Internet Appendix for
“Rents and Intangible Capital: A Q+ Framework”

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ABSTRACT

This Internet Appendix provides additional tables and figures supporting the main text.

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IA.A. Proofs of main results

IA.A.1. Lemma 1 and Result 1

Proof. The first-order necessary condition and the envelope theorem, for each capital type, are:

$$\begin{aligned} \Phi'_{n,t} &= q_{n,t} \\ \frac{\partial V^c_{t+1}}{\partial K_{n,t+1}} &= \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1} \frac{K_{n,t+2}}{K_{n,t+1}} \end{aligned} \tag{IA.1}$$

Multiplying the latter condition by $M_{t,t+1}K_{n,t+1}$, combining with the former condition, and taking expectations at time t, we obtain:

$$q_{n,t}K_{n,t+1} = \mathbb{E}_t \left[M_{t,t+1} \left(\Pi_{n,t+1}K_{n,t+1} - \Phi_{n,t+1}K_{n,t+1} \right) \right] + \mathbb{E}_t \left[M_{t,t+2}q_{n,t+1}K_{n,t+2} \right] \tag{IA.2}$$

Assuming the transversality condition $\lim_{k\to+\infty} \mathbb{E}_t \left[M_{t,t+k}q_{n,t+k-1}K_{n,t+k+1} \right] = 0$ holds for each type of capital, we can iterate forward and sum across capital types to obtain:

$$\begin{aligned} \sum_{n=1}^{N} q_{n,t}K_{n,t+1} &= \sum_{n=1}^{N} \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} \left\{ \Pi_{n,t+k}K_{n,t+k} - \Phi_{n,t+k}K_{n,t+k} \right\} \right] \\ &= \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} \sum_{n=1}^{N} \left\{ \Pi_{n,t+k}K_{n,t+k} - \Phi_{n,t+k}K_{n,t+k} \right\} \right]. \end{aligned} \tag{IA.3}$$

On the other hand, firm value excluding current distributions is given by:

$$V^e_t = \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} \left\{ \Pi_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+k}K_{n,t+k} \right\} \right].$$

Note that, given Assumption 1, we have that:

$$\begin{aligned} \Pi_{t+k} &= \mu \Pi_{K,t+k} K_{t+k} \\ &= \mu \sum_{n=1}^{N} \Pi_{K,t+k} \frac{\partial F_{t+k}}{\partial K_{n,t+k}} K_{n,t+k} = \mu \sum_{n=1}^{N} \Pi_{n,t+k} K_{n,t+k}, \end{aligned}$$

so that firm value can be rewritten as:

$$V_t^e = \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} \sum_{n=1}^{N} \{\mu \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+k} K_{n,t+k}\} \right]. \tag{IA.4}$$

Taking the difference between Equations (IA.4) and (IA.3) gives the result of Lemma 1. Result 1 follows from dividing both sides by $K_{n,t+1}$, and subtracting $q_{n,t}$. □

IA.A.2. Result 2

Proof. We start with the case of a general investment cost function:

$$\Phi_n(x) = x - 1 + \delta_n + \gamma_n \Gamma(x), \quad n = 1, ..., N.$$

The necessary first-order conditions, for each type of capital, are given by:

$$\begin{aligned} \Phi'_{n,t} &= q_{n,t}, \\ q_{n,t} &= \tfrac{1}{1+r} \left(\Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1} \frac{K_{n,t+2}}{K_{n,t+1}} \right). \end{aligned} \tag{IA.5}$$

which we can rewrite as:

$$(1+r)\Phi'_n\left(1+g_{n,t}\right) = \Pi_{n,t+1} - \Phi_n\left(1+g_{n,t+1}\right) + \Phi'_n\left(1+g_{n,t+1}\right)\left(1+g_{n,t+1}\right),$$

where $g_{n,t} \equiv \frac{K_{n,t+1}}{K_{n,t}} - 1$.

We next guess and verify that $g_{n,t} = \tilde{g}$ for $n = 1, ..., N$, for some trend growth rate $\tilde{g}$, is a solution. Substituting into the condition above, and re-arranging, we obtain:

$$R_n = \Pi_{n,t+1}, \tag{IA.6}$$

where we defined:

$$R_n \equiv (r - \tilde{g})\Phi'_n\left(1+\tilde{g}\right) + \Phi_n\left(1+\tilde{g}\right).$$

By homogeneity of degree 1 of the capital aggregator:

$$\Pi_{n,t+1} = \frac{1}{\mu} A_{t+1}^{1-\frac{1}{\mu}} K_{t+1}^{\frac{1}{\mu}-1} \frac{\partial K_{t+1}}{\partial K_{n,t+1}}, \quad n = 1, ..., N. \tag{IA.7}$$

Under our guess, the left-hand side of this expression is a constant. Moreover, the linear homogeneity of the capital aggregator implies that K_{t+1} also grows at rate $\tilde{g}$. Also, each partial derivative $\frac{\partial K_{t+1}}{\partial K_{n,t+1}}$ is homogeneous of degree 0 in each of its arguments, implying that they only depend on the $N-1$ ratios:

$$S_{m,t+1} = \frac{K_{m,t+1}}{K_{1,t+1}}, \quad m = 2, ..., N,$$

of each capital stock to the physical capital stock. Under our guess of constant growth $g_{n,t} = \tilde{g}$, these ratios must be constant. Therefore, the right-hand side of equation (IA.7) grows at rate $g^{1-1/\mu}\tilde{g}^{1/\mu}$. Given that the left-hand side is constant, this implies that it must be the case that $\tilde{g} = g$.

Taking the ratio of the first-order condition (IA.6) for $n = 2, ..., N$, to the first-order condition for $n = 1$, we obtain:

$$\frac{R_m}{R_1} = \frac{\partial K_t / \partial K_{m,t}}{\partial K_t / \partial K_{1,t}}, \quad n = 2, ..., N. \tag{IA.8}$$

This is a system of $N-1$ equations in $N-1$ variables, $\{S_{m,t+1}\}_{m=2}^{N}$. Assume that F is such that the solution $\left\{\tilde{S}_m\right\}_{m=2}^{N}$ is unique. Then, the steps above show that any solution with constant growth rates g for each capital stock, and capital ratios equal to $S_{m,t} = \tilde{S}_m$, satisfies the set of first-oder conditions (IA.5).[2]

[2]In Appendix IA.B.1, below, we provide the exact solution to the system of equations (IA.8) when the aggregator between capital types is constant elasticity of substitution (CES).

Finally, the expressions for the generalized user costs follow from the Taylor expansion:

$$
\begin{aligned}
(r-g)\Phi_n'(1+g)+\Phi_n(1+g) &= (r-g)(1+\gamma_n g+o(g))+\left(\delta_n+g+\frac{1}{2}\gamma_n g^2+o(g^2)\right)\\
&= r+\delta_n+\gamma_n rg-\frac{1}{2}\gamma_n g^2+o(g)(r-g)\\
&= r+\delta_n+\gamma_n rg+o(g).
\end{aligned}
$$

This approximation is exact when adjustment costs are given by the following functional form:

$$
\Phi_n\left(x\right)=x-1+\delta_n+\gamma_n r\left(x-1+(r-(x-1))\log\left(\frac{r-(x-1)}{r}\right)\right).
$$

It can be checked that this cost function is strictly convex and satisfies the conditions $\Phi_n(1)=\delta_n$, $\Phi_n'(1)=1$ and $\Phi_n''(1)=\gamma_n$, $n=1,2$. Additionally, this functions satisfies the relationship:

$$
(r-y)\Phi'(1+y)+\Phi(1+y)=r+\delta_n+\gamma_n ry,
$$

leading to an exact expression for the generalized user costs, $R_n=r+\delta_n+\gamma_n rg$. □

IA.A.3. Results on Total Q

Define total Q (Peters and Taylor, 2017) and total net investment as:

$$
\begin{aligned}
Q_{tot,t} &\equiv V_t^e/\sum_{n=1}^{N}K_{n,t+1},\\
g_{t,t+1} &\equiv \textstyle\sum_{n=1}^{N}s_{n,t+1}g_{n,t,t+1},
\end{aligned}
\tag{IA.9}
$$

where:

$$
s_{n,t+1}=K_{n,t+1}/\sum_{m=1}^{N}K_{m,t+1}. \tag{IA.10}
$$

In general, we have:

$$
Q_{tot,t}=\sum_{n=1}^{N}s_{n,t+1}q_{n,t}+(\mu-1)\sum_{n=1}^{N}s_{n,t+1}\sum_{k\geq 1}\mathbb{E}_t\left[M_{t,t+k}\Pi_{n,t+k}(1+g_{n,t+1,t+k})\right]. \tag{IA.11}
$$

Thus, $Q_{tot,t}$ is the sum of two terms: "total marginal q":

$$q_{tot,t} = \sum_{n=1}^{N} s_{n,t+1} q_{n,t},$$

and a term reflecting the rents generated by each type of capital. Unsurprisingly, in the presence of rents ($\mu > 1$), $Q_{tot,t}$ overstates $q_{tot,t}$, and does measure the incentive to invest.

However, even in the absence of rents ($\mu = 1$), Total Q may not be a sufficient statistic for total net investment. The total net investment rate is given by:

$$g_{t,t+1} = \sum_{n=1}^{N} s_{n,t+1} \Psi_{n,t} \left(q_{n,t} - 1 \right), \quad \Psi_{n,t} \equiv (\Phi'_{n,t})^{-1}.$$

In general, $g_{t,t+1}$ depends on each marginal q separately; it is not a monotone function of their weighted average $q_{tot,t}$. Thus, even when $\mu = 1$, and $Q_{tot,t} = q_{tot,t}$, the latter need not be a good proxy for total net investment.

When is $q_{tot,t}$ a sufficient statistic for total net investment? A first case is when adjustment costs are identical across types of capital goods, so that $\Psi_{n,t} = \Psi_t$, and when the function Ψ_t is linear. In that case, the expression above simplifies to $g_{t,t+1} = \Psi_t \left(q_{tot,t} - 1 \right)$, and total Q is indeed a sufficient statistic for total investment. Another case is if marginal q is equal across different types of capital. In this case, $q_{tot,t} = q_{n,t}$, and so $g_{t,t+1} = \sum_{n=1}^{N} s_{n,t+1} \Psi_{n,t} \left(q_{tot,t} - 1 \right)$, so that $q_{tot,t}$ is a sufficient statistic for investment.

When is marginal q equalized across types of capital? The framework studied by Peters and Taylor (2017) is an example where this is the case. That framework considers cost functions $C_1(K_{1,t}, K_{2,t})$ and $C_2(K_{1,t}, K_{2,t})$ that are not additively separable (as in this paper), but nevertheless satisfy $\frac{\partial(C_{1,t}+C_{2,t})}{\partial K_{1,t}} = \frac{\partial(C_{1,t}+C_{2,t})}{\partial K_{2,t}}$.[3] In our model, the difference between marginal

[3] As discussed in Appendix I.C of Crouzet and Eberly (2019), for the general class of cost functions of the form $C_1(K_{1,t}, K_{2,t})$ and $C_2(K_{1,t}, K_{2,t})$, a necessary condition for $q_{1,t} = q_{2,t}$ is that intangible and physical capital be perfect substitutes, and also that they depreciate at the same rates, and enter the capital aggregator with the same weights. In this sense, the conditions under which marginal q is equalized across types of capital, and thus under which $Q_{tot,t}$ and $q_{tot,t}$ are relevant to understanding the behavior of total net investment, are fairly specific.

q for two types of capital is given by:

$$q_{n,t} - q_{m,t} = \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k}(1 + g_{n,t,t+k}) \left\{ \Pi_{n,t+k} - \Pi_{m,t+k} - (\Phi_{n,t+k} - \Phi_{m,t+k}) \right\} \right].$$

Thus, a *sufficient* condition for equalized marginal qs is that (a) $\Pi_{n,t} = \Pi_{m,t}$, which, using Equation (5), implies that $\partial K_{n,t}/\partial K_{m,t} = 1$, or equivalently that capital types are perfect substitutes; and (b) adjustment costs are identical across capital types.

IA.B. Model extensions

This appendix provides more details on the extensions to the model discussed in Section E.

IA.B.1. Preliminary: partial solution with CES capital aggregator

We start by stating a preliminary result, which partially characterizes the solution to the model when capital aggregation is CES. We use this result in several different extensions to the model.

LEMMA 1: *Assume that :*

$$\begin{aligned} \Pi_t &= A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}}, \\ K_t &= \left(\sum_{n=1}^{N} \eta_n K_{n,t}^{\rho} \right)^{\frac{1}{\rho}}, \quad \rho \leq 1, \quad \sum_{n=1}^{N} \eta_n = 1. \end{aligned} \tag{IA.12}$$

Then, the solution to the model satisfies:

$$K_{n,t+1} = \xi_{n,t+1} A_{t+1}, \quad K_{t+1} = \xi_{t+1} A_{t+1},$$

where:

$$R_{t+1} \equiv \left(\sum_{n=1}^{N} \eta_n^{\frac{1}{1-\rho}} R_{n,t+1}^{-\frac{\rho}{1-\rho}} \right)^{-\frac{1-\rho}{\rho}},$$

$$\xi_{t+1} \equiv (\mu R_{t+1})^{-\frac{\mu}{\mu-1}},$$

$$\xi_{n,t+1} \equiv \left(\eta_n \frac{R_{t+1}}{R_{n,t+1}} \right)^{\frac{1}{1-\rho}} \xi_{t+1}.$$

and:

$$R_{n,t+1} \equiv \frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} + \Phi_{n,t+1}(1+g_{n,t+1}) - \Phi'_{n,t+1}(1+g_{n,t+1}) \frac{K_{n,t+2}}{K_{n,t+1}}, \quad n = 1, ..., N.$$

Proof. The necessary first-order conditions and the envelope conditions of the general model are:

$$\begin{aligned} \Phi'_{n,t}(1+g_{n,t}) &= q_{n,t} \\ \frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} &= \Pi_{n,t+1} - \Phi_{n,t+1}(1+g_{n,t+1}) + \Phi'_{n,t+1}(1+g_{n,t+1}) \frac{K_{n,t+2}}{K_{n,t+1}}, \end{aligned}$$

where: $q_{n,t} = \mathbb{E}_t \left[M_{t,t+1} \frac{\partial V_{t+1}^c}{K_{n,t+1}} \right]$. Using the first part of Assumption IA.12, the envelope conditions imply that:

$$A_{t+1}^{1-\frac{1}{\mu}} K_{t+1}^{\frac{1}{\mu}-1} \frac{\partial K_{t+1}}{\partial K_{n,t+1}} = \mu R_{n,t+1}, \quad n = 1, ..., N.$$

When the capital aggregator is CES, as in the second part of assumption IA.12, one can check that the solution to these N equations takes the form reported in Lemma 1. □

IA.B.2. Closed form solutions with uncertainty

IA.B.2.1. Result

Assume that the fundamentals process follows:

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \begin{cases} 1 + g_{t-1} & \text{w.p. } 1 - \lambda \\ 1 + \tilde{g} & \text{w.p. } \lambda \end{cases} \tag{IA.13}$$

Here, $\tilde{g}$ is drawn, at time t, from a distribution $F(.)$, which is time-invariant, and the draw is independent of past realizations of g_t. The investment gap for physical capital is then given by

$$G_{1,t} = \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_1) + S + \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_2)S. \tag{IA.14}$$

The function $\nu(g_t)$ is given by:

$$\nu(g_t) = (1 - \lambda)\mathbb{E}\left[\frac{1}{1 + \lambda(1 + \tilde{g})/(r - \tilde{g})}\right] g_t + \lambda\mathbb{E}\left[\frac{1}{1 - (1 - \lambda)(1 + \tilde{g})/(1 + r)}\tilde{g}\right] \tag{IA.15}$$

It depends on the parameters λ and on the distribution $F(.)$. When $\lambda = 0$, the firm's growth rate is constant, and $\nu(g_t) = g_t$. When $\lambda = 1$, the growth rate of the firm is *i.i.d* and $\nu(g_t) = \mathbb{E}[\tilde{g}]$. Thus, the term $\frac{1}{r - \nu(g_t)}$ is analogous to the standard Gordon growth formula, but the function $\nu(.)$ adjusts for shifts in the growth rate of fundamentals. Thus, the key insights from the discussion in the main text survive. In particular, even with stochastic growth, the two rents terms can be thought of as the present value of markups over the user costs of physical and intangible capital, respectively.

IA.B.2.2. Derivations of the result

We define the function $\nu(g_t)$ as:

$$\nu(g_t) = r - \zeta(g_t)^{-1},$$

where:

$$\zeta(g_t) \equiv E_t\left[\sum_{k \geq 1}(1 + r)^{-k}\frac{A_{t+k}}{A_{t+1}}\right].$$

To obtain the decomposition of the investment gap in closed form, we use the following lemma.

LEMMA 2: *When fundamentals follow the process (IA.13), the function $\zeta(g_t)$ is given by:*

$$\zeta(g_t) = \mathbb{E}\left[\frac{r - \tilde{g}}{r - \tilde{g} + \lambda(1 + \tilde{g})}\right]^{-1}\frac{1}{r - g_t + \lambda(1 + g_t)}.$$

It is straightforward to check that this solution for $\zeta(g_t)$ then implies expression (IA.15).

Proof. Let:

$$\zeta(g_t) \equiv \mathbb{E}_t \left[\sum_{k \geq 1} (1+r)^{-k} \frac{A_{t+k}}{A_{t+1}} \right],$$

then $\zeta(g_t)$ satisfies:

$$\zeta(g_t) = \frac{1}{1+r} \left(1 + \lambda \mathbb{E}\left[(1+\tilde{g})\zeta(\tilde{g})\right] + (1-\lambda)(1+g_t)\zeta(g_t)\right),$$

where we used the law of iterated expectations, and the law of motion for A_t. Solving for $\zeta(g_t)$:

$$\zeta(g_t) = \frac{1+\lambda X}{1+r-(1-\lambda)(1+g_t)}, \quad X \equiv \mathbb{E}\left[(1+\tilde{g})\zeta(\tilde{g})\right]$$

Multiplying by $(1+g_t)$, taking expectations on both sides, and solving for X, we obtain:

$$X = \frac{Y}{1-\lambda Y}, \quad Y \equiv \mathbb{E}\left[\frac{1+g_t}{1+r-(1-\lambda)(1+g_t)}\right].$$

After rearranging, we have:

$$\zeta(g_t) = \frac{1}{1-\lambda Y} \frac{1}{r - g_t + \lambda(1+g_t)},$$

which gives the result, using the solution for Y. □

We now derive the closed-form expressions for the decomposition of the investment gap when the fundamentals process is given by (IA.13). We rely on the general solution of the model with uncertainty and a CES aggregator described in Lemma (1). With the fundamentals process (IA.13), A_{t+1} is in the time-t information set. Using Lemma (2), since:

$$K_{n,t+1} = \xi_{n,t+1} A_{t+1},$$

and since the $\{K_{n,t+1}\}_{n=1}^{N}$ are chosen as of time t, this implies that the $\{\xi_{n,t+1}\}_{n=1}^{N}$ are also in the information set of time t. Therefore, the user costs $\{R_{n,t+1}\}_{n=1}^{N}$ are also in the time-t information set, so that, using Assumption 3:

$$\forall n = 1, ..., N, \quad \forall t, \quad \mathbb{E}_t\left[M_{t,t+1} R_{n,t+1}\right] = \mathbb{E}_t\left[M_{t,t+1}\right] R_{n,t+1} = (1+r)^{-1} R_{n,t+1}.$$

Assumption 3 also implies that:

$$q_{n,t} = 1$$

$$R_{n,t+1} = \frac{\partial V^c_{t+1}}{\partial K_{n,t+1}} - (1-\delta_n).$$

Using the relationship $\mathbb{E}_t\left[M_{t,t+1}R_{n,t+1}\right] = (1+r)^{-1}R_{n,t+1}$ and the definition of $q_{n,t}$, these equations imply that:[4]

$$R_{n,t+1} = r+\delta_n, \quad \forall n=1,...,N, \forall t.$$

This implies that:

$$\xi_{n,t+1} = \xi_n, \quad \forall n=1,...,N, \forall t.$$

Thus, $K_{n,t} = \xi_n A_t$ for all n,t. Therefore, the net growth rates of capital stocks are given by $g_{n,t} = g_t$, and the ratios $S_{n,m,t} \equiv \frac{K_{m,t+1}}{K_{n,t+1}}$ are constant.

Using these results, and the fact that $q_{n,t}=1$ and $\Pi_{n,t} = R_{n,t} = r+\delta_n$ for all n,t, we can write the investment gap decomposition for capital of type n as:

$$\begin{aligned}
Q_{n,t} - q_{n,t} &= (\mu-1)\,(r+\delta_n)\sum_{k\geq 1}\mathbb{E}_t\left[M_{t,t+k}(1+g_{t+1,t+k})\right] \\
&+ \sum_{\substack{m=1\\ m\neq n}}^{N} S_{m,n} \\
&+ (\mu-1)\sum_{\substack{m=1\\ m\neq n}}^{N} S_{m,n}(r+\delta_m)\sum_{k\geq 1}\mathbb{E}_t\left[M_{t,t+k}(1+g_{t+1,t+k})\right],
\end{aligned}$$

where we have denoted:

$$g_{t+1,t+k} \equiv \frac{K_{n,t+k}}{K_{n,t+1}} = \frac{A_{t+k}}{A_{t+1}}.$$

Using the lemma above, as well as the fact that we have assumed that $M_{t,t+k} = (1+r)^{-k}$, the present values that appear in the rents term in the investment gap above are given by:

$$\sum_{k\geq 1}\mathbb{E}_t\left[M_{t,t+k}(1+g_{t+1,t+k})\right] = \sum_{k\geq 1}\mathbb{E}_t\left[(1+r)^{-k}\frac{A_{t+k}}{A_{t+1}}\right] = \zeta(g_t) = \frac{1}{r-\nu(g_t)}.$$

[4] The result is somewhat stronger: $\frac{\partial V^c_{t+1}}{\partial K_{n,t+1}} = 1+r$, since $\frac{\partial V^c_{t+1}}{\partial K_{n,t+1}}$ is also in the information set of time t.

Thus the investment gap decomposition is given by:

$$Q_{n,t} - q_{n,t} = \frac{(\mu - 1)(r + \delta_n)}{r - \nu(g_t)} + \sum_{\substack{m=1 \\ m \neq n}}^{N} S_{m,n} + (\mu - 1) \sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{(\mu - 1)(r + \delta_m)}{r - \nu(g_t)} \times S_{m,n},$$

and the decomposition (IA.14) is a particular case of this expression for $N = 2$.

IA.B.3. Market power, decreasing returns, and rents

Proofs for key lemmas are reported at the end of this section.

IA.B.3.1. Model

Consider a monopolistic firm that uses M distinct variable inputs, $\{M_{j,t}\}_{n=1}^{N}$, as well as capital, to produce and sell output to consumers whose demand function is given by:

$$Y_t = P_t^{-\frac{\mu_S}{\mu_S - 1}} D_t,$$

where P_t is the price of the good, D_t indexes aggregate demand, and $\mu_S \geq 1$ will be the markup over the marginal cost of gross output (or sales) charged by the firm. The prices of the variable inputs are given by $\{W_{j,t}\}_{n=1}^{N}$. The total input of capital, K_t (which is made up of tangible and intangible capital), is quasi-fixed; that is, it is chosen dynamically by the firm but cannot be modified immediately. The static variable profit maximization problem of the firm is then:

$$\begin{aligned} \Pi_t \quad = \quad & \max_{\{M_{j,t}\}_{j=1}^{M}, P_t} \quad P_t^{-\frac{1}{\mu_S - 1}} D_t - \sum_{j=1}^{M} W_{j,t} M_{j,t} \\ & \text{s.t.} \qquad Z_t \left(K_t^{\alpha} \left(\prod_{j=1}^{M} M_{j,t}^{\nu_j} \right)^{1-\alpha} \right)^{\zeta} \geq P_t^{-\frac{\mu_S}{\mu_S - 1}} D_t \quad [MC_t]. \end{aligned} \tag{IA.16}$$

In addition to market power on the output market, with markup μ_S, the firm's problem features returns to scale ζ with respect to an aggregate of capital inputs K_t and variable inputs. We assume that:

$$\zeta \leq \mu_S,$$

so that the maximization problem has a unique interior solution. In general, we will be interested in cases where $\zeta \leq 1$, so that there are decreasing returns to scale, but constant or increasing returns are possible so long as the markup μ_S is sufficiently high. The aggregate of variable inputs, including labor, is given by:

$$M_t = \prod_{j=1}^{N} M_{j,t}^{\nu_j},$$

where the Cobb-Douglas shares $\{\nu_j\}_{j=1}^{M}$ are assumed to sum up to 1. The parameter α therefore represents the Cobb-Douglas elasticity of substitution between capital and aggregated variable inputs. The Lagrange multiplier MC_t measures the marginal cost of gross output. Additionally, note that $Y_t = P_t^{-\frac{\mu_S}{\mu_S - 1}} D_t$ is gross output, and $S_t = P_t Y_t = P_t^{-\frac{1}{\mu_S - 1}} D_t$ is total revenue. Finally, Π_t represents operating surplus (revenue minus variable costs):

$$\Pi_t = S_t - \sum_{j=1}^{M} W_{j,t} M_{j,t}.$$

LEMMA 3: *After minimizing variable cost and optimally choosing the price, operating surplus is given by:*

$$\Pi_t = A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}} \tag{IA.17}$$

where:

$$\begin{aligned} \mu &\equiv 1 + \frac{\chi - 1}{\alpha} \geq 1 \\ \chi &\equiv \frac{\mu_S}{\zeta} \quad \geq 1 \end{aligned} \tag{IA.18}$$

and:

$$\begin{aligned} A_t &\equiv \left(\frac{\chi}{1-\alpha}\right)^{-\frac{\chi}{\chi-1}} \left(\frac{\chi}{1-\alpha} - 1\right)^{\frac{\chi-(1-\alpha)}{\chi-1}} D_t^{\frac{\chi-\zeta}{\chi-1}} W_t^{-\frac{1-\alpha}{\chi-1}} Z_t^{\frac{1}{\zeta(\chi-1)}} \\ W_t &\equiv \prod_{j=1}^{M} \left(\frac{W_{j,t}}{\nu_j}\right)^{\nu_j} \end{aligned} \tag{IA.19}$$

There are two main points to note about Lemma 3. First, Equation (IA.17) indicates that the functional form for operating surplus as a function of capital K_t and exogenous conditions A_t is the same as the one used in the balanced growth model of Section C. Thus, the firm problem (IA.16) provides a microfoundation for the functional form assumption for operating surplus as a function of capital in that model. The exogenous process A_t can then be interpreted as reflecting simultaneously the cost of intermediate inputs W_t, demand D_t, and total factor productivity Z_t.

Second, Equation (IA.18) gives a structural interpretation of the reduced-form rents parameter μ used in our model. It indicates that the reduced-form rents parameter μ increases with markups μ_S, and decreases with the degree of returns to scale ζ and the Cobb-Douglas elasticity of capital with respect to variable inputs, α. Fixing α, rents will therefore be elevated either when markups are substantially above 1, or when returns to scale are substantially below 1, or both.

IA.B.3.2. Partial identification of pure and quasi-rents

Perhaps most importantly, Equation (IA.18) indicates that the reduced-form rents parameter μ does not vary independently with markups μ_S and returns to scale ζ. Instead, it is only a function of their *ratio* $\chi = \frac{\mu_S}{\zeta}$. As a result, separating pure rents μ_S from quasi-rents ζ is challenging, and generally requires a direct estimate of the production function. The following Lemma formalizes this point.

LEMMA 4: *Given data on the ratios of variable input costs, labor costs, capital costs, value added and operating surplus to either sales, value added, or operating surplus, the markup μ_S and the degree of returns to scale ζ cannot be separately identified.*

Without loss of generality, we assume that the first variable input is labor. In that case, value added in the model is given by:

$$\mathrm{VA}_t = S_t - (W_t M_t - W_{1,t} M_{1,t}) = \Pi_t + W_{1,t} M_{1,t}.$$

Table V reports the ratios of variable input costs, labor costs, capital costs, value added and operating surplus to either sales, value added, or operating surplus in the model above. It also reports the expressions for rents as a fraction of sales, value added or operating surplus, where rents are defined as:

$$\mathrm{Re}_t = \Pi_t - (R_{1,t} K_{1,t} + R_{2,t} K_{2,t}).$$

In order to pin down competitive payments to capital, we assume that the firm solves the same dynamic problem as in the balanced growth model of Section A. In this case, since competitive payments to capital, $R_{1,t}K_{1,t} + R_{2,t}K_{2,t}$, satisfy:

$$R_{1,t}K_{1,t} + R_{2,t}K_{2,t} = \frac{1}{\mu}\Pi_t = \frac{\alpha}{\chi - (1-\alpha)}\Pi_t,$$

Table V uses this expression to solve for the different ratios in Lemma 4 as a function of the structural parameters α, ζ and μ^S.

The main point of Table V is that none of the 18 ratios depend independently on μ_S and ζ; instead, they only depend on their ratio, the reduced-form parameter $\chi = \frac{\mu_S}{\zeta}$. Thus, manipulation of these ratios cannot be used to separately identify μ_S and ζ.

Lemma 4 is useful because a number of the approaches that have been used in the literature to estimate markups rely partially or entirely on these ratios. The implication of Lemma 4 is then that these approaches do not in fact identify markups, but rather, some function of the reduced-form parameter χ, which depends on both markups and decreasing returns. (Put differently, these approaches only identify markups under the assumption of constant returns to scale, $\zeta = 1$). We next give three examples of such approaches.

Example 1: the surplus ratio approach The surplus ratio approach is used by Baqaee and Farhi (2020). It consists of using the ratio of operating surplus to sales, $x_\Pi = \Pi_t/S_t$, net of the ratio of capital costs to sales, $x_K = (R_tK_{1,t} + R_tK_{2,t})/S_t$, to estimate markups. The results of Table V imply that, in our balanced growth model,

$$x_\Pi - x_K = \left(1 - \frac{1-\alpha}{\chi}\right) - \left(\frac{\alpha}{\chi}\right) = \frac{\chi - 1}{\chi}.$$

Thus, this approach recovers an estimate of χ, but not of μ_S and ζ separately.

Example 2: the user cost approach The user cost approach is used by Gutiérrez and Philippon (2017). It consists of computing an implied markup $\hat{\mu}$ by inverting relationship:

$$\frac{\Pi_t}{K_t} = R_t + \left(1 - \frac{1}{\hat{\mu}}\right)\frac{S_t}{K_t},$$

where K_t denotes total capital input. In our model, interpreting R_t as $R_t = R_{1,t}K_{1,t}/K_t + R_{2,t}K_{2,t}/K_t$, we can rewrite this relationship as:

$$\frac{1}{\nu_K} \;=\; 1 + \left(1 - \frac{1}{\hat{\mu}}\right)\frac{1}{s_K},$$

where:

$$\nu_K = \frac{R_{1,t}K_{1,t} + R_{2,t}K_{2,t}}{\Pi_t} \qquad \text{and } s_K = \frac{R_{1,t}K_{1,t} + R_{2,t}K_{2,t}}{S_t}$$

are the ratios of competitive payments to capital to either operating surplus, or sales. However, manipulation of the results of Table V shows that:

$$\frac{1}{\nu_K} = 1 + \left(1 - \frac{1}{\chi}\right)\frac{1}{s_K}.$$

In other words, the "implied markup" $\hat{\mu}$ derived in the user cost approach recovers, from the standpoint of our model, the reduced form parameter χ.[5]

Example 3: the cost share approach The cost share approach is used by De Loecker et al. (2020). It consists of estimating the markup using the ratio $x_M = (W_t M_t)/S_t$ of variable costs to sales. Using the results of Table V, this ratio is given by:

$$x_M = \frac{1-\alpha}{\chi}.$$

Thus, alone, this ratio does not separately identify α, μ_S, and χ. As a result, De Loecker et al. (2020) estimate, separately, the elasticity of gross output with respect to variable costs. In our model, this elasticity is given by:

$$\eta = (1-\alpha)\zeta.$$

[5]Equivalently, the expression of the user cost relationship in the context of a model that allows for decreasing returns is:

$$\frac{\Pi_t}{K_t} = R_t + \left(1 - \frac{1}{\chi}\right)\frac{S_t}{K_t} = R_t + \left(1 - \frac{\zeta}{\mu_S}\right)\frac{S_t}{K_t}.$$

One can then recover the markup μ by forming:

$$\frac{\eta}{x_M} = \frac{\mu_S(1-\alpha)\zeta}{(1-\alpha)\zeta} = \mu_S.$$

This method therefore cannot identify the markup using only the variable cost share; a separate estimate of $(1-\alpha)\zeta$ (that is, a direct estimate of the production function) must be constructed first.

We conclude by noting another consequence of the results of Table V.

LEMMA 5 (The share of rents): *The share of pure rents in sales, value added, and operating surplus are given by:*

$$\begin{aligned}
x_{Re} &= \frac{Re_t}{S_t} = \frac{\chi-1}{\chi}, \\
s_{Re} &= \frac{Re_t}{VA_t} = \frac{\chi-1}{\chi-(1-\alpha)(1-\nu_1)} = (1-s_L)\frac{\mu-1}{\mu}, \\
\nu_{Re} &= \frac{Re_t}{\Pi_t} = \frac{\chi-1}{\chi-(1-\alpha)} = \frac{\mu-1}{\mu},
\end{aligned}$$

where $s_L = (W_{1,t}M_{1,t})/VA_t$ is the labor share of value added.

Thus, the reduced-form parameter χ should be interpreted as controlling the size of rents relative to *sales*, whereas the reduced-form parameter μ controls the size of rents relative to *operating surplus*. Again, rents shares do not depend on the separate values of μ_S and ζ, but only on their ratio. This implies that the size of rents relative to sales, operating surplus, or value added does not depend on markups μ_S or decreasing returns ζ independently, but only on their ratio.[6]

[6] Additionally, this lemma indicates is that knowledge of the reduced-form parameter μ, along with a measure of the labor share, is sufficient to compute the implied ratio of rents to value added, which is a commonly used measure of the size of rents (see, e.g., Karabarbounis and Neiman 2019). We use this result in order to map our estimates of μ to a value for the share of rents as a fraction of value added.

IA.B.3.3. Proofs for results on market power and decreasing returns

Proof of Lemma 3. To make notation lighter, we rewrite the firm problem (1) as:

$$\begin{aligned}
\Pi_t &= \max_{\{M_{j,t}\}_{j=1}^{M}, P_t} \quad P_t^{-\frac{1}{\mu_S - 1}} D_t - \sum_{j=1}^{M} W_{j,t} M_{j,t} \\
&\qquad \text{s.t.} \qquad X_t \left(\prod_{j=1}^{N} M_{j,t}^{\nu_j} \right)^{\gamma} \geq P_t^{-\frac{\mu_S}{\mu_S - 1}} D_t \quad [MC_t]
\end{aligned}$$

where:

$$\gamma \equiv (1 - \alpha)\zeta, \quad X_t \equiv Z_t K_t^{\alpha\zeta}.$$

We solve this problem in two steps. The cost minimization problem is:

$$\begin{aligned}
VC_t &= \min_{\{M_{j,t}\}_{j=1}^{M}} \quad \sum_{j=1}^{M} W_{j,t} M_{j,t}, \\
&\qquad \text{s.t.} \qquad X_t \left(\prod_{j=1}^{N} M_{j,t}^{\nu_j} \right)^{\gamma} \geq Y_t \quad [\tilde{MC}_t].
\end{aligned}$$

Define:

$$W_t \equiv \prod_{j=1}^{M} \left(\frac{W_{j,t}}{\nu_j} \right)^{\nu_j}, \quad M_t \equiv \prod_{j=1}^{M} M_{j,t}^{\nu_j}.$$

Then solution to the cost minimization problem is:

$$\frac{W_{j,t} M_{j,t}}{W_t M_t} = \nu_j,$$

and:

$$\begin{aligned}
VC_t &= W_t M_t = W_t \left(\frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}} = \gamma \tilde{MC}_t Y_t, \\
\tilde{MC}_t &= \frac{W_t}{\gamma Y_t} \left(\frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}} = \frac{\partial VC_t}{\partial Y_t} = \frac{VC_t}{\gamma Y_t}, \\
M_t &= \left(\frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}}.
\end{aligned}$$

Substituting the expression for total variable cost, the price and quantity choice problem is:

$$\begin{aligned}
\Pi_t &= \max_{P_t, Y_t} \quad P_t^{-\frac{1}{\mu_S - 1}} D_t - W_t \left(\frac{Y_t}{X_t} \right)^{\frac{1}{\gamma}}, \\
&\qquad \text{s.t.} \quad Y_t \geq P_t^{-\frac{\mu_S}{\mu_S - 1}} D_t \qquad [MC_t].
\end{aligned}$$

The first-order conditions are

$$P_t = \mu_S MC_t \text{ and } MC_t = \frac{W_t}{\gamma Y_t}\left(\frac{Y_t}{X_t}\right)^{\frac{1}{\gamma}}$$

so that $\tilde{MC}_t = MC_t$, and:

$$Y_t \;=\; \left(\frac{\mu_S}{\gamma}\right)^{-\frac{\gamma\mu_S}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} D_t^{\frac{\gamma(\mu_S-1)}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} W_t^{-\frac{\gamma\mu_S}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} X_t^{\frac{\mu_S}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}}.$$

We can use the solution to the cost minimization problem to obtain:

$$VC_t = \left(\frac{\mu_S}{\gamma}\right)^{-\frac{\mu_S}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} D_t^{\frac{\mu_S-1}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} W_t^{-\frac{\gamma}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} X_t^{\frac{1}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}}.$$

Moreover, profits are given by:

$$\Pi_t = \left(\frac{\mu_S}{\gamma} - 1\right) VC_t,$$

so that:

$$\Pi_t = \left(\frac{\mu_S}{\gamma} - 1\right)\left(\frac{\mu_S}{\gamma}\right)^{-\frac{\mu_S}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} D_t^{\frac{\mu_S-1}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} W_t^{-\frac{\gamma}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}} X_t^{\frac{1}{\gamma(\mu_S-1)+(1-\gamma)\mu_S}}.$$

Using the definitions of γ and X_t, we can write this as:

$$\begin{aligned}
\Pi_t &= A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}}, \\
\mu &\equiv 1 + \frac{\mu_S/\zeta - 1}{\alpha}, \\
A_t &= \left(\left(\frac{\mu_S}{(1-\alpha)\zeta} - 1\right)\left(\frac{\mu_S}{(1-\alpha)\zeta}\right)^{-\frac{\mu_S}{\mu_S-(1-\alpha)\zeta}} D_t^{\frac{\mu_S-1}{\mu_S-(1-\alpha)\zeta}} W_t^{-\frac{(1-\alpha)\zeta}{\mu_S-(1-\alpha)\zeta}} Z_t^{\frac{1}{\mu_S-(1-\alpha)\zeta}}\right)^{\frac{\mu_S-(1-\alpha)\zeta}{\mu_S-\zeta}}.
\end{aligned}$$

Substituting $\chi = \frac{\mu_S}{\zeta}$ in these expressions gives the results of Lemma 3. □

IA.B.4. *Heterogeneous rents parameters*

IA.B.4.1. Model and assumptions

The model of the firm is a generalization of the baseline model. The firm solves:

$$V_t^c(\mathbf{K}_t) = \max_{\mathbf{K}_{t+1}} \quad \tilde{\Pi}_t(\mathbf{K}_t) - \tilde{\Phi}_t(\mathbf{K}_t, \mathbf{K}_{t+1}) + \mathbb{E}_t\Big[M_{t,t+1}V_{t+1}^c(\mathbf{K}_{t+1})\Big] \tag{IA.20}$$

where $V_t^c(.)$ is the value of the firm including distributions.

We make the same assumptions regarding adjustment costs as in the baseline model. Different from the baseline model, we replace the two assumptions 1-2 by the following assumption.

ASSUMPTION 1: *There exist real numbers* $\{\mu_n\}_{n=1}^N$, $\mu_n \geq 1\,\forall n = 1, ..., N$, *such that the function* $\tilde{\Pi}_t$ *satisfies:*

$$\forall\, \mathbf{K}_t = \{K_{n,t}\}_{n=1}^N, \quad \tilde{\Pi}_t(\mathbf{K}_t) = \sum_{n=1}^N \mu_n \tilde{\Pi}_{n,t}(\mathbf{K}_t)\, K_{n,t}, \tag{IA.21}$$

where $\Pi_{n,t}$ *is the marginal revenue product of capital of type* n*, i.e.* $\Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_{n,t}}$.

Under our baseline assumptions, the revenue function satisfies:

$$\forall\, \mathbf{K}_t = \{K_{n,t}\}_{n=1}^N, \quad \tilde{\Pi}_t = \mu \sum_{n=1}^N \tilde{\Pi}_{n,t} K_{n,t}.$$

Assumption (1) thus broadens our baseline assumptions, but allowing heterogeneity in the wedge between marginal and average revenue for each type of capital. A fairly general functional form that satisfies Assumption 1 is:

$$\begin{aligned}
\tilde{\Pi}_t(\mathbf{K}_t) \quad &= \quad G_t\left(\left\{Y_t^{(n)}(K_{n,t})\right\}_{n=1}^N\right), \quad \text{where :} \\
G_t \quad &\text{is} \quad \text{homogeneous of degree } \frac{1}{\mu}, \quad \mu \geq 1 \\
\forall\, n = 1, ..., N, \quad Y_t^{(n)} \quad &\text{is} \quad \text{homogeneous of degree } \frac{1}{\tilde{\mu}_n}, \quad \tilde{\mu}_n \geq 1
\end{aligned} \tag{IA.22}$$

In this case, it is easy to check that the rents parameters $\{\mu_n\}_{n=1}^{N}$ are given by:

$$\forall\, n = 1, ..., N, \quad \mu_n = \mu \times \tilde{\mu}_n. \tag{IA.23}$$

Our baseline model is a particular case of the functional form (IA.22), with $G(\mathbf{K}_t) \equiv \Pi_t(F_t(\mathbf{K}_t))$, and $Y_t^{(n)}(K_{n,t}) = K_{n,t}$. In this case, $\tilde{\mu}_n = 1$ for all $n = 1, ..., N$, so that $\mu_n = \mu$, that is, rents are the same across capital types. One concrete example of a function satisfying equation (IA.22) is:

$$\tilde{\Pi}_t(\mathbf{K}_t) \;=\; A_t^{1-\frac{1}{\mu}} \left(\sum_{n=1}^{N} \eta_n \left(A_{n,t}^{1-\frac{1}{\tilde{\mu}_n}} K_{n,t}^{\frac{1}{\tilde{\mu}_n}} \right)^{\rho} \right)^{\frac{1}{\mu\rho}}. \tag{IA.24}$$

Heuristically, this aggregator can be described as follows. Production of final goods in the firm takes place in two stages. In the first stage, each type of capital is used (potentially in conjunction with flexible labor, but separately from other capital types) to produce intermediate varieties. In the second stage, intermediate varieties are aggregated into a final good. In the first stage (intermediate input production), the firm has decreasing returns with respect to each type of capital: intermediate output is $Y_t^{(n)} = (A_{n,t})^{1-\frac{1}{\mu_n}} K_{n,t}^{\frac{1}{\mu_n}}$, where μ_n indexing the strength of the decreasing returns for each capital type n. In the second stage (aggregation), the firm has monopoly power in the consumer goods market. Revenue is then given by $A_t^{1-\frac{1}{\mu}} \left(\sum_{n=1}^{N} \eta_n Y_{n,t}^{\rho} \right)^{\frac{1}{\rho\mu}}$, where μ governs the firm's market power in final goods markets, and ρ the substitutability between intermediates.

IA.B.4.2. The investment gap with heterogeneous rents

RESULT 1: *Firm value can be written as:*

$$V_t^e \;=\; \sum_{n=1}^{N} q_{n,t} K_{n,t+1} \;+\; \sum_{n=1}^{N} (\mu_n - 1) \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \tilde{\Pi}_{n,t+k} K_{n,t+k} \right], \tag{IA.25}$$

and the investment gap for each capital type can be written as:

$$Q_{n,t} - q_{n,t} = (\mu_n - 1) \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \tilde{\Pi}_{n,t+k} (1 + g_{m,t+1,t+k}) \right] \tag{IA.26}$$

$$+ \sum_{\substack{m=1 \\ m \neq n}}^{N} q_{m,t} S_{m,n,t+1} \tag{IA.27}$$

$$+ \sum_{\substack{m=1 \\ m \neq n}}^{N} (\mu_m - 1) \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \tilde{\Pi}_{m,t+k} (1 + g_{m,t+1,t+k}) \right] \times S_{m,n,t+1}, \tag{IA.28}$$

where $1 + g_{n,t+1,t+k} \equiv \dfrac{K_{n,t+k}}{K_{n,t+1}}$, *and* $S_{m,n,t+1} \equiv \dfrac{K_{n,t+1}}{K_{m,t+1}}$.

Proof. Following the same steps as in proof for the baseline model, we have:

$$\begin{aligned} V_t^e - \sum_{n=1}^{N} q_{n,t} K_{n,t+1} &= \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \left\{ \tilde{\Pi}_t - \tilde{\Pi}_{n,t+k} K_{n,t+k} \right\} \right] \\ &= \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} (\mu_n - 1) \tilde{\Pi}_{n,t+k} K_{n,t+k} \right] \\ &= \sum_{n=1}^{N} (\mu_n - 1) \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \tilde{\Pi}_{n,t+k} K_{n,t+k} \right]. \end{aligned}$$

The decomposition of the investment gap follows from this expression. □

The interpretation of the investment gap decomposition is the same as for our baseline model. The terms $(\mu_n - 1) \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \tilde{\Pi}_{n,t+k} K_{n,t+k} \right]$ represent the gap between the average and marginal revenue products of capital of type n, and therefore capture rents attributable to capital of type n. Rents remain additively separable across capital types, as in the baseline decomposition. The main difference with our baseline decomposition is that the size of rents (per unit of marginal revenue product) can now differ between capital types.

IA.B.4.3. Analytical expression in balanced growth

We next given an analytical solution for a balanced growth version of the model above. Our definition of balanced growth is the same as in our baseline analysis, except that we also assume that the function $\tilde{\Pi}_t$ is given by Equation (IA.24). In this case, we have the following result, of which Result (5) is a particular case.

RESULT 2: *Let $n = 1$ denote physical capital. Along the balanced growth path, the physical investment gap is given by:*

$$Q_1 - q_1 = \sum_{m\geq 2} q_m S_m + \frac{(\mu_1 - 1)R_1}{r-g} + \sum_{m\geq 2} \frac{(\mu_m - 1)R_m}{r-g} \times S_m, \tag{IA.29}$$

where:

$$R_n = (r-g)\Phi_n'(1+g) + \Phi_n(1+g), \quad n = 1, ..., N.$$

Proof. Along the balanced growth path, the necessary first-order conditions, for capital of type n, are given by:

$$\begin{aligned} \Phi'_{n,t} &= q_{n,t}, \\ q_{n,t} &= \frac{1}{1+r}\left(\tilde{\Pi}_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1}\frac{K_{n,t+2}}{K_{n,t+1}}\right). \end{aligned} \tag{IA.30}$$

We can write these conditions as:

$$(1+r)\Phi_n'\left(1+g_{n,t}\right) = \tilde{\Pi}_{n,t+1} - \Phi_n\left(1+g_{n,t+1}\right) + \Phi_n'\left(1+g_{n,t+1}\right)\left(1+g_{n,t+1}\right),$$

where $g_{n,t} \equiv \frac{K_{n,t+1}}{K_{n,t}} - 1$.

We next guess and verify that $g_{n,t} = g$ for $n = 1, ..., N$ is a solution. Substituting into the condition above, and re-arranging, we obtain:

$$R_n \equiv (r-g)\Phi_n'\left(1+g\right) + \Phi_n\left(1+g\right) = \tilde{\Pi}_{n,t+1}.$$

Moreover, using the functional form (IA.24):

$$\tilde{\Pi}_{n,t+1} = \frac{\eta_n}{\mu\tilde{\mu}_n} A_{t+1}^{1-\frac{1}{\mu}} K_{t+1}^{\frac{1}{\mu}-\rho} A_{n,t}^{\left(1-\frac{1}{\tilde{\mu}_n}\right)\rho} K_{n,t}^{\frac{\rho}{\tilde{\mu}_n}-1}, \quad n = 1, ..., N.$$

Define the following variables detrended variables:

$$k_{n,t} = \frac{K_{n,t}}{A_{n,t}}, \quad \alpha_n = \frac{A_{n,t}}{A_t}, \quad n = 1, ..., N$$

where we have used the fact that $A_{n,t}$ and A_t grow at the same rate. With these definitions, we can write the system of first-order conditions as:

$$R_n = \frac{\eta_n}{\mu \tilde{\mu}_n} H\left(\{k_{n,t}\}_{n=1}^N\right)^{-\left(\frac{1}{\mu}-1\right)} I\left(\{k_{n,t}\}_{n=1}^N\right)^{1-\rho} k_{n,t}^{-\left(1-\frac{1}{\tilde{\mu}_n}\right)\rho} \tag{IA.31}$$

where the functions $H(.)$ and $I(.)$ are given by:

$$\begin{aligned} H(\{k_{n,t}\}_{n=1}^N) &\equiv \left(\sum_{n=1}^N \eta_n \left(\alpha_n k_{n,t}^{\frac{1}{\tilde{\mu}_n}}\right)^\rho\right)^{\frac{1}{\rho}} = \frac{K_t}{A_t}, \\ I(\{k_{n,t}\}_{n=1}^N) &\equiv \left(\sum_{n=1}^N \eta_n \left(k_{n,t}^{\frac{1}{\tilde{\mu}_n}-1}\right)^\rho\right)^{\frac{1}{\rho}} = \frac{K_t}{K_{n,t}}. \end{aligned}$$

The system (IA.31) consists of N equations in N unknowns, $\{k_{n,t}\}_{n=1}^N$. We assume that the solution to this system exists and is unique and given by $\{k_n\}_{n=1}^N$.[7] Since none of the parameters in this system of equation is time-varying, we must have $k_{n,t} = k_n$ for all n. Given that each of the $A_{n,t}$ grows at rate g, this confirms the guess $g_{n,t} = g$. This also implies that the ratios $S_{n,1,t} = K_{n,t}/K_{1,t}$ are constant along the balanced growth path. Constant shares, constant growth, and the constant discount rate, along with the relationship $\Pi_{n,t+1} = R_n$, then imply the decomposition (IA.29). □

IA.B.5. Relationship to the production-based asset pricing literature

IA.B.5.1. Definitions

Recall that our model of the firm is:

$$\begin{aligned} V_t^c(\mathbf{K}_t) &= \max_{\mathbf{K}_{t+1}} \quad \Pi_t(K_t) - \tilde{\Phi}_t(\mathbf{K}_t, \mathbf{K}_{t+1}) + \mathbb{E}_t\Big[M_{t,t+1} V_{t+1}^c(\mathbf{K}_{t+1})\Big] \\ \text{s.t.} \quad & K_t = F_t(\mathbf{K}_t), \end{aligned} \tag{IA.32}$$

[7] We have not been able to establish existence and unicity formally, except in the case $N = 2$.

where F_t is homogeneous of degree 1, and Π_t is homogeneous of degree μ, and:

$$\tilde{\Phi}_t\left(\mathbf{K}_t, \mathbf{K}_{t+1}\right) = \sum_{n=1}^{N} \Phi_{n,t}\left(\frac{K_{n,t+1}}{K_{n,t}}\right) K_{n,t}.$$

Assuming that the firm is equity-financed, realized stock returns are defined as:

$$R_{E,t+1} \equiv \frac{V_{t+1}^c}{V_t^e} = \frac{\Pi_{t+1} - \sum_{n=1}^N \Phi_{n,t+1} K_{n,t+1} + V_{t+1}^e}{V_t^e}, \quad V_t^e \equiv \mathbb{E}_t\Big[M_{t,t+1} V_{t+1}^c\left(\mathbf{K}_{t+1}\right)\Big].$$

Stock returns are defined for the firm as a whole. We define returns on investment, for each type of capital $n = 1, ..., N$, as:

$$R_{n,I,t+1} \equiv \frac{\Pi_{n,t+1} - \Phi_{n,t+1} + \frac{K_{n,t+2}}{K_{n,t+1}}\Phi'_{n,t+1}}{\Phi'_{n,t}}.$$

This definition is the same as in Cochrane (1991), Equation (12). The denominator is the cost of adding an incremental unit to $K_{n,t+1}$, which, given our assumption about the structure of investment costs, is simply $\Phi'_{n,t}$. The numerator is the marginal return. This marginal return is the sum of two terms: incremental flow profits (the term $\Pi_{n,t+1}$); and the net change in investment costs associated with the incremental unit of $K_{n,t+1}$, assuming that the stock at time $t+2$ and onward is unchanged (the term $\frac{K_{n,t+2}}{K_{n,t+1}}\Phi'_{n,t+1} - \Phi_{n,t+1}$).[8]

IA.B.5.2. Asset pricing equations

Stock returns satisfy the fundamental asset pricing relationship:

$$1 = \mathbb{E}_t\left[M_{t,t+1} R_{E,t+1}\right]. \tag{IA.33}$$

Recall that the first-order conditions and the envelope conditions for the firm problem are:

$$\begin{aligned} \Phi'_{n,t} &= q_{n,t}, \\ \frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} &= \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1}\frac{K_{n,t+2}}{K_{n,t+1}}, \quad n = 1, ..., N, \end{aligned} \tag{IA.34}$$

[8]The mapping from the model of Cochrane (1991), equations (7)-(9), to the model in our paper is, in the one-capital case, $f(K_t, L_t, s_t) = \Pi_t(K_t)$ and $g(K_t, I_t) = K_t \Phi_t^{-1}\left(I_t/K_t\right)$.

where $q_{n,t} \equiv \mathbb{E}_t\left[\frac{\partial V^c_{t+1}}{\partial K_{n,t+1}}\right] = \frac{\partial V^e_t}{\partial K_{n,t+1}}$. Using the first-order condition $q_{n,t} = \Phi'_{n,t}$, returns to investment can be rewritten as:

$$R_{n,I,t+1} \equiv \frac{\Pi_{n,t+1} - \Phi_{n,t+1} + \frac{K_{n,t+2}}{K_{n,t+1}} q_{n,t+1}}{q_{n,t}}.$$

Using the envelope condition, we then have, for each type of capital:

$$1 = \mathbb{E}_t\left[M_{t,t+1} R_{n,I,t+1}\right]. \tag{IA.35}$$

IA.B.5.3. Relationship between stock returns and returns to investment

RESULT 3: *Stock returns $R_{E,t+1}$ and the returns to investment $\{R_{n,I,t+1}\}_{n=1}^N$ satisfy:*

$$R_{E,t+1} = \sum_{n=1}^{N} \frac{q_{n,t}}{Q_{n,t}} R_{n,I,t+1} + (\mu - 1) \sum_{n=1}^{N} \frac{1}{Q_{n,t}} \left(\sum_{k \geq 1} \mathbb{E}_{t+1}\left[M_{t+1,t+k}\left\{\Pi_{n,t+k}(1 + g_{n,t+1,t+k})\right\}\right] \right), \tag{IA.36}$$

where $1 + g_{n,t+1,t+k} \equiv \frac{K_{n,t+k}}{K_{n,t+1}}$ and $Q_{n,t} \equiv \frac{V^e_t}{K_{n,t+1}}$. Moreover, for any capital type n,

$$\begin{aligned}
R_{E,t+1} - R_{n,I,t+1} &= \frac{1}{Q_{n,t}}\Big\{ \quad -(Q_{n,t} - q_{n,t}) R_{n,I,t+1} \\
&+ \quad (\mu - 1) \sum_{k \geq 1} \mathbb{E}_{t+1}\left[M_{t+1,t+k} \Pi_{n,t+k}(1 + g_{m,t+1,t+k})\right] \\
&+ \quad \sum_{\substack{m=1 \\ m \neq n}}^{N} S_{m,n,t+1} q_{m,t} R_{m,I,t+1} \\
&+ \quad (\mu - 1) \sum_{\substack{m=1 \\ m \neq n}}^{N} S_{m,n,t+1} \sum_{k \geq 1} \mathbb{E}_{t+1}\left[M_{t+1,t+k} \Pi_{m,t+k}(1 + g_{m,t+1,t+k})\right] \Big\}.
\end{aligned}$$

When $N = 1$ and $\mu = 1$, so that $Q_t = q_t$, stock returns are equalized with investment returns:

$$\forall t, \quad R_{E,t+1} = R_{I,t+1}.$$

The proof is reported below. The first part of this result gives a relationship between stock returns, and an appropriately weighted average of investment returns. The weights are the ratio of marginal $q_{n,t}$ to average $Q_{n,t}$ for each type of capital. Note that in the definition of the return to investment, the denominator is the marginal cost of investment, which is equal to marginal q. Analogously, stock returns can be written, in terms of average $Q_{n,t}$ for each type of capital, as:

$$R_{E,t+1} = \frac{\Pi_{t+1}/K_{n,t+1} - \sum_{m=1}^{N} \Phi_{m,t+1} S_{m,n,t+1} + (K_{n,t+2}/K_{n,t+1}) Q_{n,t+1}}{Q_{n,t}},$$

which helps understand why the ratio $q_{n,t}/Q_{n,t}$ is used to "weight" investment returns. Equation (IA.36) then says that the gap between stock returns and this weighted average of investment returns is exactly equal to the present value of future rents. Moreover, as in our analysis of investment gap, the present value of rents can be separated across capital types.[9]

Note that, even when there are no rents ($\mu = 1$), stock returns are not necessarily equal to the returns to investing in each capital type, since, as discussed in the main text, $q_{n,t}$ need not equal $Q_{n,t}$ even when there are no rents. However, stock returns are equalized to a weighted average of investment returns state by state, analogous to the results of Cochrane (1991) and Cochrane (1996).

The second part of the result gives a decomposition of the gap between equity returns, and returns to investment for each capital type $R_{I,n,t+1}$. The intuition for the components of this gap is analogous to the intuition for the components of the investment gap.

When there is only one type of capital and no rents ($N = 1, \mu = 1$), the difference between stock returns and investment returns is given by $-(1 - Q_{n,t}/q_{n,t}) = 0$, since in that case, average Q and marginal q are equal.

When there is only one type of capital, but the firm earns rents ($N = 1, \mu > 1$), the difference between stock returns and investment returns is given by:

$$R_{E,t+1} - \frac{q_{1,t}}{Q_{1,t}} R_{1,I,t+1} = Q_{1,t}^{-1} N_{t+1}.$$

In other words, the gap between stock and investment returns is higher, the larger the rents N_{t+1} that the firm earns.

[9]Note that using the two asset pricing conditions (IA.33) and (IA.35), Equation (IA.36) implies our main firm value decomposition, Result 1 in the main text.

When there are several types of capital, but the firm earns no rents ($N > 1, \mu = 1$), the difference between stock and investment returns can be written as:

$$\begin{aligned} R_{E,t+1} - R_{n,I,t+1} &= Q_{n,t}^{-1} \left(-(Q_{n,t} - q_{n,t}) R_{n,I,t+1} + \textstyle\sum_{\substack{m=1 \\ m \neq n}}^{N} S_{m,n,t+1} q_{m,t} R_{m,I,t+1} \right) \\ &= \sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{q_{m,t}}{Q_{m,t}} (R_{m,I,t+1} - R_{n,I,t+1}), \end{aligned}$$

where, to go from the first to the second line, we used the relationship between $Q_{n,t}$ and $q_{n,t}$ derived in the main paper for the case $N > 1$ and $\mu =$. In that case, as discussed above, stock returns are equal to an appropriately weighted sum of investment returns; so an intuition for the difference between stock returns and individual investment returns is that the gap will be larger, for capital types n whose investment returns are low relative to other investment types (and particularly relative to those with high marginal q relative to average Q).

Finally, in the general case $N > 1, \mu > 1$, the gap between stock returns and investment returns reflects a combination of three factors: the rents generated by capital of type n (analogous to the "traditional rents" in the investment gap decomposition); investment returns of capital of type n relative to other capital types (analogous to the "omitted capital effect" in the investment gap decomposition); and the interaction of the two, i.e. the effect of the rents generated by omitted capital types.

Proof of Result 3. First, using the definition of equity returns, we have:

$$\begin{aligned} R_{E,t+1} V_t^e &= \Pi_{t+1} - \textstyle\sum_{n=1}^{N} \Phi_{n,t+1} \left(\frac{K_{n,t+2}}{K_{n,t+1}} \right) K_{n,t+1} + V_{t+1}^e \\ &= \Pi_{t+1} - \textstyle\sum_{n=1}^{N} \Phi_{n,t+1} \left(\frac{K_{n,t+2}}{K_{n,t+1}} \right) K_{n,t+1} + \mathbb{E}_{t+1} \left[M_{t+1,t+2} R_{E,t+2} V_{t+1}^e \right] \end{aligned}$$

Therefore, assuming that the transversality condition $\lim_{k \to +\infty} \mathbb{E}_t \left[M_{t,t+k} R_{E,t+k} V_{t+k}^e \right] = 0$ holds, we have that:

$$R_{E,t+1} V_t^e = \sum_{k \geq 1} \mathbb{E}_{t+1} \left[M_{t+1,t+k} \left\{ \Pi_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+1} K_{n,t+k} \right\} \right]$$

Using homogeneity of degree μ of the revenue function, we can rewrite this expression as:

$$R_{E,t+1} \cdot V_t^e = \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_{t+1} \left[M_{t+1,t+k} \left\{ \mu \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+k} K_{n,t+1} \right\} \right].$$

Similarly, iterating forward the definition of investment returns, we obtain that, for each $n = 1, ..., N$:

$$R_{n,I,t+1} \cdot (q_{n,t} K_{n,t+1}) = \sum_{k \geq 1} \mathbb{E}_{t+1} \left[M_{t+1,t+k} \left\{ \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+1} K_{n,t+1} \right\} \right],$$

so that:

$$\sum_{n=1}^{N} R_{n,I,t+1} \cdot (q_{n,t} K_{n,t+1}) = \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_{t+1} \left[M_{t+1,t+k} \left\{ \Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+1} K_{n,t+1} \right\} \right].$$

Combining these two equations, we obtain:

$$R_{E,t+1} \cdot V_t^e = \sum_{n=1}^{N} R_{n,I,t+1} \cdot (q_{n,t} K_{n,t+1}) + (\mu - 1) \sum_{n=1}^{N} \sum_{k \geq 1} \mathbb{E}_{t+1} \left[M_{t+1,t+k} \Pi_{n,t+k} K_{n,t+k} \right].$$

This establishes Equation (IA.36), and the rest of the result follows. □

IA.B.5.4. Implications in balanced growth

We next briefly discuss the implications of the balanced growth model for the behavior of stock returns and investment returns. In the balanced growth case, there is no uncertainty, so the two asset pricing relationships collapse to:

$$\begin{aligned} R_{E,t+1} &= 1 + r, \quad \forall t, \\ R_{n,I,t+1} &= 1 + r, \quad n = 1, 2, \quad \forall t. \end{aligned}$$

So, along the balanced growth path, stock returns and returns to investment in either physical or intangible capital are equalized. This can also be see directly from the definition of the two returns. For stock returns:

$$\begin{aligned}
R_{E,t+1} &= \frac{\Pi_{t+1} - \sum_{n=1}^{2} \Phi_n K_{n,t+1} + V_{t+1}^e}{V_t^e} \\
&= \frac{ROA_1 - (\Phi_1 + \Phi_2) + (1+g)Q_1}{Q_1} \\
&= \frac{ROA_1 - (\iota_1 + S\iota_2) - (\gamma_1 + \gamma_2 S)g^2 + (1+g)Q_1}{Q_1} \\
&= r - g + (1+g) = 1 + r,
\end{aligned}$$

where, to go from the first to the second line, we used the notation for key ratios that are constant along the balanced growth path, to go from the second to the third line, we used the relationship $\Phi_n = \iota_n + \gamma_n g^2 + o(g)$ (implied by the functional form we use for adjustment costs), and to go from the third to the fourth line, we used the expression for Q_1 obtained in the main text.

Likewise, for returns to investment in capital of type $n = 1, 2$:

$$\begin{aligned}
R_{n,I,t+1} &= \frac{\Pi_n - \Phi_n - (1+g)q_n}{q_n} \\
&= \frac{R_n - \Phi_n - (1+g)q_n}{q_n} \\
&= \frac{r + \delta_n + \gamma_n r g - \Phi_n + (1+g)q_n}{q_n} \\
&= \frac{(1+\gamma_n g)(r-g) + (1+g)q_n}{q_n} = 1 + r,
\end{aligned}$$

where we successively used the balanced growth path optimality condition $R_n = \Pi_n$, the balanced growth user cost definition $R_n = r + \delta_n + \gamma_n r g = r - g + \iota_n + \gamma_n r g$, and the q-theory condition $q_n = 1 + \gamma_n g$.

IA.B.6. Financing frictions

IA.B.6.1. Overview of key results

In this section, we provide two key results, one for debt frictions and one for equity frictions.

RESULT 4: *Assume that shareholders can raise debt B_{t+1}, subject to a collateral constraint of the form $B_{t+1} \leq \theta K_{1,t+1}$. Define marginal $q_{n,t}$ as $q_{1,t} \equiv q_{1,t}^{(E)} + \lambda_t \theta$ for $n = 1$ and $q_{n,t} \equiv q_{n,t}^{(E)}$ for $n = 2, ..., N$, where $q_{n,t}^{(E)}$ is the marginal value of an unit of capital to shareholders, and λ_t is the Lagrange multiplier on the leverage constraint. Then, the decomposition of the enterprise investment gap for capital n is the same as in Result 1.*

This result states that a collateral constraint with respect to physical capital does not change the expression for the investment gap, so long as one focuses on the *enterprise* investment gap, defined as the gap $Q_{n,t} - q_{n,t}$.[10] The intuition is that because debt is risk-free, there is no conflict between creditors and shareholders, and the investment policy chosen by shareholders also maximizes total *enterprise* value.

RESULT 5: *Assume that the flow value of dividends to shareholders is given by $K_t f(d_t)$, where $d_t = D_t/K_t$, D_t is revenue net of investment costs, and f satisfies $f(0) = 0$, $f' > 0$, $f'(0) = 1$, and $f'' \leq 0$. Then, the investment gap has the same expression as in Result 1, replacing the discount factor $M_{t,t+k}$ with $f'(d_{t+k})M_{t,t+k}$.*

The function $f(.)$ describes equity financing frictions in a reduced-form way: the fact that $f'(d_t) < 1$ when $d_t > 0$ could capture agency costs of free cash flows (Jensen and Meckling, 1976), while the fact that $f'(d_t) > 1$ when $d_t < 0$ could capture costs of seasoned equity offerings (Altınkılıç and Hansen, 2000).[11] These frictions change the way in which shareholders

[10] Below, we show that, consistent with the prior literature, the investment gap for *shareholders*, that is, the difference between the ratio of equity value to the stock of physical capital, and $q_{n,t}^{(E)}$, has a similar expression as Result 1, with an additional, *negative* wedge, reflecting the fact that part of the marginal return to investment, for shareholders, comes from the fact that it relaxes the borrowing constraint.

[11] We follow Hennessy et al. (2007), except that we allow for $f'(d_t) < 1$ when $d_t > 0$. This makes equity financing costs matter on the balanced growth path, where $d_t = d > 0$.

value future rents, but do not affect the three main elements of the decomposition.[12]

IA.B.6.2. Frictions to equity issuance or dividend distributions

General results The firm solves:

$$\begin{aligned} V_t^c(\mathbf{K}_t) \quad &= \quad \max_{\mathbf{K}_{t+1}} \quad K_t f_t\left(\frac{D_t}{K_t}\right) + \mathbb{E}_t\Big[M_{t,t+1}V_{t+1}^c(\mathbf{K}_{t+1})\Big] \\ \text{s.t.} \quad & D_t = \Pi_t(\mathbf{K}_t) - \tilde{\Phi}_t(\mathbf{K}_t, \mathbf{K}_{t+1}), \\ & K_t = F_t(\mathbf{K}_t). \end{aligned} \tag{IA.37}$$

The only difference with our baseline model is how dividend payments are valued. The flow value of dividends is $K_t f_t(D_t/K_t)$, instead of D_t in the baseline model. (The baseline model is nested in this model, when $f_t(x) = x$.)

ASSUMPTION 2 (Frictions to equity issuance or dividend distributions): *The functions* $\{f_t : \mathbb{R} \to \mathbb{R}\}$ *are increasing, concave, twice differentiable, and satisfy:*

$$\forall t, \quad f_t(0) = 0, \quad f_t'(0) = 1.$$

The function $f_t(.)$ captures the equity issuance frictions. We require it to be smooth, so that we can continue using a first-order approach. The concavity of $f_t(.)$ implies that firms will prefer to smooth dividend distributions or equity issuances. The restriction that $f_t'(0) = 1$ additionally implies that dividend distributions are (weakly) *less* valuable, at the margin, than in the baseline model, while equity issuances are (weakly) *more* costly, at the margin, than in the baseline model. These distortions could capture direct costs of equity issuance (when $d_t < 0$) or agency costs associated with free cash flow (when $d_t > 0$).

[12] As discussed below, two definitions of marginal q are possible, depending on whether one normalizes marginal q by $f'(d_t)$ or not. Result 5 refers to an unadjusted marginal q; with the latter definition, the investment gap has an additional wedge, which we characterize below.

RESULT 6: *Firm value can be written as:*

$$
\begin{aligned}
V_t^e &= \sum_{n=1}^{N} q_{n,t} K_{n,t+1} + \sum_{n=1}^{N} (\mu - 1) \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} f'_{t+k}(d_{t+k}) \Pi_{n,t+k} K_{n,t+k} \right] \\
&= f'(d_t) \left\{ \sum_{n=1}^{N} q_{n,t}^{(a)} K_{n,t+1} + \sum_{n=1}^{N} (\mu - 1) \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k}^{(a)} \Pi_{n,t+k} K_{n,t+k} \right] \right\}
\end{aligned}
\tag{IA.38}
$$

where $q_{n,t} \equiv \mathbb{E}_t \left[M_{t,t+1} \frac{\partial V_{t+1}^{(e)}}{\partial K_{n,t+1}} \right]$, $M_{t,t+k}^{(a)} \equiv \frac{f'_{t+k}(d_{t+k})}{f'_t(d_t)} M_{t,t+k}$, *and* $q_{n,t}^{(a)} \equiv \frac{q_{n,t}}{f'_t(d_t)}$*. The gap between* $Q_{n,t}$ *and* $q_{n,t}$ *is given by the same expression as in Result (1), replacing* $M_{t,t+k}$ *with* $f'(d_{t+k}) M_{t,t+k}$*. Moreover, denote* $Q_{n,t}^{(a)} \equiv \frac{Q_{n,t}}{f'(d_t)}$*. Then we have:*

$$
\begin{aligned}
G_t^{(a)} = Q_{n,t}^{(a)} - q_{n,t}^{(a)} &= (\mu - 1) \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k}^{(a)} \Pi_{n,t+k} (1 + g_{n,t+1,t+k}) \right] \\
&+ \sum_{\substack{m=1 \\ m\neq n}}^{N} q_{m,t}^{(a)} S_{m,n,t+1} \\
&+ (\mu - 1) \sum_{\substack{m=1 \\ m\neq n}}^{N} \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k}^{(a)} \Pi_{m,t+k} (1 + g_{m,t+1,t+k}) \right] \times S_{m,n,t+1},
\end{aligned}
$$

and:

$$
G_t^{(u)} = Q_{n,t} - q_{n,t}^{(a)} = - \left(\frac{1 - f'_t(d_t)}{f'_t(d_t)} \right) Q_{n,t} + G_t^{(a)}.
$$

The proof is reported below.

When the investment gap is defined as $Q_{n,t} - q_{n,t}$, as in the baseline model, equity frictions do not change the basic insights from that model. The friction primarily appears as modification to the discount factor, and thus affects the way future rents are valued. However, an important difference with the baseline model is that $q_{n,t}$ is not a sufficient statistic for investment. Indeed, the first-order condition for investment can be written as:

$$
\Phi'_{n,t} f'(d_t) = q_{n,t}.
$$

The term $f'(d_t)$, which can be thought of as the marginal rate of substitution between "inside" cash and "outside" distributions, appears as a wedge between the marginal value of capital and the marginal cost of investment. As noted by other papers, this wedge changes the expected relationship between investment and both average Q and marginal q, even in a model without rents or intangibles. For instance, in a model with no intangibles and no rents, and with quadratic adjustment costs, we have $q_t = Q_t$, but:

$$\begin{aligned} \iota_t &= \delta + \frac{1}{\gamma}\left(\frac{q_t}{f'(d_t)} - 1\right) \\ &= \delta + \frac{1}{\gamma}\left(\frac{Q_t}{f'(d_t)} - 1\right). \end{aligned}$$

This creates an "investment gap" in the sense that ι_t is lower than predicted by values of $Q_{n,t}$ whenever $d_t < 0$ (so that $f'(d_t) > 1$).

This suggests studying an alternative decomposition based on "adjusted" average q and marginal Q, $q_{n,t}^{(a)} \equiv \frac{q_{n,t}}{f_t'(d_t)}$ and $Q_{n,t}^{(a)} \equiv \frac{Q_{n,t}}{f'(d_t)}$. The result above shows that the decomposition of the investment gap between these two quantities has the same three components as in our baseline analysis.

However, $Q_{n,t}^{(a)}$ is not directly observable. The result therefore also reports the gap between marginal q, $q_{n,t}^{(a)}$ (the correct measure of the incentive to invest) and observable average Q, $Q_{n,t}$. In this case, the gap has an additional term, which is simply equal to $Q_{n,t} - Q_{n,t}^{(a)}$. This difference is zero in the absence of equity issuance frictions, i.e. $f_t'(d_t) = 1$.

Under this last definition, there is a bias in the *level* of the investment gap, which appears even if there are no rents and no intangibles. In this case, we have:

$$Q_{n,t} = f'(d_t) q_{n,t}^{(a)}.$$

When $d_t < 0$, $f'(d_t) > 1$ and so $Q_{n,t} > q_{n,t}^{(a)}$. In other words, there is a positive "investment gap". Intuitively, $q_{n,t}^{(a)}$ is the marginal value of capital "inside" the firm (that is, expressed in units of internal cash flows), whereas $Q_{n,t}$ is the average value of capital to "outsiders" (that is, to shareholders). When the firm is issuing equity ($d_t > 0$), outside liquidity is more costly than inside liquidity, so average $Q_{n,t}$ is higher than marginal q; and conversely if $d_t < 0$.

Proof. The first-order necessary condition and the envelope theorem, for each capital type, are:

$$
\begin{aligned}
f_t'(d_t)\,\Phi_{n,t}' &= q_{n,t}, \\
\frac{\partial V_{t+1}^c}{\partial K_{n,t+1}} &= \big(f_{t+1}(d_{t+1}) - d_{t+1} f_{t+1}'(d_{t+1})\big)\, F_{n,t} \\
&+ f'(d_{t+1}) \left(\Pi_{n,t+1} - \Phi_{n,t+1} + \Phi_{n,t+1}' \frac{K_{n,t+2}}{K_{n,t+1}}\right).
\end{aligned}
$$

Multiplying the latter condition by $M_{t,t+1}K_{n,t+1}$, combining with the former condition, and taking expectations at time t, we obtain:

$$
\begin{aligned}
q_{n,t}K_{n,t+1} &= \mathbb{E}_t \Big[M_{t,t+1} \big\{\big(f_{t+1}(d_{t+1}) - d_{t+1} f_{t+1}'(d_{t+1})\big)\, F_{n,t+1} K_{n,t+1} \\
&\qquad + f_{t+1}'(d_{t+1})\left(\Pi_{n,t+1}K_{n,t+1} - \Phi_{n,t+1}K_{n,t+1}\right)\big\}\Big] \\
&+ \mathbb{E}_t \left[M_{t,t+2} q_{n,t+1} K_{n,t+2}\right]
\end{aligned}
$$

Assuming the transversality condition $\lim_{k\to+\infty} \mathbb{E}_t \left[M_{t,t+k} q_{n,t+k-1} K_{n,t+k}\right] = 0$ holds for each type of capital, we can iterate forward and sum across capital types to obtain:

$$
\begin{aligned}
\sum_{n=1}^{N} q_{n,t}K_{n,t+1} &= \sum_{k\geq 1} \mathbb{E}_t \Bigg[M_{t,t+k} \Bigg\{ f_{t+k}(d_{t+k}) K_{t+k} \\
&+ f_{t+k}'(d_{t+k}) \left(\left(\sum_{n=1}^{N} \Pi_{n,t+k}K_{n,t+k} - \Phi_{n,t+k}K_{n,t+k} \right) - D_{t+k} \right) \Bigg\}\Bigg]
\end{aligned} \tag{IA.39}
$$

where we used the homogeneity of degree 1 of the capital aggregator, which implies:

$$
K_{t+1} = \sum_{n=1}^{N} F_{n,t+1} K_{n,t+1}.
$$

Likewise, since:

$$
\Pi_t = \mu \sum_{n=1}^{N} \Pi_{n,t+1} K_{n,t+1},
$$

we have:

$$\begin{aligned}\sum_{n=1}^{N} q_{n,t} K_{n,t+1} &= \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} \left\{ f_{t+k}(d_{t+k}) K_{t+k} \right.\right. \\ &\left.\left. -(\mu-1) f'_{t+k}(d_{t+k}) \left(\sum_{n=1}^{N} \Pi_{n,t+k} K_{n,t+k} \right) \right\} \right] \\ &= \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} f_{t+k}(d_{t+k}) K_{t+k} \right] \\ &- \sum_{n=1}^{N} \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} f'_{t+k}(d_{t+k})(\mu-1) \Pi_{n,t+k} K_{n,t+k} \right]\end{aligned} \tag{IA.40}$$

On the other hand, firm value excluding current distributions is given by:

$$V_t^e = \sum_{k\geq 1} \mathbb{E}_t \left[M_{t,t+k} f_{t+k}(d_{t+k}) K_{t+k} \right]. \tag{IA.41}$$

Taking the difference between Equations (IA.41) and (IA.40) gives the result. The expression of the investment gap for the adjusted definitions of marginal q and average Q, $q_{n,t}^{(a)}$ and $Q_{n,t}^{(a)}$, follows from dividing the second part of Result 6 by $f'_t(d_t)$, and likewise for the expression of the investment gap between $Q_{n,t}$ and $q_{n,t}^{(a)}$. □

Balanced growth We next give analytical expressions for the different definitions of the investment gap above in the balanced growth version of the model. We make the same assumptions as in the main text, and moreover, we assume that $f_t(d_t) = f(d_t)$ for all $t \geq 0$.

RESULT 7: *Let $n = 1$ denote physical capital. Along the balanced growth path, the adjusted physical investment gap is given by:*

$$Q_1^{(a)} - q_1^{(a)} = \sum_{m\geq 2} q_m^{(a)} S_m + \frac{(\mu-1)R_1^{(a)}}{r-g} + \sum_{m\geq 2} \frac{(\mu-1)R_m^{(a)}}{r-g} \times S_m \tag{IA.42}$$

where:

$$\begin{aligned}\forall n = 1, ..., N, \quad q_n^{(a)} &= 1 + \gamma_n g, \\ \tilde{R}_n &= r + \delta_n + \gamma_n r g - \frac{1-\varepsilon(d)}{\varepsilon(d)} F_n d,\end{aligned}$$

$d > 0$ is the dividend to capital ratio along the balanced growth path, and $\varepsilon(d) \equiv \frac{df'(d)}{f(d)}$.

The proof is reported below. The intuition for the expressions of the different terms in the decomposition is the same as in the baseline model. The main effect of the equity frictions is to modify the relevant definition of user costs, which include an additional term: $-\frac{1-\varepsilon(d)}{\varepsilon(d)}F_n d < 0$. This term can be thought of as a "capital gains" expression: along the balanced growth path, trend growth in total capital K_t increases the flow value of dividends, $K_t f(D_t/K_t)$, when $D_t > 0$.

Proof. Along the balanced growth path, the first-order conditions for each type of capital can be written as:

$$\begin{aligned} f'\left(d_t\right)\Phi'_{n,t} &= q_{n,t}, \\ (1+r)q_{n,t} &= \left(f(d_{t+1}) - d_{t+1}f'(d_{t+1})\right)F_{n,t+1} \\ &+ f'(d_{t+1})\left(\Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1}\frac{K_{n,t+2}}{K_{n,t+1}}\right), \end{aligned}$$

which we can write as:

$$\begin{aligned} (1+r)\Phi'_n\left(1+g_{n,t}\right) &= \left(f(d_{t+1}) - d_{t+1}f'(d_{t+1})\right)F_{n,t+1} \\ &+ f'(d_{t+1})\left(\Pi_{n,t+1} - \Phi_n\left(1+g_{n,t+1}\right) + \Phi'_n\left(1+g_{n,t+1}\right)\left(1+g_{n,t+1}\right)\right), \end{aligned}$$

where $g_{n,t} \equiv \frac{K_{n,t+1}}{K_{n,t}} - 1$. We next guess and verify that $g_{n,t} = g$ for $n = 1, ..., N$ is a solution. First, note that with this guess, because of the homogeneity of degree 1 of the capital aggregator F, K_t is also growing at rate a. Therefore:

$$D_t = A_t^{1-1/\mu}K_t^{1/\mu} - \sum_{n=1}^{N}\Phi\left(\frac{K_{n,t+1}}{K_{n,t}}\right)K_{n,t}$$

is also growing at rate g. Define the following detrended variables:

$$k \equiv \frac{K_t}{A_t}, \quad d \equiv \frac{D_t}{K_t}, \quad k_{n,t} = \frac{K_{n,t}}{A_t}, \quad n = 1, ..., N,$$

which are all constant under our guess. Substituting the guess into the first-order condition above, and re-arranging, we obtain:

$$(r-g)\Phi_n'(1+g)+\Phi_n(1+g)=\left(\frac{f(d)}{f'(d)}-d\right)F_{n,t+1}+\Pi_{n,t+1}.$$

Neglecting terms of order $o(g)$ and higher, we can rewrite this as:

$$r+\delta_n+\gamma_n rg=\left(\frac{f(d)}{f'(d)}-d\right)F_{n,t+1}+\Pi_{n,t+1}=\left(\left(\frac{f(d)}{f'(d)}-d\right)+\frac{1}{\mu}\left(\frac{K_{t+1}}{A_{t+1}}\right)^{\frac{1}{\mu}-1}\right)F_{n,t+1}. \tag{IA.43}$$

Given the homogeneity of degree 1 of the capital aggregator, we have:

$$\frac{K_{t+1}}{A_{t+1}}=\frac{F(K_{1,t+1},...,K_{n,t+1})}{A_{t+1}}=F(k_1,...,k_n). \tag{IA.44}$$

and:

$$F_{n,t+1}=F_n(K_{1,t+1},...,K_{n,t+1})=F_n(k_1,...,k_n). \tag{IA.45}$$

Thus, (IA.43) is a system of N equations in N unknowns, the $\{k_n\}_{n=1}^N$. We assume that it has a unique solution $\{k_1\}_{n=1}^N$.[13] Using this solution, we can verify that the guess $K_{n,t}=k_nA_t$ indeed satisfies the necessary first-order conditions for each type of capital.

Note that, because $f(.)$ is concave and $f(0)=0$,

$$\frac{f(d)}{f'(d)}-d\geq 0.$$

If the inequality holds strictly (that is, when there is a dividend smoothing motive), there is no analytical solution to the system of equations (IA.43). Nevertheless, we can define:

$$\tilde{R}_n=r+\delta_n+\gamma_n rg-\left(\frac{f(d)}{f'(d)}-d\right)F_n;$$

we then have $\Pi_n=\tilde{R}_n$ for each $n=1,...,N$, thus establishing the result.

□

[13] We can establish this formally for the case of a CES aggregator; otherwise, we have not been able to find a general proof of existence and unicity.

IA.B.6.3. Debt issuance with collateral limit

General results Shareholders solve:

$$
\begin{aligned}
E_t^c\left(B_t, \mathbf{K}_t\right) &= \max_{\mathbf{K}_{t+1}} \quad D_t + \mathbb{E}_t\Big[M_{t,t+1} E_{t+1}^c\left(B_{t+1}, \mathbf{K}_{t+1}\right)\Big] \\
\text{s.t.} \quad & D_t = \Pi_t(\mathbf{K}_t) - \tilde{\Phi}_t\left(\mathbf{K}_t, \mathbf{K}_{t+1}\right) + B_{t+1} - (1 + r_{b,t-1}) B_t, \\
& K_t = F_t\left(\mathbf{K}_t\right), \\
& B_{t+1} \leq \theta K_{1,t+1} \quad [\lambda_t].
\end{aligned}
\tag{IA.46}
$$

Here, E_t is the (cum-dividend) value of equity, B_t is the stock of debt outstanding, and $r_{b,t}$ is the interest rate on debt, defined as:

$$
r_{b,t-1} \equiv \mathbb{E}_{t-1}\left[M_{b,t-1,t}\right]^{-1} - 1,
$$

where $M_{b,t,t+1}$ is the stochastic discount factor of debtholders. All debt is one-period. We assume that debt is collateralized using $K_{1,t+1}$; θ captures the collateral limit. In our applications, $K_{1,t+1}$ will denote the stock of physical capital, so that the assumption that the borrowing constraint only involves $K_{1,t+1}$ captures the idea that physical assets are more likely to be used as collateral in lending transactions.

The Lagrange multiplier λ_t capture the shadow value of relaxing the borrowing constraint to equityholders. The first-order condition with respect to investment in each type of capital n is given by:

$$
\begin{aligned}
q_{1,t}^E + \lambda_t \theta &= \Phi_n'\left(\frac{K_{n,t+1}}{K_{n,t}}\right) \\
q_{n,t}^E &= \Phi_n'\left(\frac{K_{n,t+1}}{K_{n,t}}\right), \quad n = 2, ..., N,
\end{aligned}
$$

where:

$$
q_{n,t}^E \equiv \left[M_{t,t+1} \frac{\partial E_{t+1}^{(c)}}{\partial K_{n,t+1}}\right], \quad n = 1, ..., N,
$$

is the marginal value of a unit of debt to shareholders, or equity's marginal q. Note, that, for physical capital, equity's marginal q is *not* a sufficient statistic for investment in physical capital. This is because another marginal benefit of increasing investment in physical capital is that it relaxes the collateral constraint. Therefore, we also define the firm's marginal q as:

$$
\begin{aligned}
q_{1,t} &\equiv q_{1,t}^{(E)} + \lambda_t \theta, \\
q_{n,t} &\equiv q_{n,t}^{(E)}, \quad n = 2, ..., N.
\end{aligned}
$$

Given the first-order conditions above, the $\{q_{n,t}\}_{n=1}^{N}$ are sufficient statistics for investment in each type of capital.

The first-order condition with respect to borrowing is:

$$
\lambda_t = 1 - \mathbb{E}_t \left[M_{t,t+1} \frac{\partial E_{t+1}^{(c)}}{\partial B_{t+1}} \right]
$$

We make the following assumption about the borrowing constraint

ASSUMPTION 3 (Binding borrowing constraint): *The stochastic discount factors $M_{b,t,t+1}$ and $M_{t,t+1}$ are such that the collateral constraint is always binding, that is:*

$$
\forall t, \quad \lambda_t > 0 \quad \textit{and} \quad B_{t+1} = \theta K_{t+1}.
$$

Our baseline model and this model will coincide whenever $\lambda_t = 0 \forall t$, a sufficient condition for which is that debtholders and shareholders have the same discount factor. We assume that the borrowing constraint is binding in order to deviate from the baseline model.[14] Under this assumption, combining the first-order condition for debt issuance with the envelope condition for the stock of debt, we obtain:

$$
\lambda_t = 1 - (1 + r_{b,t}) \mathbb{E}_t \left[M_{t,t+1} \right],
$$

[14]In the balanced growth path, this assumption will hold as long as $r > r_b$, where r is the discount rate of equityholders and r_b is the discount rate of debtholders. In general, the sufficient condition for the borrowing constraint to be binding should be:

$$
\forall t \geq 0, \quad 1 + r_{b,t} \equiv \mathbb{E}_t \left[M_{b,t,t+1} \right]^{-1} < \mathbb{E}_t \left[M_{t,t+1} \right]^{-1},
$$

though we do not have a formal proof.

that is, the shadow value of an additional unit of debt is one minus the discounted interest cost of borrowing. Thus, in this model, equityholders choose to borrow (despite frictionless equity markets) because of the positive wedge between their discount rate and debtholders' discount rate. The collateral constraint then limits their ability to do so.[15]

RESULT 8: *Define total firm value as:*

$$V_t^{(e)} = \mathbb{E}_t\left[M_{t,t+1}E^c_{t+1}\right] + \mathbb{E}_t\left[M_{b,t,t+1}(1+r_{b,t})B_{t+1}\right] = E^e_t + B_{t+1}.$$

Total firm value is given by the same expression as in the main text:

$$\begin{aligned} V_t^{(e)} &= \sum_{n=1}^{N} q^E_{n,t}K_{n,t+1} \quad + \quad (\mu-1)\sum_{n=1}^{N} E_t\left[\sum_{k\geq 1} M_{t,t+k}\Pi_{n,t+k}K_{n,t+k}\right] + \lambda_t B_{t+1} \\ &= \sum_{n=1}^{N} q_{n,t}K_{n,t+1} \quad + \quad (\mu-1)\sum_{n=1}^{N} E_t\left[\sum_{k\geq 1} M_{t,t+k}\Pi_{n,t+k}K_{n,t+k}\right] \end{aligned} \tag{IA.47}$$

The investment gap is given by the same expression as in the main text:

$$\begin{aligned} G_{n,t} = Q_{n,t} - q_{n,t} \quad &= \quad (\mu-1)\sum_{k\geq 1}\mathbb{E}_t\left[M_{t,t+k}\Pi_{n,t+k}(1+g_{n,t+1,t+k})\right] \\ &+ \quad \sum_{\substack{m=1 \\ m\neq n}}^{N} q_{m,t}S_{m,n,t+1} \\ &+ \quad (\mu-1)\sum_{\substack{m=1 \\ m\neq n}}^{N}\sum_{k\geq 1}\mathbb{E}_t\left[M_{t,t+k}\Pi_{m,t+k}(1+g_{m,t+1,t+k})\right]\times S_{m,n,t+1}. \end{aligned}$$

The proof is reported below. In this model with a borrowing constraint, the *same* decomposition of the investment gap hold as in our baseline model, where debt issuance is unconstrained. The interpretation of the components of the investment gap is the same as in that model.

A "shareholder" investment gap for physical capital can be defined as: $Q^E_{1,t} - q^E_{1,t} =$

[15] Note, in particular, that we assume unlimited liability on the part of the shareholders, that is, no option to default if the value of equity falls below zero. This assumption will not bind along the balanced growth path. In the general analysis, it helps us avoid having to deal with complications associated with pricing default risk in debt contracts and describing the resolution of default.

$E_t^{(e)}/K_{1,t+1} - q_{1,t}^E$. It satisfies $Q_{1,t}^E - q_{1,t}^E = Q_{1,t} - q_{1,t} - (1-\lambda_t)\theta$. The two coincide when $\lambda_t = 1$ (in which case equity and debt financing are perfect substitutes) or when $\theta = 0$ (in which case the firm is all equity-financed).

However, though this is not immediately visible from the decomposition, the introduction of a collateral constraint along with an incentive to use debt will change the *quantitative* implications of the model. This is because in choosing how much to investment in physical capital, shareholders take into account its beneficial effects on the borrowing constraint. This, in turn, affects the user cost of physical capital. We return to this issue in our empirical applications (Appendix IA.D.9).

Proof. In general, the necessary first-order conditions to the shareholder value maximization problem are:

$$
\begin{aligned}
\lambda_t &= 1 + \mathbb{E}_t\left[M_{t,t+1}\frac{\partial E_{t+1}^{(e)}}{\partial B_{t+1}}\right] \\
\frac{\partial E_{t+1}^{(e)}}{\partial B_{t+1}} &= -(1+r_{b,t}) + \frac{\partial \lambda_{t+1}}{\partial B_{t+1}}\left(\theta K_{1,t+2} - \theta B_{t+2}\right) \\
\Phi'_{1,t} &= \mathbb{E}_t\left[M_{t,t+1}\frac{\partial E_{t+1}^{(e)}}{\partial K_{1,t+1}}\right] + \lambda_t\theta \\
\Phi'_{n,t} &= \mathbb{E}_t\left[M_{t,t+1}\frac{\partial E_{t+1}^{(e)}}{\partial K_{n,t+1}}\right], \quad n = 2, ..., N \\
\frac{\partial E_{t+1}^{(e)}}{\partial K_{n,t+1}} &= \Pi_{n,t+1} - \Phi_{n,t+1} + \Phi'_{n,t+1}\frac{K_{n,t+2}}{K_{n,t+1}} + \frac{\partial \lambda_{t+1}}{\partial K_{n,t+1}}\left(\theta K_{1,t+2} - \theta B_{t+2}\right)
\end{aligned}
$$

When the borrowing constraint is binding, these conditions imply that:

$$
\begin{aligned}
q_{1,t+1}^E K_{1,t+1} &= \mathbb{E}_t\left[M_{t,t+1}\left(\Pi_{1,t+1}K_{1,t+1} - \Phi_{1,t+1}K_{1,t+1}\right)\right] \\
&\quad + \theta\mathbb{E}_t\left[M_{t,t+1}\lambda_{t+1}K_{1,t+2}\right] + \mathbb{E}_t\left[M_{t,t+1}q_{1,t+1}^E K_{1,t+2}\right], \\
q_{n,t+1}^E K_{n,t+1} &= \mathbb{E}_t\left[M_{t,t+1}\left(\Pi_{n,t+1}K_{n,t+1} - \Phi_{n,t+1}K_{n,t+1}\right)\right] + \mathbb{E}_t\left[M_{t,t+1}q_{n,t+1}^E K_{n,t+2}\right], \quad n = 2, ..., N,
\end{aligned}
$$

Combining these expressions, and assuming that the transversality conditions:

$$
\lim_{k\to+\infty}\mathbb{E}_t\left[M_{t,t+k}q_{n,t+k}^E K_{n,t+k+1}\right] = 0
$$

and:

$$\lim_{k \to +\infty} \mathbb{E}_t \left[M_{t,t+k} \lambda_{t+k} K_{n,t+k+1} \right] = 0$$

hold for each type of capital, we obtain:

$$\begin{aligned} \sum_{n=1}^{N} q_{n,t}^{E} K_{n,t+1} &= \mathbb{E}_t \left[\sum_{k \geq 1} \sum_{n=1}^{N} M_{t,t+k} \left(\Pi_{n,t+k} K_{n,t+k} - \Phi_{n,t+k} K_{n,t+k} \right) \right] \\ &+ \theta \mathbb{E}_t \left[\sum_{k \geq 1} M_{t,t+k} \lambda_{t+k} K_{1,t+k+1} \right]. \end{aligned}$$

Equity value is given by:

$$\begin{aligned} E_t^e &= \mathbb{E}_t \left[\sum_{k \geq 1} M_{t,t+k} \left(\Pi_{t+k} - \sum_{n=1}^{N} \Phi_{n,t+k} K_{n,t+k} \right) \right] \\ &+ \mathbb{E}_t \left[\sum_{k \geq 1} M_{t,t+k} \left(B_{t+k+1} - (1 + r_{b,t+k-1}) B_{t+k} \right) \right] \end{aligned}$$

The net present value of net proceeds from future debt issuances to the shareholders is given by:

$$\begin{aligned} & \mathbb{E}_t \left[\sum_{k \geq 1} M_{t,t+k} \left(B_{t+k+1} - (1 + r_{b,t+k-1}) B_{t+k} \right) \right] \\ &= - \mathbb{E}_t \left[M_{t,t+1} (1 + r_{b,t}) B_{t+1} \right] \\ &\quad + \mathbb{E}_t \left[\sum_{k \geq 1} M_{t,t+k} \left(1 - M_{t+k,t+k+1} (1 + r_{b,t+k}) \right) B_{t+k+1} \right] \\ &= - \mathbb{E}_t \left[M_{t,t+1} (1 + r_{b,t}) B_{t+1} \right] + \theta \mathbb{E}_t \left[\sum_{k \geq 1} M_{t,t+k} \lambda_{t+k} K_{1,t+k+1} \right] \end{aligned}$$

where, to go from the second to the third line, we used the law of conditional expectations, the fact that $B_{t+k+1} = \theta K_{1,t+k+1}$ (because the borrowing constraint is assumed to bind), and the first-order condition for borrowing when the borrowing constraint is binding:

$$\lambda_{t+k} = 1 - \mathbb{E}_{t+k} \left[M_{t+k,t+k+1} (1 + r_{b,t+k}) \right].$$

Therefore, total firm value is given by:

$$
\begin{aligned}
V_t^{(e)} &= E_t^{(e)} + B_{t+1} \\
&= \mathbb{E}_t\left[\sum_{k\geq 1} M_{t,t+k}\left(\Pi_{t+k} - \sum_{n=1}^{N}\Phi_{n,t+k}\left(\frac{K_{n,t+k+1}}{K_{n,t+k}}\right)K_{n,t+k}\right)\right] + \theta\mathbb{E}_t\left[\sum_{k\geq 1} M_{t,t+k}\lambda_{t+k}K_{t+k+1}\right] \\
&\quad + \quad \left(1 - \mathbb{E}_t\left[M_{t,t+1}(1+r_{b,t})\right]\right)B_{t+1} \\
&= \mathbb{E}_t\left[\sum_{k\geq 1} M_{t,t+k}\left(\Pi_{t+k} - \sum_{n=1}^{N}\Phi_{n,t+k}\left(\frac{K_{n,t+k+1}}{K_{n,t+k}}\right)K_{n,t+k}\right)\right] + \theta\mathbb{E}_t\left[\sum_{k\geq 1} M_{t,t+k}\lambda_{t+k}K_{t+k+1}\right] \\
&\quad + \quad \lambda_t B_{t+1} \\
&= \sum_{n=1}^{N} q_{n,t}^{E}K_{n,t+1} \quad + \quad (\mu - 1)\sum_{n=1}^{N} E_t\left[\sum_{k\geq 1} M_{t,t+k}\Pi_{n,t+k}K_{n,t+k}\right] + \lambda_t B_{t+1} \\
&= \sum_{n=1}^{N} q_{n,t}K_{n,t+1} \quad + \quad (\mu - 1)\sum_{n=1}^{N} E_t\left[\sum_{k\geq 1} M_{t,t+k}\Pi_{n,t+k}K_{n,t+k}\right]
\end{aligned}
$$

where, to go from the penultimate to the last line, we used the fact that $q_{n,t} = q_{n,t}^{(E)}$ for $n = 2, ..., N$, and $q_{1,t} = q_{1,t}^{E} + \lambda_t\theta$. The investment gap decomposition reported in Result 8 then follows. Equity value is given by:

$$
\begin{aligned}
E_t^{(e)} &= V_t^{(e)} - B_{t+1} \\
&= \left(q_{1,t}^{E} - (1-\lambda_t)\theta\right)K_{1,t+1} + \sum_{n=2}^{N} q_{n,t+1}^{E}K_{n,t+1} \\
&+ (\mu - 1)\sum_{n=1}^{N} E_t\left[\sum_{k\geq 1} M_{t,t+k}\Pi_{n,t+k}K_{n,t+k}\right],
\end{aligned}
$$

establishing the decomposition of the investment gap for shareholders. □

IA.C. Data sources and construction

IA.C.1. Sources for investment and profit rates for the NFCB sector

We use the following time series from NIPA, all for the non-financial corporate business sector (NFCB): NFCB gross value added ($Y^{(BEA)}$) (FRED series A455RC1Q027SBEA), NFCB compensation of employees ($WN^{(BEA)}$) (FRED series A460RC1Q027SBEA), NFCB taxes on production less subsidies ($T^{(BEA)}$) (FRED series W325RC1Q027SBEA), NFCB transfers ($Tr^{(BEA)}$) (FRED series W325RC1Q027SBEA). The data are annual. We use them to compute the surplus of the NFCB sector as:

$$\Pi^{(BEA)} = Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}$$

and to compute the labor share of the NFCB sector as:

$$LS = WN^{(BEA)}/(Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}).$$

We use the labor share only to translate our estimates of the model parameter governing rents, μ, into the share of rents as a fraction of value added. Additionally, we obtain current cost measures of the capital stock for the NFCB sector from the BEA fixed asset tables. We extract $K_{struct}^{(BEA)}$, $K_{equip}^{(BEA)}$ and $K_{intan}^{(BEA)}$, from BEA table 4.1; in particular, we define $K_{intan}^{(BEA)}$ as the stock of intellectual property products. We then define:

$$K_1^{(BEA)} = K_{struct}^{(BEA)} + K_{equip}^{(BEA)}, \quad K_2^{(BEA)} = K_{intan}^{(BEA)}.$$

We use table 4.7 to obtain measures of current investment for the NFCB sector, and we define $I_1^{(BEA)}$ and $I_2^{(BEA)}$ analogously to $K_1^{(BEA)}$ and $K_2^{(BEA)}$. Note that all time series from tables 4.1 and 4.7 are expressed in current dollar values; we only use them in the computation of ratios.

IA.C.2. Computation of enterprise value for the NFCB sector

Next, we consider an alternative measure of the enterprise value of the NFCB sector, that of Hall (2001). As mentioned in the main text, this measure subtracts all financial assets of the NFCB sector from gross claims, instead of subtracting only liquid financial assets, as we do in our baseline. The top panel of Figure 8 reports the time series for Q_1 obtained this way (details on data construction are below). It is lower than in our baseline, though it displays approximately the same medium and long-run trends. The bottom panel of Appendix Figure 8 then reports the investment gap obtained using this measure of Q_1.

The main difference with our baseline is in the overall level of the gap; it is about half as large. As a result, implied rents are lower than in our baseline. For instance, without adjustment costs, rents are 4.2% of value added when using this measure of Q_1, as opposed to 7.7% in our baseline measurement exercise, and their cumulative increase from 1985 to 2015 is 5 p.p., as opposed to 6.2 p.p. in our baseline measurement exercise.[16] Moreover, the direct effect of intangibles becomes larger; and overall, intangibles account for *more* of the gap with this measure of Q_1 than in our baseline. Overall, results using this alternative measure of the enterprise value of the NFCB sector suggest that intangibles play a larger role in the investment gap.

Methodology In order to construct an estimate of $Q_{1,t}$ for the NFCB sector, we require a time series for total firm value for the non-financial corporate business (NCFB) sector, V_t, for the 1947-2017 period. We measure V_t by estimating the total market value of securities outstanding from the NCFB sector. Specifically, we define:

$$V_t = MVE_t + MVD_t - L_t.$$

Here, MVE_t is the market value of equity claims on the NFCB sector, MVD_t is the market value of other financial claims (including debt liabilities) on the NFCB sector, and L_t is the book value of liquid financial assets owned by the NFCB sector.

We use measures of MVE_t and L_t provided by the Flow of Funds. For MVE_t, we use Flow

[16]With adjustment costs, the share of rents is 3.4% in 2015, and the 1985 to 2015 increase is 5.1 p.p.

of Funds series LM103164103 (nonfinancial corporate business; corporate equities; liability; Table L.103). This series is constructed by the Flow of Funds as the sum of the market value of equities of publicly traded companies, plus an estimate of the market value of equities of closely held firms, which is estimated from a variety of sources, including the Statistics of Income from the IRS.[17] For L_t, we use Flow of Funds series FL104001005 (Nonfinancial corporate business; liquid assets, broad measure; Table L.103). This series is the sum of the market values of municipal securities, commercial paper, deposits, Treasuries and agency securities, repos, money market fund shares, and corporate equities held by the NFCB sector, estimated from a variety of sources, including the Statistics of Income and the Quarterly Financial Report.[18]

The main difficulty in constructing a series for V_t is to obtain an estimate of the market value of other financial claims on the NFCB sector, MVD_t. In order to estimate this quantity, we extend the approach of Hall (2001) (whose data stops in 1999) to the 1947-2017 period. Specifically, we estimate MVD_t as:

$$MVD_t = BVD_t + (MVB_t - BVB_t),$$

where BVD_t is the book value of all non-equity claims on the NFCB sector, MVB_t is an estimate of the market value of bonds issued by the NFCB sector, and BVB_t is the book value of bonds issued by the NFCB sector. This approach therefore only imputes a market value for *bonds* issued by NFCB, as opposed to imputing a market value for all non-equity claims on the NFCB.

For BVD_t, we use Flow of Funds series FL104190005 (Nonfinancial corporate business; total liabilities; Table L.103).[19] We define BVB_t as the sum of the book value of taxable bonds (Flow of Funds series FL103163003; Nonfinancial corporate business; corporate bonds; liabil-

[17]See www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=LM103164103&t=L.103&suf=Q and www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=FL103164123&t=.

[18]See www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=FL104001005&t=L.103&suf=Q.

[19]BVB_t includes debt securities, taxes payable, trade payables, miscellaneous liabilities, and foreign direct investment. It does not include any estimate of the book value of equities. See www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=FL104190005&t=L.103&suf=Q for more details.

ity; Table L.103), plus tax-exempt bonds (Flow of Funds series FL103162000; Nonfinancial corporate business; municipal securities; liability; Table L.103).

We proceed as in Hall (2001) in order to compute MVD_t. For taxable securities, we assume a maturity at issuance of ten years. In each year, we compute gross issuance as the sum of net issuance (obtained from the Transactions table, series FA103163003), plus principal repayment from the 10-year-prior vintage. We then impute a coupon rate for this new bond vintage equal to the yield on corporate bonds multiplied by total gross issuance. Finally, in each year, we recalculate the market value of each bond vintage by discounting the remaining principal and coupon payments on all outstanding vintages at the current bond yield. We follow the same strategy for municipal securities.

For yields on taxable bonds, we use Moody's Seasoned Corporate Baa bond yield (FRED series BAA). These yields are based on securities with maturities 20 years and above, so we subtract the gap between the 20-year and the 10-year Treasury yield (FRED series GS10 and GS20). For non-taxable securities, we use the Bond Buyer Go 20-Bond Municipal Bond Index (FRED series WSLB20).[20]

Comparison with Hall (2001) Appendix Figure 21 reports the resulting time series for V_t (the black dotted line).[21] The figure also reports two other estimates of V_t. The crossed green line is the total market value of securities outstanding from the NFCB sector, V_t^{Hall}, constructed by Hall (2001), and the solid blue line is an update of this time series to 2017.[22]

The series for V_t^{Hall} is defined as:

$$V_t^{Hall} = MVE_t + MVD_t - F_t = V_t - (F_t - L_t),$$

[20]When the data are missing, and before 1986, we use the time series provided in the replication files of Hall (2001), available at web.stanford.edu/~rehall/SMCA_Data_Appendix.html. The WSLB20 series was discontinued 2016m10, so we splice it with the yield on taxable securities for the remainder of the sample.

[21]The time series are estimated at the quarterly frequency; the annual time series used in the main analysis are obtained by averaging the quarterly time series for each year.

[22]We obtained the original data (up to 1999) used in Hall (2001) from web.stanford.edu/~rehall/SMCA_Data_Appendix.html. The replication code for our paper contains an extension to 2017 of Figures 2, 3, 4, 5, 8, 9, and 10 from Hall (2001).

where MVE_t and MVD_t are defined as above, and F_t denotes total financial assets of the NFCB sector (Flow of Funds series FL104090005; Nonfinancial corporate business; total financial assets; from Table L.103).

In words, while Hall (2001) nets out all financial assets from estimates of the value of claims on the NFCB in order to arrive at an estimate of the market value of non-financial corporations, we only subtract those financial assets identified by the Flow of Funds as liquid. Note that in the Flow of Funds, $F_t \geq L_t$, and L_t is a subcomponent of F_t.

We choose to diverge from the Hall (2001) methodology in this respect two reasons. First, the purpose of netting out financial claims is to obtain an estimate of net (as opposed to gross) debt liabilities in the numerator of Tobin's Q. Standard measures of net debt, however, typically net out cash and cash equivalents, not other financial securities that may not be readily liquidated in order to honor debt commitments. Second, and more important, in the Flow of Funds, the difference between F_t and L_t is series FL103090005, Miscellaneous Financial Assets. The bulk of that series (Unidentified Miscellaneous Assets, series FL103090005) is a residual, imputed by the Flow of Funds in order to reconcile firm-level financial assets totals from the Statistics of Income and the Quarterly Financial Report, from totals obtained, at the instrument-level, from other sources.[23] So the gap between F_t and L_t is as likely to capture measurement error across different underlying data sources as it is to capture actual net financial claims. As a result, in our baseline, we exclude from the imputation of net debt in the computation of Tobin's Q.

IA.C.3. Growth rates of capital stocks

We use table 4.2 to construct estimates of the real net growth rates of the stocks of physical, intangible, and total capital. One difficulty is that the Fixed Assets tables do not report a quantity index for physical capital, but only separate quantity indices for equipment and for structures. We aggregate the growth rates in these quantity indices into a growth

[23] See www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=FL103093005&t=L.103&bc=L.103:FL103090005&suf=Q.

rate rate $g_{1,t}$ for physical capital following the BEA's own methodology:

$$g_{1,t} = \left(\frac{K_{\text{struct},t}}{K_t} (1 + g_{\text{struct},t})^{-1} + \frac{K_{\text{equip},t}}{K_t} (1 + g_{\text{equip},t})^{-1} \right)^{-1} - 1.$$

Here, $K_{\text{struct},t}$ is the current-cost net stock of non-residential structures (from Fixed assets Table 4.1), $g_{\text{struct},t}$ is the growth rate in the chain-type quantity index for the net stock of non-residential structures (Fixed assets Table 4.2), $K_{\text{equip},t}$ and $g_{\text{equip},t}$ are similarly defined, but for equipment, and $K_t = K_{\text{struct},t} + K_{\text{equip},t}$ is the current-cost total stock of physical capital. For the growth rates in the quantity of total capital g_t and intangible capital $g_{2,t}$, we directly use the growth rate of the quantity indices reported in Fixed Assets Table 4.2.

Appendix Figure 3 reports times series of these estimates.[24] Growth rates in these quantity indices approximately coincide for physical and total capital (they are 2.8% and 3.0% per year on average, respectively), while the growth rate of the quantity index of intangible stock is higher (5.6% per year, on average).

The bottom panel plots the difference in the growth rates of the physical and intangible capital stock over time.[25] In both cases, the difference between the two growth rates in quantity indices is positive, on average. Thus, the balanced growth assumption that $g_1 = g_2 = g$ does not hold strictly in the data. However, the assumption seems be plausible for certain sub-periods, such as the post-2000s, as well as the 1970-1980 period, when the difference is substantially smaller.

IA.C.4. Estimates of average depreciation rates across capital types

We next describe how we construct estimates for the average rate of economic depreciation of physical and intangible capital. In order to describe our approach, we briefly summarize the methodology behind the BEA estimates of capital stocks.[26] Throughout the paper, we

[24]For completeness, the graph also reports nominal time series for growth rates, though these are not used in our analysis.

[25]The two lines refer to the difference obtained when growth rates are measured from stocks evaluated at current cost ("nominal"), and when they are obtained from changes in real quantity indices ("real").

[26]See Evans (2003) for a detailed description, as well as a list of the economic depreciation rates used by the BEA in applying the perpetual inventory method.

measure investment rates as:

$$\iota_t = \frac{I_t^c}{K_{t-1}^c}, \tag{IA.48}$$

where I_t^c is current cost investment during year t, and K_{t-1}^c is the current-cost net stock of capital at the end of year $t-1$, and c refers to equipment, structures, or intellectual property products. We use the data provided in Fixed Assets tables 4.1 (for the net stocks at current cost) and 4.7 (for the gross investment flows at current cost). For physical capital, we use the sum of equipment and structures, and for intangible capital, we use intellectual property products.

For a particular capital aggregate c (equipment, structures or intellectual property products), the BEA's current-cost estimate of the net stock is constructed from the history of gross investment flows and assumptions on depreciation rates, as follows. First, both current-cost investment flows and current-cost net stocks are simple sums of flows and stocks at the asset-type level:

$$I_t^c = \sum_j I_{j,t}^c, \quad K_{j,t} = \sum_j K_{j,t}^c.$$

In this notation, asset types are indexed by j; for instance, for equipment, this comprises automobiles, computers, machinery, etc. At the asset-type level, the net stock is defined as:

$$K_{j,t}^c = P_{j,t} K_{j,t}^r.$$

Here, $P_{j,t}$ is a price index that is used to deflate investment in the construction of real stocks (as described below), and $K_{j,t}^r$ is a real-cost estimate of the net capital stock. The real-cost estimate of the capital stock is computed using a perpetual inventory method. Specifically:

$$K_{j,t}^r = \sum_{k\geq 0} K_{j,t,t-k}^r = \sum_{k\geq 0} (1-\nu\delta_j)(1-\delta_j)^k \frac{I_{j,t-k}^c}{P_{j,t-k}}. \tag{IA.49}$$

The price index $P_{j,t}$ is used to express current-cost gross investment in fixed dollars. $K_{j,t,t-k}^r$ represents the contribution of investment in year $t-k$ to the net stock of capital of type j at time t. This contribution depends on initial investment $I_{j,t-k}^c / P_{j,t-k}$, and economic depreciation, δ_j, which is allowed to vary across asset types, but is fixed over time for particular asset type. The contribution also depends on $\nu \in [0,1]$, which captures when the investment is assumed to be placed in service ($\nu = 0$ corresponds to the end of the year, and $\nu = 1$ corresponds to the beginning of the year; the BEA uses $\nu = 1/2$).

In order to gain some intuition on the implications of this methodology for the drivers of the investment rate time series defined in Equation (IA.48), assume that, in the underlying data, gross investment flows at current cost by asset type $I^c_{j,t}$ were growing at the constant rate g^{nom}_j, while the price indices were growing at the constant rate π_j. Then, given the methodology described above for constructing net stocks, measured investment rates, as defined in Equation (IA.48), would be constant, and given by:

$$\iota^c_{j,t} \equiv \frac{I^c_t}{K^c_{j,t-1}} = \frac{1 + g^{nom}_j - (1-\delta_j)(1+\pi_j)}{1 - \nu\delta_j} \approx g_j + \delta_j,$$

where $g_j = g^{nom}_j - \pi_j$ is the real growth rate of investment flows. Thus, measured investment rates, even on average, would not be driven only by assumptions about depreciation rates, δ_j, but also the growth rates of measured gross investment flows, g_j, which are entirely independent from δ_j.

We next describe how to construct an estimate of average economic depreciation from the data provided in the aggregate Fixed Assets tables. We define our current-cost average depreciation estimate as:

$$\delta^c_t \equiv \frac{D^c_t}{K^c_{t-1} + \nu I^c_t}, \tag{IA.50}$$

where D^c_t is the estimate of current-cost depreciation reported in Fixed Assets table 4.4, and K^c_t and I^c_t are defined as above. This data object is related to the economic depreciation rates δ_j at the asset-type level as follows:

$$\delta^c_t = \sum_j w^c_{j,t}\delta_j, \quad w^c_{j,t} \equiv \frac{(P_{j,t}/P_{j,t-1})K^c_{j,t-1} + \nu I^c_{j,t}}{K^c_{t-1} + \nu I^c_t}. \tag{IA.51}$$

This expression says that δ^c_t is weighted average of the constant, asset-specific depreciation rates, where the weights reflect the current cost value of the undepreciated stock of capital in period t.[27]

[27] These weights do not exactly add up to 1. This is because the definition of the average depreciation rate in Equation (IA.50) does not appropriately reflate the value of the current-cost stock of capital. Appropriately

The reason why δ_t^c defined in this way is a weighted average of asset-level depreciation rates is the following. First, in the BEA data, the current-cost estimate of depreciation for a particular capital aggregate is defined as $D_t^c = \sum_j D_{j,t}^c$, where $D_{j,t}^c = P_{j,t} D_{j,t}^r$ is a current-cost estimate of depreciation during year t for asset type j, and where $D_{j,t}^r$ is its real counterpart. In turn, $D_{j,t}^r$ is computed from gross investment and real net stock estimates, as:

$$D_{j,t}^r = \frac{I_{j,t}^c}{P_{j,t}} - (K_{j,t}^r - K_{j,t-1}^r).$$

In words, the real depreciation estimates $D_{j,t}^r$ are constructed as residuals that satisfy the law of motion for capital.[28] However, these asset-level depreciation flows can be used to infer depreciation rates. Using the perpetual inventory formula (IA.49), we have:

$$K_{j,t}^r = (1 - \nu\delta_j)\frac{I_{j,t}^c}{P_{j,t}} + (1 - \delta_j)K_{j,t-1}^h \implies \delta_j = \frac{D_{j,t}^r}{K_{j,t-1}^r + \nu\frac{I_{j,t}^c}{P_{j,t}}}.$$

Using this result, it is straightforward to see that Equation (IA.51) holds.[29]

reflating is difficult to do directly because the price indices $P_{j,t}$ are not reported in Fixed Assets tables 4.1-4.7. However, this only affects the interpretation of δ_t^c, not its value.

[28] Two additional points are worth making. First, as can be directly checked from the data in Fixed Assets tables 4.1, 4.4 and 4.7, the aggregate current-cost time series do not satisfy the law of motion $D_t^c = I_t^c - (K_t^c - K_{t-1}^c)$. Second, in practice, the BEA uses a slightly different deflator for translating real depreciation estimates into current cost estimates (the average of $P_{j,t}$ over the year, instead of its end-of-period value); this latter point complicates somewhat the expression for $w_{j,t}^c$, but does not change its interpretation.

[29] As an alternative to δ_t^c, we can also define an average depreciation rate based on the historical cost estimates reported in Tables 4.3 and 4.6, as follows:

$$\delta_t^h \equiv \frac{D_t^h}{K_{t-1}^h + \nu I_t^h}. \tag{IA.52}$$

Here, K_{t-1}^h and D_t^h are historical cost estimates of the net stock of capital and of depreciation, respectively. These estimates are constructed by the BEA exactly as the current-cost estimates, but assuming that the price index is $P_{j,t} = 1$ for all underlying asset types. In other words, these estimates do not account for changes in the relative price of underlying assets over time. Following similar steps as above, one can show that:

$$\delta_t^h = \sum_j w_{j,t}^h \delta_j, \quad w_{j,t} \equiv \frac{K_{j,t-1}^h + \nu I_{j,t}^h}{K_{t-1}^h + \nu I_t^h}, \tag{IA.53}$$

Appendix Figure 4 reports time-series for average depreciation rates δ_t^c, along with the gross investment rates $\iota_t^c = I_t^c/K_{t-1}^c$ used in the paper. We report this separately for physical capital (defined as the sum of equipment and structures) and intangibles (intellectual property products), consistent with our approach in the paper.[30] Note that underlying depreciation rates assumed by the BEA at the asset type level (the δ_j) are constant. The small upward trends in average depreciation rates for physical and intangible capital therefore reflect compositional changes in the underlying asset types, as opposed to changes in economic rates of depreciation of granular assets types.

IA.C.5. Compustat data

Sample selection We use the annual version of the Compustat-CRSP merged files. We apply the standard screens (indfmt=INDL, popsrc=D, consol=C, datafmt=STD). We keep firm-year observations that satisfy the following criteria: fic=USA (domestically incorporated), 2-digit SIC code (first two digits of the variable sic) not equal to 49 (utilities), not between 60 and 69 (finance and real estate), and not between 90 to 99 (public administration); 2-digit SIC code not missing; variable sale (sales) and at (assets) not missing; variables emp, sale, at, act, lct, ppent, ppegt, che, and gdwl not negative. Finally, we drop any observation which we can identify as an American Depository Institution (ADR). We use only data from 1974 onward (included), as the data prior to 1974 has incomplete coverage (a jump in the number of firms in the sample occurs from 1973 to 1974).

Variable construction For each firm, we start by constructing six time series in levels, $\{K_{1,t}, I_{1,t}, K_{2,t}, I_{2,t}, \Pi_t, V_t\}$. For physical capital investment, we use capital expenditures (capx) net of sales of property, plant and equipment (sppe); we measure the stock using gross property, plant and equipment (ppegt), for reasons we discuss in Appendix IA.C.7. We consider two definitions of intangibles: R&D capital, and organization capital. For R&D, we use

where the δ_j are the same as for current-cost estimates. We prefer using the current-cost version of δ_t because our analysis relies on current-cost estimates of net stocks. However, the two time series δ_t^c and δ_t^h are very close in trends and, for intangibles, in levels.

[30] For physical capital, following the logic above, we weigh the aggregate depreciation rate of equipment and structures by their lagged shares of undepreciated net stocks.

reported R&D expenditure (xrd), recoding missing values with 0.

We mesaure investment in organization capital as 30% of SG&A expenditures (variable xsga) net of R&D investment. For the stock of both R&D and organization capital, we use the capitalized values provided by Peters and Taylor (2017). We discuss below in more detail the sources for the imputation of investment in organization capital.

For Π_t, we use the Compustat variable oibdp. We add estimates of intangible investment expenditures to actual measures of operating income in order to obtain an adjusted operating surplus measure consistent with our model. For V_t, we use the sum of the market value of common stock and the book value of debt, net of cash and liquid securities. We then take the sum of these time series across firms either by year (when studying all publicly traded firms jointly), or by year and sector (when constructing the sectoral investment gaps.) Finally, we construct the growth rate of total capital at either the aggregate or sectoral level by subtracting from the growth rate of $K_{1,t} + K_{2,t}$ the deflator implicitly used in Section II, that is, the difference between nominal and real growth rates of total non-residential fixed assets for the NFCB sector.

Sectoral classification Appendix Tables I and II report the sectoral classification used in the analysis of Section III. In order to be able to match the data to the BLS's KLEMS data for the period over which the latter are available, we use a NAICS-based classification that maps to the BLS classification of sectors. We nevertheless aggregate the data up to sectors that are similar to the Fama-French 5 subsectors, with the main difference being that we exclude financial companies from our analysis.

Rate of imputation of investment in organization capital We follow Eisfeldt and Papanikolaou (2014) and Peters and Taylor (2017) in choosing a rate of imputation of $\lambda = 0.3$ for SG&A (net of R&D expenditures) for our measure of investment in organization capital. Peters and Taylor (2017) show that their main conclusions regarding the relationship between investment and Q hold for values of λ ranging from 0.2 to 1.0. They also attempt to estimate λ via maximum likelihood, allowing for heterogeneity across industries, and find values in the neighborhood of 0.3, though they caution against using these estimates, since they rely heavily on assuming perfect substitution between intangible and physical capital.

In both Eisfeldt and Papanikolaou (2014) and Peters and Taylor (2017), the main source for the imputation rate of $\lambda = 0.3$ is Hulten and Hao (2008). That paper uses data from financial statements of a composite of six large US pharmaceutical companies, which report expenditures on brand equity and organization capital. These expenditures account for 30% of all SG&A spending in their sample.

More recently, Ewens et al. (2020) provide estimates of λ (and the implied values for the organization capital stock) based, in part, from asset valuations of public firms that exit, either by going private, being liquidated, or being acquired. Their average estimate (Table 5 of their paper) is $\lambda = 0.43$, with values ranging from 0.24 (in Manufacturing) to 0.62 (in Healthcare). Moreover, the ratios of total intangible (R&D plus organization capital) to physical capital implied by their estimated values for λ are quantitatively similar to our estimates, in both levels and trends (see their Figure 8(a)). Their estimates rely on a different structure than the one we explore in this paper (specifically, they use an investment-Q model with no rents and perfect substitutability between physical and intangible capital). While their estimates are quantitatively close to the value of λ used in our paper and in Peters and Taylor (2017), we nevertheless think that using these estimates would not be internally consistent, given our approach in this paper. Their finding of higher values for λ in Healthcare and Tech, and lower values in the Consumer and Manufacturing sector, would likely strengthen our point on heterogeneity in the composition of the investment gap across sectors.

IA.C.6. Other data sources

We obtain the statutory corporate income tax rate form the Tax Policy Center.[31] For the cum- and ex-dividend returns to equity used to compute the PD ratio, $R^c_{E,t-1,t}$ and $R^e_{E,t-1,t}$, we use returns from the CRSP file on daily returns on the S&P 500, downloaded from WRDS.[32] We compute cumulative annual cum- and ex-dividend returns by taking the cumulative sum of the log of one plus daily returns over each year, and exponentiating end-of-year values. As a measure of the risk-free rate, we use the average annual rate of return

[31]The specific series we use is the Top Tax Bracket series at `https://www.taxpolicycenter.org/statistics/corporate-top-tax-rate-and-bracket`.

[32]We use the `dsp500` file.

on one-month Treasury bills, obtained from Kenneth French's website.[33] Finally, we use the BLS multifactor productivity database in order to obtain measures of value added as well as total factor productivity growth.[34]

IA.C.7. Robustness checks and comparisons across data sources

Residential assets Our baseline approach for the NFCB sector only includes non-residential fixed assets. As a robustness check, we obtained residential fixed assets $K_{resid}^{(BEA)}$ as the difference between the sum of the three capital stocks above, and total fixed assets of NFCB sector report in BEA fixed asset table 6.1, and likewise for investment. The top panel of Figure 21 reports time series for the ratio $\Pi^{(BEA)}/K^{(BEA)}$. The solid red and solid orange line use $\Pi^{(BEA)}$ as the numerator, and for the total capital stock, either $K = K_{struct}^{(BEA)} + K_{equip}^{(BEA)} + K_{intan}^{(BEA)}$ (as in our baseline analysis), or $K = K_{struct}^{(BEA)} + K_{equip}^{(BEA)} + K_{intan}^{(BEA)} + K_{resid}^{(BEA)}$. The two lines are almost identical. The stock of residential fixed assets in the NFCB sector thus appears to be low relative to other types of fixed assets owned by the NFCB sector, and so we abstract from it in our analysis.

Economy-wide vs. NFCB measures Finally, Figure 21 also compares our measures of Π/K for the NFCB sector, with those reported by Farhi and Gourio (2018), who study economy-wide trends, instead of the NFCB sector specifically. The rate of return on capital measured by these authors is substantially lower than our measures of rates of return for the NFCB sector (by about 5-7% throughout the sample.) Here, we briefly discuss why this is the case, as it matters for inferences about the importance of rents. These authors compute Π/K as:

$$\Pi/K = \left[(Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})/(Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})\right] \times Y/K,$$

[33] Specifically, we use the time series for R_f available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors_CSV.zip.

[34] The BLS multifactor productivity, or KLEMS, database is available at https://www.bls.gov/mfp/special_requests/klemscombinedbymeasure.xlsx.

where Y is total nominal GDP (including other sectors than the NFCB) and K is the total private capital stock (at replacement cost). This adjustment is made in order to maintain comparability with other ratios in their analysis, which has a broader scope than the NFCB. By contrast, our measures of Π/K are:

$$\Pi/K = \left[(Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})/(Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})\right]$$
$$\times (Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})/K^{(BEA)}.$$

Thus the differences between our measures of Π/K and the measures in Farhi and Gourio (2018) must be due to differences in the ratio of value added to capital between the NFCB sector and the economy as a whole. The bottom panel of Figure 21 indeed shows that the NFCB sector has a substantially higher dollar of value added per dollar of capital at current cost. The most accurate comparison is between the crossed blue line of the bottom panel, and the orange solid line, which measures K for the NFCB sector as the sum of all types of capital (residential, non-residential physical, and non-residential intangible): the value added to capital ratio is approximately 10 percentage points higher in the NFCB sector versus the economy as a whole.

Compustat nonfinancials vs. NFCB sector There are two potentially important differences between the data used in Section II and the Compustat data. First, Compustat only includes publicly traded corporations. There may be systematic differences in returns to capital and intangible intensity between privately held and publicly traded corporations. Second, the measurement of the stock of physical capital differs across sources. We next discuss these differences in more detail.

In Compustat, our baseline measure of surplus as the sum of ebitda across all observations in our sample. (Missing observations are thus treated as zeros.) We use ebitda because it is the financial statement measure most closely related to our model definition of Π_t; it is a measure of operating income before depreciation, and does not deduct costs of capital, or non-operating income, which our model does not capture.[35] The top right panel of Appendix Figure 24 report

[35]The inclusion of non-operating income makes little difference to the results.

the NFCB sector surplus measured in this manner in Compustat, and the measure from the BEA tables. The two are highly correlated, but their levels differs substantially. This reflects the fact that the BEA NFCB sector data also includes private firms. The surplus of public firms (from Compustat) represents about two thirds of the total surplus of the NFCB sector (from the BEA).

The main difficulty in the Compustat data is in computing estimates of the current-cost total stock of physical capital. A natural definition would seem to be net property, plant and equipment (variable ppent). However, measuring K_1 for the NFCB sector in Compustat leads to extremely elevated measures of Π/K_1, as reported in the bottom right panel of Appendix Figure 24. These measures are almost double the BEA-derived measures. This is primarily because the aggregate value of ppent in Compustat is only about a third of physical capital in the NFCB sector according to BEA data (top left panel of Appendix Figure 24). The reason for this gap are unclear. One hypothesis is that the surplus of Compustat firms includes income from foreign subsidiaries, and so could overestimate the true surplus of public NFCB firms. Alternatively, it could be that private firms indeed have much lower rates of return on physical capital than public firms do (though the gap would have to be very large, given the relative importance of public firms in total surplus, as indicated in the top right panel of Appendix Figure 24). The more likely reason is that the accounting treatment of depreciation may lead the (balance sheet) net stock to underestimate the true current cost stock of physical assets. The red line in the top left and bottom left panels of Appendix Figure 24 instead report measures of asset returns using aggregate gross property, plant and equipment at historical cost (deflated using the implicit deflator from the BEA fixed tables). The bottom left panel shows that this estimate of K_1 leads to values of Π/K_1 that align more closely (in levels) with those provided by the BEA data on the NFCB. In what follows, in order to align our BEA and Compustat profitability moments as closely as possible, we therefore use gross property, plant and equipment as our main measure of K_1 in Compustat data.

We measure (gross) investment in physical capital in Compustat using capital expenditures (variable capx) minus sales of property, plant and equipment (variable sppe). Appendix Figure 22, top panel, shows that physical investment, computed in this manner, accounts for about two thirds of total physical investment in the BEA NFCB sector ($I_1^{(BEA)}$, with closely related cyclical movements. For investment rates (the bottom panel of Appendix Figure 22), the data again suggest a much higher investment rate in Compustat when K_1 is measured using net book values, but investment rates are closer in levels when the capital stock is measured using gross book values.

IA.D. Additional empirical results

IA.D.1. Heterogeneous growth rates across capital stocks

While the balanced growth model imposes the restriction $g = g_1 = g_2$, we can nevertheless try to assess, heuristically, what the impact of having heterogeneous growth rates for the two capital stocks would be on our baseline decomposition.

In order to do this, we construct an "approximate" decomposition of the investment gap. This decomposition is the same as our baseline, except that when we map model and data, we allow the growth rates that enter marginal q, as well as user costs, to differ. That is, given the same data as in our baseline exercise, as well as growth rates g_1 for physical capital and g_2 for intangible capital, we construct the following variables:

$$\begin{aligned}
\widehat{q_1} &\equiv 1 + \gamma_1 g_1 \qquad \text{(IA.54)} \\
\widehat{q_2} &\equiv 1 + \gamma_2 g_2 \\
\widehat{r-g} &\equiv \frac{ROA_1 - (\iota_1 + S\iota_2)}{Q_1} - \frac{\gamma_1 g_1^2 + \gamma_2 g_2^2 S}{Q_1} \\
\widehat{R_1} &\equiv \widehat{r-g} + \iota_1 + \gamma_1(\widehat{r-g} + g)g_1 \\
\widehat{R_2} &\equiv \widehat{r-g} + \iota_2 + \gamma_1(\widehat{r-g} + g)g_2 \\
\widehat{\mu} &\equiv \frac{ROA_1}{\widehat{R_1} + S\widehat{R_2}}
\end{aligned}$$

These definitions are analogous to those derived from the balanced growth model, except that we replaced $q_i = 1 + \gamma_i g$ by $\widehat{q_i} = 1 + \gamma_i g_i$, $i = 1, 2$, and we also replaced any term of the form

$\gamma_i g$ or $\gamma_i g^2$ by $\gamma_i g_i$ or $\gamma_i g_i^2$, $i = 1, 2$ in user costs.

To the extent that $g \neq g_1 \neq g_2$, the values of $\left\{\widehat{r-g}, \widehat{R}_1, \widehat{R}_2, \widehat{\mu}, \widehat{q}_1, \widehat{q}_2\right\}$ defined above will differ from the values for the variables $\{r - g, R_1, R_2, \mu, q_1, q_2\}$ computed in our baseline approach using the same data.

It is straightforward to show that, as in our baseline decomposition, the hatted variables defined above satisfy:

$$Q_1 - \widehat{q}_1 \quad = \quad \hat{q}_2 S + \frac{(\mu - 1)\widehat{R}_1}{\widehat{r-g}} + \frac{(\mu - 1)\widehat{R}_2}{\widehat{r-g}} \times S. \tag{IA.55}$$

We can therefore compare the decomposition obtained when imposing $g = g_1 = g_2$ on the data (consistent with the model's predictions along the balanced growth path), and the more general decomposition (IA.55), which allows for $g \neq g_1 \neq g_2$ (but is not consistent with the balanced growth restrictions imposed by the model).Note, importantly, that the decomposition (IA.55) is *not* structural (since it violates $g_1 = g_2 = g$), so that comparisons with our baseline decomposition are only heuristic.

In order to measure g, g_1 and g_2, we then use the growth rate in the quantity indices for the three capital stocks reported in the middle panel of Appendix Figure 3, consistent with our measurement in the main paper.

Appendix figure 14 reports the results. The top panel is the baseline decomposition in the main paper, and the bottom panel is the decomposition using heterogeneous growth rates, as outlined above. The two are difficult to distinguish. Intuitively, heterogeneity in the measurement of the growth rates that enter marginal q have similar effects as changing adjustment costs across capital types. As we discuss in the main text, the effect of varying capital adjustment costs on the decomposition is small; the result here is consistent with that finding.

IA.D.2. Implications for the labor share

Consider the model with variable intermediate inputs, described in Appendix IA.B.3, and assume that all intermediate inputs are labor (or equivalently, that the production function is a value-added production function), and that returns to scale are constant. Then, using the results of Table V, the labor share of value added is given by:

$$LS \equiv \frac{W_t L_{j,t}}{P_{j,t} Y_t} = \frac{1-\alpha}{\mu_S}.$$

Moreover, using Lemma 3, the link between our reduced-form rents parameter μ and LS is:

$$\mu_S = \alpha(\mu - 1) + 1 = (1 - \mu_S LS)(\mu - 1) + 1,$$

and so, solving for the markup μ_S:

$$\mu_S = \frac{\mu}{\mu LS + (1 - LS)}.$$

However, this approach implicitly assumes that $1-\alpha$, the Cobb-Douglas exponent for labor in the production function, is varying over time, at least to the extent that the labor share varies. Specifically, our procedure also implies that $1-\alpha = \mu LS / \left(\mu LS + (1 - LS)\right)$. The top panel of Appendix Figure 35 shows the implied value for $1-\alpha$ in our baseline exercise. The mean is approximately 0.72. Moreover, the implied value declines from 0.74 to 0.70 during the 2000's, along with the decline in LS.

An alternative approach is to fix the Cobb-Douglas labor exponent. In that case, we do not require data on the labor share to obtain the valued-added markup $\tilde{\mu}$ implied by our estimate of the rents parameter μ; it can simply be obtained from $\tilde{\mu} = \alpha(\mu-1)+1$. The share of rents in value added, $s = 1 - 1/\mu$ obtained with this approach, reported in the middle panel of Appendix Figure 35, is very close to the share of rents of value added obtained in our baseline approach.

Additionally, this approach produces an implied labor share that is given by $LS = (1 - \alpha)/\tilde{\mu}$. The bottom panel of Appendix Figure 35 reports the path of this implied labor share, and compares it to the data. The magnitude of the decline in the implied labor is similar to the data, but the timing is somewhat different, because the rents parameter μ starts rising in the mid-80's, along with the rise in the investment gap, whereas the labor share only starts declining in the late 2000's.

IA.D.3. *No intangibles or no rents*

IA.D.3.1. No intangibles

When the firm has no intangibles, the expression for the investment gap collapses to:

$$Q_{1,t} - q_{1,t} \quad = \quad (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \Pi_{1,t+k} (1 + g_{1,t+1,t+k}) \right] \tag{IA.56}$$

In other words, the investment gap is exactly the net present of future rents generated by physical capital $K_{1,t}$, as in Lindenberg and Ross (1981). In this case, the decomposition of the investment gap in balanced growth is:

$$Q_1 - q_1 = \frac{(\mu - 1) R_1}{r - g}.$$

The variables entering the expression for the present value of rents can be constructed using the same methodology as described in Section A, setting the ratio of intangible to physical capital to $S = 0$ and the intangible investment rate to $\iota_2 = 0$.

Since by assumption, all of the investment gap is now accounted for by rents, we do not report its decomposition in this case. Instead, in Appendix Figure 12, we report the implied pure rents, expressed as a fraction of total value added, obtained using this approach. The top panel reports the estimate of this share obtained when assuming that firms do not use any intangibles in production. Ignoring intangibles, by 2015, rents account for about 14% of value added; by contrast, in our baseline estimate with intangibles measured as R&D (also reported in Figure 6), rents account for only about 8% of total value added.

The bottom panel of Figure 12 repeats this exercise, for the same sample of Compustat non-financial firms studied in the first part of Section III. In this sample, assuming firms have no intangible capital leads to an estimate of pure rents of 14% in 2015; including R&D capital lowers this estimate of 10%. Additionally, including a fraction of capitalized SG&A expenditures as a measure of organization capital further lowers estimates of pure rents to approximately 6%.

In this case (as also for the NFCB sector in the late 1940s), estimates of pure rents can be negative in the early 1980s. This reflects the combination of two effects: our estimates of

Q are strictly lower than 1 for a few years around 1980; and, as mentioned in Section III, estimates of the organization capital stock, while overall elevated, have relatively little trend upward in the Compustat sample (by contrast with R&D capital).

IA.D.3.2. No rents

When the firm earns no rents ($\mu = 0$), the decomposition of the investment gap collapses to:

$$Q_{1,t} - q_{1,t} = q_{2,t} S_t, \quad S_t \equiv \frac{K_{2,t}}{K_{1,t}}.$$

Along the balanced growth path, the same expression holds, without the time subscripts. One can invert this relationship in order to recover a value for S, the ratio of intangible to physical capital:

$$S = \frac{Q_1 - q_1}{q_2} = \frac{Q_1 - (1 + \gamma_1 g)}{(1 + \gamma_2 g)},$$

where g is the trend growth rate of the capital stock. Effectively, this amounts to backing out an implied value of the intangible capital stock from observed values of Q_1, which is possible when there are no rents. This is the approach followed by Hall (2001).

Additionally, given a value for S, the same relationship as in Section A holds for $r - g$:

$$\begin{aligned} r - g &= \frac{ROA_1 - (\iota_1 + S\iota_2)}{Q_1} - \frac{\gamma_1 + \gamma_2 S}{Q_1} g^2 \\ &= \frac{ROA_1 - (\iota_1 + S\iota_2)}{Q_1} - \frac{Q_1 - (1 + S)}{Q_1} g \end{aligned}$$

where, to go from the first to the second line, we used the expression for the investment gap along the balanced growth path when $\mu = 1$. From this expression, one can then obtain values of user costs, R_1 and R_2.[36] The implied Cobb-Douglas share of intangibles in production is

[36] Note that when there are strictly positive adjustment costs, $\gamma_1, \gamma_2 > 0$, this approach requires taking a stance on the gross intangible investment rate ι_2 and on the growth rate of the capital stock g. For g, in Figure 8, we use the growth rate of the physical capital stock. There is some tension between assuming that ι_2, the intangible investment rate, is observed, and assuming that $S_t = K_{2,t}/K_{1,t}$ is unobserved. In Figure 8, we assume that ι_2 is given by the R&D investment rate, but results are not materially different if it is assumed to be equal to 0 or, in the case of Compustat non-financial firms, to the total investment rate in R&D and organization capital.

then given by

$$\eta = 1 - \frac{1}{1 + \frac{R_2}{R_1} S}.$$

Figure 8 reports implied estimates for S and η when we assume that $\mu = 1$, that is, there are no rents. The top two panels provide results from the NFCB sector as a whole, and the bottom two panels provide results for Compustat non-financial firms.

The results show that assuming away rents will lead to high estimates of the importance of intangible capital in production, relative to physical capital. For instance, among Compustat firms, assuming no rents leads to an estimate of the ratio S of intangible to physical capital of approximately 1.2 in 2015. By contrast, in Compustat data, the magnitude of S is approximately 0.4 when including both R&D and organization capital (as indicated by the bottom right panel of Figure 8). We note, however, that values of S close to 1 seem plausible for some sectors, such as the Healthcare sector and the Consumer sector (for the latter, when organization capital is included).

An additional drawback from this approach is that the implied time-series S exhibit periods of large decline (in the 1970s, and after the burst of the dot-com bubble). This reduction in the stock intangible capital (relative to the physical capital) is difficult to reconcile with the fact that empirical measures of S have trended upward consistently throughout the post-war period.

IA.D.4. Separating R&D from SG&A capital

Methodology With more than two types of capital, the decomposition of the physical investment gap along the balanced growth path can be written as:

$$Q_1 - q_1 = \sum_{m \geq 2} S_m q_m + \frac{(\mu - 1) R_1}{r - g} + \sum_{m \geq 2} \frac{(\mu - 1) R_m S_m}{r - g}.$$

Additionally, we have:

$$(\mu - 1)\left(R_1 + \sum_{m \geq 2} S_m R_m\right) = ROA_1 - \left(R_1 + \sum_{m \geq 2} S_m R_m\right). \tag{IA.57}$$

Multiplying the expression for the investment gap by $r-g$, using the expression above, along with the fact that, neglecting terms of order $o(g)$, for each $n = 1, ..., N$:

$$\begin{aligned} R_n &= r - g + \iota_n + \gamma_n r g, \\ q_n &= 1 + \gamma_n g, \end{aligned}$$

we arrive at:

$$(r-g)Q_1 = ROA_1 - \left(R_1 + \sum_{m\geq 2} S_m R_m\right) - \left(\gamma_1 + \sum_{m>2} \gamma_m S_m\right) g^2,$$

or:

$$(r-g) = \frac{ROA_1 - \left(R_1 + \sum_{m\geq 2} S_m R_m\right)}{Q_1} - \frac{\left(\gamma_1 + \sum_{m>2} \gamma_m S_m\right)}{Q_1} g^2.$$

Given values for investment rates $\{\iota_n\}_{n=1}^N$, relative capital stocks $\{S_m\}_{m=2}^N$, adjustment costs $\{\gamma_n\}_{n=1}^N$, average Q_1, the return to physical capital ROA_1, and the growth rate of the total capital stock g, the right-hand side of this expression can be constructed in the data, implying a particular value for $r-g$. The values of user costs can then be obtained from $R_n = r - g + \iota_n + \gamma_n r g$, and the value of μ is then given by Equation (IA.57). Along with the fact that $q_n = 1 + \gamma_n g$, this is sufficient to construct all the elements in the decomposition of the investment gap.

Results The bottom panel of Appendix Figure 6 reports the results of the generalized decomposition of the physical investment gap when $R\&D$ capital and organization capital $SG\&A$ are treated as different capital inputs:

$$Q_1 - q_1 = S_2 q_2 + S_3 q_3 + \frac{(\mu-1)R_1}{r-g} + \frac{(\mu-1)R_2 S_2}{r-g} + \frac{(\mu-1)R_3 S_3}{r-g},$$

where the number 2 indexes R&D capital, and the number 3 indexes organization capital. For the adjustment cost to R&D capital, we use a value of $\gamma_2 = 12$, as in the main text, while for the adjustment cost to organization capital, we use the estimate of $\gamma_3 = 3.2$ reported in Belo et al. (2019) (Table 3) for the parameter governing the convexity of adjustment costs to brand capital. This decomposition is quantitatively very similar to the decomposition in the middle panel (which takes the simple sum of organization and R&D capital), in that it attributes approximately 40% of the physical investment gap after 2000 to the direct effect of th R&D and intangible capital stocks, and approximately 30% to the rents they generate.

An additional insight from this graph is that rents account for a bigger fraction of the part of the investment gap created by the R&D capital stock than they do for the organization capital stock. This result is driven by the fact that user costs for R&D capital implied by the decomposition, R_2, are higher than those for organization capital, R_3. In turn, this difference in user costs can be explained by the fact that investment rates for R&D capital imputed using Compustat data are on average 25% per year after 2000, whereas investment rates for organization capital are only 15%.

Recall that our decomposition implicitly infers depreciation rates from observed gross investment rates, so that high gross investment rates imply high rates of depreciation and hence high user costs. The magnitude of both R&D investment rates and the implied depreciation rates (in the order of 20%) are consistent with the evidence in Li and Hall (2020). To our knowledge, there are no direct sources for depreciation rates on organization capital, though the literature typically uses a depreciation rate of 20% (Lev and Radhakrishnan, 2005; Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Falato et al., 2020). Thus, our decomposition produces somewhat lower implicit depreciation rates for organization capital than those assumed in the literature. We note, however, that both Peters and Taylor (2017) and Ewens et al. (2020) show that estimates of the size of the organization capital stock obtained from Compustat data are not very sensitive to the choice of depreciation rate.

IA.D.5. GMM estimation on split samples

IA.D.5.1. Moment conditions

We first derive moment conditions that can be used for estimation. We use a model with uncertainty that admits a closed-form solution. Specifically, as in Lemma 1, we assume that:

$$\begin{aligned} \Pi_t &= A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}}, \\ K_t &= \left(\sum_{n=1}^{N} \eta_n K_{n,t}^{\rho} \right)^{\frac{1}{\rho}}, \quad \rho \leq 1, \quad \sum_{n=1}^{N} \eta_n = 1. \end{aligned} \tag{IA.58}$$

In order to obtained closed-form solutions, we make the following additional assumptions.

ASSUMPTION 4: *Assume that:*

(1) Adjustment costs are linear: $\Phi_{n,t}(1+g) = g + \delta_n, \quad \forall n = 1, ..., N, \forall t.$

(2) The discount rate is constant: $M_{t,t+1} = (1+r)^{-1}$.

(3) $\{A_t\}_{t \geq 0}$ *satisfies* $A_{t+1} = (1+g_t)A_t$, $g_t \sim F(.) \quad i.i.d., \quad \mathbb{E}(g_t) = \bar{g}$.

LEMMA 6: *If Assumption 4 holds, then the solution to the model satisfies:*

$$S_t = \frac{\eta}{1-\eta}\left(\frac{r+\delta_1}{r+\delta_2}\right)^{\frac{1}{1-\rho}} \tag{IA.59}$$

$$\iota_{1,t} = g_t + \delta_1 \tag{IA.60}$$

$$\iota_{2,t} = g_t + \delta_2 \tag{IA.61}$$

$$g_{K,t} = g_t \tag{IA.62}$$

$$ROA_{1,t} = \mu\left((r+\delta_1) + (r+\delta_2)S_t\right) \tag{IA.63}$$

$$Q_{1,t} = 1 + S_t + \frac{(\mu-1)}{r-\bar{g}}(r+\delta_1) + \frac{(\mu-1)}{r-\bar{g}}(r+\delta_2)\, S_t \tag{IA.64}$$

where:

$$g_{K,t} \equiv \frac{K_{1,t+1} + K_{2,t+1}}{K_{1,t} + K_{2,t}} - 1.$$

This result is a particular case of the risky balanced growth model described in Section E and Appendix IA.B.2.1, when growth in fundamentals is *i.i.d.* (which corresponds to the case $\lambda = 1$). Note that the first five expressions for key ratios do not depend on the i.i.d. growth assumption (that assumption is only used in the computation of $Q_{1,t}$), though they do depend on the assumption that A_{t+1} is in the information set of time t. Additionally, the ratios S_t, $ROA_{1,t}$ and $Q_{1,t}$ implied by the model are constant over time; random shocks g_t will only appear in measures of investment $g_{K,t}$, $\iota_{1,t}$ and $\iota_{2,t}$. We write the closed-form solution of the model as moment conditions:

$$0 = \mathbb{E}\left[S_t - \frac{\eta}{1-\eta}\left(\frac{r+\delta_1}{r+\delta_2}\right)^{\frac{1}{1-\rho}}\right] \tag{IA.65}$$

$$0 = \mathbb{E}\left[\iota_{1,t} - (\bar{g}+\delta_1)\right] \tag{IA.66}$$

$$0 = \mathbb{E}\left[\iota_{2,t} - (\bar{g}+\delta_2)\right] \tag{IA.67}$$

$$0 = \mathbb{E}\left[g_{K,t} - \bar{g}\right] \tag{IA.68}$$

$$0 = \mathbb{E}\left[ROA_{1,t} - \mu\left\{(r+\delta_1) + (r+\delta_2)S_t\right\}\right] \tag{IA.69}$$

$$0 = \mathbb{E}\left[Q_{1,t} - \left\{1 + S_t + \frac{(\mu-1)}{r-\bar{g}}(r+\delta_1) + \frac{(\mu-1)}{r-\bar{g}}(r+\delta_2)S_t\right\}\right] \tag{IA.70}$$

IA.D.5.2. Estimation approach

We fix the elasticity of substitution between intangible and physical capital, ρ, to $\rho = 0$, so that the two types of capital are Cobb-Douglas substitutes. This follows our approach in the main text. We then estimate the six structural parameters $\{\bar{g}, \delta_1, \delta_2, \mu, \eta, r\}$ using the six moment conditions above.

Our estimation method is standard: we use two-step efficient GMM (Hansen, 1982), with the identity matrix as the first-step weighting matrix. HAC standard errors are computed using a Bartlett kernel with four lags.

We report two additional sets of results from this estimation. First, we compute point estimates and standard errors for four "implied moments": user costs R_1 and R_2, the markup over value added μ_{VA}, and the share of rents as a fraction of value added s_{VA}. We obtain estimates and standard errors for these implied moments by stacking the four moment conditions to the GMM system:

$$0 = \mathbb{E}\left[R_1 - (r+\delta_1)\right] \tag{IA.71}$$

$$0 = \mathbb{E}\left[R_2 - (r+\delta_2)\right] \tag{IA.72}$$

$$0 = \mathbb{E}\left[s_{VA} - (1 - s_{L,t})\left(1 - \frac{1}{\mu}\right)\right] \tag{IA.73}$$

$$0 = \mathbb{E}\left[\mu_{VA} - \frac{1}{1 - s_{VA}}\right] \tag{IA.74}$$

where $s_{L,t}$ is the time-series for labor as a share of value added.[37] Second, we also compute the difference between point estimates across subsamples, and test for whether it is significantly different from zero. We perform this test by stacking the moment conditions for the two subsamples (with a set of structural parameter and implied moments for each subsample), and interacting the moment conditions with an indicator for each subsample. The p-values reported are for the two-sided test against the null of equality of a given structural parameter or implied moment across subsamples.

IA.D.5.3. Results

The results for this estimation approach are reported in Table 21. The table contains estimation results for the NFCB sector as a whole (the first three columns), and for the sample of Compustat non-financial firms (columns four through nine), separating the case where intangibles are measured using R&D from the case where they are measured using R&D plus organization capital. Additionally, we focus on data form 1985-2017, since this is the period of primary interest for the paper, and we report results on subsamples split around the year 2000, following again our analysis in the main text.

Before discussing the results, it is worth noting that in the simple model with i.i.d. growth, GMM estimation leads to exactly the same point estimates as one would obtain by replacing the various data series in the moment conditions above by their sample means, and then inverting the moment conditions. This is very similar to what we do in our baseline approach. The reason for the equivalence between the GMM estimation and our baseline approach is that, with the exception of the solution for investment rates, the representation of the model with i.i.d. growth and adjustment costs is exactly the same. (Moreover, investment rates are linear functions of the only source of random disturbances, g_t.) Thus, qualitatively, we

[37] Following our discussion in Section E and Appendix IA.B.3, the reduced-form moment μ_{VA} is the value-added markup under the assumption of constant returns in production.

should expect to find the same patterns as were obtained in the simpler analysis of the baseline model.

Quantitatively, there two reasons why the GMM results for the simple model with i.i.d. growth might differ from the baseline results in the paper, and in particular the results of Table 1 of the main paper. First, our baseline approach averages moments over seven-year rolling windows. By contrast, GMM estimation of the simple model uses averages over the 16-year and 17-year windows that make up the two subsamples from 1985-2017. Second, the results of Table 1 rely on the model with adjustment costs, whereas the simple model with i.i.d. growth does not allow for adjustment costs.

Given this caveat, the main interest of estimating the simple model via GMM is that it allows to assess the statistical significance of the structural changes that we documented with our baseline approach. The results of Table 21 find the same two simultaneous structural changes as in our baseline approach: an increase in the share of intangibles in production, and an increase in rents. The increase in the share of intangibles in production between the 1985-2000 period and 2001-2017 is significant at the 1% level in all three data sources, and it is quantitatively similar to the one obtained in our baseline approach.

The statistical significance of the increase in rents is somewhat more muddled. In the NFCB data, all measures of rents (the curvature parameter μ, the rent share of value added s_{VA}, and the markup over value added μ_{VA}) are statistically significant. However, in the Compustat sample, the increase in the curvature parameter μ is only significant at the 5% level when measuring intangibles as R&D, and it becomes insignificant when intangibles are measured as the sum of R&D and organization capital. The latter finding is consistent with our baseline results, which suggested that adding organization capital to the measure of intangibles tends to weaken the upward trend in rents. Likewise, in the Compustat data, the increase in μ_{VA} and s_{VA} is only significant at the 5% level when measuring intangibles as R&D, and at the 10& level when also including organization capital. Additionally, and consistent with the results in the main text, there is a decline in estimated user costs, but it is only significant in the NFCB sector.

The main issue with these estimation results is that the point estimates of the implied discount rate r are substantially higher, and more stable, for the Compustat sample than they are for the NFCB sector (in the order of 8%, instead of 5% for the NFCB sector). (Note that this was also true in our baseline approach, though we had not reported the values of the discount rate in isolation in Table 1, so that this difference between the NFCB data and the Compustat data was not clearly visible.) The empirical force driving this result is the fact that measured returns on assets are substantially higher in Compustat than they are in the NFCB sector. Recall that, in the model, the discount rate r satisfies $r - g_t = (ROA_1 - (\iota_1 + \iota_2 S))/Q_1$ — it is the wedge between flow profits in excess of investment costs, over valuations. Empirically, values of returns on assets ROA_1 are substantially higher in the Compustat data (as also reported in Table 1 in the main text), in a manner that is not fully offset by either higher investment rates, or a higher values of Q_1 among publicly traded non-financial firms.

IA.D.6. Alternative identification strategies

Along the balanced growth path, the general decomposition of the investment gap is:

$$Q_1 - q_1 = q_2 S + \frac{(\mu - 1)R_1}{r - g} + \frac{(\mu - 1)R_2}{r - g} \times S. \tag{IA.75}$$

In our baseline approach, we directly measure S. We primarily infer the Gordon growth term $r - g$ from the value of Q_1. Combining estimates of this term with investment rates, we then compute the user costs R_1 and R_2. Finally, we infer μ by combining the estimates of user costs with a measure of the average return on physical assets, ROA_1.

Here, we describe in detail alternative approaches that could be used to construct our decomposition. All these alternative approaches rely on constructing a measure of the average cost of capital, r, in order to construct the Gordon growth term $r - g$, instead of using Q_1, as we do in our baseline approach. This allows the value of Q_1 to be used to infer some other underlying structural parameter, such as intangible intensity or rents. In order to limit the size of the results, we only report them for the NFCB sector; they are qualitatively similar for the Compustat sample.

IA.D.6.1. Alternative approach 1: average cost of capital

We next describe a different identification approach, which we refer to as the "average cost of capital approach". This approach is closer to that of Barkai (2020) and Karabarbounis and Neiman (2019) (case Π). We measure the average cost of capital as the leverage weighted average of the cost of debt (obtained from average interest rates on the market value of debt of the NFCB sector), and the cost of equity (obtained from the PD ratio of public firms). We then construct the different terms on the right-hand side of Equation (19) using the same moments as in our baseline, except that we do not match the observed value of Q_1.

In this approach, the (implied) value of the investment gap (that is, the left-hand side of Equation 19) is growing faster after 1985 than the investment gap we measured in our baseline approach (that is, the left-hand side of Equation 19). By 2015, the implied investment gap is about twice as large as the measured one. This is because the discount rate r obtained using an average cost of capital approach is lower, and declining faster, than the discount rate implicit in our baseline decomposition. Consistent with Barkai (2020) and Karabarbounis and Neiman (2019) (case Π), lower discount rates also lead to a higher, and more rapidly increasing profit share (approximately 9.0 p.p. over the 1985 to 2015 period, instead of 6.2 p.p. in our baseline approach).

However, the composition of the implied investment gap remains similar to our baseline findings. The rest of this Appendix reports more detailed results, compares the discount rates implied by both approaches, and expands on the interpretation of the results in terms of implicit equity risk premia, following the discussion of Section C.

Description Let $\{D_{E,t}\}$ and $\{D_{B,t}\}$ be distributions to shareholders and debtholders. These distributions must satisfy $D_t = D_{E,t} + D_{B,t}$, where D_t are total distribution to firm owners. Moreover, let E_t^e and M_t^e be the ex-distribution values of equity and debt, which must satisfy $V_t^e = E_t^e + B_t^e$, where V_t^e is total firm value. We make the following assumptions about $\{D_{E,t}, D_{B,t}, E_t^e, B_t^e\}$.

ASSUMPTION 5 (Weighted average cost of capital): *There exists two discount factors*

$\{M_{E,t,t+1}\}$ and $\{M_{B,t,t+1}\}$, such that:

$$E_t^e = \mathbb{E}_t\left[M_{E,t,t+1}\left(D_{E,t+1} + E_{t+1}^e\right)\right], \quad B_t^e = \mathbb{E}_t\left[M_{B,t,t+1}\left(D_{B,t+1} + B_{t+1}^e\right)\right].$$

Moreover, along the balanced growth path, (a) there exist r_E and r_B such that:

$$\forall t, \quad M_{E,t,t+1}^{-1} = 1 + r_E, \quad M_{B,t,t+1}^{-1} = 1 + r_B,$$

and (b) the ratio of the market value of debt to the market value of equity is constant:

$$l_t \equiv \frac{E_t^e}{B_t^e} = l.$$

In this case, along the balanced growth path, r, r_E and r_B must satisfy:

$$r = \frac{l}{1+l} r_B + \frac{1}{1+l} r_E.$$

In other words, under Assumption 5, the firm-wide discount rate r must equal the weighted average of the shareholder and debtholder discount rates r_E and r_B. This suggests an alternative avenue to construct the decomposition, which is to use estimates of r_B and r_E in order to construct the Gordon growth term $r - g$ from the relationship:

$$r - g = \frac{l}{1+l}(r_B - g) + \frac{1}{1+l}(r_E - g).$$

This is similar to the approach followed by Barkai (2020) and in Karabarbounis and Neiman (2019) in order to estimate of the pure profit share.

Implementation In order to implement this approach, we need estimates of r_B and r_E. For the former, we use a weighted average of the different interest rates which were an input into our computation of the market value of total debt liabilities of non-financial corporations, and are described in Appendix IA.C.2. The weights are the relative market values of each types of debt liability. We then pre-multiply this interest rate by ones minus the statutory top corporate income tax rate. We denote the resulting time series by r_b^n. For the estimate of

r_E, we use the fact that along the balanced growth path, the PD ratio is constant and given by:

$$PD_t \equiv \frac{E^e_{t-1}}{D_{E,t}} = \frac{1}{r_E - g}.$$

In order to estimate the PD ratio, we use the fact that:

$$PD_t = \frac{1}{R^c_{E,t-1,t} - R^e_{E,t-1,t}}, \quad R^c_{E,t-1,t} \equiv \frac{D_{E,t} + E^e_t}{E^e_{t-1}}, \quad R^e_{E,t-1,t} \equiv \frac{E^e_t}{E^e_{t-1}}.$$

Our sources for $R^e_{E,t-1,t}$, $R^c_{E,t-1,t}$ and the corporate income tax rate are described in Appendix IA.C.6. Finally, for leverage l, we use the ratio of the market value of debt liabilities to the market value of equity, both of which we computed in order to construct the total value of the firm, V^e_t. We then obtain the Gordon growth term as:

$$r - g = \frac{l}{1+l}(r^n_B - g^n) + \frac{1}{1+l}PD^{-1}.$$

where g^n is the nominal growth rate of the total capital stock.[38] Appendix Figure 15 reports the resulting time series for the PD ratio PD_t.[39] The top panel of Appendix Figure 16 reports the resulting cost of equity, the cost of debt, and the average cost of capital.[40]

Results Figure 17 reports the results obtained in this alternative approach. The top panel of the figure shows the implied investment gap, and its decomposition. Generally, this approach implies a larger estimated investment gap, particularly so in the first half of the sample, prior to 1980. Nevertheless, targeting the PD ratio still leads to the same key insights as the baseline approach. In particular, by 2015, the rents attributable to intangibles account for 29% of the total investment gap in the PD ratio approach (compared to 25% in the baseline

[38] We use the nominal instead of the real growth rate because the time series for the cost of debt r^n_b which we construct is a nominal interest rate.

[39] The figure also reports the time series for the "leverage-adjusted" PD ratio, defined as $PD = (r-g)^{-1}$, and which, in the balanced growth path of the model, satisfies $PD = V^e_{t-1}/D_t$.

[40] All are expressed in nominal terms; for the cost of debt, this is the series r^N_B which we directly construct from the data, and for the cost of equity and the average cost of capital, we report $r^n_E \equiv = (r_E - g) + g^n$ and $r^n = (r-g) + g^n$.

approach), while the two rents-related terms together account for 33% of the total investment gap (versus 36% in the baseline approach).

This approach leads to a larger overall investment gap because the values of Tobin's Q for physical capital which it implies are far superior to those observed in the data, as reported in the bottom left panel of Figure 17. Fundamentally, this is because the average cost of capital approach leads to a lower estimate of the firm discount rate r than our baseline approach. The top panel of Figure 2 reports the estimates of (nominal) discount rates we obtain in the average cost of capital approach and in our baseline approach. The former is generally lower than the latter. The two are closest together in 1985. After 1985, the discount rate in the average cost of capital approach declines more than in our baseline approach.

Because lower discount rates imply lower user costs, the average cost of capital approach also leads to higher estimates of the pure profit share, as reported in the bottom right panel of Figure 2. The ratio is both higher on average, and increasing faster, in our approach, relative to the baseline approach. After 1985, rents in the average cost of capital approach increase by about 9.0 p.p., versus 6.2 p.p. in our baseline approach.

Why the two approaches lead to a different implied discount rate is a difficult question. As discussed in Section C, one possible interpretation is in terms of implicit risk premia on equity. To illustrate this, the bottom panel of Figure 17 constructs a measure of the risk premium implicit in the cost of equity implied by our baseline approach and by the average cost of capital approach. We define this implicit risk premium as:

$$RP \equiv (1+l)\left(r^n - \frac{l}{1+l} r_B^n\right) - r_f^n = r_E^n - r_f^n,$$

where $r^n = (r-g)+g^n$ is the nominal, firm-wide discount rate, l is leverage, r_B^n is the (nominal, after-tax) cost of debt, and r_f^n is a measure of the risk-free rate. We use the average annual return on one-month Treasury bills to measure the risk-free rate; the source is reported in Appendix IA.C.6.

The resulting time series show that the average cost of capital approach leads to a lower implicit equity risk premium than our approach. The two implicit risk premia are closest in 1985; after that year, the implicit risk premium in our approach rises somewhat more than in the average cost of capital approach.

There are also measurement issues that may contribute to the differences between our baseline approach and the average cost of capital approach. The sample underlying our measure of Q_1 (the NFCB sector) and the sample underlying our measure of the PD ratio (the S&P 500). Additionally, distributions to equityholders may not be accurately measured. Our measure is based on cash distributions, and excludes share repurchases, which became more common after the early 2000's. Generally, it is difficult to accurately match the rate of distributions to shareholders implicit in the computation of the PD ratio, and the distributions to shareholders measured in Flow of Funds data; this may contribute to further differences between the two approaches.

A different measure of r_E We also report the results of this approach when using a similar method to measure the cost of equity as Barkai (2020). Specifically, we assume that the (nominal) cost of equity is:

$$r_E^n = r_f^n + RP,$$

where RP is a constant risk premium, and r_f^n is the time-series for the risk-free rate described above. We then compute the Gordon growth term as:

$$r - g = \frac{l}{1+l}(r_B^n - g^n) + \frac{1}{1+l}(r_E^n - g^n).$$

For the constant risk premium, we use a value of $RP = 6.5\%$, in line with the long-run average estimates of equity risk premia constructed by Campbell and Thompson (2008) and extended by Martin (2017).

The bottom panel of Figure 16 reports the time series for the average cost of capital used in this approach, and Figure 2 compares it with the cost of capital in other approaches. The most notable difference with the other approaches is that the cost of capital according to this approach is substantially lower in the 1945-1985 period. In fact, in a number of years in this period, the implied discount rate is close to, or below, the growth rate of the capital stock. In turn, this implies implausibly high values of Tobin's Q_1 (in excess of 20, in particular in the early part of the sample). Therefore, we only report the results related to the investment gap obtained using this approach after 1985.

These results are reported in Figure 18. After 1985, the results share number of common features with Figure 17, where the PD ratio is used instead of the risk-free rate. The implied values for Q_1 are substantially higher than in the data. Moreover, the increase in total rents as a fraction of value added is larger than in our baseline approach. The latter effect is more muted than in alternative approach 1; this is because, as indicated in Figure 17, the discount rate in this approach is somewhat higher than when using the PD ratio.

Finally, note that in this approach, the implicit risk premium is constant. By contrast, in both our baseline approach, and the average cost of capital approach using the PD ratio, the implicit equity risk premium is rising after 1985. Additionally, in our baseline approach, the implicit equity risk premium is above 6.5% after 2003. These differences help explain why estimates of the pure profit share are lower in our baseline approach than in either of the two average cost of capital approaches.

IA.D.6.2. Alternative approach 2: inferring intangibles from the investment gap

Given a measure of the Gordon growth term $r - g$ that is independent from Q_1, the decomposition above can also be used as a way to derive an implicit stock of intangibles, as opposed to measuring it in the data. This amounts to computing the ratio of intangible to physical capital such that the model matches both measured $r - g$ and the measured investment gap $Q_1 - q_1$. Straightforward derivations show that this ratio is given by:

$$S^{\text{implied}} = \frac{(r-g)(Q_1 - q_1) - (ROA_1 - R_1)}{(r-g)q_2 - R_2}. \tag{IA.76}$$

This expression can be thought of as a generalization of the approach of Hall (2001), who derives the ratio of intangible to physical capital consistent with stock market values and measures of the physical capital stock (and therefore of Q_1). The expression is a generalization in the sense that it allows for rents. In the Hall (2001) case of no rents ($\mu = 1$), from the decomposition of the investment gap, the value of S is given by:

$$S^{\text{Hall}} = \frac{Q_1 - q_1}{q_2}.$$

When μ can be different from 1, given a value of S, the implied value for μ is[41]:

$$\mu = \frac{ROA_1}{R_1 + S^{\text{implied}} R_2}.$$

Figure 19 reports results from the decomposition constructed using this approach, rather than our baseline approach. The top panel shows the investment gap decomposition, and the bottom two panels report, respectively, the time series for S^{implied} (comparing it with the time series for S in the data, which is the one that our baseline approach matches); and the time series for the value of rents as a fraction of total value added implied by the model.

One noteworthy finding from Figure 19 is that, after 2000, $S^{implied}$ grows substantially faster than what is measured by the BEA Fixed Assets tables. By 2015, the implied intangible stock is about twice as large as its BEA counterpart. As a result of this rapid growth, the implied increase in rents is lower than in our baseline approach. Overall, as indicated by the top panel, this approach attributes a bigger share of the overall investment gap to intangibles than our baseline approach (about two-thirds, versus one-third in our baseline approach).

A potential problem with this approach, however, is that it also implies that there must have been a very large stock of intangible capital, relative to physical capital, in the 1950-1970 period (and moreover, that this stock turned negative for a few years around 1980). Mechanically, this is because the stock of intangibles account for movements in the investment gap that cannot be fully accounted for by increases in the PD ratio (and therefore declines in $r - g$). In other words, through the lens of the model, the period 1950-1970 was one where discount rates (as implied by PD ratios) were low, but not enough to explain the high investment gap, so that intangibles must have been high.

Another drawback of this approach is that it creates a mechanical negative correlation between the level of discount rates (implied by the PD ratio), and the stock of intangibles. Hall (2001) also contends with this issue, and finds the same declining intangible capital stock in the late 1970s. This "destruction" of intangible capital might be difficult to reconcile with the fact that direct measures of the capital stock instead suggest that the ratio of intangible

[41] Equation (IA.76) can be rewritten as $(r-g)\left(Q_1 - q_1 - q_2 S^{\text{implied}}\right) = ROA_1 - R_1 - S^{\text{implied}} R_2$. Therefore, $S^{\text{implied}} = S^{\text{Hall}}$, if and only if, $ROA_1 = R_1 + S^{\text{implied}} R_2$, which is the same as $\mu = 1$.

to physical capital has been continuously growing over the post-war period.

IA.D.6.3. Alternative approach 3: inferring rents from the investment gap

Another possible approach is to infer the rents parameter, μ, from the investment gap. In this approach, the rents parameter would be obtained from:

$$\mu = 1 + \frac{r-g}{R_1 + R_2 S} \left(Q_1 - q_1 - q_2 S \right). \tag{IA.77}$$

This approach will lead to positive estimates of rents whenever average Q_1 is higher than $q_1 + q_2 S$, which would be the value of Q_1 in a model without rents. Moreover, relative to the baseline approach, the model will not necessarily match measured returns to physical capital ROA_1. The model-implied value of returns to physical capital is then:

$$ROA_1^{\text{implied}} = R_1 + R_2 S + (r-g)(Q_1 - q_1 - q_2 S). \tag{IA.78}$$

The no-rents case again obtains when $Q_1 = q_1 + q_2 S$, which implies that the return to physical capital is equated to the weighted average user cost $R_1 + R_2 S$. As in the other alternative approach, this approach requires using measures of $r - g$ not derived from Tobin's Q_1; we use the average cost of capital measure, with the inverse PD ratio as a proxy for the cost of equity capital.

Figure 20 reports results in this case. The decomposition of the investment gap (top panel) is qualitatively and quantitatively close to our baseline results. As indicated by the bottom right panel of Figure 20, the size of rents is somewhat smaller in this approach alternative approach, particularly after 2000. The bottom right panel of Figure 20 reports the implied returns to physical capital, ROA_1 in this approach, and compares them with their measures in the data, which our baseline approach matches by construction. Returns to capital implied by this approach are overall lower than in the data, consistent with the fact that this approach leads to somewhat lower rents. Both in the data and in the model-implied series, there is an increase in average returns to physical capital after 1980, though it is less marked in this approach than in the data. Thus, overall, this approach leads to somewhat smaller rents than our baseline, both in levels, and in terms of their overall increase since 1985.

IA.D.7. Markups and returns to scale

IA.D.7.1. Methodology

Given the partial identification result highlighted by Lemma 4, in what follows, we compute estimates of the reduced-form parameter $\chi = \mu_S/\zeta$, and report implied values of the markup μ_S of price over the marginal cost of sales under different assumptions about decreasing returns to scale.

The results of Table V can be used to identify the reduced-form rents parameter χ and the Cobb-Douglas elasticity α separately. We use the following approach, which, as explained below, is consistent with our analysis of the balanced growth model.

LEMMA 7: *The values of α and χ can be derived from data on the ratio of operating surplus to sales, and on the ratio of capital costs to operating surplus:*

$$x_\Pi \equiv \frac{\Pi_t}{S_t}, \quad \nu_K \equiv \frac{R_{1,t}K_{1,t} + R_{2,t}K_{2,t}}{\Pi_t},$$

as follows:

$$\begin{aligned} \chi &= \frac{1}{\nu_K + (1-\nu_K)(1-x_\Pi)} \quad \geq 1, \\ \alpha &= 1 - \frac{1-x_\Pi}{\nu_K + (1-\nu_K)(1-x_\Pi)} \quad \leq 1. \end{aligned} \tag{IA.79}$$

The proof of this result follows from the expressions reported in Appendix Table V. Of course, there are other combination of the ratios reported in Appendix Table V that could also be used to identify separately χ and α. The advantage of this particular identification approach is twofold. First, given an estimate of capital costs, it can be implemented using firm accounting data, since it only requires observing total sales and total operating surplus. (In particular, no separate measure of labor costs is needed.) Second, this approach encompasses the approach used to measure the rents parameter μ in the balanced-growth model of the main text. Indeed, there, the reduced-form rents parameter is obtained from the relationship:

$$\frac{1}{\mu} = \frac{\alpha}{\chi - (1-\alpha)} = \frac{R_{1,t}K_{1,t} + R_{2,t}K_{2,t}}{\Pi_t} = \nu_K.$$

Identification of μ, α and χ can therefore be thought of as follows. First, estimate μ, using a measure of ν_K, as we do in our empirical analysis of the balanced growth model in the main text. Second, estimate α, using the expression reported in Lemma 7. Third, obtain the value of $\chi = \mu_S/\zeta$ using:

$$\mu = 1 + \frac{\chi - 1}{\alpha} \qquad \Longleftrightarrow \qquad \chi = \frac{\mu_S}{\zeta} = 1 + \alpha(\mu - 1),$$

or equivalently, using the expression reported in Lemma 7.

IA.D.7.2. Results

Figure 31 reports the results of this methodology, applied to the data from the entire Compustat Non-Financial sample, from 1974 to 2017. We focus on this data source because, to our knowledge, there is no good source on total sales (as opposed to total profits or total operating surplus) for the non-financial corporate sector.[42] The data we use is the same as in the analysis of Section III in the main text, except that we also make use of the ratio of total sales to gross operating surplus. Total sales is measured using Compustat variable `sale`.

The results of Figure 31 report the time series for the sales markup, which, following the previous discussion, is measured as:

$$\mu_S = (1 + \alpha(\mu - 1))\,\zeta = (\nu_K + (1 - \nu_K)(1 - x_\Pi))^{-1}\,\zeta.$$

Related work has taken different approaches to estimating returns to scale, depending on data availability. Where detailed cost data are available, for example from the Census of Manufacturing, returns to scale can be estimated using data on cost shares and output. Syverson (2004) develops this methodology and estimates that a benchmark of constant returns to scale is justified in his detailed industry analysis. More recently De Loecker et al. (2020) use

[42]The NIPA tables do not appear to contain information on domestic sales specifically for non-financial corporations; see https://apps.bea.gov/iTable/iTable.cfm?ReqID=13&step=1. The Flow of Funds reports a series titled Revenue from Sales of Goods and Services (FRED series BOGZ1FA106030005Q), but this series is identical to the gross value added for the non-financial corporate sector reported in NIPA table 1.14 (FRED series A455RC1Q027SBEA), indicating that the Flow of Funds series likely measures value added, not gross revenue, despite its name.

two approaches. First, using *Compustat* and hence lacking detailed cost shares, they use a demand approach and estimate slightly increasing returns to scale in their specifications. In a standard specification, similar to ours, they estimate nearly constant returns of 1.02 in 1980, rising to 1.08 by 2016. When they specify overhead in the production function, which in *Compustat* includes some intangibles, they have higher returns to scale of 1.07 initially rising to 1.13 at the end of the sample. When instead they approximate the Syverson (2004) cost share methodology, they obtain lower estimates of nearly constant returns, of 0.98 pre-1980 and 1.03 by 2010, using industry averages. In firm-level data, which have more heterogeneity, they find initially slightly more decreasing returns and a larger increase.

Given this background, we report the implied values of μ_S for three different values of the degree of returns to scale: decreasing returns ($\zeta = 0.95$); constant returns ($\zeta = 1.00$, as in our baseline analysis), and increasing returns ($\zeta = 1.05$).

In the case of decreasing returns ($\zeta = 0.95$), implied markups over sales range from *negative* (approximately 0.92-0.95) in the 1980s, to approximately 0 in the 2000s, when intangibles are measured either using R&D or using the sum of R&D and organization capital. Thus, a modest amount of decreasing returns in aggregate is sufficient to eliminate markups altogether.

On the other hand, with increasing returns ($\zeta = 1.05$), implied markups are substantially larger, reaching approximately 1.1 in the 2000s. Note that, even so, these numbers are substantially smaller than those obtained in the literature that estimates markups using a production function approach, such as for instance De Loecker et al. (2020). (The sales markups we find here are approximately one-third of the median markup documented by these authors.)

Table VI reports averages for the underlying values of μ, χ and α obtained in two subsamples, 1980-2000 and 2001-2015. That table also attempts to isolate the contribution of "pure rents" (those due to markups). The rents due to markups, as a fraction of value added, are defined as:

$$s^{\mu}_{\text{Re}} \equiv (1 - s_L)\left(\frac{\mu_S - 1}{\alpha + \mu_S - 1}\right).$$

Recall that *total* rents as fraction of value added are given by $(1 - s_L)\left(\dfrac{\mu_S/\zeta - 1}{\alpha + \mu_S/\zeta - 1}\right)$.

So, for any value of markups μ_S, s^{μ}_{Re} gives the implied rents, as a fraction of value added, assuming constant returns. The numbers reported in Table VI under "Rents due to markups" are the values of s^{μ}_{Re}, using the markups implied by different values of ζ. As discussed above, with even relatively modest decreasing returns ($\zeta = 0.95$), the value of χ implies negative markups, so that "pure rents" contribute negatively to total rents. With increasing returns, "pure rents" exceed total rents (because increasing returns implies negative "quasi-rents"). Even then, the size of rents attributable to markups is relatively small in comparison to the numbers implied by the estimate of De Loecker et al. (2020) (which are in the order of 40% of total value added).

IA.D.8. Heterogeneous rents parameters

Methodology The decomposition of the investment gap in the case of heterogeneous rents parameter, balanced growth, and $N = 2$, can be implemented as follows. First, as in our baseline model, we can recover the value of $r - g$ through:

$$r - g = \frac{ROA_1 - \iota_1 - \iota_2 S}{Q_1} - \frac{\gamma_1 + \gamma_2 S}{Q_1} g^2,$$

where all the objects on the right-hand side can be obtained from the data. Third, we can measure user costs as:

$$\begin{aligned} R_n &= r + \delta_n + \gamma_n r g \\ &= r - g + \iota_n + \gamma_n r g \\ &= r - g + \iota_n + \gamma_n g^2 + \gamma_n (r - g) g \\ &= (r - g) q_n + \iota_n + \gamma_n g^2, \qquad\qquad n = 1, 2. \end{aligned}$$

where all the objects on the right-hand side can now be measured in the data. Finally, note that the first-order conditions to the firm's problem imply that:

$$\Pi_t = \mu_1 R_1 K_{1,t} + \mu_2 R_2 K_{2,t}.$$

Define the average markup as:

$$\bar{\mu} = \frac{R_1}{R_1 + SR_2}\mu_1 + \frac{SR_2}{R_1 + SR_2}\mu_2.$$

Note that the values for R_1, R_2 and S that we derive from the data are not dependent upon whether the rents parameters μ_n, $n = 1, 2$ are heterogeneous or not. The average markup $\bar{\mu}$ will therefore always be given by:

$$\begin{aligned} \bar{\mu} &= \frac{R_1}{R_1 + SR_2}\mu_1 + \frac{SR_2}{R_1 + SR_2}\mu_2 \\ &= \frac{\Pi_t / K_{1,t}}{R_1 + SR_2} \\ &= \frac{ROA_1}{R_1 + SR_2}, \end{aligned} \tag{IA.80}$$

where the expression in the last line only depends on data objects, and does not depend on the values of μ_1 and μ_2. Note that this is the expression for the rents parameter obtained our baseline case, μ. Thus, the average of the rents parameters across capital types, weighted by their user costs, will always be equal to the rents parameter that we obtained in our baseline decomposition. In other words, the *total* contribution of rents (generated by either intangibles or by physical capital) to the decomposition is the same whether one uses baseline model with a single rent parameter, or the heterogeneous rents model; heterogeneous values for (μ_1, μ_2) only affects the distribution of rents across capital types.

In order to estimate quantify separately the two rents parameter $\{\mu_1, \mu_2\}$, we additionally assume that the operating profit function is given by:

$$\Pi_t(K_{1,t}, K_{2,t}) = \left(A_{1,t}^{1-\frac{1}{\mu_1}} K_{1,t}^{\frac{1}{\mu_1}} \right)^{1-\eta} \left(A_{2,t}^{1-\frac{1}{\mu_2}} K_{2,t}^{\frac{1}{\mu_2}} \right)^{\eta}$$

where $A_{1,t}$ and $A_{2,t}$ are both growing at rate g. The first-order conditions $R_n = \Pi_{n,t}$ can then be expressed as:

$$\begin{aligned} R_1 &= \frac{1-\eta}{\mu_1} \frac{K_{1,t}}{\Pi_t}, \\ R_2 &= \frac{\eta}{\mu_2} \frac{K_{2,t}}{\Pi_t}, \end{aligned}$$

so that the capital-specific rents parameters satisfy:

$$\mu_1 = (1-\eta)\frac{ROA_1}{R_1},$$
$$\mu_2 = \eta\frac{ROA_1}{SR_2}.$$

Results The methodology described above helps clarify that, in the more general model with heterogeneous rents parameters, our methodology identifies the user-cost weighted average rents parameter. However, the methodology has the drawback that the value of η must be known (independently from measures of S) in order to estimate μ_1 and μ_2. In other words, while it is straightforward to measure the (user-cost weighted) average rents parameter, estimating them separately from the data seem difficult.

In Figure 30, we provide bounds on the values of η in the two limit cases where rents are due either only to intangible capital, or only to physical capital. Rents will only be due to intangibles when:

$$\mu_1 = 1, \quad \mu_2 = \frac{ROA_1 - R_1}{S_2 R}, \quad \eta = \frac{ROA_1 - R_1}{ROA_1}.$$

Figure 30 shows that this would imply an increase in η has been increasing rapidly over time, reaching approximately $\eta = 0.4$ by 2015. At the other extreme, rents will only be due to physical capital when:

$$\mu_1 = \frac{ROA_1 - SR_2}{R_1}, \quad \mu_2 = 1, \quad \eta = \frac{SR_2}{ROA_1}.$$

This would imply a smaller increase in η, reaching only approximately $\eta = 0.2$ by 2015, as reported in the bottom panel of Figure 30. The bottom panel of Figure 30 also reports the implied values of the rents parameters in these two limiting cases. The case in which all rents are due to physical capital implies values for μ_1 that are close to our baseline estimates of μ, since the relative user cost R_1/SR_2 is large. On the other hand, when all rents are due to intangibles, μ_2 must be very large (in the order of $\mu_2 = 2$), again because when intangibles are measured with R&D capital, the relative weight SR_2/R_1 is small.

Separate identification of μ_1 and μ_2 without the knowledge of η is more difficult. The core issue is that without being able to separate cash flows generated by either type of capital (which, when they are not perfect substitutes, is difficult), it is also challenging to measure average returns generated by each type of capital. Even under an alternative interpretation where μ_1 and μ_2 reflect monopsony power in the market for capital inputs, separate data on rental rates intangible and physical capital would be required to identify each rents parameter separately.

IA.D.9. Financing frictions

IA.D.9.1. Equity financing frictions

Appendix Figures 10 and 11 report a simplified decomposition of the investment gap, under different assumptions about the magnitude of equity financing frictions. Specifically, using the balanced growth results of Appendix IA.B.6.2, we have that along the balanced growth path,

$$
\begin{aligned}
Q_1 - q_1 &= Q_1 - f'(d)(1+\gamma_1 g) \\
&= f'(d)\left(q_2 S + \frac{(\mu-1)\tilde{R}_1}{r-g} + \frac{(\mu-1)\tilde{R}_2}{r-g} S\right)
\end{aligned}
$$

so that total rents are given by:

$$
\text{Rents} \equiv f'(d)\left(\frac{(\mu-1)\tilde{R}_1}{r-g} + \frac{(\mu-1)\tilde{R}_2}{r-g} S\right) = Q_1 - f'(d)(1+S+(\gamma_1+\gamma_2 S)g).
$$

Appendix Figures 10 and 11 use these expressions to construct the contribution of total rents and of the omitted capital effect for different values of $f'(d)$.

The main message of these figures is that introducing equity financing frictions will in general magnify the total contribution of rents to the gap. The intuition is that *total* rents (those due to either physical capital or intangibles) are the residual after taking into account the value of the intangible capital stock. This latter value is adjusted downward with equity financing frictions, because of the wedge $f'(d_t)$ between inside and outside finance. Thus for a given (empirical) value of Q_1, rents are magnified.

Next, we briefly discuss how the results of our main empirical decomposition, which assumes no equity financing frictions, would be biased if the data had instead been generated by a model with equity frictions. Since our procedure to estimate the decomposition relies on the balanced growth model, we study this question in the context of the balanced growth model. For simplicity, we also abstract from adjustment costs to capital in this discussion.

Recall that our procedure uses five data moments, $\{S, ROA_1, \iota_1, \iota_2, Q_1\}$. We combine these five moments as follows:

$$\begin{aligned}
\widehat{r-g} &= \frac{ROA_1 - (\iota_1 + S\iota_2)}{Q_1} \\
\widehat{R}_1 &= \widehat{r-g} + \iota_1 \\
\widehat{R}_2 &= \widehat{r-g} + \iota_2 \\
\widehat{\mu} &= \frac{ROA_1}{\widehat{R}_1 + S\widehat{R}_2}
\end{aligned}$$

Moreover, in the balanced growth model with no adjustment costs, the (unadjusted) physical investment gap is given by $Q_1 - 1$. We decompose it as:

$$Q_1 - 1 \quad = \quad S \quad + \quad \frac{(\hat{\mu}-1)}{\widehat{r-g}}\widehat{R}_1 \quad + \quad \frac{(\hat{\mu}-1)}{\widehat{r-g}}\widehat{R}_2 S.$$

By contrast, the adjusted investment gap is given by:

$$Q_1^a - 1 \quad = \quad S \quad + \quad \frac{\mu-1}{r-g}R_1^{(a)} \quad + \quad \frac{\mu-1}{r-g}R_2^{(a)}S$$

Note, that if the data is generated by a model with equity frictions, there is no guarantee that $\widehat{r-g}$ will properly measure $r-g$, and likewise for $\widehat{R}_1$, $\widehat{R}_2$ and $\widehat{\mu}$. Additionally, measurable and adjusted average Q will generally differ, i.e. $Q_1 \neq Q_1^{(a)}$. Nevertheless, we can establish the following results.

RESULT 9: *Along the balanced growth path, the unadjusted investment gap is biased downward* *relative to the adjusted investment gap:*

$$Q_1 - 1 = \left(Q_1^{(a)} - 1\right) - \left(\frac{1 - f'(d)}{f'(d)}\right) Q_1 < Q_1^{(a)} - 1.$$

This bias is entirely reflected in the estimated contribution of total rents to the unadjusted investment gap, $\widehat{\text{Rents}}$*, which is biased downward relative to their contribution to the adjusted investment gap,* Rents*:*

$$\underbrace{\frac{(\hat{\mu}-1)}{\widehat{r-g}}\widehat{R}_1 \quad + \quad \frac{(\hat{\mu}-1)}{\widehat{r-g}}\widehat{R}_2 S}_{\equiv \widehat{\text{Rents}}} \quad = \quad \underbrace{\frac{(\mu-1)}{r-g}R_1^{(a)} \quad + \quad \frac{\mu-1}{r-g}R_2^{(a)} S}_{\equiv \text{Rents}} - \left(\frac{1-f'(d)}{f'(d)}\right) Q_1.$$

The ratio of intangible rents relative *to physical rents is biased upward, if and only if:*

$$\frac{F_2}{F_1} \geq \frac{\iota_2}{\iota_1},$$

or, in the case of a Cobb-Douglas capital aggregator with elasticity of substitution η*,*

$$\eta \geq \frac{\iota_2 S}{\iota_1 + \iota_2 S}. \tag{IA.81}$$

Proof. The value of $\widehat{r-g}$ implied by our empirical decomposition is:

$$\widehat{r-g} \quad = \quad \frac{ROA_1 - (\iota_1 + \iota_2 S)}{Q_1}$$

In a model with equity issuance frictions,

$$Q_1 \quad = \quad \frac{ROA_1 - (\iota_1 + \iota_2 S)}{r-g}\frac{f(d)}{d}$$

so:

$$\widehat{r-g} \quad = \quad \frac{d}{f(d)}(r-g) \equiv \frac{1}{\nu(d)}(r-g).$$

We then have:

$$\begin{aligned}
\frac{\widehat{R}_2}{\widehat{R}_1} &= \frac{\widehat{r-g}+\iota_2}{\widehat{r-g}+\iota_1} = \frac{r-g+\nu(d)\iota_2}{r-g+\nu(d)\iota_1} \\
&= \frac{R_2^{(a)}+F_2\varepsilon(d)-(1-\nu(d))\iota_2}{R_1^{(a)}+F_1\varepsilon(d)-(1-\nu(d))\iota_1} \\
&= \frac{R_2^{(a)}}{R_1^{(a)}}\frac{1+F_2/R_2^{(a)}\varepsilon(d)-(1-\nu(d))\iota_2/R_2^{(a)}}{1+F_1/R_1^{(a)}\varepsilon(d)-(1-\nu(d))\iota_1/R_1^{(a)}}
\end{aligned}$$

Along the balanced growth path, the first-order conditions of the firm's problem imply that $\dfrac{R_2^{(a)}}{R_1^{(a)}} = \dfrac{F_2}{F_1}$. Thus, $\frac{\widehat{R}_2}{\widehat{R}_1} > \frac{R_2^{(a)}}{R_1^{(a)}}$, if and only if:

$$\iota_2/\iota_1 < R_2^{(a)}/R_1^{(a)} = F_2/F_1,$$

establishing the result. □

There are two parts to this result. First, as highlighted above, within equity frictions, the *level* of the (unadjusted) investment gap is generally too low, relative to the level of the "adjusted" investment gap, reflecting the fact that Q_1 is *too low* relative to $Q_1^{(a)}$. The intuition is that along the balanced growth path where $d_t > 0$, the replacement cost of existing capital, $K_{1,t+1}$, is *too high* relative to shareholders' valuation of it, $f'(d_t)K_{1,t+1}$. The first part of the result says that, in our empirical decomposition, this bias does not affect estimates of the direct effect of intangibles on the investment gap (which are simply given by S, in both our empirical decomposition and the adjusted investment gap decomposition). Instead, the bias shows up in estimated rents, which are too small, compared to the rents in the adjusted investment gap decomposition. The ratio of true rents to estimated rents is given by:

$$\frac{\widehat{\text{Rents}}}{\text{Rents}} = \frac{Q_1-1-S}{Q_1-1-S+\frac{1-f'(d)}{f'(d)}Q_1}$$

Consistent with the intuitions developed in Appendix Figures 10 and 11, accounting for equity frictions would thus lead to higher overall rents, with the quantitative effect.

The second part of the result says that the *composition* of rents, though, need not be biased in a particular direction. The share of rents attributable to intangibles vs. physical capital, in our baseline decomposition, depends on:

$$\frac{\widehat{R}_2 S}{\widehat{R}_1},$$

whereas in the adjusted investment gap decomposition, it depends on:

$$\frac{R_2^{(a)} S}{R_1^{(a)}}.$$

The result says that along the balanced growth path, the difference between these two ratios only depends on the properties of the production function, and on (observable) rates of gross investment in each type of capital — *not* on the curvature parameter $f'(d)$.

Different from the baseline model, without knowledge of the function $f(d)$, if the data is generated by a model with equity issuance frictions, one cannot construct the different elements on the adjusted investment gap decomposition. In particular, in the case of a Cobb-Douglas capital aggregator, it is not possible to estimate the elasticity of substitution η between intangible and physical capital without knowing $f(.)$.

However, condition (IA.81) gives a lower bound on η, above which our estimate of the relative contribution of intangibles to rents would be biased upward. Appendix Figure 32 reports the times series for this lower bound, when intangibles are measured either as R&D capital, or as the sum of R&D and organization capital. This graph shows that the lower bound for the composition bias is relatively high. By 2015, η would have needed to be higher than 0.3 (with only R&D) or 0.6 (with R&D and organization capital) for our decomposition to overstate the contribution of intangibles to rents. (One can compare this to the estimates of η in Appendix Figure 7, which, though they are obtained using a model without equity frictions, are uniformly below these lower bounds.) Thus, if anything, equity frictions appear more likely to lead to understating the contributions of intangibles to total rents, rather than overstating it.

IA.D.9.2. Debt financing frictions

We next discuss how the investment gap would change, and how our results would be biased, in the presence of debt issuance subject to a collateral constraint. As for the case of equity frictions, we focus on the balanced growth model, since this is the model to which we apply our estimation approach in the main text. We start by giving the expression of the decomposition in the model with debt issuance frictions.

RESULT 10: *Let $n = 1$ denote physical capital. Along the balanced growth path, neglecting terms of order $o(g)$ and higher, the physical investment gap is given by:*

$$Q_1 - q_1 = \sum_{m\geq 2} q_m S_m + \frac{(\mu-1)R_1}{r-g} + \sum_{m\geq 2} \frac{(\mu-1)R_m}{r-g} \times S_m \tag{IA.82}$$

where:

$$\begin{aligned}
\forall n = 1, ..., N, \quad q_n &= 1 + \gamma_n g, \\
R_1 &= r + \delta_1 + \gamma_1 r g - \theta(r - r_b) \\
\forall n = 2, ..., N, \quad R_n &= r + \delta_n + \gamma_n r g.
\end{aligned}$$

With respect to our baseline decomposition, the only difference is in the expression for the user cost of physical capital. Specifically, the user cost of physical capital is *lower than* in the baseline model, at least when $\theta > 0$ and $r > r_b$. Intuitively, the user cost of physical capital is lower because there is an additional benefit from holding physical capital: it relaxes the borrowing constraint, and allows shareholders to lever up and take advantage of the wedge between their discount rate and the discount rate of debtholders.

When there are debt collateral constraint, is our empirical decomposition of the investment gap, which relies on a model without collateral constraints, biased, and if so, how? The following result provides an answer to this question. As in the case of equity frictions, we use hatted variables to denote the parameters we derive from observations of $\{Q_1, ROA_1, \iota_1, \iota_2, S\}$ from the model. We focus on the version of the model without adjustment costs and with only two capital types for simplicity. The mapping from observed moments to estimated parameters is the following:

$$\begin{aligned}
\widehat{r-g} &= \frac{ROA_1 - (\iota_1 + S\iota_2)}{Q_1} \\
\widehat{R}_1 &= \widehat{r-g} + \iota_1
\end{aligned}$$

$$
\begin{aligned}
\widehat{R}_2 &= \widehat{r-g} + \iota_2 \\
\widehat{\mu} &= \frac{ROA_1}{\widehat{R}_1 + S\widehat{R}_2}
\end{aligned}
$$

Our estimate of the decomposition is then given by:

$$
Q_1 - 1 = S + \underbrace{\frac{\widehat{\mu}-1}{\widehat{r-g}}\widehat{R}_1 + \frac{\widehat{\mu}-1}{\widehat{r-g}}S\widehat{R}_2}_{\equiv \widehat{\text{Rents}}}
$$

RESULT 11: *Let the total contribution of rents to the investment gap in the true model (i.e. the model with collateral constraints) be given by:*

$$
\text{Rents} \equiv \frac{\mu-1}{r-g}R_1 + \frac{\mu-1}{r-g}SR_2
$$

Along the balanced growth path, our approach correctly estimates the total size of rents:

$$
\text{Rents} = \widehat{\text{Rents}}.
$$

Moreover, the contribution of intangibles to rents is underestimated, and the contribution of physical capital is overestimated:

$$
\widehat{R}_1 > R_1, \quad \widehat{R}_2 < R_2.
$$

Finally, the rents parameter is overestimated, while the Gordon growth term is underestimated:

$$
\widehat{\mu} > \mu, \quad \widehat{r-g} < r-g.
$$

The fact that the total contribution of rents is correctly estimated, $\text{Rents} = \widehat{\text{Rents}}$, follows from the fact that total rents are effectively estimated as $Q_1 - 1 - S$, which is correct, from the standpoint of the model with a collateral constraint.[43]

[43]Recall that we assumed no adjustment costs, so that $q_1 = 1$; more generally, the estimate of the size of total rents is $Q_1 - \widehat{q}_1 - S = Q_1 - (1 + \gamma_1 g) - S$, which is correct in both versions of the model.

The presence of collateral constraints however biases the estimates of the user costs of physical and intangible capital relative to their true value. The estimated user cost of physical capital is too high, because it fails to take into account the fact that part of the return to holding physical capital is the shadow value of relaxing the collateral constraint. The user cost of intangibles is too low because, generally speaking, our approach estimates an implicit discount rate of shareholders, $\widehat{r} = \widehat{r-g} + g$, that is too low. The estimated shareholder discount rate obtained in our approach can be written as:

$$\widehat{r} = \left(1 - \frac{E_t^e}{V_t^e}\right) r_b + \frac{E_t^e}{V_t^e} r < r.$$

In other words, our approach recovers the weighted average cost of capital, instead of the relevant cost of capital for computing the opportunity cost of investing, which is the shareholders' discount rate.

Finally, estimated markups are generally too high, relative to their true value, because total estimated user costs in our approach are too low, relative to their true value:

$$\widehat{R}_1 + \widehat{R}_2 S > R_1 + R_2 S.$$

The effect of the overestimation of the user cost of physical capital always dominates, resulting in estimates of total user costs that are too high, and therefore estimates of μ that are too low relative to their true values.

How large are the biases likely to be? Figures 33 and 34 illustrate the potential size of the biases.

Figure 33 reports the decomposition of the investment gap obtained using the baseline model (top panel) and using the model with collateral constraints (middle and bottom panels). In order to construct the latter two decompositions, θ and $r - r_b$ are all that is needed besides the moments we already use in our baseline decomposition. We follow the model and use the empirical debt-to-physical capital ratio in order to estimate the collateral tightness parameter θ, since:

$$\theta = \frac{B_{t+1}}{K_{1,t+1}}.$$

Moreover, we assume two potential values for $r - r_b$, $r - r_b = 0.02$ and $r - r_b = 0.05$. There is no clear source for calibrating the wedge between discount factors. However, the quantitative effects are very small regardless of the value of $r - r_b$ chosen. The three panels of Figure 33 are indeed hard to distinguish. Thus the potential biases in the investment gap decomposition, though they generally would lead to a higher estimate of the importance of intangible rents and a lower estimate of the importance of physical rents, are small quantitatively. In a similar spirit, Figure 34 reports the implied markups for the baseline model and for two versions of the model with a debt collateral constraint. The differences in implied markups are very close across the different models, with a gap of 2% at most across the different models, occurring toward the late 2000s.

Overall, we therefore conclude that, while the omission of debt frictions characterized by a collateral constraint would generally lead our decomposition to produce biased estimates, the quantitative effect of these biases would almost certainly be second order.

IA.D.10. Rents and productivity

We use the disaggregated data to assess whether our measure of rents (the parameter μ) is related to productivity. A worry is that, because our approach uses average returns to capital to identify μ, a high value of μ may reflect high marginal products of capital, rather than high rents. In order to assess this possibility, we look at the correlation between measures of total factor productivity at the industry level obtained from the BLS' KLEMS data, described in Appendix IA.C, and the measure of rents which we obtain from applying the balanced growth model at the same level of sectoral aggregation.

In Figure 36, we report a scatterplot and regression results that highlight the lack of correlation between the upward trend in rents, and total factor productivity growth, across sectors. In all panels, a point represents a KLEMS industry s. Its vertical coordinate is an estimate of the time slope β of the rents parameter in the industry over the 1985-2015 period. Its horizontal coordinate is the average growth rate of multi-factor productivity in that industry over the same period. Simple regression lines by groups of industry are also reported. The different panels of the figure correspond to different industry groups or different measures of the intangible capital stock.

The main message of the figure is that the correlation between our estimates of the rents parameters and KLEMS' measure of multifactor productivity growth is either zero or negative. Visually, there is no clear correlation, and simple regressions all deliver robust t-statistics below 1 in absolute value. Thus, this simple evidence suggests that our estimates of the changes in rents do not simply (or even mainly) reflect changes in multifactor productivity.

An important caveat to this simple evidence is that, while the KLEMS multi-factor productivity growth series have the advantage of relying strictly on output measures (as opposed to revenue), they still implicitly assume perfect competition and no pure profits. If, in reality, firms earn rents, the KLEMS measures will generally be biased. We explore this issue in more detail in Crouzet and Eberly (2020); there, we argue that this bias could have led to substantially underestimated *aggregate* productivity growth, though not necessarily of *sectoral* productivity growth.

REFERENCES

Altınkılıç, Oya, and Robert S Hansen, 2000, Are there economies of scale in underwriting fees? Evidence of rising external financing costs, *The Review of Financial Studies* 13, 191–218.

Baqaee, David, and Emmanuel Farhi, 2020, Entry vs. rents, *NBER working paper* .

Barkai, Simcha, 2020, Declining labor and capital shares, *The Journal of Finance* 75, 2421–2463.

Belo, Frederico, Vito Gala, Juliana Salomao, and Maria Ana Vitorino, 2019, Decomposing firm value, *Working paper* .

Campbell, John Y, and Samuel B Thompson, 2008, Predicting excess stock returns out of sample: Can anything beat the historical average?, *The Review of Financial Studies* 21, 1509–1531.

Cochrane, John H, 1991, Production-based asset pricing and the link between stock returns and economic fluctuations, *The Journal of Finance* 46, 209–237.

Cochrane, John H, 1996, A cross-sectional test of an investment-based asset pricing model, *Journal of Political Economy* 104, 572–621.

Crouzet, Nicolas, and Janice Eberly, 2020, Intangibles, markups, and the measurement of productivity growth, *Working paper* .

Crouzet, Nicolas, and Janice C Eberly, 2019, Understanding weak capital investment: the role of market power and intangibles, *Proceedings of the 2018 Jackson Hole Symposium* 87–148.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger, 2020, The rise of market power and the macroeconomic implications, *The Quarterly Journal of Economics* 135, 561–644.

Eisfeldt, Andrea L, and Dimitris Papanikolaou, 2013, Organization capital and the cross-section of expected returns, *The Journal of Finance* 68, 1365–1406.

Eisfeldt, Andrea L, and Dimitris Papanikolaou, 2014, The value and ownership of intangible capital, *American Economic Review* 104, 189–94.

Evans, Donald L, 2003, Fixed assets and consumer durable goods in the united states, 1925-97, *Technical Report, Bureau of Economic Analysis* .

Ewens, Michael, Ryan H Peters, and Sean Wang, 2020, Measuring intangible capital with market prices, *NBER working paper* .

Falato, Antonio, Dalida Kadyrzhanova, Jae Sim, and Roberto Steri, 2020, Rising intangible capital, shrinking debt capacity, and the US corporate savings glut, *Working paper* .

Farhi, Emmanuel, and François Gourio, 2018, Accounting for macro-finance trends: Market power, intangibles, and risk premia, *Brookings Papers on Economic Activity* 147–250.

Gutiérrez, Germán, and Thomas Philippon, 2017, Investmentless Growth: An Empirical Investigation, *Brookings Papers on Economic Activity* 89–190.

Hall, Robert E, 2001, The stock market and capital accumulation, *American Economic Review* 91, 1185–1202.

Hansen, Lars Peter, 1982, Large sample properties of generalized method of moments estimators, *Econometrica: Journal of the Econometric Society* 1029–1054.

Hennessy, Christopher A, Amnon Levy, and Toni M Whited, 2007, Testing Q theory with financing frictions, *Journal of financial economics* 83, 691–717.

Hulten, Charles R, and Xiaohui Hao, 2008, What is a Company Really Worth? Intangible Capital and the "Market to Book Value" Puzzle, *NBER working paper* .

Jensen, Michael C, and William H Meckling, 1976, Theory of the firm: Managerial behavior, agency costs and ownership structure, *Journal of financial economics* 3, 305–360.

Karabarbounis, Loukas, and Brent Neiman, 2019, Accounting for factorless income, *NBER Macroeconomics Annual* 33, 167–228.

Lev, Baruch, and Suresh Radhakrishnan, 2005, The valuation of organization capital, in *Measuring capital in the new economy*, 73–110 (University of Chicago Press).

Li, Wendy CY, and Bronwyn H Hall, 2020, Depreciation of business R&D capital, *Review of Income and Wealth* 66, 161–180.

Lindenberg, Eric B, and Stephen A Ross, 1981, Tobin's Q ratio and industrial organization, *Journal of business* 1–32.

Martin, Ian, 2017, What is the expected return on the market?, *The Quarterly Journal of Economics* 132, 367–433.

Peters, Ryan H, and Lucian A Taylor, 2017, Intangible capital and the investment-Q relation, *Journal of Financial Economics* 123, 251–272.

Syverson, Chad, 2004, Market structure and productivity: A concrete example, *Journal of Political Economy* 112, 1181–1222.

Broad sector	**Subsector**	**NAICS codes**
Consumer (20%)	Crop and Animal Production†	111
	Forestry, Fishing, and Related Activities†	113
	Wholesale Trade	42
	Retail Trade	44
High-tech (11%)	Computer and Electronic Products	334
	Publishing industries, except internet (includes software)	511
	Motion picture and sound recording industries†	512
	Data processing, internet publishing, and other information services	518 to 519
	Broadcasting and telecommunications	515 to 517
	Computer Systems Design and Related Services	5415
Healthcare (10%)	Chemical Products	325
	Miscellaneous Manufacturing	339
	Ambulatory Health Care Services†	621
	Hospitals and Nursing and Residential Care Facilities	622 to 623
Manufacturing (22%)	Oil and Gas Extraction	211
	Mining, except Oil and Gas	212
	Support Activities for Mining	213
	Utilities	22
	Food and Beverage and Tobacco Products	311 to 312
	Textile Mills and Textile Product Mills†	313 to 314
	Apparel and Leather and Applied Products	315 to 316
	Wood Products	321
	Paper Products	322
	Printing and Related Support Activities†	323
	Petroleum and Coal Products	324
	Plastics and Rubber Products	326
	Nonmetallic Mineral Products	327
	Primary Metal Products	331
	Fabricated Metal Products	332
	Machinery	333
	Electrical Equipment, Appliances, and Components	335
	Transportation Equipment	336
	Furniture and Related Products	337

Table I. Composition of the Consumer, High-tech, Healthcare, and Manufacturing sectors studied in Section III. Numbers in parentheses in the first column indicate the sectors' share of 2001 total value added by private non-financial businesses, constructed using KLEMS data. These numbers do not add up to 100% because the remaining sectors, described in Table II, also contribute to total value added by private businesses. The second and third column reports reports the name of the subsectors and the corresponding NAICS codes. Subsectors are defined following the classification used by the BLS to construct the KLEMS data. Subsectors marked with † are dropped from the analysis of disaggregated subsectors because they do not have at least 10 firms in Compustat in each year from 1985 to 2015.

Broad sector	Group	Subsector	NAICS codes
Other (36%)	Services (64%)	Legal services†	5411
		Miscellaneous Professional, Scientific, and Technical Services	5412 to 5414, 5416 to 5419
		Management of companies and enterprises†	55
		Administrative and Support Services	561
		Waste Management and Remediation Services†	562
		Educational Services†	61
		Social assistance†	624
		Amusements, Gambling, and Recreation Industries	713
		Performing arts, spectator sports, museums, and related activities†	711 to 712
		Accommodation	721
		Food Services and Drinking Places	722
		Other services except Government†	81
	Construction* (23%)	Construction†	23
	Transportation and warehousing* (14%)	Truck Transportation	484
		Other Transportation and Support Activities†	487 to 488
		Air Transportation	481
		Rail Transportation	482
		Pipeline Transportation†	486
		Water Transportation†	483
		Transit and ground passenger transportation†	485
		Warehousing and storage†	493

Table II. Composition of the Services and Other sectors. The number in parentheses in the first column is the overall share of 2001 total value added by private non-financial businesses of the sectors in this table, constructed using KLEMS data. The numbers in parentheses in the second column report the share of each group in the all the sectors in the table (so that they add up to 100%). The last columns the name of the subsectors and the corresponding NAICS codes. Subsectors are defined following the classification used by the BLS to construct the KLEMS data. Subsectors are defined following the classification used by the BLS to construct the KLEMS data. Subsectors marked with † are dropped from the analysis of disaggregated subsectors because they do not have at least 10 firms in Compustat in each year from 1985 to 2015. The Construction and Transportation groups are marked with $\star$ to indicate that they are not included in the analysis of the five main sectors (Consumer, High-tech, Healthcare, Manufacturing, and Services), and that their subsectors are not included in the analysis of disaggregate subsectors.

		Compustat non-financials (Intangibles = R&D + organization capital)									
		Consumer		Services		High-tech		Healthcare		Manufacturing	
	Targeted moments	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
i_1	Physical investment rate	0.128	0.098	0.142	0.084	0.139	0.101	0.105	0.082	0.093	0.093
i_2	Intangible investment rate	0.278	0.261	0.288	0.259	0.302	0.292	0.235	0.196	0.222	0.226
S	Intangible/physical capital	0.799	0.813	0.260	0.259	0.541	0.548	0.729	1.171	0.267	0.087
ROA_1	Return on physical capital	0.489	0.485	0.329	0.309	0.443	0.478	0.448	0.588	0.262	0.222
Q_1	Av. Q for physical capital	2.672	2.651	2.517	2.587	2.937	3.261	3.064	4.306	1.743	1.743
g	Growth rate of total capital stock	0.054	0.037	0.082	0.016	0.065	0.014	0.046	0.028	0.016	0.028
	Implied moments	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
$Q_1 - q_1$	Investment gap	1.523	1.645	1.380	1.574	1.634	2.424	1.908	3.329	0.367	0.687
	% rents from physical capital	*9*	*14*	*50*	*50*	*13*	*32*	*13*	*18*	*-21*	*35*
	% intangibles	*77*	*63*	*26*	*21*	*73*	*29*	*67*	*45*	*132*	*48*
	% rents from intangibles	*14*	*23*	*24*	*29*	*14*	*39*	*20*	*38*	*-11*	*17*
η	Intangible share in production	0.606	0.627	0.325	0.371	0.507	0.546	0.580	0.683	0.373	0.333
s	Rents as a fraction of value added	0.011	0.027	0.042	0.082	0.019	0.073	0.026	0.065	-0.009	0.027
R_1	User cost of physical capital	0.185	0.167	0.190	0.149	0.197	0.174	0.171	0.150	0.173	0.158
R_2	User cost of intangible capital	0.354	0.346	0.355	0.340	0.384	0.378	0.323	0.278	0.321	0.300
μ	Curvature of operating profit function	1.040	1.086	1.163	1.317	1.075	1.273	1.103	1.240	0.977	1.088
$\tilde{\mu}$	Markup over value added	1.040	1.086	1.043	1.090	1.020	1.079	1.027	1.070	0.992	1.028

Table III. Summary of targeted and implied moments for the different sectors of the Compustat non-financial sample. All columns measure intangibles as the sum of the R&D capital stock plus the organization capital stock. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. Rents as a fraction of value added are computed as $s = (1 - s_L)(1 - 1/\mu)$, where s_L is the labor share of value added for the NFCB sector. Markups over value added are computed as $\tilde{\mu} = 1/(1 - s)$. The implied moments reported are for the model with adjustment costs; the adjustment cost values are $\gamma_1 = 3$ and $\gamma_2 = 12$. In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources and construction are described in Section III.

		Non-Financial Corporate Businesses			Compustat Non-Financials: Intangibles = R&D			Compustat Non-Financials: Intangibles = R&D + org. cap.		
	Structural parameters	**1985 - 2000**	**2001 - 2017**	**Diff.**	**1985 - 2000**	**2001 - 2017**	**Diff.**	**1985 - 2000**	**2001 - 2017**	**Diff.**
$\bar{g}$	Mean g.r. of fundamentals	0.029	0.019	-0.010	0.030	0.025	-0.005	0.031	0.024	-0.007
		[0.021,0.037]	[0.016,0.021]	(0.021)	[0.022,0.039]	[0.006,0.043]	(0.625)	[0.019,0.042]	[0.007,0.040]	(0.530)
r	Discount rate	0.067	0.056	-0.011	0.087	0.088	0.002	0.087	0.087	0.000
		[0.062,0.073]	[0.051,0.061]	(0.000)	[0.069,0.104]	[0.069,0.107]	(0.911)	[0.068,0.106]	[0.070,0.105]	(0.993)
δ_1	Phys. depreciation rate	0.070	0.068	-0.002	0.079	0.069	-0.010	0.078	0.070	-0.009
		[0.067,0.073]	[0.065,0.072]	(0.413)	[0.065,0.092]	[0.050,0.087]	(0.437)	[0.062,0.094]	[0.053,0.086]	(0.510)
δ_2	Intan. depreciation rate	0.251	0.242	-0.010	0.230	0.223	-0.007	0.221	0.221	0.000
		[0.239,0.264]	[0.234,0.250]	(0.166)	[0.209,0.251]	[0.198,0.248]	(0.674)	[0.203,0.238]	[0.197,0.245]	(0.990)
η	Cobb-Douglas intan. share	0.223	0.281	0.058	0.201	0.247	0.046	0.426	0.474	0.049
		[0.196,0.250]	[0.276,0.287]	(0.000)	[0.183,0.219]	[0.240,0.254]	(0.000)	[0.407,0.444]	[0.464,0.484]	(0.000)
μ	Curv. of profit function	1.196	1.276	0.079	1.241	1.387	0.146	1.121	1.208	0.087
		[1.157,1.235]	[1.253,1.299]	(0.002)	[1.108,1.374]	[1.357,1.418]	(0.043)	[1.016,1.226]	[1.182,1.234]	(0.122)
	Implied moments	**1985 - 2000**	**2001 - 2017**	**Diff.**	**1985 - 2000**	**2001 - 2017**	**Diff.**	**1985 - 2000**	**2001 - 2017**	**Diff.**
μ_{VA}	Markup over value added	1.051	1.079	0.028	1.061	1.105	0.044	1.033	1.062	0.029
		[1.042,1.061]	[1.069,1.090]	(0.001)	[1.032,1.091]	[1.095,1.115]	(0.012)	[1.006,1.060]	[1.054,1.070]	(0.056)
s_{VA}	Rents/value added	0.049	0.073	0.025	0.058	0.095	0.037	0.032	0.059	0.026
		[0.040,0.057]	[0.064,0.082]	(0.001)	[0.032,0.084]	[0.087,0.103]	(0.015)	[0.007,0.057]	[0.051,0.066]	(0.061)
R_1	User cost of phy. cap.	0.137	0.125	-0.013	0.165	0.157	-0.008	0.165	0.157	-0.008
		[0.133,0.142]	[0.122,0.127]	(0.000)	[0.158,0.173]	[0.146,0.168]	(0.181)	[0.158,0.173]	[0.146,0.168]	(0.181)
R_2	User cost of intan. cap.	0.319	0.298	-0.021	0.317	0.311	-0.006	0.308	0.308	0.000
		[0.307,0.331]	[0.292,0.304]	(0.002)	[0.308,0.326]	[0.302,0.320]	(0.386)	[0.300,0.316]	[0.296,0.320]	(0.963)

Table IV. GMM estimation of a version of the model with with i.i.d. shocks to the growth rate of fundamentals. The columns marked "Non-Financial Corporate Businesses" report results obtained aggregate data for the NFCB sector, while the columns marked "Compustat Non-Financials" report results using aggregated data from the sample of Compustat non-financial firms. In the columns marked "1985-2000" and "2001-2017", the numbers in brackets are $(5, 95)$ confidence intervals for the point estimates of the different parameters or implied moments, computed using HAC standard errors based on a Bartlett kernel with four lags (the data are all annual). Point estimates for implied moments are computed by stacking the moment conditions defining these additional implied moments with the rest of the GMM moment conditions. The columns marked "Diff." report the change in structural parameters or implied moments across periods; the numbers in parentheses are p-values for the two-sided test that the difference is equal to 0. The p-value is computed by re-estimating the model on all the data from 1985 to 2017, and interacting all data and estimated parameters or implied moments with a full set of dummies for each of the two sub-samples. Estimation is done using two-step efficient GMM with the identity matrix as the first-step weighting matrix. More details are provided in Appendix IA.D.5.

	Ratio to:		
	Sales S_t	Value added VA_t	Operating surplus Π_t
Variable input costs W_tM_t	$\frac{1-\alpha}{\chi}$	$\frac{1-\alpha}{\chi-(1-\alpha)(1-\nu_1)}$	$\frac{1-\alpha}{\chi-(1-\alpha)}$
Labor costs $W_{1,t}M_{1,t}$	$\frac{1-\alpha}{\chi}\nu_1$	$\frac{1-\alpha}{\chi-(1-\alpha)(1-\nu_1)}\nu_1$	$\frac{1-\alpha}{\chi-(1-\alpha)}\nu_1$
Capital costs† $R_{1,t}K_{1,t}+R_{2,t}K_{2,t}$	$\frac{\alpha}{\chi}$	$\frac{\alpha}{\chi-(1-\alpha)(1-\nu_1)}$	$\frac{\alpha}{\chi-(1-\alpha)}$
Rents† Re_t	$\frac{\chi-1}{\chi}$	$\frac{\chi-1}{\chi-(1-\alpha)(1-\nu_1)}$	$\frac{\chi-1}{\chi-(1-\alpha)}$
Value added VA_t	$1-\frac{1-\alpha}{\chi}(1-\nu_1)$	1	$1+\frac{1-\alpha}{\chi-(1-\alpha)}\nu_1$
Operating surplus Π_t	$1-\frac{1-\alpha}{\chi}$	$1-\frac{1-\alpha}{\chi-(1-\alpha)(1-\nu_1)}\nu_1$	1

Table V. Expression for the ratios of variable input costs, labor costs, capital costs, rents, value added, and operating surplus, to sales, value added, and operating surplus implied by the variable profit maximization problem described in Appendix IA.B.3. These expressions hold generally (regardless of how capital inputs are chosen), except the expressions for capital and rents shares, marked with †, which only hold in the balanced growth model of Section C of the main text. Variable input costs are defined as $\sum_j W_{j,t}M_{j,t} = W_tM_t$, where W_t and M_t are aggregate price and quantity indices defined in Appendix IA.B.3. Without loss of generality, labor is assumed to be the first variable input, entering the aggregate quantity index M_t with Cobb-Douglas share ν_1. The parameter α is the Cobb-Douglas elasticity of substitution between capital K_t and variable inputs M_t. The reduced-form parameter $\chi \geq 1$ indexes the importance of rents in the model; it is given, in terms of structural parameters, by $\zeta = \frac{\mu_S}{\zeta}$, where μ_S is the markup of the price of output over the marginal cost of output, and ζ is the degree of returns to scale to the capital-variable input bundle $K_t^{\alpha}M_t^{1-\alpha}$. Revenue is $S_t = P_tY_t$; value added is $VA_t = S_t - (W_tM_t - W_{1,t}M_{1,t}) = \Pi_t + W_{1,t}M_{1,t}$; operating surplus is $\Pi_t = VA_t - W_tM_t$. Given competitive payments to capital $R_{1,t}K_{1,t} + R_{2,t}K_{2,t}$, rents are defined as: $\text{Re}_t = \Pi_t - (R_{1,t}K_{1,t} + R_{2,t}K_{2,t})$.

		Compustat Non-Financials			
		R&D		R&D + org. cap.	
	Identified parameters	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
μ	Curvature of profit function	1.152	1.346	1.038	1.168
α	Elast. subs. btw. K_t and M_t	0.144	0.135	0.197	0.191
χ	$= \mu_S/\zeta$	1.021	1.047	1.007	1.032
	Implied markup over sales μ_S	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
μ_S	Decreasing returns ($\zeta = 0.95$)	0.970	0.994	0.957	0.980
μ_S	Constant returns ($\zeta = 1.00$)	1.021	1.047	1.007	1.032
μ_S	Increasing returns ($\zeta = 1.05$)	1.072	1.099	1.057	1.084
	Implied size of rents	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
s	Rents as a fraction of value added	0.037	0.088	0.009	0.049
	due to markups (if $\zeta = 0.95$)	-0.081	-0.015	-0.086	-0.039
	due to markups (if $\zeta = 1.00$)	0.037	0.088	0.009	0.049
	due to markups (if $\zeta = 1.05$)	0.099	0.144	0.066	0.104

Table VI. Markups and decreasing returns in the Compustat Non-Financial sample. The first panel reports estimates of three parameters of the version of the balanced growth model with an explicit microfoundation for variable input choices. α is the Cobb-Douglas elasticity of substitution between variable inputs and capital. χ is the reduced-form parameter governing the size of rents as a fraction of total sales; it is equal to the ratio of the two structural parameters μ_S (the markup of the marginal cost of variable inputs over sales) and ζ (the degree of returns to scale with respect to variable inputs and capital). The reduced-form parameter μ, which governs the size of rents as a fraction of operating surplus, is the same as in the balanced growth model of Section I, and is related to the other parameters by $\mu = 1 + \frac{\chi-1}{\alpha}$. The estimates of these parameters are obtained using the methodology described in II; in particular, the values of the rents parameter μ are the same as in Section II. The second and third panels report the values for the markup μ_S implied by different assumptions regarding the degree of returns to scale, as well as the implications for the size of rents as a fraction of value added. The "rents due to markups" are defined as $(1 - s_L)(\alpha/(\alpha + \mu_S - 1))$.

		Non-Financial Corporate Businesses				Compustat non-financials (R&D)		Compustat non-financials (R&D+ org. cap.)	
	Targeted moments	**1947 - 1965**	**1966 - 1984**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
i_1	Physical investment rate	0.089	0.108	0.099	0.087	0.097	0.090	0.097	0.090
i_2	Intangible investment rate	0.252	0.276	0.281	0.261	0.260	0.248	0.251	0.245
S	Intangible/physical capital	0.053	0.078	0.124	0.164	0.102	0.136	0.308	0.377
ROA_1	Return on physical capital	0.208	0.211	0.211	0.221	0.225	0.256	0.276	0.314
Q_1	Av. Q for physical capital	1.184	1.413	2.032	2.479	1.764	2.177	1.764	2.177
g	Growth rate of total capital stock	0.034	0.038	0.029	0.019	0.026	0.024	0.026	0.024
	Implied moments	**1947 - 1965**	**1966 - 1984**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
$Q_1 - q_1$	Investment gap	0.072	0.308	0.908	1.439	0.620	1.173	0.859	1.446
	% rents from physical capital	*69*	*41*	*61*	*71*	*71*	*66*	*36*	*33*
	% intangibles	*25*	*52*	*21*	*14*	*15*	*15*	*43*	*41*
	% rents from intangibles	*7*	*7*	*18*	*24*	*15*	*19*	*21*	*26*
η	Intangible share in production	0.099	0.145	0.227	0.286	0.179	0.222	0.392	0.440
s	Rents as a fraction of value added	-0.008	0.014	0.035	0.067	0.029	0.076	0.005	0.043
R_1	User cost of physical capital	0.193	0.171	0.143	0.128	0.164	0.155	0.163	0.155
R_2	User cost of intangible capital	0.392	0.369	0.341	0.312	0.350	0.326	0.339	0.322
μ	Rents parameter	0.984	1.051	1.136	1.244	1.117	1.290	1.023	1.145
$\tilde{\mu}$	Markup over value added	0.993	1.014	1.037	1.072	1.030	1.083	1.005	1.045

Table VII. Complete version of Table I from the main text. For Compustat non-financials, columns 6 and 7 use R&D as the measure of intangibles, and columns 8 and 9 use the sum of R&D and SG&A as the measure of intangibles. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. Rents as a fraction of value added are computed as $s = (1 - s_L)(1 - 1/\mu)$, where s_L is the labor share of value added for the NFCB sector. Markups over value added are computed as $\tilde{\mu} = 1/(1 - s)$. The implied moments reported are for the model with adjustment costs; the adjustment cost values are $\gamma_1 = 3$ and $\gamma_2 = 12$. In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources are described in Sections II and III.

		Compustat non-financials (Intangibles = R&D)									
		Consumer		Services		High-tech		Healthcare		Manufacturing	
	Targeted moments	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
i_1	Physical investment rate	0.128	0.098	0.142	0.084	0.139	0.101	0.105	0.082	0.094	0.093
i_2	Intangible investment rate	0.245	0.317	0.241	0.224	0.346	0.331	0.225	0.190	0.226	0.226
S	Intangible/physical capital	0.008	0.023	0.028	0.010	0.227	0.238	0.346	0.722	0.113	0.087
ROA_1	Return on physical capital	0.269	0.281	0.261	0.245	0.359	0.397	0.355	0.495	0.226	0.222
Q_1	Av. Q for physical capital	2.672	2.651	2.517	2.587	2.937	3.261	3.064	4.306	1.467	1.743
g	Growth rate of total capital stock	0.054	0.037	0.082	0.016	0.065	0.014	0.046	0.028	0.016	0.028
	Implied moments	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
$Q_1 - q_1$	Investment gap	1.523	1.645	1.380	1.574	1.634	2.424	1.908	3.329	0.367	0.687
	% rents from physical capital	*98*	*93*	*93*	*97*	*46*	*55*	*42*	*32*	*43*	*72*
	% intangibles	*1*	*2*	*3*	*1*	*31*	*13*	*30*	*27*	*47*	*16*
	% rents from intangibles	*1*	*6*	*4*	*2*	*23*	*32*	*28*	*41*	*9*	*12*
η	Intangible share in production	0.013	0.058	0.038	0.020	0.324	0.367	0.387	0.562	0.176	0.143
s	Rents as a fraction of value added	0.087	0.124	0.070	0.130	0.044	0.110	0.060	0.103	0.016	0.052
R_1	User cost of physical capital	0.188	0.169	0.191	0.150	0.198	0.174	0.173	0.151	0.175	0.159
R_2	User cost of intangible capital	0.319	0.418	0.310	0.306	0.429	0.420	0.316	0.271	0.332	0.305
μ	Curvature of operating profit function	1.412	1.575	1.308	1.621	1.184	1.474	1.260	1.433	1.064	1.186
$\tilde{\mu}$	Markup over value added	1.095	1.142	1.075	1.150	1.047	1.124	1.064	1.115	1.017	1.055

Table VIII. Complete version of Table II from the main text. All columns measure intangibles as the R&D capital stock. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. Rents as a fraction of value added are computed as $s = (1-s_L)(1-1/\mu)$, where s_L is the labor share of value added for the NFCB sector. Markups over value added are computed as $\tilde{\mu} = 1/(1-s)$. The implied moments reported are for the model with adjustment costs; the adjustment cost values are $\gamma_1 = 3$ and $\gamma_2 = 12$. In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources are described in Section III.

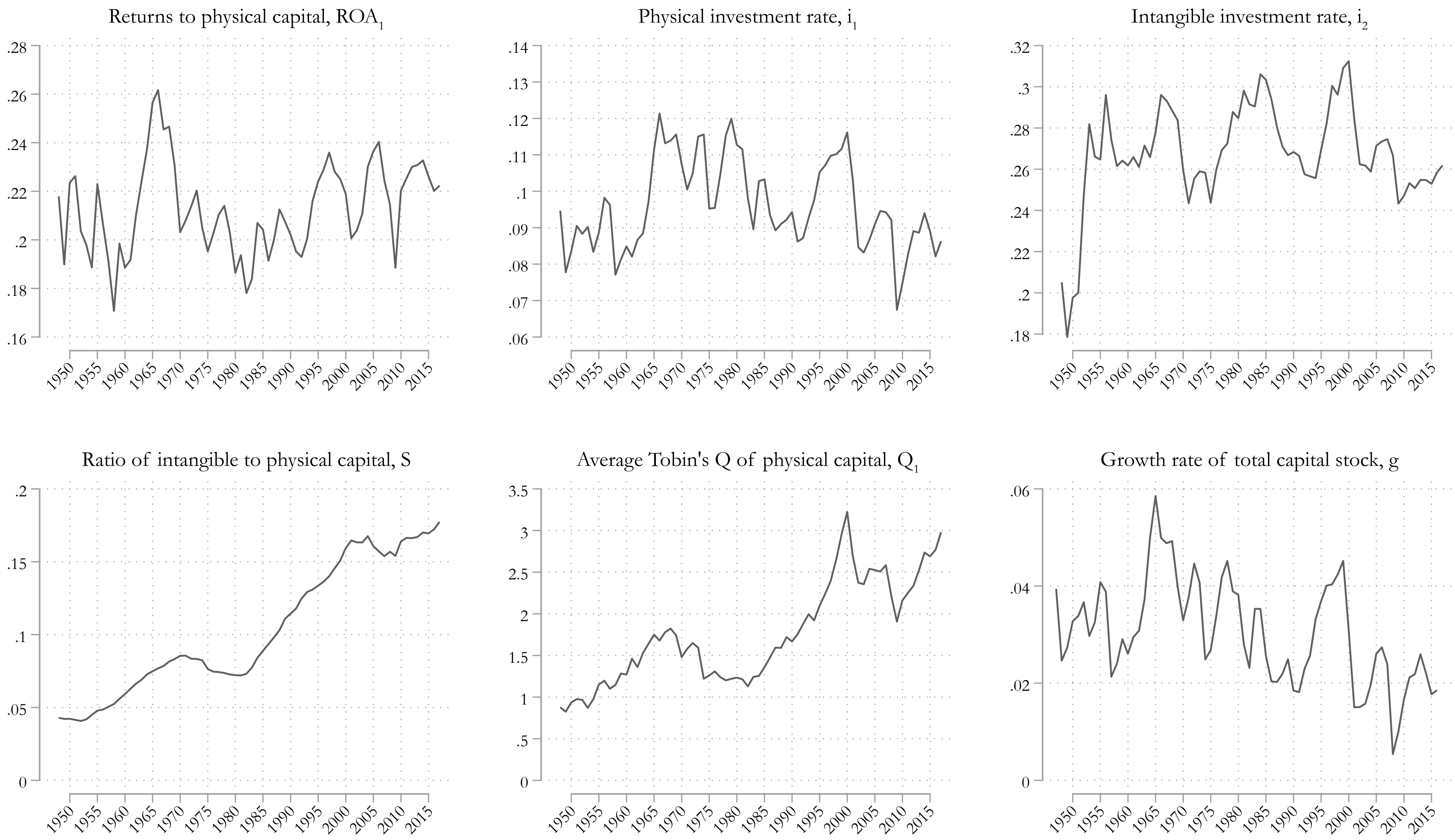


Figure 1. Time series for the moments used in the construction of the physical investment gap decomposition, Equation (19) from the main text. Returns to physical capital are defined as $\Pi_t/K_{1,t}$, where Π_t is operating surplus and $K_{1,t}$ the stock of physical capital at current cost. Investment rates are defined as $i_{n,t} = I_{n,t}/K_{n,t}$, $n = 1, 2$, where $n = 1$ indexes physical capital and $n = 2$ indexes intangible capital, $K_{2,t}$ is the stock of intangible capital at current cost, and $I_{n,t}$ are investment expenditures for each type of capital. The ratio of intangible to physical capital is $S_t = K_{2,t+1}/K_{1,t+1}$. Average Tobin's Q of physical capital is defined as $Q_{1,t} = V_t/K_{1,t+1}$, where V_t is an estimate of the total market value of net claims on the sector. The time series are the raw data; in particular, they are not averaged over seven-year windows. Data sources are described in Section II and in Appendix IA.C.

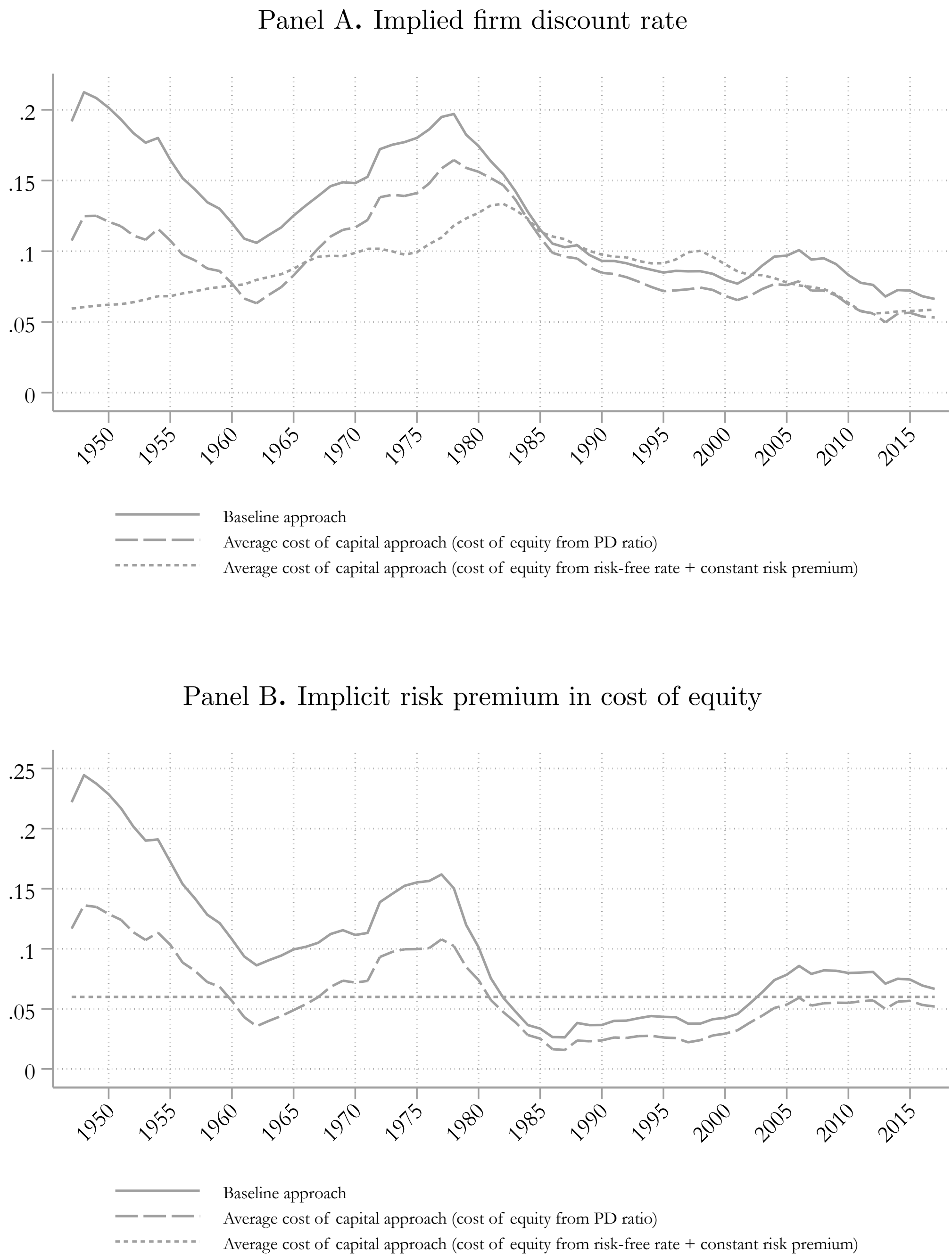


Figure 2. Firm discount rates (top panel) and implicit equity risk premium (bottom panel) across different approaches. The top panel reports the firm-wide discount rate expressed in nominal terms, defined as $r^n = (r - g) + g^n$. The bottom panel reports the risk premium implicit in the equity cost of capital across different approaches. In all three approaches approach, this is computed as $RP = (1 + l)(r^n - l/(1 + l)r_b^n) - r_f^n$, where l is the ratio of the market value of debt to the market value of equity, and r_b^n is the (after-tax, nominal) cost of debt reported in Appendix Figure 15.

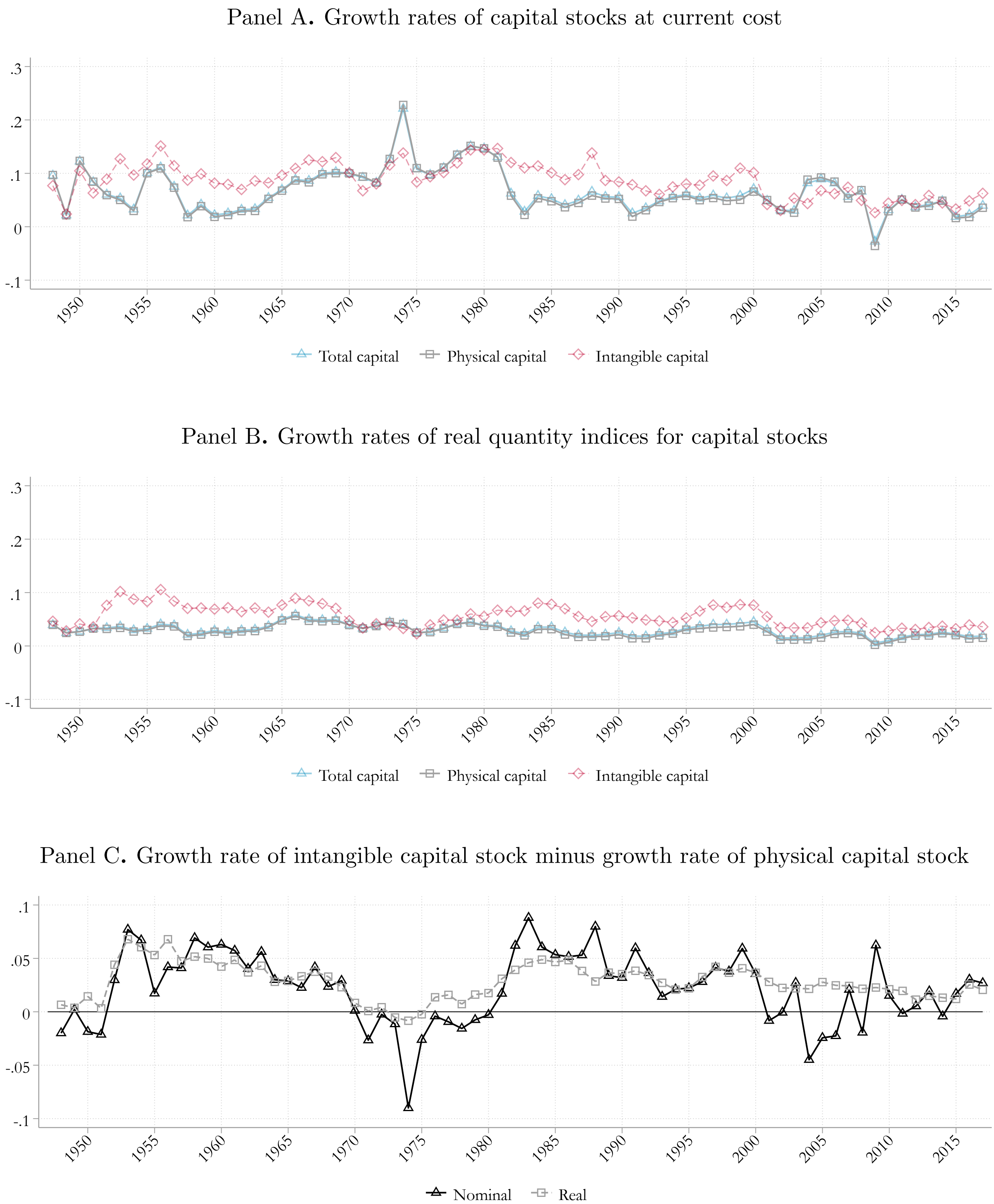
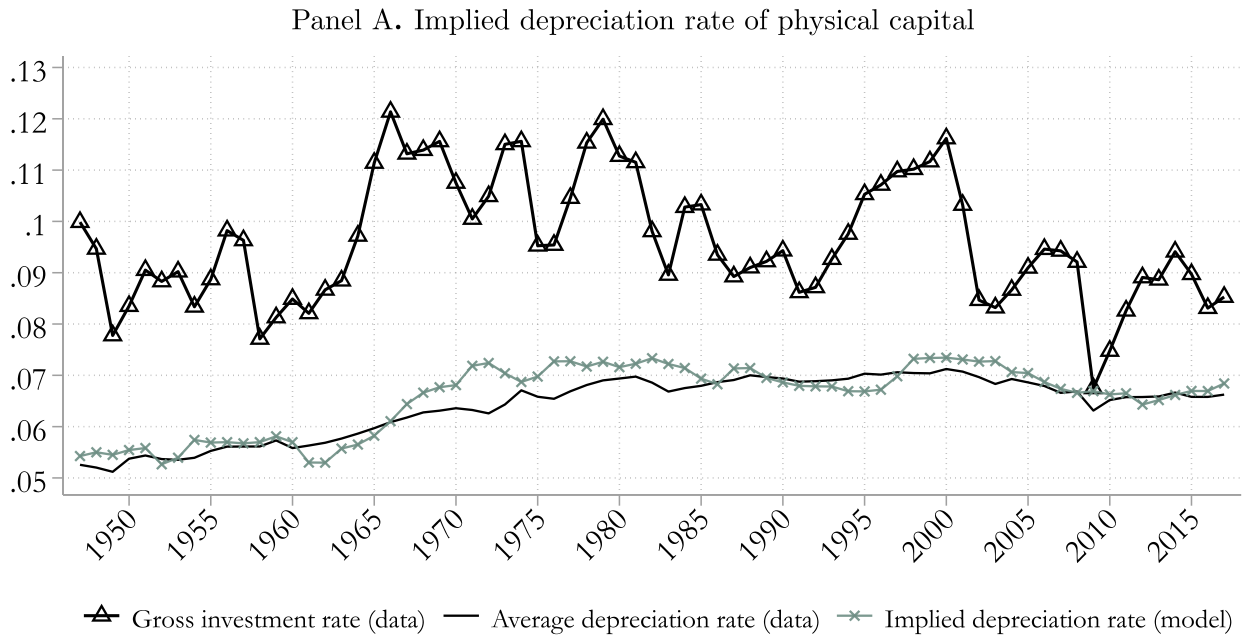


Figure 3. Growth rates for intangible, physical, and total capital stocks. The top panel reports the growth rates of net capital stocks at current cost, obtained from Fixed Assets table 4.1; physical capital is defined as the sum of equipment and structures, intangible capital is defined as intellectual property products, and total capital is the sum of the two. The medium panel reports the growth rates of real quantity indices for capital stocks. For intangible and total capital, this is obtained from Fixed Assets table 4.2; for physical capital, the growth rate is constructed as described in Appendix IA.C.3. The bottom panel reports the difference between the growth rate of intangible capital and the growth rate of physical capital, when they are measured using either stocks at current cost ("nominal") or quantity indices ("real").



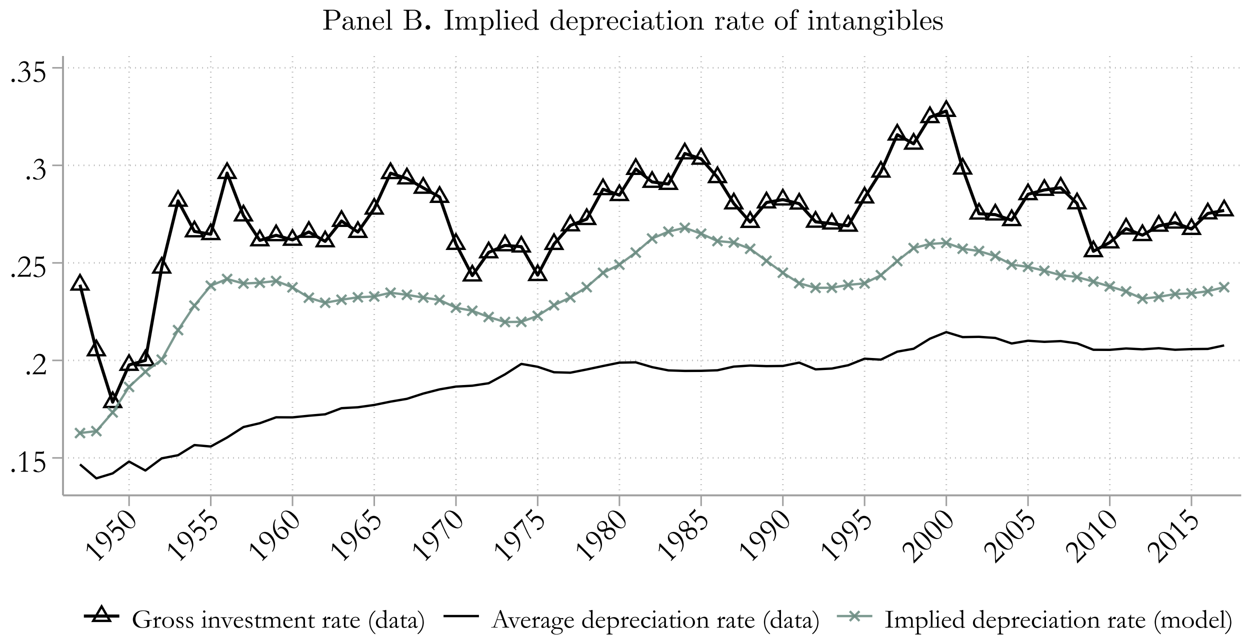


Figure 4. Gross investment rates and average depreciation rates in the data, and model-implied depreciation rates. Data is for the NFCB sector. The top panel reports data for physical capital, and the bottom panel reports data for intangible capital. Each panel reports the gross investment rate (black triangle line) and average depreciation rates (black solid line), both computed from BEA Fixed Assets tables. In particular, average depreciation rates are computed using current cost estimates of depreciation and net stocks. Model-implied depreciation rates (teal crossed lines) are computed as $\delta_i = R_i - r - \gamma_i r g = \iota_i - g$, $i = 1, 2$. Appendix IA.C.4 contains more detail on methodology and data sources.

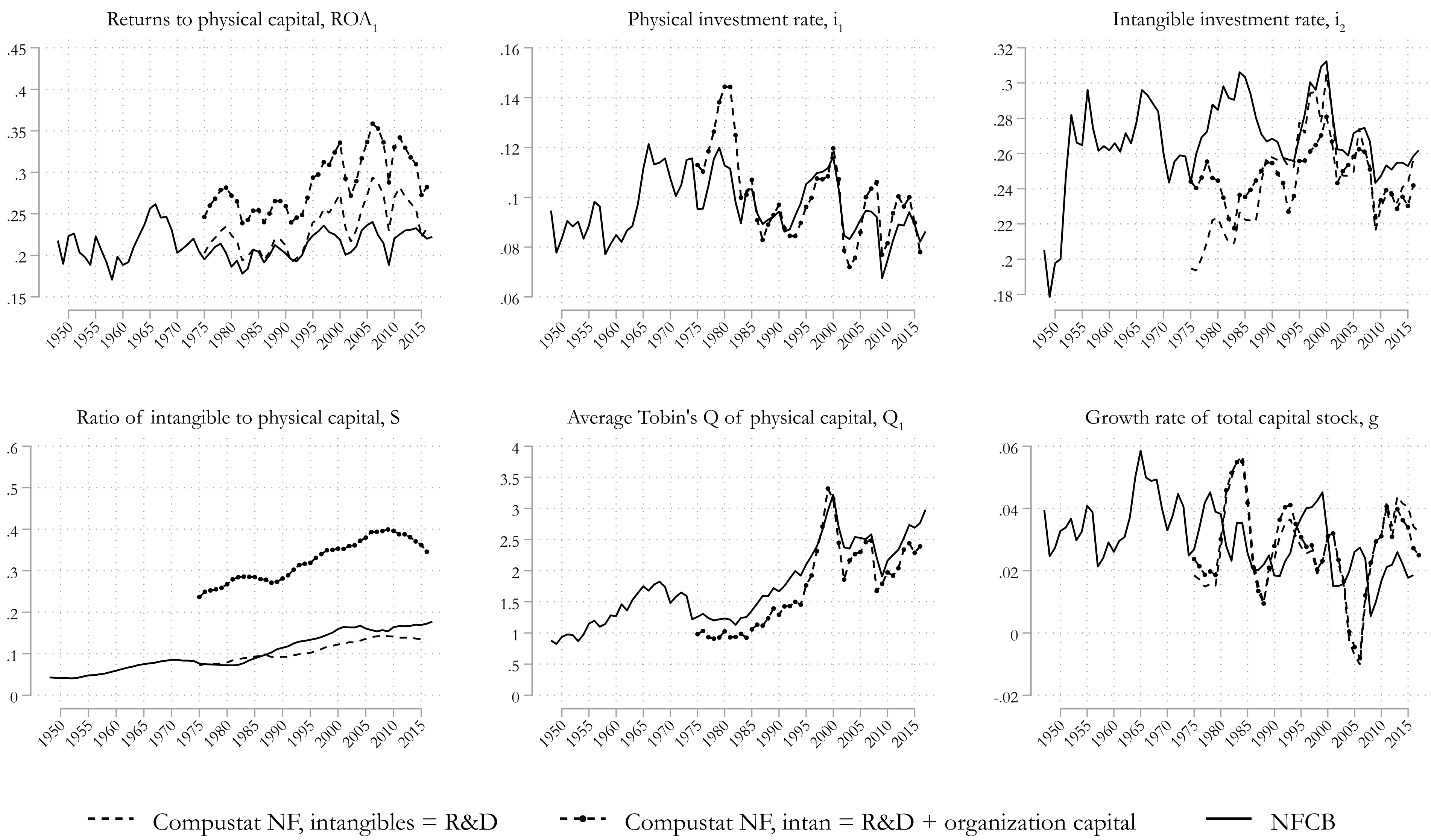


Figure 5. Time series moments for Compustat Non-Financial (NF), all sectors (aggregated). The corresponding time series moments for the aggregate non-financial corporate business (NFCB) sector are also reported, for comparison. All variables are defined as in Appendix Figure 1. Data sources for the NFCB sector are described in Section II and Appendix IA.C. Data sources are described in Section III.

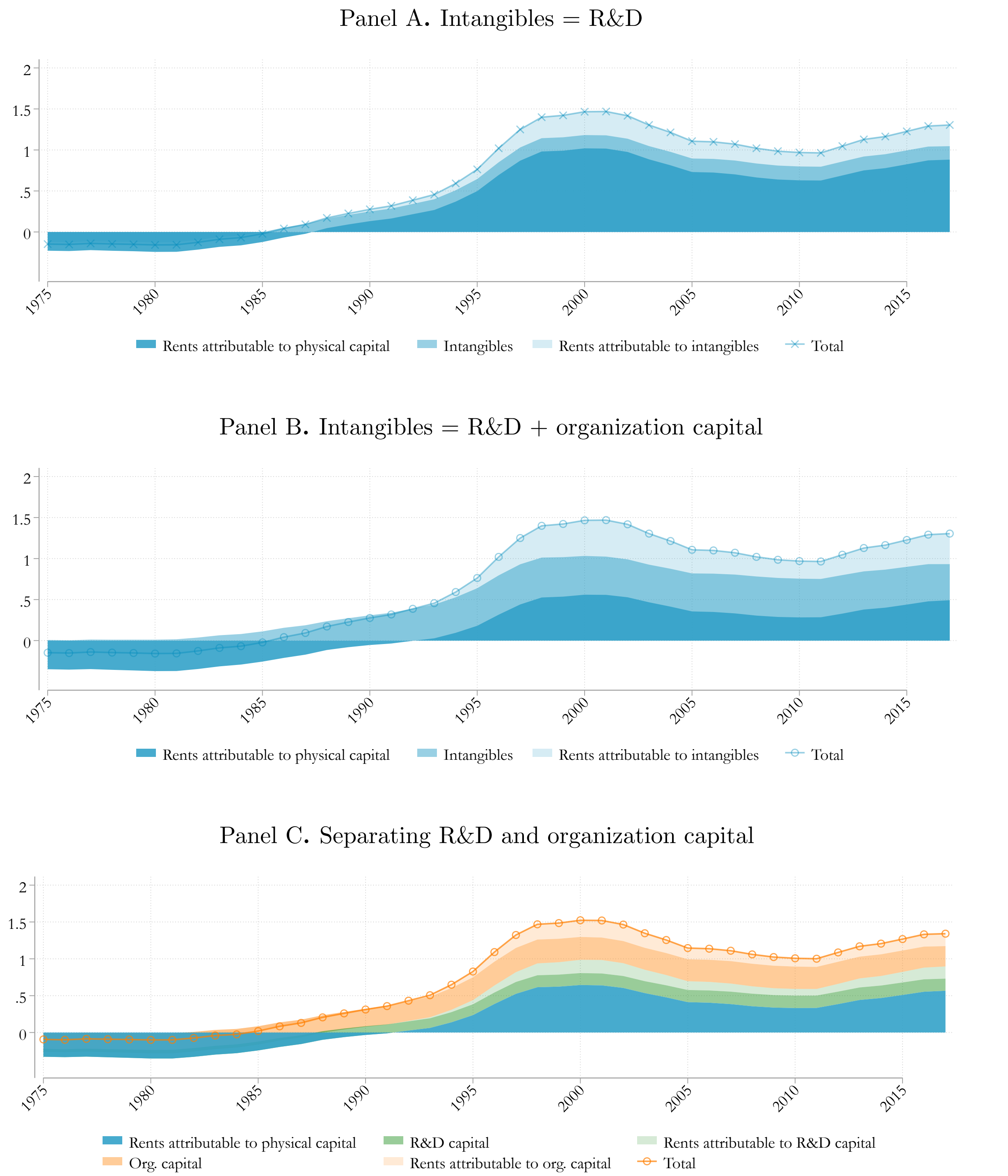


Figure 6. The investment gap $Q_1 - q_1$ for physical capital in the Compustat Non-Financial (NF) sample. The top panel reports results when only R&D is used to measure intangibles. The middle panel reports results when both R&D and organization capital are used to measure intangibles. In the top and middle panels, we use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$, in order to construct the components of the investment gap. The bottom panel reports a version of the decomposition that separates explicitly the contribution of the terms related to R&D capital (in green) and those related to organization capital (in orange). In order to construct this decomposition, we use a model with adjustment costs of $\gamma_1 = 3$ for physical capital, $\gamma_2 = 12$ for R&D capital, and $\gamma_3 = 3.2$. Our values for adjustment costs are drawn from Belo et al. (2019). Methodology and data sources are described in Section III.

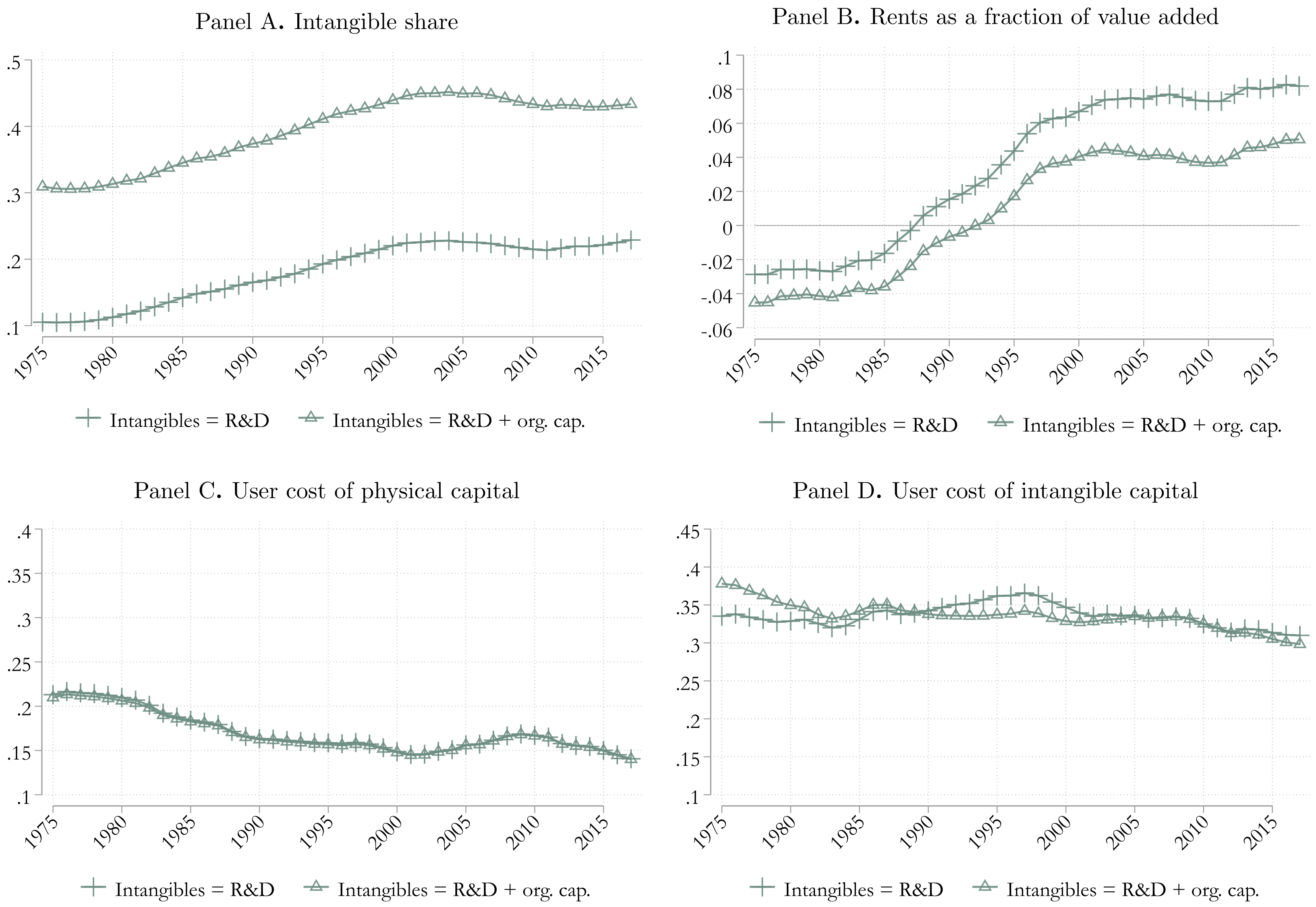
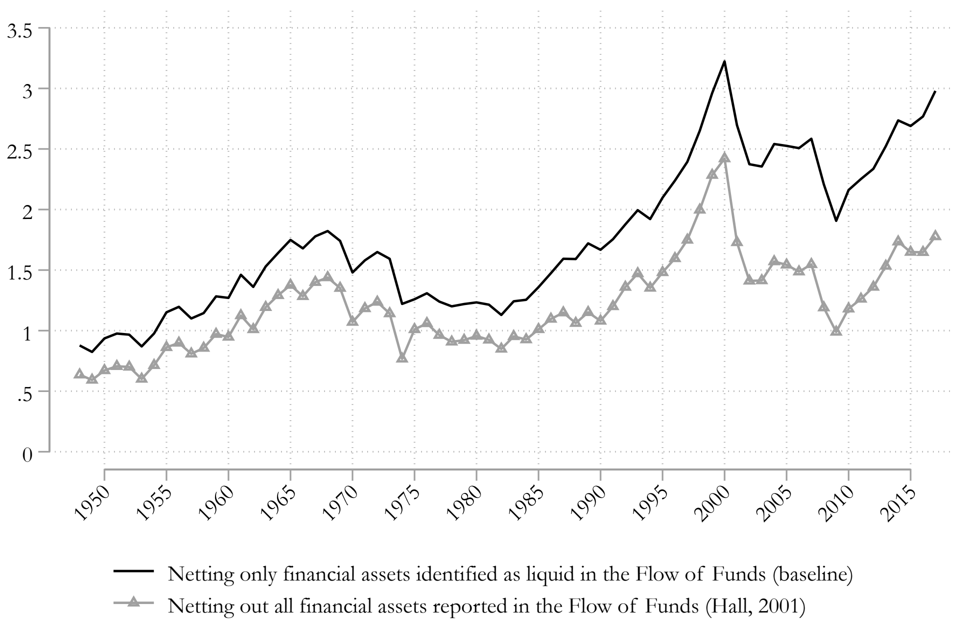


Figure 7. Other model moments for the Compustat Non-Financial (NF) sample. Panel (a) reports the share of intangibles in production, η, when the capital aggregator is assumed to be Cobb-Douglas: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. Panel (b) reports rents as a fraction of value added, s_{VA}, which is given by $s_{VA} = (1 - s_L)(1 - 1/\mu)$, where μ is the model parameter governing the size of rents, and s_L is labor's share of value added. Panels (c) and (d) report user costs for each type of capital, R_1 and R_2. We use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$, in order to construct the components of the investment gap. Methodology and data sources are described in Section III.

Panel A. Average Tobin's Q for physical capital, Q_1



Panel B. Investment gap

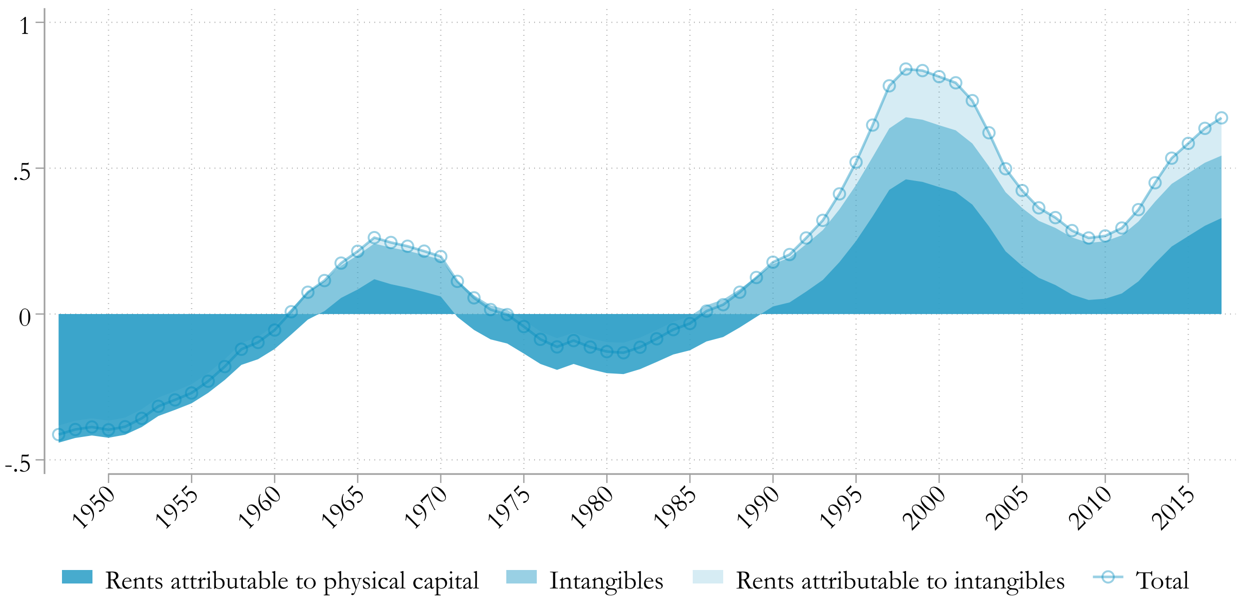


Figure 8. Robustness: the investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate (NFCB) sector when using the Hall (2001) measure of Tobin's average Q for physical assets. Panel A reports our baseline measure of average Tobin's Q for physical capital, Q_1 (black line), and an alternative measure based on Hall (2001). The difference between the two is that our baseline measure only nets out financial assets identified as liquid in the Flow of Funds in the computation of the net value of claims on the NFCB sector. Panel B reports the investment gap in the model without adjustment costs ($\gamma_1 = \gamma_2 = 0$). Details on the construction of total enterprise value are discussed in Appendix IA.C.2.

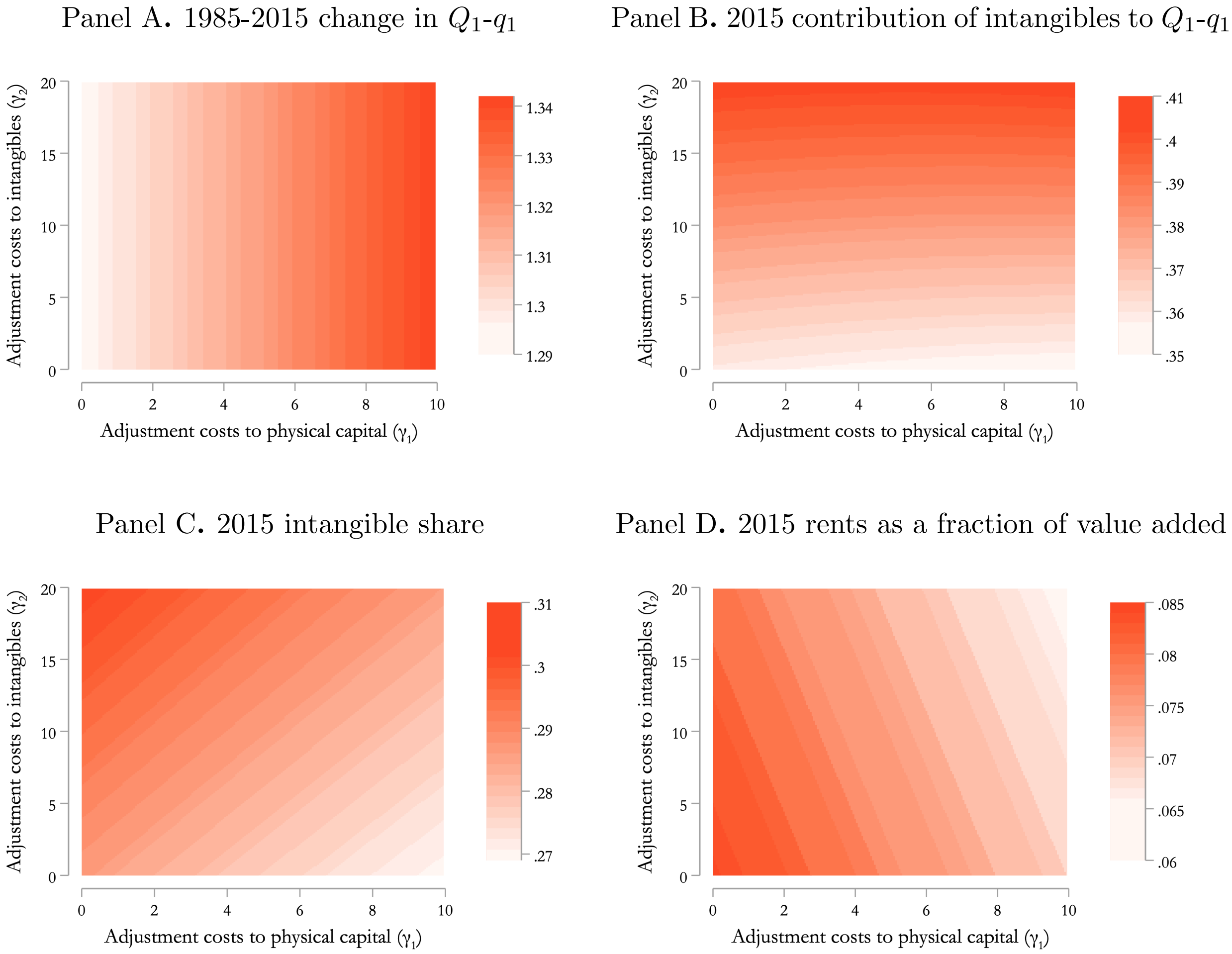
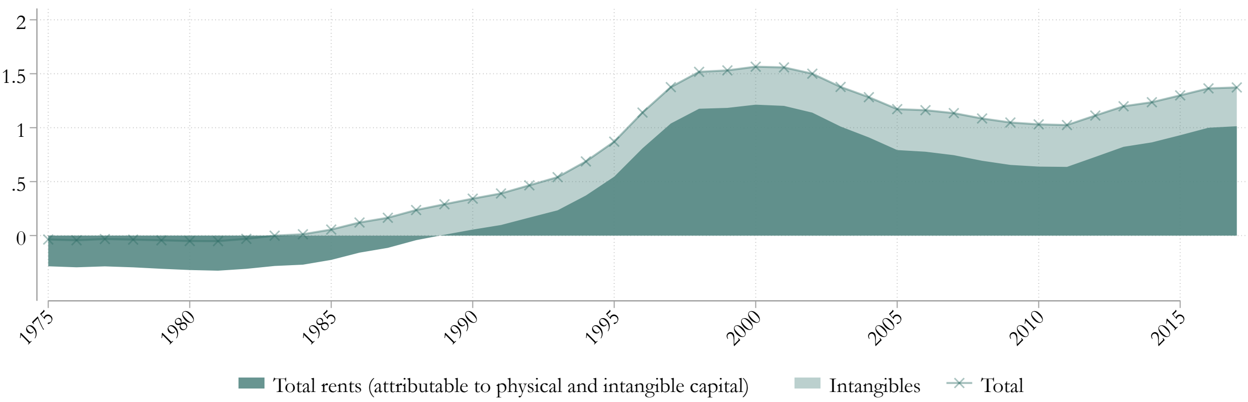
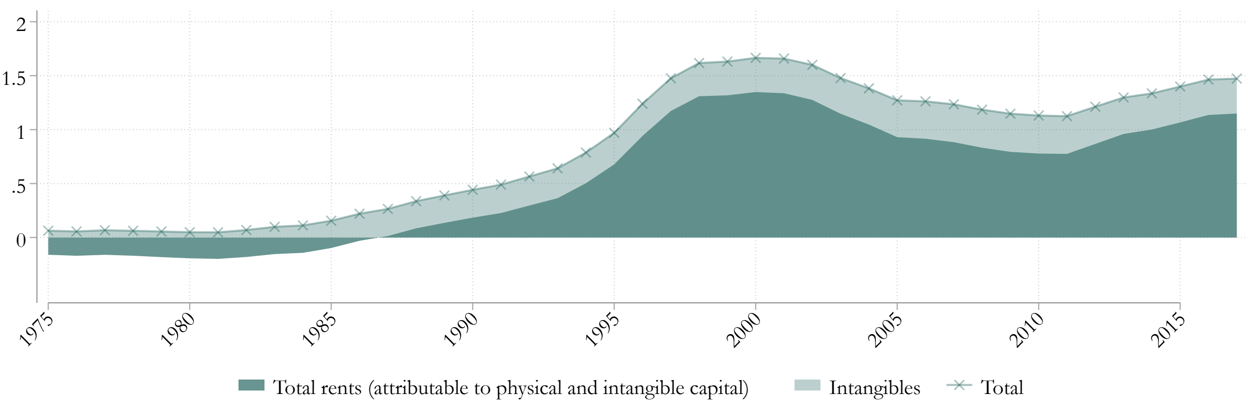


Figure 9. Robustness: adjustment costs. Each panel reports a moment from the investment gap decomposition, Equation (19), for the NFCB sector, for different combinations of adjustment costs for physical and intangible capital. In each panel, a point corresponds to a particular combination for (γ_1, γ_2), and the color corresponds to the value of the moments, with the correspondence reported on the right axis. Panel (a) reports the change in $Q_1 - q_1$ from 1985 to 2015; in our baseline results with positive adjustment costs, this moment is equal to 1.30. Panel (b) reports the contribution of intangibles to $Q_1 - q_1$ in 2015; in our baseline results, this moment is equal to 0.39 (or 39%). Panel (c) reports the implied intangible share in production in 2015; in our baseline results with positive adjustment costs, this moment is equal to 0.29. Panel (d) reports rents as a share of value added in 2015; in our baseline results with positive adjustment costs, this moment is equal to 0.063. Our baseline results with positive adjustment costs use $\gamma_1 = 3$ and $\gamma_2 = 12$. Methodology and data sources are described in Section II in the main text.

Panel A. No equity financing frictions ($f'(d) = 1$)



Panel B. Positive equity financing frictions ($f'(d) = 0.90$)



Panel C. High equity financing frictions ($f'(d) = 0.80$)

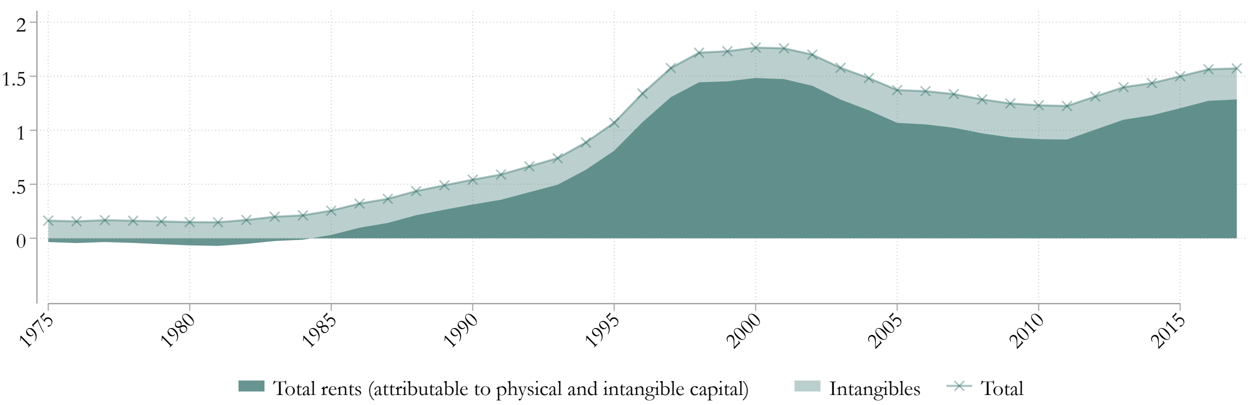
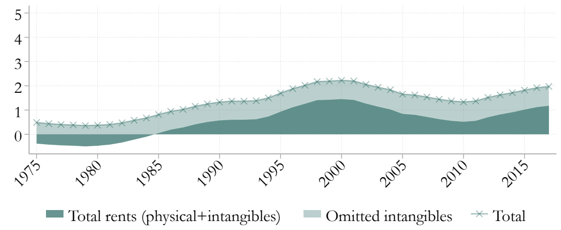
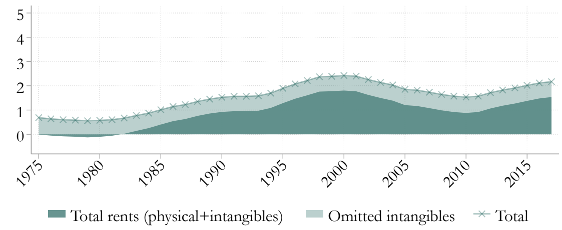


Figure 10. The investment gap for physical capital in the Compustat Non-Financial (NF) sample, for different magnitudes of equity financing frictions. In each panel, the crossed green line is an estimate of the investment gap. The shaded areas present the decomposition of the physical investment gap into two terms, corresponding to total rents (the dark region) and the omitted intangibles effect (light region). The top panel reports results with no equity financing frictions ($f'(d) = 1$); the middle panel reports results positive equity financing frictions ($f'(d) = 0.90$); and the bottom panel reports results with high equity financing frictions ($f'(d) = 0.80$). The methodology is described in Section IV in the main text and Appendix IA.D.9.1.

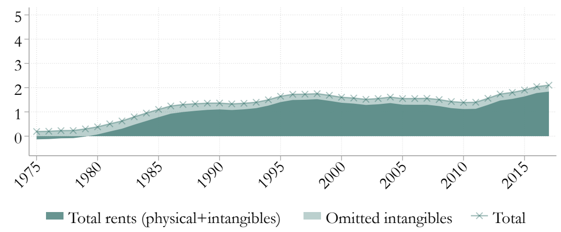
Panel A. Consumer sector, no frictions ($f'(d) = 1$)



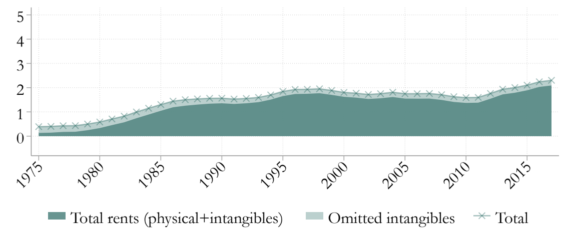
Panel B. Consumer sector, frictions ($f'(d) = 0.90$)



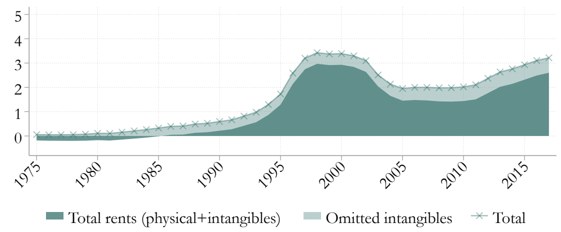
Panel C. Services sector, no frictions ($f'(d) = 1$)



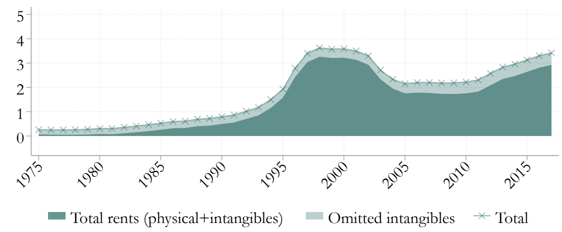
Panel D. Services sector, frictions ($f'(d) = 0.90$)



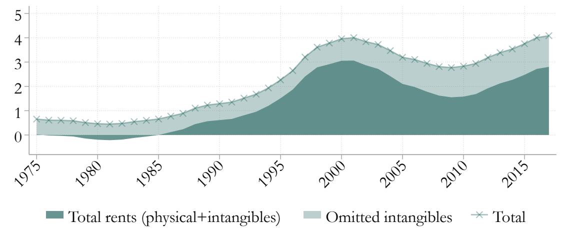
Panel E. High-tech sector, no frictions ($f'(d) = 1$)



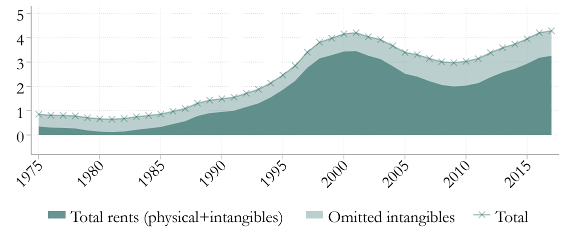
Panel F. High-tech sector, frictions ($f'(d) = 0.90$)



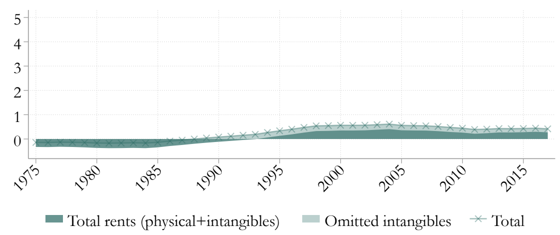
Panel G. Healthcare sector, no frictions ($f'(d) = 1$)



Panel H. Healthcare sector, frictions ($f'(d) = 0.90$)



Panel I. Manufacturing sector, no frictions ($f'(d) = 1$)



Panel J. Manufacturing sector, frictions ($f'(d) = 0.90$)

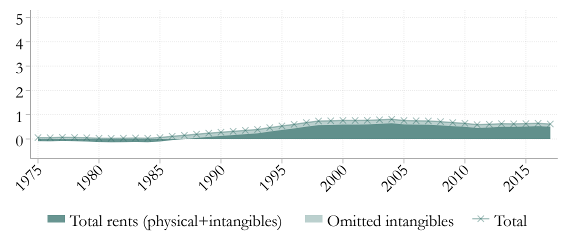
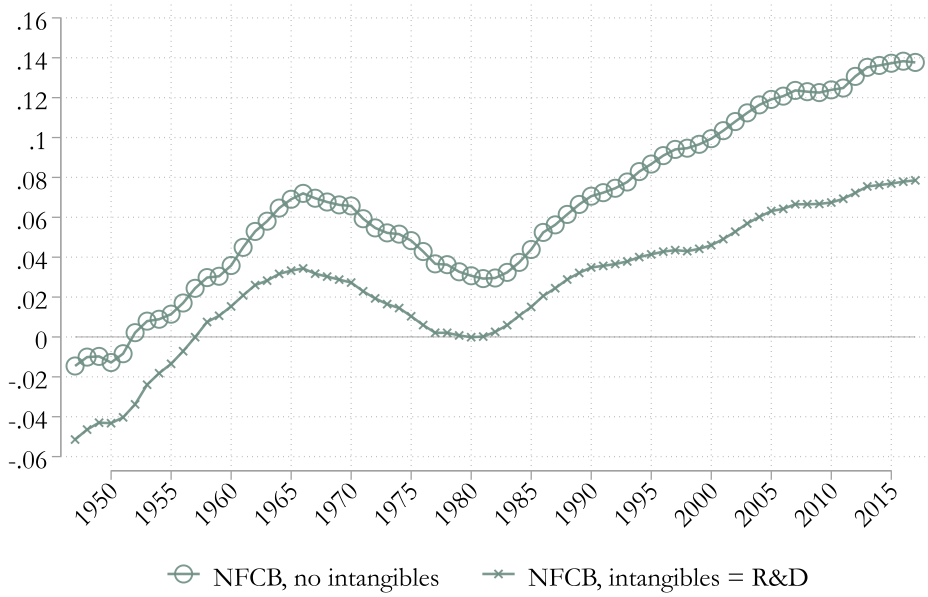


Figure 11. Sectoral investment gaps, with and without equity financing frictions. The left column reports the investment gaps obtained in our baseline approach, without equity financing frictions. The left column reports the gaps with equity financing frictions, assuming $f'(d) = 0.90$. The methodology is described in Section IV in the main text and Appendix IA.D.9.1.

Panel A. Rents as a fraction of value added: NFCB sector



Panel B. Rents as a fraction of value added: Compustat non-financial firms

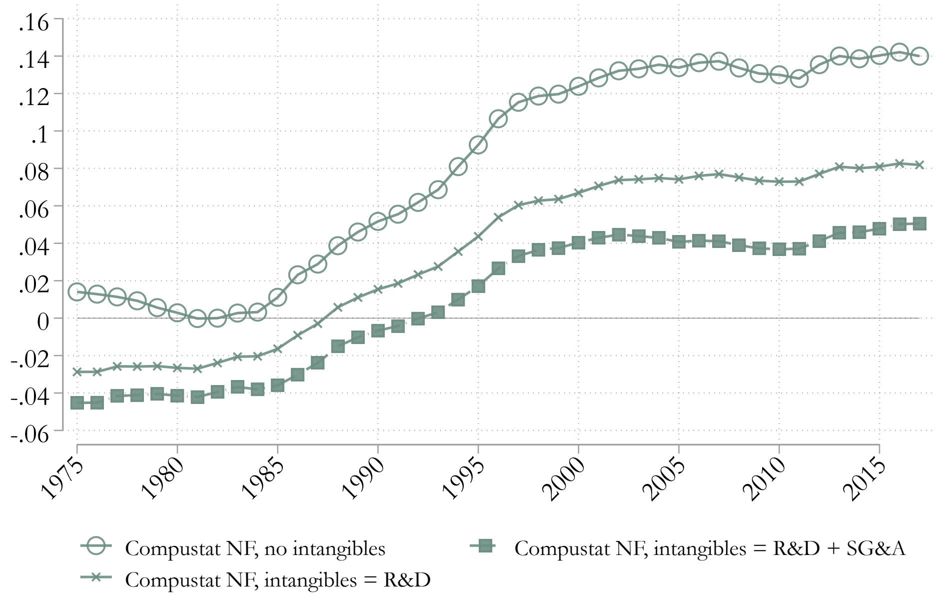
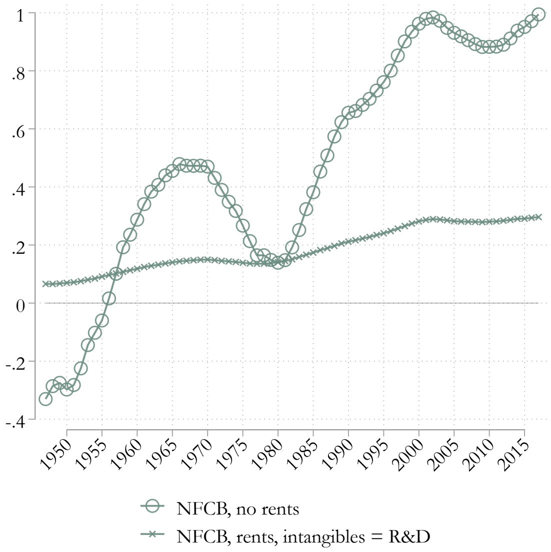
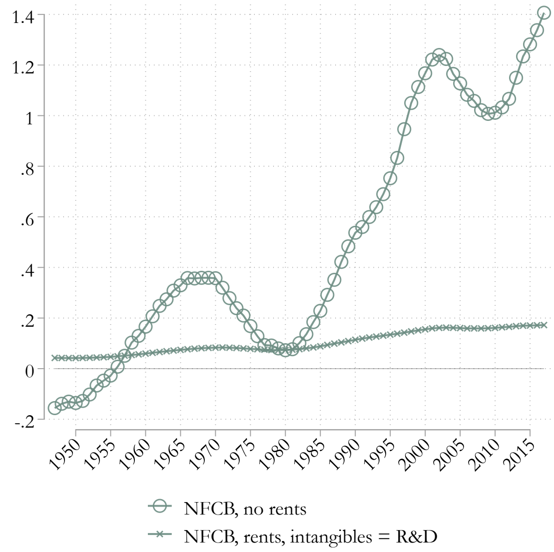


Figure 12. Rents as a fraction of value added, in the case of no intangibles versus the baseline. The top panel reports results obtained using aggregate data for the Non-Financial Corporate Business (NFCB) sector. The bottom panel reports results obtained using aggregated data for the Compustat non-financial (NF) sample. In both panels, the circled line is the trend in markups obtained when we assume that firms have no intangible assets. The other lines correspond to the cases where either capitalized R&D, or capitalized R&D plus capitalized SG&A are used to measure the intangible capital stock. In both panels, we use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$.

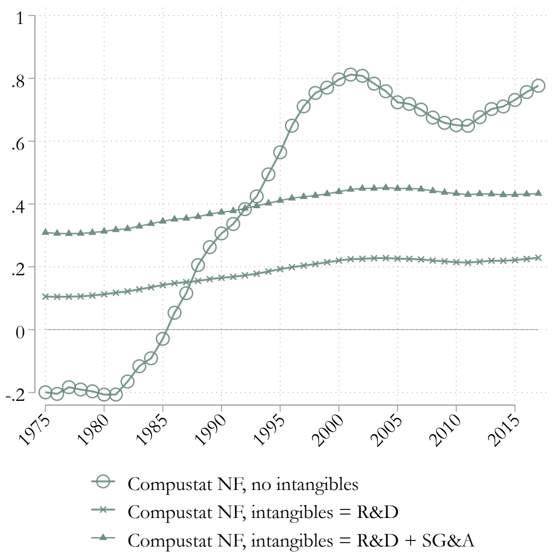
Panel A. Cobb-Douglas share of intan, NFCB sector



Panel B. Intangibles/physical capital: NFCB sector



Panel C. Cobb-Douglas share of intan: Compustat NF



Panel D. Intangibles/physical capital: Compustat NF

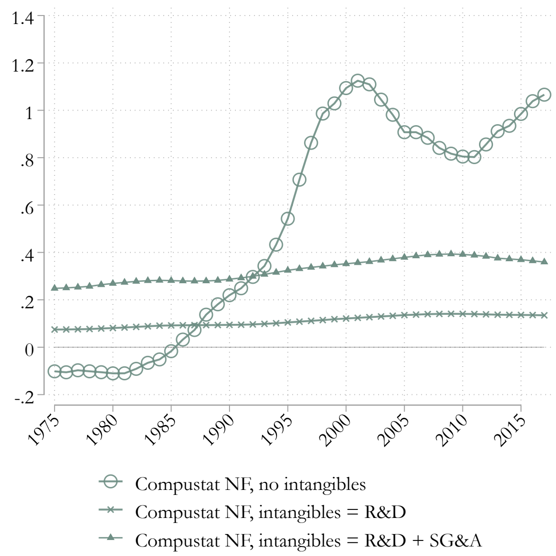
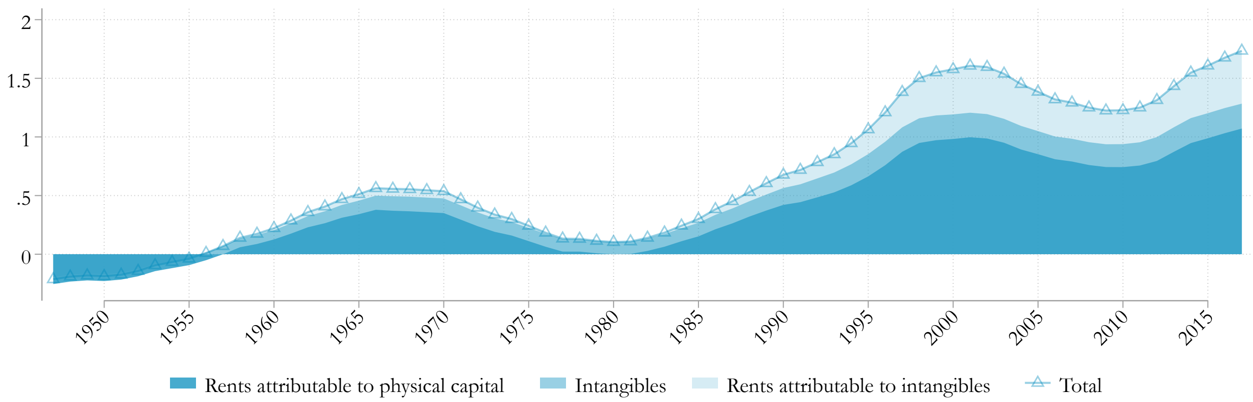


Figure 13. The importance of intangible capital in production, in the case of no rents versus the baseline. The top panels reports results obtained using data for the non-financial corporate business (NFCB) sector. The bottom panels reports results obtained using aggregated data from Compustat nonfinancials (NF). Panels in the left column report estimates of the intangible share of capital in the production function, η, assuming a Cobb-Douglas specification. Panels in the right column report estimates of the stock of intangible capital relative to physical capital, S. The two are related through $\eta/(1-\eta) = SR_2/R_1$. In all panels, the circled lines represent the implied values of η or S when we assume no rents, $\mu = 0$. The other lines correspond to the cases where either capitalized R&D, or capitalized R&D plus capitalized SG&A are used to measure the intangible capital stock. In all panels, we use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$.

Panel A. Positive adjustment costs and $g = g_1 = g_2$



Panel B. Positive adjustment costs and $g \neq g_1 \neq g_2$

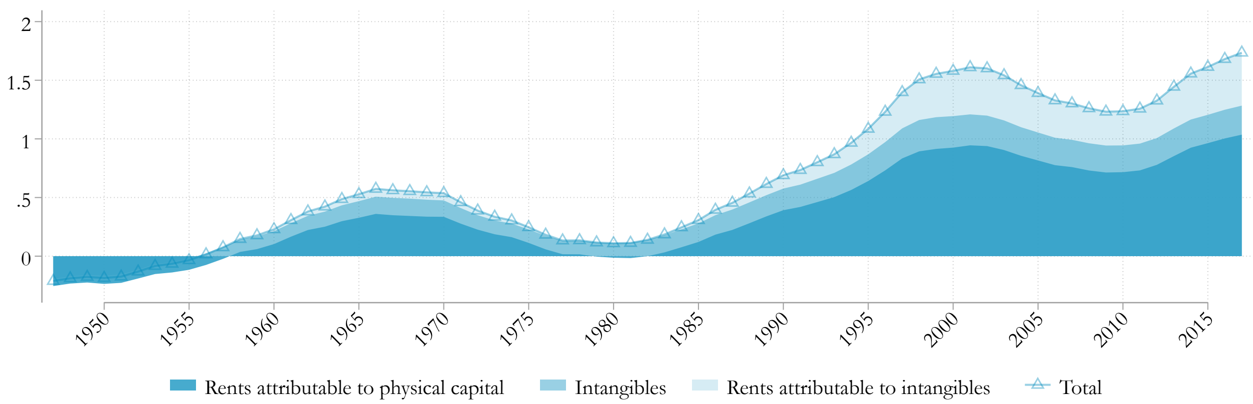


Figure 14. The investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate business (NFCB) sector, allowing for heterogeneous growth rates between physical and intangible capital. The top panel reports the investment gap in our baseline approach with positive adjustment costs ($\gamma_1 = 3$, $\gamma_2 = 12$), where the two types of capital are assumed to grow at the same rate ($g_1 = g_2 = g$), and g is measured using the growth rate of the total capital stock in the NFCB sector. The results are the same as the middle panel of Figure 1 in the main text. The bottom panel reports the investment gap decomposition when we allow for heterogeneous growth rates in intangible and physical capital, $g_1 \neq g_2$, in the measurement of marginal q for physical capital and intangible capital. The two growth rates are computed using data from the Fixed Asset Tables, as described in Appendix IA.C.3.

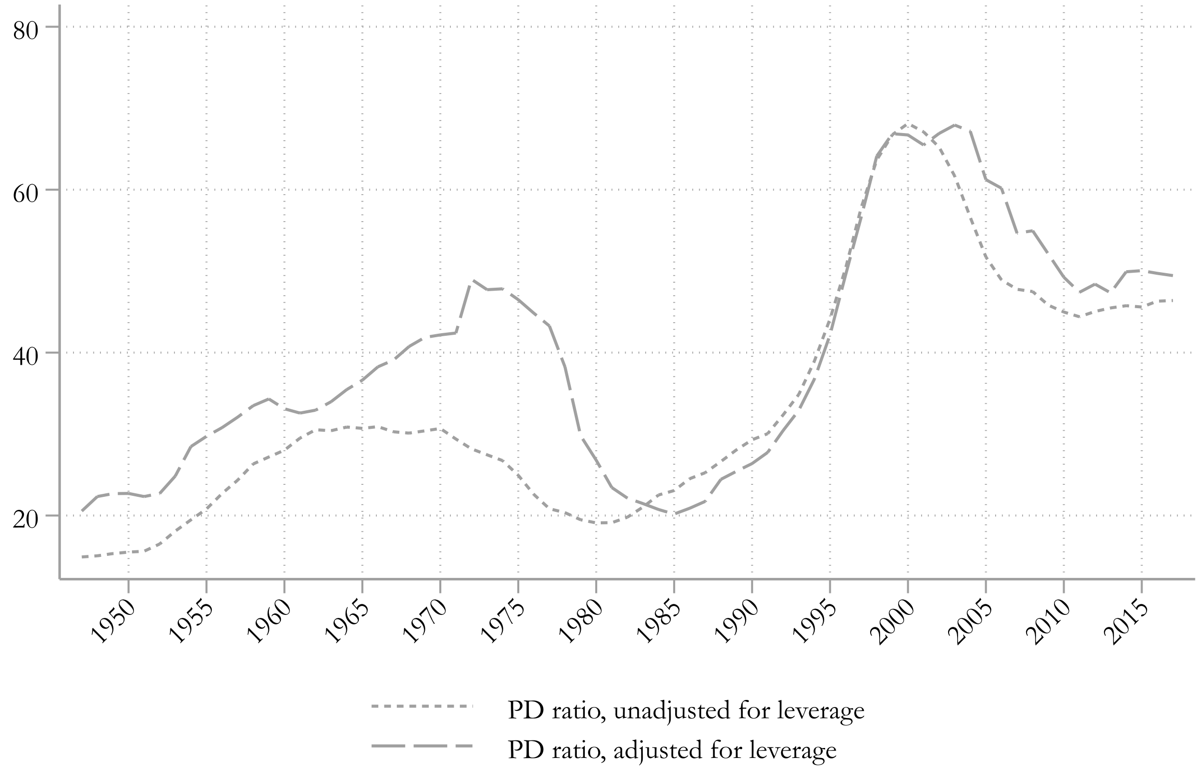
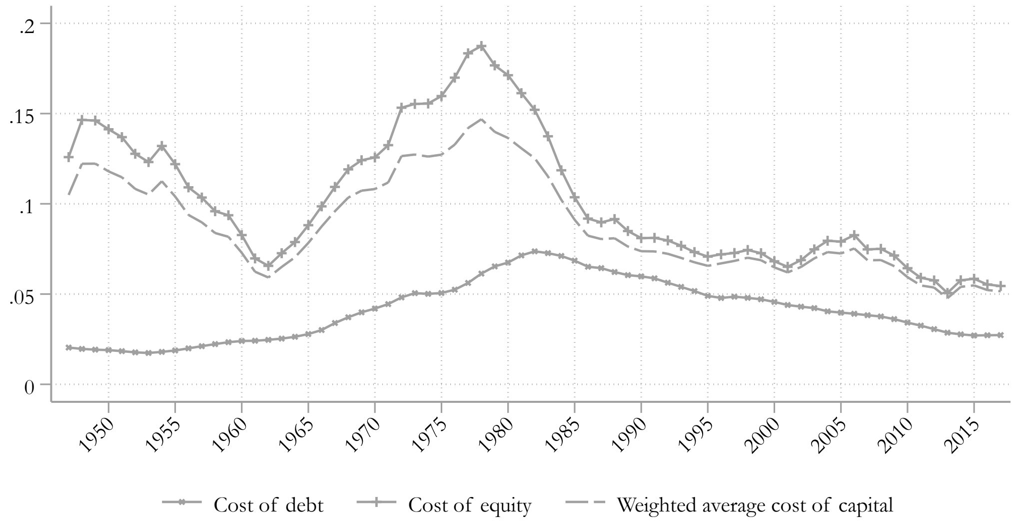


Figure 15. Time series for the PD ratio used in alternative approach 1. The short dashed line is the unadjusted PD ratio, $PD = 1/(R^c_{E,t-1,t} - R^e_{E,t-1,t})$, where $R^e_{E,t-1,t}$ and $R^c_{E,t-1,t}$ are, respectively, the cum- and ex-dividend returns on the S&P 500. The long dashed line is the PD ratio adjusted for leverage, $\tilde{PD} = (1+l)PD^E/(1+(r^n_B - g^n)lPD)$, where $l = B^e_{t-1}/E^e_{t-1}$ is market leverage, r^n_B is the (after-tax, nominal) interest rate on debt securities, and g^n is the nominal growth rate of the total capital stock. Data sources are described in Appendix IA.C.

Panel A. Using the PD ratio to measure the cost of equity



Panel B. Using risk-free rate + constant risk premium to measure the cost of equity

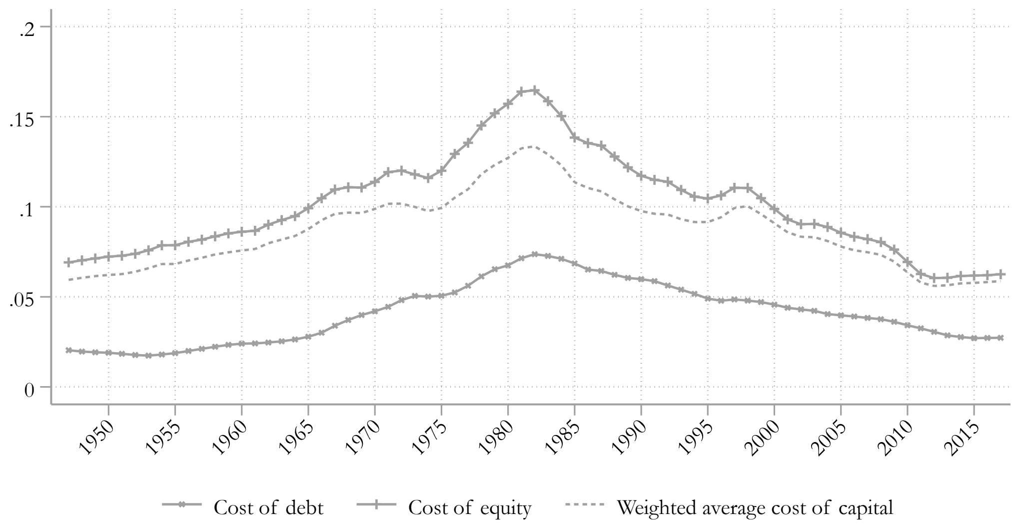


Figure 16. Components of the average cost of capital. All the costs of capital are expressed in nominal terms. Both panels report r_B^n, r_E^n and r^n, which are respectively the cost of debt, equity, and the weighted average cost of capital, where $r^n = l/(1+l)r_b^n + 1/(1+l)r_e^n$. r^n is the nominal discount rate and is related to the Gordon growth term via $r^n = (r-g) + g^n$, where g^n is the growth rate of the nominal capital stock. r_E^n is the nominal cost of equity. In the top panel, it is measured as $r_E^n = PD^{-1} + g^n$, where PD is the PD ratio reported in Appendix Figure 15. In the bottom panel, it is measured as $r_E^n = r_f^n + RP$, where r_f^n is the (nominal) rate of return on one-month T-bill, and RP is a constant risk premium of $RP = 6.5\%$. In both panels, r_b^n is an average interest rate on debt liabilities of non-financial corporate firms. More details are provided in Appendix IA.D.6.1.

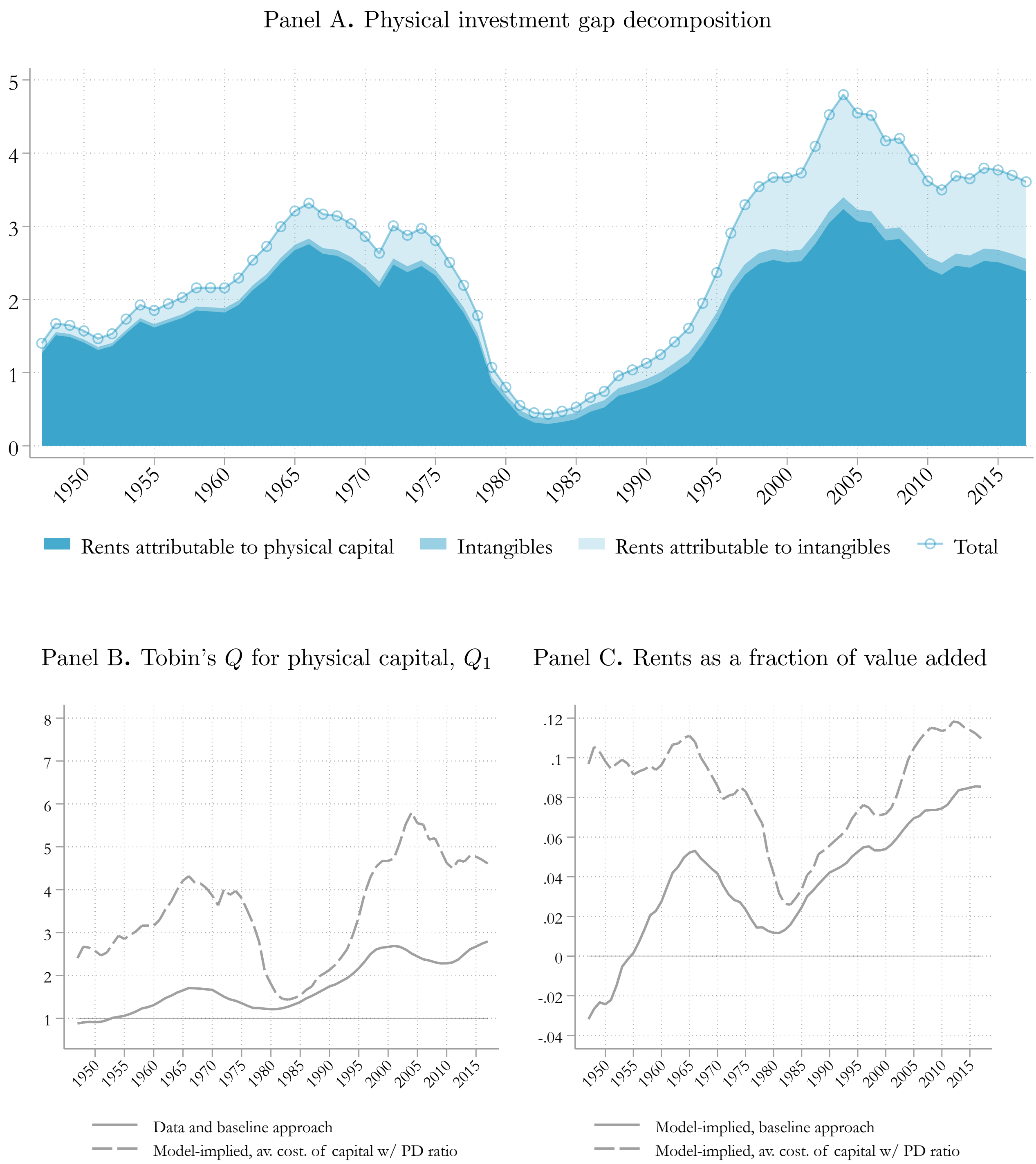


Figure 17. Results in the average cost of capital approach, when the PD ratio is used to measure the cost of equity. Data is for the non-financial corporate (NFCB) sector. The top panel reports the decomposition of the investment gap obtained obtained using this approach. The bottom panels compare the value of Q_1 (left panel) and of the share of rents as a fraction of value added $s = (1-\mu^{-1})(1-LS)$, between our baseline and this approach. Solid correspond to the baseline, which matches empirical values of Q_1, while circled lines correspond to alternative approach 1. The model without adjustment costs ($\gamma_1 = \gamma_2 = 0$) is used in both cases.

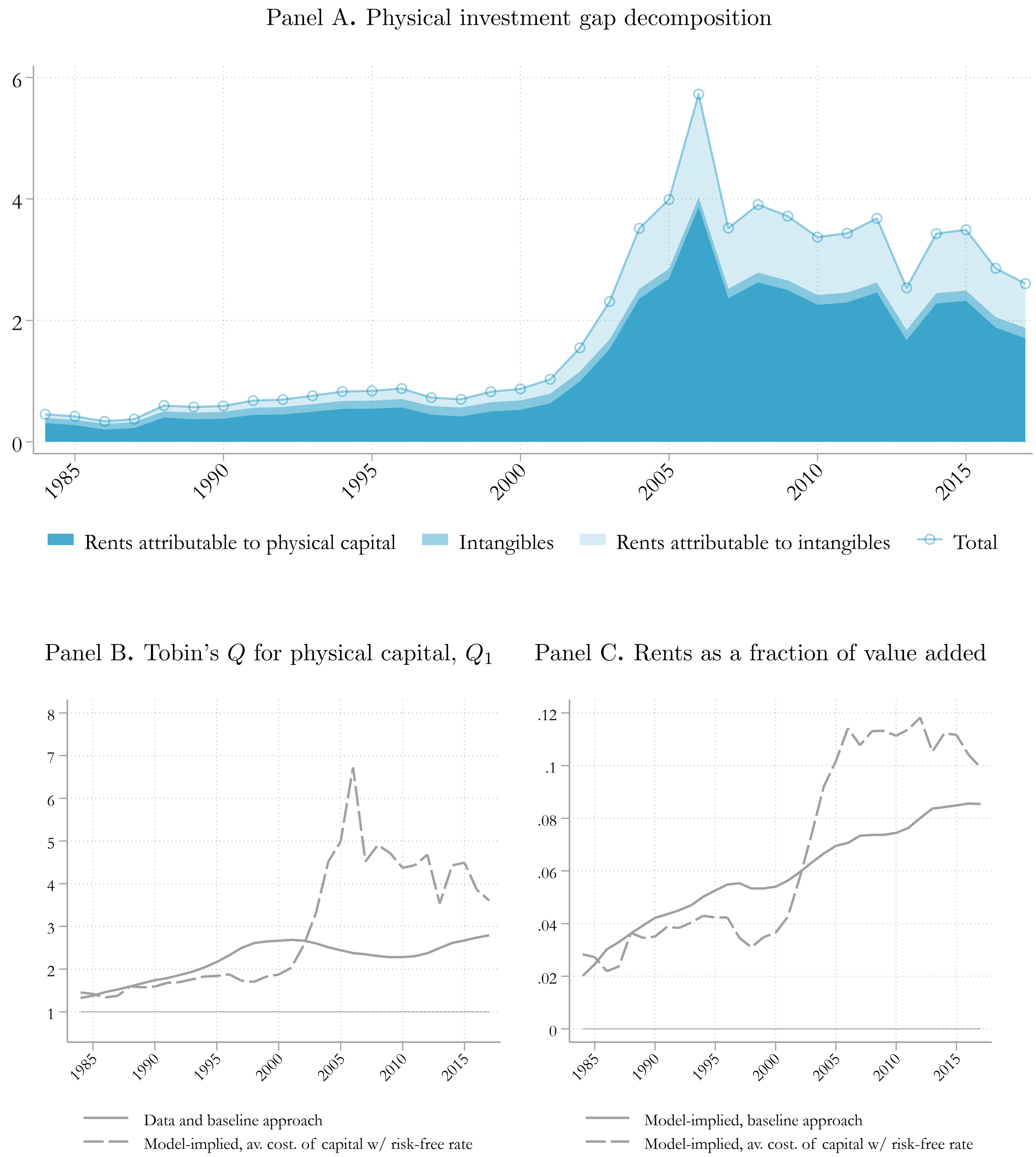


Figure 18. Results in the average cost of capital approach, when the risk-free rate plus a constant risk premium is used to measure the cost of equity. Data is for the non-financial corporate (NFCB) sector. The top panel reports the decomposition of the investment gap obtained obtained using this approach. The bottom panels compare the value of Q_1 (left panel) and of the share of rents as a fraction of value added $s = (1 - \mu^{-1})(1 - LS)$, between our baseline and this approach. Solid correspond to the baseline, which matches empirical values of Q_1, while circled lines correspond to the alternative approach. The model without adjustment costs ($\gamma_1 = \gamma_2 = 0$) is used in both cases. For Tobin's Q_1 and the investment gap, only values after 1985 are reported, because before 1985, the implied discount rate is frequently very close to or below the growth rate g, leading to implausible values for Q_1 and the investment gap.

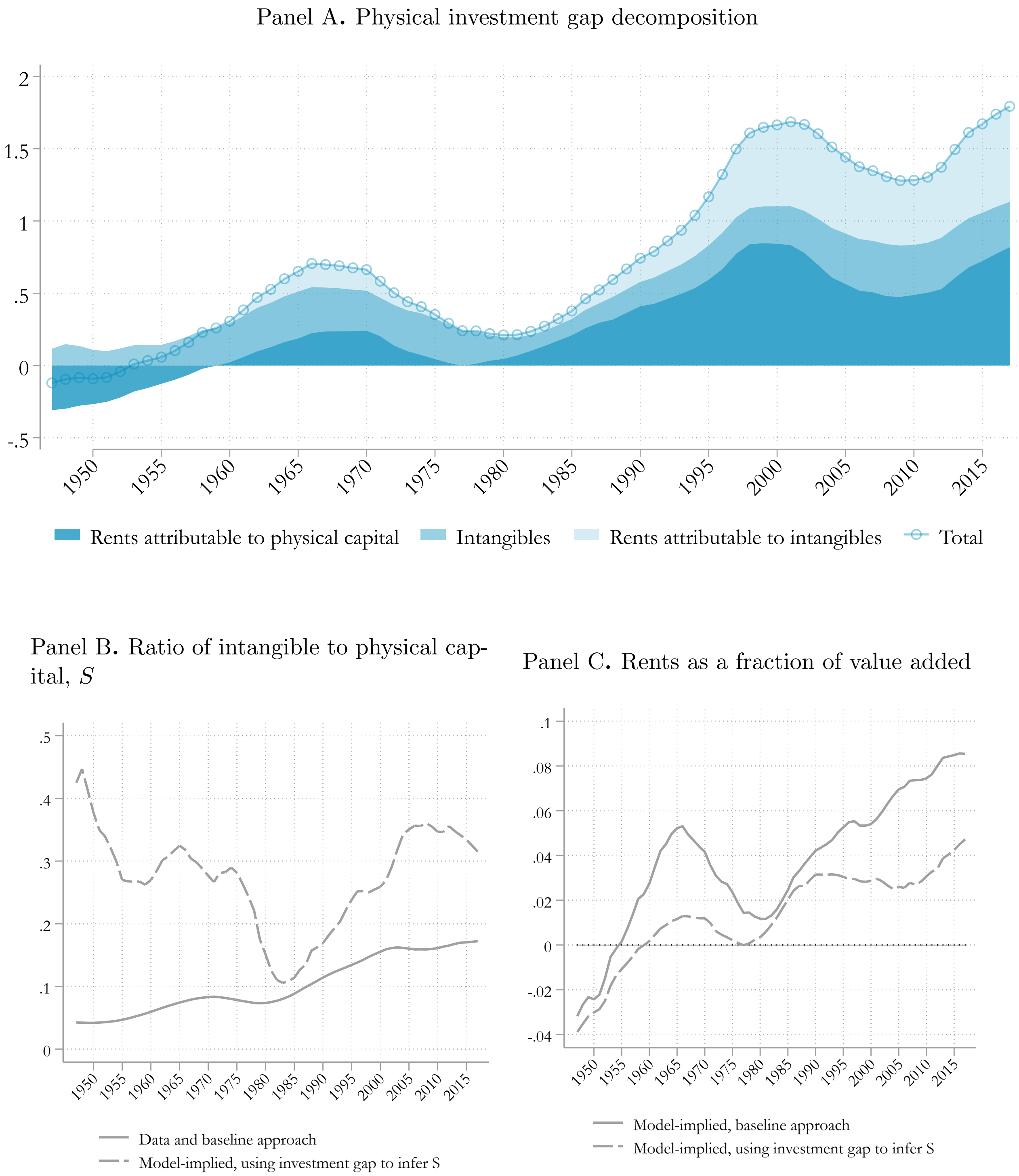


Figure 19. Results when the size of the investment gap is used to infer the ratio of intangible to physical capital, S. Data is for the non-financial corporate (NFCB) sector. The top panel reports the decomposition of the investment gap obtained obtained using this approach. The bottom panels compare the value of S (left panel) and of the share of rents as a fraction of value added $s = (1 - \mu^{-1})(1 - LS)$ (right panel), between our baseline and this approach. Solid lines correspond to the baseline, which matches empirical values of S, while dashed lines correspond to the alternative approach. The model without adjustment costs ($\gamma_1 = \gamma_2 = 0$) is used in both cases.

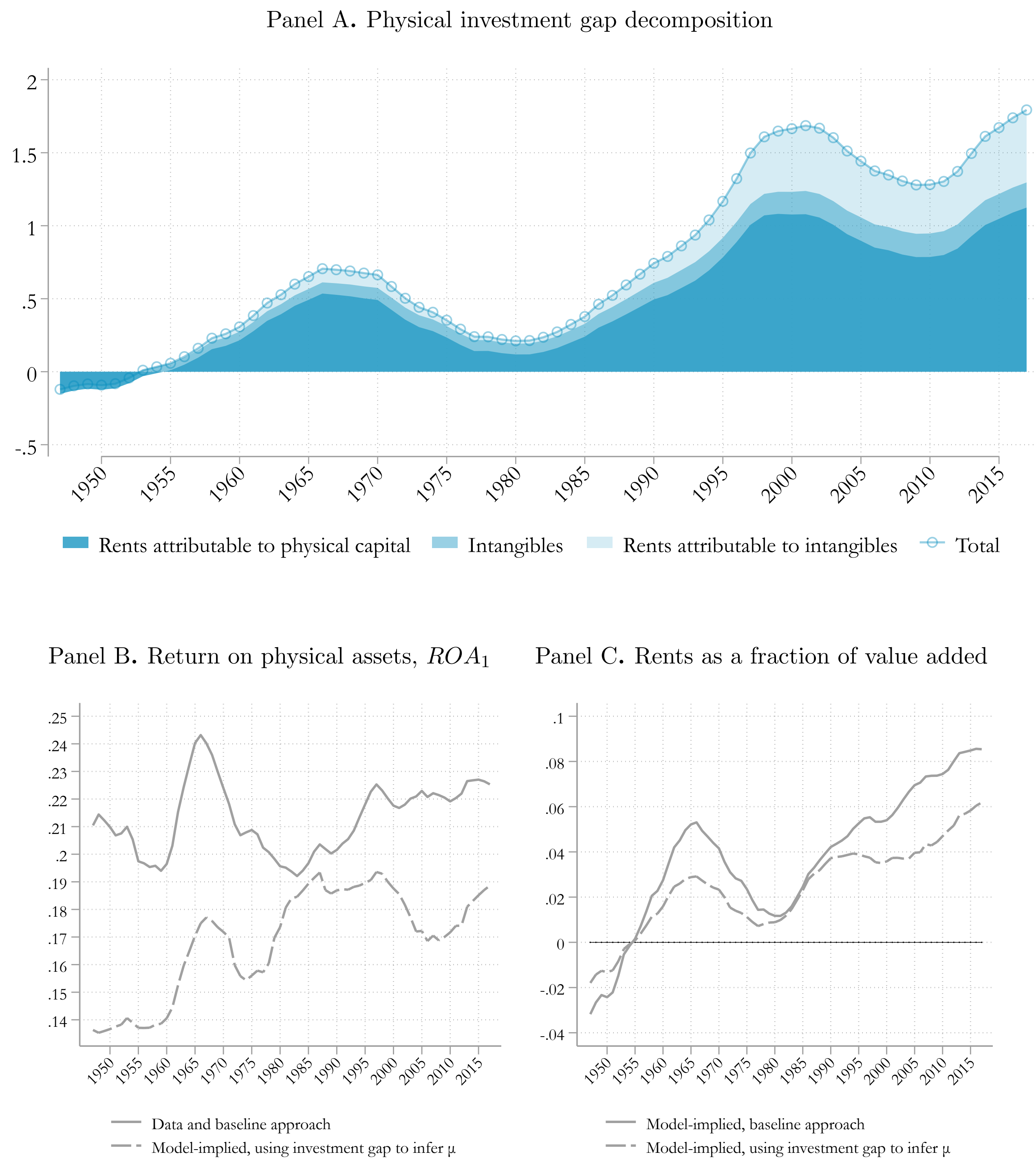


Figure 20. Results when the size of the investment gap is used to infer the size of the rents parameter μ. Data is for the non-financial corporate (NFCB) sector. The top panel reports the decomposition of the investment gap obtained obtained using this approach. The bottom panels compare the value of ROA_1 (left panel) and of the share of rents as a fraction of value added $s = (1 - \mu^{-1})(1 - LS)$ (right panel), between our baseline and this approach. Solid correspond to the baseline, which matches empirical values of ROA_1, while dashed lines correspond to the alternative approach. The model without adjustment costs is used in both cases.

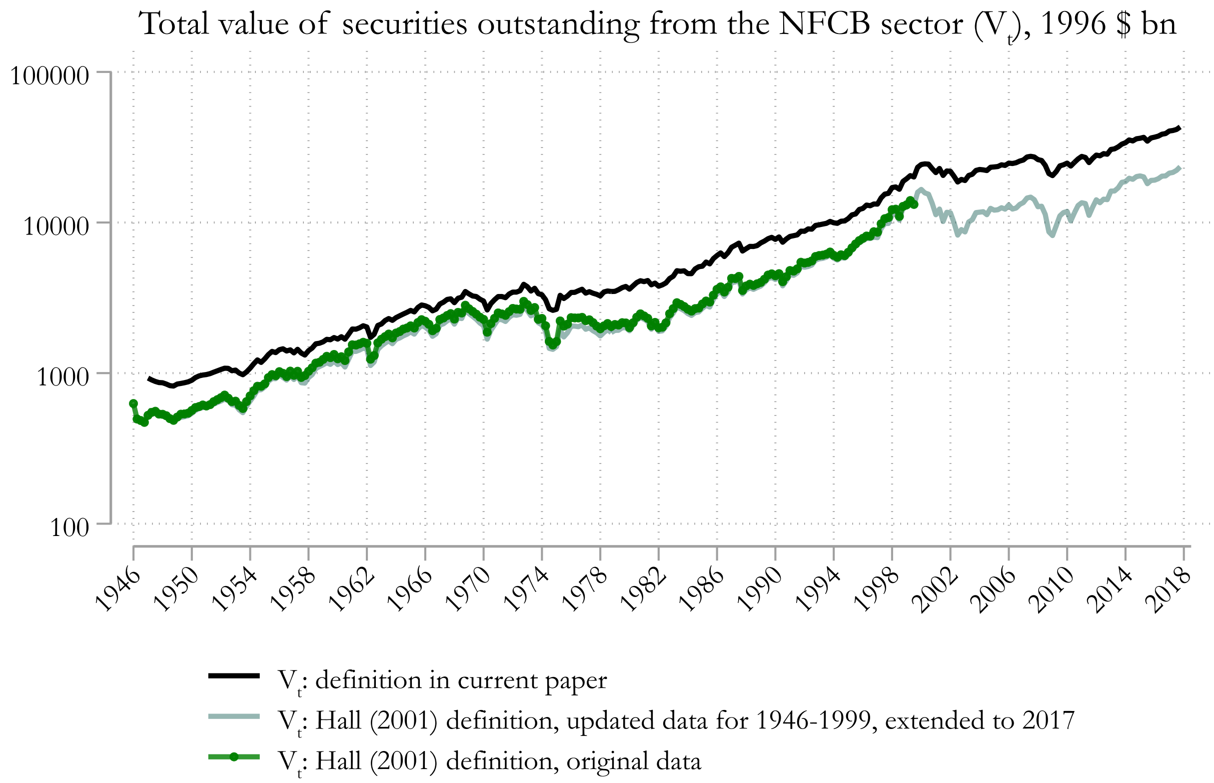


Figure 21. Total market value of securities outstanding from the non-financial corporate (NFCB) sector, V_t. V_t is defined as the sum of the market value of equity securities, plus an estimate of the market value of non-equity claims, minus financial assets. The estimate of the market value of non-equity claims is equal to their book value, plus an adjustment for the difference between the market and book value of corporate bonds and municipal securities, following Hall (2001). Data are from the Flow of Funds. The black line reports the estimate used in the main text. The green dotted line is the estimate of V_t constructed by Hall (2001), and obtained from his replication data, available at `web.stanford.edu/~rehall/SMCA_Data_Appendix.html`. The solid blue line is an extension of the Hall (2001) estimate of V_t to 2017. The differences between original and extended Hall (2001) estimates come from small differences in the updated data sources, and in the time series for the stock of municipal bonds and the issuance of corporate bonds in the 1950s and 1960s. All time series are deflated using the deflator for investment in non-residential fixed assets (FRED series A008RD3Q086SBEA) for the solid blue and black lines, and the original deflator constructed by Hall (2001) for the dotted green line. More details on data sources and methodology are reported in Appendix IA.C.1.

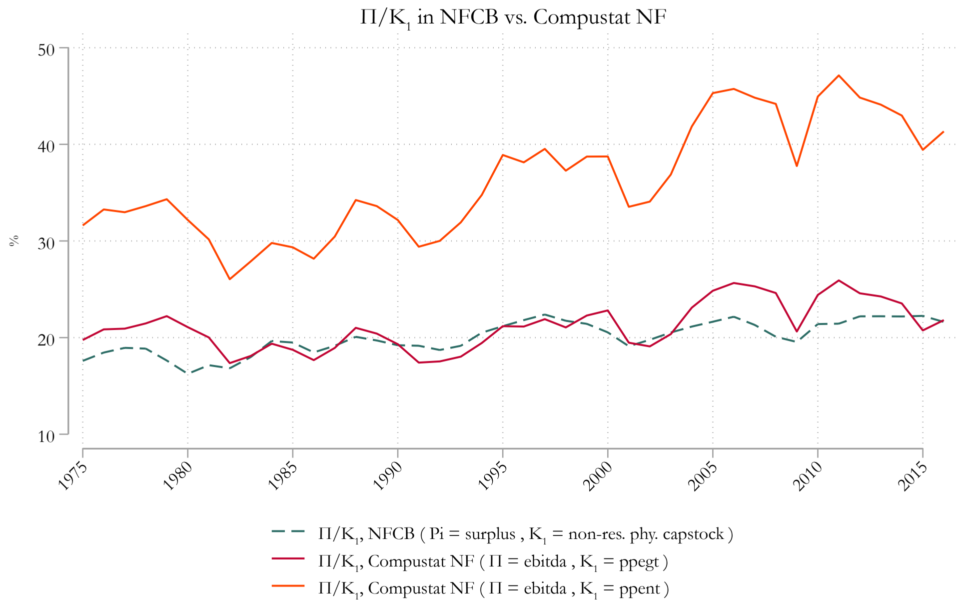


Figure 22. Measures of the dollar value of investment in the NFCB sector (top panel), and of the investment rate (bottom panel). Dashed lines are measures obtained using BEA data, while solid lines are measures obtained using Compustat data. Differences between the series are discussed in Appendix IA.C.5.

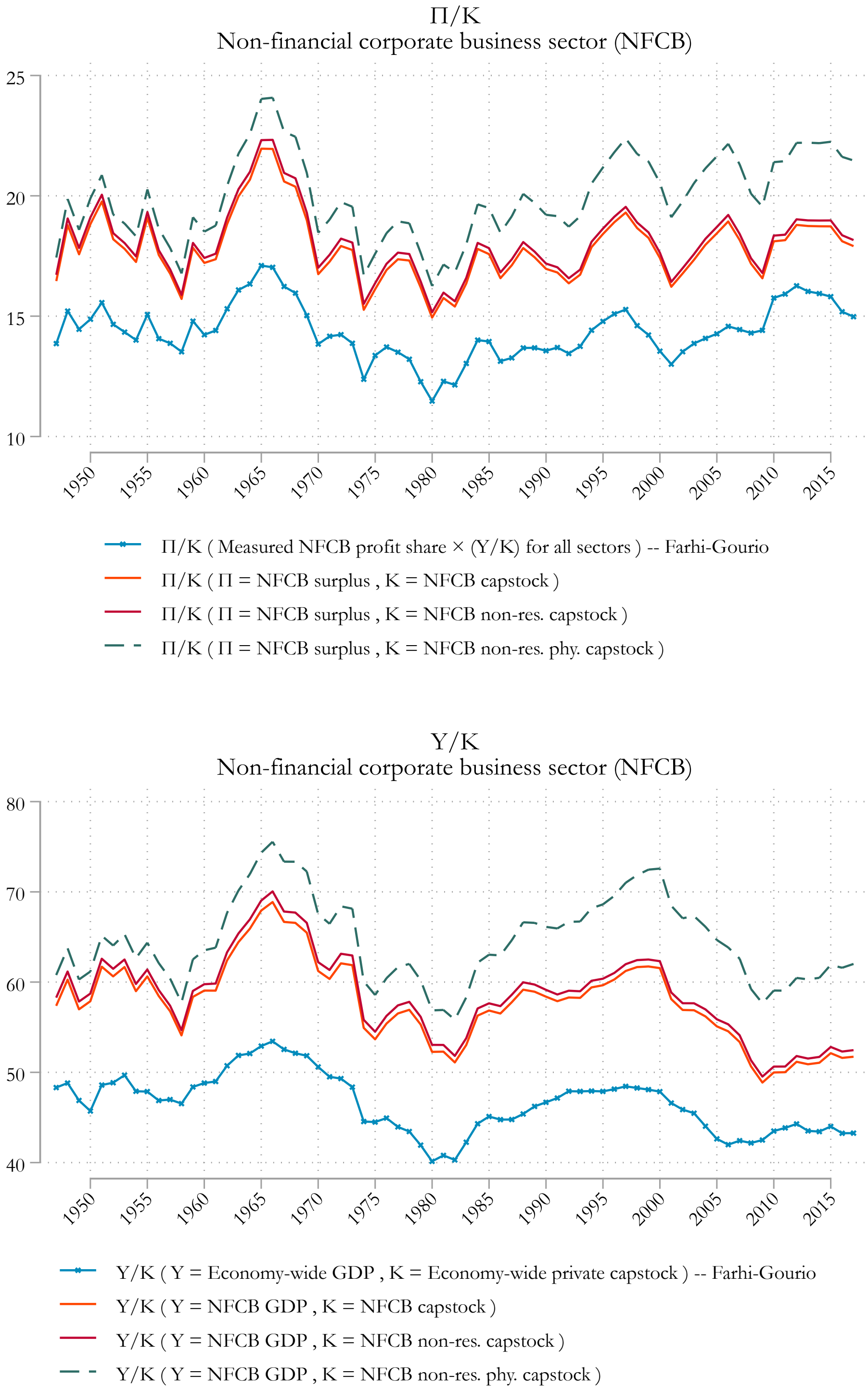


Figure 23. Comparison between alternative measures of $\Pi_t/(K_{1,t}+K_{2,t})$ (surplus per unit of total capital; top panel) and $Y_t/(K_{1,t}+K_{2,t})$ (value added per unit of capital; bottom panel) in BEA data. The construction of each time series is discussed in Appendix IA.C.1. The blue line reproduces the measures of Π/K and Y/K used in Farhi and Gourio (2018), which differ from our measures primarily because we focus only on the NFCB sector instead of the whole economy. See Appendix IA.C.1 for further details on data sources.

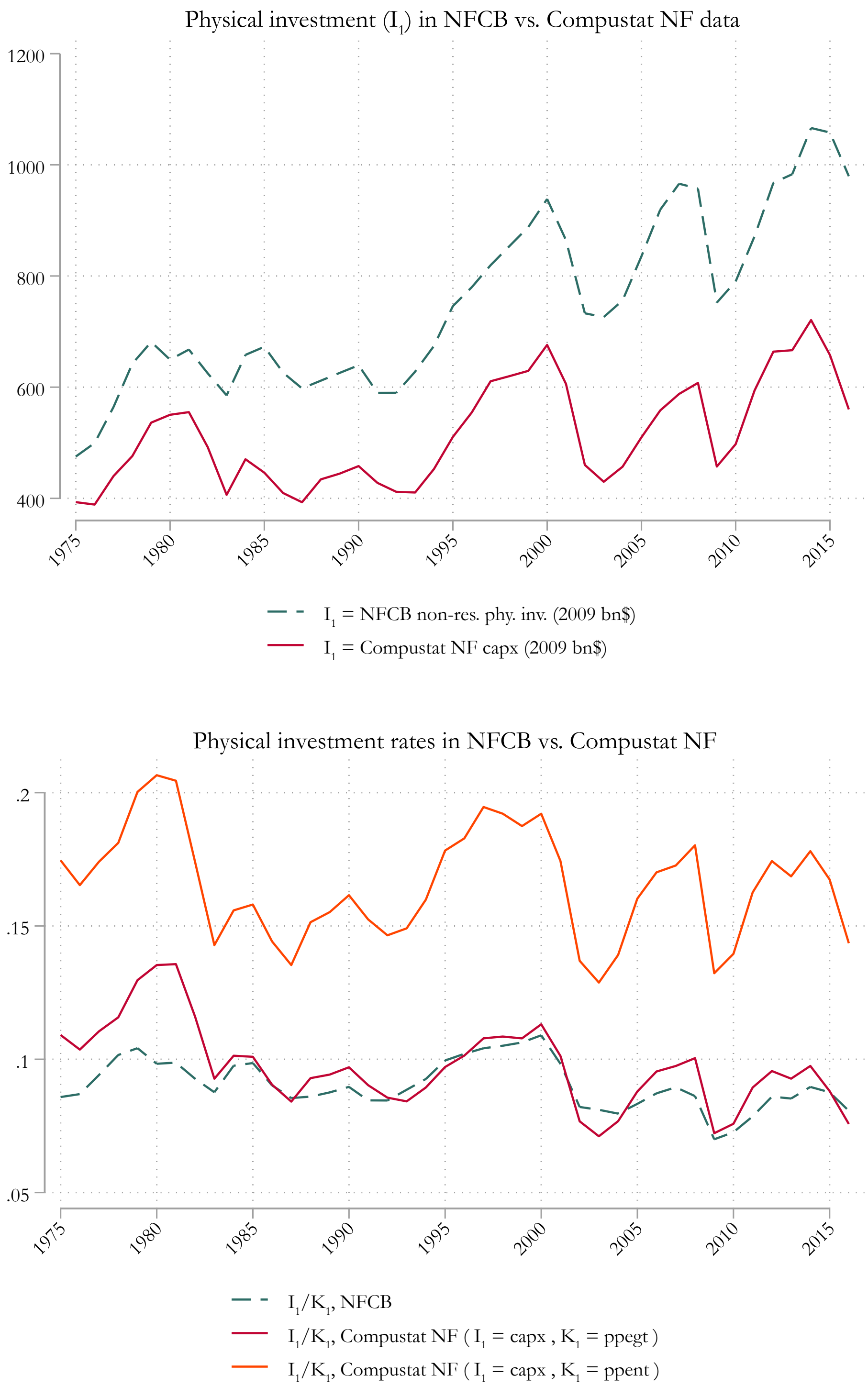


Figure 24. Measures of the total physical capital stock at current cost (K_1), of surplus (Π), and of the ratio of surplus to capital (Π/K_1) in BEA and Compustat data. All nominal data are deflated using the CPI with base 2009. Differences between the series are discussed in Appendix IA.C.5.

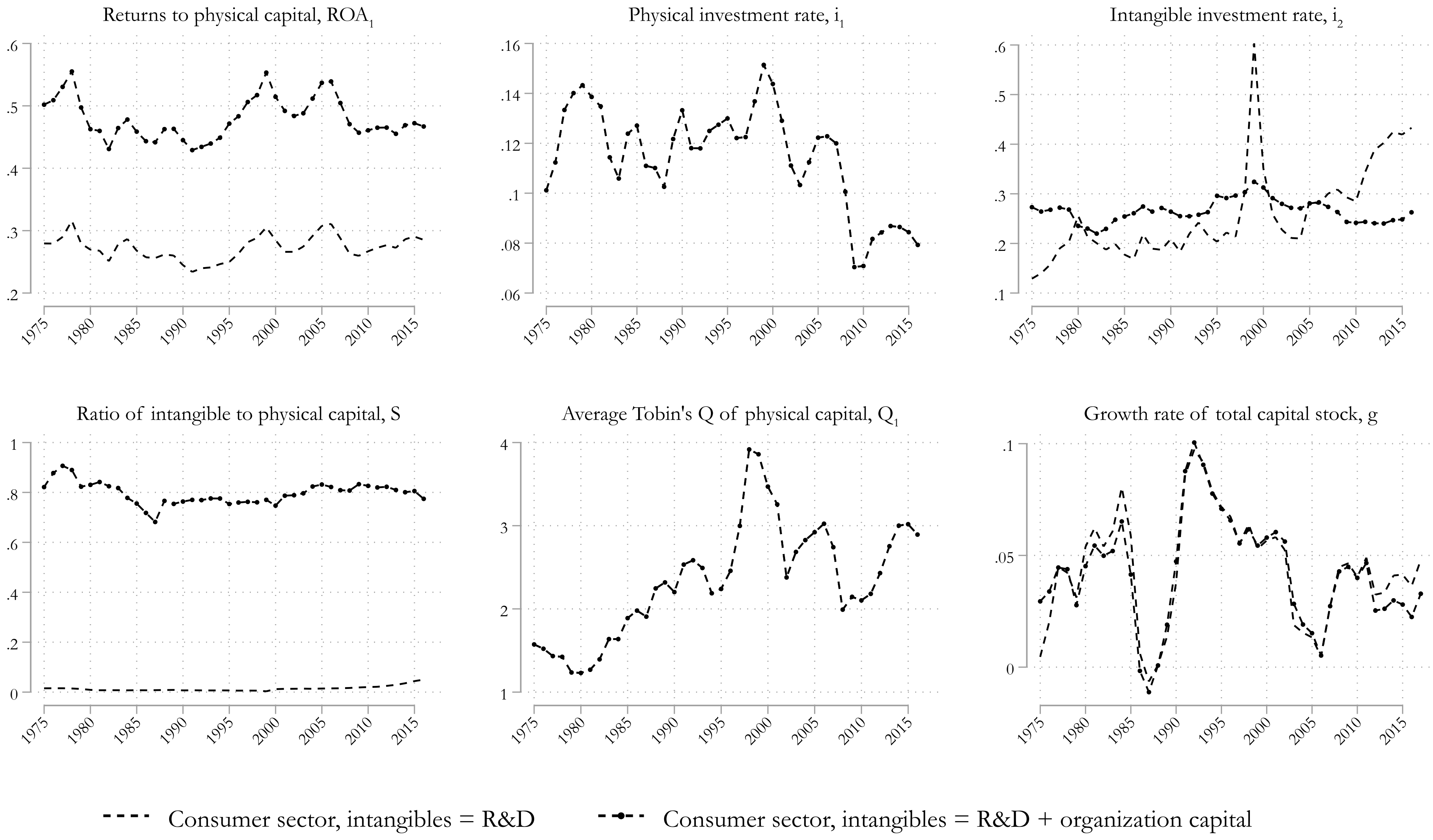


Figure 25. Time series moments for the Consumer sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section III. The sectoral classification is described in Appendix Tables I and II.

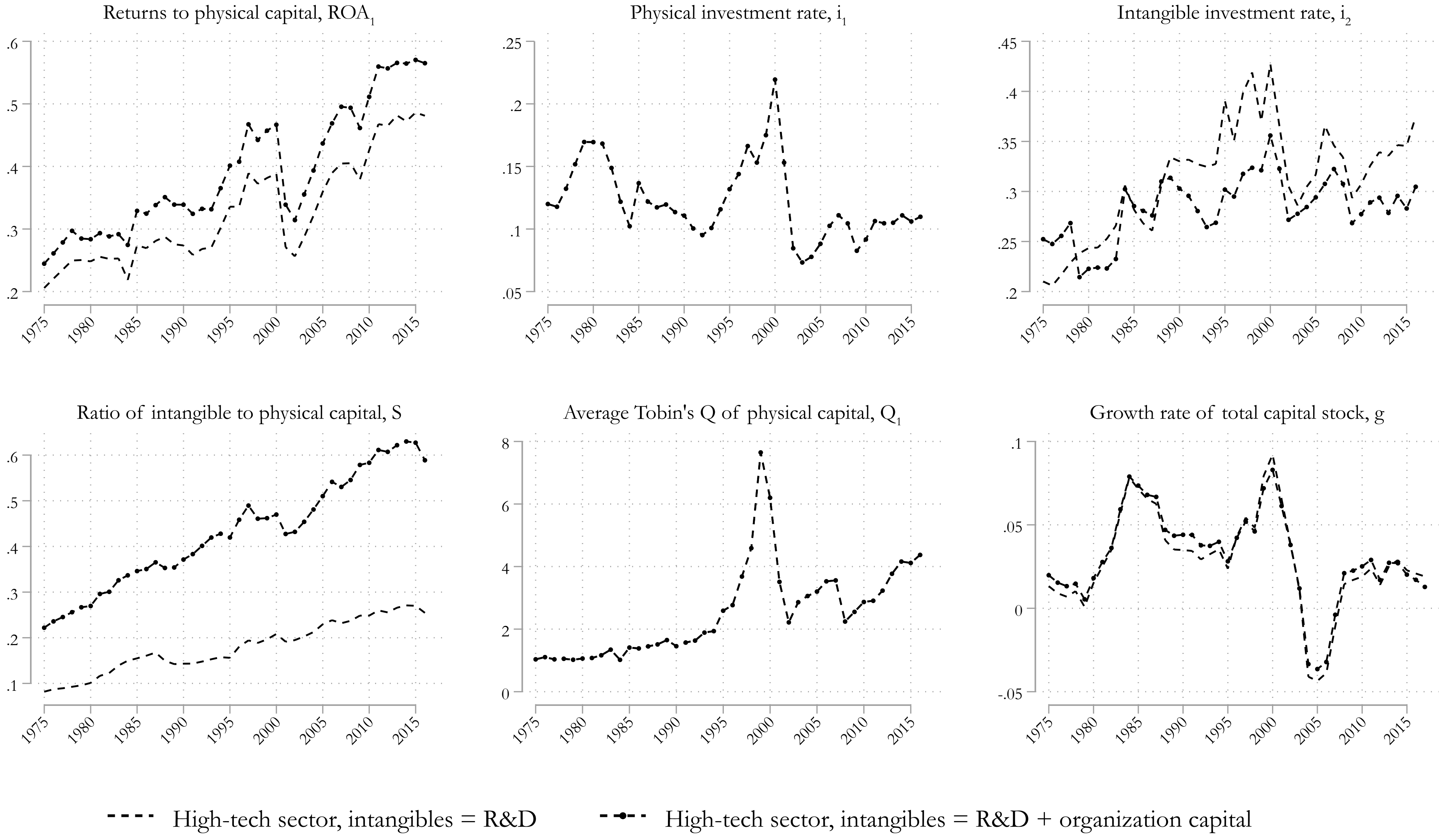


Figure 26. Time series moments for the High-Tech sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section III. The sectoral classification is described in Appendix Tables I and II.

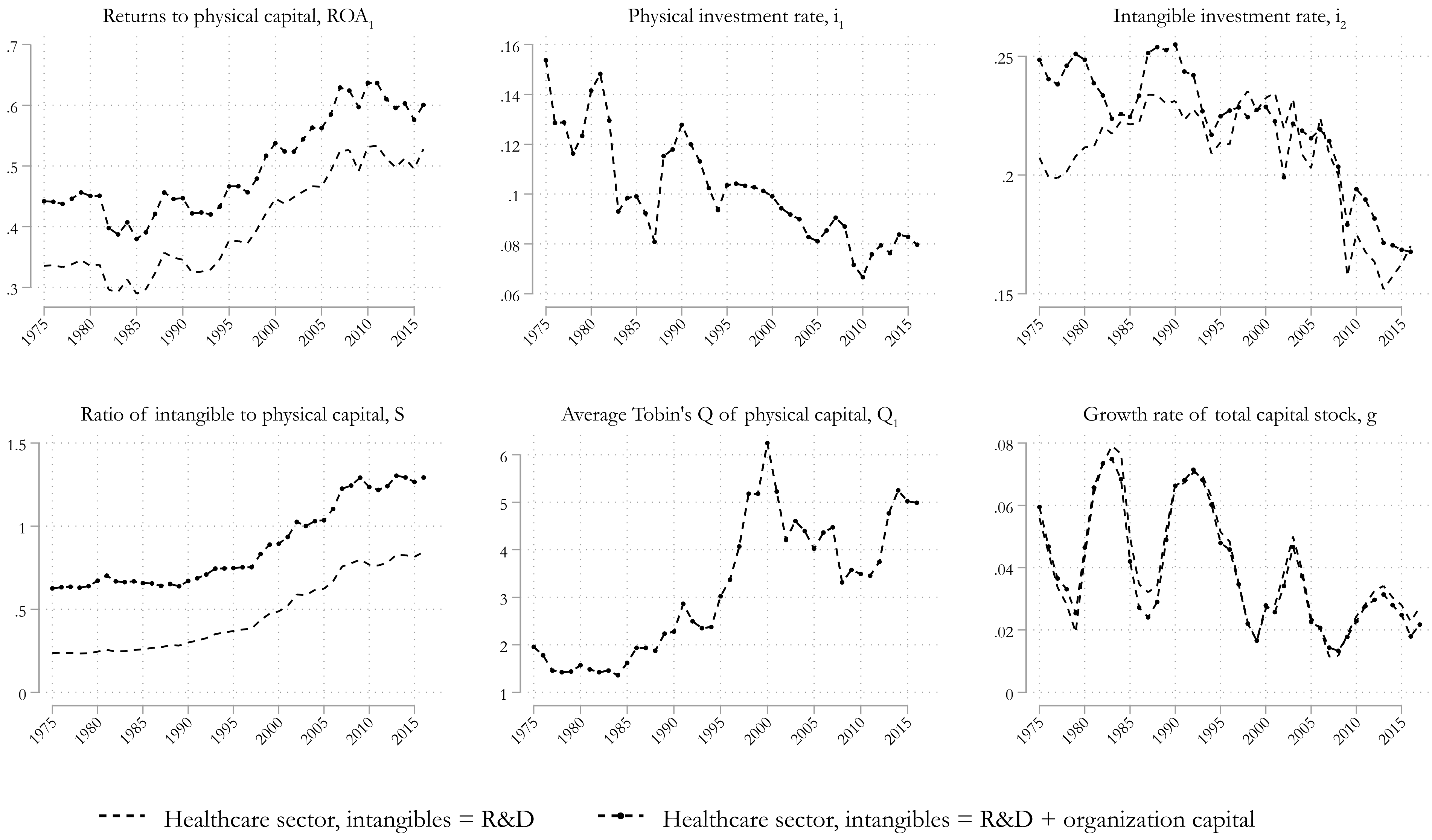


Figure 27. Time series moments for the Healthcare sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section III. The sectoral classification is described in Appendix Tables I and II.

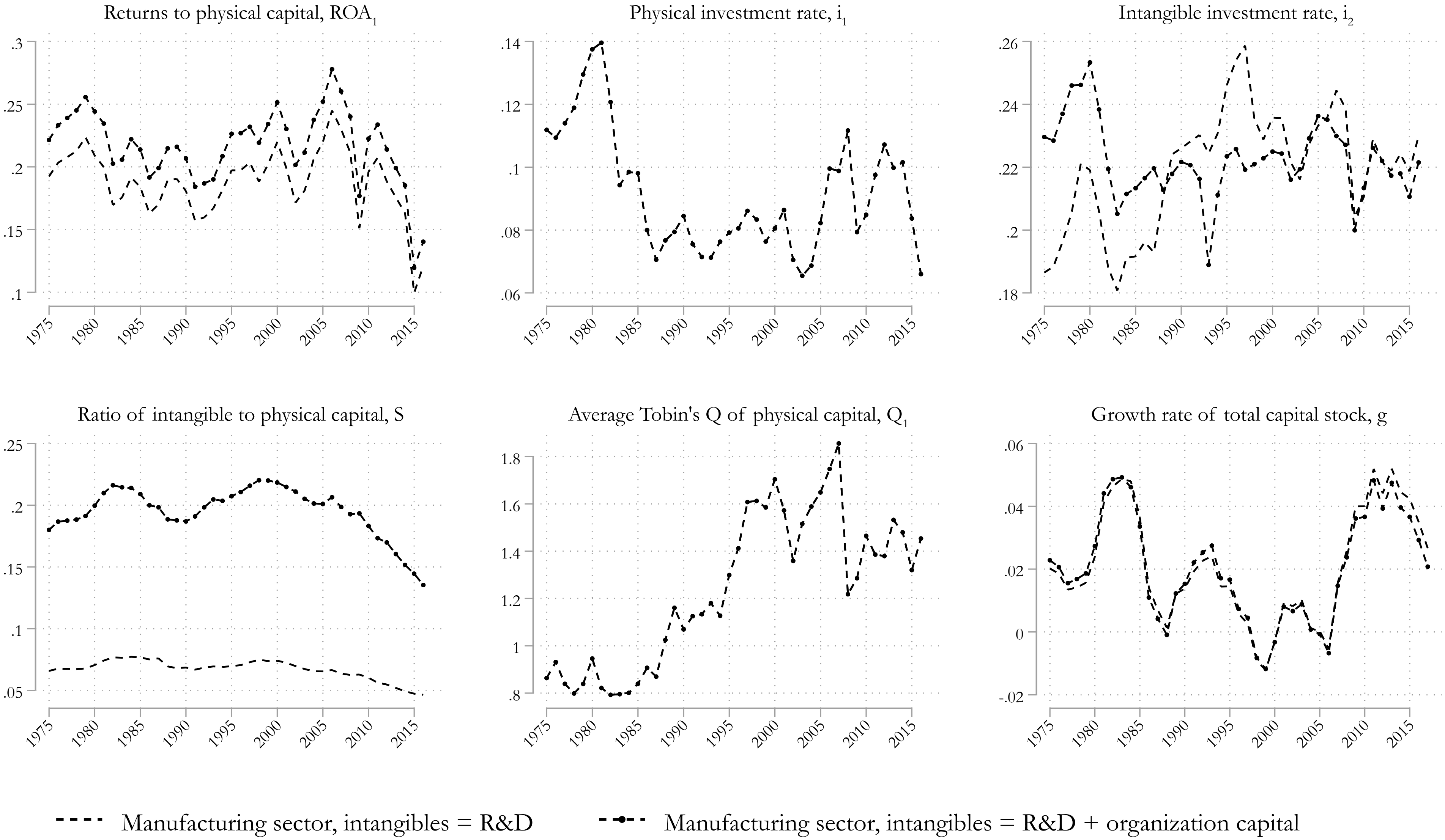


Figure 28. Time series moments for the Manufacturing sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section III. The sectoral classification is described in Appendix Tables I and II.

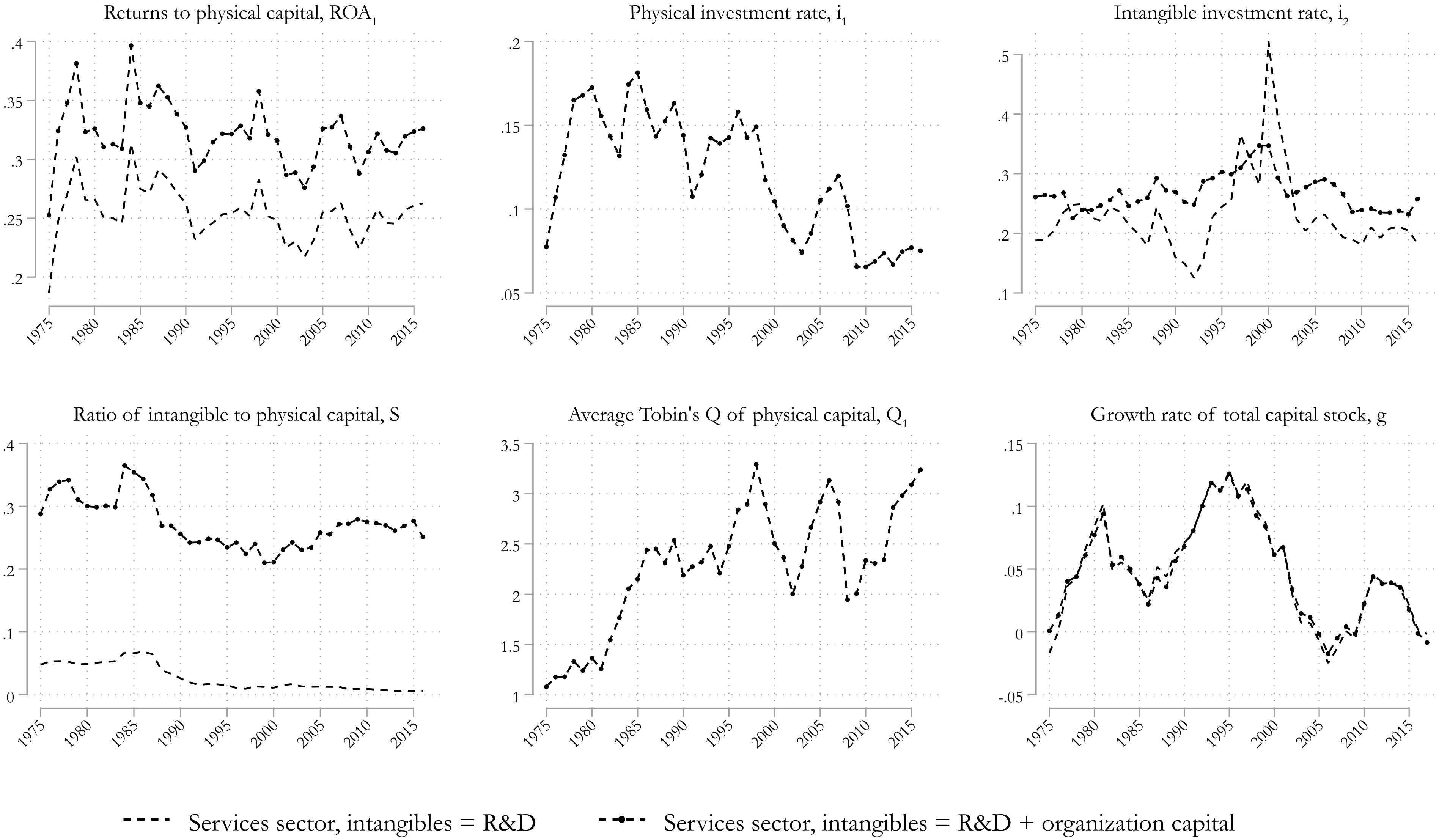
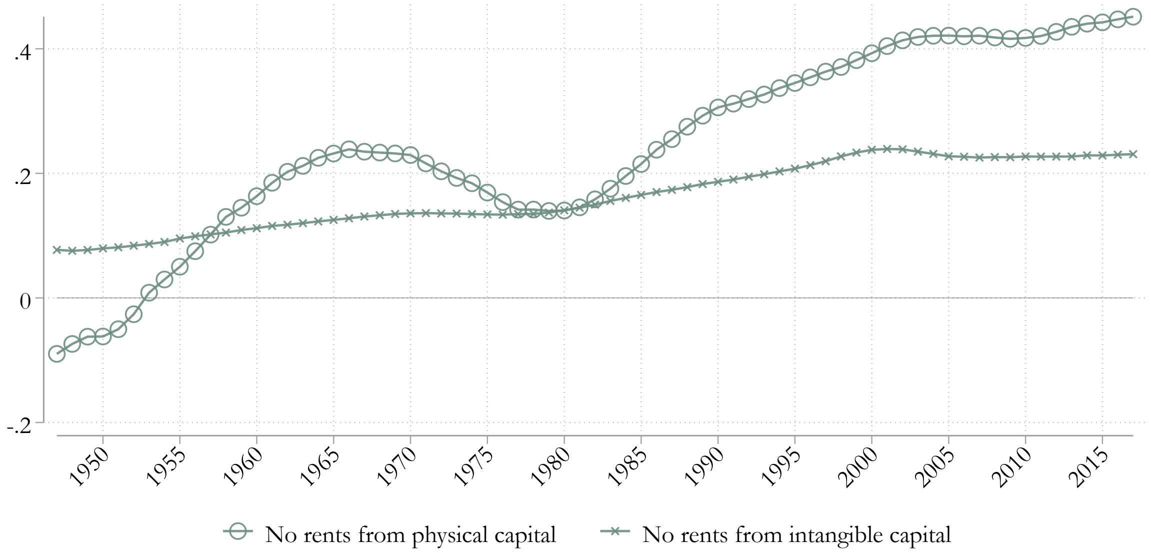


Figure 29. Time series moments for the Services sector, in the Compustat nonfinancials (NF) sample. All variables are defined as in Figure 1. Data sources for Compustat NF are described in Section III. The sectoral classification is described in Appendix Tables I and II.

Panel A. Share of intermediate goods produced using intangibles in total revenue



Panel B. Implied rents parameters

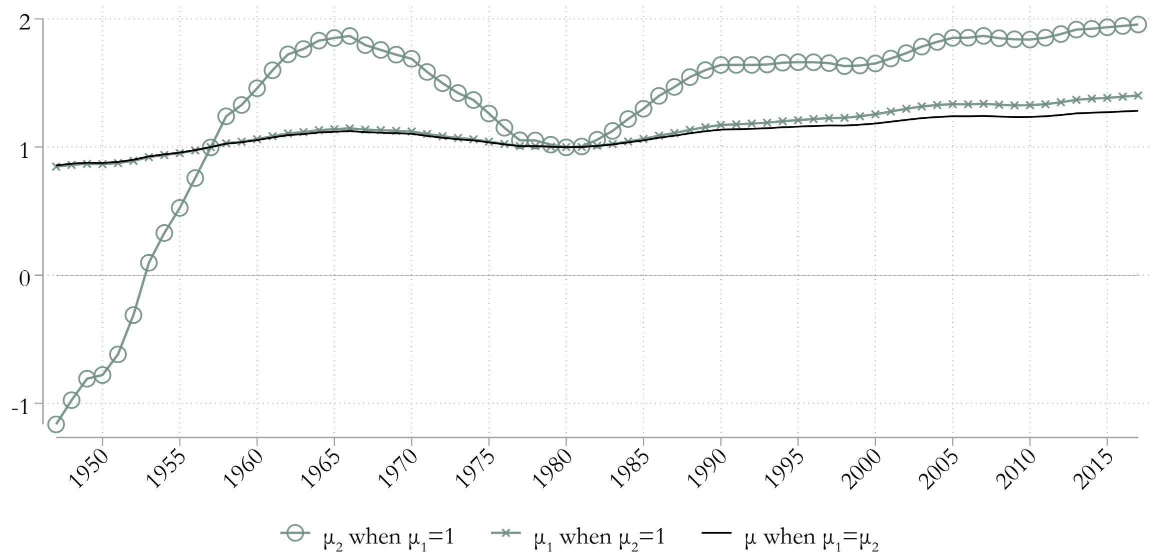
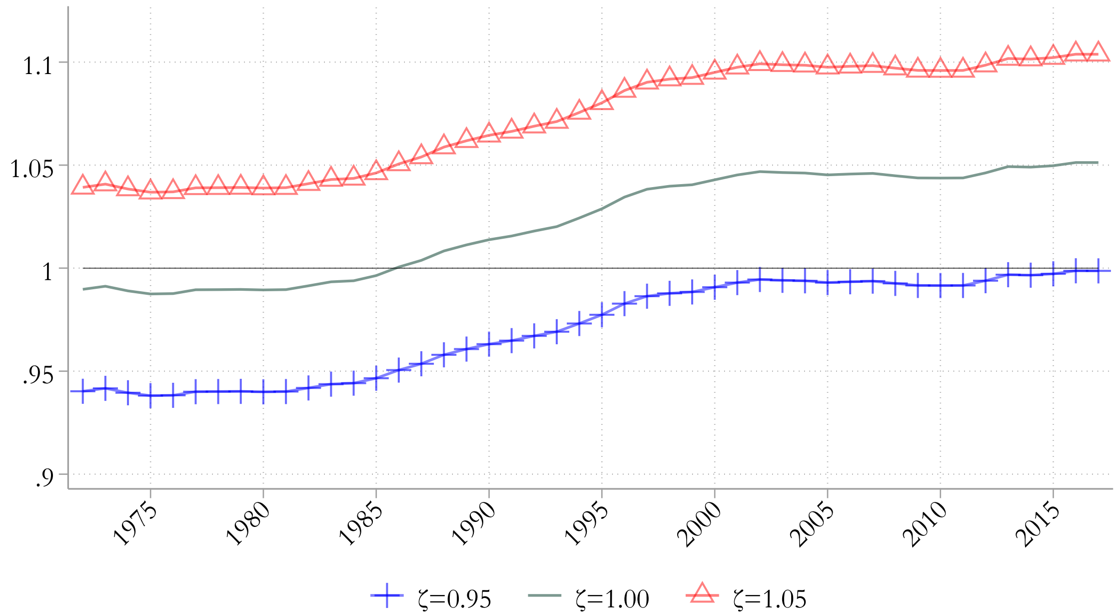


Figure 30. Share of intermediate goods produced using intangibles. The revenue function of the firm is assumed to be given by $\Pi_t = \left(A_{1,t}^{1-1/\mu_1} K_{1,t}^{1/\mu_1}\right)^{1-\eta} \left(A_{2,t}^{1-1/\mu_2} K_{2,t}^{1/\mu_2}\right)^{\eta}$, where μ_1 indexes rents generated by intangibles and μ_2 indexes rents generated by physical capital. The top panel reports the value of η when $\mu_1 = 1$ (circled line) and when $\mu_2 = 1$ (crossed line). The bottom panel reports the implied value of μ_2 when all rents are attributable to intangibles (circled line), the implied value of μ_1 when all rents are attributable to physical capital (crossed line), and user-cost weighted average rents parameter μ, which is identical in either case, and is also equal to the rents parameter μ obtained in our baseline analysis. Details of the decomposition of the investment gap, and of the derivation of η, are reported in Appendix IA.D.8.

Panel A. Markup over sales μ_S (intangibles = R&D)



Panel B. Markup over sales μ_S (intangibles = R&D + org. cap.)

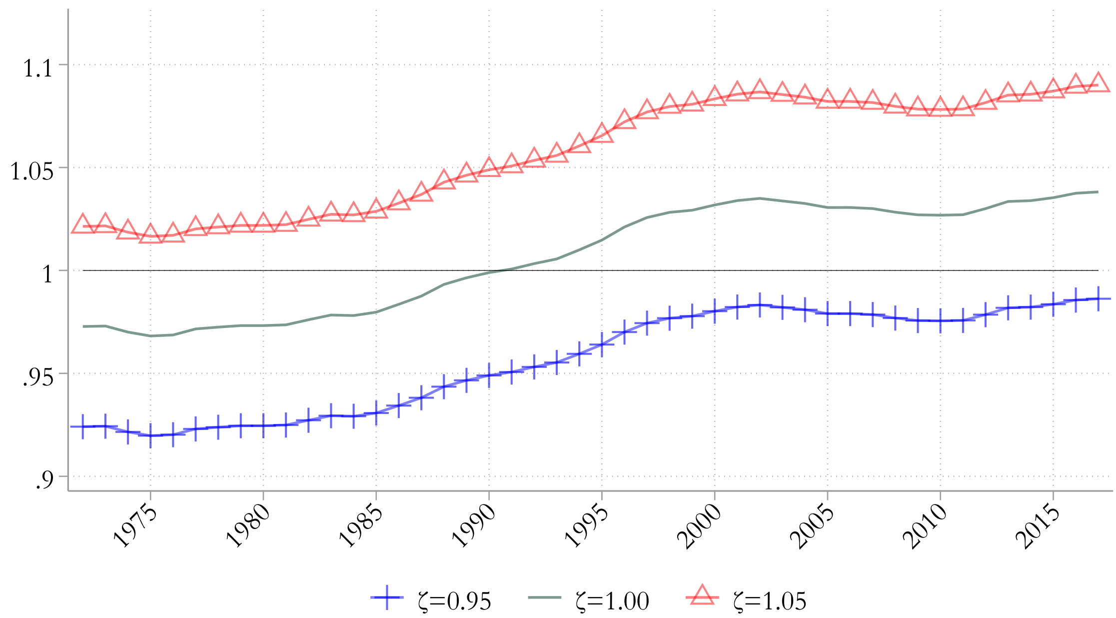


Figure 31. Markup over sales μ_S implied by different degrees of returns to scale ζ, in the Compustat non-financial (NF) sample. The top panel reports results when intangibles are defined are R&D only, and the bottom panel reports results when intangibles are defined as R&D plus organization capital. In each panel, markups over sales are computed as $\mu_S = (\nu_K + (1 - \nu_K)(1 - s_\Pi)))^{-1}\zeta$, where $\nu_K = (R_{1,t}K_{1,t} + R_{2,t}K_{2,t})/\Pi_t$ is the ratio of capital payments to operating surplus, and $s_\Pi = \Pi_t/S_t$ is the ratio of operating surplus to sales. The ratio of capital payments to operating surplus is obtained from solving the balanced growth model; each point corresponds to a value estimated over a different 7-year centered window; in the computation of μ_S, s_Π is also averaged over the same windows. See Section I in the main text for more details.

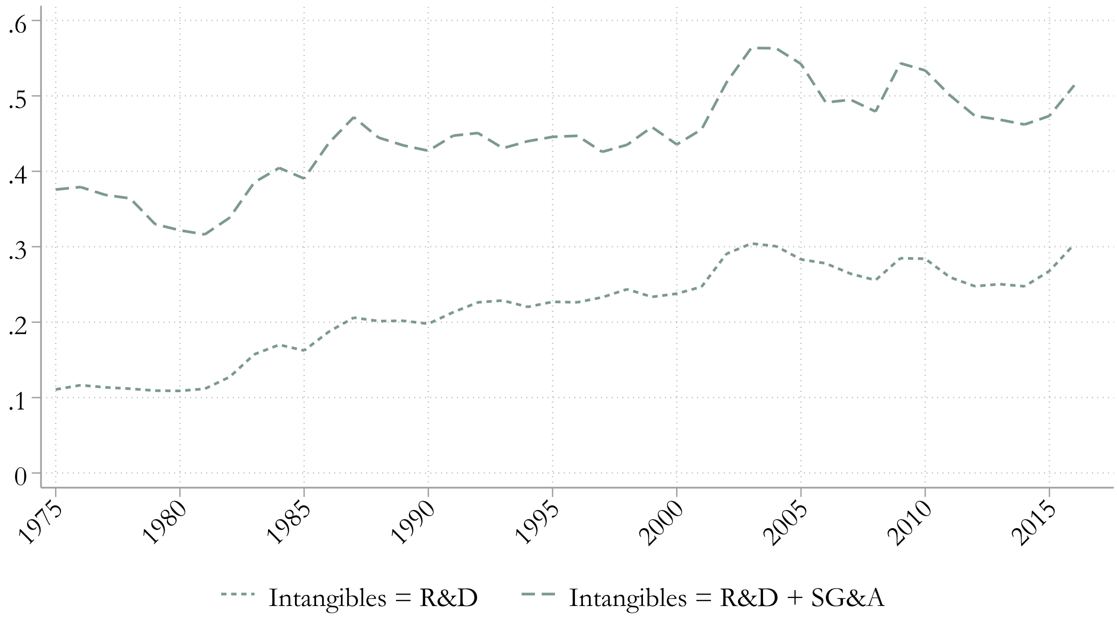


Figure 32. Composition bias with equity issuance frictions. The graph reports the time series for $\iota_2 S/(\iota_1 + \iota_2 S)$, when intangibles are measured either as R&D, or as the sum of R&D and organization capital. This ratio provides the lower bound above which omitting equity issuance frictions would bias upward the contribution of intangibles to total rents. See Appendix IA.D.9.1 for more details.

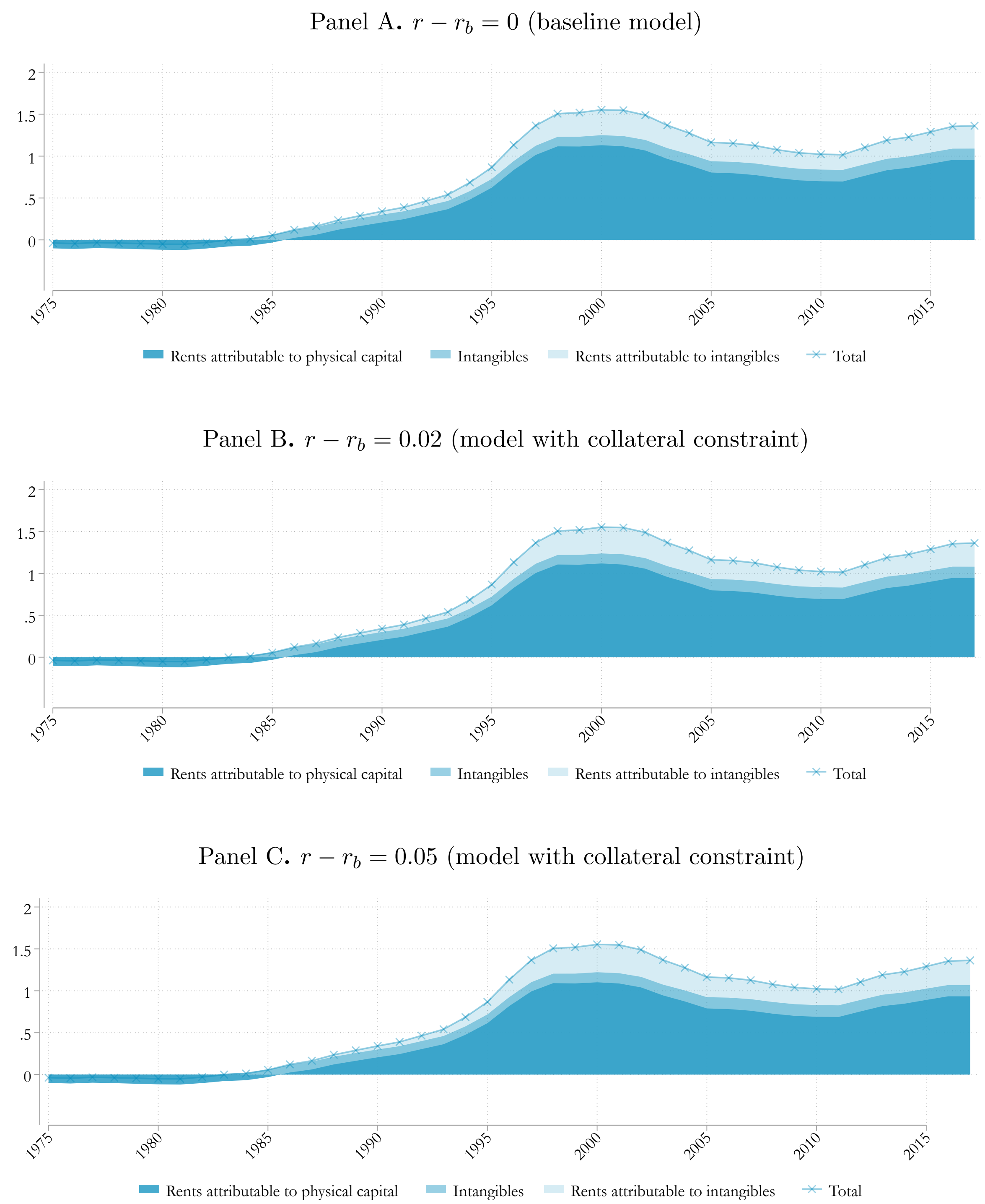


Figure 33. The physical investment gap $Q_1 - q_1$ with and without debt collateral constraints. The top panel reports the investment gap in our baseline model, where we assume no frictions in debt issuance. The middle and bottom panels report the physical investment gap in the model with a collateral constraint limiting debt issuance, under different assumptions about the wedge between shareholders' and debtholders' discount factor. In all three figures, we use the model with no adjustment costs, and R&D as a measure of intangibles.

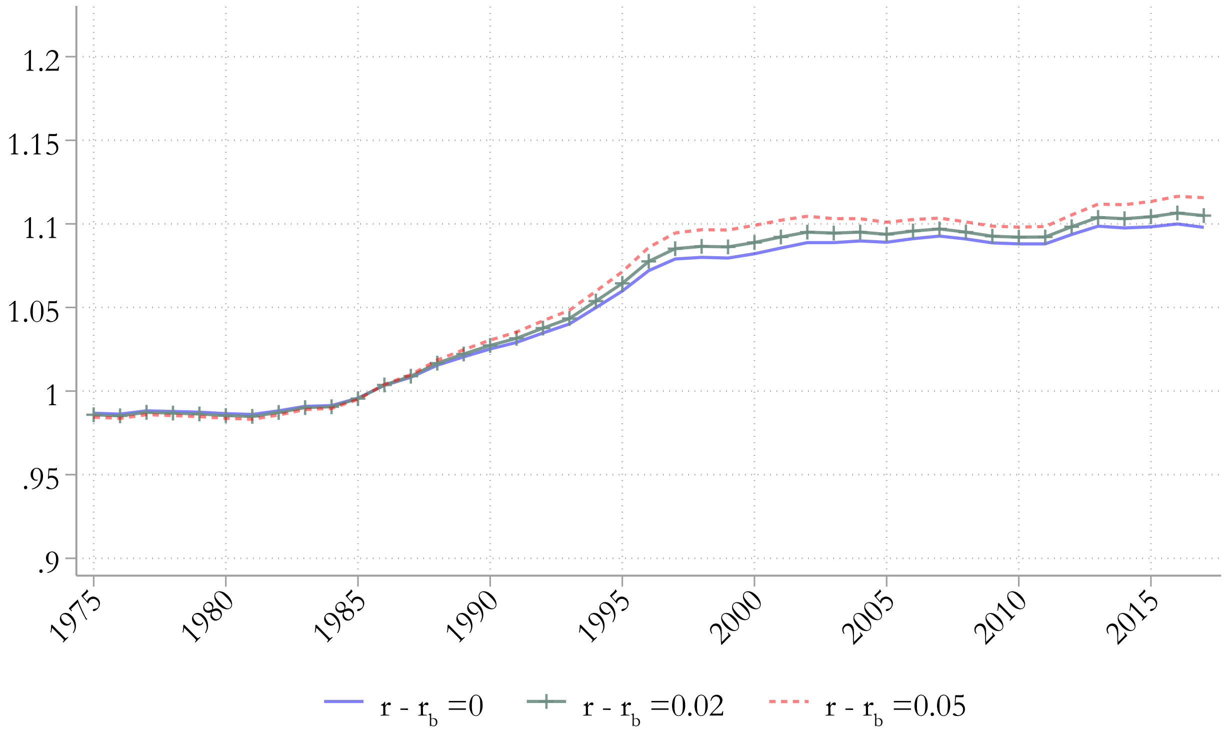


Figure 34. Implied markup over value added $\tilde{\mu}$ with and without debt collateral constraints. The solid line reports the implied markup in our baseline model, where we assume no frictions in debt issuance. The crossed and dashed line report the markup in the model with a collateral constraint limiting debt issuance, under different assumptions about the wedge between shareholders' and debtholders' discount factor. In all three lines, we use the model with no adjustment costs, and R&D as a measure of intangibles.

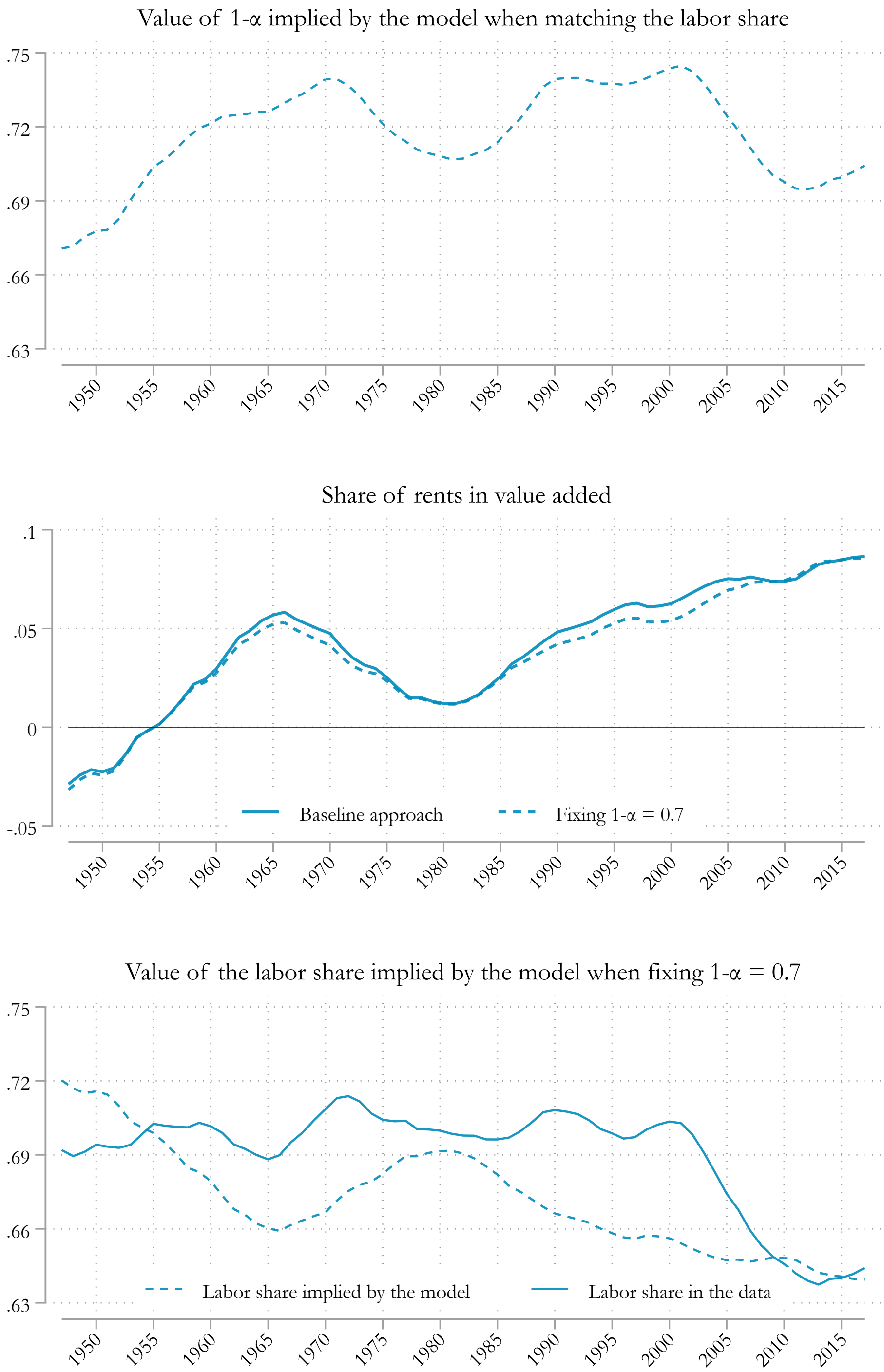


Figure 35. Implications for the labor share. The top panel reports the value of the Cobb-Douglas exponent on labor, $1-\alpha$, obtained when using the model described in Appendix IA.B.3, and assuming that all intermediate inputs are labor (or equivalently, that the production function is a value-added production funciton), and matching the labor share, as we do in our baseline analysis when translating estimates of μ into rents as a share of value added, s. The middle panel reports estimates of s in our baseline approach, and in an approach where we instead fix $1-\alpha = 0.7$ (and use no information on the labor share). Finally, the bottom panel reports the labor share obtained when we fix $1-\alpha = 0.7$, and the actual labor share in the data.

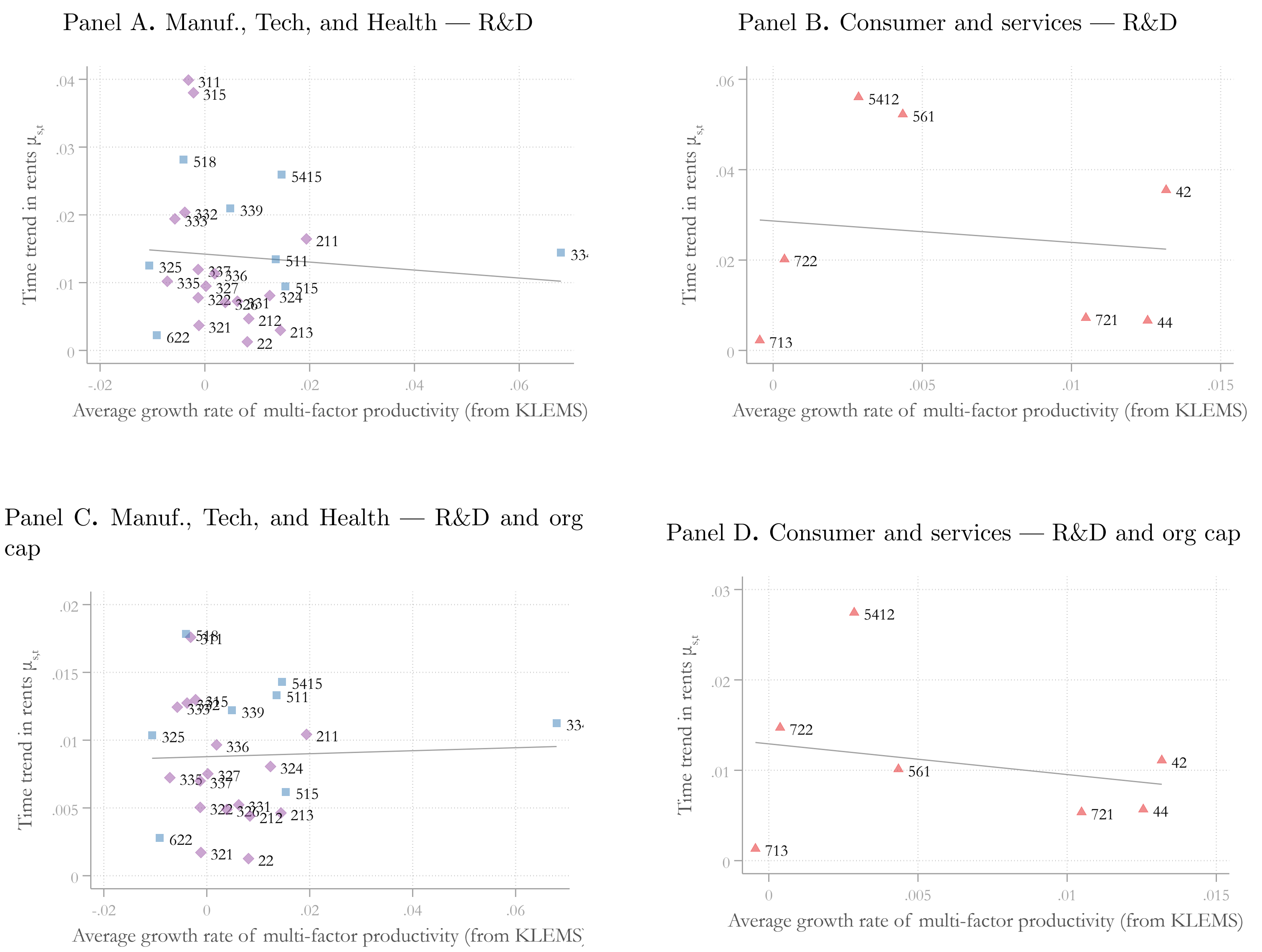


Figure 36. The relationship between rising rents and productivity growth across subsectors. Each panel reports a scatterplot of the coefficients $(\gamma_{\mu,s}, g_{Z,s})$, where s is a sector, the coefficients $\gamma_{\mu,s}$ are the estimated time trends of the rents parameters $\mu_{s,t}$ and the Cobb-Douglas intangible intensity $\eta_{s,t}$, i.e. $\mu_{s,t} = \alpha_{\mu,s} + \gamma_{\mu,s}t + \epsilon_{\mu,s,t}$, and $g_{Z,s}$ are the average growth rates of multi-factor productivity in the corresponding sector. The top left panel reports these coefficients for the Manufacturing, Healthcare, and High-tech sectors when intangibles are measured using R&D capital (the slope of the simple OLS line is -0.06, with a robust t-statistic of -0.64); the bottom left panel reports these when intangibles are measured using R&D capital plus organization capital (the slope of the simple OLS line is 0.01, with a robust t-statistic of 0.28). The top and bottom right panels are similarly constructed, but subsectors belonging to the Consumer and Services subsectors; in the top panel, the slope of the OLS line is -0.47, with a robust t-statistic of -0.31; in the bottom panel, the slope is -0.34, with a robust t-statistic of -0.58.