COMMENTS ON “DEMAND DISAGREEMENT” BY HEYERDAHL-LARSEN AND ILLEDITSCH

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The correlation puzzle

Annual stock returns and consumption growth
corr(r_t^e - r_t, \Delta c_{t+p})

Annual stock returns and output growth
corr(r_t^e - r_t, \Delta y_{t+p})

- Annual data, US, 1929-2009: no correlation btw. $r^e - r$ and $\Delta c$ and $\Delta y$
  - ... except with one-year ahead consumption growth — Parker (2001)
- Cochrane and Hansen (1992), Campbell and Cochrane (1999)
- Same in longer sample and in other countries (Albuquerque et al., 2016)
The correlation puzzle

- Inescapable:

\[ \mathbb{E} \left[ R^e_{t+1} - R_{f,t+1} \right] = -(1 + R_{f,t+1}) \text{cov} (M_{t,t+1}, R^e_{t+1} - R_{f,t+1}) \]
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- But in many representative-agent models,
  \[ \log(M_{t,t+1}) = -\rho - \gamma \Delta \log(X_{t+1}), \]
  where \( \gamma > 0 \) and
  \[ X_t = \text{aggregate consumption, aggregate output, ...} \]
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- So we should only observe an equity premium if:

\[ \text{cov} (\Delta \log(X_{t+1}), R_{t+1}^e) > 0, \]

there is some amount of (positive) comovement between \( R_{t+1}^e \) and growth in fundamentals.
Potential solutions

- We need a model where:

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\text{cov}\left(\log(M_{t,t+1}), R_{t+1}\right) \ll 0 \approx \text{cov}\left(\Delta \log(X_{t+1}), R_{t+1}\right).
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- A natural idea is:

$$\log(M_{t,t+1}) = -\rho_{t,t+1} - \gamma \Delta \log(X_{t+1}),$$

$$\Delta \log(X_{t+1}) \perp \rho_{t,t+1}.$$
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\begin{align*}
\log(M_{t,t+1}) & = -\rho_{t,t+1} - \gamma \Delta \log(X_{t+1}), \\
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\end{align*}
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- How could this generate \(\text{cov}(M_{t,t+1}, R_{t+1}^e) < 0\)?
  - \(\rho_{t,t+1} > 0 \implies M_{t,t+1} \downarrow\)
  - agents all sell the risky asset in order to consumer more today ...
  - ... so \(P_t \downarrow\) and \(R_{t+1}^e \uparrow\).
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- Turns out more is needed for this to work — Albuquerque et al. (2016)
  - Short-term risk-free asset vs. long-lived risky assets
  - Persistent \( \rho_{t,t+1} \)
  - Epstein-Zin + \( \gamma \psi > 1 \)
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- i.e. where the $\rho_{t,t+1}$ shock may be coming from
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- Otherwise, endowment economy with time-separable preferences.
Comment 1: disagreement

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- In general, in a Merton problem:

\[
\pi_{\alpha,t} \sigma_{\alpha,t} = \left( \frac{V_{W,t}^{(i)}}{-W_t^{(i)} V_{WW,t}^{(i)}} \right) \frac{\mu_{\alpha,t} - r_t}{\sigma_{\alpha,t}} + \left( \frac{V_{WS,t}^{(i)}}{-W_t^{(i)} S_t V_{WW,t}^{(i)}} \frac{\sigma_{S,t}}{\sigma_{\alpha,t}} \right)
\]

In this model, \( S_t \) would probably be the relative wealth of patient and impatient agents — determined by how cohort sizes vary over time.

Even without disagreement, if \( S_t \) affects marginal utility — \( V_{WS,t}^{(i)} \neq 0 \) — then prices should depend on it ...

... but in the case of log-utility (this paper):

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V_{(i)}(W, S) = 1 \rho_i \log(W) + f(S) = \Rightarrow V_{WS,t}^{(i)} = 0 = \Rightarrow \pi_{A,t} = \pi_{A,t} = 0.
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- How far can one go without disagreement? What does predictions disagreement generate, that the hedging motive would not?
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- What separate predictions does this story have, relative to disagreement about other fundamentals?

\[ dY_t = \mu(i) Y_t \, dt + \sigma Y_t \, dZ_{\alpha,t} \]

- Relative size of optimists/pessimists in each cohort still governed by $dZ_{\alpha,t}$, but agents agree on that.

- Would $dZ_{\alpha,t}$ still be priced in equilibrium? If so, is there any obvious way to see that disagreeing about $dZ_{\alpha,t}$ has different predictions from disagreeing about $dZ_{Y,t}$?

Predictions about volume don't seem helpful here.
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- Example:
  - Model where agents have homogeneous preferences but different beliefs about the endowment process:
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    \frac{dY_t}{Y_t} = \mu_{Y,t}^{(i)} dt + \sigma_Y dZ_{Y,t}^{(i)}
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  Predictions about volume don’t seem helpful here.
- Agents of type $i$ believe

$$dl_t = \kappa \left( l^{(i)} - l_t \right) dt + \sigma_l dZ_{\alpha,t}^{(i)}, \quad l^a > l^b,$$

i.e. they over-estimate the share of their own type.
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- Bayesian agents should be able to learn about $l$ over time, by observing how the share of agents of each type fluctuates.

- How different would the results be? Would $dZ_{\alpha,t}$ still be priced?
  - likely attenuated, especially as $\nu \to 0$
  - would this be tractable?
**Conclusion**

- Rich paper, lots of moving pieces
- Decompose the key ones for “general” audience
- More testable predictions of disagreement, and disagreement about demand!