Rents and Intangible Capital: A Q+ Framework*

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Abstract

In recent years, US investment has been lackluster, despite rising valuations. Key explanations include growing rents and growing intangibles. We propose and estimate a framework to quantify their roles. The gap between valuations — reflected in average Q — and investment — reflected in marginal q — can be decomposed into three terms: the value of installed intangibles; rents generated by physical capital; and an interaction term, measuring rents generated by intangibles. The intangible-related terms contribute significantly to the gap, particularly in fast-growing sectors. Our findings suggest care in a pure-rents interpretation, given the rising role of intangibles.

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1 Introduction

Recent research highlights two apparently contradictory, medium-run facts about the US economy: returns to business capital, and corporate profits more generally, have been either stable or growing (Gomme et al., 2011); yet investment has been lackluster, in particular relative to corporate valuations (Gutiérrez and Philippon, 2017; Alexander and Eberly, 2018). *Ceteris paribus*, investment theory would predict a rise in investment in response to higher returns to capital and corporate valuations.

In neoclassical models, the divergence between returns and investment can be cast as a rising gap between the average value of business capital, or Tobin’s *average* $Q$, and its marginal value, or Tobin’s *marginal* $q$. We directly observe rising average $Q$ in the data, via market values, while marginal $q$ is a shadow value measured implicitly by lackluster investment. A gap between the average value of capital and its marginal value can arise and grow for a number of reasons. Two leading explanations have recently emerged: intangible capital and rents.

Over the last several decades, intangible capital has grown as a share of investment and as a share of assets (Corrado et al., 2005, 2009). A shift toward intangibles in production could cause physical investment to appear low relative to valuations. Typical measures, such as Tobin’s average $Q$, increasingly underestimate the true stock of assets, and thus increasingly overstate the incentive to invest in physical capital (Gutiérrez and Philippon, 2017; Alexander and Eberly, 2018; Crouzet and Eberly, 2019).

Alternatively, the gap between average $Q$ and marginal $q$ may be explained by market power. Rising market power and its corresponding rents can account for a stable or rising rate of return on assets despite a falling user cost of capital. Rising rents also reduce the marginal return to additional capital, consistent with a weaker incentive to invest. Several recent papers indeed document a rise in the measured capital share over the last three decades, which, along with declining required returns to capital, is consistent with higher rents (Barkai, 2020; Gutiérrez and Philippon, 2018a).

From a positive perspective, both intangibles and rents have the potential to explain the divergence between returns and investment. However, the normative implications of the two mechanisms may differ. Rising intangibles reflect supply-side changes in the organization of production (Haskel and Westlake, 2018), with no clear implications for welfare. By contrast, rising rents could be associated with deadweight losses, for instance if they are due to price markups (De Loecker et al., 2020) or wage markdowns (Benmelech et al., 2018).  

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1 The normative implications of rising rents and reduced competition can however depend on the economic environment. Among many others, Aghion et al. (2005), for instance, provide an example of a model in which reduced competition may be associated with increased innovation.
Any normative or policy conclusion drawn from the divergence between investment and returns thus requires a careful assessment of which of the two mechanisms is most relevant in practice. However, most of the literature has considered each of these mechanisms in isolation, which tends to overstate their respective explanatory power. The goal of this paper is to assess them jointly, and in doing so, to provide a quantitative estimate of the role of each in the divergence between returns and investment. To do this, we extend the Q-theory model (Hayashi, 1982; Abel and Eberly, 1994) to simultaneously allow for the presence of economic rents and the accumulation of a stock of intangible assets. We call this model the "Q+" framework.

Using this framework, we make two main contributions. First, from a theoretical perspective, we show that the gap between average Q and marginal q for physical capital, which we call the "investment gap", can be decomposed into three distinct terms: a term capturing rents to physical capital, a term capturing the value of installed intangible capital, and a term capturing rents to intangible capital. The last element of this decomposition, an interaction term that is new to our analysis, is particularly important: it clarifies the fact that rising rents and rising intangibles cannot be meaningfully analyzed in isolation, as their interaction contributes to the gap between investment and returns. Moreover, this decomposition is very general, as our framework nests a number of existing investment models.

Second, we show that this interaction term is empirically important to the recent rise in the investment gap. Importantly, we show how each term in our decomposition can be quantified using data on profits, investment, valuations, and estimates of the intangible capital stock within the structure of the model. In aggregate data, the interaction term accounts for between one-quarter and one-half of the investment gap, depending on how broad the definition of intangibles is. In addition, our approach leads to lower estimates of the increase in total rents than existing work. As we show, this is equivalent to a smaller estimate of the decrease in total user costs of capital. This occurs because while including intangibles raises valuations, it also boosts the user cost of capital due to higher depreciation rates (hence reducing rents).

Finally, we move beyond the aggregate data, recognizing that economy-wide increases in rents and intangibles may be driven by composition effects across sectors. In fact, we find that the aggregate investment gap is driven by fast-growing industries, such as Healthcare and Tech. Moreover, these industries’ investment gaps are mostly explained by intangibles, even when intangibles are narrowly measured. We also show that among the subsectors of Healthcare, Tech, and Manufacturing, only a subset experienced rising rents, and those that did generally also experienced a rise in intangible intensity. Taken together, these empirical results suggest that the investment gap in these industries reflects a change in the factors of production, rather than unequivocal and broad evidence of rising market power.
In Section 2, we develop and analyze the "Q+" framework. The gap between average Q and marginal q, which we call the "investment gap", is our main focus. We show how this gap can be decomposed into three distinct terms: a term capturing rents to physical capital, a term capturing the value of installed intangibles, and a term capturing rents to intangible capital. The first two terms would obtain, respectively, in a model without rents (but with intangibles), and in a model without intangibles (but with rents). When both are present in the model, a third term appears, which captures the economic rents earned by intangible capital. The model demonstrates how these can be identified separately from rents earned by physical capital. The result is independent of the specifics of exogenous processes and of capital adjustment cost and revenue functions, so long as they satisfy simple homogeneity assumptions. We also provide versions of the framework in which each of these terms can be solved in closed form. These analytical expressions clarify the key forces driving the effects of rents, intangibles, and their interaction. In particular, rents on intangible capital are the present value of markups multiplied by an appropriately defined user cost, which takes into account adjustment costs. This user cost is large for intangible capital because intangibles depreciate quickly, foreshadowing our findings on the quantitative importance of rents generated by intangibles.

In Section 3, we apply this decomposition to aggregate data, after showing how to estimate the components of the investment gap using moments of corporate profits, investment, valuations, and estimates of the intangible stock. We begin with data from US national accounts, which are broader in coverage, but provide a narrower definition of intangibles, as they focus on R&D capital. Two periods stand out with large investment gaps: the 1965-1975 decade, and the post-1990 period. Most interestingly, the composition of the gap is different between these two periods: whereas the 1965-1975 gap is mostly driven by rents generated by physical capital, approximately 40% of the post-1990’s gap is due to the intangibles-related terms. The term capturing rents to intangibles is sizable, accounting for 25% of the gap, with the direct intangibles effect making up the other 15%. The post-1990’s change is driven by three underlying trends. First, the share of intangibles approximately doubles. Second, the user costs of intangibles are not only much higher, but also more stable than those of physical capital. We infer this from the fact that gross intangible investment rates are stable and elevated in the data, which is consistent with high and stable depreciation rates for intangibles (a finding which is borne out independently by BEA data on intangible depreciation rates). Third, overall rents increase, though they do so more moderately than suggested by other recent work. This is driven by differences in our estimates of the decline in the user cost of capital, which we explore in detail in Section 3.

Section 4 examines the investment gap using data on publicly traded firms. While narrower
in scope, these data have two advantages: we can use a broader definition of intangible capital, and we can disaggregate results by sector. When we expand intangibles to include the organization capital stock of firms (rather than just R&D) following Eisfeldt and Papanikolaou (2013), we find that by 2015, the two intangibles-related terms account for two thirds of the total investment gap. Including organization capital has relatively little impact on estimated user costs of intangibles — they remain elevated —, but it substantially increases the stock of intangibles, boosting both their direct effect on the investment gap, and the interaction term. Our estimates of rents as a share of value added are also roughly cut in half. Thus, empirically plausible amounts of intangible capital can explain the investment gap without requiring high rents.

Finally, in Section 4, we also estimate our decomposition at the sectoral level, in order to assess the extent to which the aggregate investment gap reflects composition effects. We divide our sample into five broad sectors: Consumer, Services, Tech, Healthcare, and Manufacturing. In the Manufacturing sector, the investment gap is small, and both rents and intangibles are declining. By contrast, in the Tech and Healthcare sectors, the investment gap has been growing rapidly since the 2000’s. In both sectors, the primary driver is rents to intangible capital. In the Consumer sector, results depend on the measurement of the intangible capital stock. Reported R&D is small, so there is little role for intangibles when they are measured with this proxy. However, innovation in the consumer sector is not well-measured by R&D (see Foster et al. 2006 and Crouzet and Eberly 2018). When including organization capital, most of the gap is estimated to reflect the direct effect of large investment in intangibles in that sector — rents on either physical or intangible capital appear to have only modestly increased. The Service sector is similar in some respects to Consumer, in that R&D is small, so rents explain most of the gap. However, adding organization capital does not change this view, as there has been little growth in organization capital in the Service sector; hence, rents explain most of the gap throughout. Finally, we also study the relationship between rising rents and rising intangibles across the constituent subsectors of our five broad sectors. We find that the rise in rents was heterogeneous across subsectors, and within Manufacturing, Tech and Healthcare, subsectors that experienced a rise in rents also experienced an increase in intangible intensity. These findings suggest that the rise in rents may be both narrower than aggregate estimates and also related to changes in the underlying structure of production.

Our results caution against interpreting the gap as a broad rise in market power. Our evidence shows that intangibles play a key role, and no single mechanism provides a unified account of the gap, even across broadly defined sectors. Normative implications should hence be drawn with care.
Related research and contribution  Our work first relates to the literature on the implications of rising intangible capital for macroeconomics and finance, which itself builds on work measuring intangibles and documenting their rise (Corrado et al., 2005, 2009; Eisfeldt and Papnikolaou, 2013). Closest to our approach are Hall (2001), who links the rise in intangibles to stock market valuations, and McGrattan and Prescott (2010), who examine the potential role of intangibles for macro trends in a business cycle model. Relative to these papers, we study medium-run trends, emphasize sectoral heterogeneity, and, most importantly, allow for market power within our model.

Second, our work is related to a recent literature on the size and implications of rising rents. A number of researchers have interpreted the findings of Autor et al. (2020), who show that industry concentration rose in U.S. industries after 2000, as potential evidence of market power, and examined profitability and markup data for further evidence. Most closely related to our work are Gutiérrez and Philippon (2018a) and Barkai (2020), who document a significant increase in pure profit shares and markups, especially after 2000. Barkai (2020), in particular, does not directly examine investment, but shows that the decline in the labor share is not offset by a rising capital share; he attributes the resulting gap to pure profits. Our approach, based on valuations, uncovers a more modest increase in rents than these papers, a point we expand on in Section 3. Similarly, Basu (2019) reviews the evidence from the rents literature, and argues that macro trends related to profitability are largely consistent with historical variation. He points instead to weak investment as the outlier and asks how to reconcile it with the apparently modest changes in rents. Our paper explains this apparent divergence as the combined effect of moderate rents with rising intangibles.

In recent and related research, Karabarbounis and Neiman (2019) find that the gap between measured capital income and estimates of the required compensation of capital is most likely explained by mismeasurement in the cost of capital. Our approach provides an alternative measure of the user cost of capital which further supports this view. Most closely related to our work is Farhi and Gourio (2018), who estimate the contribution of market power, risk premia, and intangibles to recent macro trends, as well as Corhay et al. (2020), who highlight the role of declining entry as a source of increasing market power. Relative to their work, our analysis focuses more specifically on investment and on the role that intangible capital plays in explaining weak investment relative to valuations.

A rich literature in corporate finance has discussed potential sources of wedges between average Q and marginal q, and the performance of investment-Q regressions. Most recently,

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2 See also Hansen et al. (2005) and Ai et al. (2013).

3 De Loecker et al. (2020) and Hall (2019) use firm-level accounting data and industry data, respectively, and find both high and rising markups.
Peters and Taylor (2017) revisit the relationship between investment and Q when intangibles are present. Belo et al. (2019) also provide decompositions of firm value across types of capital, including intangibles. We leverage the empirical results of both papers in our analysis, but also provide a more general framework than either, by allowing for rents, a key element in the relationship between investment and Q. Section 2 provides further comparisons of our framework with existing models.

Our results are also connected to recent findings documenting a decline in investment/cash-flow sensitivities and questions whether it reflects financing constraints (Chen and Chen, 2012). From the standpoint of our model, a potential interpretation of the results of Chen and Chen (2012) is that Tobin’s Q increasingly captures cash flow effects, through the growing importance of rents, particularly those associated with intangibles. Relatedly, recent research by Falato et al. (2020) studies how the growth in corporate cash holdings relates to rising intangible intensity. They argue that the reduced reliance on physical capital has shrunk corporate debt capacity, which firms offset by increasing precautionary cash holdings. We document an additional aggregate and sectoral trend, the increase in rents. This trend, by increasing the curvature of firms’ profits with respect to capital, may have exacerbated the precautionary motive, further contributing to the growth in cash holdings.

Finally, this paper is related to our own prior research, and in particular to Crouzet and Eberly (2019). Relative to that paper, the current paper differs in two important ways. First, we derive a decomposition of the investment gap that allows for both intangibles and rents. By contrast, the framework Crouzet and Eberly (2019) does not allow for rents. The addition of rents delivers one of the key insights of this paper: rents can amplify the effect of intangibles on the investment gap; or, put differently, in the more general framework studied in this paper, the slope coefficient on intangible capital is higher when rents are high. Second, on the empirical side, this paper uses the structure of the model to quantify the respective contributions of intangibles, rents, and their interaction, to the growth of the investment gap over the past three decades. We find that the contribution of the interaction term is substantial, leading us to estimate that up to 60% of the aggregate investment gap is due to the rise in intangibles. By contrast, Crouzet and Eberly (2019) provides reduced-form evidence that the investment gap is higher in industries with higher intangible intensity and higher market power, but does not allow for an interaction between the two mechanisms. As a result, Crouzet and Eberly (2019) attributes only about 30% of the gap to intangibles.5

4 Related, Andrei et al. (2019) show that the correlation between Q and investment at high frequencies has recently increased. We focus on the divergence between valuations and investment at longer horizons.

5 It is also worth noting that the approach followed in Crouzet and Eberly (2019) is not structural. The statement, in that paper, that 30% of the gap is attributable to intangibles refers to the incremental explanatory power of intangibles in reduced-form regressions.
2 Rents, intangibles, and the investment gap: theory

In this section, we derive a general decomposition of the gap between average \( Q \) and marginal \( q \). We call this the “investment gap”. For each type of capital employed by the firm, the investment gap depends on economic rents, the other forms of capital employed by the firm, and the rents they generate. We provide analytical characterizations of the gap in certain special cases, relate our results to existing work, and study extensions of our basic framework. Proofs for the results of this section are in Appendix 1.

2.1 Model

Time \( t \) is discrete. A firm uses \( N \) different capital inputs, collected in a vector \( K_t = \{K_{n,t}\}_{n=1}^N \) in production.\(^6\) The firm’s operating profits as a function of capital are \( \Pi_t(K_t) \), where \( K_t \) is an aggregate of the different types of capital, given by \( K_t = F_t(K_t) \). Total investment costs, including adjustment costs on capital, are given by \( \Phi_t(K_t, K_{t+1}) \). We index the functions \( F_t, \Pi_t \) and \( \Phi_t \) to indicate that they can depend arbitrarily on other unspecified exogenous variables. The discount factor of the firm is \( M_{t,t+1} \). Firm value satisfies:

\[
V^c_t(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \Phi_t(K_t, K_{t+1}) + E_t \left[ M_{t,t+1} V^c_{t+1}(K_{t+1}) \right] \\
\text{s.t.} \quad K_t = F_t(K_t),
\]

where \( V^c_t(\cdot) \) is the value of the firm including distributions. We make the following assumptions about the primitives of the problem.

**Assumption 1.** \( F_t(K_t) \) is homogeneous of degree 1.

**Assumption 2.** \( \Pi_t(K_t) \) is increasing, concave, and homogeneous of degree \( \frac{1}{\mu} \leq 1 \).

**Assumption 3.** Investment costs satisfy \( \Phi_t(K_t, K_{t+1}) = \sum_{n=1}^N \Phi_{n,t} \left( \frac{K_{n,t+1}}{K_{n,t}} \right) K_{n,t} \), where each function \( \Phi_{n,t} \) is strictly increasing and convex.

The parameter \( \mu \) plays a central role in our analysis: it indexes the economic rents accruing to the firm, with \( \mu = 1 \) corresponding to no rents. We discuss the link between \( \mu \) and economic rents in more detail in Section 2.2.

In Section 2.4, we provide examples of models in the literature which are particular cases of the general model just described. We also clarify which frictions this model abstracts from, some of which we tackle in the extensions described in Section 2.5.

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\(^6\) These capital inputs can be broadly understood as any quasi-fixed factor which are costly to adjust and contribute to the output of the firm over more than one period; for instance, any stock of skilled labor that is both costly to adjust and does not fully depreciate within the period.
2.2 A decomposition of the investment gap

Our main result on the investment gap uses the following lemma.

**Lemma 1.** Let:

\[ V^e_t = \mathbb{E}_t \left[ M_{t,t+1} V^e_{t+1} \right], \quad q_{n,t} = \frac{\partial V^e_t}{\partial K_{n,t}}, \quad \Pi_{n,t} = \frac{\partial \Pi_t}{\partial K_t} \frac{\partial K_t}{\partial K_{n,t}}. \]  

(2)

Firm value can be written as:

\[ V^e_t = \sum_{n=1}^{N} q_{n,t} K_{n,t+1} + (\mu - 1) \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \Pi_{n,t+k} K_{n,t+k} \right]. \]  

(3)

This lemma decomposes firm value into two parts.

The first part is the sum of the value of the installed stocks of each capital type \( n \). This value is equal to the replacement cost, \( K_{n,t+1} \), multiplied by marginal \( q \), \( q_{n,t} \); the latter will be different from 1 so long as the corresponding capital adjustment costs \( \Phi_{n,t} \) are strictly convex. This generalizes the Hayashi (1982, Proposition 1) result to multiple capital inputs; this generalization was first noted by Hayashi and Inoue (1991), in a model where \( \mu = 1 \).

For the second part of equation (3), note that when there is only one type of capital \( (N = 1) \), it boils down to a discounted sum of the terms \( (\mu - 1) \Pi_{K,t+k} K_{n,t+k} \). Moreover,

\[ (\mu - 1) \Pi_{K,t+k} = \frac{\Pi_{t+k}}{K_{t+k}} - \Pi_{K,t+k}, \]  

(4)

so that these terms are difference between the average and the marginal (revenue) product of capital. We interpret this difference as the flow value of rents. When \( N = 1 \) the second term in equation (3) is then simply the present value of future rents. This term is non-zero only when \( \mu > 1 \), as first noted by Lindenberg and Ross (1981) and Hayashi (1982, Proposition 2) in models where \( N = 1 \). The magnitude of \( \mu \) controls the overall size of rents.

When \( N > 1 \), the second term in Equation (3) is the sum of terms of the form:

\[ (\mu - 1) \Pi_{n,t+k} = \left( \frac{\Pi_{t+k}}{K_{n,t+k}} - \Pi_{K,t+k} \right) \frac{\partial F_{t+k}}{\partial K_{n,t+k}}. \]  

(5)

These terms capture the marginal contribution of capital of type \( n \) to overall rents earned by the firm. The flow value of rents is the gap between the average and the marginal (revenue) product of capital of type \( n \). The intuition from the \( N = 1 \) case thus carries through, with the added insight that total rents are additively separable across capital types, which will be useful in quantifying the contribution of each type of capital to overall rents.
Result 1. Define average $Q$ for capital of type $n$, $Q_{n,j,t}$, as:

\[ Q_{n,t} = \frac{V_t}{K_{n,t+1}}. \]

Then, the investment gap for capital of type $n$ can be written as:

\[
Q_{n,t} - q_{n,t} = (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+1,t+k}) \right] + \sum_{m=1 \atop m \neq n}^{N} S_{m,n,t+1} q_{m,t} + (\mu - 1) \sum_{m=1 \atop m \neq n}^{N} \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \Pi_{m,t+k}(1 + g_{m,t+1,t+k}) \right],
\]

where $1 + g_{n,t+1,t+k} = \frac{K_{n,t+k}}{K_{n,t+1}}$, and $S_{m,n,t+1} = \frac{K_{m,t+1}}{K_{n,t+1}}$.

The investment gap is the sum of three terms, (6), (7) and (8).

When there are no rents and a single type of capital ($\mu = 1$ and $N = 1$), these three terms are zero. Average $Q$ and marginal $q$ are equal, as in Hayashi (1982, Proposition 1), and there is no investment gap.

If there are rents but only one type of capital ($\mu > 1$ and $N = 1$), only the term (6) is nonzero. Average $Q$ will overstate marginal $q$, and the gap is equal to the present value of flow rents, that is, the term (6). This case includes the Lindenberg and Ross (1981) effect.

If there are no rents but several types of capital ($\mu = 1$ and $N > 1$), then for each type of capital, average $Q$ will still overstate marginal $q$. Average $Q$ for a specific type of capital reflects, in part, the value of other types of capital used by the firm, because these other types of capital contribute to firm value overall. It therefore overstates the true incentive to invest — the marginal $q$ — of that type of capital.\(^7\) This omitted capital effect is captured by the term (7) in the expression of the investment gap.

If there are both economic rents and several types of capital ($\mu > 1$ and $N > 1$), the rents term (6) and the omitted capital term (7) are still non-zero. But additionally, the term (8) is non-zero. It represents the interaction between the rents and the omitted capital effect. It captures how rents accruing to other types of capital affect total firm value and, through the omitted capital effect described above, add to the gap between average $Q$ and marginal $q$. This interaction term is larger, the higher the relative importance of other types of capital, and the higher the rents generated by other types of capital.

\(^7\) Crouzet and Eberly (2019) also make this point in a model with two types of capital and no rents.
2.3 Balanced growth

We now provide analytical expressions for the investment gap decomposition in a model with balanced growth. These expressions help build intuition for each of the components of the gap, and also anticipate our empirical applications.

Without loss of generality, we focus on the $N = 2$ case; $K_{1,t}$ is "physical capital," and $K_{2,t}$ is "intangible capital". We assume the profit function is $\Pi_t = A_t^{1-\frac{1}{\mu}} K_{t,1}^{\frac{1}{\mu}}$, where $\mu \geq 1$. $A_t$ is an exogenous process capturing firm fundamentals and growth such that $A_{t+1}/A_t = 1 + g$. We also assume $M_{t,t+1} = (1 + r)^{-1}$, with $g < r$. Finally, we assume that the capital aggregator and the capital adjustment costs are time-invariant, and that investment costs satisfy the standard conditions:

$$\Phi_n(1) = \delta_n, \quad \Phi'_n(1) = 1, \quad \Phi''_n(1) = \gamma_n \geq 0.$$

**Result 2.** In balanced growth, the investment gap for physical capital is given by:

$$Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + S q_2 + \frac{\mu - 1}{r - g} R_2 S,$$

$$R_n \equiv (r - g) \Phi'_n(1 + g) + \Phi_n(1 + g), \quad n = 1, 2,$$

and where marginal $q$, average $Q$, and intangible to physical capital ratio, $S$, are constant.

In order to build intuition for the elements of Equation (9), consider first the special case of linear investment costs: $\gamma_n = 0$ and $\Phi_n(x) = x - 1 + \delta_n$. In that case,

$$R_n = r + \delta_n.$$

Intuitively, without convex adjustment costs, the firm behaves as though it were renting capital in perfectly competitive markets, equating the marginal revenue product of each type of capital to its Jorgensonian user cost, $R_n$: $\Pi_{n,t} = \Pi_n = R_n = r + \delta_n$. The two rents terms in decomposition (9) then represent the net markup over the marginal (user) cost of each type of capital, discounted by the Gordon growth term $r - g$.

When investment costs are convex ($\gamma_1 > 0, \gamma_2 > 0$), the $\{R_n\}$ can be interpreted as "internal" user costs. They satisfy:

$$R_n = r + \delta_n + \gamma_n r g + o(g),$$

where $o(g)$ is the little-o Landau notation. The additional term $\gamma_n r g + o(g)$ reflect the cost of continuously adjusting capital along the firm’s growth path.
2.4 Discussion

Why is \( Q_{n,t} - q_{n,t} \) an “investment gap”? We extend the terminology “investment gap” used in Gutiérrez and Philippon (2017) and Alexander and Eberly (2018). The first-order condition for investment is:

\[
g_{n,t} = \Psi_{n,t}(q_{n,t} - \frac{1}{q_{n,t}}),
\]

where \( g_{n,t} \) is the net investment rate, and \( \Psi_{n,t}(y) \equiv (\Phi_{n,t}'(y))^{-1}(1 + y) - 1 \). When the investment gap is positive \( (Q_{n,t} > q_{n,t}) \), we have \( \Psi_{n,t}(q_{n,t} - \frac{1}{q_{n,t}}) = g_{n,t} \leq \Psi_{n,t}(Q_{n,t} - \frac{1}{Q_{n,t}}) \). Investment predicted using average \( Q \) will exceed actual investment; that is, there will appear to be a “gap” the two.

Why not use “Total \( Q \)”?

Total \( Q \) is the ratio of the value of the firm to its total (physical plus intangible) capital stock (Peters and Taylor, 2017). In our model, it is given by:

\[
Q_{tot,t} \equiv V_t^e / \sum_{n=1}^{N} K_{n,t+1} = \sum_{n=1}^{N} s_{n,t+1} q_{n,t} + (\mu - 1) \sum_{n=1}^{N} s_{n,t+1} \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+1,t+k})]
\]

where \( s_{n,t+1} = K_{n,t+1} / \sum_{n=1}^{N} K_{n,t+1} \). Define the “total \( Q \) investment gap” as \( Q_{tot,t} - q_{tot,t} \). This gap will be positive when the firm earns rents; moreover, rents can be decomposed across types of capital. However, we do not focus on \( Q_{tot,t} - q_{tot,t} \) for one main reason: \( q_{tot,t} \) is not a sufficient statistic for total investment, except in specific cases.\(^8\) As a result, there is no mapping from \( Q_{tot,t} - q_{tot,t} \) to the empirical shortfall in total investment. By contrast, \( q_{n,t} \) is a sufficient statistic for investment in capital \( n \), and so the capital-specific investment gap \( Q_{n,t} - q_{n,t} \) entirely accounts for the relationship between \( Q_{n,t} \) and investment.

How general is the model?


However, it has three limitations. First, it does not allow for non-convex adjustment costs. Second, it abstracts from financial constraints. The next subsection discusses extensions in this direction. Third, it assumes that rents, \( \mu \), are exogenous. In particular, they do not depend on past investment, in contrast, for instance, with models of customer capital.\(^9\) In this sense, our results are restricted to “neoclassical” models of the firm, and provide a benchmark against which the effects of other frictions on the investment gap can be compared.

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\(^8\) Appendix 1.3 shows that perfect substitutability and identical investment costs is one such case.

\(^9\) See, for instance, Gourio and Rudanko (2014) and Belo et al. (2014).
2.5 Extensions

Uncertainty  Closed-form solutions for the investment gap exist when fundamentals are uncertain; we discuss this in greater detail in Appendix 2.2.

Result 3. Assume that $N = 2$, $\Pi_t = A_t^{1-\frac{1}{2}} K_t^{\frac{1}{2}}$, and:

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \begin{cases} 1 + g_{t-1} & \text{w.p.} \ 1 - \lambda \\ 1 + \tilde{g} & \text{w.p.} \ \lambda \end{cases}, \quad \tilde{g} \sim G(.) \ i.i.d.$$ 

Moreover, assume that $\Phi_n(x) = x - 1 + \delta_n$, $n = 1, 2$. Then:

$$Q_{1,t} - q_{1,t} = \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_1) + S + \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_2)S,$$  \hspace{1cm} (10)

where the expression for the function $\nu(.)$ is reported in Appendix 2.2.

The resulting decomposition is similar to Result 2, except that the Gordon growth term $\frac{1}{r - \nu(g_t)}$ adjusts for the possibility of regime changes in growth rates.\(^{10}\) Key intuitions are similar to those discussed in Section 2.3: the two rents terms are equal to the present value of flow rents, with flow rents equal to the net markup over user costs. In Section 5.3, we use Result 3 in order to estimate a version of the model with uncertainty in our empirical applications.

Market power, decreasing returns, and rents  Two natural sources of rents are market power on the goods market (“pure” rents), and decreasing returns to scale (“quasi” or “Ricardian” rents). The mapping between $\mu$ and these two sources of rents is the following.

Result 4. Suppose that the firm uses flexible inputs that are Cobb-Douglas substitutes with capital, where $\alpha$ is the capital share. Let $\zeta$ index returns to scale, and let $\mu_S$ be the firm’s markup over the cost of sales. Then:

$$\mu \equiv 1 + \frac{\mu_S/\zeta - 1}{\alpha}.$$ 

Moreover, total (pure and quasi-) rents over operating surplus are given by $\frac{\mu - 1}{\mu}$.

Appendix 2.3 establishes this result. Importantly, the magnitude of $\mu$ does not depend separately on $\mu_S$ or $\zeta$, but only on their ratio. Our results can therefore be thought of through the lens of either type of rent. In Appendix 2.3, we also argue $\mu_S$ and $\zeta$ cannot be separately identified using only nominal ratios (such as cost shares, surplus ratios, user

\(^{10}\) The case $\lambda = 0$ corresponds to constant growth, as in the balanced growth model of Result 2; in that case, $\nu(g_t) = g_t$. The case $\lambda = 1$ corresponds to i.i.d. growth rates, with $\nu(g_t) = E[\tilde{g}]$. 

costs, or average returns to capital), as these ratios are all functions of $\mu_S/\zeta$ instead of either parameter independently. This point is also highlighted by Basu (2019). Therefore, in Section 5.5, we will discuss what our estimates of $\mu$ imply for the value of “pure” rents under different assumptions about returns to scale.

**Heterogeneous rents parameters**  
Our results extend to the case where the rents parameters $\mu$ is allowed to be different across types of capital, as follows.

**Result 5.** Assume that operating profits are given by a mapping $\Pi_t(K_t)$ satisfying:

$$\tilde{\Pi}_t(K_t) = \sum_{n=1}^{N} \mu_n \tilde{\Pi}_{n,t} K_{n,t}, \quad \mu_n \geq 1 \quad \forall n. \quad (11)$$

In balanced growth with $N = 2$, we have:

$$Q_1 - q_1 = \frac{\mu_1 - 1}{r - g} R_1 + S q_2 + \frac{\mu_2 - 1}{r - g} R_2 S. \quad (12)$$

Similar generalizations of Lemma 1 and Result 1 are reported in Appendix 2.4.\(^{11}\) Appendix 2.4 also characterizes a class of operating profit functions satisfying condition (11); it could capture, for instance, a firm with different revenue streams generated by independent divisions, each using a different type of capital. We focus on the version with $\mu_1 = \mu_2 = \mu$ because separate identification of each the rents parameters in the case $\mu_1 \neq \mu_2$ is more challenging: it requires data on the marginal revenue product of each type of capital separately. We return to this issue in Section 5.6.

**Link to the production-based asset pricing literature**  
In Appendix 2.5, we study the difference between stock returns and returns to investment in each type of capital, following Cochrane (1991, 1996).\(^{12}\) When $N = 1$ and $\mu = 1$, the two are equalized, as in Cochrane (1991). But when $N > 1$ or $\mu > 1$, they need not be. Moreover, their difference is driven by the same three forces as the investment gap: omitted capital; rents; and their interaction.

An important difference is that returns depend on changes in firm value, whereas average $Q$ and marginal $q$ depend on the level of firm value. As a result, the returns gap is more likely to be informative about high-frequency movements in intangible intensity and rents, while the investment gap is more likely to be informative about long-run trends.\(^{13}\) For instance, in

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\(^{11}\) The baseline model explicitly separates capital aggregation from the operating surplus function; it is a special case of this more general model, with $\tilde{\Pi}_t(K_t) = \Pi_t(F_t(K_t))$ and $\mu_n = \mu \ \forall n$.

\(^{12}\) We thank an anonymous referee for this suggestion.

\(^{13}\) This echoes Cochrane (1991, p.218): “Returns emphasize high frequency aspects of the data that the models may be better able to capture in the presence of slow moving and unobserved changes in technology.”
balanced growth, the difference between stock returns and returns to investment is zero, as the two returns are equalized to the discount rate even if $N > 1$ and $\mu > 1$. By contrast, in balanced growth the investment gap remains positive, as highlighted by Result 2. Given that trends are the focus of our paper, we choose to work with the investment gap.

**Financing frictions** A large literature has shown financial constraints can drive a wedge between average $Q$ and marginal $q$ (Whited, 1992; Gomes, 2001; Hennessy et al., 2007; Bolton et al., 2011; DeMarzo et al., 2012). However, the sign and size of this wedge is a matter of debate, particularly if the firm has market power (Cooper and Ejarque, 2003). In Appendix 2.6, we study the investment gap in versions of the model with two simple financial frictions.

**Result 6.** Assume that shareholders can raise debt $B_{t+1}$, subject to a collateral constraint of the form $B_{t+1} \leq \theta K_{1,t+1}$. Define marginal $q_{n,t}$ as $q_{1,t} \equiv q_{1,E}^{(E)} + \lambda_t \theta$ for $n = 1$ and $q_{n,t} \equiv q_{n,E}^{(E)}$ for $n = 2, ..., N$, where $q_{n,E}^{(E)}$ is the marginal value of an unit of capital to shareholders, and $\lambda_t$ is the Lagrange multiplier on the leverage constraint. Then, the decomposition of the enterprise investment gap for capital $n$, $Q_{n,t} - q_{n,t}$, is the same as in Result 1.

This result states that a collateral constraint with respect to physical capital does not change the expression for the investment gap, so long as one focuses on the enterprise investment gap, defined as the gap $Q_{n,t} - q_{n,t}$.\(^{14}\) The intuition is that because debt is risk-free, there is no conflict between creditors and shareholders, and the investment policy chosen by shareholders also maximizes total enterprise value.

**Result 7.** Assume that the flow value of dividends to shareholders is given by $K_t f(d_t)$, where $d_t = D_t/K_t$, $D_t$ is revenue net of investment costs, and $f$ satisfies $f(0) = 0$, $f' > 0$, $f''(0) = 1$, and $f'' \leq 0$. Then, the investment gap has the same expression as in Result 1, replacing the discount factor $M_{t,t+k}$ with $f'(d_{t+k}) M_{t,t+k}$.

The function $f(.)$ describes equity financing frictions in a reduced-form way: the fact that $f'(d_t) < 1$ when $d_t > 0$ could capture agency costs of free cash flows (Jensen and Meckling, 1976), while the fact that $f'(d_t) > 1$ when $d_t < 0$ could capture costs of seasoned equity offerings (Altinkılıç and Hansen, 2000).\(^{15}\) These frictions change the way in which shareholders value future rents, but do not affect the three main elements of the decomposition.\(^{16}\)

\(^{14}\)Appendix 2.6 shows that, consistent with the prior literature, the investment gap for shareholders, that is, the difference between the ratio of equity value to the stock of physical capital, and $q_{n,E}^{(E)}$, has a similar expression as Result 1, with an additional, negative wedge, reflecting the fact that part of the marginal return to investment, for shareholders, comes from the fact that it relaxes the borrowing constraint.

\(^{15}\)We follow Hennessy et al. (2007), except that we allow for $f'(d_t) < 1$ when $d_t > 0$. This makes equity financing costs matter on the balanced growth path, where $d_t = d > 0$.

\(^{16}\)As discussed in Appendix 2.6, two definitions of marginal $q$ are possible, depending on whether one normalizes marginal $q$ by $f'(d_t)$ or not. Result 7 refers to an unadjusted marginal $q$; with the latter definition, the investment gap has an additional wedge, which we characterize in Appendix 2.6.
Thus, simple frictions to either equity or debt financing do not change the basic insights of Result 1 regarding the components of the investment gap. However, they can change the magnitude of these components, as well as the size of the investment gap overall, relative to the frictionless model. We discuss this point in more detail in Section 5.7.

3 The investment gap in aggregate data

We now show that the investment gap for non-financial corporate businesses has tripled since 1985, driven by the combined effects of rising rents and rising intangibles. This section uses national accounts data, which has the most coverage, but the narrowest measure of intangible capital. We broaden the intangibles measure in the next section, drawing on firm-level data.

3.1 Methodology

We use the balanced growth model to construct the investment gap and its components in the data. We have:

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S, \]  

(13)

where recall that, neglecting terms of order \( o(g) \), \( R_n = r + \delta_n + \gamma_n rg \), and \( q_n = 1 + \gamma_n g \), \( n = 1, 2 \).

We measure \( Q_1 \) and \( S \) directly from data, as described below. However, we infer values for \{\( \mu, r - g, R_1, R_2, q_1, q_2 \} \) from the following additional observable moments: \{ROA_1, i_1, i_2, g\}, where \( ROA_1 = \Pi_t / K_{1,t} \) is average returns to physical capital, \( i_1 \) and \( i_2 \) are gross investment rates, and \( g \) is the net growth rate of total capital \( K_{1,t} + K_{2,t} \).

We proceed as follows. First, we use the fact that:

\[ \mu = \frac{ROA_1}{R_1 + SR_2}. \]  

(14)

Intuitively, rents create a wedge between average returns to physical capital and the weighted average user cost of capital. Second, we have that:

\[ R_n = r - g + i_n + \gamma_n rg, \quad n = 1, 2, \]  

(15)

where we used the fact that \( i_n = g + \delta_n \) along the balanced growth path. Finally, substituting

\[ 17 \] See Appendix 1 for a formal derivation of this relationship.
Equations (14) and (15) in the investment gap decomposition (13), we obtain:

\[ r - g = \frac{ROA_1 - (i_1 + S_i)}{Q_1} - \frac{\gamma_1 + S \gamma_2 g^2}{Q_1}. \]  

(16)

This expression for the Gordon growth term \( r - g \) only requires estimates of the adjustment cost parameters. Given the value for \( r - g \) and other data moments, values of \( R_1 \) and \( R_2 \) follow from Equation (15); and the value of \( \mu \) follows from Equation (14). Finally, \( q_1 \) and \( q_2 \) are obtained from the values of \( g \) and from calibrated values for the adjustment cost parameters \( \gamma_1 \) and \( \gamma_2 \), which we discuss below.

The most important feature of this identification approach is that it matches, by construction, the empirical value of \( Q_1 \). It infers the Gordon growth term \( r - g \) which, given other moments, ensures that the model produces a value of \( Q_1 \) consistent with the data. Our use of valuations, via \( Q_1 \), is a natural implication of the model, but also an important point of departure from the recent literature. We discuss this point in more detail Section 3.3.

Additionally, we note that our methodology does not make direct use of data estimates of economic rates of depreciation, \( \delta_n \). Instead, we substitute depreciations for gross and net investment rates, using the relationship \( \iota_n = g + \delta_n \), \( n = 1, 2 \). We choose to use investment rates in our empirical approach because our main goal is to account for their behavior relative to valuations. Below, we discuss in more detail the implications of our estimated model for depreciation rates.

### 3.2 Aggregate Data

Our sample period is 1947-2017, and we focus our analysis on the non-financial corporate business (NFCB) sector.\(^{18}\) Appendix 3 reports details on data sources and data construction. We construct time series for five of the moments used in the decomposition, \( \{i_{1,t}, i_{2,t}, S_t, ROA_{1,t}, Q_{1,t}\} \), using six times series in levels, \( \{K_{1,t}, I_{1,t}, K_{2,t}, I_{2,t}, \Pi_t, V_t\} \). These are the operating surplus of the NFCB sector, the stock of physical capital at replacement cost, investment in physical capital, the stock of intangibles at replacement cost, investment in intangibles, and the market value of claims on the NFCB sector.\(^{19}\)

We obtain measures of \( K_{1,t}, I_{1,t}, K_{2,t} \) and \( I_{2,t} \) from BEA Fixed Assets tables 4.1 and 4.7. The BEA Fixed Assets tables use perpetual inventory methods to construct the stock of three specific forms of intangible capital: R&D; own-account software; and artistic originals.\(^{20}\) To

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\(^{18}\) Appendix 3.7 shows that trends in economy-wide and NFCB average returns to capital are similar.

\(^{19}\) We use current-dollar values for all time series in levels, with the exception of our proxy for \( g_t \), the computation of which is described below.

\(^{20}\) Related to Footnote 6, we note that investment in R&D capital, in the BEA, is partly estimated using compensation to R&D workers (Corrado et al., 2005). Thus, one potential re-interpretation of the BEA’s
the extent that firms invest in other types of intangibles, results in this section should thought of as a lower bound on the overall role of intangibles. Section 4 expands the analysis to organization capital for the subset of publicly traded firms in the NFCB sector.

Operating surplus $\Pi_t$ is obtained from NIPA Table 1.14. Consistent with the model, this series represents the difference between value added and payments to labor; expenditures categorized as intangible investment are not treated as intermediates in value added.

We construct a measure of $V_t$ using Flow of Funds tables L.103 and F.103. In the model, $V_t$ represents the market value of all net claims on the NFCB sector, both debt and equity. The Flow of Funds data provide an estimate for the market value of equity of the NFCB sector, but not for debt. Our approach to estimate the latter is described in detail in Appendix 3.2. It is similar to the approach of Hall (2001), except that we do not subtract all financial assets owned by the sector from the gross market value of claims, but only financial assets identified as liquid in the Flow of Funds.\footnote{Financial assets are generally subtracted from the gross market value of claims in order to include net debt, instead of gross debt, in firm value calculations. On the other hand, financial assets can only meaningfully be counted as negative debt to the extent that they are liquid. Additionally, a large part of non-liquid financial assets in table L.103 are obtained as a residual, further complicating their interpretation.}

Section 5.1 shows that this choice affects the level of the investment gap, but not its composition.

We then construct $ROA_{1,t} = \Pi_t/K_{1,t}$, $i_{1,t} = I_{1,t}/K_{1,t}$, $i_{2,t} = I_{2,t}/K_{2,t}$, $S_t = K_{2,t}/K_{1,t}$, and $Q_{1,t} = V_t/K_{1,t}$. Additionally, $g_t$ is the annual growth rate of the quantity index for private non-residential fixed assets of the NFCB sector, provided in BEA Fixed Assets table 4.2.\footnote{Although the balanced growth model imposes identical growth rates across capital types, the growth rate of intangibles has generally been higher than that of physical capital, and therefore higher than $g_t$, as reported in Appendix Figure 3. However, as that figure also shows, they were close the 1970s to the mid-1980s, and have been close since the early 2000s. Appendix 3.3 describes these time series in more detail, and Appendix 4.1 shows that the results from main decomposition are robust to allowing for heterogeneous growth rates across capital stocks.}

We use calibrated values for adjustment costs. We consider three cases: zero adjustment costs, $\gamma_1 = \gamma_2 = 0$; positive adjustment costs; and high adjustment costs. For the positive adjustment cost case, we choose values of $\gamma_1 = 3$ and $\gamma_2 = 12$, following the estimates of Belo et al. (2019). For the high adjustment cost case, we choose values $\gamma_1 = 8$ and $\gamma_2 = 18$, at the high end of existing estimates. In Section 5.2, we discuss the effect of adjustment costs in more detail. We show that our results regarding the composition of the investment gap are robust to the choice of adjustment costs, and attribute greater importance to intangibles when adjustment costs are higher.

The time series for the resulting six moments, \{ $i_{1,t}, i_{2,t}, S_t, ROA_{1,t}, Q_{1,t}, g_t$ \} are reported in Appendix Figure 1. The key trends discussed in the introduction are visible in that figure.

\begin{footnotesize}
\begin{itemize}
  \item $\Pi_t$ is obtained from NIPA Table 1.14.
  \item $V_t$ is constructed using Flow of Funds tables L.103 and F.103.
  \item $ROA_{1,t}$ is constructed using Flow of Funds tables L.103 and F.103.
  \item $g_t$ is the annual growth rate of the quantity index for private non-residential fixed assets of the NFCB sector, provided in BEA Fixed Assets table 4.2.
  \item $\gamma_1 = \gamma_2 = 0$ for the zero adjustment cost case.
  \item $\gamma_1 = 3$ and $\gamma_2 = 12$ for the positive adjustment cost case.
  \item $\gamma_1 = 8$ and $\gamma_2 = 18$ for the high adjustment cost case.
\end{itemize}
\end{footnotesize}
The average return to physical capital increases after 1985, while the physical investment rate declines. The ratio of intangible to physical capital increases, particularly after 1985. \( Q_1 \) rises sharply after 1985, and after a peak in 2000, remains approximately double its value in the pre-1985 period.

Finally, we compute the decomposition using moving averages of moments over 7-year centered rolling windows. This treats each successive window as if it were generated by a different quantitative implementation of the model, allowing us to capture gradual changes in the investment gap.\(^{23}\) In Section 5.3, we report results obtained by estimating a version of the model with shocks (instead of the balanced growth model) on split samples using GMM.

### 3.3 Baseline results

**The investment gap and underlying structural changes** Figure 1 reports the investment gap decomposition, Equation (13), for the NFCB sector and R&D intangible capital. The decomposition emphasizes three main findings.

First, the investment gap is large during two distinct periods: 1960-1970, and after 1985. The wedge between average \( Q \) and marginal \( q \) is therefore not strictly a hallmark of the post-1980s period. Second, rents attributable to physical capital — the first term in Equation (13) — play a sizable (though somewhat declining) role in explaining the investment gap: they account 61% of it in 2015, compared to 67% in 1965.\(^{24}\) Third, rents attributable to intangibles — the third term in Equation (13) — have become markedly more important in recent years. In 2015, 25% of the investment gap reflects the combined effects of high rents and a large stock of intangibles, compared to 10% in 1965, using the BEA measure of R&D capital only, the narrow measure of intangibles available in these data.

From the standpoint of the model, these changes are driven by three underlying forces, reported in Figure 2: a greater importance of intangibles in the production function; higher rents; and a decline in user costs, more pronounced for physical than for intangible capital.

The top left panel of Figure 2 shows that even using the relatively narrow definition of intangibles in the NFCB data, the share of intangible capital in production, \( \eta \), increased substantially after 1985, from 0.17 to 0.29 in 2015.\(^{25}\) The behavior of the intangible share approximately mimics the behavior of the measured ratio of intangible to physical capital at replacement cost, which increases rapidly after 1985.

\(^{23}\) Using alternative window sizes from 3 to 9 years gives quantitatively similar results.

\(^{24}\) These numbers, and those that follow in this discussion, refer to the model with intermediate adjustment costs, \( \gamma_1 = 3 \) and \( \gamma_2 = 12 \).

\(^{25}\) This is derived assuming a Cobb-Douglas aggregator \( K_t = K_{1,t}^{1-\eta}K_{2,t}^{\eta} \). The level of intangible share is sensitive to the Cobb-Douglas assumption, but not the magnitude of the change after 1985.
The effects of the intangible share on the overall investment gap are magnified by the rise in rents after 1985. The top right panel of Figure 2 reports estimates of the rents implicit in Equation (13). In order to facilitate comparison with existing estimates, we express them as the flow value of rents relative to value added, which is related to the parameter controlling rents in the model, $\mu$, through $s = (1 - s_L)(1 - 1/\mu)$, where $s_L$ is the labor share of value added. Rents, as a fraction of value added, increase from 1.5% in 1985, to 7.7% in 2015 — a cumulative 6.2 percentage point (p.p.) change over three decades. Expressed as markups over value added, this is an increase from 1.015 in 1985, to 1.083 in 2015.

Finally, we note two other features of our time-series for the investment gap. First, the gap is elevated during the 1960s; the decomposition attributes this to a combination of low user costs (driven by the low interest rates of the period), and high rents. Second, the gap is particularly small during the 1975-1985 period. The model primarily attributes this reversal to the large increase in discount rate and the decrease in growth rates around the early 1980s, which, by reducing the present value of future rents, pushes the average value of installed capital closer to its marginal value.

Comparison to existing literature These findings are qualitatively consistent with the recent literature arguing that pure profits as fraction of value added have been growing over the last three decades (Gutiérrez and Philippon, 2017; Barkai, 2020; Karabarbounis and Neiman, 2019). However, they differ quantitatively. For instance, Barkai (2020) finds that the pure profit share rose from -5.6% in 1984 to 7.9% in 2014, an increase of 13.5 p.p. over the period. Karabarbounis and Neiman (2019), in their “case II,” find that the pure profit share must have risen by about 13 p.p. over the same period. We find an increase in rents of half that magnitude.

User costs are at the heart of this difference. Specifically, our approach leads to user costs that are initially lower, but that decline more slowly. Figure 2 reports these implied user costs. User costs for physical capital decline from 15.4% to 12.6% between 1985 and 2015, while user costs for intangibles decline from 36.8% to 30.4%; their weighted average only declines from

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26 We measure the labor share for the NFCB sector using NIPA data on labor payments for that sector, as described in Appendix 3.1. As discussed in Appendix 4.2, given our estimate of $\mu$, matching the labor share in the data implies that the Cobb-Douglas elasticity of value added with respect to labor must vary over time. Alternatively, we consider fixing the Cobb-Douglas elasticity of value added with respect to labor; the results are almost identical.

27 On the latter point, we note that, related to the recent work of Gutiérrez and Philippon (2018b), the legal literature on antitrust policy has highlighted the 1960s as a period of weak enforcement (Hovenkamp, 2018).

28 Related to this are the markup estimates of De Loecker et al. (2020) and Hall (2019). These markups, when expressed in value added terms, are much higher than ours — approximately 1.9 and 4 in 2015, respectively —, and also far outside the range typically considered reasonable in the macroeconomics literature, as discussed in detail by Basu (2019).
17.1% to 15.2%. (By contrast, Barkai (2020), for instance, finds a required rate of return on capital that falls from approximately 20% in 1985 to approximately 14% in 2014.) The smaller decline in user costs translates to higher payments to capital (particularly to intangibles), and therefore a smaller increase in rents.

The way we infer the discount factor perceived by firms from the data is key to this result. As discussed before, we rely on valuations; by contrast, the papers mentioned above generally combine risk-free rates with imputed estimates of risk premia to obtain discount rates. Appendix Figure 2 reports the discount rate $r$ implied by our approach. It declines from 7.9% to 5.6% between 1985 and 2015. This is a smaller decline than the risk-free rate over the same period of time, and is therefore consistent with a mild rise in risk premia over this period of time, as argued by Caballero et al. (2017), Farhi and Gourio (2018), and Karabarbounis and Neiman (2019) in their case $R$. In Section 5.4, we discuss the results we would obtain using a cost-of-capital approach instead of a Q approach.

**User costs and rates of depreciation** Our analysis implies that user costs for intangible capital have fallen by less than those of physical capital. This change in relative user costs explains why rents attributable to intangibles, which are the present value of net markups over their user costs, as indicated by Equation (13), account for an increasing fraction of the investment gap after 1985.

Our approach infers this from the higher gross investment rates in intangibles, which, through the relationship $\delta_n = \iota_n - g$, $n = 1, 2$, also imply high rates of depreciation. Appendix Figure 4 reports the model-implied depreciation rates, along with empirical counterparts, obtained from the BEA’s Fixed Assets tables. The model-implied and data series behave similarly; both show a marked increase in depreciation rates for intangibles. The main difference is that our implied rate of depreciation for intangible is higher, on average, than its empirical counterpart, owing to the higher net growth rate of the intangible capital stock mentioned above.

We note that depreciation estimates indirectly enter our measurement, through their impact on estimates of capital stocks. However, investment rates in BEA data are not primarily

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29 Appendix 3.4 describes the computation of the empirical counterparts to the model depreciation rates.

30 Given that the BEA’s estimates of depreciation rates are based on constant depreciation rates at the asset level, the upward trend in depreciation rates reflect a shift in the composition of the capital stock toward assets with shorter service lives.

31 As mentioned in footnote 22, these small differences in growth rates across capital stocks do not materially affect the quantitative results obtained in our decomposition.

32 In the BEA data, estimates of economic depreciation are the residuals that reconcile measured gross investment, and estimates the net stocks of capital based on perpetual inventory methods. The net stock estimates themselves rely on assuming constant rates of economic depreciation at the asset-level, the values of which are based on microeconomic studies. Appendix 3.4 describes the methodology in detail.
driven by the value of economic depreciation rates assumed by the BEA. Appendix Figure 4 also reports gross investment rates and gross depreciation rates for both physical and intangible investment. While gross physical investment rates have trended downward, depreciation rates of physical capital, for instance, have trended upward.

### 3.4 Counterfactuals

In order to further illustrate the respective roles played by intangibles and rents in our estimation, Figure 3 reports results from two counterfactual exercises.

The top panel constructs the change in the share of intangibles in production, \( \eta \), that would be necessary in order to fully account for the increase in the investment gap, assuming that rents remain fixed at their 1985 level. This change is 34 p.p., compared to 12 p.p. in our baseline results. This, in turn, implies that the ratio of intangible to total capital, at replacement cost, would need to be 30% in 2015, or approximately twice its observed value of 14% in the NFCB sector.\(^{33}\) In Section 4, we show that this magnitude is comparable to the ratio of intangible to total capital \emph{including} organization capital among publicly traded firms.\(^{34}\)

The bottom panel of Figure 3 shows the increase in rents, as a fraction of value added, which would be required in order to match the observed investment gap, assuming that both the share of intangible capital, \( R_2 \), and the intangible investment rate \( \iota_2 \), had remained fixed at their 1985 values. Instead of the 6.2 p.p. increase in rents as a fraction of value added which we estimate as our baseline, rents would have needed to increase by 8.4 p.p., reaching 10.0% of value added by 2015. The total contribution of intangibles to the investment gap would nevertheless remain elevated (approximately 31%, instead of 39% in our baseline), due to the rising rents generated by the (more moderate) fixed stock of intangibles. This is really an intermediate case, since it allows growth in intangibles, but not the acceleration seen in the data.

To show more extreme cases of these two counterfactuals, Appendix 4.3 reports results in

\(^{33}\)Simple algebra, using the results of Section 3.1, shows that the counterfactual ratio of intangible to physical capital \( \hat{S} \) under fixed rents is the smallest positive root of \( Ax^2 + Bx + C = 0 \), where \( A = \iota_2 + \gamma_2g(\iota_2 + g + \gamma_2g), B = \iota_1 + \iota_2 - \text{ROA}_1 - Q_1\iota_2 + \gamma_1g(\iota_2 + g) + \gamma_2g(\iota_2 + g - Q_1g - \text{ROA}_1) + 2\gamma_1\gamma_2g^3, C = \text{ROA}_1Q_1/\mu(1985) + \iota_1 - \text{ROA}_1 - Q_1\iota_1 + \gamma_1g(\iota_1 + g + \gamma_1g^2 - \text{ROA}_1 - Q_1g), \) and \( \mu(1985) \) is the estimated value of the rents parameter \( \mu \) in 1985 using our baseline approach. The ratio of intangible to total capital is then given by \( \hat{S}/(1 + \hat{S}) \).

\(^{34}\)This magnitude is also comparable to Karabarbounis and Neiman (2019), “case K”. These authors show that, if the profit share is assumed to be zero, then unmeasured capital would need to account for approximately 40% of all business capital after 1970 in order to explain the measured capital share. Expressed in terms of value added, our estimates imply that intangibles would need to be approximately 63% of value added in the NFCB sector; this in line with similar estimates obtained by McGrattan and Prescott (2005) under perfect competition.
versions of the model with either no intangibles ($N = 1$) or no rents ($\mu = 1$). In the case of no intangibles, the model requires a 12 p.p. increase rents (as a fraction of value added) from 1980 to 2015, reaching 14% in 2015; this is almost double the magnitude obtained in our baseline approach. In the case of no rents, the implied ratio $S$ of intangible to physical capital required to explain the level of $Q_1$ in 2015 is approximately 1, compared to 0.3 in the data. Additionally, the implied times series for $S$ exhibits periods of substantial decline, particularly in the late 1970s and in the wake of the dot-com bubble. This intangible capital “destruction” is at odds with empirical measures of $S$, which grow consistently in the data, as shown in Appendix Figure 1.35

Aside from the additional results already mentioned, Section 5 provides further robustness checks and extensions to our baseline results, including: results from an approach that infers intangibles from $Q_1$, and from an approach that infers rents from $Q_1$ (Section 5.4); a discussion of the magnitude of pure rents under different assumptions about returns to scale (Section 5.5); a discussion of the implications of the model with heterogeneous rents parameters $\{\mu_n\}$ (Section 5.6); and a discussion of how the financing frictions discussed in Section 2.5 may bias our results (Section 5.7).

Summarizing, we documented a large investment gap in the NFCB sector after 1985. This gap reflects a combination of rising rents and a growing importance intangibles in production, with the latter accounting for about one-third of the gap. Additionally, though our valuation-based approach finds rising rents, the magnitude of the increase is approximately half that of existing estimates.

4 The investment gap in firm-level data

In this section, we construct investment gaps at the sectoral level, and highlight how they change when measures of intangibles are expanded beyond R&D capital. We find substantial differences across sectors in both the level of the gap and the relative contributions of rents and intangibles. Expanding measures of intangibles beyond R&D reduces the quantitative estimates of rents, and suggests that intangibles are the dominant force behind the growth in the investment gap.

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35 The no rents approach corresponds to the method used by Hall (2001) to estimate the stock of intangible capital of non-financial businesses. He also finds a decline in the stock of intangibles in the late 1970s.
4.1 Data

We use the non-financial segment of Compustat, instead of data drawn from the National Accounts. This restricts the scope of our analysis to publicly traded firms. We choose Compustat both because, to our knowledge, there is no comprehensive sectoral data on operating surplus \( \Pi_t \) and enterprise value \( V_t \) spanning a sufficiently long time period, and because it allows for measures of intangible capital that can be expanded beyond R&D.

**Sector definitions**  Compustat is a dataset of publicly traded US firms, so that the scope of the analysis is similar to Section 3, but now excludes private corporations.\(^{36}\) We split the sample into five broad sectors: the Consumer sector (primarily retail and wholesale trade); the High-tech sector (primarily software and IT); the Healthcare sector (producers of medical devices, drug companies, and health care service companies); the Manufacturing sector; and the Service sector (professional and business services, entertainment, and hospitality services). These groups are similar to the Fama-French 5 classification, with the main difference being that we exclude financial companies from our analysis.\(^{37}\)

**Data moments**  In order to construct the key moments needed for our analysis, we proceed similarly to Section 3; Appendix 3.5 reports the details. The two main differences are as follows. First, we consider two types of intangibles: R&D, similar to the analysis of Section 3; and organization capital, which we did not observe in the aggregate data in Section 3. R&D investment is measured using reported R&D expenditures. For investment in organization capital, we follow Eisfeldt and Papanikolaou (2014) and Peters and Taylor (2017) and impute investment as 30\% of SG&A expenditures net of R&D investment.\(^{38}\) Second, for operating surplus, \( \Pi_t \), we use operating income before depreciation, but we adjust for expensing of intangible investment in accounting data, consistent with our model.

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\(^{36}\) Details on data construction are reported in Appendix 3.5. Appendix 3.7 contains a discussion of the differences between Compustat and the National Accounts data.

\(^{37}\) Appendix Tables 1 and 2 report the NAICS sectors that make up our classification. Using KLEMS data from the BLS, described in Appendix 3.6, we estimate that the sectors we study accounted for 86.0\% of total value added by private, non-financial businesses in 2001. The remaining 14.0\% are accounted for by Transportation, Warehousing, and Construction, which we also exclude from our analysis because they are not well represented in Compustat; there are fewer than 10 firm observations per year for a majority of their constituent NAICS subsectors.

\(^{38}\) The primary source for the 30\% imputation rate is the work of Hulten and Hao (2008). Appendix 3.5 discusses other existing estimates in the literature, which are generally close to this value.
4.2 The aggregate investment gap in Compustat

We start by applying our baseline analysis to pooled data from all Compustat sectors, as an initial comparison to the aggregate results. The results are summarized in Table 1. Here, we highlight the two main findings of this exercise.

First, when using only R&D capital, the same trends highlighted in the introduction are apparent in both the Compustat and the NFCB data: rising returns to physical capital, rising $Q_1$, and declining physical investment rates. Compustat moments are very close to those of the NFCB, consistent with the fact that the fixed asset tables primarily measure intangibles as capitalized R&D. The exception are returns to physical capital, which are higher among publicly traded firms. As a result, total rents as a fraction of value added are higher among Compustat firms than in the NFCB sector as a whole. The rent share of value added is about 2 percentage points higher in the post-2001 period in the Compustat sample, as indicated in Table 1. Other than this difference, when using only R&D, the implications of our analysis look similar for Compustat and the NFCB as a whole.

Second, once organization capital is included, intangibles are the dominant force behind the investment gap. With organization capital, the ratio of intangible to physical capital more than doubles. Returns to physical capital also further increase, since operating surplus rises after adjusting for the expensing of intangible investment in organization capital. However, the effect of the higher stock of intangibles dominates. After 2001, for instance, the two intangible-related terms account for 69% of the total investment gap, on average, as opposed to 39% when only including R&D. The intangible share in production approximately doubles compared to when only R&D capital is included, reaching $\eta = 0.48$ on average after 2001. Additionally, the importance of rents overall declines. The share of rents in value added falls to 4.9% of value added after 2001, compared to 8.7% when only R&D capital is included. Thus, intangible capital of an empirically plausible magnitude can account for the majority of the investment gap and reduce the role of rents substantially.

Appendix Figure 6 also reports the time series for the components of the gap obtained when explicitly separating R&D from SG&A. This decomposition is quantitatively similar to the one obtained by taking the sum of the two measures of intangibles. As discussed in Appendix 4.4, this alternative decomposition approach indicates that rents generated by R&D capital are rising somewhat faster than those generated by SG&A capital.

39 Additionally, Appendix Figure 5 reports the raw time series for the moments used in our baseline analysis, Appendix Figure 6 reports the time series for the investment gap and its decomposition, and Appendix Figure 7 reports the time series for the share of intangibles in production, the share of rents in value added, and the user costs of the two types of capital, all based on the aggregated data from the Compustat sample.
4.3 The investment gap at the sectoral level

Trends across sectors Table 2 reports averages of the six data moments used in the construction of the investment gap over two periods, 1985-2000 and 2001-2017.\textsuperscript{40} There are notable differences across sectors, even with this relatively coarse sectoral classification. High-tech and Healthcare are characterized by a combination of high asset returns and high valuations, declining physical investment, and a high (and rising) share of intangibles, consistent with the aggregate data for the NFCB sector as a whole. The Consumer and Services sectors also feature high returns and low physical investment. In these sectors, when measured as R&D, intangibles appear to be a negligible fraction of total capital. (As we discuss below, they are between one quarter and one half of total capital when organization capital is included.) Finally, Manufacturing is characterized by declining returns, declining valuations, declining physical investment, and a declining intangible share, in contrast to the other sectors.

Results using only R&D capital Figure 4 reports investment gaps and their decomposition for the five sectors of our analysis, when intangibles are measured only with R&D capital. The model used to construct this decomposition has positive adjustment costs of $\gamma_1 = 3$ and $\gamma_2 = 12$, as in the previous section. This figure shows that the level and the composition of the investment gap differs substantially across sectors.

One extreme is the Manufacturing sector. In that sector, the investment gap is small. Moreover, little of it is explained by intangibles. This is consistent with the fact that the stock of R&D capital (relative to the stock of physical capital) has been declining in manufacturing since the early 2000’s. Accordingly, the bottom panel of Table 2 indicates that intangibles’ share in the production function has decreased. Though rents have been rising in that sector — they increased by 3.8 percentage points of value added from before to after 2000, as indicated by Table 2 —, they remain small.

The other extreme is the Consumer and the Services sectors. There, the investment gap is large, in particular after 1990. However, it is almost entirely explained by rents to physical capital when using R&D capital alone — our measure of intangibles for this exercise — since measured R&D is very small.\textsuperscript{41} The combination of high returns, high valuations, and low intangibles lead to a high (and rising) share of rents in value added, reaching 12.4% in the Consumer sector and 13.0% in the Services sector after 2000, as reported in Table 2.

The Healthcare and High-tech sectors are intermediate cases. Both experienced a large increase in the physical investment gap starting in the mid-1980’s. In both cases, rents at-

\textsuperscript{40} Appendix Figures 25 to 29 report the full time series for these moments for each sector.

\textsuperscript{41} In the Consumer sector, intangibles rise slightly after the mid-2000’s, driven primarily by Amazon’s reported R&D expenditures, but remains too low to account for the physical investment gap.
tributable to physical capital have also increased. However, they only account for about one-half — in the High-tech sector — and one-third — in the Healthcare sector — of the investment gap overall. In both sectors, the key change in the composition of the investment gap after 2000 is a substantial increase in the level and rents to intangible capital. For the Healthcare sector, for instance, they account, alone, for 41% half of the total investment gap. Table 2 indicates that this is the effect of two changes: a rising intangible share; and a rise in overall rents. Rents as a fraction of value added rise by 6.6 percentage points in the High-tech sector, and 4.3 percentage points in the Healthcare sector, between the pre- and post-2000 periods. The intangible share in production also increased, particularly in the Healthcare sector, where it roughly doubles.

**Results including organization capital** The previous sectoral results were constructed using only R&D as a measure of the intangible capital stock. Expanding the definition of intangibles to include organization capital has two main effects, both of which are most clearly apparent in the Consumer sector. (Summary results are in Appendix Table 3.)

First, unsurprisingly, the implied share of intangibles in the production function increases substantially. The increase is particularly striking in the Consumer sector, where the stock of organization capital becomes comparable in magnitude to the stock of physical capital. (The increase in intangible intensity \( \eta \) is smaller, though still visible, in the Services sector.) Second, the level of implied rents declines substantially. In the Consumer sector, rents fall from 12.4% to 2.7% of value added after 2001. (In the Services sector, they fall from 13.0% to 8.2% of value added.) The combined effect of these two changes is to magnify the direct contribution of intangibles to the investment gap. The Consumer and Healthcare sectors are both particularly impacted; in both, intangibles measured in this way account for more than half of the investment gap.

It is worth noting, though, that while including organization capital leads to a substantial decrease in the level of rents, it has a more moderate impact on their trend. Figure 5 reports the cumulative change in the estimated share of rents in total value added from 1985 onward for each of the four sectors, measuring intangibles using either R&D (blue circled line) or the sum of R&D and organization capital (green crossed line). The Consumer sector is where including organization capital makes the sharpest difference: the cumulative change in rents falls by approximately one-third.\(^{42}\) In the Services sector, including organization capital also reduces the trend increase in rents, by about a fifth. In other sectors, there is little trend

\(^{42}\) Prior work (Foster et al., 2006; Crouzet and Eberly, 2018) has indeed argued that the Consumer sector relies extensively on intangible capital, particularly brand capital and, in more recent years, innovations to supply chain and logistics. Investment in these intangibles are not recorded as R&D expenditures, but instead expensed as SG&A, and so they are picked up by our measure of organization capital.
increase in organization capital relative to R&D capital after 1985, and so cumulative changes in rents are similar under the two measures.

**Counterfactuals** Figure 5 also reports a counterfactual that highlights the differential effects of the rise in intangibles across sectors. Similarly to Section 3, we compute the cumulative change in the share of rents that would have had to occur in order to explain the investment gap, had the ratio of intangible to physical capital stayed constant over the sample. In the Manufacturing sector, where intangible intensity is declining, the cumulative increase in rents would have been smaller. A similar finding holds for the Services sector, where intangible intensity is also slightly declining when using R&D capital only, as reported in Table 2. In the other sectors, it would have been larger, and in some substantially so. The Healthcare sector is the most striking example; there, the increase in rents needed to account for the investment gap without a rise in intangibles would have been about 50% (or 5 p.p.) larger. In the Consumer sector, the difference is approximately 30%, relative to the case where intangibles are measured including organization capital. Thus, in both of these sectors, a substantial part of the investment gap is due not purely to rising rents, but to the interaction of rising rents with high and growing intangibles.

### 4.4 The relationship between rents and intangibles

The previous analysis shows that sectors that experienced the sharpest increase in rents over the last three decades (Healthcare, High-Tech) were also those where intangible capital grew most rapidly. In this section, we ask whether the relationship between trends in rents and in intangible intensity is systematic, by exploring these trends at a more disaggregated level.

**Results using only R&D capital** Figure 6 summarizes the contrasting evolution of the five broad sectors of our analysis more succinctly. The top left panels of the figure reports the distribution of the rents parameter $\mu$ and the Cobb-Douglas share $\eta$ of intangibles in production as of 1980, with $\mu$ on the vertical axis and $\eta$ on the horizontal axis. The top right panel of the figure reports this distribution as of 2015.

As of 1985, rents and intangible intensity were low in all five sectors, and there was little heterogeneity across sectors — the five sectors cluster in the southwest portion of the graph. Thereafter, the five sectors diverge. In the Consumer and Services sectors, rents increased, but intangible intensity remained roughly the same — the sectors move vertically toward the northwest part of the graph. Rents and intangible intensity did not change substantially in the Manufacturing sector, which remains in the southwest corner of the graph. Finally, rents
and intangible intensity increased simultaneously in the Healthcare and High-tech sectors, which move out from the origin toward the northeast part of the graph.

Figure 6 also reports the distribution of rents $\mu$ and intangible intensity $\eta$ for the subsectors that make up each of the five sectors in our analysis. The subsectors correspond to the NAICS 2D/3D level and are those described in Appendix Tables 1 and 2. Each subsector is represented by a transparent dot (the shape of the dots match those of their parent sector). Additionally, in order to keep the graph area compact, we have not plotted six subsectors where $\mu$ exceeds 2 in 2015.

Figure 6 suggests that the evolution of the five broad sectors generally captures the more granular evolution of their subsectors. With few exceptions, subsectors are initially clustered around the southwest part of the graph, indicating limited rents and intangibles in 1985. The Consumer and Services subsectors then experienced no increase in intangible intensity but a sharp increase in rents, moving up toward the northwest. The Healthcare and High-tech subsectors also generally experienced a simultaneous increase in both rents and intangibles, moving out northeastward between the 1985 and 2015 plot.

However, the evolution of subsectors within Manufacturing seems to have been substantially more heterogeneous than the aggregate sector’s evolution would suggest. Certain subsectors experienced a large increase in both intangibles and rents, while other remained physical-capital intensive and rent-free. For instance, subsector 333 (Machinery, in which the two largest companies by book assets in 2015 were John Deere and Caterpillar) experienced both a large increase in intangibles, and a large increase in rents. On the other hand, subsector 212 (Mining excluding Oil and Gas, in which the two largest companies by book assets in 2015 were Newmont Mining and Freeport McMoRan) had stable intangible intensity and no notable increase in rents over the period. The same pattern holds in the Oil and Gas subsector (324), which also had stable intangible intensity and stable rents over the period.

As a result, within Manufacturing (as also within Healthcare and High-tech), sectors which experienced a large increase in intangible intensity also experienced a high increase in rents — as in the broad Healthcare and High-tech sectors. Aggregation however obscures this coherence between the three sectors, as Manufacturing is dominated by subsectors where

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43 In this analysis, we have dropped all the subsectors that did not have at least ten firms in each year from 1985 to 2015 in Compustat; the list of the subsectors dropped for this reason is reported in Appendix Tables 1 and 2.

44 These sectors are the following (with their NAICS code, the share of operating profits in their sector, and the implied value for $\mu$ in 2015): in High-tech, Computer Systems Designs and Related Services (5415; 2.4%; $\mu = 2.04$) in Manufacturing, Food and Beverage and Tobacco Products (311; 20%; $\mu = 2.71$), Apparel and Leather Product (315; 3.3%; $\mu = 2.95$), and Oil and Gas extraction (211; -20.2%; $\mu = 2.30$); in Services, Administrative and Support Services (561; 18.3%; $\mu = 3.25$), and Miscellaneous Professional, Scientific and Technical Services (5412; 16.0%; $\mu = 3.08$).
rents and intensity did not substantially change since 1980, while in Healthcare and High-tech, most subsectors experienced an increase in intangible intensity and rents. This pattern stands in contrast with the Consumer and Services subsectors, where rents rose in spite of little or no change in intangible intensity, at least as measured by R&D, which we generalize below.

Figure 7 expands on the differences between the Manufacturing, Healthcare, and High-tech sectors, on the one hand, and the Consumer and Services sectors, on the other. The top two panels of the figure report a scatterplot of time trends of the rents parameters $\mu_{s,t}$ and the Cobb-Douglas intangible share $\eta_{s,t}$, estimated within each of the 55 subsectors separately. These scatterplots help evaluate whether subsectors where the trend increase in intangibles was high, also experienced a high trend increase in rents, and vice-versa.

Consistent with the previous results, the scatterplots indicate that this is the case for the Manufacturing, Healthcare, and High-tech subsectors — where the correlation between the time trends in rents and intangibles is positive —, but not for the Consumer and Services subsectors — where the correlation is negative. Table 3 provides additional evidence consistent with this interpretation of Figure 7, using the simple regression framework:

$$\mu_{s,t} = \alpha_s + \beta \eta_{s,t} + \epsilon_{s,t}. \quad (17)$$

The point estimates of $\beta$ — which capture the within-subsector covariance between rents and intangibles — is positive in the sample of Manufacturing, Health and High-tech subsectors, but negative in the sample of Consumer and Services subsectors.

This supports the notion that in the Manufacturing, Health and High-tech subsectors, when rents rose, they rose in tandem with intangible (R&D) capital. By contrast, the two trends were not coincident in the Consumer and Services subsectors.

**Results including organization capital**  Figures 6 and 7, and Table 3, also report results for the case where organization capital is also used in addition to R&D capital to measure intangible intensity. The results are qualitatively similar, and quantitatively stronger, in the sample of Manufacturing, Healthcare, and High-Tech subsectors.

For Consumer and Services, the results generally still support the view that the rise in rents

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45 More precisely, Figure 7 reports the coefficients $\{\gamma_\mu, \gamma_\eta\}$ in:

$$\mu_{s,t} = \delta_{\mu,s} + \gamma_{\mu,s} t + \epsilon_{\mu,s,t},$$

$$\eta_{s,t} = \delta_{\eta,s} + \gamma_{\eta,s} t + \epsilon_{\eta,s,t}.\,$$

46 The slope of the simple OLS line in the top left panel of Figure 7 is 0.51, with a heteroskedasticity-robust t-statistic of 1.34; on the top right panel, the slope is −5.70, with a robust t-statistic of 2.84.
was not accompanied by a rise in intangible intensity in those sectors. The time trends in the Cobb-Douglas share $\eta_{s,t}$ and the rents parameters $\mu_{s,t}$ appear to be weakly negatively related. The point estimate of the coefficient $\beta$ in Equation 17 is positive, though it is smaller than for the Manufacturing, Healthcare and High-tech subsectors, and only marginally statistically significant. It should be noted that there are relatively few subsectors belonging to the Services and Consumer sectors in the Compustat Non-Financial sample (only seven in total), making it more difficult to ascertain whether rents and intangibles rose in tandem in those subsectors. Nevertheless, even including organization capital as a measure of intangibles, the evidence of positive correlation between rising rents and rising intangibles appears to be substantially less clear-cut in the Consumer and Services subsectors than in the Manufacturing, Healthcare and High-tech subsectors.

Relation to prior work and interpretation

In prior work (Crouzet and Eberly, 2019), we highlighted the fact that measures of intangible intensity and measures of markups appeared to be correlated, both in aggregate or sectoral time series, and within sector. The current analysis differs in two main ways. First, the methodology we use is different: our prior work used reduced-form proxies for intangibles and market power, while this analysis uses estimates of the rents parameter $\eta$ and the intangible intensity $\mu$ derived from our structural model. Second, the results we arrive differ in some important ways from our prior analysis. In particular, the joint increase in intangible intensity and in rents only appears to be significant in the High-Tech, Healthcare, and Manufacturing sample. By contrast, in the Consumer and Services sectors, the relationship is either significant and negative (with R&D only), or has weak statistical significance (with R&D and organization capital).

One potential interpretation of the contrasting results between High-tech, Healthcare, and Manufacturing, on the one hand, and Consumer and services, on the other, is that intangible investment has a different economic function in each of these groups of sectors. In the first group (which contains subsectors such as machinery or medical devices), intangible investment may be associated with product differentiation, which in turn might allow firms to charge higher prices and earn higher rents. On the other hand, in Consumer and Services (which contains subsectors such as retail chains), product differentiation may be weaker. There, intangible investment might instead be associated with efficiency gains and reductions in costs (for instance, through process innovation), which could in turn lead to price competition and lower rents.

Summarizing, the three main findings of this section are the following. First, a broader empirical definition of intangibles — one that includes organization capital — reduces the
contribution of rents to the investment gap, and substantially so after 2000. Second, even
across broadly defined sectors, there are large differences in the composition of the investment
gap. As a whole, the Manufacturing sector has a small investment gap, declining intangibles,
and moderate rents, at odds with aggregate trends. By contrast, the Healthcare and High-
tech sector are characterized by a larger investment gap than in aggregate, and one where
intangibles play a bigger role, particularly in the Healthcare sector. Third, the rise in rents
and the rise in intangibles are systematically correlated within the subsectors of Healthcare,
High-Tech, and Manufacturing, but not within Consumer and Services subsectors. The latter
two findings are particularly interesting: they suggest that any aggregate statement about the
investment gap may be misguided, as there is substantial heterogeneity in both the aggregate
investment gap itself and the underlying forces that explain it.

5 Robustness and additional results

In this section, we discuss robustness checks on our baseline results, as well as results related
to the extensions to our main model that were discussed in Section 2.5.

5.1 Enterprise value

We consider an alternative measure of the enterprise value of the NFCB sector, that of Hall
(2001). As mentioned above, this measure subtracts all financial assets of the NFCB sector
from gross claims, instead of subtracting only liquid financial assets, as we do in our baseline.
The top panel of Appendix Figure 8 reports the time series for $Q_1$ obtained this way (details
on data construction are reported in Appendix 3.2). It is lower than in our baseline, though it
displays approximately the same medium and long-run trends. The bottom panel of Appendix
Figure 8 then reports the investment gap obtained using this measure of $Q_1$.

The main difference with our baseline is in the overall level of the gap; it is about half
as large. As a result, implied rents are lower than in our baseline. For instance, without
adjustment costs, rents are 4.2% of value added when using this measure of $Q_1$, as opposed
to 7.7% in our baseline measurement exercise, and their cumulative increase from 1985 to
2015 is 5 p.p., as opposed to 6.2 p.p. in our baseline measurement exercise.\footnote{With adjustment costs, the share of rents is 3.4% in 2015, and the 1985-2015 increase is 5.1 p.p.} Moreover, the
direct effect of intangibles becomes larger; and overall, intangibles account for more of the gap
with this measure of $Q_1$ than in our baseline. Overall, results using this alternative measure
of the enterprise value of the NFCB sector suggest that intangibles play a larger role in the
investment gap.
5.2 Adjustment costs

The value of $\gamma_1 = 3$, which we draw from Belo et al. (2019), is in the range of typical estimates of values of the convexity parameter in quadratic adjustment investment cost functions. Given the annual calibration, the value $\gamma_1 = 3$ corresponds to a doubling time of three years, between the cases of fast adjustment (2 years) and slow adjustment (8 years) considered in Hall (2001). The value $\gamma_2 = 12$ is close to the baseline estimate for knowledge capital adjustment costs in Belo et al. (2019).

Appendix Figure 9 reports four implied moments under alternative combinations of adjustment costs for physical and intangible assets. The values considered are $\gamma_2 \in [0, 20]$ and $\gamma_1 \in [0, 10]$. The four moments are the change in the overall investment gap, $Q_1 - q_1$; the contribution of intangibles to the investment gap in 2015; the implied intangible share in 2015; and the implied share of rents in total value added in 2015. Of these moments, none display significant sensitivity to changes in user costs except the share of rents in value added. That share is highest when adjustment costs are lowest. This is because taking into account adjustment costs tends to raise user costs of capital and lower implied rents.

5.3 GMM estimation on split samples

In our baseline approach, we apply moments conditions implied by the model to seven-year averages of the underlying data in order to construct our decomposition of the investment gap. Appendix 4.5 reports results from a different methodology, which consists of estimating a version of the model with i.i.d. shocks to fundamentals and no adjustment costs using GMM on split samples (the 1985-2000 and 2001-2017 samples, respectively).\footnote{We focus on the post-1985 period because over this period, results from aggregate data and results from the Compustat sample can be compared.}

The moment conditions used in this estimation are similar to those described in Section 3.1, so the results of the estimation, which are reported in Appendix Table 4 are qualitatively in line with our baseline analysis, confirming that, in the NFCB sector, both rents and the Cobb-Douglas intangible share increased.\footnote{Quantitatively, the results are somewhat different because the underlying data used to estimate the moments conditions is not averaged over seven-year windows.}

The value added by this estimation approach over our baseline approach is that it allows to test formally whether point estimates of the structural parameters of interest (intangible intensity, the size of rents relative to value added, and user costs) changed significantly across subsamples. Appendix Table 4 shows that for the NFCB data, across the two subsamples, the increase in rents and in intangible intensity, and the decline in user costs, are all statistically significant. However, this is not the case in the Compustat sample, where changes in rents,
in particular, are not significant in the specifications where intangibles are measured using organization capital in addition to R&D capital. These results thus further support the notion that including organization capital in our analysis substantially reduces the estimated increase in rents.

5.4 Alternative identification strategies

We next discuss alternative identification strategies for constructing the various elements of the investment gap decomposition (13). These alternative approaches are described in greater detail in Appendix 4.6. They use measures of the average cost of capital in order to construct \( r \) and the Gordon growth term \( r - g \), and but do not necessarily match observed values of \( Q_1 \). By contrast, our approach measures infer the Gordon Growth term \( r - g \) from \( Q_1 \), so that it matches \( Q_1 \) by construction.

**Average cost of capital approach**  Appendix 4.6.1 reports the investment gap decomposition obtained using a first alternative identification strategy, which we call the ”average cost of capital approach”. This approach is closer to that of Barkai (2020) and Karabarbounis and Neiman (2019) (case Π). We measure the average cost of capital as the leverage-weighted average of the cost of debt (obtained from average interest rates on the market value of debt of the NFCB sector), and the cost of equity (obtained from the PD ratio of public firms). We then construct the different terms on the right-hand side of Equation (13) using the same moments as in our baseline, except that we do not match the observed value of \( Q_1 \).

In this approach, the (implied) value of the investment gap (that is, the left-hand side of Equation 13) is growing faster after 1985 than the investment gap we measured in our baseline approach (that is, the left-hand side of Equation 13). By 2015, the implied investment gap is about twice as large as the measured one. This is because the discount rate \( r \) obtained using an average cost of capital approach is lower, and declining faster, than the discount rate implicit in our baseline decomposition. Consistent with Barkai (2020) and Karabarbounis and Neiman (2019) (case Π), lower discount rates also lead to a higher, and more rapidly increasing profit share (approximately 9.0 p.p. over the 1985-2015 period, instead of 6.2 p.p. in our baseline approach).

However, the composition of the implied investment gap remains similar to our baseline findings. Appendix 4.6.1 reports more detailed results, compares the discount rates implied by both approaches, and expands on the interpretation of the results in terms of implicit equity risk premia, following the discussion of Section 3.3.
**Inferring intangibles from the investment gap** Appendix 4.6.2 describes a second alternative approach, which builds on the average cost of capital approach. In this approach, we use the values of $Q_1$ in order to infer the size of the intangible capital stock, instead of matching the times series for $S$ from the BEA data. This approach is similar to the "no rents" case discussed above in that movements in the implied intangible capital stock mirror movements in $Q_1$. It can be thought of as an extension of the analysis of Hall (2001) that allows for rents.

This approach suggests that about two-thirds of the investment gap is due to intangibles, as opposed to one-third in our baseline analysis. However, similar to the "no rents" case, this approach leads to inferring an intangible capital stock that is sharply declining in the late 1970s and early 1980s, by contrast with most empirical measures of intangibles, including the one which our baseline approach uses.

**Inferring rents from the investment gap** Finally, Appendix 4.6.2 describes a third alternative approach, which consists of inferring the rents parameter $\mu$ from the value of $Q_1$, while again using the average cost of capital approach to measure $r - g$. This approach matches all the same moments as our baseline analysis, except average returns to capital $ROA_1$. Overall, this approach leads to an investment gap decomposition that is quantitatively and qualitatively similar to our baseline analysis. The main difference is that the increase in rents is somewhat smaller than in our baseline approach. Relatedly, the implied returns to capital in this approach, are on average approximately 4 p.p. lower than in the data, and increases somewhat less after 1980.

**Summary** While these alternative approaches lead to somewhat different values of rents and of the investment gap than in our baseline analysis, the relative contribution of intangibles to the investment gap is similar (or larger) in these alternative approaches compared to our baseline approach.

We view our $Q$ approach as having two main advantages over these alternatives. First, it allows us to match simultaneously the two most natural metrics of the returns to investment, the average return to capital, $ROA_1$, and Tobin’s $Q$, $Q_1$, whereas the alternative approaches generally do not match these moments or, when they do, require large and sometimes negative changes in intangibles. Second, our baseline approach does not require information on the capital structure of the firm, other than that contained in the measurement of enterprise value. This allows us to sidestep issues related to direct measurement of the cost of equity and debt capital.
5.5 Markups and returns to scale

Appendix 4.7 discusses what our estimates of total rents imply for "pure rents" (rents attributable to pricing power or markups) and for "quasi-rents" (rents attributable to decreasing returns). Specifically, we estimate the markup \( \mu_S \) over the marginal cost of sales implied by different degrees of returns to scale \( \zeta \).

The broad conclusion from the empirical analysis is that, at the aggregate level, a relatively modest degree of decreasing returns to scale \( (\zeta = 0.95) \) is sufficient to account for most of revenue in excess of capital costs and variable input costs without having to resort to markups.\(^{50}\) Additionally, even under increasing returns to scale \( (\zeta = 1.05) \), the implied sales markup remains substantially below existing estimates. In particular, in our baseline specification, the markup over sales is 1.099 on average after 2000, increasing from 1.072 on average in the pre-2000 period. By comparison, De Loecker et al. (2020) report a revenue-weighted average markup of price over the cost of sales of approximately 1.5 after 2010.\(^{52}\)

Related work has taken different approaches to estimating returns to scale, depending on data availability. Where detailed cost data are available, for example from the Census of Manufacturing, returns to scale can be estimated using data on cost shares and output. Syverson (2004) develops this methodology and estimates that a benchmark of constant returns to scale is justified in his detailed industry analysis. More recently De Loecker et al. (2020) use two approaches. First, using Compustat and hence lacking detailed cost shares, they use a demand approach and estimate slightly increasing returns to scale in their specifications. In a standard specification, similar to ours, they estimate nearly constant returns of 1.02 in 1980, rising to 1.08 by 2016. When they specify overhead in the production function, which in Compustat includes some intangibles, they have higher returns to scale of 1.07 initially rising to 1.13 at the end of the sample. When instead they approximate the Syverson (2004) cost share methodology, they obtain lower estimates of nearly constant returns, of 0.98 pre-1980 and 1.03 by 2010, using industry averages. In firm-level data, which have more heterogeneity, they find initially slightly more decreasing returns and a larger increase.

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\(^{50}\) In order to estimate \( \mu_S \), we additionally require data on total revenue, as opposed to operating surplus. As discussed in Appendix 4.7, this data does not appear to be available for the NFCB sector in either NIPA tables or in Flow of Funds data, so we use the Compustat sample of Section 4 instead.

\(^{51}\) Recent research has argued that returns to R&D investment may have declined in recent years (Bloom et al., 2020), which would strengthen the idea that “quasi”-rents, not pure rents, may explain the growth in total rents in our estimates, particularly in R&D-intensive sectors.

\(^{52}\) See their Figure III.
5.6 Heterogeneous rents parameters

In our baseline approach, the operating profit function is homogeneous of degree $\mu$ with respect to all capital types. As discussed in Section 2, it is straightforward to relax this assumption and instead construct the investment gap with heterogeneous rents parameters $\{\mu_n\}_{n=1}^N$. However, estimating these rents parameters in the data is more challenging; separate identification of each rents parameter $\mu_n$ would essentially require measuring separately revenue generated from each type of capital, which we do not observe in the data.

In Appendix 4.8, we nevertheless make two empirical points about the model with heterogeneous rents parameters $\{\mu_n\}_{n=1}^N$. First, it can be shown that if the true model featured heterogeneous rents, then the $\mu$ measured in our baseline methodology would be the user-cost weighted average rents parameter across capital types. Second, two limit cases (all rents generated by intangibles, $\mu_1 = 1$; and all rents generated by physical capital $\mu_2 = 1$) can be identified in the data using the same moments as our baseline approach. The former case requires an intangible intensity $\eta$ in the order of $\eta = 0.5$, and a rent parameters in the order of $\mu_2 = 2$, to rationalize the data, both of which are substantially above our baseline estimates. The case $\mu_2 = 1$, by contrast, delivers predictions that are closer to our baseline, primarily because of the relative size of physical capital, the average rents parameter $\mu$ is close to the rents parameter for physical capital, $\mu_1$.

5.7 Financing frictions

As mentioned in the extensions to the model described in Section 2.5, while introducing simple financing frictions does not change the insight of Result 1, frictions can affect the overall magnitude of the gap, or its composition.

In Appendix 4.9.1, we discuss the quantitative impact of equity financing frictions on the size of the investment gap. On the balanced growth path, these frictions generally imply that the investment gap is larger than in our baseline model. With equity financing frictions, the first-order condition for firm investment is $q_{n,t} = \Phi_{n,t} f'(d_t)$, where $f'(d_t)$ captures the wedge between the marginal value of internally generated cash and external distributions to and from shareholders. Along the balanced growth path, it must be that $d_t < 0$, so $f'(d_t) < 1$, and so $q_{n,t}$ is lower than when there are no equity financing frictions $f'(d_t)$. Intuitively, the marginal returns to increasing capital are lower because of the wedge created the friction, $f'(d_t) < 1$.

Appendix Figure 10 reports the implied size of the investment gap, for different values of $f'(d_t)$, along the balanced growth path, along with the composition of the investment gap between total rents (attributable to intangibles and physical capital) and the omitted
intangibles effect. The main message of these figures is that introducing equity financing frictions will in general magnify the total contribution of rents to the gap. The intuition is that total rents (those due to either physical capital or intangibles) are the residual after taking into account the value of the intangible capital stock. This latter value is adjusted downward with equity financing frictions, because of the wedge $f'(d_t)$ between inside and outside finance. Thus for a given (empirical) value of $Q_1$, rents are magnified. Appendix Figure 11 repeats this exercise across the five sectors of the analysis of Section 4; the effects of introducing equity financing frictions are most visible for the most intangible-intensive sectors, where the omitted capital effect is initially largest. However, in general, even introducing relatively large frictions ($f'(d) = 0.80$, implying that one dollar of free cash flow after investment only raises flow payoffs to shareholders by 80 cents) does not alter our qualitative conclusions, and quantitatively, only has modest effects on the overall direct contribution of intangibles to the gap.

Finally, Appendix 4.9.2 studies how frictions to debt issuance affect our estimates. The main result is that, when the collateral constraint applies to the stock of physical capital (only), total rents remain correctly estimated in our approach, but the contribution of physical rents is underestimated, while the contribution of intangibles is overestimated. The intuition for this result is that the collateral constraint reduces the “internal” user cost of capital, because part of the return to holding physical capital is that it relaxes the borrowing constraint, and helps shareholders lever up and take advantage of the wedge between their discount factor and that of debtholders. Additionally, we show that with a collateral constraint, our approach leads to total user costs that are always larger than their true value (driven by the overestimate of the user cost of physical assets). In turn this implies that our baseline approach underestimates the rents parameter $\mu$, relative to its true value. Thus, when borrowing creates excess returns to shareholder, but is subject to a collateral constraint that applies to physical capital only, our baseline results will generally overstate the role of rents to physical capital, and understate the value of $\mu$.

However, Appendix 4.9.2 also shows that these effects are likely to be quantitatively small. For instance, even for a wedge between shareholders’ discount rate, and debtholders’ discount rate, of 5%, our estimate of total rents as a share of value added is only 2 p.p. higher than its

53 In Appendix 4.9.1, we show that the complete decomposition cannot be obtained in the model with equity issuance frictions without additional parametric assumptions about $f(\cdot)$. However, we provide conditions on the Cobb-Douglas share of intangibles in production such that our baseline estimate of the relative contribution of intangibles to total rents (which assumes no equity issuance frictions) is an underestimate of their true contribution, in the presence of equity issuance frictions.

54 This result is reminiscent of Bianchi et al. (2019), who find, using a structural approach, that equity financing shocks (as opposed to debt financing shocks) are more likely to affect R&D investment.
true value. Thus, overall, the omission of this form of financing friction from our baseline model is unlikely to substantially alter our quantitative results.

5.8 Rents and productivity

Finally, we briefly discuss the relationship between our measure of rents, and measures of total factor productivity. Our baseline analysis estimates the rents parameter \( \mu \) using the ratio of aggregate or sectoral returns to physical capital, to estimated user costs. One concern is that, if firms have heterogeneous marginal returns to capital, highly productive firms will produce more, have higher revenue, and push up average returns to physical capital. This could occur even if rents are relatively small. To alleviate this concern, in Appendix 4.10, we use the disaggregated data from Section 4, along with estimates of total factor productivity at the subsector level obtained from the BLS KLEMS tables, to study the correlation between our estimates of rents, and measures of total factor productivity. We show that growth in the two measures are uncorrelated, both within the Healthcare/High-Tech/Manufacturing sectors and within the Consumer/Services sectors. This suggests that our rents measure is not primarily driven by heterogeneity in marginal returns to capital across firms.

6 Conclusion

This research provides a general decomposition of the gap between average \( Q \) — which is observable — and marginal \( q \) — the shadow value that drives investment. This decomposition captures the effects of unmeasured capital, such as intangibles, and also the effect of rents.

We use measurement of the gap to shed light on the growing divergence between physical investment and valuations, which our approach interprets as being driven by the combined effects of growing rents and growing intangible capital. With a relatively narrow measure of intangibles (R&D capital), one-third of the investment gap reflects a combination of growth in the intangible capital stock and rents generated by intangible capital. Expanding the definition of intangibles beyond R&D increases this contribution to about two thirds. In addition to these aggregate effects, sectoral results show that rents on intangibles are largest in some of the fastest growing sectors in the economy, such as Tech and Health, and that within these sectors, rents are highest in subsectors with rapid growth in intangibles, as well.

---

55 We estimate the investment gap and its elements by matching \( \theta \) to observed values of the ratio of book debt to the replacement cost of physical assets, and by calibrating the wedge in discount rates \( r - r_b \).

56 In the limit where rents are zero, for instance because products within an industry are perfect substitutes, the most productive firms would be the only ones to produce; this is highlighted, for instance, in Autor et al. (2020).
Our analysis opens several questions for future research.

First, though our general decomposition allows for risk premia, we remained deliberately agnostic about their source in our empirical applications. A more thorough treatment of their interaction with the investment gap would be a useful next step. A particularly interesting direction to explore are priced capital quality shocks specific to intangible capital, as the rise in intangibles might then contribute to the growing wedge between the risk-free rate and the implicit firm discount rates discussed in Section 3.

Second, our decomposition holds at the firm level. Exploring the distribution of the investment gap across firms of particular sectors would both help validate our findings on the sources of the investment gap, and shed further light on the reasons for its growth over the last two decades.

Third, our decomposition suggests ways in which standard investment-Q regressions might need to be adjusted in order to take into account the possibility that firms have intangible capital and earn rents. Specifically, building on Peters and Taylor (2017), the decomposition suggests that controlling for intangible intensity may not be sufficient; an additional interaction term with empirical proxies for rents may further help improve the empirical performance of the regressions, particularly in the cross-section, a dimension we have not explored in this paper.\footnote{Relatedly, in previous work (Crouzet and Eberly, 2019), we showed that estimated time effects in a standard investment-Q panel regressions display a smaller downward trend when controlling for intangibles; the current analysis suggests expanding this with an interaction term capturing rents.}

Fourth, we noted in Section 2.5 that there is a close link between the investment gap and the gap between stock returns and returns to investment. The returns gap, in our model, is driven by the same three fundamental forces that explain the investment gap. As we argued, because it captures changes in firm value, the returns gap approach is better suited to studying short-run variation in rents and omitted factors such as intangibles. Using our framework to decompose higher-frequency data on the returns gap would help connect our $Q$ approach to the production-based asset pricing literature, and also hopefully shed light on the importance of rents and omitted factors for that literature.

Finally, and in a different vein, we have maintained a neoclassical approach to the interaction between intangibles and rents. A broader approach, however, could allow for an economic interaction; for example, investment in intangibles such as product innovation or a software platform may generate rents to the firm. These interactions would augment the neoclassical approach we take here, and could generate additional links between intangible capital and the decisions and valuation of the firm. We pursue this in future work.
References


<table>
<thead>
<tr>
<th></th>
<th>Non-Financial Corporate Businesses</th>
<th>Compustat non-financials (R&amp;D)</th>
<th>Compustat non-financials (R&amp;D+ org. cap.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1$ Physical investment rate</td>
<td>0.089</td>
<td>0.108</td>
<td>0.099</td>
</tr>
<tr>
<td>$i_2$ Intangible investment rate</td>
<td>0.252</td>
<td>0.276</td>
<td>0.281</td>
</tr>
<tr>
<td>$S$ Intangible/physical capital</td>
<td>0.053</td>
<td>0.078</td>
<td>0.124</td>
</tr>
<tr>
<td>$ROA_1$ Return on physical capital</td>
<td>0.208</td>
<td>0.211</td>
<td>0.211</td>
</tr>
<tr>
<td>$Q_1$ Av. Q for physical capital</td>
<td>1.184</td>
<td>1.413</td>
<td>2.032</td>
</tr>
<tr>
<td>$g$ Growth rate of total capital stock</td>
<td>0.034</td>
<td>0.038</td>
<td>0.029</td>
</tr>
</tbody>
</table>

| $Q_1 - q_1$ Investment gap | 0.072 | 0.308 | 0.908 | 1.439 | 0.620 | 1.173 | 0.859 | 1.446 |
| % rents from physical capital | 69   | 41   | 61   | 71   | 71   | 66   | 36   | 33   |
| % intangibles | 25   | 52   | 21   | 14   | 15   | 15   | 43   | 41   |
| % rents from intangibles | 7    | 7    | 18   | 24   | 15   | 19   | 21   | 26   |
| $\eta$ Intangible share in production | 0.099 | 0.145 | 0.227 | 0.286 | 0.179 | 0.222 | 0.392 | 0.440 |
| $s$ Rents as a fraction of value added | -0.008 | 0.014 | 0.035 | 0.067 | 0.029 | 0.076 | 0.005 | 0.043 |
| $R_1$ User cost of physical capital | 0.193 | 0.171 | 0.143 | 0.128 | 0.164 | 0.155 | 0.163 | 0.155 |
| $R_2$ User cost of intangible capital | 0.392 | 0.369 | 0.341 | 0.312 | 0.350 | 0.326 | 0.339 | 0.322 |
| $\mu$ Curvature of operating profit function | 0.984 | 1.051 | 1.136 | 1.244 | 1.117 | 1.290 | 1.023 | 1.145 |
| $\tilde{\mu}$ Markup over value added | 0.993 | 1.014 | 1.037 | 1.072 | 1.030 | 1.083 | 1.005 | 1.045 |

Table 1: Summary of targeted and implied moments, for the non-financial corporate business sector (columns 3 to 5) and for the Compustat Non-Financial sample. For Compustat non-financials, columns 6 and 7 use R&D as the measure of intangibles, and columns 8 and 9 use the sum of R&D and SG&A as the measure of intangibles. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: $K_t = K^{1-\eta}L^{\eta}$. Rents as a fraction of value added are computed as $s = (1 - s_L)(1 - 1/\mu)$, where $s_L$ is the labor share of value added for the NFCB sector. Markups over value added are computed as $\tilde{\mu} = 1/(1 - s)$. The implied moments reported are for the model with adjustment costs; the adjustment cost values are $\gamma_1 = 3$ and $\gamma_2 = 12$. In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources are described in Sections 3 and 4.
<table>
<thead>
<tr>
<th>Compustat non-financials (Intangibles = R&amp;D)</th>
<th>Consumer</th>
<th>Services</th>
<th>High-tech</th>
<th>Healthcare</th>
<th>Manufacturing</th>
</tr>
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<tbody>
<tr>
<td>(i_1) Physical investment rate</td>
<td>0.128</td>
<td>0.098</td>
<td>0.142</td>
<td>0.084</td>
<td>0.139</td>
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<tr>
<td>(i_2) Intangible investment rate</td>
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<td>0.317</td>
<td>0.241</td>
<td>0.224</td>
<td>0.346</td>
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<tr>
<td>(S) Intangible/physical capital</td>
<td>0.008</td>
<td>0.023</td>
<td>0.028</td>
<td>0.010</td>
<td>0.227</td>
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<tr>
<td>(ROA_1) Return on physical capital</td>
<td>0.269</td>
<td>0.281</td>
<td>0.261</td>
<td>0.245</td>
<td>0.359</td>
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<tr>
<td>(Q_1) Av. Q for physical capital</td>
<td>2.672</td>
<td>2.651</td>
<td>2.517</td>
<td>2.587</td>
<td>2.937</td>
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<td>(g) Growth rate of total capital stock</td>
<td>0.054</td>
<td>0.037</td>
<td>0.082</td>
<td>0.016</td>
<td>0.065</td>
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<tr>
<td>(Q_1 - q_1) Investment gap</td>
<td>1.523</td>
<td>1.645</td>
<td>1.380</td>
<td>1.574</td>
<td>1.634</td>
</tr>
<tr>
<td>(%) rents from physical capital</td>
<td>98</td>
<td>93</td>
<td>93</td>
<td>97</td>
<td>46</td>
</tr>
<tr>
<td>(%) intangibles</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>31</td>
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<tr>
<td>(%) rents from intangibles</td>
<td>1</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>(\eta) Intangible share in production</td>
<td>0.013</td>
<td>0.058</td>
<td>0.038</td>
<td>0.020</td>
<td>0.324</td>
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<tr>
<td>(s) Rents as a fraction of value added</td>
<td>0.087</td>
<td>0.124</td>
<td>0.070</td>
<td>0.130</td>
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<tr>
<td>(R_1) User cost of physical capital</td>
<td>0.188</td>
<td>0.169</td>
<td>0.191</td>
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<td>(R_2) User cost of intangible capital</td>
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<td>(\mu) Curvature of operating profit function</td>
<td>1.412</td>
<td>1.575</td>
<td>1.308</td>
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<tr>
<td>(\tilde{\mu}) Markup over value added</td>
<td>1.095</td>
<td>1.142</td>
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<td>1.150</td>
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Table 2: Summary of targeted and implied moments for the different sectors of the Compustat non-financial sample. All columns measure intangibles as the R&D capital stock. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: \(K_t = K_t^{1-\eta}K_t^\eta\). Rents as a fraction of value added are computed as \(s = (1 - s_L)(1 - 1/\mu)\), where \(s_L\) is the labor share of value added for the NFCB sector. Markups over value added are computed as \(\tilde{\mu} = 1/(1 - s)\). The implied moments reported are for the model with adjustment costs; the adjustment cost values are \(\gamma_1 = 3\) and \(\gamma_2 = 12\). In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources are described in Section 4.
(a) Manufacturing, High-tech, and Healthcare sectors

<table>
<thead>
<tr>
<th></th>
<th>Rents ((\mu_{s,t}))</th>
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<tr>
<td></td>
<td>(\text{Intangibles} = \text{R&amp;D})</td>
<td>(\text{Intangibles} = \text{R&amp;D + org. cap.})</td>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
</tr>
<tr>
<td>Cobb-Douglas intangible share ((\eta_{s,t}))</td>
<td>1.39***</td>
<td>1.39***</td>
<td>1.13***</td>
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<tr>
<td></td>
<td>(0.10)</td>
<td>(0.44)</td>
<td>(0.09)</td>
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<td>Number of observations</td>
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<td>1040</td>
<td>1040</td>
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<tr>
<td>Adjusted R-sq.</td>
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<td>0.71</td>
<td>0.53</td>
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<td>Clustering of s.e.</td>
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<td>subsector</td>
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(b) Consumer and services sectors

<table>
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<td>(\text{Intangibles} = \text{R&amp;D + org. cap.})</td>
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<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
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<td>Cobb-Douglas intangible share ((\eta_{s,t}))</td>
<td>-6.04***</td>
<td>-6.04***</td>
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<td>(0.48)</td>
<td>(1.37)</td>
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<td>Adjusted R-sq.</td>
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</tbody>
</table>

**Table 3**: The relationship between intangibles and rents at the sub-sector level. In both panels, the model estimated is \(\mu_{s,t} = \alpha_s + \beta \eta_{s,t} + \epsilon_{s,t}\), where \(s\) is a sector and \(t\) is a year. Specifications marked (1) report heteroskedasticity-consistent standard errors, while the specifications marked (2) report standard errors clustered at the sub sector level. The data is the Compustat non-financial sample, aggregated to the level of the subsectors described in Tables 1 and 2. Both \(\mu_{s,t}\) and \(\eta_{s,t}\) are winsorized at the bottom and top 1%. The top panel reports results for the subsectors belonging to the Manufacturing, High-tech and Healthcare sectors (pooled together), while the bottom panel reports the results for the subsectors belonging to the Consumer and Services subsectors.
Figure 1: The investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate (NFCB) sector for different values of adjustment costs. In each panel, the crossed blue line is an estimate of $Q_1 - q_1$ constructed using data from the Flow of Funds and from the BEA fixed asset tables. The shaded areas present the decomposition of the physical investment gap into three terms, corresponding to the effects of rents generated by physical capital, the omitted capital effect due to intangibles, and rents generated by intangibles. The decomposition is described in Equation (13). The top panel reports results with zero adjustment costs ($\gamma_1 = \gamma_2 = 0$); the middle panel reports results with positive adjustment costs ($\gamma_1 = 3$, $\gamma_2 = 12$); and the bottom panel reports results with high adjustment costs ($\gamma_1 = 8$, $\gamma_2 = 18$). Methodology and data sources are described in Section 3.
Figure 2: Other model moments for the NFCB sector. Panel (a) reports the share of intangibles in production, $\eta$, when the capital aggregator is assumed to be Cobb-Douglas: $K_t = K_{1,t}^{1-\eta}K_{2,t}^{\eta}$. Panel (b) reports rents as a fraction of value added, $s_{VA}$, which is given by $s_{VA} = (1 - s_L)(1 - 1/\mu)$, where $\mu$ is the model parameter governing the size of rents, and $s_L$ is labor’s share of value added. Panels (c) and (d) report user costs for each type of capital, $R_1$ and $R_2$. The ”zero adjustment costs” case corresponds to $\gamma_1 = \gamma_2 = 0$; the ”positive adjustment costs case” corresponds to $\gamma_1 = 3$ and $\gamma_2 = 12$; the ”high adjustment costs” case corresponds to $\gamma_1 = 8$ and $\gamma_2 = 18$. Methodology and data sources are described in Section 3.
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1 Proofs of main results

1.1 Lemma 1 and Result 1

Proof. The first-order necessary condition and the envelope theorem, for each capital type, are:

\[ \Phi'_{n,t} = q_{n,t} \]

\[ \frac{\partial V_{t+1}}{\partial K_{n,t+1}} = \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1} \frac{K_{n,t+2}}{K_{n,t+1}} \]  \hspace{1cm} (1)

Multiplying the latter condition by \( M_{t,t+1} K_{n,t+1} \), combining with the former condition, and taking expectations at time \( t \), we obtain:

\[ q_{n,t} K_{n,t+1} = \mathbb{E}_t [M_{t,t+1} (\Pi_{n,t+1} K_{n,t+1} - \Phi_{n,t+1} K_{n,t+1})] + \mathbb{E}_t [M_{t,t+2} q_{n,t+1} K_{n,t+2}] \]  \hspace{1cm} (2)

Assuming the transversality condition \( \lim_{k \to \infty} \mathbb{E}_t [M_{t,t+k} q_{n,t+k-1} K_{n,t+k+1}] = 0 \) holds for each type of capital, we can iterate forward and sum across capital types to obtain:

\[ \sum_{n=1}^{N} q_{n,t} K_{n,t+1} = \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k} \{\Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+k} K_{n,t+k}\}] \]

\[ = \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \sum_{n=1}^{N} \{\Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+k} K_{n,t+k}\} \right]. \]  \hspace{1cm} (3)

On the other hand, firm value excluding current distributions is given by:

\[ V^e_t = \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \left\{ \Pi_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+k} K_{n,t+k} \right\} \right]. \]

Note that, given Assumption 1, we have that:

\[ \Pi_{t+k} = \mu \Pi_{K,t+k} K_{t+k} \]

\[ = \mu \sum_{n=1}^{N} \Pi_{K,t+k} \frac{\partial F_{t+k}}{\partial K_{n,t+k}} K_{n,t+k} = \mu \sum_{n=1}^{N} \Pi_{n,t+k} K_{n,t+k}, \]

so that firm value can be rewritten as:

\[ V^e_t = \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \sum_{n=1}^{N} \left\{ \mu \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+k} K_{n,t+k} \right\} \right]. \]  \hspace{1cm} (4)

Taking the difference between Equations (4) and (3) gives the result of Lemma 1. Result 1 follows from dividing both sides by \( K_{n,t+1} \), and subtracting \( q_{n,t} \).

1.2 Result 2

Proof. We start with the case of a general investment cost function:

\[ \Phi_n(x) = x - 1 + \delta_n + \gamma_n \Gamma(x), \quad n = 1, ..., N. \]
The necessary first-order conditions, for each type of capital, are given by:

\[
\Phi'_{n,t} = q_{n,t},
\]

\[
q_{n,t} = \frac{1}{1+r} \left( \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1} \frac{K_{n,t+2}}{K_{n,t+1}} \right),
\]

which we can rewrite as:

\[
(1 + r)\Phi'_{n}(1 + g_{n,t}) = \Pi_{n,t+1} - \Phi_{n}(1 + g_{n,t+1}) + \Phi'_{n}(1 + g_{n,t+1})(1 + g_{n,t+1}),
\]

where \( g_{n,t} \equiv \frac{K_{n,t+1}}{K_{n,t}} - 1 \).

We next guess and verify that \( g_{n,t} = \tilde{g} \) for \( n = 1, ..., N \), for some trend growth rate \( \tilde{g} \), is a solution. Substituting into the condition above, and re-arranging, we obtain:

\[
R_{n} = \Pi_{n,t+1},
\]

where we defined:

\[
R_{n} \equiv (r - \tilde{g})\Phi'_{n}(1 + \tilde{g}) + \Phi_{n}(1 + \tilde{g}).
\]

By homogeneity of degree 1 of the capital aggregator:

\[
\Pi_{n,t+1} = \frac{1}{\mu} A^{\frac{1-\mu}{1+\mu}} K_{n,t+1}^{\frac{1}{1+\mu}} \frac{\partial K_{t+1}}{\partial K_{n,t+1}}, \quad n = 1, ..., N.
\]

Under our guess, the left-hand side of this expression is a constant. Moreover, the linear homogeneity of the capital aggregator implies that \( K_{t+1} \) also grows at rate \( \tilde{g} \). Also, each partial derivative \( \frac{\partial K_{t+1}}{\partial K_{n,t+1}} \) is homogeneous of degree 0 in each of its arguments, implying that they only depend on the \( N - 1 \) ratios:

\[
S_{m,t+1} = \frac{K_{m,t+1}}{K_{1,t+1}}, \quad m = 2, ..., N,
\]

of each capital stock to the physical capital stock. Under our guess of constant growth \( g_{n,t} = \tilde{g} \), these ratios must be constant. Therefore, the right-hand side of equation (7) grows at rate \( g^{1-1/\mu} \tilde{g}^{1/\mu} \). Given that the left-hand side is constant, this implies that it must be the case that \( \tilde{g} = g \).

Taking the ratio of the first-order condition (6) for \( n = 2, ..., N \), to the first-order condition for \( n = 1 \), we obtain:

\[
\frac{R_{m}}{R_{1}} = \frac{\partial K_{t}/\partial K_{m,t}}{\partial K_{t}/\partial K_{1,t}}, \quad n = 2, ..., N.
\]

This is a system of \( N - 1 \) equations in \( N - 1 \) variables, \( \{S_{m,t+1}\}_{m=2}^{N} \). Assume that \( F \) is such that the solution \( \{\tilde{S}_{m}\}_{m=2}^{N} \) is unique. Then, the steps above show that any solution with constant growth rates \( g \) for each capital stock, and capital ratios equal to \( S_{m,t} = \tilde{S}_{m} \), satisfies the set of first-order conditions (5).

Finally, the expressions for the generalized user costs follow from the Taylor expansion:

\[
(r - g)\Phi'_{n}(1 + g) + \Phi_{n}(1 + g) = (r - g)(1 + \gamma_{ng} + o(g)) + \left( \delta_{n} + g + \frac{1}{2} \gamma_{ng}^{2} + o(g^{2}) \right).
\]

\[\text{In Appendix 2.1, below, we provide the exact solution to the system of equations (8) when the aggregator between capital types is constant elasticity of substitution (CES).}\]
\[ r + \delta_n + \gamma_nr - \frac{1}{2} \gamma_n g^2 + o(g)(r - g) \]
\[ = r + \delta_n + \gamma_nr + o(g). \]

This approximation is exact when adjustment costs are given by the following functional form:

\[ \Phi_n(x) = x - 1 + \delta_n + \gamma_n r (x - 1 + (r - (x - 1)) \log \left( \frac{r - (x - 1)}{r} \right) ) . \]

It can be checked that this cost function is strictly convex and satisfies the conditions \( \Phi_n(1) = \delta_n, \Phi'_n(1) = 1 \) and \( \Phi''_n(1) = \gamma_n, \ n = 1, 2 \). Additionally, this function satisfies the relationship:

\[ (r - y)\Phi'(1 + y) + \Phi(1 + y) = r + \delta_n + \gamma_n ry, \]

leading to an exact expression for the generalized user costs, \( R_n = r + \delta_n + \gamma_n rg . \)

\[ \Box \]

### 1.3 Results on Total \( Q \)

Define total \( Q \) (Peters and Taylor, 2017) and total net investment as:

\[ Q_{tot,t} \equiv \frac{V^e}{\sum_{n=1}^N K_{n,t+1}}, \]
\[ g_{t+1} \equiv \sum_{n=1}^N s_{n,t+1}g_{n,t+1}, \]

where:

\[ s_{n,t+1} = K_{n,t+1}/\sum_{n=1}^N K_{m,t+1}. \]

In general, we have:

\[ Q_{tot,t} = \sum_{n=1}^N s_{n,t+1}q_{n,t} + (\mu - 1) \sum_{n=1}^N s_{n,t+1} \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+1,t+k}) \right]. \]

Thus, \( Q_{tot,t} \) is the sum of two terms: “total marginal q”:

\[ q_{tot,t} = \sum_{n=1}^N s_{n,t+1}q_{n,t}, \]

and a term reflecting the rents generated by each type of capital. Unsurprisingly, in the presence of rents \( (\mu > 1) \), \( Q_{tot,t} \) overstates \( q_{tot,t} \), and does measure the incentive to invest.

However, even in the absence of rents \( (\mu = 1) \), Total \( Q \) may not be a sufficient statistic for total net investment. The total net investment rate is given by:

\[ g_{t+1} = \sum_{n=1}^N s_{n,t+1}\Psi_{n,t}(q_{n,t} - 1), \quad \Psi_{n,t} \equiv (\Phi'_n)^{-1}. \]

In general, \( g_{t+1} \) depends on each marginal \( q \) separately; it is not a monotone function of their weighted average \( q_{tot,t} \). Thus, even when \( \mu = 1 \), and \( Q_{tot,t} = q_{tot,t} \), the latter need not be a good proxy for total net investment.

When is \( q_{tot,t} \) a sufficient statistic for total net investment? A first case is when adjustment costs are
identical across types of capital goods, so that $\Psi_{n,t} = \Psi_{t}$, and when the function $\Psi_{t}$ is linear. In that case, the expression above simplifies to $g_{t,t+1} = \Psi_{t}(q_{\text{tot},t} - 1)$, and total $Q$ is indeed a sufficient statistic for total investment. Another case is if marginal $q$ is equal across different types of capital. In this case, $q_{\text{tot},t} = q_{n,t}$, and so $g_{t,t+1} = \sum_{n=1}^{N} s_{n,t+1} \Psi_{n,t}(q_{\text{tot},t} - 1)$, so that $q_{\text{tot},t}$ is a sufficient statistic for investment.

When is marginal $q$ equalized across types of capital? The framework studied by Peters and Taylor (2017) is an example where this is the case. That framework considers cost functions $C_1(K_{1,t}, K_{2,t})$ and $C_2(K_{1,t}, K_{2,t})$ that are not additively separable (as in this paper), but nevertheless satisfy $\frac{\partial (C_1 + C_2)}{\partial K_1} = \frac{\partial (C_1 + C_2)}{\partial K_2}$. In our model, the difference between marginal $q$ for two types of capital is given by:

$$q_{n,t} - q_{m,t} = \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(1 + g_{n,t,t+k}) \{\Pi_{n,t+k} - \Pi_{m,t+k} - (\Phi_{n,t+k} - \Phi_{m,t+k})\}] .$$

Thus, a sufficient condition for equalized marginal $q$s is that (a) $\Pi_{n,t} = \Pi_{m,t}$, which, using Equation (5), implies that $\partial K_{n,t}/\partial K_{m,t} = 1$, or equivalently that capital types are perfect substitutes; and (b) adjustment costs are identical across capital types.

---

$^2$As discussed in Appendix I.C of Crouzet and Eberly (2019), for the general class of cost functions of the form $C_1(K_{1,t}, K_{2,t})$ and $C_2(K_{1,t}, K_{2,t})$, a necessary condition for $q_{1,t} = q_{2,t}$ is that intangible and physical capital be perfect substitutes, and also that they depreciate at the same rates, and enter the capital aggregator with the same weights. In this sense, the conditions under which marginal $q$ is equalized across types of capital, and thus under which $Q_{\text{tot},t}$ and $q_{\text{tot},t}$ are relevant to understanding the behavior of total net investment, are fairly specific.
2 Model extensions

This appendix provides more details on the extensions to the model discussed in Section 2.5.

2.1 Preliminary: partial solution with CES capital aggregator

We start by stating a preliminary result, which partially characterizes the solution to the model when capital aggregation is CES. We use this result in several different extensions to the model.

Lemma 2. Assume that:

\[ \Pi_t = A_t^{1-\frac{1}{p}} K_t^\frac{1}{p}, \]

\[ K_t = \left( \sum_{n=1}^{N} \eta_n K_{n,t}^\rho \right)^{\frac{1}{\rho}}, \quad \rho \leq 1, \quad \sum_{n=1}^{N} \eta_n = 1. \]  

(12)

Then, the solution to the model satisfies:

\[ K_{n,t+1} = \xi_{n,t+1} A_{t+1}, \quad K_{t+1} = \xi_{t+1} A_{t+1}, \]

where:

\[ R_{n,t+1} \equiv \left( \sum_{n=1}^{N} \eta_n^{\frac{1}{1-\rho}} R_{n,t+1}^{\frac{1}{1-\rho}} \right)^{-\frac{1-\rho}{\rho}}, \]

\[ \xi_{t+1} = (\mu R_{t+1})^{-\frac{\rho}{1-\rho}}, \]

\[ \xi_{n,t+1} = \left( \eta_n \frac{R_{t+1}}{R_{n,t+1}} \right)^{\frac{1}{\rho}} \xi_{t+1}. \]

and:

\[ R_{n,t+1} \equiv \frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} + \Phi_{n,t+1}(1 + g_{n,t+1}) - \Phi'_{n,t+1}(1 + g_{n,t+1}) K_{n,t+2} K_{n,t+1}, \quad n = 1, \ldots, N. \]

Proof. The necessary first-order conditions and the envelope conditions of the general model are:

\[ \Phi'_{n,t}(1 + g_{n,t}) = q_{n,t} \]

\[ \frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} = \Pi_{n,t+1} - \Phi_{n,t+1}(1 + g_{n,t+1}) + \Phi'_{n,t+1}(1 + g_{n,t+1}) K_{n,t+2} K_{n,t+1}, \quad n = 1, \ldots, N, \]

where: \( q_{n,t} = E_t \left[ M_{t,t+1} \frac{\partial V_{t+1}^c}{K_{n,t+1}} \right]. \) Using the first part of Assumption 12, the envelope conditions imply that:

\[ A_t^{1-\frac{1}{p}} K_t^\rho \frac{1}{\rho} \frac{\partial K_{t+1}}{\partial K_{n,t+1}} = \mu R_{n,t+1}, \quad n = 1, \ldots, N. \]

When the capital aggregator is CES, as in the second part of assumption 12, one can check that the solution to these \( N \) equations takes the form reported in Lemma 2.

\[ \square \]
2.2 Closed form solutions with uncertainty

2.2.1 Result

Assume that the fundamentals process follows:

\[
\frac{A_{t+1}}{A_t} = 1 + g_t = \begin{cases} 
1 + g_{t-1} & \text{w.p. } 1 - \lambda \\
1 + \bar{g} & \text{w.p. } \lambda 
\end{cases} \tag{13}
\]

Here, \( \bar{g} \) is drawn, at time \( t \), from a distribution \( F(.) \), which is time-invariant, and the draw is independent of past realizations of \( g_t \). The investment gap for physical capital is then given by

\[
G_{1,t} = \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_1) + S + \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_2)S. \tag{14}
\]

The function \( \nu(g_t) \) is given by:

\[
\nu(g_t) = (1 - \lambda)E_t \left[ \frac{1}{1 + \lambda(1 + \bar{g})/(r - \bar{g})} g_t + \lambda E_t \left[ \frac{1}{1 - (1 - \lambda)(1 + \bar{g})/(1 + r)\bar{g}} \right] \right]. \tag{15}
\]

It depends on the parameters \( \lambda \) and on the distribution \( F(.) \). When \( \lambda = 0 \), the firm’s growth rate is constant, and \( \nu(g_t) = g_t \). When \( \lambda = 1 \), the growth rate of the firm is i.i.d and \( \nu(g_t) = E[\bar{g}] \). Thus, the term \( \frac{1}{r - \nu(g_t)} \) is analogous to the standard Gordon growth formula, but the function \( \nu(.) \) adjusts for shifts in the growth rate of fundamentals. Thus, the key insights from the discussion in the main text survive. In particular, even with stochastic growth, the two rents terms can be thought of as the present value of markups over the user costs of physical and intangible capital, respectively.

2.2.2 Derivations of the result

We define the function \( \nu(g_t) \) as:

\[
\nu(g_t) = r - \zeta(g_t)^{-1},
\]

where:

\[
\zeta(g_t) \equiv E_t \left[ \sum_{k \geq 1} (1 + r)^{-k} \frac{A_{t+k}}{A_{t+1}} \right].
\]

To obtain the decomposition of the investment gap in closed form, we use the following lemma.

**Lemma 3.** When fundamentals follow the process (13), the function \( \zeta(g_t) \) is given by:

\[
\zeta(g_t) = E_t \left[ \frac{r - \bar{g}}{r - \bar{g} + \lambda(1 + \bar{g})} \right]^{-1} \frac{1}{r - g_t + \lambda(1 + g_t)}. \]

It is straightforward to check that this solution for \( \zeta(g_t) \) then implies expression (15).

**Proof.** Let:

\[
\zeta(g_t) \equiv E_t \left[ \sum_{k \geq 1} (1 + r)^{-k} \frac{A_{t+k}}{A_{t+1}} \right],
\]

then \( \zeta(g_t) \) satisfies:

\[
\zeta(g_t) = \frac{1}{1 + r} (1 + \lambda E[(1 + \bar{g})\zeta(\bar{g})] + (1 - \lambda)(1 + g_t)\zeta(g_t)),
\]
where we used the law of iterated expectations, and the law of motion for \( A_t \). Solving for \( \zeta(g_t) \):

\[
\zeta(g_t) = \frac{1 + \lambda X}{1 + r - (1 - \lambda)(1 + g_t)}, \quad X \equiv \mathbb{E}[(1 + g)\zeta(g)]
\]

Multiplying by \((1 + g_t)\), taking expectations on both sides, and solving for \( X \), we obtain:

\[
X = \frac{Y}{1 - \lambda Y}, \quad Y \equiv \mathbb{E}\left[\frac{1 + g_t}{1 + r - (1 - \lambda)(1 + g_t)}\right].
\]

After rearranging, we have:

\[
\zeta(g_t) = \frac{1}{1 - \lambda Y} \frac{1}{r - g_t + \lambda(1 + g_t)},
\]

which gives the result, using the solution for \( Y \).

We now derive the closed-form expressions for the decomposition of the investment gap when the fundamentals process is given by (13). We rely on the general solution of the model with uncertainty and a CES aggregator described in Lemma (2). With the fundamentals process (13), \( A_{t+1} \) is in the time-\( t \) information set. Using Lemma (2), since:

\[
K_{n,t+1} = \xi_{n,t+1}A_{t+1},
\]

and since the \( \{K_{n,t+1}\}_{n=1}^N \) are chosen as of time \( t \), this implies that the \( \{\xi_{n,t+1}\}_{n=1}^N \) are also in the information set of time \( t \). Therefore, the user costs \( \{R_{n,t+1}\}_{n=1}^N \) are also in the time-\( t \) information set, so that, using Assumption 7:

\[
\forall n = 1, ..., N, \quad \forall t, \quad \mathbb{E}_t[M_{t,t+1}R_{n,t+1}] = \mathbb{E}_t[M_{t,t+1}] R_{n,t+1} = (1 + r)^{-1}R_{n,t+1}.
\]

Assumption 7 also implies that:

\[
q_{n,t} = 1
\]

\[
R_{n,t+1} = \frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} - (1 - \delta_n).
\]

Using the relationship \( \mathbb{E}_t[M_{t,t+1}R_{n,t+1}] = (1 + r)^{-1}R_{n,t+1} \) and the definition of \( q_{n,t} \), these equations imply that:

\[
R_{n,t+1} = r + \delta_n, \quad \forall n = 1, ..., N, \forall t.
\]

This implies that:

\[
\xi_{n,t+1} = \xi_n, \quad \forall n = 1, ..., N, \forall t.
\]

Thus, \( K_{n,t} = \xi_nA_t \) for all \( n, t \). Therefore, the net growth rates of capital stocks are given by \( g_{n,t} = g_t \), and the ratios \( S_{n,m,t} \equiv \frac{K_{m,t+1}}{K_{n,t+1}} \) are constant.

Using these results, and the fact that \( q_{n,t} = 1 \) and \( \Pi_{n,t} = R_{n,t} = r + \delta_n \) for all \( n, t \), we can write the investment gap decomposition for capital of type \( n \) as:

\[
Q_{n,t} - q_{n,t} = (\mu - 1)(r + \delta_n) \sum_{k \geq 1} \mathbb{E}_t[M_{t,t+k}(1 + g_{t+1,t+k})]
\]

\[\text{A9}\]

The result is somewhat stronger: \( \frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} = 1 + r \), since \( \frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} \) is also in the information set of time \( t \).
\[ + \sum_{m=1}^{N} S_{m,n} \]
\[ + (\mu - 1) \sum_{m=1 \atop m \neq n}^{N} S_{m,n} (r + \delta_m) \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(1 + g_{t+1,t+k})], \]

where we have denoted:
\[ g_{t+1,t+k} \equiv \frac{K_{n,t+k}}{K_{n,t+1}} = \frac{A_{t+k}}{A_{t+1}}. \]

Using the lemma above, as well as the fact that we have assumed that \( M_{t,t+k} = (1 + r)^{-k} \), the present values that appear in the rents term in the investment gap above are given by:
\[ \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(1 + g_{t+1,t+k})] = \sum_{k \geq 1} \mathbb{E}_t \left[ (1 + r)^{-k} \frac{A_{t+k}}{A_{t+1}} \right] = \zeta(g_t) = \frac{1}{r - \nu(g_t)}. \]

Thus the investment gap decomposition is given by:
\[ Q_{n,t} - q_{n,t} = \frac{(\mu - 1)(r + \delta_n)}{r - \nu(g_t)} + \sum_{m=1 \atop m \neq n}^{N} S_{m,n} + (\mu - 1) \sum_{m=1 \atop m \neq n}^{N} \frac{(\mu - 1)(r + \delta_m)}{r - \nu(g_t)} \times S_{m,n}, \]

and the decomposition (14) is a particular case of this expression for \( N = 2 \).

### 2.3 Market power, decreasing returns, and rents

Proofs for key lemmas are reported at the end of this section.

#### 2.3.1 Model

Consider a monopolistic firm that uses \( M \) distinct variable inputs, \( \{M_{j,t}\}_{n=1}^{N} \), as well as capital, to produce and sell output to consumers whose demand function is given by:
\[ Y_t = P_t - \mu S_t - \frac{1}{\mu S - 1} D_t, \]

where \( P_t \) is the price of the good, \( D_t \) indexes aggregate demand, and \( \mu S \geq 1 \) will be the markup over the marginal cost of gross output (or sales) charged by the firm. The prices of the variable inputs are given by \( \{W_{j,t}\}_{n=1}^{N} \). The total input of capital, \( K_t \) (which is made up of tangible and intangible capital), is quasi-fixed; that is, it is chosen dynamically by the firm but cannot be modified immediately. The static variable profit maximization problem of the firm is then:

\[ \Pi_t = \max_{\{M_{j,t}\}_{j=1}^{M}} P_t^{-\nu S - 1} D_t - \sum_{j=1}^{M} W_{j,t} M_{j,t} \]
\[ \text{s.t. } Z_t \left( K_t^{\alpha} \left( \prod_{j=1}^{M} M_{j,t}^{\nu_j} \right)^{1-\alpha} \right) \geq P_t^{-\nu S - 1} D_t \left[ MC_t \right]. \]

\[ \text{A10} \]
In addition to market power on the output market, with markup \( \mu_S \), the firm’s problem features returns to scale \( \zeta \) with respect to an aggregate of capital inputs \( K_t \) and variable inputs. We assume that:

\[ \zeta \leq \mu_S, \]

so that the maximization problem has a unique interior solution. In general, we will be interested in cases where \( \zeta \leq 1 \), so that there are decreasing returns to scale, but constant or increasing returns are possible so long as the markup \( \mu_S \) is sufficiently high. The aggregate of variable inputs, including labor, is given by:

\[ M_t = \prod_{j=1}^{N} M_j^{\nu_j}, \]

where the Cobb-Douglas shares \( \{\nu_j\}_{j=1}^{M} \) are assumed to sum up to 1. The parameter \( \alpha \) therefore represents the Cobb-Douglas elasticity of substitution between capital and aggregated variable inputs. The Lagrange multiplier \( MC_t \) measures the marginal cost of gross output. Additionally, note that \( Y_t = P_t^{\frac{\mu_S}{\mu_S-1}} D_t \) is gross output, and \( S_t = P_t Y_t = P_t^{\frac{1}{\mu_S-1}} D_t \) is total revenue. Finally, \( \Pi_t \) represents operating surplus (revenue minus variable costs):

\[ \Pi_t = S_t - \sum_{j=1}^{M} W_{j,t} M_{j,t}. \]

**Lemma 4.** After minimizing variable cost and optimally choosing the price, operating surplus is given by:

\[ \Pi_t = A_t^{1-\frac{1}{\alpha}} K_t^{\frac{1}{\alpha}} \]

where:

\[ \mu \equiv 1 + \frac{\chi - 1}{\alpha} \geq 1 \]

\[ \chi \equiv \frac{\mu_S}{\zeta} \geq 1 \]

and:

\[ A_t \equiv \left( \frac{\chi}{1 - \alpha} \right)^{-\frac{\chi}{\alpha - 1}} \left( \frac{\chi}{1 - \alpha} - 1 \right)^{-\frac{\chi(1-\alpha)}{\alpha - 1}} D_t^\frac{\chi - \zeta}{\alpha - 1} W_t^{-\frac{1-\alpha}{1-\alpha}} Z_t^{\chi(1-\alpha)} \]

\[ W_t \equiv \prod_{j=1}^{M} \left( \frac{W_{j,t}}{\nu_j} \right)^{\nu_j} \]

There are two main points to note about Lemma 4. First, Equation (17) indicates that the functional form for operating surplus as a function of capital \( K_t \) and exogenous conditions \( A_t \) is the same as the one used in the balanced growth model of Section 2.3. Thus, the firm problem (16) provides a microfoundation for the functional form assumption for operating surplus as a function of capital in that model. The exogenous process \( A_t \) can then be interpreted as reflecting simultaneously the cost of intermediate inputs \( W_t \), demand \( D_t \), and total factor productivity \( Z_t \).

Second, Equation (18) gives a structural interpretation of the reduced-form rents parameter \( \mu \) used in our model. It indicates that the reduced-form rents parameter \( \mu \) increases with markups \( \mu_S \), and decreases with the degree of returns to scale \( \zeta \) and the Cobb-Douglas elasticity of capital with respect to variable
inputs, \( \alpha \). Fixing \( \alpha \), rents will therefore be elevated either when markups are substantially above 1, or when returns to scale are substantially below 1, or both.

### 2.3.2 Partial identification of pure and quasi-rents

Perhaps most importantly, Equation (18) indicates that the reduced-form rents parameter \( \mu \) does not vary independently with markups \( \mu_S \) and returns to scale \( \zeta \). Instead, it is only a function of their ratio \( \chi = \frac{\mu_S}{\zeta} \). As a result, separating pure rents \( \mu_S \) from quasi-rents \( \zeta \) is challenging, and generally requires a direct estimate of the production function. The following Lemma formalizes this point.

**Lemma 5.** Given data on the ratios of variable input costs, labor costs, capital costs, value added and operating surplus to either sales, value added, or operating surplus, the markup \( \mu_S \) and the degree of returns to scale \( \zeta \) cannot be separately identified.

Without loss of generality, we assume that the first variable input is labor. In that case, value added in the model is given by:

\[
VA_t = S_t - (W_t M_t - W_{1,t} M_{1,t}) = \Pi_t + W_{1,t} M_{1,t},
\]

Table 5 reports the ratios of variable input costs, labor costs, capital costs, value added and operating surplus to either sales, value added, or operating surplus in the model above. It also reports the expressions for rents as a fraction of sales, value added or operating surplus, where rents are defined as:

\[
Re_t = \Pi_t - (R_{1,t} K_{1,t} + R_{2,t} K_{2,t}).
\]

In order to pin down competitive payments to capital, we assume that the firm solves the same dynamic problem as in the balanced growth model of Section 2.3. In this case, since competitive payments to capital, \( R_{1,t} K_{1,t} + R_{2,t} K_{2,t} \), satisfy:

\[
R_{1,t} K_{1,t} + R_{2,t} K_{2,t} = \frac{1}{\mu} \Pi_t = \frac{\alpha}{\chi - (1 - \alpha)} \Pi_t,
\]

Table 5 uses this expression to solve for the different ratios in Lemma 5 as a function of the structural parameters \( \alpha \), \( \zeta \) and \( \mu_S \).

The main point of Table 5 is that none of the 18 ratios depend independently on \( \mu_S \) and \( \zeta \); instead, they only depend on their ratio, the reduced-form parameter \( \chi = \frac{\mu_S}{\zeta} \). Thus, manipulation of these ratios cannot be used to separately identify \( \mu_S \) and \( \zeta \).

Lemma 5 is useful because a number of the approaches that have been used in the literature to estimate markups rely partially or entirely on these ratios. The implication of Lemma 5 is then that these approaches do not in fact identify markups, but rather, some function of the reduced-form parameter \( \chi \), which depends on both markups and decreasing returns. (Put differently, these approaches only identify markups under the assumption of constant returns to scale, \( \zeta = 1 \)). We next give three examples of such approaches.

**Example 1: the surplus ratio approach** The surplus ratio approach is used by Baqaee and Farhi (2020). It consists of using the ratio of operating surplus to sales, \( x_{\Pi} = \Pi_t / S_t \), net of the ratio of capital
costs to sales, \( x_K = (R_tK_{1,t} + R_tK_{2,t})/S_t \), to estimate markups. The results of Table 5 imply that, in our balanced growth model,

\[
x_{\Pi} - x_K = \left(1 - \frac{1 - \alpha}{\chi}\right) - \left(\alpha\frac{\chi}{\lambda}\right) = \frac{\chi - 1}{\chi}.
\]

Thus, this approach recovers an estimate of \( \chi \), but not of \( \mu_S \) and \( \zeta \) separately.

**Example 2: the user cost approach** The user cost approach is used by Gutiérrez and Philippon (2017). It consists of computing an implied markup \( \hat{\mu} \) by inverting relationship:

\[
\frac{\Pi_t}{K_t} = R_t + \left(1 - \frac{1 - \hat{\mu}}{\hat{\mu}}\right) \frac{S_t}{K_t},
\]

where \( K_t \) denotes total capital input. In our model, interpreting \( R_t \) as \( R_t = R_{1,t}K_{1,t}/K_t + R_{2,t}K_{2,t}/K_t \), we can rewrite this relationship as:

\[
\frac{1}{\nu_K} = 1 + \left(1 - \frac{1}{\hat{\mu}}\right) \frac{1}{s_K},
\]

where:

\[
\nu_K = \frac{R_{1,t}K_{1,t} + R_{2,t}K_{2,t}}{\Pi_t} \quad \text{and} \quad s_K = \frac{R_{1,t}K_{1,t} + R_{2,t}K_{2,t}}{S_t}
\]

are the ratios of competitive payments to capital to either operating surplus, or sales. However, manipulation of the results of Table 5 shows that:

\[
\frac{1}{\nu_K} = 1 + \left(1 - \frac{1}{\chi}\right) \frac{1}{s_K}.
\]

In other words, the “implied markup” \( \hat{\mu} \) derived in the user cost approach recovers, from the standpoint of our model, the reduced form parameter \( \chi \).

**Example 3: the cost share approach** The cost share approach is used by De Loecker et al. (2020). It consists of estimating the markup using the ratio \( x_M = (W_tM_t)/S_t \) of variable costs to sales. Using the results of Table 5, this ratio is given by:

\[
x_M = \frac{1 - \alpha}{\chi}.
\]

Thus, alone, this ratio does not separately identify \( \alpha \), \( \mu_S \), and \( \chi \). As a result, De Loecker et al. (2020) estimate, separately, the elasticity of gross output with respect to variable costs. In our model, this elasticity is given by:

\[
\eta = (1 - \alpha)\zeta.
\]

One can then recover the markup \( \mu \) by forming:

\[
\frac{\eta}{x_M} = \frac{\mu_S(1 - \alpha)\zeta}{(1 - \alpha)\zeta} = \mu_S.
\]

---

\(^4\) Equivalently, the expression of the user cost relationship in the context of a model that allows for decreasing returns is:

\[
\frac{\Pi_t}{K_t} = R_t + \left(1 - \frac{1}{\chi}\right) \frac{S_t}{K_t} = R_t + \left(1 - \frac{\zeta}{\mu_S}\right) \frac{S_t}{K_t}.
\]
This method therefore cannot identify the markup using only the variable cost share; a separate estimate of \((1 - \alpha)\zeta\) (that is, a direct estimate of the production function) must be constructed first.

We conclude by noting another consequence of the results of Table 5.

**Lemma 6** (The share of rents). The share of pure rents in sales, value added, and operating surplus are given by:

\[
\begin{align*}
    x_{Re} &= \frac{Re_t}{S_t} = \frac{\chi - 1}{\chi}, \\
    s_{Re} &= \frac{Re_t}{VA_t} = \frac{\chi - 1}{\chi - (1 - \alpha)(1 - \nu_1)} = (1 - s_L)\frac{\mu - 1}{\mu}, \\
    \nu_{Re} &= \frac{Re_t}{\Pi_t} = \frac{\chi - 1}{\chi - (1 - \alpha)} = \frac{\mu - 1}{\mu},
\end{align*}
\]

where \(s_L = (W_{1,t}M_{1,t})/VA_t\) is the labor share of value added.

Thus, the reduced-form parameter \(\chi\) should be interpreted as controlling the size of rents relative to sales, whereas the reduced-form parameter \(\mu\) controls the size of rents relative to operating surplus. Again, rents shares do not depend on the separate values of \(\mu_S\) and \(\zeta\), but only on their ratio. This implies that the size of rents relative to sales, operating surplus, or value added does not depend on markups \(\mu_S\) or decreasing returns \(\zeta\) independently, but only on their ratio.\(^5\)

### 2.3.3 Proofs for results on market power and decreasing returns

**Proof of Lemma 4.** To make notation lighter, we rewrite the firm problem (1) as:

\[
\Pi_t = \max_{(M_{j,t})_{j=1}^M, \nu_1} P_t^{-\frac{1}{\mu_S-1}} D_t - \sum_{j=1}^M W_{j,t} M_{j,t} \nu_1 \\
\text{s.t. } X_t \left( \prod_{j=1}^N M_{j,t}^{\nu_j} \right)^{\gamma} \geq P_t^{-\frac{1}{\mu_S-1}} D_t \left[ MC_t \right]
\]

where:

\[
\gamma \equiv (1 - \alpha)\zeta, \quad X_t \equiv Z_t K_t^{\alpha \zeta}.
\]

We solve this problem in two steps. The cost minimization problem is:

\[
VC_t = \min_{(M_{j,t})_{j=1}^M} \sum_{j=1}^M W_{j,t} M_{j,t}, \\
\text{s.t. } X_t \left( \prod_{j=1}^N M_{j,t}^{\nu_j} \right)^{\gamma} \geq Y_t \left[ MC_t \right].
\]

Define:

\[
W_t \equiv \prod_{j=1}^M \left( \frac{W_{j,t}}{\nu_j} \right)^{\nu_j}, \quad M_t \equiv \prod_{j=1}^M M_{j,t}^{\nu_j}.
\]

\(^5\) Additionally, this lemma indicates is that knowledge of the reduced-form parameter \(\mu\), along with a measure of the labor share, is sufficient to compute the implied ratio of rents to value added, which is a commonly used measure of the size of rents (see, e.g., Karabarbounis and Neiman 2019). We use this result in order to map our estimates of \(\mu\) to a value for the share of rents as a fraction of value added.
Then solution to the cost minimization problem is:
\[
\frac{W_{j,t}M_{j,t}}{W_tM_t} = \nu_j,
\]
and:
\[
VC_t = W_tM_t = W_t \left( \frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}} = \gamma MC_t Y_t,
\]
\[
\dot{MC}_t = \frac{W_t}{\gamma Y_t} \left( \frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}} = \frac{\partial VC_t}{\partial Y_t} = \frac{VC_t}{\gamma Y_t},
\]
\[
M_t = \left( \frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}}.
\]
Substituting the expression for total variable cost, the price and quantity choice problem is:
\[
\Pi_t = \max_{P_t, Y_t} P_t^{-\mu_S^{-1}} D_t - W_t \left( \frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}},
\]
\[
\text{s.t. } Y_t \geq P_t^{-\mu_S^{-1}} D_t \quad [MC_t].
\]
The first-order conditions are
\[
P_t = \mu_S MC_t \text{ and } MC_t = \frac{W_t}{\gamma Y_t} \left( \frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}}.
\]
so that \(\dot{MC}_t = MC_t\), and:
\[
Y_t = \left( \frac{\mu_S}{\gamma} \right)^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} D_t^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} W_t^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} X_t^{\gamma \mu_S^{-1} (1-\gamma) \mu_S}.
\]
We can use the solution to the cost minimization problem to obtain:
\[
VC_t = \left( \frac{\mu_S}{\gamma} \right)^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} D_t^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} W_t^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} X_t^{\gamma \mu_S^{-1} (1-\gamma) \mu_S}.
\]
Moreover, profits are given by:
\[
\Pi_t = \left( \frac{\mu_S}{\gamma} - 1 \right) VC_t,
\]
so that:
\[
\Pi_t = \left( \frac{\mu_S}{\gamma} - 1 \right) \left( \frac{\mu_S}{\gamma} \right)^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} D_t^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} W_t^{-\gamma \mu_S^{-1} (1-\gamma) \mu_S} X_t^{\gamma \mu_S^{-1} (1-\gamma) \mu_S}.
\]
Using the definitions of \(\gamma\) and \(X_t\), we can write this as:
\[
\Pi_t = A_t^{\frac{1}{\alpha}} K_t^{\frac{1}{\alpha}},
\]
\[
\mu = 1 + \frac{\mu_S/\zeta - 1}{\alpha},
\]
\[
A_t = \left( \left( \frac{\mu_S}{1-\alpha \zeta} - 1 \right) \left( \frac{\mu_S}{(1-\alpha) \zeta} \right)^{-\frac{\mu_S}{\mu_S-1-\alpha \zeta}} D_t^{-\frac{\mu_S-1}{\mu_S-1-\alpha \zeta}} W_t^{-\frac{\mu_S-1}{\mu_S-1-\alpha \zeta}} Z_t^{\frac{1}{\mu_S-1-\alpha \zeta}} \right)^{\frac{\mu_S-1}{\mu_S-1-\alpha \zeta}}.
\]
Substituting \(\chi = \frac{\mu_S}{\zeta}\) in these expressions gives the results of Lemma 4.
2.4 Heterogeneous rents parameters

2.4.1 Model and assumptions

The model of the firm is a generalization of the baseline model. The firm solves:

\[
V^c_c(K_t) = \max_{K_{t+1}} \tilde{\Pi}_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t [M_{t,t+1} V^c_{t+1}(K_{t+1})] \tag{20}
\]

where \(V^c(\cdot)\) is the value of the firm including distributions.

We make the same assumptions regarding adjustment costs as in the baseline model. Different from the baseline model, we replace the two assumptions 1-2 by the following assumption.

**Assumption 4.** There exist real numbers \(\{\mu_n\}_{n=1}^N\), \(\mu_n \geq 1\) \(\forall n = 1, \ldots, N\), such that the function \(\tilde{\Pi}_t\) satisfies:

\[
\forall K_t = \{K_{n,t}\}_{n=1}^N, \quad \tilde{\Pi}_t(K_t) = \sum_{n=1}^N \mu_n \tilde{\Pi}_{n,t}(K_t) K_{n,t}, \tag{21}
\]

where \(\Pi_{n,t}\) is the marginal revenue product of capital of type \(n\), i.e. \(\Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_{n,t}}\).

Under our baseline assumptions, the revenue function satisfies:

\[
\forall K_t = \{K_{n,t}\}_{n=1}^N, \quad \tilde{\Pi}_t = \mu \sum_{n=1}^N \tilde{\Pi}_{n,t} K_{n,t}.
\]

Assumption (4) thus broadens our baseline assumptions, but allowing heterogeneity in the wedge between marginal and average revenue for each type of capital. A fairly general functional form that satisfies Assumption 4 is:

\[
\tilde{\Pi}_t(K_t) = G_t \left( \left\{ Y_t^{(n)}(K_{n,t}) \right\}_{n=1}^N \right), \quad \text{where}:
\]

\[
G_t \quad \text{is homogeneous of degree} \quad \frac{1}{\mu}, \quad \mu \geq 1
\]

\[
\forall n = 1, \ldots, N, \quad Y_t^{(n)} \quad \text{is homogeneous of degree} \quad \frac{1}{\tilde{\mu}_n}, \quad \tilde{\mu}_n \geq 1
\]

In this case, it is easy to check that the rents parameters \(\{\mu_n\}_{n=1}^N\) are given by:

\[
\forall n = 1, \ldots, N, \quad \mu_n = \mu \times \tilde{\mu}_n.
\]

Our baseline model is a particular case of the functional form (22), with \(G(K_t) \equiv \Pi_t(F_t(K_t))\), and \(Y_t^{(n)}(K_{n,t}) = K_{n,t}\). In this case, \(\tilde{\mu}_n = 1\) for all \(n = 1, \ldots, N\), so that \(\mu_n = \mu\), that is, rents are the same across capital types. One concrete example of a function satisfying equation (22) is:

\[
\tilde{\Pi}_t(K_t) = A^{1 - \frac{1}{\rho}} \left( \sum_{n=1}^N \eta_n \left( A^{1 - \frac{1}{\rho}} a_{n,t}^{\frac{1}{\rho}} K_{n,t}^{\frac{1}{\rho}} \right)^{\frac{1}{\rho}} \right)^{-\frac{\rho}{\rho - 1}}. \tag{24}
\]

Heuristically, this aggregator can be described as follows. Production of final goods in the firm takes place in two stages. In the first stage, each type of capital is used (potentially in conjunction with flexible labor, but separately from other capital types) to produce intermediate varieties. In the second stage, intermedi-
ate varieties are aggregated into a final good. In the first stage (intermediate input production), the firm has decreasing returns with respect to each type of capital: intermediate output is $Y_t^{(n)} = (A_{n,t})^{1-\frac{1}{\mu_n}} K_{n,t}^{\frac{1}{\mu_n}}$, where $\mu_n$ indexing the strength of the decreasing returns for each capital type $n$. In the second stage (aggregation), the firm has monopoly power in the consumer goods market. Revenue is then given by $\sum_{n=1}^{N} \eta_n Y_t^{(n)} \bar{p}_n$, where $\mu$ governs the firm’s market power in final goods markets, and $\rho$ the substitutability between intermediates.

2.4.2 The investment gap with heterogeneous rents

Result 8. Firm value can be written as:

$$V_t^e = \sum_{n=1}^{N} q_{n,t} K_{n,t+1} + \sum_{n=1}^{N} (\mu_n - 1) \sum_{k \geq 1} E_t \left[ M_{t,t+k} \Pi_{n,t+k} K_{n,t+k} \right],$$

and the investment gap for each capital type can be written as:

$$Q_{n,t} - q_{n,t} = (\mu_n - 1) \sum_{k \geq 1} E_t \left[ M_{t,t+k} \Pi_{n,t+k} (1 + g_{m,t+1,t+k}) \right]$$

$$+ \sum_{m=1}^{N} q_{m,t} S_{m,n,t+1}$$

$$+ \sum_{m \neq n}^{N} (\mu_m - 1) \sum_{k \geq 1} E_t \left[ M_{t,t+k} \Pi_{m,t+k} (1 + g_{m,t+1,t+k}) \right] \times S_{m,n,t+1},$$

where $1 + g_{n,t+1,t+k} \equiv \frac{K_{n,t+k}}{K_{n,t+1}}$, and $S_{m,n,t+1} \equiv \frac{K_{n,t+1}}{K_{m,t+1}}$.

Proof. Following the same steps as in proof for the baseline model, we have:

$$V_t^e - \sum_{n=1}^{N} q_{n,t} K_{n,t+1} = \sum_{n=1}^{N} \sum_{k \geq 1} E_t \left[ M_{t,t+k} \left( \Pi_t - \Pi_{n,t+k} K_{n,t+k} \right) \right]$$

$$= \sum_{n=1}^{N} \sum_{k \geq 1} E_t \left[ M_{t,t+k} (\mu_n - 1) \Pi_{n,t+k} K_{n,t+k} \right]$$

$$= \sum_{n=1}^{N} (\mu_n - 1) \sum_{k \geq 1} E_t \left[ M_{t,t+k} \Pi_{n,t+k} K_{n,t+k} \right].$$

The decomposition of the investment gap follows from this expression. □

The interpretation of the investment gap decomposition is the same as for our baseline model. The terms $(\mu_n - 1) \sum_{k \geq 1} E_t \left[ M_{t,t+k} \Pi_{n,t+k} K_{n,t+k} \right]$ represent the gap between the average and marginal revenue products of capital of type $n$, and therefore capture rents attributable to capital of type $n$. Rents remain additively separable across capital types, as in the baseline decomposition. The main difference with our
baseline decomposition is that the size of rents (per unit of marginal revenue product) can now differ between capital types.

2.4.3 Analytical expression in balanced growth

We next given an analytical solution for a balanced growth version of the model above. Our definition of balanced growth is the same as in our baseline analysis, except that we also assume that the function $\tilde{\Pi}_t$ is given by Equation (24). In this case, we have the following result, of which Result (5) is a particular case.

Result 9. Let $n = 1$ denote physical capital. Along the balanced growth path, the physical investment gap is given by:

$$Q_1 - q_1 = \sum_{m \geq 2} q_m S_m + \frac{(\mu_1 - 1) R_1}{r - g} + \sum_{m \geq 2} \frac{(\mu_m - 1) R_m}{r - g} \times S_m,$$

(29)

where:

$$R_n = (r - g)\Phi'_n(1 + g) + \Phi_n(1 + g), \quad n = 1, ..., N.$$

Proof. Along the balanced growth path, the necessary first-order conditions, for capital of type $n$, are given by:

$$\Phi_{n,t} = q_{n,t},$$

$$q_{n,t} = \frac{1}{1 + r} \left( \tilde{\Pi}_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1} \left( \frac{K_{n,t+2}}{K_{n,t+1}} \right) \right).$$

(30)

We can write these conditions as:

$$(1 + r)\Phi'_{n}(1 + g_{n,t}) = \tilde{\Pi}_{n,t+1} - \Phi_{n}(1 + g_{n,t+1}) + \Phi'_{n}(1 + g_{n,t+1})(1 + g_{n,t+1}),$$

where $g_{n,t} = \frac{K_{n,t+1}}{K_{n,t+1}} - 1$.

We next guess and verify that $g_{n,t} = g$ for $n = 1, ..., N$ is a solution. Substituting into the condition above, and re-arranging, we obtain:

$$R_n \equiv (r - g)\Phi'_{n}(1 + g) + \Phi_{n}(1 + g) = \tilde{\Pi}_{n,t+1}.$$

Moreover, using the functional form (24):

$$\tilde{\Pi}_{n,t+1} = \frac{\eta_n}{\mu \mu_n} A_{t+1}^{-\frac{1}{\rho}} K_{t+1}^{\frac{1}{\rho}} A_{n,t}^{-\rho} \left( \frac{1 - \rho}{\rho} \right)^{\rho} K_{n,t}^{\frac{\rho}{\rho} - 1}, \quad n = 1, ..., N.$$

Define the following variables detrended variables:

$$k_{n,t} = \frac{K_{n,t}}{A_{n,t}}, \quad \alpha_n = \frac{A_{n,t}}{A_t}, \quad n = 1, ..., N$$

where we have used the fact that $A_{n,t}$ and $A_t$ grow at the same rate. With these definitions, we can write the system of first-order conditions as:

$$R_n = \frac{\eta_n}{\mu \mu_n} H \left( \{k_{n,t}\}_{n=1}^{N} \right)^{-\left( \frac{1}{\rho} - 1 \right)} I \left( \{k_{n,t}\}_{n=1}^{N} \right)^{1-\rho} k_{n,t}^{-\frac{1}{\rho} + \frac{1}{\rho} - 1} \left( \frac{1 - \rho}{\rho} \right)^{\rho}$$

(31)
where the functions $H(.)$ and $I(.)$ are given by:

$$H\left(\{k_{n,t}\}^N_{n=1}\right) \equiv \left( \sum_{n=1}^{N} \eta_n \left( \alpha_n k_{n,t}^{\frac{1}{\rho}} \right)^{\frac{1}{\rho}} \right)^{\frac{1}{\rho}} = \frac{K_t}{A_t},$$

$$I\left(\{k_{n,t}\}^N_{n=1}\right) \equiv \left( \sum_{n=1}^{N} \eta_n \left( k_{n,t}^{\frac{1}{\rho}} - 1 \right)^{\frac{1}{\rho}} \right)^{\frac{1}{\rho}} = \frac{K_t}{K_{n,t}}.$$

The system (31) consists of $N$ equations in $N$ unknowns, $\{k_{n,t}\}^N_{n=1}$. We assume that the solution to this system exists and is unique and given by $\{k_{n}\}^N_{n=1}$. Since none of the parameters in this system of equation is time-varying, we must have $k_{n,t} = k_n$ for all $n$. Given that each of the $A_{n,t}$ grows at rate $g$, this confirms the guess $g_{n,t} = g$. This also implies that the ratios $S_{n,1,t} = K_{n,t}/K_{1,t}$ are constant along the balanced growth path. Constant shares, constant growth, and the constant discount rate, along with the relationship $\Pi_{n,t+1} = R_n$, then imply the decomposition (29).

$\Box$

### 2.5 Relationship to the production-based asset pricing literature

#### 2.5.1 Definitions

Recall that our model of the firm is:

$$V_t^c(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \Phi_t(K_t, K_{t+1}) + E_t\left[M_{t,t+1}V_{t+1}^c(K_{t+1})\right]$$

$$\text{s.t.} \quad K_t = F_t(K_t),$$

where $F_t$ is homogeneous of degree 1, and $\Pi_t$ is homogeneous of degree $\mu$, and:

$$\Phi_t(K_t, K_{t+1}) = \sum_{n=1}^{N} \Phi_{n,t} \left( \frac{K_{n,t+1}^{\rho}}{K_{n,t}^{\rho}} \right) K_{n,t}.$$

Assuming that the firm is equity-financed, realized stock returns are defined as:

$$R_{E,t+1} \equiv \frac{V_{t+1}^e}{V_t^e} = \frac{\Pi_{t+1} - \sum_{n=1}^{N} \Phi_{n,t+1}K_{n,t+1} + V_{t+1}^e}{V_t^e}, \quad V_t^e \equiv E_t\left[M_{t,t+1}V_{t+1}^c(K_{t+1})\right].$$

Stock returns are defined for the firm as a whole. We define returns on investment, for each type of capital $n = 1, ..., N$, as:

$$R_{n,t+1} \equiv \frac{\Pi_{n,t+1} - \Phi_{n,t+1} + \frac{K_{n,t+2}}{K_{n,t+1}}\Phi'_{n,t+1}}{\Phi'_{n,t}}.$$

This definition is the same as in Cochrane (1991), Equation (12). The denominator is the cost of adding an incremental unit to $K_{n,t+1}$, which, given our assumption about the structure of investment costs, is simply $\Phi'_{n,t}$.

The numerator is the marginal return. This marginal return is the sum of two terms: incremental flow profits (the term $\Pi_{n,t+1}$); and the net change in investment costs associated with the incremental unit of $K_{n,t+1}$, assuming that the stock at time $t+2$ and onward is unchanged (the term $\frac{K_{n,t+2}}{K_{n,t+1}}\Phi'_{n,t+1} - \Phi_{n,t+1}$).  

---

6 We have not been able to establish existence and unicity formally, except in the case $N = 2$.

7 The mapping from the model of Cochrane (1991), equations (7)-(9), to the model in our paper is, in the one-capital case, $f(K_t, L_t, s_t) = \Pi_t(K_t)$ and $g(K_t, I_t) = K_0\Phi^{-1}_t (I_t/K_t)$. 

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2.5.2 Asset pricing equations

Stock returns satisfy the fundamental asset pricing relationship:

\[ 1 = \mathbb{E}_t [M_{t,t+1}R_{E,t+1}] . \quad (33) \]

Recall that the first-order conditions and the envelope conditions for the firm problem are:

\[ \Phi'_{n,t} = q_{n,t}, \quad \frac{\partial V^c_{t+1}}{\partial K_{n,t+1}} = \Pi_{n,t+1} - \Phi_{n,t+1} + \frac{\Phi'_{n,t+1}}{\Phi'_{n,t+1}} K_{n,t+2}, \quad n = 1, ..., N, \quad (34) \]

where \( q_{n,t} \equiv \mathbb{E}_t \left[ \frac{\partial V^c_{t+1}}{\partial K_{n,t+1}} \right] = \frac{\partial V^e_t}{\partial K_{n,t+1}}. \) Using the first-order condition \( q_{n,t} = \Phi'_{n,t}, \) returns to investment can be rewritten as:

\[ R_{n,I,t+1} = \frac{\Pi_{n,t+1} - \Phi_{n,t+1} + \frac{K_{n,t+2}}{q_{n,t}}}{q_{n,t}}. \]

Using the envelope condition, we then have, for each type of capital:

\[ 1 = \mathbb{E}_t [M_{t,t+1}R_{n,I,t+1}] . \quad (35) \]

2.5.3 Relationship between stock returns and returns to investment

**Result 10.** Stock returns \( R_{E,t+1} \) and the returns to investment \( \{R_{n,I,t+1}\}_{n=1}^N \) satisfy:

\[ R_{E,t+1} = \sum_{n=1}^N \frac{q_{n,t}}{Q_{n,t}} R_{n,I,t+1} + (\mu - 1) \sum_{n=1}^N \frac{1}{Q_{n,t}} \left( \sum_{k \geq 1} \mathbb{E}_{t+1} [M_{t+1,t+k} \{\Pi_{n,t+k}(1 + g_{n,t+1,t+k})\}] \right), \quad (36) \]

where \( 1 + g_{n,t+1,t+k} \equiv \frac{K_{n,t+k}}{K_{n,t+1}} \) and \( Q_{n,t} \equiv \frac{V^e_t}{K_{n,t+1}}. \) Moreover, for any capital type \( n, \)

\[ R_{E,t+1} - R_{n,I,t+1} = \frac{1}{Q_{n,t}} \left\{ -(Q_{n,t} - q_{n,t})R_{n,I,t+1} + (\mu - 1) \sum_{k \geq 1} \mathbb{E}_{t+1} [M_{t+1,t+k} \Pi_{n,t+k}(1 + g_{n,t+1,t+k})] \right. \]

\[ + \left. \sum_{m=1}^N S_{m,n,t+1} q_{m,t} R_{m,I,t+1} \right\} \]

\[ + \left. (\mu - 1) \sum_{m=1}^N S_{m,n,t+1} \sum_{k \geq 1} \mathbb{E}_{t+1} [M_{t+1,t+k} \Pi_{m,t+k}(1 + g_{m,t+1,t+k})] \right\}. \]

When \( N = 1 \) and \( \mu = 1, \) so that \( Q_t = q_t, \) stock returns are equalized with investment returns:

\[ \forall t, \quad R_{E,t+1} = R_{I,t+1}. \]

The proof is reported below. The first part of this result gives a relationship between stock returns, and an appropriately weighted average of investment returns. The weights are the ratio of marginal \( q_{n,t} \)
to average $Q_{n,t}$ for each type of capital. Note that in the definition of the return to investment, the denominator is the marginal cost of investment, which is equal to marginal $q$. Analogously, stock returns can be written, in terms of average $Q_{n,t}$ for each type of capital, as:

$$R_{E,t+1} = \frac{\Pi_{t+1}/K_{n,t+1} - \sum_{m=1}^{N} \Phi_{m,t+1} S_{m,n,t+1} + (K_{n,t+2}/K_{n,t+1})Q_{n,t+1}}{Q_{n,t}},$$

which helps understand why the ratio $q_{n,t}/Q_{n,t}$ is used to “weight” investment returns. Equation (36) then says that the gap between stock returns and this weighted average of investment returns is exactly equal to the present value of future rents. Moreover, as in our analysis of investment gap, the present value of rents can be separated across capital types.\(^8\)

Note that, even when there are no rents ($\mu = 1$), stock returns are not necessarily equal to the returns to investing in each capital type, since, as discussed in the main text, $q_{n,t}$ need not equal $Q_{n,t}$ even when there are no rents. However, stock returns are equalized to a weighted average of investment returns state by state, analogous to the results of Cochrane (1991) and Cochrane (1996).

The second part of the result gives a decomposition of the gap between equity returns, and returns to investment for each capital type $R_{I,n,t+1}$. The intuition for the components of this gap is analogous to the intuition for the components of the investment gap.

When there is only one type of capital and no rents ($N = 1, \mu = 1$), the difference between stock returns and investment returns is given by $-(1 - Q_{n,t}/q_{n,t}) = 0$, since in that case, average $Q$ and marginal $q$ are equal.

When there is only one type of capital, but the firm earns rents ($N = 1, \mu > 1$), the difference between stock returns and investment returns is given by:

$$R_{E,t+1} - \frac{q_{1,t}}{Q_{1,t}} R_{1,I,t+1} = Q_{1,t}^{-1} N_{t+1}.$$

In other words, the gap between stock and investment returns is higher, the larger the rents $N_{t+1}$ that the firm earns.

When there are several types of capital, but the firm earns no rents ($N > 1, \mu = 1$), the difference between stock and investment returns can be written as:

$$R_{E,t+1} - R_{n,I,t+1} = Q_{n,t}^{-1} \left( -(Q_{n,t} - q_{n,t})R_{n,I,t+1} + \sum_{m=1}^{N} S_{m,n,t+1} q_{m,t} R_{m,I,t+1} \right)$$

$$= \sum_{m=1}^{N} \left( \frac{q_{m,t}}{Q_{m,t}} (R_{m,I,t+1} - R_{n,I,t+1}) \right),$$

where, to go from the first to the second line, we used the relationship between $Q_{n,t}$ and $q_{n,t}$ derived in the main paper for the case $N > 1$ and $\mu =$. In that case, as discussed above, stock returns are equal to an appropriately weighted sum of investment returns; so an intuition for the difference between stock returns and individual investment returns is that the gap will be larger, for capital types $n$ whose investment

\(^8\)Note that using the two asset pricing conditions (33) and (35), Equation (36) implies our main firm value decomposition, Result 1 in the main text.
returns are low relative to other investment types (and particularly relative to those with high marginal \( q \) relative to average \( Q \)).

Finally, in the general case \( N > 1, \mu > 1 \), the gap between stock returns and investment returns reflects a combination of three factors: the rents generated by capital of type \( n \) (analogous to the “traditional rents” in the investment gap decomposition); investment returns of capital of type \( n \) relative to other capital types (analogous to the “omitted capital effect” in the investment gap decomposition); and the interaction of the two, i.e. the effect of the rents generated by omitted capital types.

**Proof of Result 10.** First, using the definition of equity returns, we have:

\[
R_{E,t+1} V_t^e = \Pi_{t+1} - \sum_{n=1}^{N} \Phi_{n,t+1} \left( \frac{K_{n,t+2}}{K_{n,t+1}} \right) K_{n,t+1} + V_{t+1}^e
\]

\[
= \Pi_{t+1} - \sum_{n=1}^{N} \Phi_{n,t+1} \left( \frac{K_{n,t+2}}{K_{n,t+1}} \right) K_{n,t+1} + \mathbb{E}_{t+1} \left[ M_{t+1,t+2} R_{E,t+2} V_{t+1}^e \right]
\]

Therefore, assuming that the transversality condition \( \lim_{k \to +\infty} \mathbb{E}_t \left[ M_{t,t+k} R_{E,t+k} V_{t+k}^e \right] = 0 \) holds, we have that:

\[
R_{E,t+1} V_t^e = \sum_{k \geq 1} \mathbb{E}_{t+1} \left[ M_{t+1,t+k} \left\{ \Pi_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+1} K_{n,t+k} \right\} \right]
\]

Using homogeneity of degree \( \mu \) of the revenue function, we can rewrite this expression as:

\[
R_{E,t+1} V_t^e = \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_{t+1} \left[ M_{t+1,t+k} \left\{ \mu \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+1} K_{n,t+1} \right\} \right]
\]

Similarly, iterating forward the definition of investment returns, we obtain that, for each \( n = 1, \ldots, N \):

\[
R_{n,I,t+1} \cdot (q_{n,t} K_{n,t+1}) = \sum_{k \geq 1} \mathbb{E}_{t+1} \left[ M_{t+1,t+k} \left\{ \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+1} K_{n,t+1} \right\} \right],
\]

so that:

\[
\sum_{n=1}^{N} R_{n,I,t+1} \cdot (q_{n,t} K_{n,t+1}) = \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_{t+1} \left[ M_{t+1,t+k} \left\{ \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+1} K_{n,t+1} \right\} \right].
\]

Combining these two equations, we obtain:

\[
R_{E,t+1} V_t^e = \sum_{n=1}^{N} R_{n,I,t+1} \cdot (q_{n,t} K_{n,t+1}) + (\mu - 1) \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_{t+1} \left[ M_{t+1,t+k} \Pi_{n,t+k} K_{n,t+k} \right].
\]

This establishes Equation (36), and the rest of the result follows.

### 2.5.4 Implications in balanced growth

We next briefly discuss the implications of the balanced growth model for the behavior of stock returns and investment returns. In the balanced growth case, there is no uncertainty, so the two asset pricing relationships collapse to:

\[
R_{E,t+1} = 1 + r, \quad \forall t,
\]

\[
R_{n,I,t+1} = 1 + r, \quad n = 1, 2, \quad \forall t.
\]
So, along the balanced growth path, stock returns and returns to investment in either physical or intangible capital are equalized. This can also be seen directly from the definition of the two returns. For stock returns:

\[
R_{E,t+1} = \frac{\Pi_{t+1} - \sum_{n=1}^{2} \Phi_n K_{n,t+1} + V_{t+1}^c}{V_t^c}
\]

\[
= \frac{ROA_1 - (\Phi_1 + \Phi_2) + (1 + g)Q_1}{Q_1}
\]

\[
= \frac{ROA_1 - (\nu_1 + S\nu_2) - (\gamma_1 + \gamma_2 S)g^2 + (1 + g)Q_1}{Q_1}
\]

\[
= r - g + (1 + g) = 1 + r,
\]

where, to go from the first to the second line, we used the notation for key ratios that are constant along the balanced growth path, to go from the second to the third line, we used the relationship \( \Phi_n = \nu_n + \gamma_n g^2 + o(g) \) (implied by the functional form we use for adjustment costs), and to go from the third to the fourth line, we used the expression for \( Q_1 \) obtained in the main text.

Likewise, for returns to investment in capital of type \( n = 1, 2 \):

\[
R_{n,I,t+1} = \frac{\Pi_n - \Phi_n - (1 + g)q_n}{q_n}
\]

\[
= \frac{R_n - \Phi_n - (1 + g)q_n}{q_n}
\]

\[
= \frac{r + \delta_n + \gamma_n rg - \Phi_n + (1 + g)q_n}{q_n}
\]

\[
= \frac{(1 + \gamma_n g)(r - g) + (1 + g)q_n}{q_n} = 1 + r,
\]

where we successively used the balanced growth path optimality condition \( R_n = \Pi_n \), the balanced growth user cost definition \( R_n = r + \delta_n + \gamma_n rg = r - g + \nu_n + \gamma_n rg \), and the q-theory condition \( q_n = 1 + \gamma_n g \).

### 2.6 Financing frictions

#### 2.6.1 Frictions to equity issuance or dividend distributions

**General results** The firm solves:

\[
V_t^c(K_t) = \max_{K_{t+1}} K_t f_t \left( \frac{D_t}{K_t} \right) + E_t \left[ M_{t,t+1} V_{t+1}^c(K_{t+1}) \right]
\]

s.t. \( D_t = \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) \),

\[
K_t = F_t(K_t).
\]

\[ (37) \]

The only difference with our baseline model is how dividend payments are valued. The flow value of dividends is \( K_t f_t(D_t/K_t) \), instead of \( D_t \) in the baseline model. (The baseline model is nested in this model, when \( f_t(x) = x \).)
Assumption 5 (Frictions to equity issuance or dividend distributions). The functions \( \{f_t : \mathbb{R} \to \mathbb{R}\} \) are increasing, concave, twice differentiable, and satisfy:

\[
\forall t, \quad f_t(0) = 0, \quad f'_t(0) = 1.
\]

The function \( f_t(.) \) captures the equity issuance frictions. We require it to be smooth, so that we can continue using a first-order approach. The concavity of \( f_t(.) \) implies that firms will prefer to smooth dividend distributions or equity issuances. The restriction that \( f'_t(0) = 1 \) additionally implies that dividend distributions are (weakly) less valuable, at the margin, than in the baseline model, while equity issuances are (weakly) more costly, at the margin, than in the baseline model. These distortions could capture direct costs of equity issuance (when \( d_t < 0 \)) or agency costs associated with free cash flow (when \( d_t > 0 \)).

Result 11. Firm value can be written as:

\[
V_t^e = \sum_{n=1}^{N} q_{n,t} K_{n,t+1} + \sum_{n=1}^{N} (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} f'_t(d_{t+k}) \Pi_{n,t+k} K_{n,t+k} \right]
\]

\[
= f'(d_t) \left\{ \sum_{n=1}^{N} q_{n,t}^{(a)} K_{n,t+1} + \sum_{n=1}^{N} (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k}^{(a)} \Pi_{n,t+k} K_{n,t+k} \right] \right\}
\]

where \( q_{n,t} \equiv \mathbb{E}_t \left[ M_{t,t+1} \frac{\partial V_{t+1}^{(e)}}{\partial q_{n,t+1}} \right] \), \( M_{t,t+k}^{(a)} = \frac{f'_t(d_{t+k})}{f'_t(d_t)} M_{t,t+k} \), and \( q_{n,t}^{(a)} = \frac{q_{n,t}}{f'_t(d_t)} \). The gap between \( Q_{n,t} \) and \( q_{n,t} \) is given by the same expression as in Result (1), replacing \( M_{t,t+k} \) with \( f'(d_{t+k})M_{t,t+k} \). Moreover, denote \( Q_{n,t}^{(a)} = \frac{Q_{n,t}}{f'(d_t)} \). Then we have:

\[
G_t^{(a)} = Q_{n,t}^{(a)} - q_{n,t}^{(a)} = (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k}^{(a)} \Pi_{n,t+k} (1 + g_{n,t+1,t+k}) \right]
\]

\[
+ \sum_{m=1}^{N} \sum_{m \neq n} q_{m,t}^{(a)} S_{m,n,t+1}
\]

\[
+ (\mu - 1) \sum_{m=1}^{N} \sum_{m \neq n} \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k}^{(a)} \Pi_{m,t+k} (1 + g_{m,t+1,t+k}) \right] \times S_{m,n,t+1},
\]

and:

\[
G_t^{(u)} = Q_{n,t} - q_{n,t}^{(a)} = - \left( \frac{1 - f'_t(d_t)}{f'_t(d_t)} \right) Q_{n,t} + G_t^{(a)}.
\]

The proof is reported below.

When the investment gap is defined as \( Q_{n,t} - q_{n,t} \), as in the baseline model, equity frictions do not change the basic insights from that model. The friction primarily appears as modification to the discount factor, and thus affects the way future rents are valued. However, an important difference with the baseline model is that \( q_{n,t} \) is not a sufficient statistic for investment. Indeed, the first-order condition for investment can be written as:

\[
\Phi'_{n,t} f'_t(d_t) = q_{n,t}.
\]
The term $f'(d_t)$, which can be thought of as the marginal rate of substitution between “inside” cash and “outside” distributions, appears as a wedge between the marginal value of capital and the marginal cost of investment. As noted by other papers, this wedge changes the expected relationship between investment and both average $Q$ and marginal $q$, even in a model without rents or intangibles. For instance, in a model with no intangibles and no rents, and with quadratic adjustment costs, we have $q_t = Q_t$, but:

$$t_t = \delta + \frac{1}{\gamma} \left( \frac{q_{n,t}}{f'(d_t)} - 1 \right)$$

$$= \delta + \frac{1}{\gamma} \left( \frac{Q_{n,t}}{f'(d_t)} - 1 \right).$$

This creates an “investment gap” in the sense that $t_t$ is lower than predicted by values of $Q_{n,t}$ whenever $d_t < 0$ (so that $f'(d_t) > 1$).

This suggests studying an alternative decomposition based on “adjusted” average $q$ and marginal $Q$, $q^{(a)}_{n,t} \equiv \frac{q_{n,t}}{f'(d_t)}$ and $Q^{(a)}_{n,t} \equiv \frac{Q_{n,t}}{f'(d_t)}$. The result above shows that the decomposition of the investment gap between these two quantities has the same three components as in our baseline analysis.

However, $Q^{(a)}_{n,t}$ is not directly observable. The result therefore also reports the gap between marginal $q$, $q^{(a)}_{n,t}$ (the correct measure of the incentive to invest) and observable average $Q$, $Q_{n,t}$. In this case, the gap has an additional term, which is simply equal to $Q_{n,t} - Q^{(a)}_{n,t}$. This difference is zero in the absence of equity issuance frictions, i.e. $f'(d_t) = 1$.

Under this last definition, there is a bias in the level of the investment gap, which appears even if there are no rents and no intangibles. In this case, we have:

$$Q_{n,t} = f'(d_t)q^{(a)}_{n,t}.$$  

When $d_t < 0$, $f'(d_t) > 1$ and so $Q_{n,t} > q^{(a)}_{n,t}$. In other words, there is a positive “investment gap”.

Intuitively, $q^{(a)}_{n,t}$ is the marginal value of capital “inside” the firm (that is, expressed in units of internal cash flows), whereas $Q_{n,t}$ is the average value of capital to “outsiders” (that is, to shareholders). When the firm is issuing equity ($d_t > 0$), outside liquidity is more costly than inside liquidity, so average $Q_{n,t}$ is higher than marginal $q$; and conversely if $d_t < 0$.

Proof. The first-order necessary condition and the envelope theorem, for each capital type, are:

$$f'_t(d_t) \Phi'_{n,t} = q_{n,t},$$

$$\frac{\partial V_{n,t+1}}{\partial K_{n,t+1}} = (f_{t+1}(d_{t+1}) - d_{t+1}f'_{t+1}(d_{t+1})) F_{n,t} + f'(d_{t+1}) \left( \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1} \frac{K_{n,t+1}}{K_{n,t+1}} \right).$$

Multiplying the latter condition by $M_{t,t+1} K_{n,t+1}$, combining with the former condition, and taking expectations at time $t$, we obtain:

$$q_{n,t} K_{n,t+1} = \mathbb{E}_t \left[ M_{t,t+1} \left( (f_{t+1}(d_{t+1}) - d_{t+1}f'_{t+1}(d_{t+1})) F_{n,t+1} K_{n,t+1} \right. \right.$$

$$+ f'_{t+1}(d_{t+1}) (\Pi_{n,t+1} K_{n,t+1} - \Phi_{n,t+1} K_{n,t+1}) \left. \right) \right]$$

$$+ \mathbb{E}_t \left[ M_{t,t+2} q_{n,t+1} K_{n,t+2} \right]$$

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Assuming the transversality condition \( \lim_{k \to \infty} \mathbb{E}_t [M_{t,t+k}q_{n,t+k-1}K_{n,t+k}] = 0 \) holds for each type of capital, we can iterate forward and sum across capital types to obtain:

\[
\sum_{n=1}^{N} q_{n,t}K_{n,t+1} = \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \left\{ f_{t+k}(d_{t+k})K_{t+k} \right. \right.
\]
\[
\left. \left. + f'_{t+k}(d_{t+k}) \left( \left( \sum_{n=1}^{N} \Pi_{n,t+k}K_{n,t+k} - \Phi_{n,t+k}K_{n,t+k} \right) - D_{t+k} \right) \right\} \right] \tag{39}
\]

where we used the homogeneity of degree 1 of the capital aggregator, which implies:

\[
K_{t+1} = \sum_{n=1}^{N} F_{n,t+1}K_{n,t+1}.
\]

Likewise, since:

\[
\Pi_t = \mu \sum_{n=1}^{N} \Pi_{n,t+1}K_{n,t+1},
\]

we have:

\[
\sum_{n=1}^{N} q_{n,t}K_{n,t+1} = \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k} \left\{ f_{t+k}(d_{t+k})K_{t+k} \right. \right.
\]
\[
\left. \left. \cdot \left( \sum_{n=1}^{N} \Pi_{n,t+k}K_{n,t+k} - (\mu - 1)\Pi_{n,t+k}K_{n,t+k} \right) \right\} \right] \tag{40}
\]

On the other hand, firm value excluding current distributions is given by:

\[
V^e_t = \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}f_{t+k}(d_{t+k})K_{t+k}] . \tag{41}
\]

Taking the difference between Equations (41) and (40) gives the result. The expression of the investment gap for the adjusted definitions of marginal \( q \) and average \( Q \), \( q^{(a)}_{n,t} \) and \( Q^{(a)}_{n,t} \), follows from dividing the second part of Result 11 by \( f'_t(d_t) \), and likewise for the expression of the investment gap between \( Q_{n,t} \) and \( q^{(a)}_{n,t} \).

**Balanced growth** We next give analytical expressions for the different definitions of the investment gap above in the balanced growth version of the model. We make the same assumptions as in the main text, and moreover, we assume that \( f_t(d_t) = f(d_t) \) for all \( t \geq 0 \).

**Result 12.** Let \( n = 1 \) denote physical capital. Along the balanced growth path, the adjusted physical investment gap is given by:

\[
Q^{(a)}_{1} - q^{(a)}_{1} = \sum_{m \geq 2} q^{(a)}_{m}S_{m} + \frac{(\mu - 1)R^{(a)}_{1}}{r - g} \sum_{m \geq 2} \frac{(\mu - 1)R^{(a)}_{m}}{r - g} \times S_{m} \tag{42}
\]

\( \square \)
where:

$$\forall n = 1, \ldots, N, \quad q_n^{(a)} = 1 + \gamma_n g,$$

$$\tilde{R}_n = r + \delta_n + \gamma_n r g - \frac{1 - \varepsilon(d)}{\varepsilon(d)} F_n d,$$

$d > 0$ is the dividend to capital ratio along the balanced growth path, and $\varepsilon(d) \equiv \frac{df}{f^2}(d)$. The proof is reported below. The intuition for the expressions of the different terms in the decomposition is the same as in the baseline model. The main effect of the equity frictions is to modify the relevant definition of user costs, which include an additional term: $-\frac{1 - \varepsilon(d)}{\varepsilon(d)} F_n d < 0$. This term can be thought of as a “capital gains” expression: along the balanced growth path, trend growth in total capital $K_t$ increases the flow value of dividends, $K_t f(D_t/K_t)$, when $D_t > 0$.

Proof. Along the balanced growth path, the first-order conditions for each type of capital can be written as:

$$f'(d_t) \Phi'_n = q_{n,t},$$

$$(1 + r) q_{n,t} = (f(d_{t+1}) - d_{t+1} f'(d_{t+1})) F_{n,t+1} + f'(d_{t+1}) \left( \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_n(1 + g_{n,t+1}) + K_{n,t+1}^{K_{n,t+2}} \frac{K_{n,t+2}}{K_{n,t+1}} \right),$$

which we can write as:

$$(1 + r) \Phi'_n (1 + g_{n,t}) = (f(d_{t+1}) - d_{t+1} f'(d_{t+1})) F_{n,t+1} + f'(d_{t+1}) (\Pi_{n,t+1} - \Phi_n (1 + g_{n,t+1}) + \Phi'_n (1 + g_{n,t+1}) (1 + g_{n,t+1})),$$

where $g_{n,t} \equiv \frac{K_{n,t+1}}{K_{n,t}} - 1$. We next guess and verify that $g_{n,t} = g$ for $n = 1, \ldots, N$ is a solution. First, note that with this guess, because of the homogeneity of degree 1 of the capital aggregator $F$, $K_t$ is also growing at rate $a$. Therefore:

$$D_t = A_t^{1-1/\mu} K_t^{1/\mu} - \sum_{n=1}^{N} \Phi_n \left( \frac{K_{n,t+1}}{K_{n,t}} \right) K_{n,t}$$

is also growing at rate $g$. Define the following detrended variables:

$$k \equiv \frac{K_t}{A_t}, \quad d \equiv \frac{D_t}{K_t}, \quad k_{n,t} = \frac{K_{n,t}}{A_t}, \quad n = 1, \ldots, N,$$

which are all constant under our guess. Substituting the guess into the first-order condition above, and re-arranging, we obtain:

$$(r - g) \Phi'_n (1 + g) + \Phi_n (1 + g) = \left( \frac{f(d)}{f'(d)} - d \right) F_{n,t+1} + \Pi_{n,t+1}.$$

Neglecting terms of order $o(g)$ and higher, we can rewrite this as:

$$r + \delta_n + \gamma_n r g = \left( \frac{f(d)}{f'(d)} - d \right) F_{n,t+1} + \Pi_{n,t+1} = \left( \frac{f(d)}{f'(d)} - d \right) + \frac{1}{\mu} \left( \frac{K_{t+1}}{K_{t+1}} \right)^{\frac{1}{\mu} - 1} F_{n,t+1}, \quad n = 1, \ldots, N.$$

(43)
Given the homogeneity of degree 1 of the capital aggregator, we have:

\[
\frac{K_{t+1}}{A_{t+1}} = \frac{F(K_{1,t+1}, \ldots, K_{n,t+1})}{A_{t+1}} = F(k_1, \ldots, k_n).
\]  
(44)

and:

\[
F_{n,t+1} = F_n(K_{1,t+1}, \ldots, K_{n,t+1}) = F_n(k_1, \ldots, k_n).
\]  
(45)

Thus, (43) is a system of \(N\) equations in \(N\) unknowns, the \(\{k_n\}_{n=1}^N\). We assume that it has a unique solution \(\{k_n\}_{n=1}^N\). Using this solution, we can verify that the guess \(K_{n,t} = k_n A_t\) indeed satisfies the necessary first-order conditions for each type of capital.

Note that, because \(f(.)\) is concave and \(f(0) = 0\),

\[
\frac{f(d)}{f'(d)} - d \geq 0.
\]

If the inequality holds strictly (that is, when there is a dividend smoothing motive), there is no analytical solution to the system of equations (43). Nevertheless, we can define:

\[
\tilde{R}_n = r + \delta_n + \gamma_n rg - \left(\frac{f(d)}{f'(d)} - d\right) F_n;
\]

we then have \(\Pi_n = \tilde{R}_n\) for each \(n = 1, \ldots, N\), thus establishing the result.

\[\square\]

2.6.2 Debt issuance with collateral limit

**General results**  Shareholders solve:

\[
E_t^c(B_t, K_t) = \max_{K_{t+1}} D_t + \mathbb{E}_t \left[ M_{t,t+1} E_{t+1}^c(B_{t+1}, K_{t+1}) \right]
\]

s.t.  \(D_t = \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + B_{t+1} - (1 + r_{b,t-1})B_t\),

\[
K_t = F_t(K_t),
\]

\[
B_{t+1} \leq \theta K_{1,t+1} [\lambda_t]_t.
\]  
(46)

Here, \(E_t\) is the (cum-dividend) value of equity, \(B_t\) is the stock of debt outstanding, and \(r_{b,t}\) is the interest rate on debt, defined as:

\[
r_{b,t-1} = \mathbb{E}_{t-1} [M_{b,t-1,t}]^{-1} - 1,
\]

where \(M_{b,t,t+1}\) is the stochastic discount factor of debtholders. All debt is one-period. We assume that debt is collateralized using \(K_{1,t+1}\); \(\theta\) captures the collateral limit. In our applications, \(K_{1,t+1}\) will denote the stock of physical capital, so that the assumption that the borrowing constraint only involves \(K_{1,t+1}\) captures the idea that physical assets are more likely to be used as collateral in lending transactions.

The Lagrange multiplier \(\lambda_t\) capture the shadow value of relaxing the borrowing constraint to equity-
holders. The first-order condition with respect to investment in each type of capital \( n \) is given by:

\[
q_{1,t}^E + \lambda_t \theta = \Phi_n \left( \frac{K_{n,t+1}}{K_{n,t}} \right),
\]

\[
q_{n,t}^E = \Phi_n \left( \frac{K_{n,t+1}}{K_{n,t}} \right), \quad n = 2, \ldots, N,
\]

where:

\[
q_{n,t}^E \equiv \left[ M_{t,t+1} \frac{\partial E^{(c)}_{t+1}}{\partial K_{n,t+1}} \right], \quad n = 1, \ldots, N,
\]

is the marginal value of a unit of debt to shareholders, or equity’s marginal \( q \). Note, that, for physical capital, equity’s marginal \( q \) is not a sufficient statistic for investment in physical capital. This is because another marginal benefit of increasing investment in physical capital is that it relaxes the collateral constraint. Therefore, we also define the firm’s marginal \( q \) as:

\[
q_{1,t} \equiv q_{1,t}^{(E)} + \lambda_t \theta,
\]

\[
q_{n,t} \equiv q_{n,t}^{(E)}, \quad n = 2, \ldots, N.
\]

Given the first-order conditions above, the \( \{q_{n,t}\}_{n=1}^N \) are sufficient statistics for investment in each type of capital.

The first-order condition with respect to borrowing is:

\[
\lambda_t = 1 - \mathbb{E}_t \left[ M_{t,t+1} \frac{\partial E^{(c)}_{t+1}}{\partial B_{t+1}} \right],
\]

We make the following assumption about the borrowing constraint

**Assumption 6** (Binding borrowing constraint). The stochastic discount factors \( M_{b,t,t+1} \) and \( M_{t,t+1} \) are such that the collateral constraint is always binding, that is:

\[
\forall t, \quad \lambda_t > 0 \quad \text{and} \quad B_{t+1} = \theta K_{t+1}.
\]

Our baseline model and this model will coincide whenever \( \lambda_t = 0 \forall t \), a sufficient condition for which is that debtholders and shareholders have the same discount factor. We assume that the borrowing constraint is binding in order to deviate from the baseline model.\(^{10}\) Under this assumption, combining the first-order condition for debt issuance with the envelope condition for the stock of debt, we obtain:

\[
\lambda_t = 1 - (1 + r_{b,t}) \mathbb{E}_t [M_{t,t+1}],
\]

that is, the shadow value of an additional unit of debt is one minus the discounted interest cost of borrowing.

\(^{10}\) In the balanced growth path, this assumption will hold as long as \( r > r_b \), where \( r \) is the discount rate of equityholders and \( r_b \) is the discount rate of debtholders. In general, the sufficient condition for the borrowing constraint to be binding should be:

\[
\forall t \geq 0, \quad 1 + r_{b,t} = \mathbb{E}_t [M_{b,t,t+1}]^{-1} < \mathbb{E}_t [M_{t,t+1}]^{-1},
\]

though we do not have a formal proof.
Thus, in this model, equityholders choose to borrow (despite frictionless equity markets) because of the positive wedge between their discount rate and debtholders’ discount rate. The collateral constraint then limits their ability to do so.\textsuperscript{11}

\textbf{Result 13.} Define total firm value as:

\[ V_t^{(e)} = E_t [M_{t,t+1}E_{t+1}^e] + E_t [M_{b,t,t+1}(1 + r_{b,t})B_{t+1}] = E_t^e + B_{t+1}. \]

Total firm value is given by the same expression as in the main text:

\[ V_t^{(e)} = \sum_{n=1}^{N} q_{n,t}K_{n,t+1} + (\mu - 1) \sum_{n=1}^{N} E_t \left[ \sum_{k \geq 1} M_{t,t+k} \Pi_{n,t+k}K_{n,t+k} \right] + \lambda_t B_{t+1} \]

\[ = \sum_{n=1}^{N} q_{n,t}K_{n,t+1} + (\mu - 1) \sum_{n=1}^{N} E_t \left[ \sum_{k \geq 1} M_{t,t+k} \Pi_{n,t+k}K_{n,t+k} \right]. \]

(47)

The investment gap is given by the same expression as in the main text:

\[ G_{n,t} = Q_{n,t} - q_{n,t} = (\mu - 1) \sum_{k \geq 1} E_t [M_{t,t+k} \Pi_{n,t+k}(1 + g_{n,t+1,t+k})] \]

\[ + \sum_{m=1}^{N} q_{m,t}S_{m,n,t+1} \]

\[ + (\mu - 1) \sum_{m \neq n}^{N} \sum_{k \geq 1} E_t [M_{t,t+k} \Pi_{m,t+k}(1 + g_{m,t+1,t+k})] \times S_{m,n,t+1}. \]

The proof is reported below. In this model with a borrowing constraint, the same decomposition of the investment gap hold as in our baseline model, where debt issuance is unconstrained. The interpretation of the components of the investment gap is the same as in that model.

A “shareholder” investment gap for physical capital can be defined as: \( Q_{t}^{E} - q_{t}^{E} = E_{t}^{(e)} / K_{t+1} - q_{t}^{E} \). It satisfies \( Q_{1,t}^{E} - q_{1,t}^{E} = Q_{1,t} - q_{1,t} - (1 - \lambda_t) \theta \). The two coincide when \( \lambda_t = 1 \) (in which case equity and debt financing are perfect substitutes) or when \( \theta = 0 \) (in which case the firm is all equity-financed).

However, though this is not immediately visible from the decomposition, the introduction of a collateral constraint along with an incentive to use debt will change the quantitative implications of the model. This is because in choosing how much to investment in physical capital, shareholders take into account its beneficial effects on the borrowing constraint. This, in turn, affects the user cost of physical capital. We return to this issue in our empirical applications (Appendix 4.9).

\textsuperscript{11} Note, in particular, that we assume unlimited liability on the part of the shareholders, that is, no option to default if the value of equity falls below zero. This assumption will not bind along the balanced growth path. In the general analysis, it helps us avoid having to deal with complications associated with pricing default risk in debt contracts and describing the resolution of default.
Proof. In general, the necessary first-order conditions to the shareholder value maximization problem are:

\[
\lambda_t = 1 + \mathbb{E}_t \left[ M_{t,t+1} \frac{\partial E^{(e)}_{t+1}}{\partial B_{t+1}} \right]
\]

\[
\frac{\partial E^{(e)}_{t+1}}{\partial B_{t+1}} = -(1 + r_{b,t}) + \frac{\partial \lambda_{t+1}}{\partial B_{t+1}} (\theta K_{1,t+2} - \theta B_{t+2})
\]

\[
\Phi'_{1,t} = \mathbb{E}_t \left[ M_{t,t+1} \frac{\partial E^{(e)}_{t+1}}{\partial K_{1,t+1}} \right] + \lambda_t \theta
\]

\[
\Phi'_{n,t} = \mathbb{E}_t \left[ M_{t,t+1} \frac{\partial E^{(e)}_{t+1}}{\partial K_{n,t+1}} \right], \quad n = 2, ..., N
\]

\[
\frac{\partial E^{(e)}_{t+1}}{\partial K_{n,t+1}} = \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1} \frac{K_{n,t+2}}{K_{n,t+1}} + \frac{\partial \lambda_{t+1}}{\partial K_{n,t+1}} (\theta K_{1,t+2} - \theta B_{t+2}), \quad n = 1, ..., N
\]

When the borrowing constraint is binding, these conditions imply that:

\[
q^{E}_{1,t+1} K_{1,t+1} = \mathbb{E}_t [M_{t,t+1} (\Pi_{1,t+1} K_{1,t+1} - \Phi_{1,t+1} K_{1,t+1})]
\]

\[
+ \theta \mathbb{E}_t [M_{t,t+1} \lambda_{t+1} K_{1,t+2}] + \mathbb{E}_t [M_{t,t+1} q^{E}_{1,t+1} K_{1,t+2}]
\]

\[
q^{E}_{n,t+1} K_{n,t+1} = \mathbb{E}_t [M_{t,t+1} (\Pi_{n,t+1} K_{n,t+1} - \Phi_{n,t+1} K_{n,t+1})] + \mathbb{E}_t [M_{t,t+1} q^{E}_{n,t+1} K_{n,t+2}], \quad n = 2, ..., N
\]

Combining these expressions, and assuming that the transversality conditions:

\[
\lim_{k \to +\infty} \mathbb{E}_t [M_{t,t+k} q^{E}_{n,t+k} K_{n,t+k+1}] = 0
\]

and:

\[
\lim_{k \to +\infty} \mathbb{E}_t [M_{t,t+k} \lambda_{t+k} K_{n,t+k+1}] = 0
\]

hold for each type of capital, we obtain:

\[
\sum_{n=1}^{N} q^{E}_{n,t} K_{n,t+1} = \mathbb{E}_t \left[ \sum_{k \geq 1} \sum_{n=1}^{N} M_{t,t+k} (\Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+k} K_{n,t+k}) \right] + \theta \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} \lambda_{t+k} K_{1,t+k+1} \right]
\]

Equity value is given by:

\[
E^{e}_t = \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} (\Pi_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+k} K_{n,t+k}) \right] + \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} (B_{t+k+1} - (1 + r_{b,t+k-1}) B_{t+k}) \right]
\]

The net present value of net proceeds from future debt issuances to the shareholders is given by:

\[
\mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} (B_{t+k+1} - (1 + r_{b,t+k-1}) B_{t+k}) \right]
\]

\[
= - \mathbb{E}_t [M_{t,t+1} (1 + r_{b,t}) B_{t+1}]
\]

\[
+ \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} (1 - M_{t+k,t+k+1} (1 + r_{b,t+k})) B_{t+k+1} \right]
\]
\[ - \mathbb{E}_t [M_{t,t+1}(1 + r_{b,t})B_{t+1}] + \theta \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} \lambda_{t+k} K_{1,t+k+1} \right] \]

where, to go from the second to the third line, we used the law of conditional expectations, the fact that \( B_{t+k+1} = \theta K_{1,t+k+1} \) (because the borrowing constraint is assumed to bind), and the first-order condition for borrowing when the borrowing constraint is binding:

\[ \lambda_{t+k} = 1 - \mathbb{E}_{t+k} [M_{t+k,t+k+1}(1 + r_{b,t+k})] \]

Therefore, total firm value is given by:

\[ V_t^{(e)} = E_t^{(e)} + B_{t+1} \]

\[ = \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} \left( \Pi_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+k} \left( \frac{K_{n,t+k+1}}{K_{n,t+k}} \right) K_{n,t+k} \right) \right] + \theta \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} \lambda_{t+k} K_{t+k+1} \right] \]

\[ + \ (1 - \mathbb{E}_t [M_{t,t+1}(1 + r_{b,t})]) B_{t+1} \]

\[ = \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} \left( \Pi_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+k} \left( \frac{K_{n,t+k+1}}{K_{n,t+k}} \right) K_{n,t+k} \right) \right] + \theta \mathbb{E}_t \left[ \sum_{k \geq 1} M_{t,t+k} \lambda_{t+k} K_{t+k+1} \right] \]

\[ + \lambda_t B_{t+1} \]

\[ = \sum_{n=1}^{N} q_{n,t} E_{n,t+1} + (\mu - 1) \sum_{n=1}^{N} E_t \left[ \sum_{k \geq 1} M_{t,t+k} \Pi_{n,t+k} K_{n,t+k} \right] + \lambda_t B_{t+1} \]

\[ = \sum_{n=1}^{N} q_{n,t} K_{n,t+1} + (\mu - 1) \sum_{n=1}^{N} E_t \left[ \sum_{k \geq 1} M_{t,t+k} \Pi_{n,t+k} K_{n,t+k} \right] \]

where, to go from the penultimate to the last line, we used the fact that \( q_{n,t} = q_{n,t}^{(E)} \) for \( n = 2, \ldots, N \), and \( q_{1,t} = q_{1,t}^{E} + \lambda_t \theta \). The investment gap decomposition reported in Result 13 then follows. The value of equity is given by:

\[ E_t^{(e)} = V_t^{(e)} - B_{t+1} \]

\[ = (q_{1,t}^{E} - (1 - \lambda_t)\theta) K_{1,t+1} + \sum_{n=2}^{N} q_{n,t+1}^{E} K_{n,t+1} + (\mu - 1) \sum_{n=1}^{N} E_t \left[ \sum_{k \geq 1} M_{t,t+k} \Pi_{n,t+k} K_{n,t+k} \right] \]

establishing the decomposition of the investment gap for shareholders. \( \square \)
3 Data sources and construction

3.1 Sources for investment and profit rates for the NFCB sector

We use the following time series from NIPA, all for the non-financial corporate business sector (NFCB): NFCB gross value added \((Y^{(BEA)})\) (FRED series A455RC1Q027SBEA), NFCB compensation of employees \((WN^{(BEA)})\) (FRED series A460RC1Q027SBEA), NFCB taxes on production less subsidies \((T^{(BEA)})\) (FRED series W325RC1Q027SBEA), NFCB transfers \((Tr^{(BEA)})\) (FRED series W325RC1Q027SBEA). The data are annual. We use them to compute the surplus of the NFCB sector as:

\[
\Pi^{(BEA)} = Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}
\]

and to compute the labor share of the NFCB sector as:

\[
LS = WN^{(BEA)} / (Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}).
\]

We use the labor share only to translate our estimates of the model parameter governing rents, \(\mu\), into the share of rents as a fraction of value added. Additionally, we obtain current cost measures of the capital stock for the NFCB sector from the BEA fixed asset tables. We extract \(K^{(BEA)}_{\text{struct}}\), \(K^{(BEA)}_{\text{equip}}\), \(K^{(BEA)}_{\text{intan}}\), from BEA table 4.1; in particular, we define \(K^{(BEA)}_{\text{intan}}\) as the stock of intellectual property products. We then define:

\[
K^{(BEA)}_1 = K^{(BEA)}_{\text{struct}} + K^{(BEA)}_{\text{equip}}, \quad K^{(BEA)}_2 = K^{(BEA)}_{\text{intan}}.
\]

We use table 4.7 to obtain measures of current investment for the NFCB sector, and we define \(I^{(BEA)}_1\) and \(I^{(BEA)}_2\) analogously to \(K^{(BEA)}_1\) and \(K^{(BEA)}_2\). Note that all time series from tables 4.1 and 4.7 are expressed in current dollar values; we only use them in the computation of ratios.

3.2 Computation of enterprise value for the NFCB sector

**Methodology**  In order to construct an estimate of \(Q_{1,t}\) for the NFCB sector, we require a time series for total firm value for the non-financial corporate business (NCFB) sector, \(V_t\), for the 1947-2017 period. We measure \(V_t\) by estimating the total market value of securities outstanding from the NCFB sector. Specifically, we define:

\[
V_t = MVE_t + MVD_t - L_t.
\]

Here, \(MVE_t\) is the market value of equity claims on the NFCB sector, \(MVD_t\) is the market value of other financial claims (including debt liabilities) on the NFCB sector, and \(L_t\) is the book value of liquid financial assets owned by the NFCB sector.

We use measures of \(MVE_t\) and \(L_t\) provided by the Flow of Funds. For \(MVE_t\), we use Flow of Funds series LM103164103 (nonfinancial corporate business; corporate equities; liability; Table L.103). This series is constructed by the Flow of Funds as the sum of the market value of equities of publicly traded companies, plus an estimate of the market value of equities of closely held firms, which is estimated from
a variety of sources, including the Statistics of Income from the IRS. For \( L_t \), we use Flow of Funds series FL104001005 (Nonfinancial corporate business; liquid assets, broad measure; Table L.103). This series is the sum of the market values of municipal securities, commercial paper, deposits, Treasuries and agency securities, repos, money market fund shares, and corporate equities held by the NFCB sector, estimated from a variety of sources, including the Statistics of Income and the Quarterly Financial Report.

The main difficulty in constructing a series for \( V_t \) is to obtain an estimate of the market value of other financial claims on the NFCB sector, \( MVD_t \). In order to estimate this quantity, we extend the approach of Hall (2001) (whose data stops in 1999) to the 1947-2017 period. Specifically, we estimate \( MVD_t \) as:

\[
MVD_t = BVD_t + (MVB_t - BVB_t),
\]

where \( BVD_t \) is the book value of all non-equity claims on the NFCB sector, \( MVB_t \) is an estimate of the market value of bonds issued by the NFCB sector, and \( BVB_t \) is the book value of bonds issued by the NFCB sector. This approach therefore only imputes a market value for bonds issued by NFCB, as opposed to imputing a market value for all non-equity claims on the NFCB.

For \( BVD_t \), we use Flow of Funds series FL104190005 (Nonfinancial corporate business; total liabilities; Table L.103). We define \( BVB_t \) as the sum of the book value of taxable bonds (Flow of Funds series FL103163003; Nonfinancial corporate business; corporate bonds; liability; Table L.103), plus tax-exempt bonds (Flow of Funds series FL103162000; Nonfinancial corporate business; municipal securities; liability; Table L.103).

We proceed as in Hall (2001) in order to compute \( MVD_t \). For taxable securities, we assume a maturity at issuance of ten years. In each year, we compute gross issuance as the sum of net issuance (obtained from the Transactions table, series FA103163003), plus principal repayment from the 10-year-prior vintage. We then impute a coupon rate for this new bond vintage equal to the yield on corporate bonds multiplied by total gross issuance. Finally, in each year, we recalculate the market value of each bond vintage by discounting the remaining principal and coupon payments on all outstanding vintages at the current bond yield. We follow the same strategy for municipal securities.

For yields on taxable bonds, we use Moody’s Seasoned Corporate Baa bond yield (FRED series BAA). These yields are based on securities with maturities 20 years and above, so we subtract the gap between the 20-year and the 10-year Treasury yield (FRED series GS10 and GS20). For non-taxable securities, we use the Bond Buyer Go 20-Bond Municipal Bond Index (FRED series WSLB20).

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\(^{13}\) See www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=FL104001005&t=L.103&suf=Q.

\(^{14}\) \( BVB_t \) includes debt securities, taxes payable, trade payables, miscellaneous liabilities, and foreign direct investment. It does not include any estimate of the book value of equities. See www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=FL104190005&t=L.103&suf=Q for more details.

\(^{15}\) When the data are missing, and before 1986, we use the time series provided in the replication files of Hall (2001), available at web.stanford.edu/~rehall/SMCA_Data_Appendix.html. The WSLB20 series was discontinued 2016m10, so we splice it with the yield on taxable securities for the remainder of the sample.
Comparison with Hall (2001) Appendix Figure 21 reports the resulting time series for $V_t$ (the black dotted line). The figure also reports two other estimates of $V_t$. The crossed green line is the total market value of securities outstanding from the NFCB sector, $V_{t}^{Hall}$, constructed by Hall (2001), and the solid blue line is an update of this time series to 2017.

The series for $V_{t}^{Hall}$ is defined as:

$$V_{t}^{Hall} = MVE_t + MVD_t - F_t = V_t - (F_t - L_t),$$

where $MVE_t$ and $MVD_t$ are defined as above, and $F_t$ denotes total financial assets of the NFCB sector (Flow of Funds series FL104090005; Nonfinancial corporate business; total financial assets; from Table L.103).

In words, while Hall (2001) nets out all financial assets from estimates of the value of claims on the NFCB in order to arrive at an estimate of the market value of non-financial corporations, we only subtract those financial assets identified by the Flow of Funds as liquid. Note that in the Flow of Funds, $F_t \geq L_t$, and $L_t$ is a subcomponent of $F_t$.

We choose to diverge from the Hall (2001) methodology in this respect two reasons. First, the purpose of netting out financial claims is to obtain an estimate of net (as opposed to gross) debt liabilities in the numerator of Tobin’s Q. Standard measures of net debt, however, typically net out cash and cash equivalents, not other financial securities that may not be readily liquidated in order to honor debt commitments. Second, and more important, in the Flow of Funds, the difference between $F_t$ and $L_t$ is series FL103090005, Miscellaneous Financial Assets. The bulk of that series (Unidentified Miscellaneous Assets, series FL103090005) is a residual, imputed by the Flow of Funds in order to reconcile firm-level financial assets totals from the Statistics of Income and the Quarterly Financial Report, from totals obtained, at the instrument-level, from other sources. So the gap between $F_t$ and $L_t$ is as likely to capture measurement error across different underlying data sources as it is to capture actual net financial claims. As a result, in our baseline, we exclude from the imputation of net debt in the computation of Tobin’s Q.

3.3 Growth rates of capital stocks

We use table 4.2 to construct estimates of the real net growth rates of the stocks of physical, intangible, and total capital. One difficulty is that the Fixed Assets tables do not report a quantity index for physical capital, but only separate quantity indices for equipment and for structures. We aggregate the growth rates in these quantity indices into a growth rate rate $g_{1,t}$ for physical capital following the BEA’s own

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16 The time series are estimated at the quarterly frequency; the annual time series used in the main analysis are obtained by averaging the quarterly time series for each year.

17 We obtained the original data (up to 1999) used in Hall (2001) from web.stanford.edu/~rehall/SMCA_Data_Appendix.html. The replication code for our paper contains an extension to 2017 of Figures 2, 3, 4, 5, 8, 9, and 10 from Hall (2001).

methodology:

\[ g_{1,t} = \left( \frac{K_{\text{struct},t}}{K_t} (1 + g_{\text{struct},t})^{-1} + \frac{K_{\text{equip},t}}{K_t} (1 + g_{\text{equip},t})^{-1} \right)^{-1} - 1. \]

Here, \( K_{\text{struct},t} \) is the current-cost net stock of non-residential structures (from Fixed assets Table 4.1), \( g_{\text{struct},t} \) is the growth rate in the chain-type quantity index for the net stock of non-residential structures (Fixed assets Table 4.2), \( K_{\text{equip},t} \) and \( g_{\text{equip},t} \) are similarly defined, but for equipment, and \( K_t = K_{\text{struct},t} + K_{\text{equip},t} \) is the current-cost total stock of physical capital. For the growth rates in the quantity of total capital \( g_t \) and intangible capital \( g_{2,t} \), we directly use the growth rate of the quantity indices reported in Fixed Assets Table 4.2.

Appendix Figure 3 reports times series of these estimates. Growth rates in these quantity indices approximately coincide for physical and total capital (they are 2.8% and 3.0% per year on average, respectively), while the growth rate of the quantity index of intangible stock is higher (5.6% per year, on average).

The bottom panel plots the difference in the growth rates of the physical and intangible capital stock over time. In both cases, the difference between the two growth rates in quantity indices is positive, on average. Thus, the balanced growth assumption that \( g_{1} = g_{2} = g \) does not hold strictly in the data. However, the assumption seems be plausible for certain sub-periods, such as the post-2000s, as well as the 1970-1980 period, when the difference is substantially smaller.

### 3.4 Estimates of average depreciation rates across capital types

We next describe how we construct estimates for the average rate of economic depreciation of physical and intangible capital. In order to describe our approach, we briefly summarize the methodology behind the BEA estimates of capital stocks. Throughout the paper, we measure investment rates as:

\[ \iota_t = \frac{I_c}{K_{t-1}^c}, \quad (48) \]

where \( I_c^t \) is current cost investment during year \( t \), and \( K_{t-1}^c \) is the current-cost net stock of capital at the end of year \( t - 1 \), and \( c \) refers to equipment, structures, or intellectual property products. We use the data provided in Fixed Assets tables 4.1 (for the net stocks at current cost) and 4.7 (for the gross investment flows at current cost). For physical capital, we use the sum of equipment and structures, and for intangible capital, we use intellectual property products.

For a particular capital aggregate \( c \) (equipment, structures or intellectual property products), the BEA’s current-cost estimate of the net stock is constructed from the history of gross investment flows and assumptions on depreciation rates, as follows. First, both current-cost investment flows and current-cost

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19 For completeness, the graph also reports nominal time series for growth rates, though these are not used in our analysis.

20 The two lines refer to the difference obtained when growth rates are measured from stocks evaluated at current cost (“nominal”), and when they are obtained from changes in real quantity indices (“real”).

21 See Evans (2003) for a detailed description, as well as a list of the economic depreciation rates used by the BEA in applying the perpetual inventory method.
net stocks are simple sums of flows and stocks at the asset-type level:

\[ I_c^t = \sum_j I_{j,t}, \quad K_c^t = \sum_j K_{j,t}^c. \]

In this notation, asset types are indexed by \( j \); for instance, for equipment, this comprises automobiles, computers, machinery, etc. At the asset-type level, the net stock is defined as:

\[ K_{j,t}^c = P_{j,t} K_{j,t}^r. \]

Here, \( P_{j,t} \) is a price index that is used to deflate investment in the construction of real stocks (as described below), and \( K_{j,t}^r \) is a real-cost estimate of the net capital stock. The real-cost estimate of the capital stock is computed using a perpetual inventory method. Specifically:

\[ K_{j,t}^r = \sum_{k \geq 0} K_{j,t-k}^r = \sum_{k \geq 0} (1 - \nu \delta_j)(1 - \delta_j)^k \frac{I_{j,t-k}}{P_{j,t-k}}. \] (49)

The price index \( P_{j,t} \) is used to express current-cost gross investment in fixed dollars. \( K_{j,t-k}^r \) represents the contribution of investment in year \( t-k \) to the net stock of capital of type \( j \) at time \( t \). This contribution depends on initial investment \( I_{j,t-k}/P_{j,t-k} \), and economic depreciation, \( \delta_j \), which is allowed to vary across asset types, but is fixed over time for particular asset type. The contribution also depends on \( \nu \in [0, 1] \), which captures when the investment is assumed to be placed in service (\( \nu = 0 \) corresponds to the end of the year, and \( \nu = 1 \) corresponds to the beginning of the year; the BEA uses \( \nu = 1/2 \)).

In order to gain some intuition on the implications of this methodology for the drivers of the investment rate time series defined in Equation (48), assume that, in the underlying data, gross investment flows at current cost by asset type \( I_{j,t}^c \) were growing at the constant rate \( g_j^{nom} \), while the price indices were growing at the constant rate \( \pi_j \). Then, given the methodology described above for constructing net stocks, measured investment rates, as defined in Equation (48), would be constant, and given by:

\[ \iota_{j,t}^c \equiv \frac{I_{j,t}^c}{K_{j,t-1}^c} = \frac{1 + g_j^{nom} - (1 - \delta_j)(1 + \pi_j)}{1 - \nu \delta_j} \approx g_j + \delta_j, \]

where \( g_j = g_j^{nom} - \pi_j \) is the real growth rate of investment flows. Thus, measured investment rates, even on average, would not be driven only by assumptions about depreciation rates, \( \delta_j \), but also the growth rates of measured gross investment flows, \( g_j \), which are entirely independent from \( \delta_j \).

We next describe how to construct an estimate of average economic depreciation from the data provided in the aggregate Fixed Assets tables. We define our current-cost average depreciation estimate as:

\[ \delta_c^t \equiv \frac{D_c^t}{K_{t-1}^c + \nu I_c^t}, \] (50)

where \( D_c^t \) is the estimate of current-cost depreciation reported in Fixed Assets table 4.4, and \( K_c^t \) and \( I_c^t \) are defined as above. This data object is related to the economic depreciation rates \( \delta_j \) at the asset-type level as follows:

\[ \delta_c^t = \sum_j w_{j,t} \delta_j, \quad w_{j,t}^c = \frac{(P_{j,t}/P_{j,t-1}) K_{j,t-1}^c + \nu I_{j,t-1}^c}{K_{t-1}^c + \nu I_c^t}. \] (51)
This expression says that \( \delta^c_t \) is weighted average of the constant, asset-specific depreciation rates, where the weights reflect the current cost value of the undepreciated stock of capital in period \( t \).\(^{22}\)

The reason why \( \delta^c_t \) defined in this way is a weighted average of asset-level depreciation rates is the following. First, in the BEA data, the current-cost estimate of depreciation for a particular capital aggregate is defined as \( D^c_t = \sum_j D^c_{j,t} \), where \( D^c_{j,t} = P^c_{j,t} D^r_{j,t} \) is a current-cost estimate of depreciation during year \( t \) for asset type \( j \), and where \( D^r_{j,t} \) is its real counterpart. In turn, \( D^r_{j,t} \) is computed from gross investment and real net stock estimates, as:

\[
D^r_{j,t} = \frac{I^r_{j,t}}{P^c_{j,t}} - (K^r_{j,t} - K^r_{j,t-1}).
\]

In words, the real depreciation estimates \( D^r_{j,t} \) are constructed as residuals that satisfy the law of motion for capital.\(^{23}\) However, these asset-level depreciation flows can be used to infer depreciation rates. Using the perpetual inventory formula (49), we have:

\[
K^r_{j,t} = (1 - \nu \delta^h_j) \frac{I^c_{j,t}}{P^c_{j,t}} + (1 - \delta^h_j)K^h_{j,t-1} \quad \Rightarrow \quad \delta^h_j = \frac{D^r_{j,t}}{K^r_{j,t-1} + \nu I^h_{j,t}}.
\]

Using this result, it is straightforward to see that Equation (51) holds.\(^{24}\)

Appendix Figure 4 reports time-series for average depreciation rates \( \delta^c_t \), along with the gross investment rates \( I^c_t / K^c_{t-1} \) used in the paper. We report this separately for physical capital (defined as the sum of equipment and structures) and intangibles (intellectual property products), consistent with our approach in the paper.\(^{25}\) Note that underlying depreciation rates assumed by the BEA at the asset type level (the

\(^{22}\) These weights do not exactly add up to 1. This is because the definition of the average depreciation rate in Equation (50) does not appropriately reflate the value of the current-cost stock of capital. Appropriately reflating is difficult to do directly because the price indices \( P^c_{j,t} \) are not reported in Fixed Assets tables 4.1-4.7. However, this only affects the interpretation of \( \delta^c_t \), not its value.

\(^{23}\) Two additional points are worth making. First, as can be directly checked from the data in Fixed Assets tables 4.1, 4.4 and 4.7, the aggregate current-cost time series do not satisfy the law of motion \( D^c_t = I^c_t - (K^c_t - K^c_{t-1}) \). Second, in practice, the BEA uses a slightly different deflator for translating real depreciation estimates into current cost estimates (the average of \( P^c_{j,t} \) over the year, instead of its end-of-period value); this latter point complicates somewhat the expression for \( w^c_{j,t} \), but does not change its interpretation.

\(^{24}\) As an alternative to \( \delta^c_t \), we can also define an average depreciation rate based on the historical cost estimates reported in Tables 4.3 and 4.6, as follows:

\[
\delta^h_t = \frac{D^h_t}{K^h_{t-1} + \nu I^h_t}.
\]

Here, \( K^h_{t-1} \) and \( D^h_t \) are historical cost estimates of the net stock of capital and of depreciation, respectively. These estimates are constructed by the BEA exactly as the current-cost estimates, but assuming that the price index is \( P^c_{j,t} = 1 \) for all underlying asset types. In other words, these estimates do not account for changes in the relative price of underlying assets over time. Following similar steps as above, one can show that:

\[
\delta^h_t = \sum_j w^h_{j,t} \delta^h_j, \quad w^h_{j,t} = \frac{K^h_{j,t-1} + \nu I^h_{j,t}}{K^h_{t-1} + \nu I^h_t},
\]

where the \( \delta^h_j \) are the same as for current-cost estimates. We prefer using the current-cost version of \( \delta^c_t \) because our analysis relies on current-cost estimates of net stocks. However, the two time series \( \delta^c_t \) and \( \delta^h_t \) are very close in trends and, for intangibles, in levels.

\(^{25}\) For physical capital, following the logic above, we weigh the aggregate depreciation rate of equipment and structures by their lagged shares of undepreciated net stocks.
δ_j) are constant. The small upward trends in average depreciation rates for physical and intangible capital therefore reflect compositional changes in the underlying asset types, as opposed to changes in economic rates of depreciation of granular assets types.

3.5 Compustat data

Sample selection We use the annual version of the Compustat-CRSP merged files. We apply the standard screens (indfmt=INDL, popsrc=D, consol=C, datafmt=STD). We keep firm-year observations that satisfy the following criteria: fic=USA (domestically incorporated), 2-digit SIC code (first two digits of the variable sic) not equal to 49 (utilities), not between 60 and 69 (finance and real estate), and not between 90 to 99 (public administration); 2-digit SIC code not missing; variable sale (sales) and at (assets) not missing; variables emp, sale, at, act, lct, ppent, ppegt, che, and gdwl not negative. Finally, we drop any observation which we can identify as an American Depository Institution (ADR). We use only data from 1974 onward (included), as the data prior to 1974 has incomplete coverage (a jump in the number of firms in the sample occurs from 1973 to 1974).

Variable construction For each firm, we start by constructing six time series in levels, \{K_{1,t}, I_{1,t}, K_{2,t}, I_{2,t}, \Pi_t, V_t\}. For physical capital investment, we use capital expenditures (capx) net of sales of property, plant and equipment (sppe); we measure the stock using gross property, plant and equipment (ppegt), for reasons we discuss in Appendix 3.7. We consider two definitions of intangibles: R&D capital, and organization capital. For R&D, we use reported R&D expenditure (xrd), recoding missing values with 0. We measure investment in organization capital as 30% of SG&A expenditures (variable xsga) net of R&D investment. For the stock of both R&D and organization capital, we use the capitalized values provided by Peters and Taylor (2017). We discuss below in more detail the sources for the imputation of investment in organization capital.

For \Pi_t, we use the Compustat variable oibdp. We add estimates of intangible investment expenditures to actual measures of operating income in order to obtain an adjusted operating surplus measure consistent with our model. For \Pi_t, we use the sum of the market value of common stock and the book value of debt, net of cash and liquid securities. We then take the sum of these time series across firms either by year (when studying all publicly traded firms jointly), or by year and sector (when constructing the sectoral investment gaps.) Finally, we construct the growth rate of total capital at either the aggregate or sectoral level by subtracting from the growth rate of \(K_{1,t} + K_{2,t}\) the deflator implicitly used in Section 3, that is, the difference between nominal and real growth rates of total non-residential fixed assets for the NFCB sector.

Sectoral classification Appendix Tables 1 and 2 report the sectoral classification used in the analysis of Section 4. In order to be able to match the data to the BLS’s KLEMS data for the period over which the latter are available, we use a NAICS-based classification that maps to the BLS classification of sectors. We nevertheless aggregate the data up to sectors that are similar to the Fama-French 5 subsectors, with
the main difference being that we exclude financial companies from our analysis.

Rate of imputation of investment in organization capital We follow Eisfeldt and Papanikolaou (2014) and Peters and Taylor (2017) in choosing a rate of imputation of $\lambda = 0.3$ for SG&A (net of R&D expenditures) for our measure of investment in organization capital. Peters and Taylor (2017) show that their main conclusions regarding the relationship between investment and $Q$ hold for values of $\lambda$ ranging from 0.2 to 1.0. They also attempt to estimate $\lambda$ via maximum likelihood, allowing for heterogeneity across industries, and find values in the neighborhood of 0.3, though they caution against using these estimates, since they rely heavily on assuming perfect substitution between intangible and physical capital.

In both Eisfeldt and Papanikolaou (2014) and Peters and Taylor (2017), the main source for the imputation rate of $\lambda = 0.3$ is Hulten and Hao (2008). That paper uses data from financial statements of a composite of six large US pharmaceutical companies, which report expenditures on brand equity and organization capital. These expenditures account for 30% of all SG&A spending in their sample.

More recently, Ewens et al. (2020) provide estimates of $\lambda$ (and the implied values for the organization capital stock) based, in part, from asset valuations of public firms that exit, either by going private, being liquidated, or being acquired. Their average estimate (Table 5 of their paper) is $\lambda = 0.43$, with values ranging from 0.24 (in Manufacturing) to 0.62 (in Healthcare). Moreover, the ratios of total intangible (R&D plus organization capital) to physical capital implied by their estimated values for $\lambda$ are quantitatively similar to our estimates, in both levels and trends (see their Figure 8(a)). Their estimates rely on a different structure than the one we explore in this paper (specifically, they use an investment-$Q$ model with no rents and perfect substitutability between physical and intangible capital). While their estimates are quantitatively close to the value of $\lambda$ used in our paper and in Peters and Taylor (2017), we nevertheless think that using these estimates would not be internally consistent, given our approach in this paper. Their finding of higher values for $\lambda$ in Healthcare and Tech, and lower values in the Consumer and Manufacturing sector, would likely strengthen our point on heterogeneity in the composition of the investment gap across sectors.

3.6 Other data sources

We obtain the statutory corporate income tax rate form the Tax Policy Center.\textsuperscript{26} For the cum- and ex-dividend returns to equity used to compute the PD ratio, $R_{E,t-1,t}$ and $R_{E,t-1,t}$, we use returns from the CRSP file on daily returns on the S&P 500, downloaded from WRDS.\textsuperscript{27} We compute cumulative annual cum- and ex-dividend returns by taking the cumulative sum of the log of one plus daily returns over each year, and exponentiating end-of-year values. As a measure of the risk-free rate, we use the average annual rate of return on one-month Treasury bills, obtained from Kenneth French’s website.\textsuperscript{28} Finally, we use the

\textsuperscript{26} The specific series we use is the Top Tax Bracket series at https://www.taxpolicycenter.org/statistics/corporate-top-tax-rate-and-bracket.
\textsuperscript{27} We use the dsp500 file.
\textsuperscript{28} Specifically, we use the time series for $R_f$ available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors_CSV.zip.
BLS multifactor productivity database in order to obtain measures of value added as well as total factor productivity growth.\textsuperscript{29}

\subsection*{3.7 Robustness checks and comparisons across data sources}

\textbf{Residential assets} Our baseline approach for the NFCB sector only includes non-residential fixed assets. As a robustness check, we obtained residential fixed assets \( K_{\text{resid}}^{(\text{BEA})} \) as the difference between the sum of the three capital stocks above, and total fixed assets of NFCB sector report in BEA fixed asset table 6.1, and likewise for investment. The top panel of Figure 21 reports time series for the ratio \( \Pi^{(\text{BEA})}/K^{(\text{BEA})} \). The solid red and solid orange line use \( \Pi^{(\text{BEA})} \) as the numerator, and for the total capital stock, either \( K = K^{(\text{BEA})}_{\text{struct}} + K^{(\text{BEA})}_{\text{equip}} + K^{(\text{BEA})}_{\text{intan}} \) (as in our baseline analysis), or \( K = K^{(\text{BEA})}_{\text{struct}} + K^{(\text{BEA})}_{\text{equip}} + K^{(\text{BEA})}_{\text{intan}} + K^{(\text{BEA})}_{\text{resid}} \). The two lines are almost identical. The stock of residential fixed assets in the NFCB sector thus appears to be low relative to other types of fixed assets owned by the NFCB sector, and so we abstract from it in our analysis.

\textbf{Economy-wide vs. NFCB measures} Finally, Figure 21 also compares our measures of \( \Pi/K \) for the NFCB sector, with those reported by Farhi and Gourio (2018), who study economy-wide trends, instead of the NFCB sector specifically. The rate of return on capital measured by these authors is substantially lower than our measures of rates of return for the NFCB sector (by about 5-7\% throughout the sample.) Here, we briefly discuss why this is the case, as it matters for inferences about the importance of rents. These authors compute \( \Pi/K \) as:

\[ \Pi/K = \left[ \frac{(Y^{(\text{BEA})} - WN^{(\text{BEA})} - T^{(\text{BEA})} - Tr^{(\text{BEA})})}{(Y^{(\text{BEA})} - T^{(\text{BEA})} - Tr^{(\text{BEA})})} \right] \times \frac{Y}{K}, \]

where \( Y \) is total nominal GDP (including other sectors than the NFCB) and \( K \) is the total private capital stock (at replacement cost). This adjustment is made in order to maintain comparability with other ratios in their analysis, which has a broader scope than the NFCB. By contrast, our measures of \( \Pi/K \) are:

\[ \Pi/K = \left[ \frac{(Y^{(\text{BEA})} - WN^{(\text{BEA})} - T^{(\text{BEA})} - Tr^{(\text{BEA})})}{(Y^{(\text{BEA})} - T^{(\text{BEA})} - Tr^{(\text{BEA})})} \right] \times \frac{(Y^{(\text{BEA})} - T^{(\text{BEA})} - Tr^{(\text{BEA})})}{K^{(\text{BEA})}}. \]

Thus the differences between our measures of \( \Pi/K \) and the measures in Farhi and Gourio (2018) must be due to differences in the ratio of value added to capital between the NFCB sector and the economy as a whole. The bottom panel of Figure 21 indeed shows that the NFCB sector has a substantially higher dollar of value added per dollar of capital at current cost. The most accurate comparison is between the crossed blue line of the bottom panel, and the orange solid line, which measures \( K \) for the NFCB sector as the sum of all types of capital (residential, non-residential physical, and non-residential intangible): the value added to capital ratio is approximately 10 percentage points higher in the NFCB sector versus the economy as a whole.

\textsuperscript{29} The BLS multifactor productivity, or KLEMS, database is available at \url{https://www.bls.gov/mfp/special_requests/klemscombinedbymeasure.xlsx}. 
Compustat nonfinancials vs. NFCB sector There are two potentially important differences between the data used in Section 3 and the Compustat data. First, Compustat only includes publicly traded corporations. There may be systematic differences in returns to capital and intangible intensity between privately held and publicly traded corporations. Second, the measurement of the stock of physical capital differs across sources. We next discuss these differences in more detail.

In Compustat, our baseline measure of surplus as the sum of ebitda across all observations in our sample. (Missing observations are thus treated as zeros.) We use ebitda because it is the financial statement measure most closely related to our model definition of $\Pi_t$; it is a measure of operating income before depreciation, and does not deduct costs of capital, or non-operating income, which our model does not capture.\(^{30}\) The top right panel of Appendix Figure 24 report the NFCB sector surplus measured in this manner in Compustat, and the measure from the BEA tables. The two are highly correlated, but their levels differs substantially. This reflects the fact that the BEA NFCB sector data also includes private firms. The surplus of public firms (from Compustat) represents about two thirds of the total surplus of the NFCB sector (from the BEA).

The main difficulty in the Compustat data is in computing estimates of the current-cost total stock of physical capital. A natural definition would seem to be net property, plant and equipment (variable ppent). However, measuring $K_1$ for the NFCB sector in Compustat leads to extremely elevated measures of $\Pi/K_1$, as reported in the bottom right panel of Appendix Figure 24. These measures are almost double the BEA-derived measures. This is primarily because the aggregate value of ppent in Compustat is only about a third of physical capital in the NFCB sector according to BEA data (top left panel of Appendix Figure 24). The reason for this gap are unclear. One hypothesis is that the surplus of Compustat firms includes income from foreign subsidiaries, and so could overestimate the true surplus of public NFCB firms. Alternatively, it could be that private firms indeed have much lower rates of return on physical capital than public firms do (though the gap would have to be very large, given the relative importance of public firms in total surplus, as indicated in the top right panel of Appendix Figure 24). The more likely reason is that the accounting treatment of depreciation may lead the (balance sheet) net stock to underestimate the true current cost stock of physical assets. The red line in the top left and bottom left panels of Appendix Figure 24 instead report measures of asset returns using aggregate gross property, plant and equipment at historical cost (deflated using the implicit deflator from the BEA fixed tables). The bottom left panel shows that this estimate of $K_1$ leads to values of $\Pi/K_1$ that align more closely (in levels) with those provided by the BEA data on the NFCB. In what follows, in order to align our BEA and Compustat profitability moments as closely as possible, we therefore use gross property, plant and equipment as our main measure of $K_1$ in Compustat data.

We measure (gross) investment in physical capital in Compustat using capital expenditures (variable capx) minus sales of property, plant and equipment (variable sppe). Appendix Figure 22, top panel, shows that physical investment, computed in this manner, accounts for about two thirds of total physical investment in the BEA NFCB sector ($I_1^{BEA}$), with closely related cyclical movements. For investment

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\(^{30}\) The inclusion of non-operating income makes little difference to the results.
rates (the bottom panel of Appendix Figure 22), the data again suggest a much higher investment rate in Compustat when $K_1$ is measured using net book values, but investment rates are closer in levels when the capital stock is measured using gross book values.
4 Additional empirical results

4.1 Heterogeneous growth rates across capital stocks

While the balanced growth model imposes the restriction \( g = g_1 = g_2 \), we can nevertheless try to assess, heuristically, what the impact of having heterogeneous growth rates for the two capital stocks would be on our baseline decomposition.

In order to do this, we construct an “approximate” decomposition of the investment gap. This decomposition is the same as our baseline, except that when we map model and data, we allow the growth rates that enter marginal \( q_i \), as well as user costs, to differ. That is, given the same data as in our baseline exercise, as well as growth rates \( g_1 \) for physical capital and \( g_2 \) for intangible capital, we construct the following variables:

\[
\hat{q}_1 \equiv 1 + \gamma_1 g_1 \\
\hat{q}_2 \equiv 1 + \gamma_2 g_2 \\
\hat{r} - g = \frac{ROA}{Q_1} - (\iota_1 + S\iota_2) - \frac{\gamma_1 g_1^2 + \gamma_2 g_2^2 S}{Q_1} \\
\hat{R}_1 \equiv \hat{r} - g + \iota_1 + \gamma_1 (\hat{r} - g + g) g_1 \\
\hat{R}_2 \equiv \hat{r} - g + \iota_2 + \gamma_1 (\hat{r} - g + g) g_2 \\
\hat{\mu} \equiv \frac{ROA_1}{R_1 + SR_2}
\]

These definitions are analogous to those derived from the balanced growth model, except that we replaced \( q_i = 1 + \gamma_i g \) by \( \hat{q}_i = 1 + \gamma_i g_i \), \( i = 1, 2 \), and we also replaced any term of the form \( \gamma_i g \) or \( \gamma_i g^2 \) by \( \gamma_i g_i \) or \( \gamma_i g_i^2 \), \( i = 1, 2 \) in user costs.

To the extent that \( g \neq g_1 \neq g_2 \), the values of \( \{ \hat{r} - g, \hat{R}_1, \hat{R}_2, \hat{\mu}, \hat{q}_1, \hat{q}_2 \} \) defined above will differ from the values for the variables \( \{ r - g, R_1, R_2, \mu, q_1, q_2 \} \) computed in our baseline approach using the same data.

It is straightforward to show that, as in our baseline decomposition, the hatted variables defined above satisfy:

\[
Q_1 - \hat{q}_1 = \hat{q}_2 S + \frac{(\mu - 1)\hat{R}_1}{\hat{r} - g} + \frac{(\mu - 1)\hat{R}_2}{\hat{r} - g} \times S.
\]

We can therefore compare the decomposition obtained when imposing \( g = g_1 = g_2 \) on the data (consistent with the model’s predictions along the balanced growth path), and the more general decomposition (55), which allows for \( g \neq g_1 \neq g_2 \) (but is not consistent with the balanced growth restrictions imposed by the model). Note, importantly, that the decomposition (55) is not structural (since it violates \( g_1 = g_2 = g \)), so that comparisons with our baseline decomposition are only heuristic.

In order to measure \( g \), \( g_1 \) and \( g_2 \), we then use the growth rate in the quantity indices for the three capital stocks reported in the middle panel of Appendix Figure 3, consistent with our measurement in the
Appendix figure 14 reports the results. The top panel is the baseline decomposition in the main paper, and the bottom panel is the decomposition using heterogeneous growth rates, as outlined above. The two are difficult to distinguish. Intuitively, heterogeneity in the measurement of the growth rates that enter marginal $q$ have similar effects as changing adjustment costs across capital types. As we discuss in the main text, the effect of varying capital adjustment costs on the decomposition is small; the result here is consistent with that finding.

### 4.2 Implications for the labor share

Consider the model with variable intermediate inputs, described in Appendix 2.3, and assume that all intermediate inputs are labor (or equivalently, that the production function is a value-added production function), and that returns to scale are constant. Then, using the results of Table 5, the labor share of value added is given by:

$$LS = \frac{W_i L_{jt}}{P_{jt} Y_{jt}} = 1 - \frac{\alpha}{\mu_S}. $$

Moreover, using Lemma 4, the link between our reduced-form rents parameter $\mu$ and $LS$ is:

$$\mu_S = \alpha(\mu - 1) + 1 = (1 - \mu_S LS)(\mu - 1) + 1,$$

and so, solving for the markup $\mu_S$:

$$\mu_S = \frac{\mu}{\mu LS + (1 - LS)}. $$

However, this approach implicitly assumes that $1 - \alpha$, the Cobb-Douglas exponent for labor in the production function, is varying over time, at least to the extent that the labor share varies. Specifically, our procedure also implies that $1 - \alpha = \mu LS / (\mu LS + (1 - LS))$. The top panel of Appendix Figure 35 shows the implied value for $1 - \alpha$ in our baseline exercise. The mean is approximately 0.72. Moreover, the implied value declines from 0.74 to 0.70 during the 2000’s, along with the decline in $LS$.

An alternative approach is to fix the Cobb-Douglas labor exponent. In that case, we do not require data on the labor share to obtain the valued-added markup $\tilde{\mu}$ implied by our estimate of the rents parameter $\mu$; it can simply be obtained from $\tilde{\mu} = \alpha(\mu - 1) + 1$. The share of rents in value added, $s = 1 - 1/\mu$ obtained with this approach, reported in the middle panel of Appendix Figure 35, is very close to the share of rents of value added obtained in our baseline approach.

Additionally, this approach produces an implied labor share that is given by $LS = (1 - \alpha)/\tilde{\mu}$. The bottom panel of Appendix Figure 35 reports the path of this implied labor share, and compares it to the data. The magnitude of the decline in the implied labor is similar to the data, but the timing is somewhat different, because the rents parameter $\mu$ starts rising in the mid-80’s, along with the rise in the investment gap, whereas the labor share only starts declining in the late 2000’s.
4.3 No intangibles or no rents

4.3.1 No intangibles

When the firm has no intangibles, the expression for the investment gap collapses to:

\[ Q_{1,t} - q_{1,t} = (\mu - 1) \sum_{k \geq 1} E_t [M_{t,t+k} \Pi_{1,t+k}(1 + g_{1,t+1,t+k})] \]  

(56)

In other words, the investment gap is exactly the net present of future rents generated by physical capital \( K_{1,t} \), as in Lindenberg and Ross (1981). In this case, the decomposition of the investment gap in balanced growth is:

\[ Q_1 - q_1 = (\mu - 1) \frac{R_1}{r - g}. \]

The variables entering the expression for the present value of rents can be constructed using the same methodology as described in Section 3.1, setting the ratio of intangible to physical capital to \( S = 0 \) and the intangible investment rate to \( \nu_2 = 0 \).

Since by assumption, all of the investment gap is now accounted for by rents, we do not report its decomposition in this case. Instead, in Appendix Figure 12, we report the implied pure rents, expressed as a fraction of total value added, obtained using this approach. The top panel reports the estimate of this share obtained when assuming that firms do not use any intangibles in production. Ignoring intangibles, by 2015, rents account for about 14% of value added; by contrast, in our baseline estimate with intangibles measured as R&D (also reported in Figure 6), rents account for only about 8% of total value added.

The bottom panel of Figure 12 repeats this exercise, for the same sample of Compustat non-financial firms studied in the first part of Section 4. In this sample, assuming firms have no intangible capital leads to an estimate of pure rents of 14% in 2015; including R&D capital lowers this estimate of 10%. Additionally, including a fraction of capitalized SG&A expenditures as a measure of organization capital further lowers estimates of pure rents to approximately 6%.

In this case (as also for the NFCB sector in the late 1940s), estimates of pure rents can be negative in the early 1980s. This reflects the combination of two effects: our estimates of \( Q \) are strictly lower than 1 for a few years around 1980; and, as mentioned in Section 4, estimates of the organization capital stock, while overall elevated, have relatively little trend upward in the Compustat sample (by contrast with R&D capital).

4.3.2 No rents

When the firm earns no rents \( (\mu = 0) \), the decomposition of the investment gap collapses to:

\[ Q_{1,t} - q_{1,t} = q_{2,t} S_t, \quad S_t = \frac{K_{2,t}}{K_{1,t}}. \]

Along the balanced growth path, the same expression holds, without the time subscripts. One can invert this relationship in order to recover a value for \( S \), the ratio of intangible to physical capital:

\[ S = \frac{Q_1 - q_1}{q_2} = \frac{Q_1 - (1 + \gamma_1 g)}{(1 + \gamma_2 g)}, \]
where $g$ is the trend growth rate of the capital stock. Effectively, this amounts to backing out an implied value of the intangible capital stock from observed values of $Q_1$, which is possible when there are no rents. This is the approach followed by Hall (2001).

Additionally, given a value for $S$, the same relationship as in Section 3.1 holds for $r - g$:

$$r - g = \frac{ROA_1 - (\iota_1 + S\iota_2)}{Q_1} - \frac{\gamma_1 + \gamma_2 S}{Q_1}g^2$$

$$= \frac{ROA_1 - (\iota_1 + S\iota_2)}{Q_1} - \frac{Q_1 - (1 + S)}{Q_1}g$$

where, to go from the first to the second line, we used the expression for the investment gap along the balanced growth path when $\mu = 1$. From this expression, one can then obtain values of user costs, $R_1$ and $R_2$.

The implied Cobb-Douglas share of intangibles in production is then given by

$$\eta = 1 - \frac{1}{1 + \frac{R_2}{R_1}S}.$$  

Figure 8 reports implied estimates for $S$ and $\eta$ when we assume that $\mu = 1$, that is, there are no rents. The top two panels provide results from the NFCB sector as a whole, and the bottom two panels provide results for Compustat non-financial firms.

The results show that assuming away rents will lead to high estimates of the importance of intangible capital in production, relative to physical capital. For instance, among Compustat firms, assuming no rents leads to an estimate of the ratio $S$ of intangible to physical capital of approximately 1.2 in 2015. By contrast, in Compustat data, the magnitude of $S$ is approximately 0.4 when including both R&D and organization capital (as indicated by the bottom right panel of Figure 8). We note, however, that values of $S$ close to 1 seem plausible for some sectors, such as the Healthcare sector and the Consumer sector (for the latter, when organization capital is included).

An additional drawback from this approach is that the implied time-series $S$ exhibit periods of large decline (in the 1970s, and after the burst of the dot-com bubble). This reduction in the stock intangible capital (relative to the physical capital) is difficult to reconcile with the fact that empirical measures of $S$ have trended upward consistently throughout the post-war period.

### 4.4 Separating R&D from SG&A capital

**Methodology** With more than two types of capital, the decomposition of the physical investment gap along the balanced growth path can be written as:

$$Q_1 - q_1 = \sum_{m \geq 2} S_m q_m + \frac{(\mu - 1)R_1}{r - g} + \sum_{m \geq 2} \frac{(\mu - 1)R_m S_m}{r - g}.$$ 

Note that when there are strictly positive adjustment costs, $\gamma_1, \gamma_2 > 0$, this approach requires taking a stance on the gross intangible investment rate $\iota_2$ and on the growth rate of the capital stock $g$. For $g$, in Figure 8, we use the growth rate of the physical capital stock. There is some tension between assuming that $\iota_2$, the intangible investment rate, is observed, and assuming that $S_t = K_{2,t}/K_{1,t}$ is unobserved. In Figure 8, we assume that $\iota_2$ is given by the R&D investment rate, but results are not materially different if it is assumed to be equal to 0 or, in the case of Compustat non-financial firms, to the total investment rate in R&D and organization capital.
Additionally, we have:

\[(\mu - 1) \left( R_1 + \sum_{m \geq 2} S_m R_m \right) = ROA_1 - \left( R_1 + \sum_{m \geq 2} S_m R_m \right). \tag{57} \]

Multiplying the expression for the investment gap by \( r - g \), using the expression above, along with the fact that, neglecting terms of order \( o(g) \), for each \( n = 1, \ldots, N \):

\[ R_n = r - g + \iota_n + \gamma_n rg, \]
\[ q_n = 1 + \gamma_n g, \]

we arrive at:

\[ (r - g)Q_1 = ROA_1 - \left( R_1 + \sum_{m \geq 2} S_m R_m \right) - \left( \gamma_1 + \sum_{m > 2} \gamma_m S_m \right) g^2, \]

or:

\[ (r - g) = \frac{ROA_1 - \left( R_1 + \sum_{m \geq 2} S_m R_m \right)}{Q_1} - \left( \frac{\gamma_1 + \sum_{m > 2} \gamma_m S_m}{Q_1} \right) g^2. \]

Given values for investment rates \( \{ \iota_n \}_{n=1}^N \), relative capital stocks \( \{ S_m \}_{m=2}^N \), adjustment costs \( \{ \gamma_n \}_{n=1}^N \), average \( Q_1 \), the return to physical capital \( ROA_1 \), and the growth rate of the total capital stock \( g \), the right-hand side of this expression can be constructed in the data, implying a particular value for \( r - g \). The values of user costs can then be obtained from \( R_n = r - g + \iota_n + \gamma_n rg \), and the value of \( \mu \) is then given by Equation (57). Along with the fact that \( q_n = 1 + \gamma_n g \), this is sufficient to construct all the elements in the decomposition of the investment gap.

**Results** The bottom panel of Appendix Figure 6 reports the results of the generalized decomposition of the physical investment gap when \( R&D \) capital and organization capital \( SGA & A \) are treated as different capital inputs:

\[ Q_1 - q_1 = S_2 q_2 + S_3 q_3 + \frac{(\mu - 1) R_1}{r - g} + \frac{(\mu - 1) R_2 S_2}{r - g} + \frac{(\mu - 1) R_3 S_3}{r - g}, \]

where the number 2 indexes R&D capital, and the number 3 indexes organization capital. For the adjustment cost to R&D capital, we use a value of \( \gamma_2 = 12 \), as in the main text, while for the adjustment cost to organization capital, we use the estimate of \( \gamma_3 = 3.2 \) reported in Belo et al. (2019) (Table 3) for the parameter governing the convexity of adjustment costs to brand capital. This decomposition is quantitatively very similar to the decomposition in the middle panel (which takes the simple sum of organization and R&D capital), in that it attributes approximately 40% of the physical investment gap after 2000 to the direct effect of the R&D and intangible capital stocks, and approximately 30% to the rents they generate.

An additional insight from this graph is that rents account for a bigger fraction of the part of the investment gap created by the R&D capital stock than they do for the organization capital stock. This result is driven by the fact that user costs for R&D capital implied by the decomposition, \( R_2 \), are higher than those for organization capital, \( R_3 \). In turn, this difference in user costs can be explained by the fact that investment rates for R&D capital imputed using Compustat data are on average 25% per year after 2000, whereas investment rates for organization capital are only 15%.
Recall that our decomposition implicitly infers depreciation rates from observed gross investment rates, so that high gross investment rates imply high rates of depreciation and hence high user costs. The magnitude of both R&D investment rates and the implied depreciation rates (in the order of 20%) are consistent with the evidence in Li and Hall (2020). To our knowledge, there are no direct sources for depreciation rates on organization capital, though the literature typically uses a depreciation rate of 20% (Lev and Radhakrishnan, 2005; Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Falato et al., 2020). Thus, our decomposition produces somewhat lower implicit depreciation rates for organization capital than those assumed in the literature. We note, however, that both Peters and Taylor (2017) and Ewens et al. (2020) show that estimates of the size of the organization capital stock obtained from Compustat data are not very sensitive to the choice of depreciation rate.

4.5 GMM estimation on split samples

4.5.1 Moment conditions

We first derive moment conditions that can be used for estimation. We use a model with uncertainty that admits a closed-form solution. Specifically, as in Lemma 2, we assume that:

$$
\Pi_t = A_t^{1-\frac{1}{\rho}} K_t^{\frac{1}{\rho}},
$$

$$
K_t = \left( \sum_{n=1}^{N} \eta_n K_{n,t}^{\rho} \right)^{\frac{1}{\rho}}, \quad \rho \leq 1, \quad \sum_{n=1}^{N} \eta_n = 1.
$$

In order to obtained closed-form solutions, we make the following additional assumptions.

**Assumption 7.** Assume that:

1. Adjustment costs are linear: $\Phi_{n,t}(1+g) = g + \delta_n, \quad \forall n = 1, ..., N, \forall t.$

2. The discount rate is constant: $M_{t,t+1} = (1+r)^{-1}.$

3. $\{A_t\}_{t \geq 0}$ satisfies $A_{t+1} = (1+g_t)A_t, \quad g_t \sim F(\cdot)$ i.i.d., $E(g_t) = \bar{g}.$

**Lemma 7.** If Assumption 7 holds, then the solution to the model satisfies:

$$
S_t = \frac{\eta}{1-\eta} \left( \frac{r + \delta_1}{r + \delta_2} \right)^{\frac{1}{1-\rho}}
$$

$$
\iota_{1,t} = g_t + \delta_1
$$

$$
\iota_{2,t} = g_t + \delta_2
$$

$$
gK_t = g_t
$$

$$
ROA_{1,t} = \mu ((r + \delta_1) + (r + \delta_2)S_t)
$$

$$
Q_{1,t} = 1 + S_t + \frac{(\mu - 1)}{r - \bar{g}} (r + \delta_1) + \frac{(\mu - 1)}{r - \bar{g}} (r + \delta_2) S_t
$$
where:
\[ g_{K,t} \equiv \frac{K_{1,t+1} + K_{2,t+1}}{K_{1,t} + K_{2,t}} - 1. \]

This result is a particular case of the risky balanced growth model described in Section 2.5 and Appendix 2.2.1, when growth in fundamentals is i.i.d. (which corresponds to the case \( \lambda = 1 \)). Note that the first five expressions for key ratios do not depend on the i.i.d. growth assumption (that assumption is only used in the computation of \( Q_{1,t} \)), though they do depend on the assumption that \( A_{t+1} \) is in the information set of time \( t \). Additionally, the ratios \( S_t, ROA_{1,t} \) and \( Q_{1,t} \) implied by the model are constant over time; random shocks \( g_t \) will only appear in measures of investment \( g_{K,t}, \nu_{1,t} \) and \( \nu_{2,t} \). We write the closed-form solution of the model as moment conditions:

\[
0 = E \left[ S_t - \eta \left( \frac{r + \delta_1}{r + \delta_2} \right)^{1/\rho} \right] \tag{65}
\]

\[
0 = E \left[ \nu_{1,t} - (\bar{g} + \delta_1) \right] \tag{66}
\]

\[
0 = E \left[ \nu_{2,t} - (\bar{g} + \delta_2) \right] \tag{67}
\]

\[
0 = E \left[ g_{K,t} - \bar{g} \right] \tag{68}
\]

\[
0 = E \left[ ROA_{1,t} - \mu \left\{ (r + \delta_1) + (r + \delta_2)S_t \right\} \right] \tag{69}
\]

\[
0 = E \left[ Q_{1,t} - \left\{ 1 + S_t + \frac{\mu - 1}{r - \bar{g}} (r + \delta_1) + \frac{\mu - 1}{r - \bar{g}} (r + \delta_2) S_t \right\} \right] \tag{70}
\]

### 4.5.2 Estimation approach

We fix the elasticity of substitution between intangible and physical capital, \( \rho \), to \( \rho = 0 \), so that the two types of capital are Cobb-Douglas substitutes. This follows our approach in the main text. We then estimate the six structural parameters \( \{ \bar{g}, \delta_1, \delta_2, \mu, \eta, r \} \) using the six moment conditions above.

Our estimation method is standard: we use two-step efficient GMM (Hansen, 1982), with the identity matrix as the first-step weighting matrix. HAC standard errors are computed using a Bartlett kernel with four lags.

We report two additional sets of results from this estimation. First, we compute point estimates and standard errors for four “implied moments”: user costs \( R_1 \) and \( R_2 \), the markup over value added \( \mu_{VA} \), and the share of rents as a fraction of value added \( s_{VA} \). We obtain estimates and standard errors for these implied moments by stacking the four moment conditions to the GMM system:

\[
0 = E \left[ R_1 - (r + \delta_1) \right] \tag{71}
\]

\[
0 = E \left[ R_2 - (r + \delta_2) \right] \tag{72}
\]

\[
0 = E \left[ s_{VA} - (1 - s_{L,t}) \left( 1 - \frac{1}{\mu} \right) \right] \tag{73}
\]
\[ 0 = \mathbb{E} \left[ \mu_{VA} - \frac{1}{1 - s_{VA}} \right] \tag{74} \]

where \( s_{L,t} \) is the time-series for labor as a share of value added.\(^\text{32}\) Second, we also compute the difference between point estimates across subsamples, and test for whether it is significantly different from zero. We perform this test by stacking the moment conditions for the two subsamples (with a set of structural parameter and implied moments for each subsample), and interacting the moment conditions with an indicator for each subsample. The p-values reported are for the two-sided test against the null of equality of a given structural parameter or implied moment across subsamples.

### 4.5.3 Results

The results for this estimation approach are reported in Table 21. The table contains estimation results for the NFCB sector as a whole (the first three columns), and for the sample of Compustat non-financial firms (columns four through nine), separating the case where intangibles are measured using R\&D from the case where they are measured using R\&D plus organization capital. Additionally, we focus on data form 1985-2017, since this is the period of primary interest for the paper, and we report results on subsamples split around the year 2000, following again our analysis in the main text.

Before discussing the results, it is worth noting that in the simple model with i.i.d. growth, GMM estimation leads to exactly the same point estimates as one would obtain by replacing the various data series in the moment conditions above by their sample means, and then inverting the moment conditions. This is very similar to what we do in our baseline approach. The reason for the equivalence between the GMM estimation and our baseline approach is that, with the exception of the solution for investment rates, the representation of the model with i.i.d. growth and adjustment costs is exactly the same. (Moreover, investment rates are linear functions of the only source of random disturbances, \( g_t \).) Thus, qualitatively, we should expect to find the same patterns as were obtained in the simpler analysis of the baseline model.

Quantitatively, there two reasons why the GMM results for the simple model with i.i.d. growth might differ from the baseline results in the paper, and in particular the results of Table 1 of the main paper. First, our baseline approach averages moments over seven-year rolling windows. By contrast, GMM estimation of the simple model uses averages over the 16-year and 17-year windows that make up the two subsamples from 1985-2017. Second, the results of Table 1 rely on the model with adjustment costs, whereas the simple model with i.i.d. growth does not allow for adjustment costs.

Given this caveat, the main interest of estimating the simple model via GMM is that it allows to assess the statistical significance of the structural changes that we documented with our baseline approach. The results of Table 21 find the same two simultaneous structural changes as in our baseline approach: an increase in the share of intangibles in production, and an increase in rents. The increase in the share of intangibles in production between the 1985-2000 period and 2001-2017 is significant at the 1% level in all three data sources, and it is quantitatively similar to the one obtained in our baseline approach.

\(^{32}\) Following our discussion in Section 2.5 and Appendix 2.3, the reduced-form moment \( \mu_{VA} \) is the value-added markup under the assumption of constant returns in production.
The statistical significance of the increase in rents is somewhat more muddled. In the NFCB data, all measures of rents (the curvature parameter $\mu$, the rent share of value added $s_{VA}$, and the markup over value added $\mu_{VA}$) are statistically significant. However, in the Compustat sample, the increase in the curvature parameter $\mu$ is only significant at the 5% level when measuring intangibles as R&D, and it becomes insignificant when intangibles are measured as the sum of R&D and organization capital. The latter finding is consistent with our baseline results, which suggested that adding organization capital to the measure of intangibles tends to weaken the upward trend in rents. Likewise, in the Compustat data, the increase in $\mu_{VA}$ and $s_{VA}$ is only significant at the 5% level when measuring intangibles as R&D, and at the 10% level when also including organization capital. Additionally, and consistent with the results in the main text, there is a decline in estimated user costs, but it is only significant in the NFCB sector.

The main issue with these estimation results is that the point estimates of the implied discount rate $r$ are substantially higher, and more stable, for the Compustat sample than they are for the NFCB sector (in the order of 8%, instead of 5% for the NFCB sector). (Note that this was also true in our baseline approach, though we had not reported the values of the discount rate in isolation in Table 1, so that this difference between the NFCB data and the Compustat data was not clearly visible.) The empirical force driving this result is the fact that measured returns on assets are substantially higher in Compustat than they are in the NFCB sector. Recall that, in the model, the discount rate $r$ satisfies $r - g = (ROA_1 - (\eta_1 + \eta_2 S))/Q_1$ — it is the wedge between flow profits in excess of investment costs, over valuations. Empirically, values of returns on assets $ROA_1$ are substantially higher in the Compustat data (as also reported in Table 1 in the main text), in a manner that is not fully offset by either higher investment rates, or a higher values of $Q_1$ among publicly traded non-financial firms.

4.6 Alternative identification strategies

Along the balanced growth path, the general decomposition of the investment gap is:

$$Q_1 - q_1 = q_2 S + \frac{(\mu - 1) R_1}{r - g} + \frac{(\mu - 1) R_2}{r - g} \times S. \quad (75)$$

In our baseline approach, we directly measure $S$. We primarily infer the Gordon growth term $r - g$ from the value of $Q_1$. Combining estimates of this term with investment rates, we then compute the user costs $R_1$ and $R_2$. Finally, we infer $\mu$ by combining the estimates of user costs with a measure of the average return on physical assets, $ROA_1$.

Here, we describe in detail alternative approaches that could be used to construct our decomposition. All these alternative approaches rely on constructing a measure of the average cost of capital, $r$, in order to construct the Gordon growth term $r - g$, instead of using $Q_1$, as we do in our baseline approach. This allows the value of $Q_1$ to be used to infer some other underlying structural parameter, such as intangible intensity or rents. In order to limit the size of the results, we only report them for the NFCB sector; they are qualitatively similar for the Compustat sample.
4.6.1 Alternative approach 1: average cost of capital

**Description** Let \(\{D_{E,t}\}\) and \(\{D_{B,t}\}\) be distributions to shareholders and debtholders. These distributions must satisfy \(D_t = D_{E,t} + D_{B,t}\), where \(D_t\) are total distribution to firm owners. Moreover, let \(E_t^e\) and \(M_t^e\) be the ex-distribution values of equity and debt, which must satisfy \(V_t^e = E_t^e + B_t^e\), where \(V_t^e\) is total firm value. We make the following assumptions about \(\{D_{E,t}, D_{B,t}, E_t^e, B_t^e\}\).

**Assumption 8** (Weighted average cost of capital). There exists two discount factors \(\{M_{E,t,t+1}\}\) and \(\{M_{B,t,t+1}\}\), such that:

\[
E_t^e = E_t \left[ M_{E,t,t+1} (D_{E,t+1} + E_{t+1}^e) \right], \quad B_t^e = E_t \left[ M_{B,t,t+1} (D_{B,t+1} + B_{t+1}^e) \right].
\]

Moreover, along the balanced growth path, (a) there exist \(r_E\) and \(r_B\) such that:

\[
\forall t, \quad M_{E,t,t+1}^{-1} = 1 + r_E, \quad M_{B,t,t+1}^{-1} = 1 + r_B,
\]

and (b) the ratio of the market value of debt to the market value of equity is constant:

\[
l_t \equiv \frac{E_t}{B_t} = l.
\]

In this case, along the balanced growth path, \(r\), \(r_E\) and \(r_B\) must satisfy:

\[
r = \frac{l}{1+l} r_B + \frac{1}{1+l} r_E.
\]

In other words, under Assumption 8, the firm-wide discount rate \(r\) must equal the weighted average of the shareholder and debtholder discount rates \(r_E\) and \(r_B\). This suggests an alternative avenue to construct the decomposition, which is to use estimates of \(r_B\) and \(r_E\) in order to construct the Gordon growth term \(r - g\) from the relationship:

\[
r - g = \frac{l}{1+l} (r_B - g) + \frac{1}{1+l} (r_E - g).
\]

This is similar to the approach followed by Barkai (2020) and in Karabarbounis and Neiman (2019) in order to estimate of the pure profit share.

**Implementation** In order to implement this approach, we need estimates of \(r_B\) and \(r_E\). For the former, we use a weighted average of the different interest rates which were an input into our computation of the market value of total debt liabilities of non-financial corporations, and are described in Appendix 3.2. The weights are the relative market values of each types of debt liability. We then pre-multiply this interest rate by ones minus the statutory top corporate income tax rate. We denote the resulting time series by \(r_b^n\). For the estimate of \(r_E\), we use the fact that along the balanced growth path, the PD ratio is constant and given by:

\[
PD_t \equiv \frac{E_{t-1}^e}{D_{E,t}} = \frac{1}{r_E - g}.
\]

In order to estimate the PD ratio, we use the fact that along the balanced growth path:

\[
PD_t = \frac{1}{R_{E,t-1,t}^e - R_{E,t-1,t}^e}, \quad R_{E,t-1,t}^e = \frac{D_{E,t} + E_{t}^e}{E_{t-1}^e}, \quad R_{E,t-1,t}^e = \frac{E_{t}^e}{E_{t-1}^e}.
\]
Our sources for $R_{E,t-1,t}^c$, $R_{E,t-1,t}^c$ and the corporate income tax rate are described in Appendix 3.6. Finally, for leverage $l$, we use the ratio of the market value of debt liabilities to the market value of equity, both of which we computed in order to construct the total value of the firm, $V_t^e$. We then obtain the Gordon growth term as:

$$r - g = \frac{l}{1+l}(r^B_n - g^n) + \frac{1}{1+l}PD^{-1}.$$ 

where $g^n$ is the nominal growth rate of the total capital stock.\(^{33}\) Appendix Figure 15 reports the resulting time series for the PD ratio $PD_t$.\(^{34}\) The top panel of Appendix Figure 16 reports the resulting cost of equity, the cost of debt, and the average cost of capital.\(^{35}\)

**Results** Figure 17 reports the results obtained in this alternative approach. The top panel of the figure shows the implied investment gap, and its decomposition. Generally, this approach implies a larger estimated investment gap, particularly so in the first half of the sample, prior to 1980. Nevertheless, targeting the PD ratio still leads to the same key insights as the baseline approach. In particular, by 2015, the rents attributable to intangibles account for 29% of the total investment gap in the PD ratio approach (compared to 25% in the baseline approach), while the two rents-related terms together account for 33% of the total investment gap (versus 36% in the baseline approach).

This approach leads to a larger overall investment gap because the values of Tobin’s $Q$ for physical capital which it implies are far superior to those observed in the data, as reported in the bottom left panel of Figure 17. Fundamentally, this is because the average cost of capital approach leads to a lower estimate of the firm discount rate $r$ than our baseline approach. The top panel of Figure 2 reports the estimates of (nominal) discount rates we obtain in the average cost of capital approach and in our baseline approach. The former is generally lower than the latter. The two are closest together in 1985. After 1985, the discount rate in the average cost of capital approach declines more than in our baseline approach.

Because lower discount rates imply lower user costs, the average cost of capital approach also leads to higher estimates of the pure profit share, as reported in the bottom right panel of Figure 2. The ratio is both higher on average, and increasing faster, in our approach, relative to the baseline approach. After 1985, rents in the average cost of capital approach increase by about 9.0 p.p., versus 6.2 p.p. in our baseline approach.

Why the two approaches lead to a different implied discount rate is a difficult question. As discussed in Section 3.3, one possible interpretation is in terms of implicit risk premia on equity. To illustrate this, the bottom panel of Figure 17 constructs a measure of the risk premium implicit in the cost of equity implied by our baseline approach and by the average cost of capital approach. We define this implicit risk

---

\(^{33}\) We use the nominal instead of the real growth rate because the time series for the cost of debt $r^B_n$ which we construct is a nominal interest rate.

\(^{34}\) The figure also reports the time series for the “leverage-adjusted” PD ratio, defined as $PD = (r - g)^{-1}$, and which, in the balanced growth path of the model, satisfies $PD = V^e_{t-1}/D_t$.

\(^{35}\) All are expressed in nominal terms; for the cost of debt, this is the series $r^B_t$ which we directly construct from the data, and for the cost of equity and the average cost of capital, we report $r^E_t \equiv (r - g) + g^n$ and $r^n = (r - g) + g^n$. 

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premium as:

\[ RP \equiv (1 + l) \left( r^n - \frac{l}{1 + l} r^n_B \right) - r^n_f = r^n_E - r^n_f, \]

where \( r^n = (r - g) + g^n \) is the nominal, firm-wide discount rate, \( l \) is leverage, \( r^n_B \) is the (nominal, after-tax) cost of debt, and \( r^n_f \) is a measure of the risk-free rate. We use the average annual return on one-month Treasury bills to measure the risk-free rate; the source is reported in Appendix 3.6.

The resulting time series show that the average cost of capital approach leads to a lower implicit equity risk premium than our approach. The two implicit risk premia are closest in 1985; after that year, the implicit risk premium in our approach rises somewhat more than in the average cost of capital approach.

There are also measurement issues that may contribute to the differences between our baseline approach and the average cost of capital approach. The sample underlying our measure of \( Q_1 \) (the NFCB sector) and the sample underlying our measure of the PD ratio (the S&P 500). Additionally, distributions to equityholders may not be accurately measured. Our measure is based on cash distributions, and excludes share repurchases, which became more common after the early 2000’s. Generally, it is difficult to accurately match the rate of distributions to shareholders implicit in the computation of the PD ratio, and the distributions to shareholders measured in Flow of Funds data; this may contribute to further differences between the two approaches.

**A different measure of \( r_E \)** We also report the results of this approach when using a similar method to measure the cost of equity as Barkai (2020). Specifically, we assume that the (nominal) cost of equity is:

\[ r^n_E = r^n_f + RP, \]

where \( RP \) is a constant risk premium, and \( r^n_f \) is the time-series for the risk-free rate described above. We then compute the Gordon growth term as:

\[ r - g = \frac{l}{1 + l} (r^n_B - g^n) + \frac{1}{1 + l} (r^n_E - g^n). \]

For the constant risk premium, we use a value of \( RP = 6.5\% \), in line with the long-run average estimates of equity risk premia constructed by Campbell and Thompson (2008) and extended by Martin (2017).

The bottom panel of Figure 16 reports the time series for the average cost of capital used in this approach, and Figure 2 compares it with the cost of capital in other approaches. The most notable difference with the other approaches is that the cost of capital according to this approach is substantially lower in the 1945-1985 period. In fact, in a number of years in this period, the implied discount rate is close to, or below, the growth rate of the capital stock. In turn, this implies implausibly high values of Tobin’s \( Q_1 \) (in excess of 20, in particular in the early part of the sample). Therefore, we only report the results related to the investment gap obtained using this approach after 1985.

These results are reported in Figure 18. After 1985, the results share number of common features with Figure 17, where the PD ratio is used instead of the risk-free rate. The implied values for \( Q_1 \) are substantially higher than in the data. Moreover, the increase in total rents as a fraction of value added is larger than in our baseline approach. The latter effect is more muted than in alternative approach 1; this
is because, as indicated in Figure 17, the discount rate in this approach is somewhat higher than when using the PD ratio.

Finally, note that in this approach, the implicit risk premium is constant. By contrast, in both our baseline approach, and the average cost of capital approach using the PD ratio, the implicit equity risk premium is rising after 1985. Additionally, in our baseline approach, the implicit equity risk premium is above 6.5% after 2003. These differences help explain why estimates of the pure profit share are lower in our baseline approach than in either of the two average cost of capital approaches.

4.6.2 Alternative approach 2: inferring intangibles from the investment gap

Given a measure of the Gordon growth term \( r - g \) that is independent from \( Q_1 \), the decomposition above can also be used as a way to derive an implicit stock of intangibles, as opposed to measuring it in the data. This amounts to computing the ratio of intangible to physical capital such that the model matches both measured \( r - g \) and the measured investment gap \( Q_1 - q_1 \). Straightforward derivations show that this ratio is given by:

\[
S_{\text{implied}} = \frac{(r - g)(Q_1 - q_1) - (ROA_1 - R_1)}{(r - g)q_2 - R_2}.
\] (76)

This expression can be thought of as a generalization of the approach of Hall (2001), who derives the ratio of intangible to physical capital consistent with stock market values and measures of the physical capital stock (and therefore of \( Q_1 \)). The expression is a generalization in the sense that it allows for rents. In the Hall (2001) case of no rents (\( \mu = 1 \)), from the decomposition of the investment gap, the value of \( S \) is given by:

\[
S_{\text{Hall}} = \frac{Q_1 - q_1}{q_2}.
\]

When \( \mu \) can be different from 1, given a value of \( S \), the implied value for \( \mu \) is\(^{36}\):

\[
\mu = \frac{ROA_1}{R_1 + S_{\text{implied}}R_2}.
\]

Figure 19 reports results from the decomposition constructed using this approach, rather than our baseline approach. The top panel shows the investment gap decomposition, and the bottom two panels report, respectively, the time series for \( S_{\text{implied}} \) (comparing it with the time series for \( S \) in the data, which is the one that our baseline approach matches); and the time series for the value of rents as a fraction of total value added implied by the model.

One noteworthy finding from Figure 19 is that, after 2000, \( S_{\text{implied}} \) grows substantially faster than what is measured by the BEA Fixed Assets tables. By 2015, the implied intangible stock is about twice as large as its BEA counterpart. As a result of this rapid growth, the implied increase in rents is lower than in our baseline approach. Overall, as indicated by the top panel, this approach attributes a bigger share of the overall investment gap to intangibles than our baseline approach (about two-thirds, versus one-third in our baseline approach).

\(^{36}\) Equation (76) can be rewritten as \((r - g)(Q_1 - q_1 - q_2S_{\text{implied}}) = ROA_1 - R_1 - S_{\text{implied}}R_2\). Therefore, \( S_{\text{implied}} = S_{\text{Hall}} \), if and only if, \( ROA_1 = R_1 + S_{\text{implied}}R_2 \), which is the same as \( \mu = 1 \).
A potential problem with this approach, however, is that it also implies that there must have been a very large stock of intangible capital, relative to physical capital, in the 1950-1970 period (and moreover, that this stock turned negative for a few years around 1980). Mechanically, this is because the stock of intangibles account for movements in the investment gap that cannot be fully accounted for by increases in the PD ratio (and therefore declines in \( r - g \)). In other words, through the lens of the model, the period 1950-1970 was one where discount rates (as implied by PD ratios) were low, but not enough to explain the high investment gap, so that intangibles must have been high.

Another drawback of this approach is that it creates a mechanical negative correlation between the level of discount rates (implied by the PD ratio), and the stock of intangibles. Hall (2001) also contends with this issue, and finds the same declining intangible capital stock in the late 1970s. This “destruction” of intangible capital might be difficult to reconcile with the fact that direct measures of the capital stock instead suggest that the ratio of intangible to physical capital has been continuously growing over the post-war period.

4.6.3 Alternative approach 3: inferring rents from the investment gap

Another possible approach is to infer the rents parameter, \( \mu \), from the investment gap. In this approach, the rents parameter would be obtained from:

\[
\mu = 1 + \frac{r - g}{R_1 + R_2 S} (Q_1 - q_1 - q_2 S). \tag{77}
\]

This approach will lead to positive estimates of rents whenever average \( Q_1 \) is higher than \( q_1 + q_2 S \), which would be the value of \( Q_1 \) in a model without rents. Moreover, relative to the baseline approach, the model will not necessarily match measured returns to physical capital \( ROA_1 \). The model-implied value of returns to physical capital is then:

\[
ROA_{1\text{implied}} = R_1 + R_2 S + (r - g)(Q_1 - q_1 - q_2 S). \tag{78}
\]

The no-rents case again obtains when \( Q_1 = q_1 + q_2 S \), which implies that the return to physical capital is equated to the weighted average user cost \( R_1 + R_2 S \). As in the other alternative approach, this approach requires using measures of \( r - g \) not derived from Tobin’s \( Q_1 \); we use the average cost of capital measure, with the inverse PD ratio as a proxy for the cost of equity capital.

Figure 20 reports results in this case. The decomposition of the investment gap (top panel) is qualitatively and quantitatively close to our baseline results. As indicated by the bottom right panel of Figure 20, the size of rents is somewhat smaller in this approach alternative approach, particularly after 2000. The bottom right panel of Figure 20 reports the implied returns to physical capital, \( ROA_1 \) in this approach, and compares them with their measures in the data, which our baseline approach matches by construction. Returns to capital implied by this approach are overall lower than in the data, consistent with the fact that this approach leads to somewhat lower rents. Both in the data and in the model-implied series, there is an increase in average returns to physical capital after 1980, though it is less marked in this approach than in the data. Thus, overall, this approach leads to somewhat smaller rents than our baseline, both in
levels, and in terms of their overall increase since 1985.

4.7 Markups and returns to scale

4.7.1 Methodology

Given the partial identification result highlighted by Lemma 5, in what follows, we compute estimates of the reduced-form parameter \( \chi = \mu_S/\zeta \), and report implied values of the markup \( \mu_S \) of price over the marginal cost of sales under different assumptions about decreasing returns to scale.

The results of Table 5 can be used to identify the reduced-form rents parameter \( \chi \) and the Cobb-Douglas elasticity \( \alpha \) separately. We use the following approach, which, as explained below, is consistent with our analysis of the balanced growth model.

**Lemma 8.** The values of \( \alpha \) and \( \chi \) can be derived from data on the ratio of operating surplus to sales, and on the ratio of capital costs to operating surplus:

\[
x_{\Pi} \equiv \frac{\Pi_t}{S_t}, \quad \nu_K \equiv \frac{R_{1,t}K_{1,t} + R_{2,t}K_{2,t}}{\Pi_t},
\]

as follows:

\[
\chi = \frac{1}{\nu_K + (1-\nu_K)(1-x_{\Pi})} \geq 1,
\]

\[
\alpha = 1 - \frac{1-x_{\Pi}}{\nu_K + (1-\nu_K)(1-x_{\Pi})} \leq 1.
\] (79)

The proof of this result follows from the expressions reported in Table 5. Of course, there are other combinations of the ratios reported in Table 5 that could also be used to identify separately \( \chi \) and \( \alpha \). The advantage of this particular identification approach is twofold. First, given an estimate of capital costs, it can be implemented using firm accounting data, since it only requires observing total sales and total operating surplus. (In particular, no separate measure of labor costs is needed.) Second, this approach encompasses the approach used to measure the rents parameter \( \mu \) in the balanced-growth model of the main text. Indeed, there, the reduced-form rents parameter is obtained from the relationship:

\[
\frac{1}{\mu} = \frac{\alpha}{\chi - (1-\alpha)} = \frac{R_{1,t}K_{1,t} + R_{2,t}K_{2,t}}{\Pi_t} = \nu_K.
\]

Identification of \( \mu \), \( \alpha \) and \( \chi \) can therefore be thought of as follows. First, estimate \( \mu \), using a measure of \( \nu_K \), as we do in our empirical analysis of the balanced growth model in the main text. Second, estimate \( \alpha \), using the expression reported in Lemma 8. Third, obtain the value of \( \chi = \mu_S/\zeta \) using:

\[
\mu = 1 + \frac{\chi - 1}{\alpha} \iff \chi = \frac{\mu_S}{\zeta} = 1 + \alpha(\mu - 1),
\]
or equivalently, using the expression reported in Lemma 8.

4.7.2 Results

Figure 31 reports the results of this methodology, applied to the data from the entire Compustat Non-Financial sample, from 1974 to 2017. We focus on this data source because, to our knowledge, there is
no good source on total sales (as opposed to total profits or total operating surplus) for the non-financial corporate sector. The data we use is the same as in the analysis of Section 4 in the main text, except that we also make use of the ratio of total sales to gross operating surplus. Total sales is measured using Compustat variable sale.

The results of Figure 31 report the time series for the sales markup, which, following the previous discussion, is measured as:

$$\mu_S = (1 + \alpha(\mu - 1)) \zeta = (\nu_K + (1 - \nu_K)(1 - x_{\Pi}))^{-1} \zeta.$$  

We report the implied values of $\mu_S$ for three different values of the degree of returns to scale: decreasing returns ($\zeta = 0.95$); constant returns ($\zeta = 1.00$, as in our baseline analysis), and increasing returns ($\zeta = 1.05$).

In the case of decreasing returns ($\zeta = 0.95$), implied markups over sales range from negative (approximately 0.92-0.95) in the 1980s, to approximately 0 in the 2000s, when intangibles are measured either using R&D or using the sum of R&D and organization capital. Thus, a modest amount of decreasing returns in aggregate is sufficient to eliminate markups altogether. On the other hand, with increasing returns, implied markups are substantially larger, reaching approximately 1.1 in the 2000s. Note that, even so, these numbers are substantially smaller than those obtained in the literature that estimates markups using a production function approach, such as for instance De Loecker et al. (2020). (The sales markups we find here are approximately one-third of the median markup documented by these authors.)

Table 6 reports averages for the underlying values of $\mu$, $\chi$ and $\alpha$ obtained in two sub-samples, 1980-2000 and 2001-2015. That table also attempts to isolate the contribution of ”pure rents” (those due to markups). The rents due to markups, as a fraction of value added, are defined as:

$$s_{Re}^\mu \equiv (1 - s_L) \left( \frac{\mu_S - 1}{\alpha + \mu_S - 1} \right).$$

Recall that total rents as fraction of value added are given by $(1 - s_L) \left( \frac{\mu_S/\zeta - 1}{\alpha + \mu_S/\zeta - 1} \right)$. So, for any value of markups $\mu_S$, $s_{Re}^\mu$ gives the implied rents, as a fraction of value added, assuming constant returns. The numbers reported in Table 6 under ”Rents due to markups” are the values of $s_{Re}^\mu$, using the markups implied by different values of $\zeta$. As discussed above, with even relatively modest decreasing returns ($\zeta = 0.95$), the value of $\chi$ implies negative markups, so that ”pure rents” contribute negatively to total rents. With increasing returns, “pure rents” exceed total rents (because increasing returns implies negative “quasi-rents”). Even then, the size of rents attributable to markups is relatively small in comparison to the numbers implied by the estimate of De Loecker et al. (2020) (which are in the order of 40% of total value added).

The NIPA tables do not appear to contain information on domestic sales specifically for non-financial corporations; see https://apps.bea.gov/iTable/iTable.cfm?ReqID=13&step=1. The Flow of Funds reports a series titled Revenue from Sales of Goods and Services (FRED series BOGZ1FA106030005Q), but this series is identical to the gross value added for the non-financial corporate sector reported in NIPA table 1.14 (FRED series A455RC1Q027SBEA), indicating that the Flow of Funds series likely measures value added, not gross revenue, despite its name.

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4.8 Heterogeneous rents parameters

Methodology The decomposition of the investment gap in the case of heterogeneous rents parameter, balanced growth, and $N = 2$, can be implemented as follows. First, as in our baseline model, we can recover the value of $r - g$ through:

$$r - g = \frac{ROA_1 - \iota_1 - \iota_2 S}{Q_1} - \frac{\gamma_1 + \gamma_2 S}{Q_1} g^2,$$

where all the objects on the right-hand side can be obtained from the data. Third, we can measure user costs as:

$$R_n = r + \delta_n + \gamma_n rg,$$

$$= r - g + \iota_n + \gamma_n rg,$$

$$= r - g + \iota_n + \gamma_n g^2 + \gamma_n (r - g) g$$

$$= (r - g)q_n + \iota_n + \gamma_n g^2,$$

where all the objects on the right-hand side can now be measured in the data. Finally, note that the first-order conditions to the firm’s problem imply that:

$$\Pi_t = \mu_1 R_1 K_{1,t} + \mu_2 R_2 K_{2,t}.$$

Define the average markup as:

$$\bar{\mu} = \frac{R_1}{R_1 + SR_2} \mu_1 + \frac{SR_2}{R_1 + SR_2} \mu_2.$$

Note that the values for $R_1$, $R_2$ and $S$ that we derive from the data are not dependent upon whether the rents parameters $\mu_n$, $n = 1, 2$ are heterogeneous or not. The average markup $\bar{\mu}$ will therefore always be given by:

$$\bar{\mu} = \frac{R_1}{R_1 + SR_2} \mu_1 + \frac{SR_2}{R_1 + SR_2} \mu_2$$

$$= \frac{\Pi_t}{K_{1,t}}$$

$$= \frac{ROA_1}{R_1 + SR_2},$$

where the expression in the last line only depends on data objects, and does not depend on the values of $\mu_1$ and $\mu_2$. Note that this is the expression for the rents parameter obtained our baseline case, $\mu$. Thus, the average of the rents parameters across capital types, weighted by their user costs, will always be equal to the rents parameter that we obtained in our baseline decomposition. In other words, the total contribution of rents (generated by either intangibles or by physical capital) to the decomposition is the same whether one uses baseline model with a single rent parameter, or the heterogeneous rents model; heterogeneous values for $(\mu_1, \mu_2)$ only affects the distribution of rents across capital types.

In order to estimate quantify separately the two rents parameter $\{\mu_1, \mu_2\}$, we additionally assume that the operating profit function is given by:

$$\Pi_t(K_{1,t}, K_{2,t}) = \left( A_{1,t}^{1 - \frac{1}{\mu_1}} K_{1,t}^{\frac{1}{\mu_1}} \right)^{1 - \eta} \left( A_{2,t}^{1 - \frac{1}{\mu_2}} K_{2,t}^{\frac{1}{\mu_2}} \right)^{\eta}.$$
where \( A_{1,t} \) and \( A_{2,t} \) are both growing at rate \( g \). The first-order conditions \( R_n = \Pi_{n,t} \) can then be expressed as:

\[
R_1 = \frac{1 - \eta K_{1,t}}{\mu_1 \Pi_t}, \\
R_2 = \frac{\eta K_{2,t}}{\mu_2 \Pi_t},
\]

so that the capital-specific rents parameters satisfy:

\[
\mu_1 = \frac{(1 - \eta) \text{ROA}_1}{R_1}, \\
\mu_2 = \frac{\eta \text{ROA}_1}{SR_2}.
\]

**Results** The methodology described above helps clarify that, in the more general model with heterogeneous rents parameters, our methodology identifies the user-cost weighted average rents parameter. However, the methodology has the drawback that the value of \( \eta \) must be known (independently from measures of \( S \)) in order to estimate \( \mu_1 \) and \( \mu_2 \). In other words, while it is straightforward to measure the (user-cost weighted) average rents parameter, estimating them separately from the data seem difficult.

In Figure 30, we provide bounds on the values of \( \eta \) in the two limit cases where rents are due either only to intangible capital, or only to physical capital. Rents will only be due to intangibles when:

\[
\mu_1 = 1, \quad \mu_2 = \frac{\text{ROA}_1 - R_1}{SR_2 R}, \quad \eta = \frac{\text{ROA}_1 - R_1}{\text{ROA}_1}.
\]

Figure 30 shows that this would imply an increase in \( \eta \) has been increasing rapidly over time, reaching approximately \( \eta = 0.4 \) by 2015. At the other extreme, rents will only be due to physical capital when:

\[
\mu_1 = \frac{\text{ROA}_1 - SR_2}{R_1}, \quad \mu_2 = 1, \quad \eta = \frac{SR_2}{\text{ROA}_1}.
\]

This would imply a smaller increase in \( \eta \), reaching only approximately \( \eta = 0.2 \) by 2015, as reported in the bottom panel of Figure 30. The bottom panel of Figure 30 also reports the implied values of the rents parameters in these two limiting cases. The case in which all rents are due to physical capital implies values for \( \mu_1 \) that are close to our baseline estimates of \( \mu \), since the relative user cost \( R_1/\text{SR}_2 \) is large. On the other hand, when all rents are due to intangibles, \( \mu_2 \) must be very large (in the order of \( \mu_2 = 2 \)), again because when intangibles are measured with R&D capital, the relative weight \( \text{SR}_2/R_1 \) is small.

Separate identification of \( \mu_1 \) and \( \mu_2 \) without the knowledge of \( \eta \) is more difficult. The core issue is that without being able to separate cash flows generated by either type of capital (which, when they are not perfect substitutes, is difficult), it is also challenging to measure average returns generated by each type of capital. Even under an alternative interpretation where \( \mu_1 \) and \( \mu_2 \) reflect monopsony power in the market for capital inputs, separate data on rental rates intangible and physical capital would be required to identify each rents parameter separately.
4.9 Financing frictions

4.9.1 Equity financing frictions

Appendix Figures 10 and 11 report a simplified decomposition of the investment gap, under different assumptions about the magnitude of equity financing frictions. Specifically, using the balanced growth results of Appendix 2.6.1, we have that along the balanced growth path,

\[ Q_1 - q_1 = Q_1 - f'(d)(1 + \gamma_1 g) \]

\[ = f'(d) \left( q_2 S + \frac{(\mu - 1) \tilde{R}_1}{r - g} + \frac{(\mu - 1) \tilde{R}_2}{r - g} S \right) \]

so that total rents are given by:

\[ \text{Rents} \equiv f'(d) \left( \frac{(\mu - 1) \tilde{R}_1}{r - g} + \frac{(\mu - 1) \tilde{R}_2}{r - g} S \right) = Q_1 - f'(d)(1 + S + (\gamma_1 + \gamma_2 S)g). \]

Appendix Figures 10 and 11 use these expressions to construct the contribution of total rents and of the omitted capital effect for different values of \( f'(d) \).

Next, we briefly discuss how the results of our main empirical decomposition, which assumes no equity financing frictions, would be biased if the data had instead been generated by a model with equity frictions. Since our procedure to estimate the decomposition relies on the balanced growth model, we study this question in the context of the balanced growth model. For simplicity, we also abstract from adjustment costs to capital in this discussion.

Recall that our procedure uses five data moments, \( \{S, ROA_1, \tau_1, \tau_2, Q_1\} \). We combine these five moments as follows:

\[ \hat{r} - g = \frac{ROA_1 - (\tau_1 + S \tau_2)}{Q_1} \]

\[ \tilde{R}_1 = \hat{r} - g + \tau_1 \]

\[ \tilde{R}_2 = \hat{r} - g + \tau_2 \]

\[ \hat{\mu} = \frac{ROA_1}{\tilde{R}_1 + S \tilde{R}_2} \]

Moreover, in the balanced growth model with no adjustment costs, the (unadjusted) physical investment gap is given by \( Q_1 - 1 \). We decompose it as:

\[ Q_1 - 1 = S + \frac{(\hat{\mu} - 1)}{r - g} \tilde{R}_1 + \frac{(\hat{\mu} - 1)}{r - g} \tilde{R}_2 S. \]

By contrast, the adjusted investment gap is given by:

\[ Q^a_1 - 1 = S + \frac{\mu - 1}{r - g} R^{(a)}_1 + \frac{\mu - 1}{r - g} R^{(a)}_2 S \]
Along the balanced growth path, the first-order conditions of the firm’s problem imply that
\[ R \eta \]
or, in the case of a Cobb-Douglas capital aggregator with elasticity of substitution \( \eta \),
\[
\text{Proof.} \quad \text{The value of } R \text{ implied by our empirical decomposition is:}
\[
\frac{\hat{R}_2}{\hat{R}_1} = \frac{\hat{r} - \hat{g} + \hat{\mu}}{\hat{r} - \hat{g} + \hat{\mu}} = \frac{\hat{r} - g + \nu(d) \hat{\mu}}{\hat{r} - g + \nu(d) \hat{\mu}}
\]
\[
= \frac{R_2^{(a)} + F_2 \varepsilon(d) - (1 - \nu(d)) \hat{\mu}}{R_1^{(a)} + F_1 \varepsilon(d) - (1 - \nu(d)) \hat{\mu}}
\]
\[
= \frac{R_2^{(a)} + F_2 / R_2^{(a)} \varepsilon(d) - (1 - \nu(d)) \hat{\mu} / R_2^{(a)}}{R_1^{(a)} + F_1 / R_1^{(a)} \varepsilon(d) - (1 - \nu(d)) \hat{\mu} / R_1^{(a)}}
\]
Along the balanced growth path, the first-order conditions of the firm’s problem imply that
\[
\frac{R_2^{(a)}}{R_1^{(a)}} = \frac{F_2}{F_1},
\]
Thus, \( \frac{\hat{R}_2}{\hat{R}_1} > \frac{R_2^{(a)}}{R_1^{(a)}} \), if and only if:
\[
\frac{\nu_2}{\nu_1} < \frac{R_2^{(a)}}{R_1^{(a)}} = \frac{F_2}{F_1},
\]
This bias is entirely reflected in the estimated contribution of total rents to the unadjusted investment gap, \( \hat{\text{Rents}} \), which is biased downward relative to their contribution to the adjusted investment gap, \( \hat{\text{Rents}} \):
\[
\left( \hat{\mu} - 1 \right) \frac{\hat{R}_1}{r - g} + \left( \hat{\mu} - 1 \right) \frac{\hat{R}_2 S}{r - g} = \left( \mu - 1 \right) \frac{R_1^{(a)}}{r - g} + \frac{\mu - 1}{r - g} R_2^{(a)} S - \left( \frac{1 - f'(d)}{f'(d)} \right) \hat{\text{Rents}}.
\]
\[
The ratio of intangible rents relative to physical rents is biased upward, if and only if:
\[
\frac{F_2}{F_1} \geq \frac{\nu_2}{\nu_1}.
\]
or, in the case of a Cobb-Douglas capital aggregator with elasticity of substitution \( \eta \),
\[
\eta \geq \frac{\nu_2 S}{\nu_1 + \nu_2 S}.
\]
Result 14. Along the balanced growth path, the unadjusted investment gap is biased downward relative to the adjusted investment gap:
\[
Q_1 - 1 = \left( Q_1^{(a)} - 1 \right) - \left( \frac{1 - f'(d)}{f'(d)} \right) Q_1 < Q_1^{(a)} - 1.
\]
establishing the result.

There are two parts to this result. First, as highlighted above, within equity frictions, the level of the (unadjusted) investment gap is generally too low, relative to the level of the “adjusted” investment gap, reflecting the fact that \( Q_1 \) is too low relative to \( Q_1^{(a)} \). The intuition is that along the balanced growth path where \( d_t > 0 \), the replacement cost of existing capital, \( K_{1,t+1} \), is too high relative to shareholders’ valuation of it, \( f'(d_t)K_{1,t+1} \). The first part of the result says that, in our empirical decomposition, this bias does not affect estimates of the direct effect of intangibles on the investment gap (which are simply given by \( S \), in both our empirical decomposition and the adjusted investment gap decomposition). Instead, the bias shows up in estimated rents, which are too small, compared to the rents in the adjusted investment gap decomposition. The ratio of true rents to estimated rents is given by:

\[
\frac{\text{Rents}}{\hat{\text{Rents}}} = \frac{Q_1 - 1 - S}{Q_1 - 1 - S + \frac{1-f'(d_t)}{f'(d)}Q_1}
\]

Consistent with the intuitions developed in Appendix Figures 10 and 11, accounting for equity frictions would thus lead to higher overall rents, with the quantitative effect.

The second part of the result says that the composition of rents, though, need not be biased in a particular direction. The share of rents attributable to intangibles vs. physical capital, in our baseline decomposition, depends on:

\[
\frac{\hat{R}_2 S}{\hat{R}_1},
\]

whereas in the adjusted investment gap decomposition, it depends on:

\[
\frac{R_2^{(a)} S}{R_1^{(a)}}.
\]

The result says that along the balanced growth path, the difference between these two ratios only depends on the properties of the production function, and on (observable) rates of gross investment in each type of capital — not on the curvature parameter \( f'(d) \).

Different from the baseline model, without knowledge of the function \( f(d) \), if the data is generated by a model with equity issuance frictions, one cannot construct the different elements on the adjusted investment gap decomposition. In particular, in the case of a Cobb-Douglas capital aggregator, it is not possible to estimate the elasticity of substitution \( \eta \) between intangible and physical capital without knowing \( f(.) \).

However, condition (81) gives a lower bound on \( \eta \), above which our estimate of the relative contribution of intangibles to rents would be biased upward. Appendix Figure 32 reports the times series for this lower bound, when intangibles are measured either as R&D capital, or as the sum of R&D and organization capital. This graph shows that the lower bound for the composition bias is relatively high. By 2015, \( \eta \) would have needed to be higher than 0.3 (with only R&D) or 0.6 (with R&D and organization capital) for our decomposition to overstate the contribution of intangibles to rents. (One can compare this to the estimates of \( \eta \) in Appendix Figure 7, which, though they are obtained using a model without equity frictions, are uniformly below these lower bounds.) Thus, if anything, equity frictions appear more likely
to lead to understating the contributions of intangibles to total rents, rather than overstating it.

4.9.2 Debt financing frictions

We next discuss how the investment gap would change, and how our results would be biased, in the presence of debt issuance subject to a collateral constraint. As for the case of equity frictions, we focus on the balanced growth model, since this is the model to which we apply our estimation approach in the main text. We start by giving the expression of the decomposition in the model with debt issuance frictions.

**Result 15.** Let \( n = 1 \) denote physical capital. Along the balanced growth path, neglecting terms of order \( o(g) \) and higher, the physical investment gap is given by:

\[
Q_1 - q_1 = \sum_{m \geq 2} q_m S_m + \frac{(\mu - 1)R_1}{r - g} + \sum_{m \geq 2} \frac{(\mu - 1)R_m}{r - g} \times S_m
\]

where:

\[
\forall n = 1, \ldots, N, \quad q_n = 1 + \gamma_n g,
\]

\[
R_1 = r + \delta_1 + \gamma_1 rg - \theta(r - r_b)
\]

\[
\forall n = 2, \ldots, N, \quad R_n = r + \delta_n + \gamma_n rg.
\]

With respect to our baseline decomposition, the only difference is in the expression for the user cost of physical capital. Specifically, the user cost of physical capital is lower than in the baseline model, at least when \( \theta > 0 \) and \( r > r_b \). Intuitively, the user cost of physical capital is lower because there is an additional benefit from holding physical capital: it relaxes the borrowing constraint, and allows shareholders to lever up and take advantage of the wedge between their discount rate and the discount rate of debtholders.

When there are debt collateral constraint, is our empirical decomposition of the investment gap, which relies on a model without collateral constraints, biased, and if so, how? The following result provides an answer to this question. As in the case of equity frictions, we use hatted variables to denote the parameters we derive from observations of \( \{Q_1, ROA_1, \iota_1, \iota_2, S\} \) from the model. We focus on the version of the model without adjustment costs and with only two capital types for simplicity. The mapping from observed moments to estimated parameters is the following:

\[
\hat{\mu} = \frac{ROA_1}{\hat{R}_1 + S\hat{R}_2}
\]

Our estimate of the decomposition is then given by:

\[
Q_1 - 1 = S + \frac{\hat{\mu} - 1}{r - g}\hat{R}_1 + \frac{\hat{\mu} - 1}{r - g}S\hat{R}_2 \equiv \text{Rents}
\]
Result 16. Let the total contribution of rents to the investment gap in the true model (i.e. the model with collateral constraints) be given by:

\[
\text{Rents} \equiv \frac{\mu - 1}{r - g} R_1 + \frac{\mu - 1}{r - g} S R_2
\]

Along the balanced growth path, our approach correctly estimates the total size of rents:

\[
\hat{\text{Rents}} = \text{Rents}.
\]

Moreover, the contribution of intangibles to rents is underestimated, and the contribution of physical capital is overestimated:

\[
\hat{R}_1 > R_1, \quad \hat{R}_2 < R_2.
\]

Finally, the rents parameter is overestimated, while the Gordon growth term is underestimated:

\[
\hat{\mu} > \mu, \quad \hat{r} - g < r - g.
\]

The fact that the total contribution of rents is correctly estimated, \(\text{Rents} = \hat{\text{Rents}}\), follows from the fact that total rents are effectively estimated as \(Q_1 - 1 - S\), which is correct, from the standpoint of the model with a collateral constraint.\(^{38}\)

The presence of collateral constraints however biases the estimates of the user costs of physical and intangible capital relative to their true value. The estimated user cost of physical capital is too high, because it fails to take into account the fact that part of the return to holding physical capital is the shadow value of relaxing the collateral constraint. The user cost of intangibles is too low because, generally speaking, our approach estimates an implicit discount rate of shareholders, \(\hat{r} = r - g + g\), that is too low. The estimated shareholder discount rate obtained in our approach can be written as:

\[
\hat{r} = \left(1 - \frac{E^e_t V^e_t}{V^e_t}\right) r_b + \frac{E^e_t V^e_t}{V^e_t} r < r.
\]

In other words, our approach recovers the weighted average cost of capital, instead of the relevant cost of capital for computing the opportunity cost of investing, which is the shareholders’ discount rate.

Finally, estimated markups are generally too high, relative to their true value, because total estimated user costs in our approach are too low, relative to their true value:

\[
\hat{R}_1 + \hat{R}_2 S > R_1 + R_2 S.
\]

The effect of the overestimation of the user cost of physical capital always dominates, resulting in estimates of total user costs that are too high, and therefore estimates of \(\mu\) that are too low relative to their true values.

How large are the biases likely to be? Figures 33 and 34 illustrate the potential size of the biases.

Figure 33 reports the decomposition of the investment gap obtained using the baseline model (top panel) and using the model with collateral constraints (middle and bottom panels). In order to construct

\(^{38}\) Recall that we assumed no adjustment costs, so that \(q_1 = 1\); more generally, the estimate of the size of total rents is \(Q_1 - \hat{g}_1 - S = Q_1 - (1 + \gamma_1 g) - S\), which is correct in both versions of the model.
the latter two decompositions, $\theta$ and $r - r_b$ are all that is needed besides the moments we already use in our baseline decomposition. We follow the model and use the empirical debt-to-physical capital ratio in order to estimate the collateral tightness parameter $\theta$, since:

$$\theta = \frac{B_{t+1}}{K_{1,t+1}}.$$ 

Moreover, we assume two potential values for $r - r_b$, $r - r_b = 0.02$ and $r - r_b = 0.05$. There is no clear source for calibrating the wedge between discount factors. However, the quantitative effects are very small regardless of the value of $r - r_b$ chosen. The three panels of Figure 33 are indeed hard to distinguish. Thus the potential biases in the investment gap decomposition, though they generally would lead to a higher estimate of the importance of intangible rents and a lower estimate of the importance of physical rents, are small quantitatively. In a similar spirit, Figure 34 reports the implied markups for the baseline model and for two versions of the model with a debt collateral constraint. The differences in implied markups are very close across the different models, with a gap of 2% at most across the different models, occurring toward the late 2000s.

Overall, we therefore conclude that, while the omission of debt frictions characterized by a collateral constraint would generally lead our decomposition to produce biased estimates, the quantitative effect of these biases would almost certainly be second order.

### 4.10 Rents and productivity

We use the disaggregated data to assess whether our measure of rents (the parameter $\mu$) is related to productivity. A worry is that, because our approach uses average returns to capital to identify $\mu$, a high value of $\mu$ may reflect high marginal products of capital, rather than high rents. In order to assess this possibility, we look at the correlation between measures of total factor productivity at the industry level obtained from the BLS’ KLEMS data, described in Appendix 3, and the measure of rents which we obtain from applying the balanced growth model at the same level of sectoral aggregation.

In Figure 36, we report a scatterplot and regression results that highlight the lack of correlation between the upward trend in rents, and total factor productivity growth, across sectors. In all panels, a point represents a KLEMS industry $s$. Its vertical coordinate is an estimate of the time slope $\beta$ of the rents parameter in the industry over the 1985-2015 period. Its horizontal coordinate is the average growth rate of multi-factor productivity in that industry over the same period. Simple regression lines by groups of industry are also reported. The different panels of the figure correspond to different industry groups or different measures of the intangible capital stock.

The main message of the figure is that the correlation between our estimates of the rents parameters and KLEMS’ measure of multifactor productivity growth is either zero or negative. Visually, there is no clear correlation, and simple regressions all deliver robust t-statistics below 1 in absolute value. Thus, this simple evidence suggests that our estimates of the changes in rents do not simply (or even mainly) reflect changes in multifactor productivity.

An important caveat to this simple evidence is that, while the KLEMS multi-factor productivity growth
series have the advantage of relying strictly on output measures (as opposed to revenue), they still implicitly assume perfect competition and no pure profits. If, in reality, firms earn rents, the KLEMS measures will generally be biased. We explore this issue in more detail in Crouzet and Eberly (2020); there, we argue that this bias could have led to substantially underestimated aggregate productivity growth, though not necessarily of sectoral productivity growth.
## Appendix Table 1: Composition of the Consumer, High-tech, Healthcare, and Manufacturing sectors studied in Section 4.

Numbers in parentheses in the first column indicate the sectors’ share of 2001 total value added by private non-financial businesses, constructed using KLEMS data. These numbers do not add up to 100% because the remaining sectors, described in Table 2, also contribute to total value added by private businesses. The second and third column reports the name of the subsectors and the corresponding NAICS codes. Subsectors are defined following the classification used by the BLS to construct the KLEMS data. Subsectors marked with a † are dropped from the analysis of disaggregated subsectors because they do not have at least 10 firms in Compustat in each year from 1985 to 2015.

<table>
<thead>
<tr>
<th>Broad sector</th>
<th>Subsector</th>
<th>NAICS codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer (20%)</td>
<td>Crop and Animal Production†</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>Forestry, Fishing, and Related Activities†</td>
<td>113</td>
</tr>
<tr>
<td></td>
<td>Wholesale Trade</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>Retail Trade</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Computer and Electronic Products</td>
<td>334</td>
</tr>
<tr>
<td></td>
<td>Publishing industries, except internet (includes software)</td>
<td>511</td>
</tr>
<tr>
<td></td>
<td>Motion picture and sound recording industries†</td>
<td>512</td>
</tr>
<tr>
<td></td>
<td>Data processing, internet publishing, and other information services</td>
<td>518 to 519</td>
</tr>
<tr>
<td></td>
<td>Broadcasting and telecommunications</td>
<td>515 to 517</td>
</tr>
<tr>
<td></td>
<td>Computer Systems Design and Related Services</td>
<td>5415</td>
</tr>
<tr>
<td></td>
<td>Chemical Products</td>
<td>325</td>
</tr>
<tr>
<td></td>
<td>Miscellaneous Manufacturing</td>
<td>339</td>
</tr>
<tr>
<td></td>
<td>Ambulatory Health Care Services†</td>
<td>621</td>
</tr>
<tr>
<td></td>
<td>Hospitals and Nursing and Residential Care Facilities</td>
<td>622 to 623</td>
</tr>
<tr>
<td>Healthcare (10%)</td>
<td>Oil and Gas Extraction</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>Mining, except Oil and Gas</td>
<td>212</td>
</tr>
<tr>
<td></td>
<td>Support Activities for Mining</td>
<td>213</td>
</tr>
<tr>
<td></td>
<td>Utilities</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>Food and Beverage and Tobacco Products</td>
<td>311 to 312</td>
</tr>
<tr>
<td></td>
<td>Textile Mills and Textile Product Mills†</td>
<td>313 to 314</td>
</tr>
<tr>
<td></td>
<td>Apparel and Leather and Applied Products</td>
<td>315 to 316</td>
</tr>
<tr>
<td></td>
<td>Wood Products</td>
<td>321</td>
</tr>
<tr>
<td></td>
<td>Paper Products</td>
<td>322</td>
</tr>
<tr>
<td>Manufacturing (22%)</td>
<td>Printing and Related Support Activities†</td>
<td>323</td>
</tr>
<tr>
<td></td>
<td>Petroleum and Coal Products</td>
<td>324</td>
</tr>
<tr>
<td></td>
<td>Plastics and Rubber Products</td>
<td>326</td>
</tr>
<tr>
<td></td>
<td>Nonmetallic Mineral Products</td>
<td>327</td>
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<tr>
<td></td>
<td>Primary Metal Products</td>
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<td>Fabricated Metal Products</td>
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<tr>
<td></td>
<td>Machinery</td>
<td>333</td>
</tr>
<tr>
<td></td>
<td>Electrical Equipment, Appliances, and Components</td>
<td>335</td>
</tr>
<tr>
<td></td>
<td>Transportation Equipment</td>
<td>336</td>
</tr>
<tr>
<td></td>
<td>Furniture and Related Products</td>
<td>337</td>
</tr>
</tbody>
</table>
Appendix Table 2: Composition of the Services sector. The number in parentheses in the first column is the overall share of 2001 total value added by private non-financial businesses of the sectors in this table, constructed using KLEMS data. The numbers in parentheses in the second column report the share of each group in the all the sectors in the table (so that they add up to 100%). The last columns the name of the subsectors and the corresponding NAICS codes. Subsectors are defined following the classification used by the BLS to construct the KLEMS data. Subsectors are defined following the classification used by the BLS to construct the KLEMS data. Subsectors marked with a † are dropped from the analysis of disaggregated subsectors because they do not have at least 10 firms in Compustat in each year from 1985 to 2015. The Construction and Transportation groups are marked with * to indicate that they are not included in the analysis of the five main sectors (Consumer, High-tech, Healthcare, Manufacturing, and Services), and that their subsectors are not included in the analysis of disaggregate subsectors.

<table>
<thead>
<tr>
<th>Broad sector</th>
<th>Group</th>
<th>Subsector</th>
<th>NAICS codes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Legal services†</td>
<td>5411</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Miscellaneous Professional, Scientific, and Technical Services</td>
<td>5412 to 5414, 5416 to 5419</td>
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<tr>
<td></td>
<td></td>
<td>Management of companies and enterprises†</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Administrative and Support Services</td>
<td>561</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Waste Management and Remediation Services†</td>
<td>562</td>
</tr>
<tr>
<td>Services</td>
<td>(64%)</td>
<td>Educational Services†</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Social assistance†</td>
<td>624</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amusements, Gambling, and Recreation Industries</td>
<td>713</td>
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<td></td>
<td></td>
<td>Performing arts, spectator sports, museums, and related activities†</td>
<td>711 to 712</td>
</tr>
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<td>Other</td>
<td>(36%)</td>
<td>Accommodation</td>
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<td>Food Services and Drinking Places</td>
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</tr>
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<td></td>
<td></td>
<td>Other services except Government†</td>
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<tr>
<td>Construction*</td>
<td>(23%)</td>
<td>Construction†</td>
<td>23</td>
</tr>
<tr>
<td>Transportation and warehousing*</td>
<td>(14%)</td>
<td>Truck Transportation</td>
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<td></td>
<td></td>
<td>Other Transportation and Support Activities†</td>
<td>487 to 488</td>
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<td></td>
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<td>Air Transportation</td>
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<td>Rail Transportation</td>
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<td>Pipeline Transportation†</td>
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<td>Water Transportation†</td>
<td>483</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Transit and ground passenger transportation†</td>
<td>485</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Warehousing and storage†</td>
<td>493</td>
</tr>
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</table>
Appendix Table 3: Summary of targeted and implied moments for the different sectors of the Compustat non-financial sample. All columns measure intangibles as the sum of the R&D capital stock plus the organization capital stock. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: $K_t = K_{1,t}^{1-\eta}K_{2,t}^{\eta}$. Rents as a fraction of value added are computed as $s = (1 - s_L)(1 - 1/\mu)$, where $s_L$ is the labor share of value added for the NFCB sector. Markups over value added are computed as $\bar{\mu} = 1/(1 - s)$. The implied moments reported are for the model with adjustment costs; the adjustment cost values are $\gamma_1 = 3$ and $\gamma_2 = 12$. In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources and construction are described in Section 4.
### Structural parameters

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<tbody>
<tr>
<td>$\hat{g}$</td>
<td>Mean g.r. of fundamentals</td>
<td>0.029</td>
<td>0.019</td>
<td>-0.010</td>
<td>0.030</td>
<td>0.025</td>
<td>-0.005</td>
<td>0.031</td>
<td>0.024</td>
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<tr>
<td>$r$</td>
<td>Discount rate</td>
<td>0.067</td>
<td>0.056</td>
<td>-0.011</td>
<td>0.087</td>
<td>0.088</td>
<td>0.002</td>
<td>0.087</td>
<td>0.087</td>
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<tr>
<td>$\delta_1$</td>
<td>Phys. depreciation rate</td>
<td>0.070</td>
<td>0.068</td>
<td>-0.002</td>
<td>0.079</td>
<td>0.069</td>
<td>-0.010</td>
<td>0.078</td>
<td>0.070</td>
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<tr>
<td>$\delta_2$</td>
<td>Intan. depreciation rate</td>
<td>0.251</td>
<td>0.242</td>
<td>-0.010</td>
<td>0.230</td>
<td>0.223</td>
<td>-0.007</td>
<td>0.221</td>
<td>0.221</td>
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<tr>
<td>$\eta$</td>
<td>Cobb-Douglas intan. share</td>
<td>0.223</td>
<td>0.281</td>
<td>0.058</td>
<td>0.201</td>
<td>0.247</td>
<td>0.046</td>
<td>0.426</td>
<td>0.474</td>
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<tr>
<td>$\mu$</td>
<td>Curv. of profit function</td>
<td>1.196</td>
<td>1.276</td>
<td>0.080</td>
<td>1.241</td>
<td>1.387</td>
<td>0.146</td>
<td>1.121</td>
<td>1.208</td>
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### Implied moments

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<tbody>
<tr>
<td>$\mu_{VA}$</td>
<td>Markup over value added</td>
<td>1.051</td>
<td>1.079</td>
<td>0.028</td>
<td>1.061</td>
<td>1.105</td>
<td>0.044</td>
<td>1.033</td>
<td>1.062</td>
</tr>
<tr>
<td>$s_{VA}$</td>
<td>Rents/value added</td>
<td>0.049</td>
<td>0.073</td>
<td>0.025</td>
<td>0.058</td>
<td>0.095</td>
<td>0.037</td>
<td>0.032</td>
<td>0.059</td>
</tr>
<tr>
<td>$R_1$</td>
<td>User cost of phy. cap.</td>
<td>0.137</td>
<td>0.125</td>
<td>-0.013</td>
<td>0.165</td>
<td>0.157</td>
<td>-0.008</td>
<td>0.165</td>
<td>0.157</td>
</tr>
<tr>
<td>$R_2$</td>
<td>User cost of intan. cap.</td>
<td>0.319</td>
<td>0.298</td>
<td>-0.021</td>
<td>0.317</td>
<td>0.311</td>
<td>-0.006</td>
<td>0.308</td>
<td>0.308</td>
</tr>
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</table>

Appendix Table 4: GMM estimation of a version of the model with with i.i.d. shocks to the growth rate of fundamentals. The columns marked “Non-Financial Corporate Businesses” report results obtained aggregate data for the NFCB sector, while the columns marked “Compustat Non-Financials” report results using aggregated data from the sample of Compustat non-financial firms. In the columns marked “1985-2000” and “2001-2017”, the numbers in brackets are (5, 95) confidence intervals for the point estimates of the different parameters or implied moments, computed using HAC standard errors based on a Bartlett kernel with four lags (the data are all annual). Point estimates for implied moments are computed by stacking the moment conditions defining these additional implied moments with the rest of the GMM moment conditions. The columns marked ”Diff.” report the change in structural parameters or implied moments across periods; the numbers in parentheses are p-values for the two-sided test that the difference is equal to 0. The p-value is computed by re-estimating the model on all the data from 1985 to 2017, and interacting all data and estimated parameters or implied moments with a full set of dummies for each of the two sub-samples. Estimation is done using two-step efficient GMM with the identity matrix as the first-step weighting matrix. More details are provided in Appendix 4.5.
Ratio to:

<table>
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<tr>
<th>Variable input costs</th>
<th>Sales $S_t$</th>
<th>Value added $VA_t$</th>
<th>Operating surplus $\Pi_t$</th>
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<tr>
<td>$W_t M_t$</td>
<td>$\frac{1-\alpha}{\chi}$</td>
<td>$\frac{1-\alpha}{\chi - (1-\alpha)(1-\nu_1)}$</td>
<td>$\frac{1-\alpha}{\chi - (1-\alpha)}$</td>
</tr>
<tr>
<td>Labor costs</td>
<td>$W_{1,t} M_{1,t}$</td>
<td>$\frac{1-\alpha}{\chi}$</td>
<td>$\frac{1-\alpha}{\chi - (1-\alpha)(1-\nu_1)}$</td>
</tr>
<tr>
<td>Capital costs†</td>
<td>$R_{1,t} K_{1,t} + R_{2,t} K_{2,t}$</td>
<td>$\frac{\alpha}{\chi}$</td>
<td>$\frac{\alpha}{\chi - (1-\alpha)(1-\nu_1)}$</td>
</tr>
<tr>
<td>Rents†</td>
<td>$R_{e,t}$</td>
<td>$\frac{\chi - 1}{\chi}$</td>
<td>$\frac{\chi - 1}{\chi - (1-\alpha)(1-\nu_1)}$</td>
</tr>
</tbody>
</table>

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| Value added $VA_t$ | 1 - $\frac{1-\alpha}{\chi}$ $(1-\nu_1)$ | 1 | 1 + $\frac{1-\alpha}{\chi - (1-\alpha)^\nu_1}$ |
| Operating surplus $\Pi_t$ | 1 - $\frac{1-\alpha}{\chi}$ | $\frac{1-\alpha}{\chi - (1-\alpha)(1-\nu_1)^\nu_1}$ | 1 |

**Appendix Table 5:** Expression for the ratios of variable input costs, labor costs, capital costs, rents, value added, and operating surplus to sales, value added, and operating surplus implied by the variable profit maximization problem described in Appendix 2.3. These expressions hold generally (regardless of how capital inputs are chosen), except the expressions for capital and rents shares, marked with †, which only hold in the balanced growth model of Section 2.3 of the main text. Variable input costs are defined as $\sum_j W_{j,t} M_{j,t} = W_t M_t$, where $W_t$ and $M_t$ are aggregate price and quantity indices defined in Appendix 2.3. Without loss of generality, labor is assumed to be the first variable input, entering the aggregate quantity index $M_t$ with Cobb-Douglas share $\nu_1$. The parameter $\alpha$ is the Cobb-Douglas elasticity of substitution between capital $K_t$ and variable inputs $M_t$. The reduced-form parameter $\chi \geq 1$ indexes the importance of rents in the model; it is given, in terms of structural parameters, by $\chi = \frac{\mu S}{\zeta}$, where $\mu_S$ is the markup of the price of output over the marginal cost of output, and $\zeta$ is the degree of returns to scale to the capital-variable input bundle $K_t^\alpha M_t^{1-\alpha}$. Revenue is $S_t = P_t Y_t$; value added is $VA_t = S_t - (W_t M_t - W_{1,t} M_{1,t}) = \Pi_t + W_{1,t} M_{1,t}$; operating surplus is $\Pi_t = VA_t - W_t M_t$. Given competitive payments to capital $R_{1,t} K_{1,t} + R_{2,t} K_{2,t}$, rents are defined as: $R_{e,t} = \Pi_t - (R_{1,t} K_{1,t} + R_{2,t} K_{2,t})$. 

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### Appendix Table 6: Markups and decreasing returns in the Compustat Non-Financial sample. The first panel reports estimates of three parameters of the version of the balanced growth model with an explicit microfoundation for variable input choices. \( \alpha \) is the Cobb-Douglas elasticity of substitution between variable inputs and capital. \( \chi \) is the reduced-form parameter governing the size of rents as a fraction of total sales; it is equal to the ratio of the two structural parameters \( \mu_S \) (the markup of the marginal cost of variable inputs over sales) and \( \zeta \) (the degree of returns to scale with respect to variable inputs and capital). The reduced-form parameter \( \mu \), which governs the size of rents as a fraction of operating surplus, is the same as in the balanced growth model of Section 2, and is related to the other parameters by \( \mu = 1 + \frac{\chi-1}{\alpha} \). The estimates of these parameters are obtained using the methodology described in 3; in particular, the values of the rents parameter \( \mu \) are the same as in Section 3. The second and third panels report the values for the markup \( \mu_S \) implied by different assumptions regarding the degree of returns to scale, as well as the implications for the size of rents as a fraction of value added. The “rents due to markups” are defined as \( (1-s_L)(\alpha/(\alpha + \mu_S - 1)) \).
Appendix Figure 1: Time series for the moments used in the construction of the physical investment gap decomposition, Equation (13). Returns to physical capital are defined as $\Pi_t/K_{1,t}$, where $\Pi_t$ is operating surplus and $K_{1,t}$ the stock of physical capital at current cost. Investment rates are defined as $i_{n,t} = I_{n,t}/K_{n,t}$, $n = 1, 2$, where $n = 1$ indexes physical capital and $n = 2$ indexes intangible capital, $K_{2,t}$ is the stock of intangible capital at current cost, and $I_{n,t}$ are investment expenditures for each type of capital. The ratio of intangible to physical capital is $S_t = K_{2,t+1}/K_{1,t+1}$. Average Tobin’s $Q$ of physical capital is defined as $Q_{1,t} = V_t/K_{1,t+1}$, where $V_t$ is an estimate of the total market value of net claims on the sector. The time series are the raw data; in particular, they are not averaged over seven-year windows. Data sources are described in Section 3 and in Appendix 3.
Appendix Figure 2: Firm discount rates (top panel) and implicit equity risk premium (bottom panel) across different approaches. The top panel reports the firm-wide discount rate expressed in nominal terms, defined as $r^u = (r - g) + g^n$. The bottom panel reports the risk premium implicit in the equity cost of capital across different approaches. In all three approaches approach, this is computed as $RP = (1 + l)(r^u - l/(1 + l)r^d) - r^n$, where $l$ is the ratio of the market value of debt to the market value of equity, and $r^d$ is the (after-tax, nominal) cost of debt reported in Figure 15.
Appendix Figure 3: Growth rates for intangible, physical, and total capital stocks. The top panel reports the growth rates of net capital stocks at current cost, obtained from Fixed Assets table 4.1; physical capital is defined as the sum of equipment and structures, intangible capital is defined as intellectual property products, and total capital is the sum of the two. The medium panel reports the growth rates of real quantity indices for capital stocks. For intangible and total capital, this is obtained from Fixed Assets table 4.2; for physical capital, the growth rate is constructed as described in Appendix 3.3. The bottom panel reports the difference between the growth rate of intangible capital and the growth rate of physical capital, when they are measured using either stocks at current cost ("nominal") or quantity indices ("real").
Appendix Figure 4: Gross investment rates and average depreciation rates in the data, and model-implied depreciation rates. Data is for the NFCB sector. The top panel reports data for physical capital, and the bottom panel reports data for intangible capital. Each panel reports the gross investment rate (black triangle line) and average depreciation rates (black solid line), both computed from BEA Fixed Assets tables. In particular, average depreciation rates are computed using current cost estimates of depreciation and net stocks. Model-implied depreciation rates (teal crossed lines) are computed as $\delta_i = R_i - r - \gamma_i r g = \iota_i - g, i = 1, 2$. Appendix 3.4 contains more detail on methodology and data sources.
Appendix Figure 5: Time series moments for Compustat Non-Financial (NF), all sectors (aggregated). The corresponding time series moments for the aggregate non-financial corporate business (NFCB) sector are also reported, for comparison. All variables are defined as in Figure 1. Data sources for the NFCB sector are described in Section 3 and Appendix 3. Data sources are described in Section 4.
Appendix Figure 6: The investment gap $Q_1 - q_1$ for physical capital in the Compustat Non-Financial (NF) sample. The top panel reports results when only R&D is used to measure intangibles. The middle panel reports results when both R&D and organization capital are used to measure intangibles. In the top and middle panels, we use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$, in order to construct the components of the investment gap. The bottom panel reports a version of the decomposition that separates explicitly the contribution of the terms related to R&D capital (in green) and those related to organization capital (in orange). In order to construct this decomposition, we use a model with adjustment costs of $\gamma_1 = 3$ for physical capital, $\gamma_2 = 12$ for R&D capital, and $\gamma_3 = 3.2$. Our values for adjustment costs are drawn from Belo et al. (2019). Methodology and data sources are described in Section 4.
Appendix Figure 7: Other model moments for the Compustat Non-Financial (NF) sample. Panel (a) reports the share of intangibles in production, \( \eta \), when the capital aggregator is assumed to be Cobb-Douglas: \( K_t = K_1^{1-\eta}K_2^\eta \). Panel (b) reports rents as a fraction of value added, \( s_{VA} \), which is given by \( s_{VA} = (1 - s_L)(1 - 1/\mu) \), where \( \mu \) is the model parameter governing the size of rents, and \( s_L \) is labor’s share of value added. Panels (c) and (d) report user costs for each type of capital, \( R_1 \) and \( R_2 \). We use the version of model with adjustment costs \( \gamma_1 = 3 \) and \( \gamma_2 = 12 \), in order to construct the components of the investment gap. Methodology and data sources are described in Section 4.
Appendix Figure 8: Robustness: the investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate (NFCB) sector when using the Hall (2001) measure of Tobin’s average $Q$ for physical assets. The top panel reports our baseline measure of average Tobin’s $Q$ for physical capital, $Q_1$ (black line), and an alternative measure based on Hall (2001). The difference between the two is that our baseline measure only nets out financial assets identified as liquid in the Flow of Funds in the computation of the net value of claims on the NFCB sector. The bottom panel reports the investment gap in the model without adjustment costs ($\gamma_1 = \gamma_2 = 0$). Details on the construction of total enterprise value are discussed in Appendix 3.2.
Appendix Figure 9: Robustness: adjustment costs. Each panel reports a moment from the investment gap decomposition, Equation (13), for the NFCB sector, for different combinations of adjustment costs for physical and intangible capital. In each panel, a point corresponds to a particular combination for \((\gamma_1, \gamma_2)\), and the color corresponds to the value of the moments, with the correspondence reported on the right axis. Panel (a) reports the change in \(Q_1 - q_1\) from 1985 to 2015; in our baseline results with positive adjustment costs, this moment is equal to 1.30. Panel (b) reports the contribution of intangibles to \(Q_1 - q_1\) in 2015; in our baseline results, this moment is equal to 0.39 (or 39%). Panel (c) reports the implied intangible share in production in 2015; in our baseline results with positive adjustment costs, this moment is equal to 0.29. Panel (d) reports rents as a share of value added in 2015; in our baseline results with positive adjustment costs, this moment is equal to 0.063. Our baseline results with positive adjustment costs use \(\gamma_1 = 3\) and \(\gamma_2 = 12\). Methodology and data sources are described in Section 3.
Appendix Figure 10: The investment gap for physical capital in the Compustat Non-Financial (NF) sample, for different magnitudes of equity financing frictions. In each panel, the crossed green line is an estimate of the investment gap. The shaded areas present the decomposition of the physical investment gap into two terms, corresponding to total rents (the dark region) and the omitted intangibles effect (light region). The top panel reports results with no equity financing frictions ($f'(d) = 1$); the middle panel reports results with positive equity financing frictions ($f'(d) = 0.90$); and the bottom panel reports results with high equity financing frictions ($f'(d) = 0.80$). The methodology is described in Section 5 and Appendix 4.9.1.
**Appendix Figure 11**: Sectoral investment gaps, with and without equity financing frictions. The left column reports the investment gaps obtained in our baseline approach, without equity financing frictions. The left column reports the gaps with equity financing frictions, assuming $f'(d) = 0.90$. The methodology is described in Section 5 and Appendix 4.9.1.
Appendix Figure 12: Rents as a fraction of value added, in the case of no intangibles versus the baseline. The top panel reports results obtained using aggregate data for the Non-Financial Corporate Business (NFCB) sector. The bottom panel reports results obtained using aggregated data for the Compustat non-financial (NF) sample. In both panels, the circled line is the trend in markups obtained when we assume that firms have no intangible assets. The other lines correspond to the cases where either capitalized R&D, or capitalized R&D plus capitalized SG&A are used to measure the intangible capital stock. In both panels, we use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$. 

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Appendix Figure 13: The importance of intangible capital in production, in the case of no rents versus the baseline. The top panels report results obtained using data for the non-financial corporate business (NFCB) sector. The bottom panels report results obtained using aggregated data from Compustat nonfinancials (NF). Panels in the left column report estimates of the intangible share of capital in the production function, $\eta$, assuming a Cobb-Douglas specification. Panels in the right column report estimates of the stock of intangible capital relative to physical capital, $S$. The two are related through $\eta/(1-\eta) = SR_2/R_1$. In all panels, the circled lines represent the implied values of $\eta$ or $S$ when we assume no rents, $\mu = 0$. The other lines correspond to the cases where either capitalized R&D, or capitalized R&D plus capitalized SG&A are used to measure the intangible capital stock. In all panels, we use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$. 
Appendix Figure 14: The investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate business (NFCB) sector, allowing for heterogeneous growth rates between physical and intangible capital. The top panel reports the investment gap in our baseline approach with positive adjustment costs ($\gamma_1 = 3$, $\gamma_2 = 12$), where the two types of capital are assumed to grow at the same rate ($g_1 = g_2 = g$), and $g$ is measured using the growth rate of the total capital stock in the NFCB sector. The results are the same as the middle panel of Figure 1. The bottom panel reports the investment gap decomposition when we allow for heterogeneous growth rates in intangible and physical capital, $g_1 \neq g_2$, in the measurement of marginal $q$ for physical capital and intangible capital. The two growth rates are computed using data from the Fixed Asset Tables, as described in Appendix 3.3.
Appendix Figure 15: Time series for the PD ratio used in alternative approach 1. The short dashed line is the unadjusted PD ratio, $PD = 1/(R_{E,t-1,t}^c - R_{E,t-1,t}^e)$, where $R_{E,t-1,t}^c$ and $R_{E,t-1,t}^e$ are, respectively, the cum- and ex-dividend returns on the S&P 500. The long dashed line is the PD ratio adjusted for leverage, $\tilde{PD} = (1 + l)PD^E/(1 + (r^n_B - g^n)LPD)$, where $l = B_{t-1}/E_{t-1}$ is market leverage, $r^n_B$ is the (after-tax, nominal) interest rate on debt securities, and $g^n$ is the nominal growth rate of the total capital stock. Data sources are described in Appendix 3.
Appendix Figure 16: Components of the average cost of capital. All the costs of capital are expressed in nominal terms. Both panels report $r^n_B$, $r^n_E$ and $r^n$, which are respectively the cost of debt, equity, and the weighted average cost of capital, where $r^n = l/(1+l)r^n_B + 1/(1+l)r^n_E$. $r^n$ is the nominal discount rate and is related to the Gordon growth term via $r^n = (r - g) + g^n$, where $g^n$ is the growth rate of the nominal capital stock. $r^n_E$ is the nominal cost of equity. In the top panel, it is measured as $r^n_E = PD^{-1} + g^n$, where $PD$ is the PD ratio reported in Appendix Figure 15. In the bottom panel, it is measured as $r^n_E = r^n_f + RP$, where $r^n_f$ is the (nominal) rate of return on one-month T-bill, and $RP$ is a constant risk premium of $RP = 6.5\%$. In both panels, $r^n_B$ is an average interest rate on debt liabilities of non-financial corporate firms. More details are provided in Appendix 4.6.1.
Appendix Figure 17: Results in the average cost of capital approach, when the PD ratio is used to measure the cost of equity. Data is for the non-financial corporate (NFCB) sector. The top panel reports the decomposition of the investment gap obtained using this approach. The bottom panels compare the value of $Q_1$ (left panel) and of the share of rents as a fraction of value added $s = (1 - \mu^{-1})(1 - LS)$, between our baseline and this approach. Solid correspond to the baseline, which matches empirical values of $Q_1$, while circled lines correspond to alternative approach 1. The model without adjustment costs ($\gamma_1 = \gamma_2 = 0$) is used in both cases.
Appendix Figure 18: Results in the average cost of capital approach, when the risk-free rate plus a constant risk premium is used to measure the cost of equity. Data is for the non-financial corporate (NFCB) sector. The top panel reports the decomposition of the investment gap obtained using this approach. The bottom panels compare the value of $Q_1$ (left panel) and of the share of rents as a fraction of value added between our baseline and this approach. Solid correspond to the baseline, which matches empirical values of $Q_1$, while circled lines correspond to the alternative approach. The model without adjustment costs ($\gamma_1 = \gamma_2 = 0$) is used in both cases. For Tobin’s $Q_1$ and the investment gap, only values after 1985 are reported, because before 1985, the implied discount rate is frequently very close to or below the growth rate $g$, leading to implausible values for $Q_1$ and the investment gap.
Appendix Figure 19: Results when the size of the investment gap is used to infer the ratio of intangible to physical capital, $S$. Data is for the non-financial corporate (NFCB) sector. The top panel reports the decomposition of the investment gap obtained using this approach. The bottom panels compare the value of $S$ (left panel) and of the share of rents as a fraction of value added $s = (1 - \mu^{-1})(1 - LS)$ (right panel), between our baseline and this approach. Solid lines correspond to the baseline, which matches empirical values of $S$, while dashed lines correspond to the alternative approach. The model without adjustment costs ($\gamma_1 = \gamma_2 = 0$) is used in both cases.
Appendix Figure 20: Results when the size of the investment gap is used to infer the size of the rents parameter $\mu$. Data is for the non-financial corporate (NFCB) sector. The top panel reports the decomposition of the investment gap obtained using this approach. The bottom panels compare the value of $ROA_1$ (left panel) and of the share of rents as a fraction of value added $s = (1 - \mu^{-1})(1 - LS)$ (right panel), between our baseline and this approach. Solid correspond to the baseline, which matches empirical values of $ROA_1$, while dashed lines correspond to the alternative approach. The model without adjustment costs is used in both cases.
Appendix Figure 21: Total market value of securities outstanding from the non-financial corporate (NFCB) sector, $V_t$. $V_t$ is defined as the sum of the market value of equity securities, plus an estimate of the market value of non-equity claims, minus financial assets. The estimate of the market value of non-equity claims is equal to their book value, plus an adjustment for the difference between the market and book value of corporate bonds and municipal securities, following Hall (2001). Data are from the Flow of Funds. The black line reports the estimate used in the main text. The green dotted line is the estimate of $V_t$ constructed by Hall (2001), and obtained from his replication data, available at web.stanford.edu/~rehall/SMCA_Data_Appendix.html. The solid blue line is an extension of the Hall (2001) estimate of $V_t$ to 2017. The differences between original and extended Hall (2001) estimates come from small differences in the updated data sources, and in the time series for the stock of municipal bonds and the issuance of corporate bonds in the 1950s and 1960s. All time series are deflated using the deflator for investment in non-residential fixed assets (FRED series A008RD3Q086SBEA) for the solid blue and black lines, and the original deflator constructed by Hall (2001) for the dotted green line. More details on data sources and methodology are reported in Appendix 3.1.
Appendix Figure 22: Measures of the dollar value of investment in the NFCB sector (top panel), and of the investment rate (bottom panel). Dashed lines are measures obtained using BEA data, while solid lines are measures obtained using Compustat data. Differences between the series are discussed in Appendix 3.5.
Appendix Figure 23: Comparison between alternative measures of $\Pi_t/(K_{1,t} + K_{2,t})$ (surplus per unit of total capital; top panel) and $Y_t/(K_{1,t} + K_{2,t})$ (value added per unit of capital; bottom panel) in BEA data. The construction of each time series is discussed in Appendix 3.1. The blue line reproduces the measures of $\Pi/K$ and $Y/K$ used in Farhi and Gourio (2018), which differ from our measures primarily because we focus only on the NFCB sector instead of the whole economy. See Appendix 3.1 for further details on data sources.
Appendix Figure 24: Measures of the total physical capital stock at current cost ($K_1$), of surplus ($\Pi$), and of the ratio of surplus to capital ($\Pi/K_1$) in BEA and Compustat data. All nominal data are deflated using the CPI with base 2009. Differences between the series are discussed in Appendix 3.5.
Appendix Figure 25: Time series moments for the Consumer sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section 4. The sectoral classification is described in Appendix Tables 1 and 2.
Appendix Figure 26: Time series moments for the High-Tech sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section 4. The sectoral classification is described in Appendix Tables 1 and 2.
Appendix Figure 27: Time series moments for the Healthcare sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section 4. The sectoral classification is described in Appendix Tables 1 and 2.
Appendix Figure 28: Time series moments for the Manufacturing sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section 4. The sectoral classification is described in Appendix Tables 1 and 2.
Appendix Figure 29: Time series moments for the Services sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section 4. The sectoral classification is described in Appendix Tables 1 and 2.
Appendix Figure 30: Share of intermediate goods produced using intangibles. The revenue function of the firm is assumed to be given by \( \Pi_t = \left( A_{1,t}^{1-1/\mu_1} K_{1,t}^{1/\mu_1} \right)^{1-\eta} \left( A_{2,t}^{1-1/\mu_2} K_{2,t}^{1/\mu_2} \right)^{\eta} \), where \( \mu_1 \) indexes rents generated by intangibles and \( \mu_2 \) indexes rents generated by physical capital. The top panel reports the value of \( \eta \) when \( \mu_1 = 1 \) (circled line) and when \( \mu_2 = 1 \) (crossed line). The bottom panel reports the implied value of \( \mu_2 \) when all rents are attributable to intangibles (circled line), the implied value of \( \mu_1 \) when all rents are attributable to physical capital (crossed line), and user-cost weighted average rents parameter \( \mu \), which is identical in either case, and is also equal to the rents parameter \( \mu \) obtained in our baseline analysis. Details of the decomposition of the investment gap, and of the derivation of \( \eta \), are reported in Appendix 4.8.
Appendix Figure 31: Markup over sales $\mu_S$ implied by different degrees of returns to scale $\zeta$, in the Compustat non-financial (NF) sample. The top panel reports results when intangibles are defined as R&D only, and the bottom panel reports results when intangibles are defined as R&D plus organization capital. In each panel, markups over sales are computed as $\mu_S = \left(\nu_K + (1 - \nu_K)(1 - s_{\Pi})\right)^{-1} \zeta$, where $\nu_K = \left(R_{1,t}K_{1,t} + R_{2,t}K_{2,t}\right)\Pi_t$ is the ratio of capital payments to operating surplus, and $s_{\Pi} = \Pi_t/S_t$ is the ratio of operating surplus to sales. The ratio of capital payments to operating surplus is obtained from solving the balanced growth model; each point corresponds to a value estimated over a different 7-year centered window; in the computation of $\mu_S$, $s_{\Pi}$ is also averaged over the same windows. See Section 2 for more details.
Appendix Figure 32: Composition bias with equity issuance frictions. The graph reports the time series for $\nu_2 S / (\nu_1 + \nu_2 S)$, when intangibles are measured either as R&D, or as the sum of R&D and organization capital. This ratio provides the lower bound above which omitting equity issuance frictions would bias upward the contribution of intangibles to total rents. See Appendix 4.9.1 for more details.
Appendix Figure 33: The physical investment gap $Q_1 - q_1$ with and without debt collateral constraints. The top panel reports the investment gap in our baseline model, where we assume no frictions in debt issuance. The middle and bottom panels report the physical investment gap in the model with a collateral constraint limiting debt issuance, under different assumptions about the wedge between shareholders’ and debtholders’ discount factor. In all three figures, we use the model with no adjustment costs, and R&D as a measure of intangibles.
Appendix Figure 34: Implied markup over value added $\tilde{\mu}$ with and without debt collateral constraints. The solid line reports the implied markup in our baseline model, where we assume no frictions in debt issuance. The crossed and dashed line report the markup in the model with a collateral constraint limiting debt issuance, under different assumptions about the wedge between shareholders’ and debtholders’ discount factor. In all three lines, we use the model with no adjustment costs, and R&D as a measure of intangibles.
Appendix Figure 35: Implications for the labor share. The top panel reports the value of the Cobb-Douglas exponent on labor, $1 - \alpha$, obtained when using the model described in Appendix 2.3, and assuming that all intermediate inputs are labor (or equivalently, that the production function is a value-added production function), and matching the labor share, as we do in our baseline analysis when translating estimates of $\mu$ into rents as a share of value added, $s$. The middle panel reports estimates of $s$ in our baseline approach, and in an approach where we instead fix $1 - \alpha = 0.7$ (and use no information on the labor share). Finally, the bottom panel reports the labor share obtained when we fix $1 - \alpha = 0.7$, and the actual labor share in the data.
Appendix Figure 36: The relationship between rising rents and productivity growth across subsectors. Each panel reports a scatterplot of the coefficients \((\gamma_{\mu,s}, g_{Z,s})\), where \(s\) is a sector, the coefficients \(\gamma_{\mu,s}\) are the estimated time trends of the rents parameters \(\mu_{s,t}\) and the Cobb-Douglas intangible intensity \(\eta_{s,t}\), i.e. \(\mu_{s,t} = \alpha_{\mu,s} + \gamma_{\mu,s}t + \epsilon_{\mu,s,t}\), and \(g_{Z,s}\) are the average growth rates of multi-factor productivity in the corresponding sector. The top left panel reports these coefficients for the Manufacturing, Healthcare, and High-tech sectors when intangibles are measured using R&D capital (the slope of the simple OLS line is \(-0.06\), with a robust \(t\)-statistic of \(-0.64\)); the bottom left panel reports these when intangibles are measured using R&D capital plus organization capital (the slope of the simple OLS line is \(0.01\), with a robust \(t\)-statistic of \(0.28\)). The top and bottom right panels are similarly constructed, but subsectors belonging to the Consumer and Services subsectors; in the top panel, the slope of the OLS line is \(-0.47\), with a robust \(t\)-statistic of \(-0.31\); in the bottom panel, the slope is \(-0.34\), with a robust \(t\)-statistic of \(-0.58\).