Comments on "Demand Disagreement" by Heyerdahl-Larsen and Illeditsch

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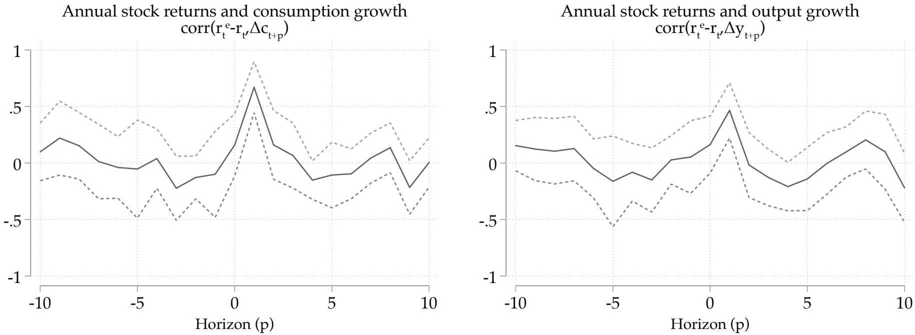
The correlation puzzle



- Annual data, US, 1929-2009: no correlation btw. $r^e - r$ and Δc and Δy
  - ... except with one-year ahead consumption growth — Parker (2001)
- Cochrane and Hansen (1992), Campbell and Cochrane (1999)
- Same in longer sample and in other countries (Albuquerque et al., 2016)

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- So we should only observe an equity premium if:

$$\mathbf{cov}\left(\Delta log(X_{t+1}), R^e_{t+1}\right) > 0,$$

  there is some amount of (positive) comovement between R^e_{t+1} and growth in fundamentals.

Potential solutions

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- How **could** this generate $\mathbf{cov}\left(M_{t,t+1}, R^e_{t+1}\right) < 0$?
  - $\rho_{t,t+1} > 0 \implies M_{t,t+1} \downarrow$
  - agents all sell the risky asset in order to consumer more today ...
  - ... so $P_t \downarrow$ and $R^e_{t+1} \uparrow$.

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- Turns out more is needed for this to work — Albuquerque et al. (2016)
  - Short-term risk-free asset vs. long-lived risky assets
  - Persistent $\rho_{t,t+1}$
  - Epstein-Zin + $\gamma\psi > 1$

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- One more ingredient: agents disagree about the long-run mean of l_t.
- Otherwise, endowment economy with time-separable preferences.

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- How far can one go without disagreement? What does predictions disagreement generate, that the hedging motive would not?

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  Predictions about volume don't seem helpful here

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$$dl_t = \kappa \left(l^{(i)} - l_t \right) dt + \sigma_l dZ^{(i)}_{\alpha,t}, \quad l^a > l^b,$$

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- How different would the results be? Would $dZ_{\alpha,t}$ still be priced?
  - likely attenuated, especially as $\nu \to 0$
  - would this be tractable?

Conclusion

- Rich paper, lots of moving pieces
- Decompose the key ones for “general” audience
- More testable predictions of disagreement, and disagreement about demand!