Intangible Capital, Non-Rivalry, and Growth

Nicolas Crouzet, Janice Eberly, Andrea Eisfeldt, and Dimitris Papanikolaou

Northwestern and UCLA
The importance of intangible assets

Intangible assets now account for a large part of the capital stock of businesses
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- IT-related assets (software, data)
- Intellectual property (patents, trademarks)
- Organization capital (production/distribution systems, firm-specific processes)
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What is special about intangible assets, relative to physical assets?
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Hard to measure?
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What is special about intangible assets, relative to physical assets?

- Hard to measure?
- Special economic characteristics?
The economic properties of intangible assets
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Intangibles are assets that are replicable, but hard to exclude.
The economic properties of intangible assets

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Replicability

Excludability
The economic properties of intangible assets

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Information on products, processes, organization, customers

Technology determines how easy replication is — e.g. language, writing, digital

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Value as an asset comes from restricting use by other firms

Institutions determine how easy exclusion is — e.g. patent system
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Replicability  —  \( \rho \)

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Excludability  —  \( \delta \)

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Different types of intangible assets $\leftrightarrow$ different $(\rho, \delta)$
The economic properties of intangibles

Limited Excludability ($\delta$) vs. Replicability ($\rho$)
The economic properties of intangibles

Limited Excludability ($\delta$)

Physical Capital (K)

Replicability ($\rho$)
The economic properties of intangibles

Limited Excludability ($\delta$)
Physical Capital ($K$)

Patented Innovation
Replicability ($\rho$)
The economic properties of intangibles

- Limited Excludability ($\delta$)
- Physical Capital ($K$)
- Management Process
- Patented Innovation
- Replicability ($\rho$)
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The economic properties of intangibles

Limited Excludability ($\delta$)

Replicability ($\rho$)

Physical Capital ($K$)

Patented Innovation

Management

Process

Data
This paper

Question Implications for growth?

Findings

$\rho$ = $\Rightarrow$

- $\uparrow$ scale economies
- $\uparrow$ spillovers to future entrants
- $\uparrow$ spillovers to existing competitors

If negative competitive effect dominates (high $\rho$)

$\downarrow$ growth, investment, entry

$\uparrow$ profits, valuations, concentration
This paper

<table>
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Question  Implications for growth?

Contrib.  Macro model w/intangibles
          Formalize replicability ($\rho$) and excludability ($\delta$)

Findings  $\uparrow \rho$
Growth rate $g$

Replicability $\rho$
Intan $\sim K$

Solow model

Growth rate $g$

Replicability $\rho$
This paper

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Formalize replicability ($\rho$) and excludability ($\delta$)

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\begin{align*}
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This paper

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If negative competitive effect dominates (high $\rho$)
- $\downarrow$ growth, investment, entry
- $\uparrow$ profits, valuations, concentration
Related literature

Macro and financial implications of rising intangibles

Contribution: formalize replicability and limited excludability

Endogenous technological change
Lucas and Moll (2014), Stokey (2015); Jones and Tonetti (2020), Farboodi and Veldkamp (2022)

Contribution: replicability facilitates imitation; not limited to data

Competition and returns to innovation
Aghion, Bloom, Blundell, Griffith, Howitt (2005), Aghion, Bergeaud, Boppart, Klenow, Li (2022)

Contribution: replicability creates both returns to scale and competitive risk
Roadmap

1. Economic environment

2. The effects of replicability
1. Economic environment
Overview

Household → Production labor → Creators → Imitators → Projects

Consumption goods

Entrepreneurial labor

Project = \{ product streams \in [0, x_t] \} \times t: project "span"
Overview

Household

Consumption goods

Production labor

Creators

Entrepreneurial labor

Imitators

Spillovers

Projects

\[
\text{project} = \{ \text{product streams} \}^s_{t \in [0, x_t]} : \text{project} \text{ span}
\]
Overview

Household → Creators

Production labor → Creators

Entrepreneurial labor

Consumption goods

Imitators

Spillovers

Projects

\[ \text{project} = \{ \text{product streams} \in [0,x_t] \} x_t \]

\text{span}
project = \{ \text{product streams } s \in [0, x_t] \}

x_t: \text{project ”span”}
Allocating intangible capital within a project

\[ \Pi(x_t, N_t) = \max_{\{N(s), L(s)\}, L_t} \int_0^{x_t} N(s)^{1-\zeta} L(s)^{\zeta} ds - W_t L_t \]
Allocating intangible capital within a project

\[
\Pi(x_t, N_t) = \max \left\{ N(s), L(s) \right\} L_t \int_0^{x_t} N(s)^{1-\zeta} L(s)^\zeta ds - W_t L_t
\]

s.t.

\[
\int_0^{x_t} L(s) ds \leq L_t
\]

\[
\left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} ds \right)^{1-\rho} \leq N_t \quad \rho \in [0, 1]
\]
Allocating intangible capital within a project

$$\Pi(x_t, N_t) = \max \left\{ N(s), L(s) \right\}, L_t \int_0^{x_t} N(s)^{1-\xi} L(s) \xi ds - W_t L_t$$

s.t. \( \int_0^{x_t} L(s) ds \leq L_t \)

\( \left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} ds \right)^{1-\rho} \leq N_t \) \quad \rho \in [0, 1]

$$\Pi(x_t, N_t) \propto x_t^\rho N_t$$
What does $\rho$ capture?

\[
\left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} \, ds \right)^{1-\rho} \leq N_t
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What does $\rho$ capture?

$$\left( \int_0^{\mathbf{x}_t} N(s) \frac{1}{1-\rho} \, ds \right)^{1-\rho} \leq N_t$$

$\rho = 0$
What does $\rho$ capture?

$$\int_{0}^{x_{t}} N(s) \, ds \leq N_{t}$$

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increasing $N(s)$ requires reducing $N(-s)$ one-for-one
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intangible capital is non-replicable within the project
What does $\rho$ capture?

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intangible capital is non-replicable within the project

e.g. leasehold rights to airport gates

allocating a gate to a route makes it unavailable to other routes
What does \( \rho \) capture?

\[
\left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} \, ds \right)^{1-\rho} \leq N_t
\]

\( \rho = 1 \)
What does $\rho$ capture?

$$\max_{s \in [0, x_t]} N(s) \leq N_t$$

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increasing $N(s)$ doesn’t require reducing $N(-s)$ at all.

intangible capital is fully replicable within the project.
What does $\rho$ capture?

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$\rho = 1$

increasing $N(s)$ doesn’t require reducing $N(-s)$ at all

intangible capital is fully replicable within the project

e.g. a patent for a steel alloy

using it in one mill does not reduce its availability to other mills
What does $\rho$ capture?

$$\max_{s \in [0, x_t]} N(s) \leq N_t$$

$$\rho \in (0, 1)$$
What does $\rho$ capture?

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$\rho \in (0, 1)$

increasing $N(s)$ requires reducing $N(-s)$, but less than one-for-one

intangible capital is imperfectly replicable within the project
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intangible capital is imperfectly replicable within the project

e.g. an inventory management process for an online retailer
deploying it in a new warehouse requires managerial resources
takes managerial time away from other warehouses
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replicability of intangibles ($\rho$) $\leftrightarrow$ returns to scale within firm
What does $\rho$ capture?

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replicability of intangibles ($\rho$) $\leftrightarrow$ returns to scale within firm

$$\Pi_t \propto x_t^\rho N_t$$

if $\rho > 0$, $N_t$ raises marginal returns to $x_t$
Imperfect excludability and spillovers

Imperfect excludability: ownership of each stream is lost w.p. $\delta dt$
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$$dx_t = -\delta x_t dt$$
Imperfect excludability and spillovers

Imperfect excludability: ownership of each stream is lost w.p. $\delta dt$

\[
dx_t = -\delta x_t dt
\]

\[
\Rightarrow dN_t = -\delta (1 - \rho) N_t dt
\]
Classifying intangibles

Excludability ($\delta$)

Replicability ($\rho$)

- Patents
- Distribution systems
- Resource rights
- Customer lists
- Software, data
Classifying intangibles

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Spillovers: $S_t$; benefit new projects and imitators
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Spillovers: $S_t$; benefit new projects and imitators

Initial intangible stock $= N_\tau = \left( N_t^{\frac{1}{1-\rho}} + S_t^{\frac{1}{1-\rho}} \right)^{1-\rho}$
Higher $\rho$ accelerates spillovers
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Higher $\rho$ accelerates spillovers

The diagram shows the function $N_\tau$ over time intervals $t = \tau$, $t = \tau + 5$, and $t = \tau + 10$. The curve $S_t$, with $\rho = 0.5$, increases smoothly as time progresses.
Higher $\rho$ accelerates spillovers
Higher $\rho$ accelerates spillovers

$N_\tau$

$t = \tau$

$t = \tau + 5$

$t = \tau + 10$

$S_t, \quad \rho = 1.0$
New projects

Initial span: $x_\tau$. 

Assume: $\delta = \gamma(x_\tau)$, $\gamma$ increasing and convex.

Value of project to creator:

$$V_\tau(N_\tau) \propto \max x_\tau N_\tau x_\rho r + \gamma(x_\tau) - (-\zeta g) \text{(scale) (limited excludability)}$$

New project requires 1 unit of labor, and starts with intangible stock:

$$N_\tau = \nu \int_0^\tau (i) \leq \tau S_i, \tau d_i$$
New projects

Initial span: $x_\tau$. Assume:

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New projects

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New project requires 1 unit of labor, and starts with intangible stock:

\[
N_\tau = \nu \int_{\tau(i) \leq \tau} S_{i,\tau} di
\]
Imitators

Imitators take over expropriated product streams
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Produce using labor, $S_{\tau,t}$
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Imitators take over expropriated product streams

Produce using labor, $S_{\tau,t}$

$$V_t \equiv \text{Total project value} \propto \frac{N_t x_t^p}{r + \zeta g}$$
Imitators

Imitators take over expropriated product streams

Produce using labor, $S_{\tau,t}$

$$V_t \equiv \text{Total project value} \propto \frac{N_t x_t^p}{r + \zeta g}$$

Creator's share $= \frac{V_t^e}{V_t} = \frac{r + \zeta g}{r + \delta + \zeta g} \equiv \theta$
Imitators

Imitators take over expropriated product streams

Produce using labor, $S_{\tau,t}$

$V_t \equiv \text{Total project value} \propto \frac{N_t x_t^p}{r + \zeta g}$

Creator’s share $= \frac{V_t^e}{V_t} = \frac{r + \zeta g}{r + \delta + \zeta g} \equiv \theta$

Imitators’ share $= 1 - \theta$
Labor markets and equilibrium

Free-entry

\[ V_t^e(x_t, N_t) = W_t \]
Labor markets and equilibrium

Free-entry

\[ V_t^e(x_t, N_t) = W_t \]

Labor market clearing

\[ L_e,t + L_{p,t} = 1 \]

#new projects

Result 1 (Balanced growth path)

For any \( \rho \in [0,1] \), if \( \nu \) is sufficiently high, there exists a unique equilibrium where \((x_t, L_e,t)\) are constant and \((S_t, N_t)\) grow at the same constant rate \(g\).
Labor markets and equilibrium

Free-entry

Labor market clearing

\[ V^e_t(x_t, N_t) = W_t \]

\[ L_{e,t} + L_{p,t} = 1 \]

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Result 1 (Balanced growth path)

For any \( \rho \in [0, 1] \), if \( \nu \) is sufficiently high, there exists a unique equilibrium where \((x_t, L_{e,t})\) are constant and \((S_t, N_t)\) grow at the same constant rate \(g\).
2. The Effects of Replicability
The effects of replicability

\[ N_t = \nu \bar{S}_t \]
The effects of replicability

\[ g = n(g; \rho)L_e \]
The effects of replicability

\[ g = n(g; \rho)L_e \]

\[ \rho = 0: \text{ Solow model} \]
The effects of replicability

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The effects of replicability

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\( g = 0 \)

\( \rho = 1: \) Romer model
The effects of replicability

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The effects of replicability

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\( \rho = 0: \) Solow model
\[ n = 0 \]
\[ g = 0 \]

\( \rho = 1: \) Romer model
\[ n = \nu \]
\[ g = \nu L_e \]
Equilibrium growth

\[ L_e(\rho) = L_e(\rho - \delta) = g_n(\rho + \delta, \delta - \rho) = 1 - \zeta \]
Equilibrium growth

"Demand" for projects

\[ L_e(g) = n(g; \rho + \delta) \]

"Supply" of projects (free-entry)

\[ L_s(g) = 1 - \zeta_1 - \zeta_r + \delta + \zeta_g n(g; \rho + \delta) \]
Equilibrium growth

”Demand” for projects

\[ g = n(g; \rho)L_e \]
Equilibrium growth

"Demand" for projects

\[ g = n(g; \rho)L_e \]

\[ L_e^{(d)}(g; \rho) = \frac{g}{n(g; \rho)} \]
Equilibrium growth

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”Supply” of projects (free-entry)
Equilibrium growth

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\[ g = n(g; \rho) L_e \]

\[ L_e^{(d)}(g; \rho) = \frac{g}{n(g; \rho)} \]

"Supply" of projects (free-entry)

\[ L_e^{(s)}(g; \rho, \delta) = 1 - \frac{\zeta (r + \delta + \zeta g)}{1 - \zeta} \frac{1}{n(g; \rho)} \]
Equilibrium growth

"Demand" for projects

\[ g = n(g; \rho) L_e \]

\[ L_e^{(d)}(g; \rho) = \frac{g}{n(g; \rho)} \]

"Supply" of projects (free-entry)

\[ L_e^{(s)}(g; \rho, \delta) = 1 - \frac{\zeta}{1 - \zeta} \frac{r + \delta + \zeta g}{n(g; \rho)} \]
Equilibrium growth

"Demand" for projects

\[ g = n(g; \rho) L_e \]

\[ L_e^{(d)}(g; \rho) = g \frac{n(g; \rho)}{n(g; \rho)_+} \]

"Supply" of projects (free-entry)

\[ L_e^{(s)}(g; \rho, \delta) = 1 - \frac{\zeta}{1 - \zeta} \frac{r + \delta + \zeta g}{n(g; \rho)_+} \]
What happens when $\rho$ increases?
What happens when $\rho$ increases?

\[ \rho \uparrow \implies \hat{g} \]

[Diagram showing the relationship between $L_e^{(d)}$ and $L_e^{(s)}$ with $g$ on the x-axis and $L_e$ on the y-axis.]
What happens when $\rho$ increases?

$\rho \uparrow \implies \uparrow n$
What happens when $\rho$ increases?

\[ \rho \uparrow \implies \uparrow n \implies \downarrow L_e^{(d)}, \uparrow L_e^{(s)} \]
What happens when $\rho$ increases?

\[
\rho \uparrow \implies \uparrow n \implies \downarrow L_e^{(d)}, \; \uparrow L_e^{(s)}
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What happens when \( \rho \) increases?

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\]

\[
\implies \downarrow L_e^{(d)}, \quad \uparrow L_e^{(s)}
\]
What happens when $\rho$ increases?

\[ \hat{g} \uparrow \implies \uparrow n \]

\[ \implies \downarrow L_e^{(d)}, \quad \uparrow L_e^{(s)} \]

\[ \implies \uparrow \hat{g} \]
What happens when $\rho$ increases?

$\rho \uparrow \implies \uparrow n$

$\implies \downarrow L_e^{(d)}, \uparrow L_e^{(s)}$

$\implies \uparrow \hat{g}$
What happens when $\rho$ increases?

\[
\begin{align*}
\rho \uparrow & \implies \uparrow n \\
& \implies \downarrow L_e^{(d)}, \uparrow L_e^{(s)} \\
& \implies \uparrow \hat{g}
\end{align*}
\]

\[
\begin{align*}
\rho \uparrow & \implies \uparrow x \implies \uparrow \delta = \gamma(x)
\end{align*}
\]
What happens when $\rho$ increases?

\[ \rho \uparrow \implies \uparrow n \]
\[ \implies \downarrow L_e^{(d)}, \quad \uparrow L_e^{(s)} \]
\[ \implies \uparrow \hat{g} \]

\[ \rho \uparrow \implies \uparrow x \quad \implies \quad \uparrow \delta = \gamma(x) \]
\[ \implies \downarrow L_e^{(s)} \]
What happens when $\rho$ increases?

$\rho \uparrow \quad \Rightarrow \quad \uparrow n$

$\Rightarrow \quad \downarrow L_e^{(d)}$, $\uparrow L_e^{(s)}$

$\Rightarrow \quad \uparrow \hat{g}$

$\rho \uparrow \quad \Rightarrow \quad \uparrow x \quad \Rightarrow \quad \uparrow \delta = \gamma(x)$

$\Rightarrow \quad \downarrow L_e^{(s)}$

$\Rightarrow \quad \downarrow \hat{g}$
The effects of replicability

Growth rate $g$

Replicability $\rho$
The effects of replicability

Growth rate $g$

Spillover intensity $n$

Replicability $\rho$
The effects of replicability

Growth rate $g$

Optimal span $x$

Replicability $\rho$
The effects of replicability

Growth rate $g$

Expropriation risk $\delta = \gamma(x)$
The effects of replicability

Growth rate $g$

Share retained by creators $\theta$

Replicability $\rho$
The effects of replicability

Growth rate $g$

Entry value $n \theta \left( \frac{V_t}{N_t} \right)$

[excludability]
When is there an inverse-U shaped relationship?

\[ \gamma(z) \equiv \frac{1}{\lambda} (z - 1)^{1+\alpha} \]
When is there an inverse-U shaped relationship?

\[ \gamma(z) \equiv \frac{1}{\lambda}(z - 1)^{1+\alpha} \implies \delta(\lambda) \]
When is there an inverse-U shaped relationship?

\[ \gamma(z) \equiv \frac{1}{\lambda} (z - 1)^{1+\alpha} \quad \Rightarrow \quad \delta(\lambda) \]

Result 2 (Non-monotonicity)

There exists \( \lambda \) such that for all \( \lambda \geq \lambda \), growth is maximized at \( \hat{\rho} \in (0, 1) \).
When is there an inverse-U shaped relationship?

\[ \gamma(z) \equiv \frac{1}{\lambda} (z - 1)^{1+\alpha} \implies \delta(\lambda) \]

Result 2 (Non-monotonicity)

There exists \( \lambda \) such that \( \forall \lambda \geq \lambda \), growth is maximized at \( \hat{\rho} \in (0, 1) \).

When \( \lambda \) is large enough, spillovers to imitators \( \gg \) spillovers to new firms at \( \rho = 1 \).
When is there an inverse-U shaped relationship?

The graph shows the relationship between spillover intensity $\nu$ and limits to excludability $\lambda$. The shaded regions represent different conditions:

- $\hat{\rho} = 1$
- $\hat{\rho} \in (0, 1)$
- No BGP

The axes are labeled as follows:

- Spillover intensity $\nu$
- Limits to excludability $\lambda$
Returns to capital and Tobin’s Q

\[ V_t = V_t^c + (1 - \theta) V_t \]

\( V_t^c \) creators

\( (1 - \theta) V_t \) imitators
Returns to capital and Tobin’s Q

\[ V_t = V^e_t + (1 - \theta)V_t \]

Transfers to capital

\[ Y_t = W_t L_t + R_{N,t} \times (p_{N,t} N_{tot,t}) + (1 - \theta)Y_t \]
Returns to capital and Tobin’s Q

\[ V_t = \left( V_t^c \right)_{\text{creators}} + \left( 1 - \theta \right) V_t \]

Returns to capital

\[ Y_t = \left( W_t L_t \right)_{\text{labor}} + R_{N,t} \times \left( p_{N,t,\tilde{N}_{tot,t}} \right) + (1 - \theta) Y_t \]

Tobin’s Q

\[ Q_t^c \equiv \frac{V_t^c}{p_{N,t,\tilde{N}_{tot,t}}} = 1 \]

\[ Q_t \equiv \frac{V_t}{p_{N,t,\tilde{N}_{tot,t}}} = \frac{1}{\theta} > 1 \]
Returns to capital and valuations

Growth rate $g$

Pure profit share

Replicability $\rho$
Returns to capital and valuations

Growth rate $g$

Aggregate $Q$

Replicability $\rho$
Concentration

Sales share for project $i$

$$s_{i,t} = n \times e^{-g(t - \tau(i))}$$

Stronger spillovers ($n$) makes the relative size of new projects larger.
Concentration

Sales share for project $i$

$$s_{i,t} = n \times e^{-g(t - \tau(i))}$$

Stronger spillovers ($n$) makes the relative size of new projects larger

Herfindhal of sales across projects

$$H_t = \int_{\tau(i) \leq t} s_{i,t}^2 di = \frac{n}{2}$$
Concentration

Growth rate $g$

Equilibrium concentration

- Among projects
- Among entrepreneurs

Replicability $\rho$

Non-rivalry $\rho$
Conclusion
Q: Unlike $K$, intangible assets are replicable. Does that matter for growth?

scale economies + spillovers to future entrants vs. spillovers to competitors

$\implies$ non-monotonic relationship btw. $\rho$ and growth

Next:

Transitional dynamics

Estimation of $(\rho, \delta)$

Implications of replicability for capital structure and for firm boundaries
The ratio of $N/K$
Output growth $dY/Y$ and intangible capital growth $dN/N$
Intangible intensity and concentration

Crouzet, Eberly, 2019

[Crouzet, Eberly, 2019]
# Intangible intensity and market share

<table>
<thead>
<tr>
<th></th>
<th>Market share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
</tr>
<tr>
<td><strong>Compustat intangible share</strong></td>
<td>0.1308***</td>
</tr>
<tr>
<td></td>
<td>(17.69)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>98520</td>
</tr>
<tr>
<td><strong>Industry × year f.e.</strong></td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Firm f.e.</strong></td>
<td>No</td>
</tr>
<tr>
<td><strong>Year f.e.</strong></td>
<td>No</td>
</tr>
</tbody>
</table>

[Crouzet, Eberly, 2019]
# Examples

<table>
<thead>
<tr>
<th></th>
<th>Storage Medium</th>
<th>Property-Rights Institution</th>
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<tbody>
<tr>
<td>Patents</td>
<td>Patent application</td>
<td>Patent system</td>
</tr>
<tr>
<td>Software</td>
<td>Computers</td>
<td>Copyright system</td>
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<tr>
<td>Production/distribution systems</td>
<td>Key talent, manuals</td>
<td>Non-compete clauses, trade secrets</td>
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<tr>
<td>Brands</td>
<td>Consumers</td>
<td>Trademark system (c)</td>
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<tr>
<td>Video and audio material</td>
<td>Audiovisual media</td>
<td>Copyright system</td>
</tr>
<tr>
<td>Franchise agreements</td>
<td>Contract</td>
<td>Contract enforcement</td>
</tr>
<tr>
<td>Customer lists</td>
<td>Digital media</td>
<td>Contract enforcement</td>
</tr>
</tbody>
</table>
The effects of excludability

Growth rate $g$

Spillover intensity $n$

Limits to excludability $\delta_0$
The effects of excludability

Growth rate $g$

Expropriation risk $\delta$

Limits to excludability $\delta_0$
The effects of excludability

Growth rate $g$

Share retained by creators $\theta$

Limits to excludability $\delta_0$

Limits to excludability $\delta_0$
The effects of excludability