“Volatility and Financial Structure of Firms” by Dinlersoz, Kalemli-Ozcam, Penciakova and Quadrini

Nicolas Crouzet

Kellogg School of Management, Northwestern University

AEA 2021

This paper

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  - this paper: equity [Cooley and Quadrini (2001)!]

Roadmap

1. two small empirical suggestions
2. review model intuition
3. discuss mapping from model to data

Some small empirical suggestions

1. $vol_{i,t} = |\text{annual revenue growth}_{i,t}|$
   - left-skewed growth rate distribution
   - negative first moment shocks that precede deleveraging?
   - suggestion : relationship btw $vol_{i,t}$ and vol of equity returns in Compustat?

2. What does short-term debt capture?
   - floating rate debt? bank loans?
   - how much debt granularity does Orbis provide?
   - suggestion : match to EU segment of Capital IQ [Darmouni et al., 2020]

The backbone of the model

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V(z_t, K_t, B_t) \quad &= \quad \max_{B_{t+1}, I_t} \quad D_t + \frac{1}{1+r} \mathbb{E}_t \left[V(z_{t+1}, K_{t+1}, B_{t+1}) \right] \\
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- but no predictions for short- vs. long-term borrowing

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- Similar broad intuition as for the overall debt constraint:

$$\zeta_t > 0 \quad \Longrightarrow \quad \frac{\partial \lambda_{t+1}}{\partial \sigma} = 0$$
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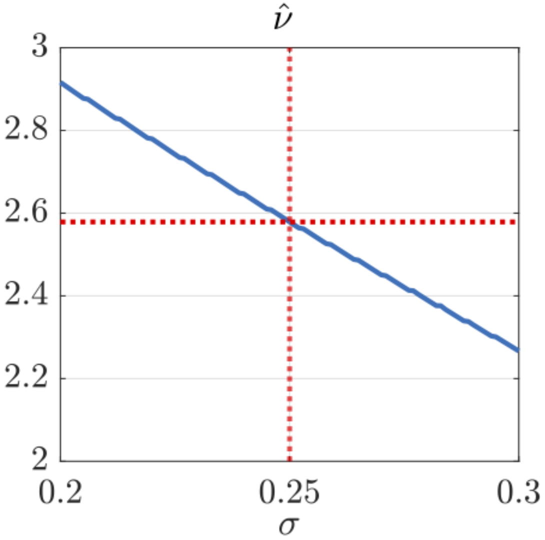

[Crouzet and Tourre, 2020]

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3. What does adding short-term debt change in the model?
    - Other than endogenizing the term structure of debt? (e.g. for investment?)

Mapping model and data

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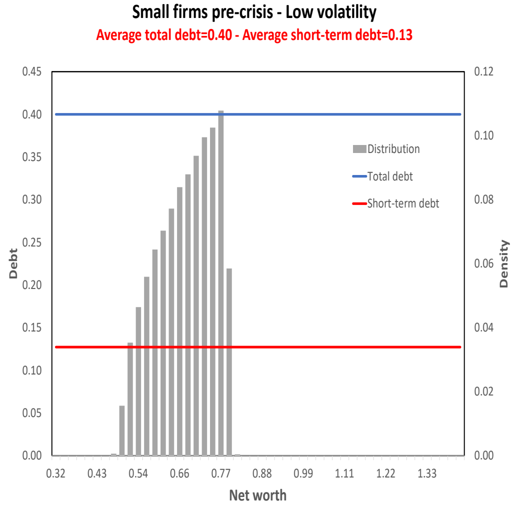

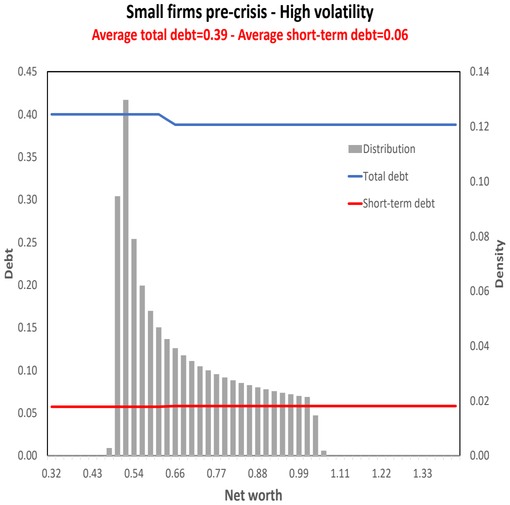

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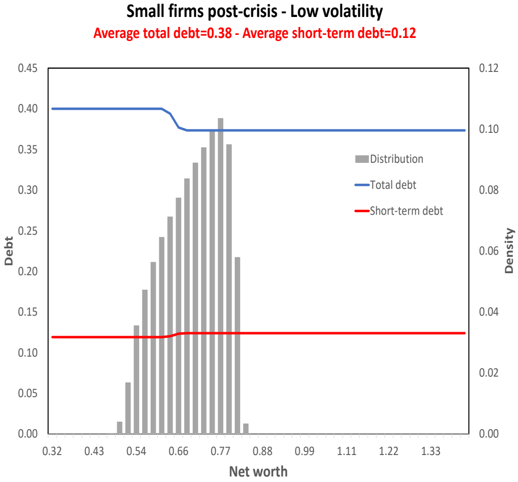

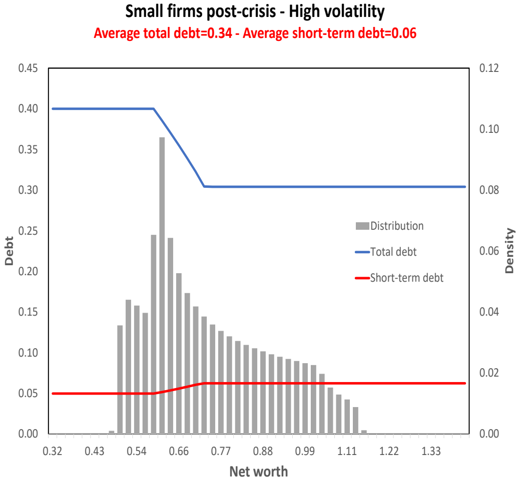

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2. what do (increasing) equity issuance costs stand for?

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    - higher equity issuance costs $\implies$ higher vol firms choose lower leverage
    - but is the lower leverage **primarily** driven by **less short-term debt**?

      <u>model:</u> Δ in **short-term** leverage btw. high- and low-vol firms seems similar in low- and high-equity cost calibrations

      <u>data:</u> Δ in **short-term** leverage btw. high- and low-vol firms increases after '07

2. what do (increasing) equity issuance costs stand for?

    - not IPO or SEO costs (little evidence that those have changed, anyway)

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2. what do (increasing) equity issuance costs stand for?

    - not IPO or SEO costs (little evidence that those have changed, anyway)
    - emergency loans from owners? from banks? loans facilitated by PE?
    - can those be measured in ORBIS data? (+ test model implications?)

Other small stuff for the authors

- Is the firm fixed effect different in each sub-sample (pre- and post-07)? Does this matter?
- Why not directly regress leverage on $|RG_{i,t}|$? (Why do the "first-stage"? The intuition wasn't clear to me.)
- It would be helpful to discuss more what the magnitude of the coefficients mean.
- Are there other outcome variables (esp. investment or employment) that behave differently/consistently with the model among high-vol/small firms?
- Typo p.12: it should be "need to pay $[r(\lambda_{t+1}) + \lambda_{t+1}]B_{t+1}$
- In the "frictionless model" ($\kappa = 0$ and $\tau = 0$), the first-order condition for borrowing is:

  $$1 - \beta R(\lambda_{t+1}) = \mu_t + \lambda_{t+1}\zeta_t.$$

  If $\lambda_{t+1} = \chi$ and $\zeta_t = \overline{p}$, then the condition for $\mu_t > 0$ is:

  $$\beta^{-1} > \frac{R(\chi)}{1 - \chi\overline{p}}.$$

  This seems potentially more restrictive than $1 > R(0)\beta$ (which I think is what the text assumes and is what is discussed in Appendix.) So is $1 > R(0)\beta$ always sufficient to guarantee that the debt limit is binding? (My math may be wrong here.)

Other small stuff for the authors

- In the model, does high κ and high τ imply low size (e.g. net worth or capital)?
- In Figure 4, it looks like leverage is constant for high-net-worth firms, not for low net-worth ones
- If my intution is correct, the firm that are most responsive to changes in volatility (in the comparative statics) are also those for which the constraints do not bind. For the total debt constraint, this will be the firms with high net worth — i.e. the larger/less constrained ones. This is maybe problematic when comparing model and data (where small firms are presumably more constrained but also more responsive to vol).
- Figures 5 and 7 seem to be very close — in particular leverage is similar, whereas in the data it seems leverage is somewhat lower among small firms.

Conclusion

- A different perspective on how financial frictions work
    - volatility + equity issuance costs (+ short-term debt subject to "illiquidity")
- Many moving pieces — simplify, without losing the message
    - is short-term debt essential?
- What do equity issuance costs stand for?