Rents and Intangible Capital: A Q+ Framework

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\textsuperscript{1}Kellogg and Chicago Fed
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Question

Why is PPE investment weak?
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PPE investment is weak

Sectoral data
Question

Why is PPE investment weak?

- ... despite high returns?
PPE investment is weak despite high returns
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- rents reduce the incentive to increase scale at the margin
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Explanation 2: *intangibles* (Crouzet and Eberly, 2018, 2019)
The growing importance of intangible capital

$K_2/K_1$ (Compustat, aggregate) \quad \dashdot \quad K_2/K_1$ (BEA)

$K_1 = $ PPE and $K_2 = $ R&D capital.
Question

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Explanation 1: economic rents (Barkai, 2017; Gutierrez and Philippon, 2018)

- rents reduce the incentive to increase scale at the margin

Explanation 2: intangibles (Crouzet and Eberly, 2018, 2019)

- omitting intangibles biases upward measured returns
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Why care? Explanation 1 has strong *policy* implications
This paper

What we do:
- Neo-classical investment model with rents + intangibles
- Quantify role of each in aggregate and sectoral data

What we find:
- "Investment gap" ≡ $Q - q = \text{Rents} \rightarrow \text{physical capital} + \text{Omitted capital effect} + (\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect})$
- Intangibles account for 1/3 of $Q - q$ in aggregate
- Large sectoral heterogeneity (2/3 in Health and Tech) ⇒ no macro story or remedy
- Increase in rents: $\Delta s \leq 0.06, \text{vs. } \Delta s \approx 0.12$ in existing work
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  + Omitted capital effect
  + $(\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect})$
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Related literature

1. Aggregate implications of rising rents:

   This paper: investment-\( Q \); new approach for estimating of rents; sectoral heterogeneity

2. Q theory and firm value:

   This paper: general decomposition of \( Q - q \), including market power
1. Theory
A (fairly) general $Q$-theory model
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A firm chooses investment according to:

$$V^c_t (K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t \left[ M_{t,t+1} V^c_{t+1} (K_{t+1}) \right]$$

s.t. $K_t = F_t(K_t), \quad K_t = \left\{ K_{n,t} \right\}_{n=1}^N$
A (fairly) general $Q$-theory model

A firm chooses investment according to:

$$V_c^c(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t \left[ M_{t,t+1} V_{t+1}^c(K_{t+1}) \right]$$

s.t. $K_t = F_t(K_t)$, $K_t = \left\{ K_{n,t} \right\}_{n=1}^N$

- $F_t(.)$ is homogeneous of degree 1
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- $F_t(.)$ is homogeneous of degree 1
- $\Pi_t(.)$ is homogeneous of degree $\frac{1}{\mu}, \quad \mu \geq 1$
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subject to:

$$K_t = F_t(K_t), \quad K_t = \left\{K_{n,t}\right\}_{n=1}^{N}$$

- $F_t(.)$ is homogeneous of degree 1
- $\Pi_t(.)$ is homogeneous of degree $\frac{1}{\mu}$, $\mu \geq 1$
- Adjustment costs satisfy:

$$\tilde{\Phi}_t(K_t, K_{t+1}) = \sum_{n=1}^{N} \Phi_{n,t} \left(\frac{K_{n,t+1}}{K_{n,t}}\right) K_{n,t}, \quad \Phi_{n,t} \text{ increasing and convex.}$$
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How general is this model?
The value of the firm

Lemma (Firm value decomposition)

\[ V^e_t = \sum_{n=1}^{N} q_{n,t} K_{n,t+1} + \sum_{n=1}^{N} \sum_{k \geq 1} E_t [M_{t,t+k} (\mu - 1) \Pi_{n,t+k} K_{n,t+k}] \]

where:

\[ V^e_t = E_t [M_{t,t+1} V^c_{t+1}] , \quad q_{n,t} \equiv \frac{\partial V^e_t}{\partial K_{n,t+1}} , \quad \Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_t} \frac{\partial K_t}{\partial K_{n,t}} . \]
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- \( \mu = 1 \):
  \[ V_t^e = \sum_n q_{n,t} K_{n,t+1} \]
  (Hayashi and Inoue, 1991)
The value of the firm

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where:

\[ V_t^e = \mathbb{E}_t [M_{t,t+1}V_{t+1}^c], \quad q_{n,t} \equiv \frac{\partial V_t^e}{\partial K_{n,t+1}}, \quad \Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_t} \frac{\partial K_t}{\partial K_{n,t}}. \]

- \( \mu = 1 \):
  \( V_t^e = \sum_n q_{n,t}K_{n,t+1} \) \hspace{1cm} (Hayashi and Inoue, 1991)

- \( \mu > 1 \):
  \( V_t^e = \sum_n q_{n,t}K_{n,t+1} + \text{rents} \) \hspace{1cm} (Lindenberg and Ross, 1981)
## The value of the firm

### Lemma (Firm value decomposition)

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V_t^e = \sum_{n=1}^{N} q_{n,t} K_{n,t+1} + \sum_{n=1}^{N} \sum_{k \geq 1} E_t \left[ M_{t,t+k} (\mu - 1) \Pi_{n,t+k} K_{n,t+k} \right]
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where:

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\[
(\mu - 1) \Pi_{n,t} = \left( \frac{\Pi_t}{K_t} - \frac{\partial \Pi_t}{\partial K_t} \right) \times \frac{\partial K_t}{\partial K_{n,t}}
\]

*gap btw. average and marginal product*
The investment gap

- The **investment gap** is the gap between average $Q$ and marginal $q$:

$$G_{n,t} \equiv Q_{n,t} - q_{n,t},$$

$$Q_{n,t} \equiv \frac{V^e_t}{K_{n,t+1}}, \quad q_{n,t} \equiv \frac{\partial V^e_t}{\partial K_{n,t+1}}.$$
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- With quadratic adjustment costs:

$$i_{n,t} = \delta_{n,t+1}^n + \gamma_{n,t}(q_{n,t} - 1) = \delta_{n,t+1}^n + \gamma_{n,t}(Q_{n,t} - 1) - 1 \gamma_{n,t} G_{n,t},$$

Investment is low relative to $Q$ iff $G_{n,t} > 0$
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\]

Investment is low relative to \( Q \) **iff** \( G_{n,t} > 0 \)
A decomposition of the investment gap

Lemma

\[ G_{n,t} = 0 \]

\( N = 1, \quad \mu = 1: \) no investment gap, \( Q = q \)  

(Hayashi, 1982)
A decomposition of the investment gap

Lemma

\[ G_{n,t} = (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+k})] \]

where:

\[ g_{n,t+k} = \frac{K_{n,t+k}}{K_{n,t+1}}, \]

\( N = 1, \mu > 1: \) investment gap due to rents

\( \text{(Lindenberg and Ross, 1981)} \)
A decomposition of the investment gap

Lemma

\[ G_{n,t} = \sum_{m \neq n} q_{m,t} S_{m,n,t+1} \]

where:

\[ S_{m,n,t+1} = \frac{K_{m,t+1}}{K_{n,t+1}} \]

\( N > 1, \mu = 1: \) investment gap due to omitted capital

(Hayashi, 1991)
A decomposition of the investment gap

Lemma

\[
G_{n,t} = (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+k})]
\]

\[
+ \sum_{m \neq n} q_{m,t} S_{m,n,t+1}
\]

where: \( g_{n,t+k} = \frac{K_{n,t+k}}{K_{n,t+1}} \), \( S_{m,n,t+1} = \frac{K_{m,t+1}}{K_{n,t+1}} \)

\[N > 1, \quad \mu > 1:\]
A decomposition of the investment gap

Lemma

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G_{n,t} = (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+k})]
+ \sum_{m \neq n} q_{m,t} S_{m,n,t+1}
+ (\mu - 1) \sum_{m \neq n} \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{m,t+k}(1 + g_{m,t+k})] S_{m,n,t+1}
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where: \( g_{n,t+k} = \frac{K_{n,t+k}}{K_{n,t+1}} \), \( S_{m,n,t+1} = \frac{K_{m,t+1}}{K_{n,t+1}} \)

\( N > 1, \mu > 1 \): additional term: rents \( \times \) omitted capital
Analytical example (1/2)

Assumption 1: Adjustment costs are given by:

$$\Phi_{n,t}\left(\frac{K_{n,t+1}}{K_{n,t}}\right)K_{n,t} = K_{n,t+1} - (1 - \delta_n)K_{n,t} + \gamma_n\Gamma\left(\frac{K_{n,t+1}}{K_{n,t}}\right)K_{n,t},$$

$$\Gamma(1) = 0, \quad \Gamma'(1) = 0, \quad \Gamma''(1) = 1.$$

$n = 1$: physical capital

$n = 2$: intangible capital

Assumption 2: The profit function is:

$$\Pi_t = A_{t}^{\frac{1}{\mu}} K_{t}^{\frac{1}{\mu}}, \quad \frac{A_{t+1}}{A_{t}} = 1 + g.$$

Additionally, $M_{t,t+1} = (1 + r)^{-1}$ for some $r > g$. 
Analytical example (2/2)

\[ G_{1,t} = (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{1,t+k}(1 + g_{1,t+k})] \]  
(Rents \to \text{physical capital})

+ \quad q_{2,t} S_{t+1} \]  
(Ommitted capital effect)

+ \quad (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{2,t+k}(1 + g_{2,t+k})] S_{t+1} \]  
(Rents \to \text{intangibles}) \times 
(Ommitted capital effect)
Analytical example (2/2)

\[ G_{1,t} = (\mu - 1) \sum_{k \geq 1} E_t [M_{t,t+k} \Pi_1, t+k (1 + g_{1,t+k})] \]  

(Rents → physical capital)

\[ \text{+ } q_{2,t} S_{t+1} \]  

(Ommitted capital effect)

\[ \text{+ } (\mu - 1) \sum_{k \geq 1} E_t [M_{t,t+k} \Pi_2, t+k (1 + g_{2,t+k})] S_{t+1} \]  

(Rents → intangibles) \times  

(Ommitted capital effect)

Optimal investment requires:

\[ q_{n,t} = 1, \quad n = 1, 2, \]

when adjustment costs are linear (\(\gamma_n = 0\)).
Analytical example (2/2)

\[ G_{1,t} = (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{1,t+k} (1 + g_{1,t+k})] \]  
\[ + S_{t+1} \]  
\[ + (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{2,t+k} (1 + g_{2,t+k})] S_{t+1} \]  

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(Rents → physical capital)  
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(Ommitted capital effect)

The envelope condition of the firm requires that:

\[ \Pi_{n,t+k} = r + \delta_n \equiv R_n, \quad n = 1, 2, \]

i.e. marginal revenue product = user cost.
Analytical example (2/2)

\[ G_{1,t} = (\mu - 1) \sum_{k \geq 1} E_t [M_{t,t+k}R_1(1 + g_{1,t+k})] \]  

\[ + S_{t+1} \]  

\[ + (\mu - 1) \sum_{k \geq 1} E_t [M_{t,t+k}R_2(1 + g_{2,t+k})] S_{t+1} \]  

(Rents → physical capital)

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\[ G_{1,t} = (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}R_1(1 + g_{1,t+k})] \]

\[ + \quad S_{t+1} \]

\[ + \quad (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}R_2(1 + g_{2,t+k})] S_{t+1} \]

(Rents → physical capital)

(Ommitted capital effect)

(Rents → intangibles) \times

(Ommitted capital effect)

The homogeneity of the capital aggregator \( F_t(.) \) requires that:

\[ g_{n,t} = g, \quad n = 1, 2, \]

i.e. capital stocks grow at the same rate, and so \( S_{t+1} = S = \frac{K_{2,t+1}}{K_{1,t+1}} \).
Analytical example (2/2)

\[ G_1 = \frac{(\mu - 1)}{r - g} R_1 \]  
(Rents → physical capital)

[ \]

\[ + S \]
( omission capital effect)

\[ + \frac{(\mu - 1)}{r - g} R_2 S \]
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Assume strictly convex adjustment costs:

\[ \gamma_n > 0. \]
Analytical example (2/2)

\[ G_1 = \frac{(\mu - 1)}{r - g} R_1 \]  
(Rents \rightarrow \text{physical capital})

\[ + \quad q_2 S \]  
(Ommitted capital effect)

\[ + \quad \frac{(\mu - 1)}{r - g} R_2 S \]  
(Rents \rightarrow \text{intangibles}) \times \quad \text{(Ommitted capital effect)}

Assume strictly convex adjustment costs:

\[ \gamma_n > 0. \]

\[ q_2 = 1 + \gamma_2 (i_2 - \delta_2) > 1. \]
Analytical example (2/2)

\[ G_1 = \frac{(\mu - 1)}{r - g} (R_1 + \gamma_1 rg) \]  
\[ + q_2 S \]  
\[ + \frac{(\mu - 1)}{r - g} (R_2 + \gamma_2 rg) S \]  

(Rents → physical capital) 

(Ommitted capital effect) 

(Rents → intangibles) \times 

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Assume strictly convex adjustment costs:

\[ \gamma_n > 0. \]

\[ \text{MRPK}_n = \Pi_{n,t} = r + \delta_n + \gamma_n rg = R_n + \gamma_n rg = \text{“adjusted” user cost} \]
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Theory: recap

In general,

\[
\text{Investment gap} = Q - q
\]

\[
= \text{Rents} \rightarrow \text{physical capital}
\]

\[
+ \text{Omitted capital effect}
\]

\[
+ (\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect})
\]
Theory: recap

In general,

\[
\text{Investment gap} = Q - q \\
= \text{Rents} \rightarrow \text{physical capital} \\
+ \text{Omitted capital effect} \\
+ (\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect})
\]

Simple formulas for specific cases:

\[
\text{Rents} \rightarrow K_n \propto \frac{\mu - 1}{r - g} \times \text{user cost}_n
\]
2. The investment gap in aggregate data
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Which moments do we need to construct this decomposition?
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Which moments do we need to construct this decomposition?

\{ S, \}

Ratio of intangible to physical capital

\[ S = \frac{K_{2,t+1}}{K_{1,t+1}} \]
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Which moments do we need to construct this decomposition?

\{ S, ROA_1, \}

“Markup” \( \mu \)

\[ \mu = \frac{ROA_1}{R_1 + SR_2} \]

\[ ROA_1 = \frac{\Pi_t}{K_{1,t}} \]
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Which moments do we need to construct this decomposition?

\{ S, ROA_1, i_1, i_2, \}

User costs \( R_1, R_2 \)

\[ R_n = r + \delta_n + \gamma_n rg \]

\[ = r - g + g + \delta_n + \gamma_n rg \]

\[ = r - g + i_n + \gamma_n rg \]

\( i_n \): gross investment rate for capital of type \( n \).
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Which moments do we need to construct this decomposition?

\{S, ROA_1, i_1, i_2, Q_1, g\}

Gordon growth term \( r - g \):

\[ r - g = \frac{ROA_1 - (i_1 + S_i_2)}{Q_1} - \frac{\gamma_1 + S\gamma_2}{Q_1} g^2 \]
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Which moments do we need to construct this decomposition?

\{ S, ROA_1, i_1, i_2, Q_1, g \}

When adjustment costs are positive:

\[ q_2 = 1 + \gamma_2 g \]
Data sources

Scope: non-financial corporate business (NFCB) sector, 1947-2017
Data sources

Scope: non-financial corporate business (NFCB) sector, 1947-2017

Obtain moments from six time series in levels: \{K_{1,t}, K_{2,t}, I_{1,t}, I_{2,t}, \Pi_t, V_t\}.
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- \(K_{1,t}, K_{2,t}, I_{1,t}, I_{2,t}\)  
  intangibles: R&D, own-account software, and artistic originals  

BEA fixed asset tables
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- \( \Pi_t \)
  - gross value added minus compensation of employees
  - intangible investment not imputed as intermediate

BEA fixed asset tables

NIPA operating surplus

Flow of Funds

MV equity + MV debt - liquid financial assets (Hall, 2001)
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- $\Pi_t$
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- \( V_t \)
  
  MV equity + MV debt − liquid financial assets  
  
  Flow of Funds  
  
  (Hall, 2001)
The investment gap in the non-financial sector

(adj. costs = 0)

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

\[ \gamma_1 = 3, \gamma_2 = 12 \] 

(Belo et al., 2019)
The investment gap in the non-financial sector

(adj. costs = 0)

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

77%
64%
23%
26%
10%

γ₁ = 3, γ₂ = 12

(Belo et al., 2019)
The investment gap in the non-financial sector

(adj. costs = 0)
The investment gap in the non-financial sector

(adj. costs = 0)

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

γ₁ = 3, γ₂ = 12
(Belo et al., 2019)
The investment gap in the non-financial sector

(adj. costs > 0)

\[ \gamma_1 = 3, \quad \gamma_2 = 12 \]  
(Belo et al., 2019)
Underlying structural changes

Cobb-Douglas intan share \( K_t = K_{1,t}^{1-\eta}K_{2,t}^\eta \)

Rents/v.a. \( s = (1 - WL/PY)(1 - \frac{1}{\mu}) \)

- zero adjustment costs
- positive adjustment costs
Relation to existing estimates of the profit share

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- Due to smaller decline in **user costs**, particularly after 1985
User costs \( R_n = r + \delta_n + \gamma_n rg \)

### Physical capital

- zero adjustment costs
- positive adjustment costs

### Intangible capital

- zero adjustment costs
- positive adjustment costs

Implied \( \delta_1, \delta_2 \), discount rate, and PD ratio
Relation to existing estimates of the profit share

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- Due to smaller decline in **user costs**, particularly after 1985

- Mild discount rate decline (7.9% → 5.6%), consistent with rising risk premia

  Caballero, Gourinchas and Farhi (2017), Farhi and Gourio (2018)

- Rise in relative user cost of intan \( \Rightarrow \) higher contribution of intan × rents to \( Q_1 - q_1 \)
Counterfactual: intan share $\eta$ with no change in rents

$$S^c_{t} = \left(\frac{K_{2,t}}{K_{1,t}}\right)^c : 9\% \rightarrow 39\%, \text{ vs. } 9\% \rightarrow 17\% \text{ in the R&D data}$$
Robustness

Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$.

Ajdustment costs $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents.

Alternative measure of net claims on NFCB sector using net NFCB claims lower $Q_1$; lower rents; same contribution of intan to $Q_1 - q_1$.

Match PD ratio $= (r - g) - 1$ instead of $Q_1$ matching PD ratio larger investment gap, particularly 1965-1975; same contribution of intan; higher rents.

Implications for the labor share

Labor share implied labor share 0.69 → 0.64, but earlier than in the data.
Robustness

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Using net NFCB claims

Adjustment costs
Robustness

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  lower $Q_1$;
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- Implications for the labor share
  
  implied labor share $0.69 \rightarrow 0.64$, but earlier than in the data
Aggregate data: recap


2. Post-1985: about 1/3 due to intangibles
   - rise in rents, though smaller than other existing estimates
   - rise of intan share
   - rise of relative intan user costs
   \[=\]  \[\Rightarrow\] larger contribution of intan to the investment gap
Aggregate data: recap


2. Post-1985: about 1/3 due to intangibles
Aggregate data: recap


2. Post-1985: about 1/3 due to intangibles
   - rise in rents, though smaller than other existing estimates
   - rise of intan share
   - rise of relative intan user costs

⇒ larger contribution of intan to the investment gap
3. The investment gap using firm-level data
Data

Scope: publicly traded, non-financial corporations, 1975-2017
Data

Scope: publicly traded, non-financial corporations, 1975-2017

- \( K_{1,t}, K_{2,t}, I_{1,t}, I_{2,t} \) (balance sheet + income statement)
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  R&D: capitalized $x_{rd}$

(balance sheet + income statement)
Data

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- $K_{1,t}, K_{2,t}, I_{1,t}, I_{2,t}$ (balance sheet + income statement)

  - R&D: capitalized $x_{rd}$

  - **Organization capital**: capitalized $0.3 \times (x_{sga} - x_{rd})$  
    
    Eisfeldt and Papanikolaou (2013)
Data

Scope: publicly traded, non-financial corporations, 1975-2017

- $K_{1,t}, K_{2,t}, I_{1,t}, I_{2,t}$
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  Eisfeldt and Papanikolaou (2013)

- $\Pi_t$
  (income statement)
  
  operating income before depreciation + intangible investment

- $V_t$
  (balance sheet)
  
  MV equity + BV debt − liquid financial assets
The investment gap in Compustat

(intan = R&D)

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

Structural changes
The investment gap in Compustat (intan = R&D)

- Rents attributable to physical capital
- Intangibles
- Rents attributable to intangibles
- Total

Structural changes

64% in 2015
15% in 2015
21% in 2015
The investment gap in Compustat

(intan = R&D + org. cap.)

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

Structural changes

30%
37%
33%
Sectoral heterogeneity

1. High-tech sector — software, IT
   
   high $ROA_1$, rising $Q_1$, declining $i_1$, rising $S$

2. Healthcare sector — medical devices, drug companies, healthcare services
   
   similar to High-tech

3. Consumer sector — retail and wholesale trade
   
   high $ROA_1$, rising $Q_1$, declining $i_1$, but stable $S$

4. Manufacturing sector — consumer durables, business equipment
   
   declining $ROA_1$, $Q_1$, $i_1$, $S$
The investment gap across sectors

(intan = R&D)
### Rents vs. intangibles by sector

<table>
<thead>
<tr>
<th></th>
<th>Consumer</th>
<th>High-tech</th>
<th>Healthcare</th>
<th>Manufacturing</th>
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<tbody>
<tr>
<td>Intan share ((\eta; 2015))</td>
<td>0.11</td>
<td>0.39</td>
<td>0.57</td>
<td>0.12</td>
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<td>Rents/v.a. ((s; 2015))</td>
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- Intangibles = R&D
## Rents vs. intangibles by sector

| Intan share  
\( \eta; \) 2015 | Consumer | High-tech | Healthcare | Manufacturing |
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- Intangibles = R&D + org. cap.
Counterfactual: rents as a fraction of value added

- **Consumer**
- **High-tech**
- **Healthcare**
- **Manufacturing**

- **Intangibles = R&D**
- **Intangibles = R&D + org. cap.**
- **Counterfactual: no increase in intangibles**
Firm-level data: recap

1. Expanded definition of intangibles: up 2/3 of investment gap due to intangibles.

2. Sectoral gaps are different — aggregate gap driven by composition effects:
   - High-tech, Healthcare: large gap, 2/3 driven by (R&D) intangibles.
   - Manufacturing, Consumer: smaller gaps; larger contribution of rents.

⇒ policy remedies, if needed, should probably not be uniform across sectors.
Firm-level data: recap

1. Expanded definition of intangibles: up 2/3 of investment gap due to intan
Firm-level data: recap

1. Expanded definition of intangibles: up $2/3$ of investment gap due to intangible assets.

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Conclusion
Conclusion

Findings:

1. General decomposition of investment gap:

\[ Q_1 - q_1 = \text{Rents} \rightarrow \text{physical capital} \]

\[ + \text{Omitted capital effect} \]

\[ + (\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect}) \]

2. Aggregate: intan is 1/3 of \( Q_1 - q_1 \); implies \( \Delta s = 0.06 \) instead of 0.12

3. Sectoral differences — intan is 2/3 of the gap in Health, Tech
Conclusion

Findings:

1. General decomposition of investment gap:

\[ Q_1 - q_1 = \text{Rents} \rightarrow \text{physical capital} + \text{Omitted capital effect} + (\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect}) \]

2. Aggregate: intan is \(1/3\) of \(Q_1 - q_1\); implies \(\Delta s = 0.06\) instead of 0.12

3. Sectoral differences — intan is \(2/3\) of the gap in Health, Tech

Next:

a. Risk premia
b. Within-firm changes of \(Q_1 - q_1\) vs. reallocation
c. Intangible investment \(\rightarrow \Delta \mu\)
Additional slides
PPE investment is weak: sectoral data

Aggregate data

\[ \frac{I_t}{K_t} \text{ (Compustat, aggregate)} \quad \text{vs.} \quad \frac{I_t}{K_t} \text{ (BEA)} \]
PPE investment is weak despite high returns: sectoral data

![Graphs showing PPE investment trends in various sectors from 1985 to 2015.](Aggregate data)
Investment is weak relative to Q

\[ i_{j,t} = \alpha_j + \gamma_t + \delta Q_{j,t} + \beta CF_{j,t} + \epsilon_{j,t} \]
Investment is weak relative to Q: sectoral data

\[ i_{j,t} = \alpha_j + \gamma_t + \delta Q_{j,t} + \beta CF_{j,t} + \epsilon_{j,t} \]
The growing importance of intangibles: sectoral data

Aggregate data

$K_1 = \text{PPE and } K_2 = \text{R&D capital.}$
How general is this model?

- No restrictions on exogenous shifters to $\Pi_t, F_t, \text{and } \Phi_{n,t}$

- Particular cases of this framework:

- What about labor?
  - The model can accommodate any flexible input: $\mu = \frac{\bar{\mu} - \alpha}{1 - \alpha}$

- Which cases does this model not fit?
  - Non-homogeneous and/or non-smooth adjustment costs
  - Endogenous markups
  - Financial frictions
The investment gap in the general case

The first-order condition for investment is:

\[ g_{n,t+1} = \Psi_{n,t} (q_{n,t} - 1) \]

where:

\[ \Psi_{n,t}(y) = \left( \Phi_{n,t}' \right)^{-1} (1 + y) - 1. \]

Since \( \Phi_{n,t} \) is convex, \( \Psi_{n,t} \) is strictly increasing. Therefore:

\[ g_{n,t+1} = \Psi_{n,t} (q_{n,t} - 1) \]

\[ = \Psi_{n,t} (Q_{n,t} - 1 - G_{n,t}) \]

\[ < \Psi_{n,t} (Q_{n,t} - 1) \quad \text{iff} \quad G_{n,t} > 0 \]
Define the total investment rate as:

\[ i_{t}^{(\text{tot})} = \frac{\sum_{n=1}^{N} I_{n,t}}{\sum_{n=1}^{N} K_{n,t}} = \sum_{n=1}^{N} \omega_{n,t} i_{n,t}. \]

In the quadratic adj. cost case:

\[ i_{t}^{(\text{tot})} = \tilde{\delta}_{t} + \sum_{n=1}^{N} \frac{\omega_{n,t}}{\gamma_{n}} (q_{n,t} - 1), \quad \tilde{\delta}_{t} = \sum_{n=1}^{N} \omega_{n,t} \delta_{n}. \]

Let \( Q_{t}^{(\text{tot})} \equiv \frac{V_{t}^{e}}{\sum_{n=1}^{N} K_{n,t+1}}. \) Then:

\[ i_{t}^{(\text{tot})} = \tilde{\delta}_{t} + \frac{1}{\gamma} \left( Q_{t}^{(\text{tot})} - 1 \right) \]

if and only if \( \mu = 1 \), and:

- \( \gamma_{n} = \gamma \) for all \( n \);
- or, \( q_{n,t} = q_{t} \) for all \( n \).
Suppose $A_t$ follows the "regime-switching process":

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \begin{cases} 
1 + g_{t-1} & \text{w.p. } (1 - \lambda) \\
1 + \tilde{g} & \text{w.p. } \lambda 
\end{cases}, \quad \tilde{g} \sim F(\cdot).$$

Then:

$$G_{1,t} = \frac{(\mu - 1)}{r - \nu(g_t)} R_1$$

(Rents $\rightarrow$ physical capital)

$$+ S$$

(Ommitted capital effect)

$$+ \frac{(\mu - 1)}{r - \nu(g_t)} R_2 S$$

(Rents $\rightarrow$ intangibles)

where:

$$\frac{1}{r - \nu(g_t)} = \frac{1}{r - \mathbb{E}(\tilde{g})} \left( 1 + \frac{g_t - \mathbb{E}(\tilde{g})}{1 + r} \right) \quad \text{if } \lambda = 1.$$
The expression for $\nu(\cdot)$ is:

$$
\nu(g_t) = g_t + \lambda(1 + g_t) \frac{(r - g_t)\zeta^* - (1 + r)}{(1 + r) + \lambda(1 + g_t)\zeta^*}
$$

where $\zeta^*$ is a constant that only depends on $F(\cdot)$, $\lambda$ and $r$. 
A microfoundation for Example 1 (1/2)

Representative household:

\[ U_t = \max \left( \frac{C_t^{1-\sigma}}{1-\sigma} + \beta U_{t+1} \right), \]  

implying \( M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \).

Final goods producer

\[ Y_t = \left( \int_0^1 Y_{j,t} \text{d}j \right)^{\tilde{\mu}}, \quad \tilde{\mu} > 1. \]  

Intermediate goods producer: \( Y_{j,t} = Z_{j,t} K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \), implying the profit function:

\[ \Pi_{j,t} = A_{j,t}^{\mu-1} K_{j,t}^{\mu}, \quad \mu = 1 + \frac{\tilde{\mu} - 1}{\alpha}, \quad A_{j,t} = (\alpha + \tilde{\mu} - 1)^{1+\frac{\alpha}{\tilde{\mu}-1}} \tilde{\mu}^{-\frac{\tilde{\mu}}{\tilde{\mu}-1}} (1 - \alpha)^{\frac{1-\alpha}{\tilde{\mu}-1}} D_t W_t^{\frac{1-\alpha}{\tilde{\mu}-1}} Z_{j,t}^{\frac{1}{\tilde{\mu}-1}}, \]

\[ D_t \equiv P_t^{\tilde{\mu}-1} Y_t. \]
A microfoundation for Example 1 (2/2)

Rest of the solution to the problem is:

\[ P_{j,t} = \tilde{\mu}MC_{j,t} \]

\[ L_{j,t} = \left( \frac{(1 - \alpha)MC_{j,t}Z_j}{W_t} \right) \frac{1}{\alpha} K_{j,t} \]

\[ MC_{j,t} = (1 - \alpha) - \frac{(1 - \alpha)(\tilde{\mu} - 1)}{\mu - 1 + \alpha} - \frac{\alpha \tilde{\mu}}{\mu - 1 + \alpha} D_t \frac{\alpha(\tilde{\mu} - 1)}{\mu - 1 + \alpha} W_t \frac{(1 - \alpha)(\tilde{\mu} - 1)}{\mu - 1 + \alpha} Z_j - \frac{\tilde{\mu} - 1}{\mu - 1 + \alpha} K_{j,t} - \frac{(\tilde{\mu} - 1)\alpha}{\mu - 1 + \alpha} \].

This implies:

\[ LS_{j,t} \equiv \frac{W_t L_{j,t}}{P_{j,t} Y_t} = \frac{1 - \alpha}{\tilde{\mu}}. \]

We have:

\[ \tilde{\mu} = \alpha (\mu - 1) + 1 = (1 - \tilde{\mu}LS_{i,t})(\mu - 1) + 1, \]

and so, solving for \( \tilde{\mu} \):

\[ \tilde{\mu} = \frac{\mu}{\mu LS_{j,t} + (1 - LS_{j,t})}. \]
Returns to physical capital, ROA₁

Physical investment rate, i₁

Intangible investment rate, i₂

Ratio of intangible to physical capital, S

Average Tobin's Q of physical capital, Q₁

Growth rate of total capital stock, g

Data sources
1985-2015 change in $Q_1-q_1$

2015 contribution of intangibles to $Q_1-q_1$

2015 intangible share

2015 rents as a fraction of value added

Robustness
Netting out all financial assets  (Hall, 2001)
Matching the PD ratio

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

Robustness
Implications for the labor share (1/2)

Value of $1-\alpha$ implied by the model when matching the labor share

Robustness
Implications for the labor share (2/2)

Value of the labor share implied by the model when setting $1-\alpha = 0.7$

Robustness
Returns to physical capital, ROA

Physical investment rate, $i_1$

Intangible investment rate, $i_2$

Ratio of intangible to physical capital, $S$

Average Tobin’s Q of physical capital, $Q_1$

Growth rate of total capital stock, $g$

--- Compustat NF, intangibles = R&D
--- Compustat NF, intan = R&D + organization capital
--- NFCB

Data sources
Consumer sector

Returns to physical capital, ROA

Physical investment rate, $i$

Intangible investment rate, $i^2$

Ratio of intangible to physical capital, $S$

Average Tobin's Q of physical capital, $Q$

Growth rate of total capital stock, $g$

Consumer sector, intangibles = R&D

Consumer sector, intangibles = R&D + organization capital

Sectoral heterogeneity
Sectoral heterogeneity
Healthcare sector

Returns to physical capital, ROA,

Physical investment rate, i,

Intangible investment rate, i,

Ratio of intangible to physical capital, S

Average Tobin's Q of physical capital, Q,

Growth rate of total capital stock, g

Healthcare sector, intangibles = R&D

Healthcare sector, intangibles = R&D + organization capital

Sectoral heterogeneity
Manufacturing sector

- Manufacturing sector, intangibles = R&D
- Manufacturing sector, intangibles = R&D + organization capital

Sectoral heterogeneity
The consumer sector: intangibles or rents?

- organization capital: no discernible trend, but high level
- still, including organization capital $\Rightarrow$ smaller markup trend after 1985