Intangible capital, non-rivalry, and growth

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Introduction

Intangible assets are an important factor of production
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- IT-related assets [software, databases]
- Intellectual property assets [patents, trademarks]
- Organization capital [managerial know-how, production processes]
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**Question:** What is special about intangible assets, relative to physical assets?
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1. Intangibles are simply hard to identify and measure?
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1. Intangibles are simply hard to identify and measure?
2. Intangibles have distinct economic characteristics?
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[software, databases]  [patents, trademarks] [managerial know-how, production processes]

Question: What is special about intangible assets, relative to physical assets?

1. Intangibles are simply hard to identify and measure?
2. Intangibles have distinct economic characteristics?

This paper: Model emphasizing 2, with an application to long-run growth
Key characteristics of intangible assets

- **Non-rivalry** ($\rho$): Can use multiple copies of the asset at the same time.
  - Technology determines how easy reproduction is — e.g. writing vs. digital.

- **Limited excludability** ($\delta$): The asset only has private value if its use by others can be restricted.
  - Property rights determine how easy exclusion is — e.g. patent system.

Different types of intangible assets ↔ different ($\rho$, $\delta$)
Key characteristics of intangible assets

Knowledge assets that can be **non-rival**, but **hard to exclude**.
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Non-rivalry

Limited excludability
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Different types of intangible assets $\leftrightarrow$ different $(\rho, \delta)$
Classifying intangibles

Limited Excludability ($\delta$)

Non-rivalry ($\rho$)
Classifying intangibles

- Limited Excludability ($\delta$)
- Non-rivalry ($\rho$)

Physical Capital ($K$)

TFP ($Z$)

Patented drug Database

Production process
Classifying intangibles

Limited Excludability ($\delta$)

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Physical Capital (K)

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Database
Classifying intangibles

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- Physical Capital (K)
- TFP (Z)
- Production process
- Database
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- 0
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Long-run growth implications

Q: How does growth change when $\uparrow \rho$?

Replication of knowledge assets becomes easier
Oral $\rightarrow$ Writing $\rightarrow$ Digital ....

A form of technological ($\rho$) change, keeping property rights ($\delta$) fixed

Bench marks:
Physical capital $\rho = 0$ (fully rival) $\rightarrow$ no growth (Solow)
Aggregate TFP $\rho = 1$ (fully non-rival) $\rightarrow$ perpetual growth (Romer)

Naive

A: Long-run growth increases with $\rho$.

No!

Real

A: Non-monotonic relationship between $\rho$ and growth
Long-run growth implications

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Real A: Non-monotonic relationship between $\rho$ and growth
Intan $\sim K$

Solow model
Growth rate \( g \)

Non-rivalry \( \rho \)

Intan \( \sim K \) Solow model

Intan \( \sim Z \) Romer model
Main Mechanism

$\uparrow \rho$

Competing forces:
- Entrants are larger, have more intangibles to build on ($\uparrow$ incentive to enter)
- Entrants appropriate lower share of surplus created ($\downarrow$ incentive to enter)

Implications:
- $\uparrow$ profits, valuations, concentration
- $\downarrow$ entry and investment

Why is this interesting?
Main Mechanism

$\uparrow \rho \implies$

- Scale economies
- Spillovers to future entrants
- Spillovers to existing competitors

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\[ \rho \uparrow \Rightarrow \begin{cases} 
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\[ \uparrow \rho \implies \left\{ \begin{array}{l} \uparrow \text{scale economies} \\ \uparrow \text{spillovers to future entrants} \\ \uparrow \text{spillovers to existing competitors} \end{array} \right. \]

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Why is this interesting?
Market valuations have increased

\[ Q_{K,t} = \frac{V_t}{K_{t+1}} \quad \text{(NFCB)} \]

\[ Q_{K,t} = \frac{V_t}{K_{t+1}} \quad \text{(Compustat)} \]
Corporate profits as a share of GDP have increased
Concentration has been increased

Compustat; NAICS-3D sectors weighted by sales.
New entry has declined

Figure 4 shows entrepreneurship rates in the high-tech and the private sector as a whole. The entrepreneurship rate is defined as the number of startups and young firms (up to five years old) over the total number of firms. The entrepreneurship rate in the high-tech sector has declined significantly despite the actual increase in absolute numbers during the same period. The high-tech entrepreneurship rate fell from a high of nearly 60 percent in 1982 to a low of 38 percent by 2011. However, the decline has not been monotonic, with a rise in the entrepreneurship rate in the second half of the 1990s, which was followed by the dot-com bust. Perhaps even more relevant is the continued decline in the entrepreneurship rate in the post-2002 period. The latter occurs at a pace that even exceeds the decline in entrepreneurship for the private sector as a whole during the same period.

Why entrepreneurial activity has been so anemic in the high-tech sector post-2002 is an open question. The overall economy has been exhibiting a declining trend in entrepreneurial activity over a much longer period, but now, even the highly dynamic and entrepreneurial high-tech sector is becoming less so.

The patterns for the overall economy are consistent with recent findings for the whole economy by Decker, Haltiwanger, Jarmin, and Miranda (2014), "Entrepreneurship and Job Creation in the U.S.," in process. We also have found the patterns of Figure 4 by examining the share of employment accounted for by young firms.

Anecdotal and empirical evidence suggests that high-tech entrepreneurship may have experienced a rebound in the years since our data were collected in March 2011. See for example: PricewaterhouseCoopers (2013), MoneyTree Report, Historical Trend Data; CB Insights (2013), Venture Capital Activity Report; Silicon Valley Bank, Angel Resource Institute, and CB Insights (2013), 2012 Halo Report: Angel Group Activity Year in Review; Silicon Valley Bank, Angel Resource Institute, and CB Insights (2013), Halo Report: Angel Group Update: Q3 2013; Silicon Valley Bank (2012, 2013), Startup Outlook.

[Haltiwanger, Hathaway, Miranda, 2014]
Related literature

Macro and financial implications of rising intangibles

Contribution: formalize non-rivalry and limited excludability

Endogenous technological change
Lucas and Moll (2014), Stokey (2015); Jones and Tonetti (2020), Farboodi and Veldkamp (2022)

Contribution: non-rivalry facilitates imitation; not limited to data

Competition and returns to innovation
Aghion, Bloom, Blundell, Griffith, Howitt (2005), Aghion, Bergeaud, Boppart, Klenow, Li (2022)

Contribution: non-rivalry creates both returns to scale and competitive risk
Roadmap

1. Economic environment

2. The effects of non-rivalry on growth

3. Other macro implications
1. Economic environment
Overview

Household

- Production labor

Entrepreneur (E)

- Entrepreneurial labor

Imitators

Projects

Consumption goods

Spillovers

\[ \text{project} = \{ \text{product streams} \in [0, x_t] \} \times x_t \]
Overview

Household ➔ Production labor ➔ Entrepreneur (E) ➔ Imitators

Consumption goods ➔ Entrepreneur (E)

Entrepreneurial labor ➔ Imitators

Projects ➔ Spillovers ➔ Entrepreneur (E)

Project = {product streams ∈ [0, x_t]}
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Projects

project = \{ \text{product streams } s \in [0, x_i] \}

x_i: project "span"
Allocating intangible capital within a project

\[
\Pi(x_t, N_t) = \max_{\{N(s), L(s)\}, L_t} \int_0^{x_t} N(s)^{1-\zeta} L(s)^{\zeta} ds - W_t L_t
\]
Allocating intangible capital within a project

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s.t. \[\int_0^{x_t} L(s) ds \leq L_t\]
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\]

s.t.

\[
\int_0^{x_t} L(s) ds \leq L_t
\]

\[
\left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} ds \right)^{1-\rho} \leq N_t \quad \rho \in [0, 1]
\]
Allocating intangible capital within a project

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\[
\text{s.t. } \int_0^{x_t} L(s) ds \leq L_t
\]

\[
\left(\int_0^{x_t} N(s)^{\frac{1}{1-\rho}} ds\right)^{1-\rho} \leq N_t \quad \rho \in [0, 1]
\]

\[
\Pi(x_t, N_t) \propto x_t^\rho N_t
\]
What does $\rho$ capture?

\[
\left( \int_0^{x_t} N(s) \frac{1}{1-\rho} \, ds \right)^{1-\rho} \leq N_t
\]
What does $\rho$ capture?

$$\left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} \, ds \right)^{1-\rho} \leq N_t$$

$\rho = 0$
What does $\rho$ capture?

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\int_0^{x_t} N(s) \, ds \leq N_t
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$\rho = 0$
What does $\rho$ capture?

$$\int_0^{x_t} N(s) \, ds \leq N_t$$

$\rho = 0$

increasing $N(s)$ requires reducing $N(-s)$ one-for-one
What does $\rho$ capture?

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$N$ is rival within the project
What does $\rho$ capture?

$$\int_0^{x_t} N(s) \, ds \leq N_t$$

$\rho = 0$

increasing $N(s)$ requires reducing $N(-s)$ one-for-one

$N$ is rival within the project

e.g. machines, structures
What does $\rho$ capture?

\[
\left( \int_0^{x_t} \frac{N(s)}{1-\rho} \, ds \right)^{1-\rho} \leq N_t
\]

$\rho = 1$
What does \( \rho \) capture?

\[
\max_{s \in [0, x_t]} N(s) \leq N_t
\]

\( \rho = 1 \)
What does $\rho$ capture?

$$\max_{s \in [0, x_t]} N(s) \leq N_t$$

$\rho = 1$

increasing $N(s)$ doesn’t require reducing $N(-s)$ at all

$N$ is non-rival within the project
What does $\rho$ capture?

$$\max_{s \in [0,x_t]} N(s) \leq N_t$$

$\rho = 1$

increasing $N(s)$ doesn’t require reducing $N(-s)$ at all

$N$ is non-rival within the project

e.g. a patent for a touchscreen

using it for one product not reduce its availability for other products
What does $\rho$ capture?

$$\max_{s \in [0, x_t]} N(s) \leq N_t$$

$\rho \in (0, 1)$
What does $\rho$ capture?

$$\left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} \, ds \right)^{1-\rho} \leq N_t$$

$\rho \in (0, 1)$
What does $\rho$ capture?

$$\left( \int_{0}^{x_t} N(s) \frac{1}{1-\rho} \, ds \right)^{1-\rho} \leq N_t$$

$\rho \in (0, 1)$

increasing $N(s)$ requires reducing $N(-s)$, but less than one-for-one

$N$ is imperfectly rival within the project
What does $\rho$ capture?

$$\left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} ds \right)^{1-\rho} \leq N_t$$

$\rho \in (0, 1)$

increasing $N(s)$ requires reducing $N(-s)$, but less than one-for-one.

$N$ is imperfectly rival within the project.

e.g. an inventory management process for an online retailer.

implementation in a new warehouse may be imperfect.

it may require allocating.
What does $\rho$ capture?

$$
\left( \int_0^{x_t} N(s)^{\frac{1}{1-\rho}} ds \right)^{1-\rho} \leq N_t
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$\rho \in (0, 1)$

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non-rivalry of intangibles ($\rho$) ↔ returns to scale
What does $\rho$ capture?

\[
\left( \int_0^{x_t} N(s) \frac{1}{1-\rho} \, ds \right)^{1-\rho} \leq N_t
\]

non-rivalry of intangibles ($\rho$) $\leftrightarrow$ returns to scale

$\Pi_t \propto x_t^\rho N_t$

if $\rho > 0$, $N_t$ raises marginal returns to $x_t$
Imperfect excludability and spillovers

Imitators progressively appropriate streams initially created by $E$.

Imperfect excludability: $E$ loses each stream $\sim \delta dN_t = \Rightarrow dN_t = -\sim \delta (1 - \rho) N_t dt$

Spillovers: Spillovers $S_t = \text{intangibles in expropriated streams}$

Initial intangible stock $= N_t = (N_1 - \rho t + S_t 1 - \rho)^{1 - \rho}$
Imperfect excludability and spillovers

Imitators progressively appropriate streams initially created by $E$
Imperfect excludability and spillovers

Imitators progressively appropriate streams initially created by $E$

Imperfect excludability: $E$ loses each stream $\delta dt$
$N(s)$

Streams $s$ owned by E
$N(s)$

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$1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ I$
Streams $s$ owned by $E$

$N(s)$
Imperfect excludability and spillovers

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Imperfect excludability: $E$ loses each stream $\tilde{\delta} dt$

$$dx_t = -\tilde{\delta} x_t dt$$

$$\implies dN_t = -\tilde{\delta}(1 - \rho) N_t dt$$
Imperfect excludability and spillovers

Imitators progressively appropriate streams initially created by $E$

**Imperfect excludability**: $E$ loses each stream $\tilde{\delta} dt$

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**Spillovers**: Spillovers $S_t = \text{intangibles in expropriated streams}$
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dx_t = -\tilde{\delta} x_t dt
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\[
\implies dN_t = -\tilde{\delta}(1 - \rho) N_t dt
\]

**Spillovers:** Spillovers $S_t = \text{intangibles in expropriated streams}$

Initial intangible stock $= N_\tau = \left( N_t^{\frac{1}{1-\rho}} + S_t^{\frac{1}{1-\rho}} \right)^{1-\rho}$
\(\rho\) determines how fast spillovers accumulate.
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determines how fast spillovers accumulate
\( \rho \) determines how fast spillovers accumulate
New projects

Initial span: $x_\tau$. 
New projects

Initial span: $x_{\tau}$. Assume:

$$\tilde{\delta} = \delta(x_{\tau}), \quad \delta \text{ increasing and (sufficiently) convex.}$$
New projects

Initial span: $x_\tau$. Assume:

$\tilde{\delta} = \delta(x_\tau), \quad \delta$ increasing and (sufficiently) convex.

Value of project to E:

$$V^e_\tau(N_\tau) \propto \max_{x_\tau} \frac{N_\tau x_\tau^p}{r + \delta(x_\tau) - (-\zeta g)}$$
New projects

Initial span: $x_\tau$. Assume:

\[ \tilde{\delta} = \delta(x_\tau), \quad \delta \text{ increasing and (sufficiently) convex}. \]

Value of project to E:

\[
V^e_{\tau}(N_\tau) \propto \max_{x_\tau} \frac{N_\tau x_\tau^p}{r + \delta(x_\tau) - (-\zeta g)} \quad \text{(scale)}
\]

\[
\quad \text{(limited excludability)}
\]
New projects

Initial span: $x_{\tau}$. Assume:

\[
\tilde{\delta} = \delta(x_{\tau}), \quad \delta \text{ increasing and (sufficiently) convex}.
\]

Value of project to E:

\[
V_e^e(N_{\tau}) \propto \max_{x_{\tau}} \frac{N_{\tau} x_{\tau}^p}{r + \delta(x_{\tau}) - (-\zeta_g)} \quad \text{(scale)} \quad \text{(limited excludability)}
\]
New projects

Initial span: $x_\tau$. Assume:

$$\tilde{\delta} = \delta(x_\tau), \quad \delta \text{ increasing and (sufficiently) convex.}$$

Value of project to E:

$$V^e_\tau(N_\tau) \propto \max_{x_\tau} \frac{N_\tau x_\tau^p}{r + \delta(x_\tau) - (-\zeta g)} \quad \text{(scale)} \quad \text{(limited excludability)}$$

New project requires 1 unit of labor, and starts with intangible stock:
New projects

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New project requires 1 unit of labor, and starts with intangible stock:

$$N_\tau = v \int_{\tau(i) \leq \tau} S_{i,\tau} di$$

$$\tilde{S}_\tau$$
Imitators take over expropriated product streams from a particular entrepreneur
Imitators

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Produce using labor, and existing stock of spillovers
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\[ V_\tau \equiv \text{Total project value} \propto \frac{N_\tau x_\tau^p}{r - (-\zeta g)} \]
Imitators

Imitators take over expropriated product streams from a particular entrepreneur

Produce using labor, and existing stock of spillovers

\[ V_\tau \equiv \text{Total project value} \propto \frac{N_\tau x_\tau^0}{r - (-\zeta_g)} \]

Entrepreneur’s share

\[
\frac{V^e_\tau}{V_\tau} = \frac{r + \zeta_g}{r + \bar{\delta} - (-\zeta_g)} \equiv \theta
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Imitators’ share

\[ 1 - \theta \]
Labor markets and equilibrium

Free-entry

\[ V_t^e(x_t, N_t) = W_t \]
Labor markets and equilibrium

Free-entry

\[ V^e_t(x_t, N_t) = W_t \]

Labor market clearing

\[ L_{e,t} + L_{p,t} = 1 \]

#new projects
Labor markets and equilibrium

Free-entry

$$ V^e_t(x_t, N_t) = W_t $$

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#new projects

Result 1 (Balanced growth path)

For any \( \rho \in [0, 1] \), if \( \nu \) is sufficiently high, there exists a unique equilibrium where \((x_t, L_{e,t})\) are constant and \((S_t, N_t)\) grow at the same constant rate \(g\).
2. The Effects of Non-Rivalry
The effects of non-rivalry

\[ N_t = \nu \bar{S}_t \]

\[ g = \frac{n(g; \rho)}{L_e} \times \text{Return to Investment} \]

\[ \text{Investment} \]
The effects of non-rivalry

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\( \rho = 0: \) Solow model
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  - \( n = 0 \)
  - \( g = 0 \)

- \( \rho = 1: \) Romer model
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\( n = 0 \)
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\( \rho = 1: \) Romer model
\( n = \nu \)
The effects of non-rivalry

\[ N_t = \nu \bar{S}_t \]

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\( \rho = 0: \text{Solow model} \)

\[ n = 0 \]

\[ g = 0 \]

\( \rho = 1: \text{Romer model} \)

\[ n = \nu \]

\[ g = \nu L_e \]
The effects of non-rivalry

Growth rate $g$

Optimal span $x$

Non-rivalry $\rho$
The effects of non-rivalry

**Growth rate** $g$

**Spillover intensity** $n$
The effects of non-rivalry

Growth rate $g$

Expropriation risk $\delta$

Non-rivalry $\rho$
The effects of non-rivalry
The effects of non-rivalry

Growth rate $g$

Entrepreneurial labor $L_e$
When is there an inverse-U shaped relationship?

\[ \delta(z) \equiv \frac{1}{\lambda} (z - 1)^{1+\alpha} \]
When is there an inverse-U shaped relationship?

\[ \delta(z) \equiv \frac{1}{\lambda}(z - 1)^{1+\alpha} \quad \Rightarrow \quad \delta(\lambda) \]
When is there an inverse-U shaped relationship?

\[
\delta(z) \equiv \frac{1}{\lambda} (z - 1)^{1+\alpha} \quad \implies \quad \tilde{\delta}(\lambda)
\]

Result 2 (Non-monotonicity)

There exists \( \lambda \) such that \( \forall \lambda \geq \lambda, \) growth is maximized at \( \hat{\rho} \in (0, 1). \)
When is there an inverse-U shaped relationship?

\[ \delta(z) \equiv \frac{1}{\lambda} (z - 1)^{1+\alpha} \implies \tilde{\delta}(\lambda) \]

Result 2 (Non-monotonicity)
There exists \( \lambda \) such that \( \forall \lambda \geq \lambda \), growth is maximized at \( \hat{\rho} \in (0, 1) \).

When \( \lambda \) is large enough, spillovers to imitators \( \gg \) spillovers to new firms at \( \rho = 1 \)
When is there an inverse-U shaped relationship?
3. Model Implications
Valuations and profits

Valuations

\[ V_t = V^e_t + (1 - \theta) V_t \]

creators

imitators

Profits

\[ Y_t = \text{labor} W_t L_t + \text{capital} R_{N, t} \times (p_{N, t \text{ tot}}, t) + (1 - \theta) Y_t \]
Valuations and profits

Valuations

\[ V_t = V^e_t + (1 - \theta) V_t \]

\[ Q^e_t \equiv \frac{V^e_t}{p_{N,t} \overline{N}_{tot,t}} = 1 \]

\[ Q_t \equiv \frac{V_t}{p_{N,t} \overline{N}_{tot,t}} = \frac{1}{\theta} > 1 \]
Valuations and profits

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Profits

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Valuations and profits in the model

Growth rate $g$

Aggregate $Q$
Valuations and profits in the model

Growth rate $g$

Pure profit share
Concentration

Sales share for project $i$

$$s_{i,t} = n \times e^{-g(t - \tau(i))}$$

Stronger spillovers ($n$) makes the relative size of new projects larger
Concentration

Sales share for project $i$

\[ s_{i,t} = n \times e^{-g(t - \tau(i))} \]

Stronger spillovers ($n$) makes the relative size of new projects larger

Herfindhal of sales across projects

\[ H_t = \int_{\tau(i) \leq t} s_{i,t}^2 di = \frac{n}{2} \]
Concentration

Growth rate $g$

Equilibrium concentration

- Among projects
- Among entrepreneurs

Non-rivalry $\rho$
The effects of excludability

Growth rate $g$

Expropriation risk $\delta$

Limits to excludability $\delta_0$
The effects of excludability

Growth rate $g$

Share retained by creators $\theta$

Limits to excludability $\delta_0$
The effects of excludability

Growth rate $g$

Spillover intensity $n$

Limits to excludability $\delta_0$
The effects of excludability

Growth rate $g$

Entrepreneurial labor $L_e$
Conclusions

Q: Intangibles can be non-rival within firm. Does that matter for growth?

- Scale + spillovers to new firms vs. spillovers to imitators
- Non-monotonic relationship btw. \( \rho \) and growth

Next:

- Transitional dynamics
- Estimation of \((\rho, \delta)\)