“Volatility and Financial Structure of Firms”
by Dinlersoz, Kalemli-Ozcam, Penciakova and Quadrini

Nicolas Crouzet
Kellogg School of Management, Northwestern University

AEA 2021
This paper
This paper

- Data

[ORBIS, EU]
This paper

- Data

\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for large firms} \]
This paper

- Data

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for large firms}
\]

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for small firms before 07}
\]
This paper

- Data

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for large firms}
\]
\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for small firms before 07}
\]
\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \text{ for small firms after 07}
\]
This paper

- Data

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for large firms}
\]

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for small firms before 07}
\]

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \quad \text{for small firms after 07}
\]
This paper

- Data

\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for large firms} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for small firms} \quad \text{before 07} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \quad \text{for small firms} \quad \text{after 07} \]

- Model

[ORBIS, EU]
This paper

- Data

\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for large firms} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for small firms} \quad \text{before 07} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \quad \text{for small firms} \quad \text{after 07} \]

- Model  
  debt limit
This paper

- Data

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for large firms}
\]

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for small firms before 07}
\]

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \quad \text{for small firms after 07}
\]

- Model  
  
  \text{debt limit + short-term borrowing}
This paper

- Data

\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for large firms} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for small firms before 07} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \quad \text{for small firms after 07} \]

- Model 
  debt limit + short-term borrowing + equity issuance costs
This paper

- Data

\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for large firms} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for small firms before 07} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \text{ for small firms after 07} \]

- Model  
  debt limit + short-term borrowing + equity issuance costs

- Data vs. model
This paper

- Data

\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for large firms} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \quad \text{for small firms before 07} \]
\[ \text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \quad \text{for small firms after 07} \]

- Model  
  debt limit + short-term borrowing + equity issuance costs

- Data vs. model

\[ \mathbb{E} \left[ \frac{\partial \text{lev}_t}{\partial \sigma} \right] \approx 0 \quad \text{w/ small equity issuance costs} \]
This paper

- **Data**

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for large firms}
\]
\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for small firms before 07}
\]
\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \text{ for small firms after 07}
\]

- **Model**  
  debt limit + short-term borrowing + **equity issuance costs**

- **Data vs. model**

\[
\mathbb{E} \left[ \frac{\partial \text{lev}_t}{\partial \sigma} \right] \approx 0 \text{ w/ small equity issuance costs}
\]
\[
\mathbb{E} \left[ \frac{\partial \text{lev}_t}{\partial \sigma} \right] < 0 \text{ w/ large equity issuance costs}
\]
This paper

- Data

\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for large firms}
\]
\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) \approx 0 \text{ for small firms before 07}
\]
\[
\text{cov}(\sigma_{i,t-1}, \text{lev}_{i,t}) < 0 \text{ for small firms after 07}
\]

- Model debt limit + short-term borrowing + equity issuance costs

- Data vs. model

\[
\mathbb{E} \left[ \frac{\partial \text{lev}_{t}}{\partial \sigma} \right] \approx 0 \text{ w/ small equity issuance costs}
\]
\[
\mathbb{E} \left[ \frac{\partial \text{lev}_{t}}{\partial \sigma} \right] < 0 \text{ w/ large equity issuance costs}
\]
Why should we care?

- In the data, for which firms do financial constraints matter most?
  - most papers: size, age, tangibility
  - this paper: size + vol [Alfaro Bloom Lin (2019)]

- In our models, how should we introduce financial constraints?
  - most papers: debt [ST vs. LT; loans vs. bonds; collateral vs. income limit]
  - this paper: equity [Cooley and Quadrini (2001)!!]
Why should we care?

- In the data, for which firms do financial constraints matter most?
  - most papers: size, age, tangibility
    - too many to cite!
  - this paper: size + vol
    - Alfaro Bloom Lin (2019)

- In our models, how should we introduce financial constraints?
  - most papers: debt
    - ST vs. LT; loans vs. bonds; collateral vs. income limit
  - this paper: equity
    - Cooley and Quadrini (2001)!
Why should we care?

before/after GR

· most papers: size, age, tangibility [too many to cite!]
  · this paper: size + vol [Alfaro Bloom Lin (2019)]

· most papers: debt [ST vs. LT; loans vs. bonds; collateral vs. income limit]
  · this paper: equity [Cooley and Quadrini (2001)]
Why should we care?

before/after GR  \times  constrained/unconstrained
Why should we care?

before/after GR  \times  constrained/unconstrained  \times  high/low \sigma

In the data, for which firms do financial constraints matter most?

- most papers: size, age, tangibility
- this paper: size + vol
  [Alfaro Bloom Lin (2019)]

In our models, how should we introduce financial constraints?

- most papers: debt
  [ST vs. LT; loans vs. bonds; collateral vs. income limit]
- this paper: equity
  [Cooley and Quadrini (2001)!]
Why should we care?

before/after GR \times \text{ constrained/unconstrained} \times \text{ high/low } \sigma

- In the data, for which firms do financial constraints matter most?
Why should we care?

before/after GR $\times$ constrained/unconstrained $\times$ high/low $\sigma$

- In the data, for which firms do financial constraints matter most?
  
  · most papers: size, age, tangibility

- In our models, how should we introduce financial constraints?
  
  · most papers: debt [ST vs. LT; loans vs. bonds; collateral vs. income limit]
  
  · this paper: equity [Cooley and Quadrini (2001)]
Why should we care?

before/after GR $\times$ constrained/unconstrained $\times$ high/low $\sigma$

- In the data, for which firms do financial constraints matter most?
  
  - most papers: size, age, tangibility

[too many to cite!]
Why should we care?

before/after GR  ×  constrained/unconstrained  ×  high/low $\sigma$

- In the data, for which firms do financial constraints matter most?
  
  · most papers: size, age, tangibility  
  · this paper: size + vol

[too many to cite!]
Why should we care?

- In the data, for which firms do financial constraints matter most?
  - most papers: size, age, tangibility [too many to cite!]
  - this paper: size + vol [Alfaro Bloom Lin (2019)]

before/after GR  ×  constrained/unconstrained  ×  high/low $\sigma$
Why should we care?

before/after GR $\times$ constrained/unconstrained $\times$ high/low $\sigma$

- In the data, for which firms do financial constraints matter most?
  - most papers: size, age, tangibility
  - this paper: size + vol

[too many to cite!]
[Alfaro Bloom Lin (2019)]

- In our models, how should we introduce financial constraints?
Why should we care?

before/after GR  \times  \text{constrained/unconstrained}  \times  \text{high/low } \sigma

- In the data, for which firms do financial constraints matter most?
  - most papers: size, age, tangibility [too many to cite!]
  - this paper: size + vol [Alfaro Bloom Lin (2019)]

- In our models, how should we introduce financial constraints?
  - most papers: debt
Why should we care?

- In the data, for which firms do financial constraints matter most?
  
  · most papers: size, age, tangibility
    
    · this paper: size + vol

  [too many to cite!]
  [Alfaro Bloom Lin (2019)]

- In our models, how should we introduce financial constraints?
  
  · most papers: debt

  [ST vs. LT; loans vs. bonds; collateral vs. income limit]
**Why should we care?**

- In the data, for which firms do financial constraints matter most?
  - most papers: size, age, tangibility
  - this paper: size + vol

- In our models, how should we introduce financial constraints?
  - most papers: debt
  - this paper: equity
Why should we care?

before/after GR \times \text{constrained/unconstrained} \times \text{high/low } \sigma

- In the data, for which firms do financial constraints matter most?
  
  - most papers: size, age, tangibility \[\text{[too many to cite!]}\]
  
  - this paper: size + vol \[\text{[Alfaro Bloom Lin (2019)]}\]

- In our models, how should we introduce financial constraints?
  
  - most papers: debt \[\text{[ST vs. LT; loans vs. bonds; collateral vs. income limit]}\]
  
  - this paper: equity \[\text{[Cooley and Quadrini (2001)!]}\]
Roadmap

1. two small empirical suggestions
2. review model intuition
3. discuss mapping from model to data
Some small empirical suggestions

1. $vol_{i,t} = |\text{annual revenue growth}_{i,t}|$
   - left-skewed growth rate distribution
   - negative first moment shocks that precede deleveraging?
   - suggestion: relationship btw $vol_{i,t}$ and vol of equity returns in Compustat?

2. What does short-term debt capture?
   - floating rate debt? bank loans?
   - how much debt granularity does Orbis provide?
   - suggestion: match to EU segment of Capital IQ [Darmouni et al., 2020]
The backbone of the model

\[
V(z_t, K_t, B_t) = \max_{B_{t+1}, I_t} D_t + \frac{1}{1 + r} \mathbb{E}_t [V(z_{t+1}, K_{t+1}, B_{t+1})]
\]

s.t. \( K_{t+1} = (1 - \delta)K_t + I_t \)

\[
D_t = z_t K_t + B_{t+1} - (1 + r_b)B_t - \left( I_t + \Gamma \left( \frac{l_t}{K_t} \right) K_t \right)
\]

\[
B_{t+1} \leq \eta K_{t+1} \quad [\mu_t]
\]
The backbone of the model

\[ V(z_t, K_t, B_t) = \max_{B_{t+1}, I_t} D_t + \frac{1}{1 + r} \mathbb{E}_t[V(z_{t+1}, K_{t+1}, B_{t+1})] \]

s.t. \[ K_{t+1} = (1 - \delta)K_t + I_t \]

\[ D_t = z_tK_t + B_{t+1} - (1 + r_b)B_t - \left( I_t + \Gamma \left( \frac{I_t}{K_t} \right) K_t \right) \]

\[ B_{t+1} \leq \eta K_{t+1} \quad [\mu_t] \]

If \( r > r_b \),

\[ \mu_t = \frac{r - r_b}{1 + r} > 0 \]
The backbone of the model

\[ V(z_t, K_t, B_t) = \max_{B_{t+1}, I_t} D_t + \frac{1}{1 + r} E_t [V(z_{t+1}, K_{t+1}, B_{t+1})] \]

s.t. \[ K_{t+1} = (1 - \delta) K_t + I_t \]

\[ D_t = z_t K_t + B_{t+1} - (1 + r_b) B_t - \left( I_t + \Gamma \left( \frac{I_t}{K_t} \right) K_t \right) \]

\[ B_{t+1} \leq \eta K_{t+1} \quad [\mu_t] \]

If \( r > r_b \),

\[ \mu_t = \frac{r - r_b}{1 + r} > 0 \]

\[ b_t = \frac{B_{t+1}}{K_{t+1}} = \eta \]
The backbone of the model

\[ V(z_t, K_t, B_t) = \max_{B_{t+1}, I_t} \left( D_t + \frac{1}{1 + r} \mathbb{E}_t [V(z_{t+1}, K_{t+1}, B_{t+1})] \right) \]

s.t.  \[ K_{t+1} = (1 - \delta) K_t + I_t \]

\[ D_t = z_t K_t + B_{t+1} - (1 + r_b) B_t - \left( I_t + \Gamma \left( \frac{I_t}{K_t} \right) K_t \right) \]

\[ B_{t+1} \leq \eta K_{t+1} \quad [\mu_t] \]

If \( r > r_b \),

\[ \mu_t = \frac{r - r_b}{1 + r} > 0 \]

\[ b_t \equiv \frac{B_{t+1}}{K_{t+1}} = \eta \quad \Rightarrow \quad \frac{\partial b_t}{\partial \sigma} = 0 \]
Adding equity issuance costs

\[ V(z_t, K_t, B_t) = \max_{B_{t+1}, I_t} \left\{ u \left( \frac{D_t}{K_t} \right) K_t + \frac{1}{1 + r} \mathbb{E}_t [V(z_{t+1}, K_{t+1}, B_{t+1})] \right\} \]

s.t.

\[ K_{t+1} = (1 - \delta)K_t + I_t \]

\[ D_t = z_tK_t + B_{t+1} - (1 + r_b)B_t - \left( I_t + \Gamma \left( \frac{l_t}{K_t} \right) K_t \right) \]

\[ B_{t+1} \leq \eta K_{t+1} \quad [\mu_t] \]
Adding equity issuance costs

\[ V(z_t, K_t, B_t) = \max_{B_{t+1}, I_t} u\left(\frac{D_t}{K_t}\right) K_t + \frac{1}{1 + r} \mathbb{E}_t [V(z_{t+1}, K_{t+1}, B_{t+1})] \]

s.t. \[ K_{t+1} = (1 - \delta) K_t + I_t \]

\[ D_t = z_t K_t + B_{t+1} - (1 + r_b) B_t - \left( I_t + \Gamma \left( \frac{I_t}{K_t} \right) K_t \right) \]

\[ B_{t+1} \leq \eta K_{t+1} \quad [\mu_t] \]

If \( u(x) = x - \tau x^2 1 \{x < 0\}, \)
Adding equity issuance costs

\[ V(z_t, K_t, B_t) = \max_{B_{t+1}, I_t} u \left( \frac{D_t}{K_t} \right) K_t + \frac{1}{1 + r} \mathbb{E}_t \left[ V(z_{t+1}, K_{t+1}, B_{t+1}) \right] \]

s.t. \[ K_{t+1} = (1 - \delta)K_t + I_t \]

\[ D_t = z_t K_t + B_{t+1} - (1 + r_b)B_t - \left( I_t + \Gamma \left( \frac{I_t}{K_t} \right) K_t \right) \]

\[ B_{t+1} \leq \eta K_{t+1} \quad [\mu_t] \]

If \( u(x) = x - \tau x^2 \mathbb{1} \{x < 0\} \),

\[ \mu_t = \frac{r - r_b}{1 + r} + 2\tau \left( \frac{1 + r_b}{1 + r} \mathbb{E}_t \left[ \frac{D_{t+1}}{K_{t+1}} \right] - \frac{D_t}{K_t} \right) \geq 0 \]
The effect of volatility

- no closed form ...

- low net worth $\Rightarrow \mu_t > 0$, $b_t = \eta$, $= \Rightarrow \frac{\partial b_t}{\partial \sigma} = 0$

- high net worth $\Rightarrow \mu_t = 0$, $b_t \leq \eta$, $= \Rightarrow \frac{\partial b_t}{\partial \sigma} \leq 0$

- so, with $\tau > 0$, higher vol leads to lower leverage (as in the post-07 data)

- but no predictions for short- vs. long-term borrowing
The effect of volatility

- no closed form ...
- ... but, roughly:

\[
\begin{align*}
\text{low net worth} & \Rightarrow \mu_t > 0, \quad b_t = \eta, \quad \Rightarrow \frac{\partial b_t}{\partial \sigma} = 0 \\
\text{high net worth} & \Rightarrow \mu_t = 0, \quad b_t \leq \eta, \quad \Rightarrow \frac{\partial b_t}{\partial \sigma} \leq 0
\end{align*}
\]

- so, with \( \tau > 0 \), higher vol leads to lower leverage (as in the post-07 data)
- but no predictions for short- vs. long-term borrowing
The effect of volatility

- no closed form ...
- ... but, roughly:

\[
\text{low net worth} \implies \mu_t > 0, \quad b_t = \eta, \quad \implies \frac{\partial b_t}{\partial \sigma} = 0
\]
The effect of volatility

- no closed form ...

- ... but, roughly:

  low net worth \( \implies \mu_t > 0, \quad b_t = \eta, \quad \implies \frac{\partial b_t}{\partial \sigma} = 0 \)

  high net worth \( \implies \mu_t = 0, \quad b_t \leq \eta, \quad \implies \frac{\partial b_t}{\partial \sigma} \leq 0 \)
The effect of volatility

- no closed form ...

- ... but, roughly:

  low net worth \( \implies \mu_t > 0, \ b_t = \eta, \implies \frac{\partial b_t}{\partial \sigma} = 0 \)

  high net worth \( \implies \mu_t = 0, \ b_t \leq \eta, \implies \frac{\partial b_t}{\partial \sigma} \leq 0 \)

- so, with \( \tau > 0 \), higher vol leads to lower leverage (as in the post-07 data)
The effect of volatility

- no closed form ...

- ... but, roughly:

\[
\text{low net worth } \implies \mu_t > 0, \quad b_t = \eta, \quad \implies \frac{\partial b_t}{\partial \sigma} = 0
\]

\[
\text{high net worth } \implies \mu_t = 0, \quad b_t \leq \eta, \quad \implies \frac{\partial b_t}{\partial \sigma} \leq 0
\]

- so, with \( \tau > 0 \), higher vol leads to lower leverage (as in the post-07 data)

- but no predictions for short- vs. long-term borrowing
Adding short-term debt

- Assume a fraction $\lambda_{t+1}$ of debt is:
Adding short-term debt

- Assume a fraction $\lambda_{t+1}$ of debt is:
  - cheaper — say $r^S_b < r_b$

- Choose $\lambda_{t+1}$ subject to:
  $$\lambda_{t+1} B_{t+1} + K_{t+1} \leq \eta \chi \left[ \zeta_t \right]$$

- Similar broad intuition as for the overall debt constraint:
  $$\zeta_t > 0 \Rightarrow \partial \lambda_{t+1} \partial \sigma = 0$$
  $$\zeta_t = 0 \Rightarrow \partial \lambda_{t+1} \partial \sigma \leq 0$$
Adding short-term debt

- Assume a fraction $\lambda_{t+1}$ of debt is:
  
  - cheaper — say $r^S_b < r_b$
  
  - but potentially subject to a penalty, which increases with the shortfall of revenue relative to short-term debt payments ("illiquidity cost")
Adding short-term debt

- Assume a fraction $\lambda_{t+1}$ of debt is:
  
  · cheaper — say $r^S_b < r_b$
  
  · but potentially subject to a penalty, which increases with the shortfall of revenue relative to short-term debt payments ("illiquidity cost")

- Choose $\lambda_{t+1}$ subject to: $\lambda_{t+1} \frac{B_{t+1}}{K_{t+1}} \leq \eta \chi$ [$\zeta_t$]
Adding short-term debt

- Assume a fraction $\lambda_{t+1}$ of debt is:
  - cheaper — say $r^S_b < r_b$
  - but potentially subject to a penalty, which increases with the shortfall of revenue relative to short-term debt payments ("illiquidity cost")

- Choose $\lambda_{t+1}$ subject to: $\lambda_{t+1} B_{t+1} \leq \eta \chi [\zeta_t]$

- Similar broad intuition as for the overall debt constraint:
  \[
  \zeta_t > 0 \implies \frac{\partial \lambda_{t+1}}{\partial \sigma} = 0 \\
  \zeta_t = 0 \implies \frac{\partial \lambda_{t+1}}{\partial \sigma} \leq 0
  \]
Comments on the model

1. Debt is always sold at par
   
   Alternatively, net cash flow from debt financing activities could be:

   \[ Q_t (B_t + 1 - (1 - \lambda_t + 1) B_t) - (\lambda_t (1 + r_s) + (1 - \lambda_t) r_b) B_t \]

   \[ Q_t = \frac{1}{1 + r_b \{ \lambda_t + 1 (1 + r_s) + (1 - \lambda_t) (r_b + E_t [Q_t + 1]) \}} \leq 1 \]

2. There is no limited liability/default
   
   Negative equity values are possible (though a tight constraint may prevent this)

   With default, \( \partial \text{lev} / \partial \sigma > 0 \) even without equity issuance costs

3. What does adding short-term debt change in the model?
   
   Other than endogenizing the term structure of debt? (e.g. for investment?)
Comments on the model

1. Debt is always sold at par
Comments on the model

1. Debt is always sold at par
   · Alternatively, net cash flow from debt financing activities could be:

\[
Q_t (B_{t+1} - (1 - \lambda_{t+1})B_t) - (\lambda_t (1 + r_b^s) + (1 - \lambda_t) r_b)B_t
\]

\[
Q_t = \frac{1}{1 + r_b} \{ \lambda_{t+1}(1 + r_b^s) + (1 - \lambda_{t+1}) (r_b + \mathbb{E}_t [Q_{t+1}]) \} \leq 1
\]
Comments on the model

1. Debt is always sold at par
   · Alternatively, net cash flow from debt financing activities could be:

   $Q_t \left( B_{t+1} - (1 - \lambda_{t+1})B_t \right) - \left( \lambda_t (1 + r^s_b) + (1 - \lambda_t) r_b \right) B_t$

   $$Q_t = \frac{1}{1 + r_b} \left\{ \lambda_{t+1}(1 + r^s_b) + (1 - \lambda_{t+1}) \left( r_b + \mathbb{E}_t [Q_{t+1}] \right) \right\} \leq 1$$

2. There is no limited liability/default
   · Negative equity values are possible (though a tight constraint may prevent this)
   · With default, $\partial \text{lev} / \partial \sigma > 0$ even without equity issuance costs
Comments on the model

1. Debt is always sold at par
   - Alternatively, net cash flow from debt financing activities could be:
     \[
     Q_t = \frac{1}{1 + r_b} \left\{ \lambda_{t+1}(1 + r_b^s) + (1 - \lambda_{t+1})(r_b + \mathbb{E}_t[Q_{t+1}]) \right\} \leq 1
     \]

2. There is no limited liability/default
   - Negative equity values are possible (though a tight constraint may prevent this)
   - With default, \( \frac{\partial lev}{\partial \sigma} > 0 \) even without equity issuance costs
Comments on the model

1. Debt is always sold at par
   · Alternatively, net cash flow from debt financing activities could be:
   $$Q_t (B_{t+1} - (1 - \lambda_{t+1})B_t) - (\lambda_t (1 + \xi_t) + (1 - \lambda_t)r_b)B_t$$
   $$Q_t = \frac{1}{1 + r_b} \left\{ \lambda_{t+1}(1 + r_b^\xi) + (1 - \lambda_{t+1}) (r_b + E_t [Q_{t+1}]) \right\} \leq 1$$

2. There is no limited liability/default
   · Negative equity values are possible (though a tight constraint may prevent this)
   · With default, \( \frac{\partial \text{lev}}{\partial \sigma} > 0 \) even without equity issuance costs

3. What does adding short-term debt change in the model?
   · Other than endogenizing the term structure of debt? (e.g. for investment?)
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs

higher equity issuance costs ⇒ higher vol firms choose lower leverage

but is the lower leverage primarily driven by less short-term debt?

model: \( \Delta \) in short-term leverage btw. high- and low-vol firms seems similar in low- and high-equity cost calibrations.

data: \( \Delta \) in short-term leverage btw. high- and low-vol firms increases after '07

2. what do (increasing) equity issuance costs stand for?

· not IPO or SEO costs (little evidence that those have changed, anyway)

· emergency loans from owners? from banks? loans facilitated by PE?

· can those be measured in ORBIS data? (+ test model implications?)
How do we reconcile, theoretically, the fact that for small firms total leverage became sensitive to volatility after the crisis? The model can generate this result if equity funding for small firms became more costly after the crisis. This is captured in the model by an increase in the value of the parameter $\gamma$.

Figure 5 plots total debt and short-term debt as a function of net worth, for firms with low cost of equity ($\gamma = 0.5$) and high cost of illiquidity ($\alpha = 0.5$). The left-hand-side panel is for firms facing low volatility ($\rho = 0.5$) while the right-hand-side panel is for firms facing high volatility ($\rho = 0.5$). By comparing the two graphs we see how volatility affects total leverage and the term structure of the debt when the cost of equity is low but the cost of illiquidity is sizable. We think of this environment as capturing the financial conditions of small firms before the financial crisis. The two plots also report the invariant distribution for the two types of firms (low volatility firms and high volatility firms).

The key insight provided by Figure 5 is that volatility leads to a very...
small change in total leverage (the average for all firms with low volatility is 0.4 and for firms with high volatility is 0.39). However, short-term debt is very different between low and high volatility firms. For low volatility firms the average value of short-term debt is 0.13 while for high volatility firms is 0.06. If we think that small firms do not face very strict financial conditions on the equity side but still face significant costs of illiquidity, then volatility is important for determining the term structure of the debt but it is marginal in affecting total leverage. We can think of this as capturing the financial conditions of small firms before the 2008 financial crisis.

Now imagine that, after the financial crisis, the cost of external equity financing increased. We change $\bar{\sigma}$ from 0.5 to 2.5, but we use the same cost of illiquidity ($\bar{\sigma} = 0.5$). Figure 6 shows the new financial policies after the increase in the cost of equity issuance.

Small firms post-crisis - Low volatility
Average total debt=0.38 - Average short-term debt=0.12

Small firms post-crisis - High volatility
Average total debt=0.34 - Average short-term debt=0.06

When we compare the left-hand-side panel (low volatility) to the right-hand-side panel (high volatility), we can see that both total debt and short-term debt are affected by volatility. The average leverage for low volatility
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs
   - higher equity issuance costs $\implies$ higher vol firms choose lower leverage

2. what do (increasing) equity issuance costs stand for?
   - not IPO or SEO costs (little evidence that those have changed, anyway)
   - emergency loans from owners? from banks? loans facilitated by PE?
   - can those be measured in ORBIS data? (+ test model implications?)
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs
   
   · higher equity issuance costs $\Rightarrow$ higher vol firms choose lower leverage
   
   · but is the lower leverage **primarily** driven by less short-term debt?
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs

   · higher equity issuance costs $\implies$ higher vol firms choose lower leverage
   · but is the lower leverage \textbf{primarily} driven by less \textbf{short-term debt}?

   \textbf{model:} $\Delta$ in \textbf{short-term} leverage btw. high- and low-vol firms seems similar in low- and high-equity cost calibrations
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs
   - higher equity issuance costs $\Rightarrow$ higher vol firms choose lower leverage
   - but is the lower leverage primarily driven by less short-term debt?

   **model:** $\Delta$ in short-term leverage btw. high- and low-vol firms seems similar in low- and high-equity cost calibrations
   **data:** $\Delta$ in short-term leverage btw. high- and low-vol firms increases after ’07
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs
   · higher equity issuance costs $\implies$ higher vol firms choose lower leverage
   · but is the lower leverage primarily driven by less short-term debt?
     model: $\Delta$ in short-term leverage btw. high- and low-vol firms seems similar in low- and high-equity cost calibrations
     data: $\Delta$ in short-term leverage btw. high- and low-vol firms increases after ’07

2. what do (increasing) equity issuance costs stand for?
   · not IPO or SEO costs (little evidence that those have changed, anyway)
   · emergency loans from owners? from banks? loans facilitated by PE?
   · can those be measured in ORBIS data? (+ test model implications?)
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs
   - higher equity issuance costs $\Rightarrow$ higher vol firms choose lower leverage
   - but is the lower leverage primarily driven by less short-term debt?

   **model:** $\Delta$ in short-term leverage btw. high- and low-vol firms seems similar in low- and high-equity cost calibrations

   **data:** $\Delta$ in short-term leverage btw. high- and low-vol firms increases after ’07

2. what do (increasing) equity issuance costs stand for?
   - not IPO or SEO costs (little evidence that those have changed, anyway)
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs
   - higher equity issuance costs $\implies$ higher vol firms choose lower leverage
   - but is the lower leverage primarily driven by less short-term debt?
     - model: $\Delta$ in short-term leverage btw. high- and low-vol firms seems similar in low- and high-equity cost calibrations
     - data: $\Delta$ in short-term leverage btw. high- and low-vol firms increases after ’07

2. what do (increasing) equity issuance costs stand for?
   - not IPO or SEO costs (little evidence that those have changed, anyway)
   - emergency loans from owners? from banks? loans facilitated by PE?
Mapping model and data

1. small firms = high illiquidity + rising equity issuance costs
   - higher equity issuance costs $\Rightarrow$ higher vol firms choose lower leverage
   - but is the lower leverage primarily driven by less short-term debt?
     model: $\Delta$ in short-term leverage btw. high- and low-vol firms seems similar in low- and high-equity cost calibrations
     data: $\Delta$ in short-term leverage btw. high- and low-vol firms increases after ’07

2. what do (increasing) equity issuance costs stand for?
   - not IPO or SEO costs (little evidence that those have changed, anyway)
   - emergency loans from owners? from banks? loans facilitated by PE?
   - can those be measured in ORBIS data? (+ test model implications?)
Other small stuff for the authors

- Is the firm fixed effect different in each sub-sample (pre- and post-07)? Does this matter?
- Why not directly regress leverage on $|RG_{i,t}|$? (Why do the "first-stage"? The intuition wasn’t clear to me.)
- It would be helpful to discuss more what the magnitude of the coefficients mean.
- Are there other outcome variables (esp. investment or employment) that behave differently/consistently with the model among high-vol/small firms?
- Typo p.12: it should be "need to pay $[r(\lambda_{t+1}) + \lambda_{t+1}]B_{t+1}$
- In the “frictionless model” ($\kappa = 0$ and $\tau = 0$), the first-order condition for borrowing is:

$$1 - \beta R(\lambda_{t+1}) = \mu_t + \lambda_{t+1} \zeta_t.$$ 

If $\lambda_{t+1} = \chi$ and $\zeta_t = \bar{p}$, then the condition for $\mu_t > 0$ is:

$$\beta^{-1} > \frac{R(\chi)}{1 - \chi \bar{p}}.$$ 

This seems potentially more restrictive than $1 > R(0) \beta$ (which I think is what the text assumes and is what is discussed in Appendix.) So is $1 > R(0) \beta$ always sufficient to guarantee that the debt limit is binding? (My math may be wrong here.)
Other small stuff for the authors

- In the model, does high $\kappa$ and high $\tau$ imply low size (e.g. net worth or capital)?

- In Figure 4, it looks like leverage is constant for high-net-worth firms, not for low net-worth ones.

- If my intuition is correct, the firm that are most responsive to changes in volatility (in the comparative statics) are also those for which the constraints do not bind. For the total debt constraint, this will be the firms with high net worth — i.e. the larger/less constrained ones. This is maybe problematic when comparing model and data (where small firms are presumably more constrained but also more responsive to vol).

- Figures 5 and 7 seem to be very close — in particular leverage is similar, whereas in the data it seems leverage is somewhat lower among small firms.
Conclusion

- A different perspective on how financial frictions work
  - volatility + equity issuance costs (+ short-term debt subject to “illiquidity”)

- Many moving pieces — simplify, without losing the message
  - is short-term debt essential?

- What do equity issuance costs stand for?