Rents and Intangible Capital: A $Q+$ Framework

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Abstract

Recent work on capital accumulation emphasizes lackluster aggregate investment since 2000, despite apparently strong fundamentals. Potential explanations include the rising importance of intangible capital and increasing economic rents. Firms may be shifting investment toward intangible, rather than physical, capital, and firms with market power have less incentive to increase scale at the margin. These explanations work through different mechanisms and have strong implications for other trends, including economy-wide markups, valuations, and the incentive to invest. In order to evaluate these hypotheses empirically, we extend the investment-$Q$ model (Hayashi, 1982; Abel and Eberly, 1994) to include economic rents (market power) and intangible capital. The gap between observable "average $Q$" and the marginal $q$ that drives investment can be characterized analytically as a function of these rents and intangible capital. We calculate the roles of rents, intangibles, and (importantly) their interaction in explaining the investment gap, taking into account returns to capital, profitability, and valuations. Both market power and intangibles play a role, but their interaction is a dominant force. This occurs even when intangibles are a small share of capital, since they earn a high return; this puts a higher weight on intangibles and any rents they generate.

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1 Introduction

Recent research highlights two apparently contradictory, medium-run facts about the US economy: returns to business capital, and corporate profits more generally, have been either stable or growing (Gomme et al., 2011); yet investment has been lackluster, in particular relative to corporate valuations (Gutiérrez and Philippon, 2017; Alexander and Eberly, 2018). These facts are particularly puzzling because over the same period of time, measures of the risk-free interest rate have been decreasing (Caballero et al., 2008). In the face of an exogenous decline in risk-free rates, standard models would predict an increase in investment and a decline in the rate of return on assets — both at odds with the data.

In neoclassical models, these facts are reflected in a rising gap between average, or Tobin’s $Q$, driven up by higher profits and a lower cost of capital on one hand, versus an implicitly lower payoff to investment, or marginal $q$, on the other hand. We directly observe the former in the data via market values, and the latter is a shadow value measured implicitly by lackluster investment.

A gap between the average value of capital and its marginal value can arise and grow for a number of reasons. Two have recently received particular attention. First, intangible capital may not be well-accounted-for, making valuations appear high relative to realized investment. Second, there may be curvature in the value function, arising from market power, for example.

Over the last several decades, intangible capital has been growing as a share of investment and as a share of assets (Corrado et al., 2005, 2009). A change in the share of intangible capital in production can explain the contradictory trends described above. Low physical investment relative to valuations could arise because typical measures of corporate valuations, such as Tobin’s $Q$, underestimate the true stock of assets, and thus overstate the incentive to invest in physical capital (Gutiérrez and Philippon, 2017; Alexander and Eberly, 2018; Crouzet and Eberly, 2019).

Alternatively, several recent papers argue that there has been a coincident increase in economic rents, evidenced by rising markups, over the last several decades in the U.S. (Barkai, 2019; Gutiérrez and Philippon, 2018). Rising rents can account for a stable or rising rate of return on assets despite a falling user cost of capital. Rising rents also reduce the marginal return to additional capital, consistent with a weaker incentive to invest.

Other recent work building on these contributions points out that a large economy-wide increase in economic rents has strong implications for other aspects of the economy. Basu (2019) argues that high economic rents would depress returns and demand for both capital and labor, in addition to acting as a cost shock, increasing inflation. More generally, Basu concludes with the challenge of how estimated economic rents can be so high without observing correspondingly large distortions across many markets. Farhi and Gourio (2018) point out that rising markups along with the declining real rate of interest should increase valuations far more than seen in the data. For this reason, they include valuations as a metric of model performance, in addition to macroeconomic trends. These papers both emphasize the problems in examining one potential explanation at a time, since the magnitudes necessary to match one phenomenon tend to exacerbate the emergence of other empirical puzzles.
The goal of this paper is to assess the importance of both rents and intangibles for the decline in investment and the rise in returns to capital. It is important to (i) consider them jointly, as they interact and are not observationally equivalent, and (ii) to take financial market valuations into account to discipline the implied economy-wide changes. To do this, we extend the Q-theory model (Hayashi, 1982; Abel and Eberly, 1994) to include two features: the presence of economic rents, and the accumulation of a stock of intangible assets. We call this model the "Q+" framework.

We show that, in this framework, the gap between average $Q$ and marginal $q$ can be decomposed into a term capturing rents to physical capital, a term capturing intangible capital, and a term capturing rents to intangible capital. Because our framework nests a number of existing investment models, the decomposition applies very generally. Moreover, each term in the decomposition of the investment gap can be quantified using data on profits, investment, valuations, and estimates of the intangible capital stock.

We then estimate this decomposition in the data. Using aggregate data, we find that the term capturing rents to intangible capital — novel to this analysis — can account for between one third and one half of the apparent gap between $Q$ and physical investment, depending on how broad the definition of intangibles is. In addition, we move beyond the aggregate data, recognizing that implied economy-wide increases in rents and intangibles likely mask variation across sectors, and subsectors of the economy may undergo substantial changes in rents and intangible capital without generating puzzling trends in the macroeconomy. We find that, indeed, experiences differ dramatically across industries. Fast-growing industries, such as Healthcare and Tech, exhibit a larger gap between average $Q$ and marginal $q$ than in aggregate; but their gap is mostly explained by intangibles, even when intangibles are narrowly measured.

In Section 2, we describe and analyze the "Q+" framework. The gap between average $Q$ and marginal $q$, which we call the "investment gap", is the main focus of our analysis. As mentioned above, this gap can be decomposed into three terms: a term capturing rents to physical capital, a term for the direct effect of intangibles, and a term capturing rents to intangible capital. The first two terms would obtain, respectively, in a model without rents (but with intangibles), and in a model without intangibles (but with rents). When both are present in the model, a third term appears, which captures the economic rents earned by intangible capital. These can be identified separately from rents earned by physical capital. The result is independent of the specifics of exogenous processes and of capital adjustment cost and revenue functions, so long as they satisfy simple homogeneity assumptions. We also provide versions of the framework in which each of these terms can be solved in closed form. These analytical expressions clarify the key forces driving the effects of rents, intangibles, and their interaction. In particular, rents on intangible capital are given the present value of markups multiplied by an appropriately defined user cost, which takes into account adjustment costs. This user cost is large for intangible capital because intangibles depreciate quickly, foreshadowing some of our findings on the quantitative importance of rents generated by intangibles.

In Section 3, we apply this decomposition to aggregate data, after showing how to estimate the components of the investment gap using moments of corporate profits, investment, valuations,
and estimates of the intangible stock. We begin with data from US national accounts, which are broader in coverage, but provide a narrower definition of intangibles, as they focus on R&D capital. Two periods stand out with large investment gaps: the 1965-1975 decade, and the post-1990 period. Most interestingly, the composition of the gap is different between these two periods: whereas the 1965-1975 gap is mostly driven by rents generated by physical capital, approximately 40% of the post-1990’s gap is due to the intangibles-related terms. The term capturing rents to intangibles becomes sizable, accounting for 25% of the gap, with the direct intangibles effect making up the other 15%. The post-1990’s change is driven by three underlying trends. First, the share of intangibles in the production function approximately doubles. Second, user costs of intangibles are not only much higher, but more stable than those of physical capital. Third, overall rents increase, though they do so more moderately than suggested by other recent work. These three effects combine to boost the contribution of rents generated by intangibles to the investment gap.

Section 4 provides a different perspective on these results, using data on publicly traded firms. These data allow us to both adopt a broader definition of intangible capital, and to disaggregate results by sector. When we expand intangibles to include the organization capital stock of firms, following Eisfeldt and Papanikolaou (2013), we find that by 2015, the two intangibles-related terms account for two thirds of the total investment gap. Including organization capital has relatively little impact on estimated user costs of intangibles — they remain elevated —, but it substantially increases the total stock of intangibles relative to physical capital, which increases both the direct effect of intangibles on the investment gap, and the interaction term. Additionally, our estimates of the overall level of rents declines; as a share of value added, they are roughly cut in half. Thus, empirically plausible amounts of intangible capital can explain the level of the investment gap without requiring high rents.

An additional advantage of the firm-level data is to implement our decomposition at different levels of aggregation. This is useful to understand how the distribution of rents and intangibles across sectors affects the aggregate investment gap. In Section 4, we divide our sample into four broad sectors: Consumer, High-tech, Healthcare, and Manufacturing. In the Manufacturing sector, the investment gap is small, and both rents and intangibles are declining. By contrast, in the High-tech and Healthcare sectors, the investment gap has been growing rapidly since the 2000’s. In both sectors, the primary driver is rents to intangible capital. Finally, in the Consumer sector, results depend on the measurement of the intangible capital stock. Reported R&D is small, so there is little role for intangibles when they are measured with this proxy. However, innovation in the consumer sector is not well-measured in R&D (see Foster et al. 2006 and Crouzet and Eberly 2018). When including organization capital, most of the gap is estimated to reflect the direct effect of large investment in intangibles in that sector — rents on either physical or intangible capital appear to have only modestly increased.

Overall, our results paint a clear picture of the growing gap between physical investment and valuations. Intangibles play a central role; and, though rents are rising, this rise is moderate, unequally distributed across industries, and interacts with intangibles in meaningful ways.
Related research and contribution Our work first relates to the literature on the implications of rising intangible capital for macroeconomics and finance, which itself builds on work measuring this rise (Corrado et al., 2005, 2009; Eisfeldt and Papanikolaou, 2013). Closest to our approach are Hall (2001), who links the rise in intangibles to stock market valuations, and McGrattan and Prescott (2010), who examine the potential role of intangibles for macro trends in a business cycle model. Relative to these papers, we study medium-run trends, emphasize sectoral heterogeneity, and, most importantly, allow for market power in our model.

Second, our work is related to a recent literature on the size and implications of rising rents. A number of researchers have interpreted the findings of Autor et al. (2017), who show that industry concentration rose in U.S. industries after 2000, as potential evidence of market power, and examined profitability and markup data for further evidence. Most closely related to our work are Gutiérrez and Philippon (2018) and Barkai (2019), who document a significant increase in pure profit shares and markups, especially after 2000. Barkai (2019), in particular, does not directly examine investment, but shows that the decline in the labor share is not offset by a rising capital share; he attributes the resulting gap to pure profits. Our approach, based on valuations, uncovers a more modest increase in rents than these papers, a point we expand on in Section 3. Basu (2019) reviews the evidence from that literature, and argues that macro trends related to profitability are largely consistent with historical variation. He points instead to investment as the outlier and asks how to reconcile it with the apparently modest changes in rents. Our paper explains this apparent divergence as the combined effect of moderate rents with rising intangibles.

Other related work on the rising capital share explores methodologies not specifically focused on markups. Karabarbounis and Neiman (2019) find that the gap between measured capital income and estimates of the required compensation of capital is most likely explained by mismeasurement in the cost of capital. Consistent with this, our results indicate that the cost of capital has not fallen as much as relying on imputed risk premia would suggest. Most closely related to our work is Farhi and Gourio (2018), who estimate the contribution of market power, risk premia, and intangibles to recent macro trends. Relative to their work, our analysis focuses more specifically on investment, and on the role that intangible capital plays in explaining its low level relative to valuations.

Finally, a rich literature in corporate finance has discussed potential sources of wedges between average $Q$ and marginal $q$, and the performance of investment-$Q$ regressions. We discuss how our framework relates to that literature in Section 2. Most recently, Peters and Taylor (2017) revisit the relationship between investment and $Q$ when intangibles are present. Belo et al. (2018) also provide decompositions of firm value across types of capital, including intangibles. We leverage the empirical results of both papers in our analysis, but also provide a more general framework than either, by allowing for rents, a key element in the relationship between investment and $Q$.

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1 See also Hansen et al. (2005) and Ai et al. (2013).
2 De Loecker and Eeckhout (2017) and Hall (2018) use firm-level accounting data and industry data, respectively, and find both high and rising markups.
3 Corhay et al. (2018) highlight the more specific role of declining entry as a source of increasing market power.
4 Related, Andrei et al. (2019) study the correlation between $Q$ and investment at higher frequencies, and find it has recently increased; by contrast, we focus on the medium-run divergence in the level of valuations and investment.
2 Rents, intangibles, and the investment gap: theory

In this section, we derive a general decomposition of the gap between average $Q$ and marginal $q$, which we refer to as the "investment gap". For each type of capital employed by the firm, the gap depends not only on economic rents, but also on other forms of capital employed by the firm, and on the rents they generate. We also relate this decomposition to existing results in the literature, and provide analytical results that allow for clearer economic interpretations.

2.1 Model

Time $t$ is discrete. A firm uses $n = 1, \ldots, N$ different capital inputs in production. The firm’s operating profits as a function of capital are $\Pi_t(K_t)$, where $K_t$ is an aggregate of the different types of capital given by:

$$K_t = F_t(K_t), \quad K_t = \left\{K_{n,t}\right\}_{n=1}^{N}. \tag{1}$$

Investment is subject to adjustment costs given by:

$$\tilde{\Phi}_t(K_t, K_{t+1}). \tag{2}$$

We index the operating profit, production, and adjustment cost functions to indicate these functions can arbitrarily depend on exogenous variables, which we do not specify explicitly. The discount factor of the firm is $M_{t,t+1}$. Firm value satisfies:

$$V_c^t(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t \left[M_{t,t+1} V_c^{t+1}(K_{t+1})\right] \tag{3}$$

$$\text{s.t.} \quad K_t = F_t(K_t),$$

where $V_c^t(.)$ is the value of the firm including distributions. We make the following assumptions about the primitives of the problem.

Assumption 1. The function $F_t(K_t)$ is homogeneous of degree 1.

Assumption 2. The function $\Pi_t(K_t)$ is increasing, concave, and homogeneous of degree $\frac{1}{\mu} \leq 1$.

Assumption 3. Adjustment costs satisfy $\tilde{\Phi}_t(K_t, K_{t+1}) = \sum_{n=1}^{N} \Phi_{n,t} \left(\frac{K_{n,t+1}}{K_{n,t}}\right) K_{n,t}$. where each function $\Phi_{n,t}$ is strictly increasing and convex.

The parameter $\mu$ plays a central role in our discussion: it captures the economic rents accruing to the firm, with $\mu = 1$ corresponding to no rents. In Section 2.4, we list examples of models in the literature which are particular cases of the general model just described. We also highlight frictions from which this model abstracts.

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5The firm may also use any other fully flexible inputs, such as labor. Appendix A.1.5 provides an example with variable labor. The operating profit function used here assumes that these flexible inputs have been optimized out.
2.2 A decomposition of the investment gap

Our main result on the investment gap uses the following expression for firm value, which is proved in Appendix A.1.

Lemma 1. Let

\[ V^e_t = E_t \left[ M_{t,t+1} V^e_{t+1} \right], \quad q_{n,t} \equiv \frac{\partial V^e_t}{\partial K_{n,t+1}}, \quad \Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_t} \frac{\partial K_{n,t}}{\partial K_{n,t}}. \] (4)

Then, firm value can be written as:

\[ V^e_t = \sum_{n=1}^{N} q_{n,t} K_{n,t+1} + (\mu - 1) \sum_{n=1}^{N} \sum_{k \geq 1} E_t \left[ M_{t,t+k} \Pi_{n,t+k} K_{n,t+k} \right]. \] (5)

This Lemma decomposes firm value into two parts. The first part is the sum of the value of the installed stocks of capital. The value of each type of installed capital is equal to its replacement cost, \( K_{n,t+1} \), multiplied its the marginal \( q_{n,t} \). The marginal \( q \) of each capital type will be different from 1 so long as its adjustment costs are strictly convex. This part of firm value is non-zero even when \( \mu = 1 \), that is, when profits exhibit constant returns to scale with respect to capital. It generalizes the Hayashi (1982) result to multiple capital inputs; this generalization was first noted by Hayashi and Inoue (1991), in a framework where \( \mu = 1 \).

In order to interpret the second part of this firm value decomposition, note that when there is only one type of capital, the expression boils down to the discounted sum of the terms \((\mu - 1)\Pi_{n,t+k} K_{n,t+k}\). These terms capture the gap between average and marginal products. Note that:

\[ (\mu - 1)\Pi_{K,t+k} = \frac{\Pi_{t+k}}{K_{t+k}} - \Pi_{K,t+k}. \] (6)

In the one-capital case, the second term in decomposition (5) is simply the present value of the gap between average and marginal products, which we interpret as the present value of economic rents. This gap is positive only when \( \mu > 1 \), as first noted by Lindenberg and Ross (1981) in the case of a firm employing a single type capital.

When there are multiple types of capital, the second term in Equation (5) is the sum of terms of the form:

\[ (\mu - 1)\Pi_{n,t+k} = \left( \frac{\Pi_{t+k}}{K_{n,t+k}} - \Pi_{K,t+k} \right) \frac{\partial F_{t+k}}{\partial K_{n,t+k}}. \] (7)

These terms capture the marginal contribution of capital of type \( n \) to overall rents earned by the firm. Rents themselves depend on the gap between the average and marginal product of capital of type \( n \), as in the one-capital case. The intuition from the one-capital case thus carries through with multiple types of capital. The added insight is that total rents are additively separable across capital types; more specifically, they are the sum of rents attributable to each capital type weighted by its marginal contribution to total capital, \( \frac{\partial F_{t+k}}{\partial K_{n,t+k}} \).
Result 1. Define average $Q$ for capital of type $n$, $Q_{n,t}$, as:

$$ Q_{n,t} = \frac{V_{t}^{e}}{K_{n,t+1}}. $$

(8)

Then, the investment gap for capital of type $n$ can be written as:

$$ Q_{n,t} - q_{n,t} = (\mu - 1) \sum_{k \geq 1} E_{t} [M_{t,t+k} \Pi_{n,t+k}(1 + g_{m,t+1,t+k})] $$

$$ + \sum_{\substack{m=1 \atop m \neq n}}^{N} S_{m,n,t+1} q_{m,t} $$

(9)

$$ + (\mu - 1) \sum_{\substack{m=1 \atop m \neq n}}^{N} S_{m,n,t+1} \sum_{k \geq 1} E_{t} [M_{t,t+k} \Pi_{m,t+k}(1 + g_{m,t+1,t+k})] $$

(10)

$$ + (\mu - 1) \sum_{\substack{m=1 \atop m \neq n}}^{N} S_{m,n,t+1} \sum_{k \geq 1} E_{t} [M_{t,t+k} \Pi_{m,t+k}(1 + g_{m,t+1,t+k})] $$

(11)

where $1 + g_{n,t+1,t+k} = \frac{K_{n,t+k}}{K_{n,t+1}}$, and $S_{m,n,t+1} = \frac{K_{n,t+1}}{K_{m,t+1}}$.

This result decomposes the investment gap into three terms, (9), (10) and (11). These three terms can be interpreted as follows.

When there are no rents and a single type of capital ($\mu = 1$ and $N = 1$), average $Q$ and marginal $q$ are equal, as in the standard model of Hayashi (1982). In this case, the terms (9), (10) and (11) are zero, and the investment gap is zero.

If there are rents but only one type of capital ($\mu > 1$ and $N = 1$), average $Q$ will overstate marginal $q$. The positive investment gap is then equal to the present value of the difference between average and marginal products of capital, that is, the term (9). This case includes the Lindenberg and Ross (1981) effect.

If there are no rents but several types of capital ($\mu = 1$ and $N > 1$), then for each type of capital, average $Q$ will still overstate marginal $q$. Average $Q$ for a specific type of capital reflects, in part, the value of other types of capital used by the firm, because these other types of capital contribute to firm value overall. It therefore overstates the true incentive to invest — the marginal $q$ — of that type of capital. This omitted capital effect is captured by the term (10) in the expression of the investment gap.

If there are both economic rents and several types of capital ($\mu > 1$ and $N > 1$), the rents term (9) and the omitted capital term (10) are still non-zero. But additionally, the term (11) is non-zero. This term represents the interaction between the rents and the omitted capital effects: it captures how the present value of the rents accruing to other types of capital affects total firm, value and, through the omitted capital effect described above, add to the gap between average $Q$ and marginal $q$. This interaction term is larger, the higher the relative importance of other types of capital, and the higher the rents generated by other types of capital.

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6This point is made in a model with two types of capital and no rents in Crouzet and Eberly (2019).
2.3 Analytical example

We now provide an analytical example of the investment gap decomposition. The expressions we obtain help build intuition for each of its components, and also anticipate the empirical applications. Without loss of generality, we focus on the case of two types of capital; \(K_{1,t}\) is "physical capital," and \(K_{2,t}\) is "intangible capital." Additionally, we assume the profit function is given by \(\Pi_t = A_t \mu_t K_{1,t} / \mu_t\), where \(\mu \geq 1\) and \(A_t\) is an exogenous process capturing firm fundamentals.\(^7\) Firm fundamentals grow at a constant rate: 
\[A_{t+1}/A_t = 1 + g.\]  
We also assume 
\[M_{t,t+1} = (1 + r)^{-1} - 1,\]  
with \(g < r\). Finally, let adjustment costs be given by: 
\[\Phi_n(x) = x - 1 + \delta_n + \gamma_n (x - 1 + (r - (x - 1)) \log (r - (x - 1)/r)).\]  
(12)

This adjustment cost is increasing and strictly convex; moreover, it satisfies \(\Phi_n(1) = \delta_n\), \(\Phi'_n(1) = 1\) and \(\Phi''_n(1) = \gamma_n\). We choose the functional form so as to obtain simple analytical expressions. Appendix A.1 shows that: 
\[Q_1 - q_1 = \mu - 1 \frac{r + \delta_1 + \gamma_1 r g}{r - g} + S q_2 + \mu - 1 \frac{r + \delta_2 + \gamma_2 r g}{r - g} S,\]  
(13)

where all ratios (marginal \(Q\), average \(q\), and the ratio of intangible to physical capital) are constant.

Following Result 1, the investment gap for physical capital has three components: the rents attributable to physical capital; the omitted capital term; and the rents attributable to intangible capital. In order to build more intuition, consider first the special case of linear adjustment costs, \(\gamma_1 = \gamma_2 = 0\). Marginal \(q\) for both types of capital is then equal to 1, and so:
\[Q_1 - q_1 = Q_1 - 1 = \mu - 1 \frac{r + \delta_1}{r - g} + S + \mu - 1 \frac{r + \delta_2}{r - g} S.\]  
(14)

When adjustment costs are linear, the firm behaves as though it was renting capital in perfectly competitive markets, equating the marginal revenue product of each type of capital to its Jorgensonian user cost: \(\Pi_{n,t} = r + \delta_n\). The two rents terms in decomposition (14) can then be thought of as a markup on the marginal (user) cost of each of these two inputs, discounted by the Gordon growth term \(r - g\). The relative magnitude of the two terms capturing rents will then depend on how intangible-intensive the firm is (that is, on \(S\)), and on how high intangible user costs are, relative to physical user costs.

When adjustment costs are positive (\(\gamma_1 > 0, \gamma_2 > 0\)), the main difference is in the expression for user costs. The typical Jorgensonian user cost \(r + \delta_n\) which appeared in decomposition (14) is replaced with \(r + \delta_n + \gamma_n r g\). The additional term increases user costs; it reflects compensation for the cost of continuously adjusting capital along the firm’s growth path.

\(^7\)Appendix A.1.5 describes a general equilibrium model in which monopolistically competitive firms also use labor in production, and shows that they effectively face this operating profit function.

\(^8\)Appendix A.1.4 provides analytical expressions for a version of the model with stochastic growth rates; these expressions are very similar to Equation (14), and key insights from that decomposition are preserved.
2.4 Discussion

Why is \( Q_{n,t} - q_{n,t} \) an “investment gap”? We extend the terminology “investment gap” used in Gutiérrez and Philippon (2017) and Alexander and Eberly (2018). The first-order condition for investment can be written as:

\[
g_{n,t} = \Psi_{n,t}(q_{n,t} - 1),
\]

where \( g_{n,t} \) is the net investment rate in capital \( K_{n,t} \), and \( \Psi_{n,t}(y) \equiv (\Phi_{n,t}^{-1}(1 + y) - 1 \) is a strictly increasing function capturing the magnitude of investment adjustment costs. Whenever the investment gap is positive (\( Q_{n,t} > q_{n,t} \)), we have \( \Psi_{n,t}(q_{n,t} - 1) = g_{n,t} < \Psi_{n,t}(Q_{n,t} - 1) \). Therefore, investment predicted using average \( Q \) instead of using marginal \( q \) will always exceed actual investment. That is, there will appear to be a “gap” between actual investment and observed \( Q \) values.

Why not use Total \( Q \)? Total \( Q \) (Peters and Taylor, 2017) is defined as:

\[
Q_{\text{tot},t} = \frac{V_t}{\sum_{n=1}^{N} K_{n,t+1}}.
\]

Total \( Q \) in our model is given by:

\[
Q_{\text{tot},t} = \sum_{n=1}^{N} s_{n,t+1}q_{n,t} + (\mu - 1) \sum_{n=1}^{N} s_{n,t+1} \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+1,t+k})]
\]

where \( s_{n,t+1} = K_{n,t+1}/\sum_{n=1}^{N} K_{n,t+1} \). Define the “total investment gap” as the difference between \( Q_{\text{tot},t} \) and \( q_{\text{tot},t} = \sum_{n=1}^{N} s_{n,t+1}q_{n,t} \). This gap will be positive, if and only if, the firm earns rents; moreover, rents can be decomposed across types of capital, as described above. However, we do not focus on this “total investment gap” for one main reason: \( q_{\text{tot},t} \) is not a sufficient statistic for total investment, except in specific cases.\(^9\) This implies that there is no one-to-one mapping between the gap \( Q_{\text{tot},t} - q_{\text{tot},t} \) and the relationship between total investment and \( Q_{\text{tot},t} \). By contrast, \( q_{n,t} \) is always a sufficient statistic for investment in \( K_{n,t} \), and so the capital-specific investment gap \( Q_{n,t} - q_{n,t} \) entirely accounts for the relationship between \( Q_{n,t} \) and investment in \( K_{n,t} \).

How general is this model? Other than Assumptions 1-3, the model puts no restrictions on the functional forms for \( F_t, \Pi_t \) and \( \Phi_{n,t} \), and therefore nests a number of existing models.\(^10\) However, it has three limitations. First, it does not allow for non-convex adjustment costs. Second, it abstracts from financial constraints.\(^11\) Finally, it assumes that rents, \( \mu \), are exogenous. In particular, they do not depend on past investment, in contrast, for instance, with models of customer capital.\(^12\) In this sense, our results are restricted to “neoclassical” models of the firm, and provide a benchmark against which the effects of other frictions on the investment gap can be compared.

\(^9\)Appendix A.1.3 discusses this issue in more detail. Examples include the case of perfectly substitutable capital types and identical adjustment cost functions.


\(^11\)A large literature has shown financial constraints can drive a wedge between average \( Q \) and marginal \( q \) (Whited, 1992; Gomes, 2001; Hennessy et al., 2007; Bolton et al., 2011; DeMarzo et al., 2012). The size this wedge is a matter of debate, particularly if the firm has market power (Cooper and Ejarque, 2003).

\(^12\)See, for instance, Gourio and Rudanko (2014) and Belo et al. (2014).
3 The investment gap in aggregate data

We now show that the investment gap for non-financial corporate businesses has tripled since 1985, driven by the combined effects of rising rents and a growing importance of intangibles. This section uses aggregate data, which has the most coverage, but the narrowest measure of intangible capital. We broaden this measure in the next section, drawing on firm-level data.

3.1 Methodology

We use the constant growth version of the model, described in Section 2.3, in order to construct the investment gap and its components in the data. In that version of the model, we have:

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g}R_1 + q_2S + \frac{\mu - 1}{r - g}R_2S, \tag{15} \]

where recall that \( R_n = r + \delta_n + \gamma_nrg \), and \( q_n = 1 + \gamma_ng \), \( n = 1, 2 \). We assume \( Q_1 \) and \( S \) are measured. Thus, we need values for \( \{ \mu, r - g, R_1, R_2, q_1, q_2 \} \). We derive these values from the following observable moments: \( \{ ROA_1, i_1, i_2, g \} \), where \( ROA_1 = \Pi_t/K_{1,t} \) is average returns to physical capital, \( i_1 \) and \( i_2 \) are gross investment rates, and \( g \) is the net growth rate of the total capital stock \( K_{1,t} + K_{2,t} \). First, we use the fact that:

\[ \mu = \frac{ROA_1}{R_1 + SR_2}. \tag{16} \]

Intuitively, rents imply a wedge between average returns to physical capital and the weighted average user cost of capital. Second, we have that:

\[ R_n = r - g + i_n + \gamma_nrg, \quad n = 1, 2, \tag{17} \]

where we used the fact that \( i_n = g + \delta_n \) along the balanced growth path. Finally, substituting Equations (16) and (17) in the investment gap decomposition (15), we obtain:

\[ r - g = \frac{ROA_1 - (i_1 + Si_2)}{Q_1} - \frac{\gamma_1 + S\gamma_2}{Q_1}g^2. \tag{18} \]

This expression for the Gordon growth term \( r - g \) only depends on observable moments and values of the adjustment cost parameters. Given the value for \( r - g \) and other data moments, values of \( R_1 \) and \( R_2 \) follow from Equation (17); and the value of \( \mu \) follows from Equation (16). Finally, \( q_1 \) and \( q_2 \) are obtained from the values of \( g \) and of the adjustment cost parameters.

This approach matches, by construction, the empirical value of \( Q_1 \). That is, it infers the Gordon growth term \( r - g \) which, given other moments, ensures that the model produces a value of \( Q_1 \) consistent with the data. Our use of valuations is a natural implication of the model, but also an important point of departure from the recent literature. We come back to this point in Section 3.3.

\[^{13}\text{See Appendix A.1 for a formal derivation of this relationship.}\]
3.2 Data

Our sample period is 1947-2017. We construct time series for five of the moments used in the decomposition, \{i_1,t, i_2,t, S_t, ROA_{1,t}, Q_{1,t}\}, using six times series in levels, \{K_{1,t}, I_1,t, K_{2,t}, I_2,t, \Pi_t, V_t\}. These are, respectively, the operating surplus of the NFCB sector, the stock of physical capital at replacement cost, investment in physical capital, the stock of intangibles at replacement cost, investment in intangibles, and the market value of claims on the NFCB sector.\(^{14}\)

We obtain measures of \(K_{1,t}, I_1,t, K_{2,t}, I_2,t\) from BEA fixed asset tables 4.1 and 4.7. The BEA fixed asset tables use perpetual inventory methods to construct the stock of three specific forms of intangible capital: R&D; own-account software; and artistic originals. To the extent that firms invest in other types of intangibles, results in this section should thought of as a lower bound on the overall role of intangibles. Section 4 expands the analysis to organization capital for the subset of publicly traded firms in the NFCB sector.

Operating surplus \(\Pi_t\) is obtained from NIPA Table 1.14. Consistent with the model, this series represents the difference between value added and payments to labor, and does not subtract from revenue expenditures categorized as intangible investment when estimating value added.

We construct a measure of \(V_t\) using Flow of Funds tables L.103 and F.103. In the model, \(V_t\) represents the market value of all net claims on the NFCB sector, both debt and equity. The Flow of Funds data provide an estimate for the market value of equity of the NFCB sector, but not for debt. We follow an approach analogous to Hall (2001) to estimate the latter. Importantly, and different from Hall (2001), we do not subtract all financial assets owned by the sector from the gross market value of claims, but only financial assets identified as liquid in the Flow of Funds.\(^{15}\) Section 3.3 shows that this choice affects the level of the investment gap, but not its composition.

We then construct \(ROA_{1,t} = \Pi_t/K_{1,t}, \ i_1,t = I_1,t/K_{1,t}, \ i_2,t = I_2,t/K_{2,t}, \ S_t = K_{2,t}/K_{1,t}, \) and \(Q_{1,t} = V_t/K_{1,t}\). Finally, we compute \(g_t\) as the annual growth rate of the chain-type quantity index for private non-residential fixed assets of the NFCB sector, provided in BEA fixed asset table 4.2. The time series for the resulting six moments, \{\(i_1,t, i_2,t, S_t, ROA_{1,t}, Q_{1,t}, g_t\}\) are reported in Appendix Figure A1. The key trends discussed in the introduction are visible in that figure. The average return to physical capital increases after 1985, while the physical investment rate declines. The ratio of intangible to physical capital increases, particularly after 1985. \(Q_1\) rises sharply after 1985, and after a peak in 2000, remains approximately double its value in the pre-1985 period.

Finally, we compute the decomposition using moving averages of moments over 7-year centered rolling windows. This treats each successive window as if it were generated by a different quantitative implementation of the model, allowing us to capture gradual changes in the investment gap.\(^{16}\)

\(^{14}\)We use current-dollar values for all time series in levels. This is with the exception of our proxy for \(g_t\), the computation of which is described below. Detailed steps for the data construction are reported in Appendix A.2.

\(^{15}\)Financial assets are generally subtracted from the gross market value of claims in order to include net debt, instead of gross debt, in firm value calculations. On the other hand, financial assets can only meaningfully be counted as negative debt to the extent that they are liquid. Additionally, a large part of non-liquid financial assets in table L103 are obtained as a residual, further complicating their interpretation.

\(^{16}\)Using alternative window sizes from 3 to 9 years gives quantitatively similar results.
3.3 Results

Baseline results  Figure 1 reports the investment gap decomposition, Equation (15), for the NFCB sector and R&D intangible capital. We consider two cases: zero adjustment costs, $\gamma_1 = \gamma_2 = 0$; and positive adjustment costs. We choose values of $\gamma_1 = 3$ and $\gamma_2 = 12$ for the latter case following (Belo et al., 2018), but also show, below, that our results are generally robust to the choice of adjustment costs.\footnote{For physical adjustment costs, this value is in the lower end of existing estimates in the literature, which range from 1 to 32 (Hall, 2001). For intangible capital, we use a value close to the recent estimate of 12.5 obtained by Belo et al. (2018) for R&D capital in a panel of publicly traded firms.}

We highlight three main findings. First, the investment gap is large during two distinct periods: 1960-1970, and after 1985. The wedge between average $Q$ and marginal $q$ is therefore not strictly a hallmark of the post-1980s period. Second, rents attributable to physical capital — the first term in Equation (15) — play a sizeable (though somewhat declining) role in explaining the investment gap: they account 61% of it in 2015, compared to 67% in 1965.\footnote{These numbers, and those that follow in this discussion, refer to the model with adjustment costs.} Third, rents attributable to intangibles — the third term in Equation (15) — have become markedly more important in recent years. In 2015, 25% of the investment gap reflects the combined effects of high rents and a large stock of intangibles, compared to 10% in 1965, using the BEA measure of R&D capital only.

From the standpoint of the model, these changes are driven by three underlying forces, reported in Figure 2: a greater importance of intangibles in the production function; higher rents; and a decline in user costs, more pronounced for physical than for intangible capital.

The top left panel of Figure 2 shows that even using the relatively narrow definition of intangibles in the NFCB data, the share of intangible capital in production increased substantially after 1985, from 0.17 to 0.29 in 2015.\footnote{This number is derived assuming that the capital aggregator is Cobb-Douglas. The level of intangible share is sensitive to this assumption, but not the magnitude of the change after 1985.} The behavior of the intangible share approximately mimics the behavior of the measured ratio of intangible to physical capital at replacement cost, which increases rapidly after 1985.

The effects of the intangible share on the overall investment gap are magnified by the rise in rents after 1985. The top right panel of Figure 2 reports estimates of the rents implicit in Equation (15). In order to facilitate comparison with existing estimates of the profit share, we express them as the flow value of rents relative to value added, which is related to the parameter controlling rents in the model, $\mu$, through $s = (1 - s_L)(1 - 1/\mu)$, where $s_L$ is the labor share of value added.\footnote{We measure the labor share for the NFCB sector using NIPA data on labor payments for that sector, as described in Appendix A.2.1. Appendix A.3.1 discusses the implications of our analysis the labor share in more detail, using a model with variable labor.} Total rents, as a fraction of value added, increase from 0.015 in 1985, to 0.077 in 2015 — a cumulative 6.2 percentage point change over three decades. Expressed in markup terms, this is an increase from 1.015 in 1985, to 1.083 in 2015.

These findings are qualitatively consistent with the recent literature arguing that pure profits as fraction of value added have been growing over the last three decades (Gutiérrez and Philippon, 2017).
However, they differ quantitatively. For instance, Barkai (2019) finds that the pure profit share rose from -5.6% in 1984 to 7.9% in 2014, an increase of 13.5 percentage points over the period. Karabarbounis and Neiman (2019), in their “case Π,” find that the pure profit share must have risen by about 13% over the same period. We find an increase in rents of half that magnitude.\footnote{Related to this are the markup estimates of De Loecker and Eeckhout (2017) and Hall (2018). These markups, when expressed in value added terms, are much higher than ours — approximately 1.9 and 4 in 2015, respectively —, and also far outside the range typically considered reasonable in the macroeconomics literature, as discussed in detail by Basu (2019).}

User costs are at the heart of this difference. Specifically, our approach leads to user costs that are initially lower, but that decline more slowly. Figure 3 reports these implied user costs. User costs for physical capital decline from 15.4% to 12.6% between 1985 and 2015, while user costs for intangibles decline from 36.8% to 30.4%; their weighted average only declines from 17.1% to 15.2%. (By contrast, Barkai (2019), for instance, finds a required rate of return on capital that falls from approximately 20% in 1985 to approximately 14% in 2014.) The lower decline in user costs translates to higher payments to capital (particularly to intangibles), and therefore a lower increase in rents.

The way we infer the discount factor perceived by firms from the data is central to this result. As discussed before, we rely on valuations; by contrast, the papers mentioned above generally combine risk-free rates with imputed estimates of risk premia to obtain discount rates. Appendix Figure A2 reports the implied discount rate implied by our approach. It declines from 7.9% to 5.6% between 1985 and 2015. This is a smaller decline than the risk-free rate over the same period of time, and is therefore consistent with a mild rise in risk premia over this period of time, as argued by Caballero et al. (2017), Farhi and Gourio (2018), and Karabarbounis and Neiman (2019) in their case R.

Finally, it is worth noting that our analysis implies that user costs for intangible capital have fallen by less than those of physical capital. Our approach infers this from the higher gross investment rates in intangibles, but this finding is also supported by the data, which shows that economic depreciation rates for intangibles have risen over the past three decades, as reported in Appendix Figure A2. This change in relative user costs explains why rents attributable to intangibles, which are the present value of net markups over their user costs, as indicated by Equation (15), account for an increasing fraction of the investment gap after 1985.

**Counterfactuals** In order to further illustrate the respective roles played by intangibles and rents in our estimation, Figure 3 reports results from two counterfactual exercises.

The top panel constructs the change in the share of intangibles in production that would be necessary in order to fully account for the increase in the investment gap, assuming that rents remain fixed at their 1985 level. This change is 34 percentage points, compared to 12 percentage points in our baseline results. This, in turn, implies that the ratio of intangible to total capital, at replacement cost, would need to be 30% in 2015, or approximately twice it observed value of 14% for R&D capital in the NFCB.\footnote{Simple algebra, using the results of Section 3.1, shows that the counterfactual ratio of intangible to physical capital is 30%, or approximately twice the observed value of 14% for R&D capital in the NFCB.} In Section 4, we show that this magnitude is comparable to...
the ratio of intangible to total capital including organization capital among publicly traded firms, measured as in Eisfeldt and Papanikolaou (2013)\(^{23}\).

The bottom panel of Figure 3 shows the increase in rents, as a fraction of value added, which would be required in order to match the observed investment gap, assuming that both the share of intangible capital, \(R_2\), and the intangible investment rate \(\iota_2\), had remained fixed at their 1985 values. Instead of the 6.2 percentage point increase in rents as a fraction of value added which we estimate as our baseline, rents would have needed to increase by 8.4 percentage points, or approximately 35% more, reaching 10.0% of value added by 2015. The total contribution of intangibles to the investment gap would nevertheless remain elevated (approximately 31%, instead of 39% in our baseline), due to the rising rents generated by the (moderate) stock of intangibles.

**Robustness** We conclude with three robustness exercises regarding these results.

Appendix Figure A4 reports four implied moments under alternative combinations of adjustment costs for physical and intangible assets. The values considered are \(\gamma_2 \in [0, 20]\) and \(\gamma_1 \in [0, 10]\). The four moments are the change in the overall investment gap, \(Q_1 - q_1\); the contribution of intangibles to the investment gap in 2015; the implied intangible share in 2015; and the implied share of rents in total value added in 2015. Of these moments, none display significant sensitivity to changes in user costs except the share of rents in value added. That share is highest when adjustment costs are lowest, consistent with the intuition, from Section 2.3, that taking into account adjustment costs tends to raise user costs of capital and lower implied rents. Nevertheless, estimates of the profit share in 2015 with zero adjustment costs are 8.5%, still moderate, compared to some of the recent results discussed above.

Next, we consider an alternative measure of the net value of claims on the NFCB sector, that of Hall (2001)\(^{24}\). As mentioned above, this measure subtracts all financial assets of the NFCB sector from gross claims, instead of subtracting only liquid financial assets, as we do in our baseline. Appendix Figure A3, panel (a), reports the time series for \(Q_1\) obtained this way. It is lower than in our baseline, though it displays approximately the same medium and long-run trends. Appendix Figure A5 then reports the investment gap obtained using this measure of \(Q_1\). The main difference with our baseline is in the overall level of the gap; it is about half as large. As a result, implied rents are lower than in our baseline. However, the direct effect of intangibles becomes larger; and overall, intangibles account for more of the gap with this measure of \(Q_1\) than in our baseline.

Finally, we consider an alternative to \(Q_1\) for inferring the value of \(r - g\): measures of the price-capital \(\hat{S}\) under fixed rents is the smallest positive root of \(Ax^2 + Bx + C = 0\), where \(A = \iota_2 + \gamma_2 g(\iota_2 + g + \gamma_2 g), B = \iota_1 + \iota_2 - ROA_1 - Q_1 t_2 + \gamma_1 g(t_2 + g) + \gamma_2 g(\iota_2 + g - Q_1 g - ROA_1) + 2\gamma_1 \gamma_2 g^2, C = ROA_1 Q_1 / \mu^{(1985)} + \iota_1 - ROA_1 - Q_1 t_1 + \gamma_1 g(t_1 + g + \gamma_1 g^2 - ROA_1 - Q_1 g), and \(\mu^{(1985)}\) is the estimated value of the rents parameter \(\mu\) in 1985 using our baseline approach. The ratio of intangible to total capital is then given by \(\hat{S}/(1 + \hat{S})\).\(^{23}\)

\(^{23}\)This magnitude is also comparable to Karabarbounis and Neiman (2019), “case K”. These authors show that, if the profit share is assumed to be zero, then unmeasured capital would need to account for approximately 40% of all business capital after 1970 in order to explain the measured capital share. Expressed in terms of value added, our estimates imply that intangibles would need to be approximately 63% of value added in the NFCB sector; this in line with similar estimates obtained by McGrattan and Prescott (2005) under perfect competition.

\(^{24}\)This measure is also used in the recent work of Andrei et al. (2019).
dividend (PD) ratio. Indeed, in the version of the model we use in this section, the PD ratio is simply given by $PD = (r - g)^{-1}$. Appendix Figure A3 reports two empirical measures of the PD ratio for the NFCB sector, and Appendix Figure A6 reports the investment gap decomposition obtained when inferring $r - g$ directly from the PD ratio. Generally, this approach implies a larger estimated investment gap, particularly so in the first half of the sample, prior to 1980. This reflects the fact that empirical measures of the PD ratio are generally larger than implied by our baseline, particularly in pre-1980 period. (The flipside is that, when using PD ratios as a proxy for $r - g$, implied measures of $Q_1$ exceed their empirical counterparts). However, targeting the PD ratio still leads to the same basic insights as the baseline approach. In particular, by 2015, the rents attributable to intangibles account for 28% of the total investment gap in the PD ratio approach (compared to 25% in our baseline approach).

Summarizing, we documented a large investment gap in the NFCB sector after 1985. This gap reflects a combination of rising rents and a growing importance intangibles in production, with the latter accounting for about one-third of the gap. Additionally, though our valuation-based approach finds rising rents, the magnitude of the increase is approximately half that of existing estimates. These results were obtained using aggregate data. This data obscures potential differences across sectors, and also focuses on R&D intangible capital, which is a subset of intangible capital emphasized in the measurement literature. We turn now to another data source in order to address these issues.

4 A different empirical perspective on the investment gap

In this section, we construct investment gaps at the sectoral level, and highlight how they change when measures of intangibles are expanded beyond R&D capital. We find substantial differences across sectors in both the level of the gap and the relative contributions of rents and intangibles. Expanding measures of intangibles beyond R&D reduces the quantitative estimates of rents, and suggests that intangibles are the dominant force behind the growth in the investment gap.

4.1 Data

We use the non-financial segment of Compustat, instead of data drawn from the National Accounts. This restricts the scope of our analysis to publicly traded firms. We choose Compustat both because, to our knowledge, there is no comprehensive sectoral data on operating surplus ($\Pi_t$, in our notation)

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25 We estimate a price-dividend ratio for equity, using cum- and ex-dividend returns on the S&P500, and adjust these estimates for leverage (at market values) and interest payments, estimated from Flow of Funds data.

26 These discrepancies may be driven by differences the sample underlying our measure of $Q_1$ (the NFCB sector) and the sample underlying our measure of the PD ratio (the S&P 500). Indeed, our estimates of the physical investment gap using the Compustat non-financial sample, presented in Section 4, lead to somewhat higher estimates of $Q_1$, and as a result of the physical investment gap, than for the NFCB sector as a whole. An additional reason may be mis-measurement of distributions to equityholders. Our measure is based on cash distributions, and excludes share repurchases, which became more common after the early 2000’s.
spanning a sufficiently long time period, and because it allows for measures intangible capital that can be expanded beyond R&D.

**Sector definitions**  We focus on domestically incorporated, publicly traded US firms not in the financial sector, so that the scope of the analysis is similar to Section 3 (but now excludes private corporations). We split the sample into four broad sectors: the Consumer sector (primarily retail and wholesale trade); the High-tech sector (primarily software and IT); the Healthcare sector (producers of medical devices, drug companies, and healthcare service companies); and the Manufacturing sector. These groups are similar to the first four groups of the Fama-French 5 classification. We omit a fifth group from our analysis, which primarily contains service industries, including professional and business services, and hospitality services.27

**Data moments**  In order to construct the key moments needed for our analysis, we proceed similarly to Section 3; Appendix A.2 reports the details. The two main differences are as follows. First, we consider two types of intangibles: R&D, similar to analysis of Section 3; and organization capital, which we did not measure in Section 3. R&D investment is measured using reported R&D expenditures. For investment in organization capital, we follow Eisfeldt and Papanikolaou (2013) and impute investment as 30% of SG&A expenditures net of R&D investment. Second, for operating surplus, \( \Pi_t \), we use operating income before depreciation, but we adjust it for the expensing of intangible investment in accounting data, consistent with our model.

### 4.2 The aggregate investment gap in Compustat

We start by applying our baseline analysis to pooled data from all Compustat sectors. The results are summarized in Table 1.28 Here, we highlight the two main findings of this exercise.

First, when using only R&D capital, the same trends highlighted in the introduction are apparent in both the Compustat and the NFCB data: rising returns to physical capital, rising \( \tilde{Q}_1 \), and declining physical investment rates. Compustat moments are very close to those of the NFCB, consistent with the fact that the fixed asset tables primarily measure intangibles as capitalized R&D. The exception are returns to physical capital, which are higher among publicly traded firms. As a result, total rents as a fraction of value added are estimated to be higher among Compustat firms than in the NFCB sector as a whole. The rent share of value added is about 2 percentage points higher in the post-2001 period in the Compustat sample, as indicated in Table 1. Other than this difference, when using only R&D, the implications of our analysis look similar for Compustat and the NFCB as a whole.

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27Appendix Tables A1 and A2 report the list of SIC and NAICS codes that make up each sector. Using the KLEMS data, the four sectors we study account for 54.4% of total value added by private businesses in 2001.

28Additionally, Appendix Figure A8 reports the raw time series for the moments used in our baseline analysis, Appendix Figure A9 reports the time series for the investment gap and its decomposition, and Appendix Figure A10 reports the time series for the share of intangible in production, the share of rents in value added, and the user costs of the two types of capital, all based on the aggregated data from the Compustat sample.
Second, once organization capital is included, results for Compustat suggest that intangibles are the dominant force behind the investment gap. When including organization capital, the ratio of intangible to physical capital more than doubles. Returns to physical capital also further increase, since operating surplus must now be adjusted for the expensing of intangible investment in organization capital. However, the effect of the higher stock of intangibles dominates. After 2001, for instance, the two intangible-related terms account for 69% of the total investment gap, on average, as opposed to 39% when only including R&D. The intangible share in production approximately doubles compared to when only R&D capital is included, reaching 0.48, on average after 2001. Additionally, the importance of rents overall declines. The share of rents in value added falls to 4.9% of value added after 2001, compared to 8.7% when only R&D capital is included. This result is consistent with the intuition developed using counterfactuals in Section 3: unmeasured intangible capital of an empirically plausible magnitude can account for the majority of the investment gap, and reduce the role of rents substantially.

4.3 Sectoral results

**Trends across sectors** The top panel of Table 2 reports averages of the six data moments used in the construction of the investment gap over two periods, 1985-2000 and 2001-2017. There are notable differences across sectors, even with the coarse industry classification. High-tech and Healthcare are characterized by a combination of high asset returns and high valuations, declining physical investment, and a high (and rising) intangible share, consistent with the aggregate data for the NFCB sector as a whole. The Consumer sector also features high returns and low physical investment. In that sector, when measured as R&D, intangibles appear to be a negligible fraction of total capital, while they are about half of total capital when organization capital is included, which we discuss further below. Finally, Manufacturing is characterized by declining returns, declining valuations, declining physical investment, and a declining intangible share, in contrast to the other sectors.

**Results using only R&D** Figure 4 reports investment gaps and their decomposition for the four sectors of our analysis, when intangibles are measured only with R&D capital. The model used to construct this decomposition has positive adjustment costs of $\gamma_1 = 3$ and $\gamma_2 = 12$, as in the previous section. This figure shows that the level and the composition of investment gaps differs substantially across sectors.

One extreme is the Manufacturing sector. In that sector, the investment gap is small. Moreover, little of it is explained by intangibles. This is consistent with the fact that the stock of R&D capital (relative to the stock of physical capital) has been declining in manufacturing since the early 2000’s. Accordingly, the bottom panel of Table 2 indicates that intangibles’ share in the production function has decreased. Though rents have been rising in that sector — they increased by 3.8 percentage points of value added from before to after 2000, as indicated by Table 2) —, they remain small.

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29Appendix Figures A11 to A14 report the full time series for these moments.
The other extreme is the Consumer sector. There, the investment gap is large, in particular after 1990. However, it is almost entirely explained by rents to physical capital when using R&D capital alone — our measure of intangibles for this exercise — since measured R&D is very small. The combination of high returns, high valuations, and low intangibles lead to a high (and rising) share of rents in value added, reaching 12.7% after 2000, as reported in Table 2.

The Healthcare and High-tech sectors are intermediate cases. Both experienced a large increase in the physical investment gap starting in the mid-1980’s. In both cases, rents attributable to physical capital have also increased. However, they only account for about one-half — in the High-tech sector — and one-third — in the Healthcare sector — of the investment gap overall. In both sectors, the key change in the composition of the investment gap after 2000 is a substantial increase in rents to intangible capital. For the Healthcare sector, for instance, they account, alone, for 41% half of the total investment gap. Table 2 indicates that this is the effect of two changes: a rising intangible share; and a rise overall rents. Rents as a fraction of value added rise by 6.6 percentage points in the High-tech sector, and 4.3 percentage points in the Healthcare sector, between the pre- and post-2000 periods. The intangible share in production also increases, particularly in the Healthcare sector, where it roughly doubles.

Results including organization capital The previous sectoral results were constructed using only R&D as a measure of the intangible capital stock. Expanding the definition of intangibles to include organization capital has two main effects, both of which are most clearly apparent in the Consumer sector. First, unsurprisingly, the implied share of intangibles in the production function increases substantially. The increase is particularly striking in the Consumer sector, where the stock of organization capital becomes comparable to the stock of physical capital. Second, the level of implied rents declines substantially. In the Consumer sector, rents fall from 12.4% to 2.7% of value added after 2001. The combined effect of these two changes is to magnify the direct contribution of intangibles to the investment gap. The Consumer and the Healthcare sector are both particularly impacted; in both, intangibles measured in this way account for more than half of the investment gap.

It is worth noting, though, that while including organization capital leads to a substantial decrease in the level of rents, it has a more moderate impact on their trend. Figure 5 reports the cumulative change in the estimated share of rents in total value added from 1985 onward for each of the four sectors, measuring intangibles using either R&D (blue circled line) or the sum of R&D and organization capital (green crossed line). The Consumer sector is where including organization capital makes the sharpest difference: the cumulative change in rents falls by approximately one-third. In other sectors, there is little trend increase in organization capital relative to R&D capital.

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30It rises slightly after the mid-2000’s, driven primarily by Amazon’s reported R&D expenditures, but remains too low to account for the physical investment gap.
31Sectoral moments obtained using this definition of intangibles are reported in Appendix Table A3.
32Prior work (Foster et al., 2006; Crouzet and Eberly, 2018) has indeed argued that the Consumer sector relies extensively on intangible capital, particularly brand capital and, in more recent years, innovations to supply chain and logistics. Investment in these intangibles are not recorded as R&D expenditures, but instead expensed as SG&A,
after 1985, and so cumulative changes in rents are similar under the two measures.

**Counterfactuals**  Figure 5 also reports a counterfactual that help highlight the differential effects of the rise in intangibles across sectors. Similarly to Section 3, we compute the cumulative change in the share of rents that would have had to occur in order to explain the investment gap, had the ratio of intangible to physical capital stayed constant over the sample. In the manufacturing sector, where intangible intensity is declining, the cumulative increase in rents would have been smaller. In the three other sectors, it would have been larger, and in some substantially so. The Healthcare sector is the most striking example; there, the increase in rents needed to account for the investment gap without a rise in intangibles would have been about 50% (or 5 p.p.) larger. In the consumer sector, the difference is approximately 30%, relative to the case where intangibles are measured including organization capital. Thus, in both of these sectors, a substantial part of the investment gap is due not purely to rising rents, but to the interaction of rising rents with high and growing intangibles.

Summarizing, the two main findings of this section are the following. First, any aggregate statement about the investment gap is likely to be misleading, as there is substantial heterogeneity, even across broadly defined sectors. The Manufacturing sector has a small investment gap, declining intangibles, and moderate rents, at odds with aggregate trends. By contrast, the Healthcare and High-tech sector are characterized by a larger investment gap than in aggregate, and one where intangibles play a bigger role, particularly in the Healthcare sector. Second, a broader empirical definition of intangibles — one that includes organization capital — generally reduces the contribution of rents to the investment gap, and particularly so in the Consumer sector, where the investment gap becomes primarily driven by intangibles.

## 5 Conclusion

This research provides a general decomposition of the gap between average $Q$ — which is observable — and marginal $q$ — the shadow value that drives movements in investment — into components reflecting the effects of rents and the effects of omitted capital. Our decomposition shows that, far from amounting to measurement error, the investment gap provides a window into the sources of firm value and their changes over time.

We used this approach to shed light on the growing divergence between physical investment and valuations, which our approach interprets as being driven by the combined effects of growing rents and growing intangibles. With a relatively narrow measure of intangibles (R&D capital), one-third of the investment gap reflects a combination of growth in the intangible capital stock and rents generated by intangible capital. Expanding the definition of intangibles beyond R&D increases this

and so they are picked up by our measure of organization capital.
contribution to about two thirds. We also argued that, because of the substantial heterogeneity across sectors, statements about the aggregate investment gap may be misleading.

Our analysis opens several important questions for future research. First, though our decomposition allows for risk premia, we remained deliberately agnostic about them in our empirical applications. A more thorough treatment of their interaction with the investment gap might be useful, particularly in understanding short-run movements in the gap. Second, though we did not explore heterogeneity within sectors, our decomposition holds at the firm level. Exploring the distribution of the investment gap across firms of particular sectors would both help validate our findings on the sources of the investment gap, and shed further light on the reasons for its growth over the last two decades. Finally, and in a different vein, we have maintained a neoclassical approach to the interaction between intangibles and rents. A broader approach, however, could allow for an economic interaction; for example, investment in intangibles such as product innovation or a software platform may generate rents to the firm. These interactions would augment the neoclassical approach we take here, and could generate additional links between intangible capital and the decisions and valuation of the firm. We pursue this in future work.
References


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<th>Targeted moments</th>
<th>Non-Financial Corporate Business sector</th>
<th>Compustat non-financials (R&amp;D)</th>
<th>Compustat non-financials (R&amp;D+ org. cap.)</th>
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<tr>
<td></td>
<td>1965</td>
<td>1984</td>
<td>2000</td>
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<tr>
<td>1. Physical investment rate</td>
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<td>0.108</td>
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<td>2. Intangible investment rate</td>
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<td>3. Intangible/physical capital</td>
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<td>4. ROA</td>
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<td>5. Q for physical capital</td>
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<td>Q1−q1</td>
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<td>s</td>
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<td>μ̃</td>
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Table 1: Summary of targeted and implied moments, for the non-financial corporate business sector (columns 3 to 5) and for the Compustat Non-Financial sample. For Compustat non-financials, columns 6 and 7 use R&D as the measure of intangibles, and columns 8 and 9 use the sum of R&D and SG&A as the measure of intangibles. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: \( K_t = K_1(1-\eta)K_2^\eta \). Rents as a fraction of value added are computed as \( s = (1-s_L)(1-1/\mu) \), where \( s_L \) is the labor share of value added for the NFCB sector. Markups over value added are computed as \( \tilde{\mu} = 1/(1-s) \). The implied moments reported are for the model with adjustment costs; the adjustment cost values are \( \gamma_1 = 3 \) and \( \gamma_2 = 12 \). In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources and construction are described in Sections 3, 4, and Appendix A.2.
Table 2: Summary of targeted and implied moments for the different sectors of the Compustat non-financial sample. All columns measure intangibles as the R&D capital stock. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: \( K_t = K_{1,t}^{1-\eta}K_{2,t}^{\eta} \). Rents as a fraction of value added are computed as \( s = (1 - s_L)(1 - 1/\mu) \), where \( s_L \) is the labor share of value added for the NFCB sector. Markups over value added are computed as \( \tilde{\mu} = 1/(1 - s) \). The implied moments reported are for the model with adjustment costs; the adjustment cost values are \( \gamma_1 = 3 \) and \( \gamma_2 = 12 \). In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources and construction are described in Section 4 and Appendix A.2.
Figure 1: The investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate (NFCB) sector. The crossed blue line is an estimate of $Q_1 - q_1$ constructed using data from the Flow of Funds and from the BEA fixed asset tables. The shaded areas present the decomposition of the physical investment gap into three terms, corresponding to the effects of rents generated by physical capital, the omitted capital effect due to intangibles, and rents generated by intangibles. The decomposition is described in Equation (15). The top panel reports results when we assume zero adjustment costs in intangible and physical capital ($\gamma_1 = \gamma_2 = 0$); the bottom panel reports results when we assume $\gamma_1 = 3$ and $\gamma_2 = 12$. Methodology and data sources are described in Section 3.
Figure 2: Other model moments for the NFCB sector. Panel (a) reports the share of intangibles in production, $\eta$, when the capital aggregator is assumed to be Cobb-Douglas: $K_t = K_{1,t}^{1-\eta}K_{2,t}^\eta$. Panel (b) reports rents as a fraction of value added, $s_{VA}$, which is given by $s_{VA} = (1 - s_L)(1 - 1/\mu)$, where $\mu$ is the model parameter governing the size of rents, and $s_L$ is labor’s share of value added. Panels (c) and (d) report user costs for each type of capital, $R_1$ and $R_2$. Methodology and data sources are described in Section 3.
Figure 3: Counterfactual exercises for the NFCB sector. The top panel reports the change in the change in the intangible share in production, $\eta$, from 1985 to 2017, when the capital aggregator is assumed to be Cobb-Douglas: $K_t = K_{1,t}^{1-\eta}K_{2,t}^{\eta}$. The blue lines report the change in the baseline decomposition; see panel (a) of Figure 2. The orange lines report the change when the parameter controlling rents, $\mu$, is set to its estimated value in 1985. The bottom panel reports the change in rents as a fraction of value added from 1985 to 2017; rents as a fraction of value added are given by $s = (1 - s_L)(1 - 1/\mu)$, where $\mu$ is the model parameter governing the size of rents, and $s_L$ is labor’s share of value added. The blue lines report the change in the baseline decomposition; see panel (b) of Figure 2. The orange lines report the change when the ratio of intangible to physical capital, $S$, and the intangible investment rate, $\iota_2$, are fixed to their 1985 values. Methodology and data sources are described in Section 3.
Figure 4: The investment gap $Q_1 - q_1$ for physical capital across sectors, using R&D as a measure of intangibles. Data is from the Compustat Non-Financial (NF) sample. We use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$, in order to construct the components of the investment gap. Methodology and data sources are described in Section 4.
Figure 5: Rents as a fraction value added in the Compustat non-financial sample. Each panel reports the change in the change in the intangible share in production, $\eta$, from 1985 to 2017, when the capital aggregator is assumed to be Cobb-Douglas: $K_t = K_{1,t}^{1-\eta} K_{2,t}^\eta$. The blue circled lines report the change obtained in the baseline exercise, using R&D as the measure of intangible capital. The green crossed line reports the change obtained when also including organization capital. Finally, the orange line with triangles reports the counterfactual change necessary to match the investment gap when the parameter controlling rents, $\mu$, is kept equal to its estimated value in 1985, in the case where R&D only is used to measure intangible capital. Methodology and data sources are described in Section 4 and Appendix A.2.
Appendix

A.1 Proofs and additional theoretical results

A.1.1 Derivation of Lemma 1

The first-order necessary condition and the envelope theorem, for each capital type, are:

\[
\Phi'_{n,t} = q_{n,t} \frac{\partial V}{\partial c_{t+1}} + \frac{\partial K_{n,t+1}}{K_{n,t+1}}
\]

Multiplying the latter condition by \( M_{t,t+1} K_{n,t+1} \), combining with the former condition, and taking expectations at time \( t \), we obtain:

\[
q_{n,t} K_{n,t+1} = \mathbb{E}_t [ M_{t,t+1} (\Pi_{n,t+1} K_{n,t+1} - \Phi_{n,t+1} K_{n,t+1}) ] + \mathbb{E}_t [ M_{t,t+2} q_{n,t+1} K_{n,t+2} ]
\]

(19)

Assuming the transversality condition \( \lim_{k \to +\infty} \mathbb{E}_t M_{t,t+k} q_{t+k-1} K_{t+k} = 0 \) holds, we can iterate forward and sum across capital types to obtain:

\[
\sum_{n=1}^{N} q_{n,t} K_{n,t+1} = \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_t [ M_{t,t+k} \{ \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+k} K_{n,t+k} \} ]
\]

(20)

(21)

On the other hand, firm value excluding distributions is given by:

\[
V^e_t = \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \left\{ \Pi_{t+k} K_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+k} K_{n,t+k} \right\} \right].
\]

(22)

Taking the difference between Equations (22) and (21) gives the result.
A.1.2 Analytical expressions

We assume that the adjustment cost function, for each type of capital, is given by:

$$\Phi_n(x) = x - 1 + \delta_n + \gamma_n r \left( x - 1 + \left( r - (x - 1) \right) \log \left( \frac{r - (x - 1)}{r} \right) \right), \quad n = 1, 2.$$  

It can be checked that this cost function is strictly convex and satisfies the standard conditions $\Phi_n(1) = \delta_n$, $\Phi'_n(1) = 1$ and $\Phi''_n(1) = \gamma_n$, $n = 1, 2$. Additionally, these functions satisfy the relationship:

$$(r - y)\Phi'(1 + y) + \Phi(1 + y) = r + \delta_n + \gamma_n ry.$$  

The case $\gamma_n = 0$, $n = 1, 2$, is the case of linear adjustment costs. The necessary first-order conditions, for each type of capital, are given by:

$$\Phi'_{n,t} = q_{n,t},$$

$$q_{n,t} = \frac{1}{1 + r} \left( \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1} \frac{K_{n,t+1}}{K_{n,t+1}} \right).$$  

Moreover, recall that $A_t$ is growing at the exogenous rate $g$. We can use this to rewrite the necessary two first-order conditions as:

$$(1 + r)\Phi'_n (1 + g_{n,t}) = \Pi_{n,t+1} - \Phi_n (1 + g_{n,t+1}) + \Phi'_n (1 + g_{n,t+1}) (1 + g_{n,t+1}),$$

where $g_{n,t} \equiv \frac{K_{n,t+1}}{K_{n,t}} - 1$. We next guess and verify that $g_{n,t} = g$ for $n = 1, 2$ is a solution. Substituting into the condition above, and re-arranging, we obtain:

$$(r - g)\Phi'_n (1 + g) + \Phi_n (1 + g) = \Pi_{n,t+1}. $$  

Using the functional form for $\Phi_n$, we can rewrite this as:

$$r + \delta_n + \gamma_n rg = \Pi_{n,t+1}. $$  

Moreover:

$$\Pi_{n,t+1} = \frac{1}{\mu} A_{t+1}^{\frac{1}{\mu} - \frac{1}{n'}} K_{t+1}^{\frac{1}{n'} - 1} \frac{\partial K_{t+1}}{\partial K_{n,t+1}}.$$  

Given our guess and the linear homogeneity of the capital aggregator, $K_{t+1}$ also grows at rate $g$. Moreover, each partial derivative $\frac{\partial K_{t+1}}{\partial K_{n,t+1}}$ is homogeneous of degree 0 in each of its arguments, implying that they only depend on the ratio $S_{t+1} = \frac{K_{2,t+1}}{K_{1,t+1}}$ of the two capital stocks. This ratio, given our guess, is constant. Hence, the right-hand side of (24), is constant. The ratio of the two capital stocks, as well as the ratio $K_t/A_t$, adjust so that the two first-order conditions (24) hold.
In particular, taking the ratio of these two first-order conditions, we obtain:

$$\frac{r + \delta_2 + \gamma_2 rg}{r + \delta_1 + \gamma_1 rg} = \frac{\partial K_{2,t}}{\partial K_{1,t}},$$

so that the ratio of the two capital stocks (which is determined entirely \(\frac{\partial K_{2,t}}{\partial K_{1,t}}\), because of the homogeneity of degree 0 of the aggregator), is constant: \(S_t = S\). Finally, note that in this model, the investment-q relationship \(\Phi'_n(1 + g_{n,t}) = q_n\), can be approximated, up to first order, by:

$$1 + \gamma_n g = 1 + \gamma_n(i_n - \delta_n) = q_n,$$

given the fact that \(\Phi'_n(x) = 1 + \gamma_n(x - 1) + o((x - 1)^2)\). Additionally, note that using Equation (24), we obtain:

$$\sum_{n=1}^N R_n K_{n,t} = \sum_{n=1}^N \Pi_{n,t} K_{n,t} = \frac{1}{\mu} \Pi_t,$$

where \(R_n \equiv \delta_n + \gamma_n rg\). Therefore, for any \(n\),

$$\frac{\Pi_t/K_{n,t}}{R_n + \sum_{m \neq n} S_{m,n} R_m} = \frac{ROA_n}{R_n + \sum_{m \neq n} S_{m,n} R_m} = \mu, \quad (25)$$

where \(S_{m,n} = K_{m,t}/K_{n,t}\).

### A.1.3 Total Q

One might wonder whether, in a model with homogeneity like the one studied in this paper, average \(Q\) for some measure of the total stock of capital properly captures the overall incentive to invest. Define total \(Q\) (Peters and Taylor, 2017) and total net investment as:

$$Q_{tot,t} \equiv V_t^e / \sum_{n=1}^N K_{n,t+1}, \quad g_{j_t,t+1} = \sum_{n=1}^N s_{n,t+1} q_{n,t+1,t+k}, \quad s_{n,t+1} = K_{n,t+1} / \sum_{m=1}^N K_{m,t+1} \quad (26)$$

In general, we have:

$$Q_{tot,t} = \sum_{n=1}^N s_{n,t+1} q_{n,t} + (\mu - 1) \sum_{n=1}^N s_{n,t+1} \sum_{k \geq 1} E_t [M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+1,t+k})]. \quad (27)$$

Thus, \(Q_{tot,t}\) is the sum of two terms: a “total marginal \(q\)” \(q_{tot,t} = \sum_{n=1}^N s_{n,t+1} q_{n,t}\), and a term reflecting the rents generated by each type of capital. Unsurprisingly, in the presence of rents \((\mu > 1)\), \(Q_{tot,t}\) overstates \(q_{tot,t}\), and does not provide a good measure of the incentive to invest.

However, even in the absence of rents \((\mu = 1)\), Total \(Q\) may not be a sufficient statistic for total
net investment. The total net investment rate is given by:

$$g_{t,t+1} = \sum_{n=1}^{N} s_{n,t+1} \Psi_{n,t} (q_{n,t} - 1),$$

In general, $g_{t,t+1}$ depends on each marginal $q$ separately; it is not a monotone function of their weighted average $q_{tot,t}$. Thus, even when $\mu = 1$, and $Q_{tot,t} = q_{tot,t}$, the latter need not be a good proxy for total net investment.

When is $q_{tot,t}$ a sufficient statistic for total net investment? A first case is when adjustment costs are identical across types of capital goods, $\Psi_{n,t} = \Psi_t$ and when the function $\Psi_t$ is linear. In that the expression above simplifies to $g_{n,t,t+1} = \Psi_t (q_{tot,t} - 1)$, and total $Q$ is indeed a sufficient statistic for total investment.

Another case is if marginal $q$ is equal across different types of capital. In this case, $q_{tot,t} = q_{n,t}$, and so $g_{t,t+1} = \sum_{n=1}^{N} s_{n,t+1} \Psi_{n,t} (q_{tot,t} - 1)$, so that $q_{tot,t}$ is a sufficient statistic for investment.

When are is marginal $q$ equalized across types of capital? The framework studied by Peters and Taylor (2017) is an example where this is the case. That framework considers cost functions belonging to a slightly more general class, $C_1(K_{1,t}, K_{2,t})$ and $C_2(K_{1,t}, K_{2,t})$. They are not additively separable, as in this paper, but they nevertheless satisfy $\frac{\partial(C_1(t) + C_2(t))}{\partial K_{1,t}} = \frac{\partial(C_1(t) + C_2(t))}{\partial K_{2,t}}$. As discussed in Appendix I.C of Crouzet and Eberly (2019), for this class of cost functions, a necessary condition for $q_{1,t} = q_{2,t}$ is intangible and physical capital are perfect substitutes, and also that they depreciate at the same rates and enter the capital aggregator with the same weights. In this sense, the conditions under which marginal $q$ is equalized across types of capital, and thus under which $Q_{tot,t}$ and $q_{tot,t}$ are relevant to understanding the behavior of total net investment, are fairly specific.

In our model, the difference between marginal $q$ for two types of capital is given by:

$$q_{n,t} - q_{m,t} = \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} (1 + g_{n,t,t+k}) \{\Pi_{n,t+k} - \Pi_{m,t+k} - (\Phi_{n,t+k} - \Phi_{m,t+k})\}].$$

Thus, a sufficient condition for equalized marginal $q$s is that (a) $\Pi_{n,t} = \Pi_{m,t}$, which, using Equation (7), implies that $\partial K_{n,t}/\partial K_{m,t} = 1$, or equivalently that capital types are perfect substitutes; and (b) adjustment costs are identical across capital types.

### A.1.4 An example with stochastic growth

**Result** Assume that the fundamentals process follows:

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \begin{cases} 1 + g_{t-1} & \text{w.p. } 1 - \lambda \\ 1 + \tilde{g} & \text{w.p. } \lambda \end{cases}$$

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Here, \( \tilde{g} \) is drawn, at time \( t \), from a distribution \( F(\cdot) \), which is time-invariant, and the draw is independent of past realizations of \( g_t \). The investment gap for physical capital is then given by

\[
G_{1,t} = \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_1) + S + \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_2)S. \tag{28}
\]

The function \( \nu(g_t) \), the expression of which is reported below, depends on the parameters \( \lambda \) and \( F(\cdot) \). When \( \lambda = 0 \), the firm’s growth rate is constant, and \( \nu(x) = x \). When \( \lambda = 1 \), the growth rate of the firm is i.i.d. In this case, letting \( \bar{g} = \mathbb{E}[\tilde{g}] \), \( \nu(x) \) satisfies

\[
\frac{1}{r - \nu(x)} = \frac{1}{r - \bar{g}} \left( 1 + \frac{x - \bar{g}}{1 + r} \right). \tag{29}
\]

The term \( \frac{1}{r - \nu(g_t)} \) is analogous to the standard Gordon growth formula, but the function \( \nu(\cdot) \) adjusts for the possibility of future “regime changes” in firm-level growth. Moreover, when growth is i.i.d., these expressions extend to the case of convex adjustment costs, with similar change to those in equation (13). Thus, the key insights from the discussion in the main text survive. In particular, even with stochastic growth, the two rents terms can be thought of as the present value of markups over the user costs of physical and intangible capital, respectively.

**Derivations of the result** We use the following lemma, adapted from Abel and Eberly (2011).

**Lemma 2.** We have:

\[
\mathbb{E}_t \left[ \sum_{k \geq 0} (1 + r)^{-k} \frac{A_{t+k}}{A_t} \right] = \frac{1 + r + \lambda(1 + g_t)}{r - g_t + \lambda(1 + g_t)} \mathbb{E} \left[ \frac{1 + r}{r - \bar{g} + \lambda(1 + \bar{g})} \right] \mathbb{E} \left[ \frac{r - \bar{g}}{r - \bar{g} + \lambda(1 + \bar{g})} \right]^{-1}. \]

**Proof.** Let:

\[
\zeta(g_t) \equiv \mathbb{E}_t \left[ \sum_{k \geq 0} (1 + r)^{-k} \frac{A_{t+k}}{A_t} \right],
\]

then \( \zeta(g_t) \) satisfies:

\[
\zeta(g_t) = 1 + \frac{1 + g_t}{1 + r} \left[ \lambda \zeta^* + (1 - \lambda) \zeta(g_t) \right],
\]

where we used the law of iterated expectations, and the fact that \( \frac{A_{t+1}}{A_t} = 1 + g_t \) is known at time \( t \). Here, we have denoted:

\[
\zeta^* = \mathbb{E}_{\tilde{g}} \left[ \zeta(\tilde{g}) \right].
\]

Solving for \( \zeta(g_t) \), we obtain:

\[
\zeta(g_t) = \frac{1 + r + \lambda(1 + g_t)}{r - g_t + \lambda(1 + g_t)} \zeta^*.
\]
Taking expectations on both sides,
\[ \zeta^* = \mathbb{E} \left[ \frac{1 + r + \lambda (1 + \hat{g}) \zeta^*}{r - \hat{g} + \lambda (1 + \hat{g})} \right] . \]

Re-arranging,
\[ \mathbb{E} \left[ \frac{r - \hat{g}}{r - \hat{g} + \lambda (1 + \hat{g})} \right] \zeta^* = \mathbb{E} \left[ \frac{1 + r}{r - \hat{g} + \lambda (1 + \hat{g})} \right] , \]

which gives the result.

The necessary first-order conditions to the firm’s problem (taking expectations at time \( t \), and using the fact that \( A_{t+1} \) is known at \( t \)) are:
\[ q_{n,t} = \frac{1}{1 + r + \lambda (1 + \hat{g})} \left( \frac{1}{\mu} A_t^{1 - \frac{1}{\mu}} K_t^{\frac{1}{\mu} - 1} \frac{\partial K_t}{\partial K_{n,t}} + 1 - \delta_n \right) , \]
\[ 1 = q_{n,t} . \]

We can rewrite this as:
\[ \mu(r + \delta_n) = A_t^{1 - \frac{1}{\mu}} K_t^{\frac{1}{\mu} - 1} \frac{\partial K_t}{\partial K_{n,t}} . \]

(29)

Tedious computation shows that these first-order conditions, in combination with the aggregator defining \( K_t \), can be written as:
\[ K_{1,t} = \left( \frac{r + \bar{\delta}}{r + \bar{\delta}_1} \right)^{\frac{1}{1 - \rho}} \eta^{\frac{1}{1 - \rho}} \left( \mu(r + \bar{\delta}) \right)^{\frac{-\mu}{\mu - 1}} A_t , \]
\[ K_{2,t} = \left( \frac{r + \bar{\delta}}{r + \bar{\delta}_2} \right)^{\frac{1}{1 - \rho}} (1 - \eta)^{\frac{1}{1 - \rho}} \left( \mu(r + \bar{\delta}) \right)^{\frac{-\mu}{\mu - 1}} A_t , \]
\[ K_t = [\mu(r + \bar{\delta})]^{\frac{-\mu}{\mu - 1}} A_t , \]

where \( r + \bar{\delta} \equiv \left( \eta^{\frac{1}{1 - \rho}} (r + \delta_1)^{-\frac{\mu}{1 - \rho}} + (1 - \eta)^{\frac{1}{1 - \rho}} (r + \delta_2)^{-\frac{\mu}{1 - \rho}} \right)^{-\frac{1 - \rho}{\rho}} \). The expression for the value of the firm reported in the main text, (5), can then be written as:
\[ V_t^e = \sum_{n=1}^{2} q_{n,t} K_{n,t+1} + (\mu - 1) \sum_{n=1}^{2} \mathbb{E}_t \left[ \sum_{k \geq 1} (1 + r)^{-k} \left( \frac{1}{\mu} A_{t+k}^{1 - \frac{1}{\mu}} K_{t+k}^{\frac{1}{\mu} - 1} \frac{\partial K_{t+k}}{\partial K_{n,t+k}} K_{n,t+k} \right) \right] \]
\[ = \sum_{n=1}^{2} q_{n,t} K_{n,t+1} + (\mu - 1) \sum_{n=1}^{2} \mathbb{E}_t \left[ \sum_{k \geq 1} (1 + r)^{-k} (r + \delta_n) K_{n,t+k} \right] \]
\[ = \sum_{n=1}^{2} q_{n,t} K_{n,t+1} + (\mu - 1) \sum_{n=1}^{2} \alpha_n (r + \delta_n) \mathbb{E}_t \left[ \sum_{k \geq 1} (1 + r)^{-k} A_{t+k} \right] \]
\[
\sum_{n=1}^{2} q_n,t K_{n,t+1} + (\mu - 1) \sum_{n=1}^{2} \alpha_n (r + \delta_n) A_{t+1} \zeta(g_t) \\
= \sum_{n=1}^{2} q_n,t K_{n,t+1} + (\mu - 1) \sum_{n=1}^{2} \frac{1}{r - \nu(g_t)} (r + \delta_n) \alpha_n A_{t+1} \\
= \sum_{n=1}^{2} q_n,t K_{n,t+1} + (\mu - 1) \sum_{n=1}^{2} \frac{1}{r - \nu(g_t)} (r + \delta_n) K_{n,t+1}.
\]

Here, using conditions (29), we defined \( \alpha_1 \) and \( \alpha_2 \) such that \( K_{1,t} = \alpha_1 A_t \) and \( K_{2,t} = \alpha_2 A_t \), and we used lemma 2. Additionally, by analogy with the case of constant growth rates, we define:

\[
\zeta(g_t) = \frac{1 + r}{r - \nu(g_t)},
\]

or equivalently:

\[
\nu(g_t) = g_t + \lambda (1 + g_t) \frac{(r - g_t) \zeta^* - (1 + r)}{(1 + r) + \lambda (1 + g_t) \zeta^*}.
\]

### A.1.5 An example with variable labor

Consider the following model. A representative household chooses consumption, \( C_t \), to solve:

\[
U_t = \max \left( \frac{(C_t)^{1-\sigma}}{1-\sigma} + \beta U_{t+1} \right),
\]

which implies the discount rate \( M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \). Labor supply by the household is exogenous and fixed. Final goods are produced and sold by a perfectly competitive firm; total final output is given by:

\[
Y_t = \left( \int_0^1 Y_{j,t}^{\hat{\mu}} dj \right)^{\hat{\mu}},
\]

with \( \hat{\mu} > 1 \). This leads to the demand curve \( Y_{j,t} = P_{j,t} Y_{t}^{\hat{\mu}} Y_t^{1-\hat{\mu}} \), and a price index is given by \( P_t = \left( \int_0^1 P_{j,t}^{\hat{\mu}} dj \right)^{1-\hat{\mu}} \). Each firm \( j \) produces an intermediate variety, with a production function taking labor and total capital as inputs:

\[
Y_{j,t} = Z_{j,t} K_{j,t}^{\alpha} L_{j,t}^{1-\alpha},
\]

where total capital is an aggregate of physical and intangible capital, as described in the main text. Here, \( Z_{j,t} \) is an exogenous process capturing firm-level or aggregate total factor productivity. Finally, the goods market clears:

\[
P_t Y_t = P_t C_t + \sum_{n=1}^{2} \int_0^1 \Phi \left( \frac{K_{n,j,t+1}}{K_{n,j,t}} \right) K_{n,j,t} dj.
\]
In this model, labor is flexible in the short-run, so each firm solves:

$$\Pi_{j,t} = \max P_{j,t} \left( \frac{P_{j,t}}{P_t} \right)^{-\frac{\bar{\mu}}{\mu-1}} Y_t - W_t L_{j,t}.$$ 

After some computation, the solution to this problem can be written as:

$$\Pi_{j,t} = A_{j,t}^{\frac{1}{\mu-1}} K_{j,t}^\frac{1}{\bar{\mu}}$$

where:

$$\mu = 1 + \frac{\bar{\mu} - 1}{\alpha},$$

and:

$$A_{j,t} = (\alpha + \bar{\mu} - 1)^{1+\frac{\alpha}{\mu-1}} \bar{\mu}^{-\frac{\mu}{\mu-1}} (1 - \alpha)^{\frac{1-\alpha}{\mu-1}} D_t W_t^{\frac{1-\alpha}{\mu-1}} Z_{j,t}^{\frac{1}{\bar{\mu}}},$$

$$D_t \equiv P_t^{\frac{\bar{\mu}}{\mu-1}} Y_t.$$ 

This example thus leads to a representation of the profit function that is the same as the one studied in the model of Section (2.3). More generally, firms in this model solve a particular case of the general model of Section 2.1. We use this model to discuss the implications of our findings for the labor share in Section (A.3.1).

### A.2 Data sources and data construction

#### A.2.1 National accounts and fixed asset tables

**Baseline** We use the following time series from NIPA, all for the non-financial corporate business sector (NFCB): NFCB gross value added ($Y^{(BEA)}$) (FRED series A455RC1Q027SBEA), NFCB compensation of employees ($WN^{(BEA)}$) (FRED series A460RC1Q027SBEA), NFCB taxes on production less subsidies ($T^{(BEA)}$) (FRED series W325RC1Q027SBEA), NFCB transfers ($Tr^{(BEA)}$) (FRED series W325RC1Q027SBEA). The data are annual. We use them to compute the surplus of the NFCB sector as:

$$\Pi^{(BEA)} = Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}$$

and to compute the labor share of the NFCB sector as:

$$LS = WN^{(BEA)} / (Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}).$$

We use the labor share only to translate our estimates of the model parameter governing rents, $\mu$, into the share of rents as a fraction of value added. Additionally, we obtain current cost measures of the capital stock for the NFCB sector from the BEA fixed asset tables. We extract $K_{struct}^{(BEA)}$, $K_{equip}^{(BEA)}$ and $K_{intan}^{(BEA)}$, from BEA table 4.1; in particular, we define $K_{intan}^{(BEA)}$ as the stock of intellectual
property products.\footnote{The tables are available at \url{https://apps.bea.gov/iTable/iTable.cfm?ReqID=10&step=2}.} We then define:

\[
K_1^{(BEA)} = K_{\text{struct}}^{(BEA)} + K_{\text{equip}}^{(BEA)}, \quad K_2^{(BEA)} = K_{\text{intan}}^{(BEA)}.
\]

We use table 4.7 to obtain measures of current investment for the NFCB sector, and we define \(I_1^{(BEA)}\) and \(I_2^{(BEA)}\) analogously to \(K_1^{(BEA)}\) and \(K_2^{(BEA)}\). Note that all time series from tables 4.1 and 4.7 are expressed in current dollar values; we only use them in the computation of ratios. We use table 4.2 to obtain the chain-type quantity index for total non-residential capital for the NFCB sector; we measure \(g_t\) as the annual growth rate in this quantity index. Finally, in Appendix Figure A2, we also report the ratios of historical-cost depreciation rates over current cost measures of the capital stock, for physical and intangible capital separately; depreciation costs obtained from BEA fixed asset table 4.6.

**Residential assets** Our baseline approach only includes non-residential fixed assets. As a robustness check, we obtained residential fixed assets \(K_{\text{resid}}^{(BEA)}\) as the difference between the sum of the three capital stocks above, and total fixed assets of NFCB sector report in BEA fixed asset table 6.1, and likewise for investment. The top panel of Figure A16 reports time series for the ratio \(\Pi^{(BEA)}/K^{(BEA)}\). The solid red and solid orange line use \(\Pi^{(BEA)}\) as the numerator, and for the total capital stock, either \(K = K_{\text{struct}}^{(BEA)} + K_{\text{equip}}^{(BEA)} + K_{\text{intan}}^{(BEA)}\) (as in our baseline analysis), or \(K = K_{\text{struct}}^{(BEA)} + K_{\text{equip}}^{(BEA)} + K_{\text{intan}}^{(BEA)} + K_{\text{resid}}^{(BEA)}\). The two lines are almost identical. The stock of residential fixed assets in the NFCB sector thus appears to be low relative to other types of fixed assets owned by the NFCB sector, and so we abstract from it in our analysis.

**Economy-wide vs. NFCB measures** Finally, Figure A16 also compares our measures of \(\Pi/K\) with those reported by Farhi and Gourio (2018), who study economy-wide trends, instead of the NFCB sector specifically. The rate of return on capital measured by these authors is substantially lower than our measures of rates of return for the NFCB sector (by about 5-7% throughout the sample.) Here, we briefly discuss why this is the case, as it matters for inferences about the importance of rents. These authors compute \(\Pi/K\) as:

\[
\Pi/K = \left( (Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}) / (Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}) \right) \times Y/K,
\]

where \(Y\) is total nominal GDP (including other sectors than the NFCB) and \(K\) is the total private capital stock (at replacement cost). This adjustment is made in order to maintain comparability with other ratios in their analysis, which has a broader scope than the NFCB. By contrast, our measures of \(\Pi/K\) are:

\[
\Pi/K = \left( (Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}) / (Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}) \right)
\]
\[ \times \left( Y^{(BEA)} - T^{(BEA)} - T_r^{(BEA)} \right) / K^{(BEA)} \].

Thus the differences between our measures of \( \Pi / K \) and the measures in Farhi and Gourio (2018) must be due to differences in the ratio of value added to capital between the NFCB sector and the economy as a whole. The bottom panel of Figure A16 indeed shows that the NFCB sector has a substantially higher dollar of value added per dollar of capital at current cost. The most accurate comparison is between the crossed blue line of the bottom panel, and the orange solid line, which measures \( K \) for the NFCB sector as the sum of all types of capital (residential, non-residential physical, and non-residential intangible): the value added to capital ratio is approximately 10 percentage points higher in the NFCB sector versus the economy as a whole.

### A.2.2 Compustat non-financial

#### Sample selection
We use the annual version of the Compustat-CRSP merged files. We apply the standard screens (indfmt=INDL, popsrc=D, consol=C, datafmt=STD). We keep firm-year observations that satisfy the following criteria: fic=USA (domestically incorporated), 2-digit SIC code (first two digits of the variable sic) not equal to 49 (utilities), not between 60 and 69 (finance and real estate), and not between 90 to 99 (public administration); 2-digit SIC code not missing; variable sale (sales) and at (assets) not missing; variables emp, sale, at, act, lct, ppent, ppegt, che, and gdwl not negative. Finally, we drop any observation which we can identify as an American Depository Institution (ADR). We use only data from 1974 onward (included), as the data prior to 1974 has incomplete coverage (a jump in the number of firms in the sample occurs from 1973 to 1974.)

#### Variable construction
As for NFCB sector, for each firm, we start by constructing six time series in levels, \( \{K_{1,t}, I_{1,t}, K_{2,t}, I_{2,t}, \Pi_t, V_t\} \). For physical capital investment, we use capital expenditures (capx) net of sales of property, plant and equipment (sppe); we measure the stock using gross property, plant and equipment (ppegt), for reasons discussed below. We consider two definitions of intangibles: R&D capital, and organization capital. For R&D, we use reported R&D expenditure (xrd), recoding missing values with 0. For investment in organization capital, we follow Eisfeldt and Papanikolaou (2013) and use 30% of SG&A expenditures (variable xsga) net of R&D investment. For the stock of both R&D and organization capital, we used the capitalized values provided by Peters and Taylor (2017). A limitation of this approach is that it does not allow for either rates of depreciation, or for the imputation of investment in organization capital, to vary across industries. This can be done for R&D capital by using the industry-level depreciation rate produced by the BEA for the fixed asset tables, as in Belo et al. (2018). No similar data source exists for organization capital, however. For \( \Pi_t \), we use the Compustat variable oibdp. We add estimates of intangible investment expenditures to actual measures of operating income in order to obtain an adjusted operating surplus measure consistent with our model. For \( V_t \), we use the sum of the market value of common stock and the book value of debt, net of cash and liquid securities. We then take the
sum of these time series across firms either by year (when studying all publicly traded firms jointly), or by year and sector (when constructing the sectoral investment gaps.) Finally, we construct the growth rate of total capital at either the aggregate or sectoral level by subtracting from the growth rate of $K_{1,t} + K_{2,t}$ the deflator implicitly used in Section 3, that is, the difference between nominal and real growth rates of total non-residential fixed assets for the NFCB sector.

**Relationship between NFCB sector data and Compustat NF data**  There are two potentially important differences between the data used in Section 3 and the Compustat data. First, Compustat only includes publicly traded corporations. There may be systematic differences in returns to capital and intangible intensity between privately held and publicly traded corporations. Second, the measurement of the stock of physical capital differs across sources. We next discuss these differences in more detail.

In Compustat, our baseline measure of surplus as the sum of ebitda across all observations in our sample. (Missing observations are thus treated as zeros.) We use ebitda because it is the financial statement measure most closely related to our model definition of $\Pi$; it measures of operating income before depreciation, and does not deduct costs of capital, or non-operating income, which our model does not capture. The top right panel of figure A17 report the NFCB sector surplus measured in this manner in Compustat, and the measure from the BEA tables. The two are highly correlated, but their levels differs substantially. This reflects the fact that the BEA NFCB sector data also includes private firms. The surplus of public firms (from Compustat) represents about two thirds of the total surplus of the NFCB sector (from the BEA).

The main difficulty in the Compustat data is in computing estimates of the current-cost total stock of physical capital. A natural definition would seem to be net property, plant and equipment (variable ppent). However, measuring $K_1$ for the NFCB sector in Compustat leads to extremely elevated measures of $\Pi/K_1$, as reported in the bottom right panel of figure A17. These measures are almost double the BEA-derived measures. This is primarily because the aggregate value of ppent in Compustat is only about a third of physical capital in the NFCB sector according to BEA data (top left panel of figure A17). The reason for this gap are unclear. One hypothesis is that the surplus of Compustat firms includes income from foreign subsidiaries, and so could overestimate the true surplus of public NFCB firms. Alternatively, it could be that private firms indeed have much lower rates of return on physical capital than public firms do (though the gap would have to be very large, given the relative importance of public firms in total surplus, as indicated in the top right panel of figure A17). The more likely reason is that the accounting treatment of depreciation may lead the (balance sheet) net stock to underestimate the true current cost stock of physical assets. The red line in the top left and bottom left panels of figure A17 instead report measures of asset returns using aggregate gross property, plant and equipment at historical cost (deflated using the implicit deflator from the BEA fixed tables). The bottom left panel shows that this estimate of $K_1$ leads to values of $\Pi/K_1$ that align more closely (in levels) with those provided by the BEA data.

34The inclusion of non-operating income makes little difference to the results.
on the NFCB. In what follows, in order to align our BEA and Compustat profitability moments as closely as possible, we therefore use gross property, plant and equipment as our main measure of $K_1$ in Compustat data.

We measure (gross) investment in physical capital in Compustat using capital expenditures (variable capx) minus sales of property, plant and equipment (variable sppe). Figure A18, top panel, shows that physical investment, computed in this manner, accounts for about two thirds of total physical investment in the BEA NFCB sector ($I_{1}^{(BEA)}$), with closely related cyclical movements. For investment rates (the bottom panel of figure A18), the data again suggest a much higher investment rate in Compsutat when $K_1$ is measured using net book values, but investment rates are closer in levels when the capital stock is measured using gross book values.

### A.3 Additional empirical results

#### A.3.1 Implied markups and the labor share

In the model with labor, described in appendix A.1.5, labor demand is given by:

\[
P_{j,t} = \tilde{\mu}MC_{j,t}
\]

\[
MC_{j,t} = (1 - \alpha)^{\frac{(1-\alpha)(\tilde{\mu}-1)}{\mu-1+\alpha}}W_t^{\frac{\alpha(\tilde{\mu}-1)}{\mu-1+\alpha}}K_{j,t}^{\frac{(\tilde{\mu}-1)\alpha}{\mu-1+\alpha}}
\]

\[
L_{j,t} = \left(\frac{(1-\alpha)MC_{j,t}Z_j}{W_t}\right)^{\frac{1}{\alpha}}K_{j,t}
\]

where:

\[
MC_{j,t} = (1 - \alpha)^{\frac{(1-\alpha)(\tilde{\mu}-1)}{\mu-1+\alpha}}W_t^{\frac{\alpha(\tilde{\mu}-1)}{\mu-1+\alpha}}K_{j,t}^{\frac{(\tilde{\mu}-1)\alpha}{\mu-1+\alpha}}
\]

Since prices are given by: $P_{j,t} = \tilde{\mu}MC_{j,t}$, the labor demand curve implies that:

\[
LS = \frac{W_tL_{j,t}}{P_{j,t}Y_t} = \frac{1 - \alpha}{\tilde{\mu}}.
\]

In order to map this model to the data on labor shares, a first approach consists in deriving $\mu$, our rents parameter (which is, more specifically, the firm’s markup over its operating surplus), from data on return on assets and user costs, as in our baseline, and then use $LS$, the labor share of value added, to recover the corresponding value of $\tilde{\mu}$, the markup over value added. Specifically:

\[
\tilde{\mu} = \alpha(\mu - 1) + 1 = (1 - \tilde{\mu}LS)(\mu - 1) + 1,
\]

and so, solving for the markup for value added:

\[
\tilde{\mu} = \frac{\mu}{\mu LS + (1 - LS)}.
\]
However, this approach implicitly assumes that $1 - \alpha$, the Cobb-Douglas exponent for labor in the production function, is varying over time, at least to the extent that the labor share varise. Specifically, our procedure also implies that $1 - \alpha = \mu LS / (\mu LS + (1 - LS))$. The top panel of figure A7 shows the implied value for $1 - \alpha$ in our baseline exercise. The mean is approximately 0.72. Moreover, the implied value declines from 0.74 to 0.70 during the 2000’s, along with the decline in LS.

An alternative approach is to fix the Cobb-Douglas labor exponent. In that case, we do not require data on the labor share to obtain the valued-added markup $\tilde{\mu}$ implied by our estimate of the rents parameter $\mu$; it can simply be obtained from $\tilde{\mu} = \alpha(\mu - 1) + 1$. This value-added markup (not reported) it virtually indistinguishable from the markup implied by the first approach described above. Additionally, approach produces an implied labor share that is given by $LS = (1 - \alpha) / \tilde{\mu}$. The bottom panel of figure A7 reports the path of this implied labor share, and compares it to the data. The magnitude of the decline in the implied labor is similar to its empirical counterpart, but the timing is somewhat different. The reason is that the rents parameter $\mu$ starts rising in the mid-80’s, along with the rise in the investment gap, whereas the labor share only starts declining in the late 2000’s.
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</tr>
<tr>
<td></td>
<td>Truck Transportation</td>
<td>4840</td>
<td>484</td>
<td>484</td>
</tr>
<tr>
<td>Other</td>
<td>Other Transportation and Support Activities</td>
<td>487S</td>
<td>487 to 488, 492</td>
<td>487 to 488</td>
</tr>
<tr>
<td></td>
<td>Air Transportation</td>
<td>4810</td>
<td>481</td>
<td>481</td>
</tr>
<tr>
<td></td>
<td>Rail Transportation</td>
<td>4820</td>
<td>482</td>
<td>482</td>
</tr>
<tr>
<td></td>
<td>Pipeline Transportation</td>
<td>4860</td>
<td>486</td>
<td>486</td>
</tr>
<tr>
<td></td>
<td>Water Transportation</td>
<td>4830</td>
<td>483</td>
<td>483</td>
</tr>
</tbody>
</table>

Table A1: Industry classification used in the sectoral analysis of Section 4.
### Table A2: Industries excluded from the sectoral analysis of Section 4.

<table>
<thead>
<tr>
<th>Subsector</th>
<th>BEA</th>
<th>KLEMS/BLS</th>
<th>NAICS</th>
<th>Reason for excluding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Estate</td>
<td>5310</td>
<td>531</td>
<td>531</td>
<td>FIRE</td>
</tr>
<tr>
<td>Rental and Leasing Services and Lessor of Intangible Assets</td>
<td>5320</td>
<td>532 to 533</td>
<td>532 to 533</td>
<td>FIRE</td>
</tr>
<tr>
<td>Federal Reserve Banks, Credit Intermediation, and Related Activities</td>
<td>5210 and 5220</td>
<td>521 to 522</td>
<td>521 to 522</td>
<td>FIRE</td>
</tr>
<tr>
<td>Insurance Carriers and Related Activities</td>
<td>5240</td>
<td>524</td>
<td>524</td>
<td>FIRE</td>
</tr>
<tr>
<td>Securities, Commodity Contracts, and Investments</td>
<td>5230</td>
<td>523</td>
<td>523</td>
<td>FIRE</td>
</tr>
<tr>
<td>Funds, trusts, and other financial vehicles</td>
<td>5250</td>
<td>525</td>
<td>525</td>
<td>FIRE</td>
</tr>
<tr>
<td>Forestry, Fishing, and Related Activities</td>
<td></td>
<td></td>
<td>113 to 115</td>
<td>Not enough Compustat observations</td>
</tr>
<tr>
<td>Transit and ground passenger transportation</td>
<td>4850</td>
<td>485</td>
<td>485</td>
<td>Not enough Compustat observations</td>
</tr>
<tr>
<td>Warehousing and storage</td>
<td>4930</td>
<td>493</td>
<td>493</td>
<td>Not enough Compustat observations</td>
</tr>
<tr>
<td>Legal services</td>
<td>5411</td>
<td>5411</td>
<td>5411</td>
<td>Not enough Compustat observations</td>
</tr>
<tr>
<td>Management of companies and enterprises</td>
<td>5500</td>
<td>55</td>
<td>55</td>
<td>Not enough Compustat observations</td>
</tr>
<tr>
<td>Social assistance</td>
<td>6240</td>
<td>624</td>
<td>624</td>
<td>Not enough Compustat observations</td>
</tr>
<tr>
<td>Performing arts, spectator sports, museums, and related activities</td>
<td>711A</td>
<td>711 to 712</td>
<td>711 to 712</td>
<td>Not enough Compustat observations</td>
</tr>
</tbody>
</table>
### Compustat non-financials (Intangibles = R&D + organization capital)

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_1 )</td>
<td>Physical investment rate</td>
<td>0.128 0.098</td>
<td>0.139 0.101</td>
<td>0.105 0.082</td>
</tr>
<tr>
<td>( i_2 )</td>
<td>Intangible investment rate</td>
<td>0.278 0.261</td>
<td>0.302 0.292</td>
<td>0.235 0.196</td>
</tr>
<tr>
<td>( S )</td>
<td>Intangible/physical capital</td>
<td>0.799 0.813</td>
<td>0.541 0.548</td>
<td>0.729 1.171</td>
</tr>
<tr>
<td>( ROA_1 )</td>
<td>Return on physical capital</td>
<td>0.489 0.485</td>
<td>0.443 0.478</td>
<td>0.448 0.588</td>
</tr>
<tr>
<td>( Q_1 )</td>
<td>Av. Q for physical capital</td>
<td>2.672 2.651</td>
<td>2.937 3.261</td>
<td>3.064 4.306</td>
</tr>
<tr>
<td>( g )</td>
<td>Growth rate of total capital stock</td>
<td>0.054 0.037</td>
<td>0.065 0.014</td>
<td>0.046 0.028</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_1 - q_1 )</td>
<td>Investment gap</td>
<td>1.523 1.645</td>
<td>1.634 2.424</td>
<td>1.908 3.329</td>
</tr>
<tr>
<td>% rents from physical capital</td>
<td>9 14</td>
<td>13 32</td>
<td>13 18</td>
<td>-21 35</td>
</tr>
<tr>
<td>% intangibles</td>
<td>77 63</td>
<td>73 29</td>
<td>67 45</td>
<td>132 48</td>
</tr>
<tr>
<td>% rents from intangibles</td>
<td>14 23</td>
<td>14 39</td>
<td>20 38</td>
<td>-11 17</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Intangible share in production</td>
<td>0.606 0.627</td>
<td>0.507 0.546</td>
<td>0.580 0.683</td>
</tr>
<tr>
<td>( s )</td>
<td>Rents as a fraction of value added</td>
<td>0.011 0.027</td>
<td>0.019 0.073</td>
<td>0.026 0.065</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>User cost of physical capital</td>
<td>0.185 0.167</td>
<td>0.197 0.174</td>
<td>0.171 0.150</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>User cost of intangible capital</td>
<td>0.354 0.346</td>
<td>0.384 0.378</td>
<td>0.323 0.278</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Curvature of operating profit function</td>
<td>1.040 1.086</td>
<td>1.075 1.273</td>
<td>1.103 1.240</td>
</tr>
<tr>
<td>( \tilde{\mu} )</td>
<td>Markup over value added</td>
<td>1.011 1.028</td>
<td>1.020 1.079</td>
<td>1.027 1.070</td>
</tr>
</tbody>
</table>

Table A3: Summary of targeted and implied moments for the different sectors of the Compustat non-financial sample. All columns measure intangibles as the sum of the R&D capital stock and the organization capital stock. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: \( K_t = K_{1,t}^{1-\eta} K_{2,t}^\eta \). Rents as a fraction of value added are computed as \( s = (1-s_L)(1-1/\mu) \), where \( s_L \) is the labor share of value added for the NFCB sector. Markups over value added are computed as \( \tilde{\mu} = 1/(1-s) \). The implied moments reported are for the model with adjustment costs; the adjustment cost values are \( \gamma_1 = 3 \) and \( \gamma_2 = 12 \). In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources and construction are described in Section 4 and Appendix A.2.
Figure A1: Time series for the moments used in the construction of the physical investment gap decomposition, Equation (15). Returns to physical capital are defined as $\Pi_t / K_{1,t}$, where $\Pi_t$ is operating surplus and $K_{1,t}$ the stock of physical capital at current cost. Investment rates are defined as $i_{n,t} = I_{n,t} / K_{n,t}$, $n = 1, 2$, where $n = 1$ indexes physical capital and $n = 2$ indexes intangible capital, $K_{2,t}$ is the stock of intangible capital at current cost, and $I_{n,t}$ are investment expenditures for each type of capital. The ratio of intangible to physical capital is $S_t = K_{2,t} / K_{1,t}$. Average Tobin’s $Q$ of physical capital is defined as $Q_{1,t} = V_t / K_{1,t}$, where $V_t$ is an estimate of the total market value of net claims on the sector. The time series are the raw data; in particular, they are not averaged over seven-year windows. Data sources are described in Section 3 and in Appendix A.2.
Figure A2: Implied moments for the NFCB sector. Panels (a) and (b) reports implied depreciation rates, computed as $\delta_i = R_i - r - \gamma_i r g$, $i = 1, 2$. Additionally, the panels report depreciation as a fraction of the stock of capital, from the BEA fixed asset tables. Panel (c) reports the discount rate implied by our approach, computed as $r = (r - g) + g$, where $g$ is the real growth rate of the capital stock. Also reported in panel (c) is a measure of the risk-free real interest rate, equal to the interest rate on ten-year Treasury constant maturity rate minus the growth rate of the GDP deflator prior to 2003, and the interest rate on the ten-year Treasury inflation-index security after 2003. Finally, panel (d) reports the model-implied PD ratio, along with the PD ratio estimated for the S&P500, adjusting or not for leverage. None of the data moments reported in this graph are used in the construction of the investment gap. Methodology and data sources are described in Section 3 and Appendix A.2.
Figure A3: Additional data sources used in the robustness checks for the non-financial corporate (NFCB) sector. The top panel reports our baseline measure of average Tobin’s Q for physical capital, $Q_1$ (black line), and an alternative measure based on Hall (2001). The difference between the two is that our baseline measure only nets out financial assets identified as liquid in the Flow of Funds in the computation of the net value of claims on the NFCB sector. The bottom panel reports two measures of the PD ratio for the NFCB sector, both using the cum- and ex-dividend returns on the S&P500. The grey squared line does not adjust for leverage, and the black line adjusts for leverage, using data from the Flow of Funds on the NCFB sector. Data sources and methodology for the construction of the empirical price-dividend ratio measures are reported in Appendix A.2.
Figure A4: Robustness: adjustment costs. Each panel reports a moment from the investment gap decomposition, Equation (15), for the NFCB sector, for different combinations of adjustment costs for physical and intangible capital. In each panel, a point corresponds to a particular combination for \((\gamma_1, \gamma_2)\), and the color corresponds to the value of the moments, with the correspondence reported on the right axis. Panel (a) reports the change in \(Q_1 - q_1\) from 1985 to 2015; in our baseline results, this moment is equal to 1.30. Panel (b) reports the contribution of intangibles to \(Q_1 - q_1\) in 2015; in our baseline results, this moment is equal to 0.39 (or 39%). Panel (c) reports the implied intangible share in production in 2015; in our baseline results, this moment is equal to 0.29. Panel (d) reports rents as a share of value added in 2015; in our baseline moment is equal to 0.063. Our baseline results use \(\gamma_1 = 3\) and \(\gamma_2 = 12\). Methodology and data sources are described in Section 3.
Figure A5: Robustness: the investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate (NFCB) sector when using the Hall (2001) measure of Tobin’s average $Q$ for physical assets. The top panel reports results when we assume zero adjustment costs in intangible and physical capital ($\gamma_1 = \gamma_2 = 0$); the bottom panel reports results when we assume $\gamma_1 = 3$ and $\gamma_2 = 12$. The series for $Q_1$ from Hall (2001) is reported in panel (a) of Figure A3. Methodology and data sources are described in Section 3.
(a) Zero adjustment costs

(b) Positive adjustment costs

**Figure A6**: Robustness: the investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate (NFCB) sector when targeting the PD ratio, instead of Tobin’s average $Q$ for physical assets. The top panel reports results when we assume zero adjustment costs in intangible and physical capital ($\gamma_1 = \gamma_2 = 0$); the bottom panel reports results when we assume $\gamma_1 = 3$ and $\gamma_2 = 12$. The series used for the PD ratio is the based on cum- and ex-dividend returns for the S&P500, and adjusted for leverage; it is reported in panel (b) of Figure A3. Methodology and data sources for constructing the decomposition are described in Section 3, and methodology and data sources for the construction of the empirical price-dividend ratio measure are reported in Appendix A.2.
Figure A7: Cobb-Douglas exponent on labor and labor share implied by two alternative approaches. The top panel reports the value of the Cobb-Douglas exponent on labor, $1 - \alpha$, obtained when using the model described in appendix A.1.5 and matching the labor share. The bottom panel reports the labor share obtained by fixing the parameter $\alpha$ to 0.3 and inferring markups from user cost data.
**Figure A8:** Time series moments for Compustat Non-Financial (NF), all sectors (aggregated). The corresponding time series moments for the aggregate non-financial corporate business (NFCB) sector are also reported, for comparison. All variables are defined as in Figure A1. Data sources for the NFCB sector are described in Section 3 and Appendix A.2. Data sources for Compustat NF are described in Section 4 and Appendix A.2.
Figure A9: The investment gap $Q_1 - q_1$ for physical capital in the Compustat Non-Financial (NF) sample. In this figure, data from all sectors is pooled and aggregated. The top panel reports results when only R&D is used to measure intangibles. The bottom panel reports results when both R&D and organization capital are used to measure intangibles, where organization capital is measured as in Eisfeldt and Papanikolaou (2013). We use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$, in order to construct the components of the investment gap. Methodology and data sources are described in Section 4.
Figure A10: Other model moments for the Compustat Non-Financial (NF) sample. Panel (a) reports the share of intangibles in production, $\eta$, when the capital aggregator is assumed to be Cobb-Douglas: $K_t = K_{1,t}^{1-\eta}K_{2,t}^{\eta}$. Panel (b) reports rents as a fraction of value added, $s_{VA}$, which is given by $s_{VA} = (1 - s_L)(1 - 1/\mu)$, where $\mu$ is the model parameter governing the size of rents, and $s_L$ is labor’s share of value added. Panels (c) and (d) report user costs for each type of capital, $R_1$ and $R_2$. We use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$, in order to construct the components of the investment gap. Methodology and data sources are described in Section 4.
Figure A11: Time series moments for the Consumer sector, in the Compustat non-financials sample. All variables are defined as in Figure A1. Data sources for Compustat NF are described in Section 4 and Appendix A.2. The sectoral classification is described in Appendix Tables A1 and A2.
Figure A12: Time series moments for the High-tech sector, in the Compustat non-financials sample. All variables are defined as in Figure A1. Data sources for Compustat NF are described in Section 4 and Appendix A.2. The sectoral classification is described in Appendix Tables A1 and A2.
Figure A13: Time series moments for the Healthcare sector, in the Compustat non-financials sample. All variables are defined as in Figure A1. Data sources for Compustat NF are described in Section 4 and Appendix A.2. The sectoral classification is described in Appendix Tables A1 and A2.
Figure A14: Time series moments for the Manufacturing sector, in the Compustat non-financials sample. All variables are defined as in Figure A1. Data sources for Compustat NF are described in Section 4 and Appendix A.2. The sectoral classification is described in Appendix Tables A1 and A2.
Figure A15: The investment gap $Q_1 - q_1$ for physical capital across sectors, using the sum of R&D and organization capital to measure intangibles. Data is from the Compustat Non-Financial (NF) sample. We use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$, in order to construct the components of the investment gap. Methodology and data sources are described in Section 4.
Figure A16: Comparison between alternative measures of $\Pi_t/(K_1,t + K_2,t)$ (surplus per unit of total capital) and $Y_t/(K_1,t + K_2,t)$ (value added per unit of capital) in BEA data. The construction of each time series is discussed in Appendix A.2.1. The blue line reproduces the measures of $\Pi/K$ and $Y/K$ used in Farhi and Gourio (2018), which differ from our measures primarily because we focus only on the NFCB sector, as discussed in Appendix A.2.1.
Figure A17: Measures of the total physical capital stock at current cost ($K_1$), of surplus ($\Pi$), and of the ratio of surplus to capital ($\Pi/K_1$) in BEA and Compustat data. All nominal data are deflated using the CPI with base 2009. Differences between the series are discussed in appendix A.2.
Figure A18: Measures of the dollar value of investment in the NFCB sector (top panel), and of the investment rate (bottom panel). Dashed lines are measures obtained using BEA data, while solide lines are measures obtained using Compustat data. Differences between the series are discussed in appendix A.2.