does little to improve the performance of the equation aggregated in this way, as shown in the second row of the table.

The performance of this aggregated investment equation can be immediately improved by using the first through third moments of the underlying distribution of firm-level values of $q$ (rather than the aggregated value) as independent variables. The third and fourth rows of Table 7 report the results of regressing the same independent variable, $\Delta \log(\frac{\sum_i I_i}{\sum_i K_i})$, on the first moment alone and then the first three moments of this distribution. The results in the third row show that using the first moment alone has little explanatory power for the aggregated investment ratio - performing approximately as well as the aggregate ratio $\frac{\sum_i V_i}{\sum_i K_i}$ (reported in the first row). However, in combination with the second and third moments, the coefficient on the first moment is more than 50 percent larger and statistically significant. The effect of the standard deviation is positive, but insignificant, while the effect of skewness is negative and statistically significant. The $R^2$ increases to 0.08 when all three moments are included. Thus, information from the moments of the distribution of $q$ can be utilized to predict the log change in the aggregated investment rate, $\Delta \log(\frac{\sum_i I_i}{\sum_i K_i})$. This measure of the change in "average investment" in the economy, however, is subject to aggregation bias owing to the logarithmic transformation of the aggregated data. In the next set of regressions, we correct this aggregation bias by performing the logarithmic transformation before aggregating the investment rate across firms. While National Accounts data report arithmetically aggregated values (as examined above), rather than the geometrically aggregated data utilized below, it is nonetheless of interest to examine the effects of this aggregation method on
econometric results. As we see below, the choice of aggregation method can have substantial implications when testing econometric models. Furthermore, where data are available to calculate higher moments of \( q \) across firms, they are also often available to calculate geometric, rather than arithmetic, averages of investment rates across firms. If the arithmetic average (or total capital accumulation) is what is truly of interest to the econometrician, this can be calculated from predicted geometric average if the econometrician has information about the size distribution of investments (which is typically available from the same sources as the distribution of Tobin's \( q \)).

The second aggregation method directly computes a weighted-average of the firm-level investment rates and values of Tobin's \( q \). Thus the aggregate log investment rate is \( \sum w_i \log(I_i/K_i) \), where \( w_i \) is the weight of firm \( i \), based on its 1991 capital stock. Similarly, aggregated (log) Tobin's \( q \) is the weighted average of the firm-level values. The fifth row of Table 7 reports the results of a linear estimation of the aggregated investment equation using these variables. Notice that the coefficient on Tobin's \( q \) rises and its standard error falls, and the effect of the tax-adjusted price of capital is now negative but still not statistically significant. Notice, however, that the \( R^2 \) rises from -0.01 in the first row to 0.26, simply by aggregating in a manner more closely related to the underlying model of the firm. The addition of the cross-section standard deviation and skewness of Tobin's \( q \), in row 6, increase the \( R^2 \) by 12 percentage points, to 0.38, while further increasing the magnitude and significance of the effects of Tobin's \( q \) and the price of capital on investment. Moreover, the sign of the effect of the higher moments of \( q \) on investment is highly suggestive. The results of estimating the isoelastic model \( (\beta > 1) \) along with Jensen's inequality, suggest that increased dispersion of fundamentals should increase investment rates ceteris paribus. However, the heterogeneous capital model suggests that there should be limits to this effect if the investment function eventually becomes concave. The findings in Table 7 suggest precisely this combination of effects: the standard deviation of Tobin's \( q \) has a positive effect on investment rates, but this effect is tempered by skewness, which has a negative effect.

8. Conclusions

This study finds statistically and economically significant evidence of a nonlinear relationship between investment and fundamentals, consistent with the presence of fixed costs or other non-quadratic costs of investment. This being said, there is also evidence that in this stable sample of relatively large firms, the linear

---

16 The weights are calculated as a firm's 1991 capital stock, relative to the total capital stock. The year 1991 was chosen since this year was common to virtually all of the firms in all the countries.
model often performs rather well. Using the isoelastic specification, the linear case cannot be confidently rejected for 5 of the 11 countries, though in 3 of these 5 countries the data favor a more general nonlinear (heterogeneous capital) specification over the linear model. On aggregate economic grounds, the nonlinearities again appear to be important for understanding investment in about half the countries. A clear pattern emerges in these results, however; the countries with weak nonlinearities (Belgium, the Netherlands, Spain, Sweden, and Switzerland) all have less than half the observations of the smallest sample in the remaining countries (France, Germany, Japan, the United Kingdom, and the United States). In the extreme, the U.S. sample is 40 times as large as the Belgian sample of firms. This raises questions of power in detecting nonlinearities in the smaller samples; it may be that the smaller samples do not report enough data to confidently infer the presence of nonlinearities. This possibility is bolstered by the results in Table 4 (the isoelastic model), where the point estimates for the curvature ($\beta$) of the investment function differ from one by similar amounts in both the "large" and "small" sample countries, but the standard errors are generally larger when the sample is smaller.

The findings here also provide an interesting comparison to those of Abel and Eberly (1995), who estimate the same models on an unbalanced panel of U.S. Compustat firms. That sample contains approximately as many firm-year observations as the U.S. sample of Global Vantage data, but exhibits a substantial amount of entry and exit. The Compustat evidence suggests more concavity in the U.S. investment function than found here. Together, these findings suggest that the unbalanced Compustat panel identifies the "upper arm" of the sigmoidal (heterogeneous capital) investment function much more successfully than the balanced Global Vantage panel, while the balanced panel focuses the data in the "middle" of the investment function, where it is convex. In linear models, the distribution of values of $q$ is less important, because one should estimate the same relation between investment and fundamentals regardless of this distribution. With nonlinear models, however, one's conclusions may depend on where one looks, so sample selection is a much more critical issue. These results emphasize this distinction and the importance of estimating nonlinear models on varied and large samples.

A related explanation for the differences between the Compustat and the international results is the presence of binding liquidity constraints on the investment of firm. If the firms entering and exiting the Compustat sample are relatively more constrained, then their investment may be less sensitive to fundamentals for

---

17 For example, in the Compustat sample, the isoelastic specification estimates $\beta$ significantly less than one. In addition, in the aggregated investment equation, skewness has a negative effect on investment (consistent with Table 7 above), but the standard deviation does not have an independent positive effect, as convexity would predict.
this reason. While this may very well explain some of the differences between the two studies, this should be viewed as a complement to the nonlinearities found in both studies, since the nonlinearities are significant even in the balanced panel. Moreover, using the same dataset used here, Cummins et al. (1995) detect little evidence of excess sensitivity of investment to cash flow outside the United States, so liquidity constraints are unlikely to be the sole explanation for the findings reported here.

Acknowledgements

The author thanks the National Science Foundation and a Sloan Foundation Fellowship for financial support. Two anonymous referees, as well as the organizers, discussants, and participants in the 1996 International seminar on Macroeconomics provided helpful comments.

Appendix A. Data

All firm-level data are from the Global Vantage industrial data base. The 11 countries considered and the sample size for each country are listed in Table 1. From the initial data set, firms were initially selected based on the availability of a complete balance sheet, income statement, flow-of-funds statement, and ISIC code. Each observation was then required to have non-missing values for all of the following variables: total equity, total debt, capital stock, and capital expenditure. The replacement value of capital was calculated from the book value using the method of Salinger and Summers (1983) (see the calculation below). Only positive investment rates are considered in this study, and to reduce the impact of measurement error, only investment rates less than 5 are considered. Capital expenditure is reported to be zero for a significant number of observations, but in many cases these are indistinguishable from missing data; hence, only positive values are considered. Values of Tobin’s \( q \) are restricted to be positive and but less than 15; while the lower bound can be defended theoretically the upper bound is arbitrarily chosen to reduce the potential impact of measurement error. Extending the upper bound tends to strengthen our results, while reducing it to 5 (as in Abel and Eberly (1995)) has little quantitative effect.

- Firm-level variables are based on the following definitions:
  - \( I \): capital expenditure
  - inventories: total inventories, end of period
  - \( R \): total revenue
  - debt: long-term debt, end of period
  - PPE: book value of property, plant, and equipment, end of period
  - equity: market value of equity, end of period
• *L*: useful life of capital goods, by two digit industry, calculated from *Compu-stat* industrial data base for industry *j* according to 

\[ L_j = (1/N_j) \sum_{i \in j} (\text{PPE}_{i,t-1} + \text{DEPR}_{i,t-1} + I_{i,t-1})/\text{DEPR}_{i,t}, \]

where *i* indexes firms and DEPR is depreciation. Using the double-declining balance method, the industry depreciation rate, \( \delta_j \), is \( 2/L_j \).

• *K*: replacement value of the capital stock, end of period. This is calculated from the reported book values using the recursion 

\[ K_{i,t} = [K_{i,t-1}(P_{k,t}/P_{k,t-1}) + I_{i,t}](1 - \delta_j) \]

• *q*: Tobin’s *q*, calculated as 

\[ q_{i,t} = (\text{debt}_{i,t-1} + \text{equity}_{i,t-1})/(K_{t-1} + \text{inventories}_{t-1}) \]

• Country-specific variables:

• *P*<sub>k</sub>: relative price of capital goods. Calculated as the ratio of the implicit price deflator for non-residential investment to the industrial or producer price index. Sources: OECD Quarterly National Accounts and IMF International Financial Statistics.

• \( \tau \): statutory marginal corporate income tax rate. Source: Cummins et al. (1995) summary of International Bureau of Fiscal Documentation (IFBD) publications, augmented by Cooper’s and Lybrand’s *International Tax Summaries* and *International Accounting Summaries* and Price Waterhouse’s *Corporate Taxes - A Worldwide Summary*.

• \( u \): investment tax credit or deduction. Source: as above.

• \( z \): value of depreciation allowances, calculated by 51 asset classes and aggregated to 2-digit industries using the BEA historical cost capital flow matrix. Applied only to countries where such depreciation allowances are incorporated into the tax schedules as reported in the above sources.

• *p*: tax-adjusted relative price of capital. Calculated as 

\[ p = (1 - u - \tau z)P_k. \]

**References**


Bertola, G., 1987, Irreversible investment, unpublished manuscript (Massachusetts Institute of Technology, Cambridge, MA), July.
Pindyck, R.S., 1988, Irreversible investment, capacity choice, and the value of the firm, American Economic Review 78, no. 5, 969–985.