Online appendix to "What do inventories tell us about news-driven business cycles?"

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Abstract

This online appendix reports results on the baseline stock-elastic model with price adjustment costs, establishes key results discussed in section 5 of the main text regarding the stockout avoidance demand model, and reports the details of the stock-elastic demand model used in the Bayesian estimation exercise of section 7.

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A The stock-elastic demand model with price adjustment costs

A.1 Model

We add price adjustment costs to the baseline stock-elastic demand model following the approach of Rotemberg (1982). There are three modifications to the model:

1. The period nominal profit of the firm is now given by:

$$\pi_t(j) = p_t(j)s_t(j) - W_t n_t(j) - R_t k_t(j) - \frac{\phi}{2}\left(\frac{p_t(j)}{p_{t-1}(j)\pi} - 1\right)^2 P_t x_t.$$

   Price adjustment costs are proportional to the square deviation of firm $j's$ price from steady-state inflation π. $\phi > 0$ determines the magnitude of price adjustment costs.

2. Risk-free one-period nominal bonds have a return i_t, which must satisfy the no-arbitrage condition:

$$i_t \mathbb{E}_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\right] = 1. \tag{1}$$

   where $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the (gross) inflation rate.

3. The monetary authority sets short-term interest rates according to the following simple Taylor rule:

$$\frac{i_t}{i} = \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}, \tag{2}$$

   where π denotes steady-state inflation, i denotes the steady-state level of the nominal interest rate, and $\phi_\pi > 1$ captures the stance of monetary policy.

The resulting competitive equilibrium is still symmetric in prices, so that $p_t(j) = P_t$ and firm-level and aggregate inflation coincide.

The set of conditions characterizing the competitive equilibrium of the model are similar to those reported in the Appendix of the main text. Equations (30)-(43) from the main text are unchanged. Equations (44) and (45) from the main text are replaced, respectively, by:

$$\phi\left[\left(\frac{\pi_t}{\pi}-1\right)\left(\frac{\pi_t}{\pi}\right)x_t-\mathbb{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}x_{t+1}\right]\right]=\theta s_t\left(\frac{1}{\mu_t}-\frac{\theta-1}{\theta}\right) \tag{3}$$

$$x_t\left[1+\frac{\phi}{2}\left(\frac{\pi_t}{\pi}-1\right)^2\right]=s_t \tag{4}$$

The price-setting decision, equation (3), now reflects a trade-off between gains from monopolistic price-setting, and dynamic costs of adjusting prices. The budget constraint, (4), reflects the fact that nominal profits are lower because of price adjustment costs. Finally, the set of equilibrium conditions now additionally contains equations (1) and (2).

A.2 Impulse responses

We solve the model up to a first-order approximation around a steady-state in which inflation is 2% at an annual rate ($\pi = (1.02)^{1/4}$). The coefficient of the Taylor rule is set to $\phi_\pi = 1.5$. The real allocation in the steady-state of this model is identical to that of the model without price adjustment costs. We choose ϕ, the parameter capturing the magnitude of price adjustment costs, so that slope of the New Keynesian Philips Curve implied by the model is equal to that obtaining in a standard New Keynesian model with a Calvo probability of adjustment of χ per quarter. The mapping between the two parameters is: $\phi = \frac{(\theta-1)(1-\chi)}{\chi(1-(1-\chi)\beta)}$. We report impulse responses for two values of χ, $\chi = 0.1$ (implying an average duration of prices of 10 qarters and corresponding to high price adjustment costs) and $\chi = 0.5$ (implying an average price duration of 2 quarters and corresponding to low adjustment costs). These parameter values are at the extremes of the range of estimates of quarterly price adjustment frequencies in the empirical literature on price adjustment at the microeconomic level.

Figure 1 reports the impulse responses to a 4-quarter ahead innovation to TFP in three models: flexible prices (the baseline model), low price adjustment costs and high price adjustment costs. Inventories and sales display persistent negative comovement, amplified by the negative response of markups in the models with price adjustment costs. Figure 2 reports the impulse responses of these models to a surprise TFP innovation. Inventories and sales comove positively on impact and at least 6 quarters ahead, and inventories only decline (relative to steady-state) after 7 quarters and when price rigidity is sufficiently high. Thus, whether for high or low degree of price rigidity,

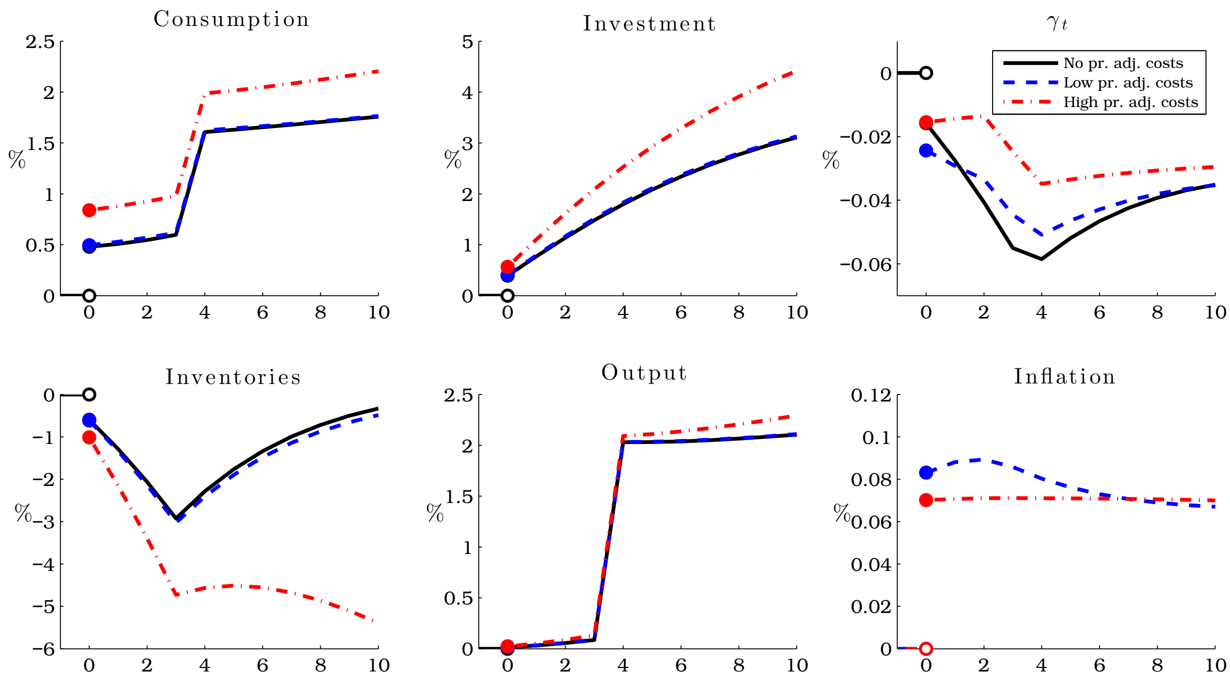

Figure 1: Impulse responses to news shocks in the stock-elastic demand model, with and without price adjustment costs. The exogenous shock is a 4-quarter ahead increase in TFP, identical to figure (2) in the main text. Solid black line: baseline model (no price adjustment costs); dashed blue line: low price adjustment costs (equivalent to 50 % Calvo probability of adjustment per quarter); dashed blue line: high price adjustment costs (equivalent to 10 % Calvo probability of adjustment per quarter). The time unit is a quarter. Impulse responses are reported in terms of percent deviation from steady-state values.

in a variant of the baseline model with price adjustment costs, negative comovement of inventories and sales on impact characterizes news shocks relative to surprise shocks.

B News shocks in the stockout-avoidance inventory model

In this appendix, we describe a Real Business Cycle version of the stockout-avoidance models of Kahn (1987) and Kryvtsov and Midrigan (2013), and analyze its impact response to news shocks.

B.1 Model description

The economy consists of a representative household and monopolistically competitive firms, where again firms produce storable goods. We start with the household problem. Since many aspects of the model are similar to the stock-elastic model, we will frequently refer to equations in

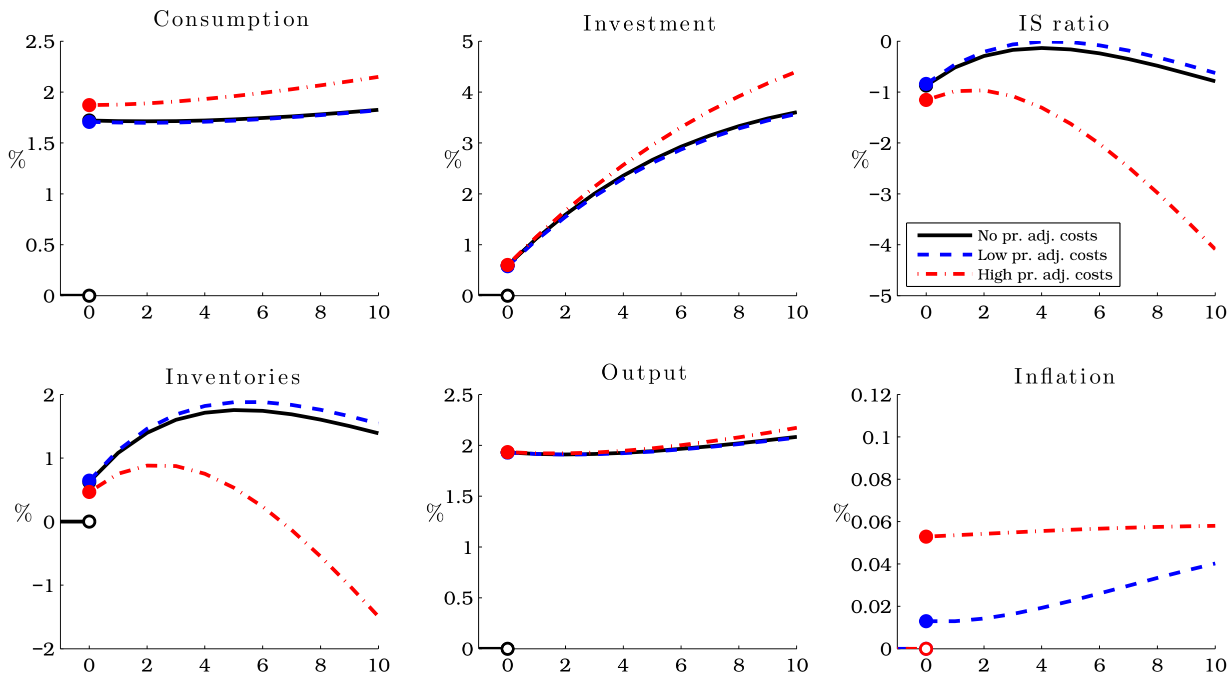

Figure 2: Impulse responses to surprise shocks in the stock-elastic demand model, with and without price adjustment costs. The exogenous shock is a contemporaneous increase in TFP, identical to figure (5) in the main text. Solid black line: baseline model (no price adjustment costs); dashed blue line: low price adjustment costs (equivalent to 50 % Calvo probability of adjustment per quarter); dashed blue line: high price adjustment costs (equivalent to 10 % Calvo probability of adjustment per quarter). The time unit is a quarter. Impulse responses are reported in terms of percent deviation from steady-state values.

the main text. We use the reference (M.xx) for equation (xx) in the main text.

Household problem A representative household maximizes (M.1), subject to the household budget constraint (M.2), capital accumulation rule (M.3), and the resource constraint (M.4). The aggregation of goods $\{s_t(j)\}_{j\in[0,1]}$ into x_t is given by (M.5), where $v_t(j)$ is the taste-shifter for product j in period t.

In stockout-avoidance models, in contrast to the stock-elastic demand models, this taste-shifter is assumed to be exogenous. In particular, we assume it is identically distributed across firms and over time according to a cumulative distribution function $F(\cdot)$ with a support $\Omega(\cdot)$:

$$v_t(j) \sim F, \qquad v_t(j) \in \Omega. \tag{5}$$

For each product j, households cannot buy more than the goods on-shelf $a_t(j)$, which is chosen by firms:

$$s_t(j) \leq a_t(j), \qquad \forall j \in [0,1]. \tag{6}$$

Although (6) also holds for the stock-elastic model, it has not been mentioned since it was never binding. Households observe these shocks, and the amount of goods on shelf $a_t(j)$, before making their purchase decisions. Firms, however, do not observe the shock $v_t(j)$ when deciding upon the amount $a_t(j)$ of goods that are placed on shelf, so that (6) occasionally binds, resulting in a stockout.

Again, a demand function and a price aggregator can be obtained from the expenditure minimization problem of the household. The demand function for product j becomes

$$s_t(j) = \min\left\{ v_t(j) \left(\frac{p_t(j)}{P_t}\right)^{-\theta} x_t,\ a_t(j) \right\}, \tag{7}$$

which states that when $v_t(j)$ is high enough so that demand is higher than the amount of on-shelf goods, a stockout occurs and demand is truncated at $a_t(j)$. The price aggregator P_t is given by:

$$P_t = \left(\int_0^1 v_t(j)\tilde{p}_t(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}. \tag{8}$$

The variable $\tilde{p}_t(j)$ is the Lagrange multiplier on constraint (6). It reflects the household's shadow valuation of goods of variety j. For varieties that do not stock out, $\tilde{p}_t(j) = p_t(j)$, whereas for varieties that do stock out, $\tilde{p}_t(j) > p_t(j)$.

Firm problem Each monopolistically competitive firm $j \in [0,1]$ maximizes (1.7) with $\pi_t(j)$ defined as

$$\pi_t(j) = p_t(j)\tilde{s}_t(j) - W_t n_t(j) - R_t k_t(j). \tag{9}$$

As explained before, firms do not observe the exogenous taste-shifter $v_t(j)$ and hence their demand $s_t(j)$ when making their price and quantity decisions. Therefore, they will have to form expectations on sales $s_t(j)$, conditional on all variables except $\nu_t(j)$. This conditional expectation is denoted by $\tilde{s}_t(j)$.

The constraints on the firm are (M.9), (M.10), (M.11) and the demand function (7) with a known distribution for the taste-shifter $v_t(j)$ in (5). Notice that this distribution is identical across all firms and invariant to aggregate conditions. By the law of large numbers, firms observe P_t and x_t in their demand function. Therefore, $\tilde{s}_t(j)$ in (9) is given by:

$$\tilde{s}_t(j) = \int_{v \in \Omega(v)} \min \left\{ v \left(\frac{p_t(j)}{P_t} \right)^{-\theta} x_t,\ a_t(j) \right\} dF(v). \tag{10}$$

Market clearing The market clearing conditions for labor, capital, and bond markets are identical to the stock-elastic model and are given by (M.13), (M.14) and (M.15). Sales of goods also clear by the demand function for each variety.

B.2 Equilibrium

A market equilibrium of the stockout-avoidance model is defined as follows.

Definition 1 (Market equilibrium of the stockout-avoidance model) *A market equilibrium in the stockout-avoidance model is a set of stochastic processes:*

$$c_t, n_t, k_{t+1}, i_t, B_{t+1}, x_t, \{a_t(j)\}, \{v_t(j)\}, \{s_t(j)\}, \{\tilde{s}_t(j)\}, \{y_t(j)\}, \{inv_t(j)\}, \{p_t(j)\}, W_t, R_t, P_t, Q_{t,t+1}$$

such that, given the exogenous stochastic process z_t and initial conditions k_0, B_0, and $\{inv_{-1}(j)\}$:

- *households maximize (M.1) subject to (M.2) - (M.4), (5) - (6), and a no-Ponzi condition,*
- *each firm $j \in [0,1]$ maximizes (M.7) subject to (M.9) - (M.11), (9) - (10),*
- *markets clear according to (M.13) - (M.15).*

In what follows, we use the following notation for aggregate output, sales, and inventories:

$$y_t = \int_0^1 y_t(j)dj, \qquad s_t = \int_0^1 s_t(j)dj, \qquad inv_t = \int_0^1 inv_t(j)dj. \tag{11}$$

In stockout-avoidance models, a market equilibrium is not symmetric across firms. Indeed, because of the idiosyncratic taste shocks $\{\nu_t(j)\}$, realized sales $\{s_t(j)\}$, and therefore end of period inventories $\{inv_t(j)\}$ differ across firms. However, it can be shown that all firms make identical *ex-ante* choices. That is, firms' choice of price $p_t(j)$ and amount of on-shelf goods $a_t(j)$ depends only on aggregate variables, and not on the inventory inherited from the past period $inv_{t-1}(j)$. We therefore denote $p_t = p_t(j)$ and $a_t = a_t(j)$. The *ex-ante* symmetric choices of price and on-shelf goods imply that there is a unique threshold of the taste shock, common across firms, above which firms stock out. Using (7), this threshold is given by:

$$\nu_t^*(j) = \nu_t^* = \left(\frac{p_t}{P_t}\right)^\theta \frac{a_t}{x_t}.$$

B.3 The stockout wedge and firm-level markups

The fact that those firms with a taste shifter $\nu_t(j) \geq \nu_t^*$ run out of goods to sell implies that $p_t \neq P_t$. Indeed, as emphasized in (8), the aggregate price level P_t depends on the household's marginal value of good j, $\tilde{p}_t(j)$. This marginal value equals the (symmetric) sales price p_t for all varieties that do not stockout. However, for varieties that do stock out, firms would like to purchase more of the good than what is on sale. Therefore, the household's marginal value of the good is higher than their market price: $\tilde{p}_t(j) > p_t$. Thus, the standard aggregation relation $P_t = p_t$ fails to hold, and instead, $P_t > p_t$. In what follows, we denote:

$$d_t = \frac{p_t}{P_t}.$$

The relative price can be thought of as a stockout wedge. It is smaller when the household's valuation of the aggregate bundle of goods is large relative to the market price of varieties, that is, when stockouts are more likely. Formally, it can be shown that the wedge d_t is a strictly increasing function of ν_t^*, and therefore a decreasing function of the probability of stocking out, $1 - F(\nu_t^*)$.

Due to the stockout wedge, firm-level markup μ_t^F differs from the definition of aggregate markup μ_t defined in the main text. Indeed, since $\mu_t^F = \frac{p_t}{P_t}\mu_t$, so that:

$$\mu_t^F = d_t \mu_t. \tag{12}$$

B.4 An alternative log-linearized framework

There are two important differences between stock-out avoidance models and the stock-elastic demand model described in section 3 of the main text. The first difference is the occurence of stockouts, which implies the existence of the stockout wedge and hence the departure of firm-level and aggregate markups as described above. The second difference is that, even in our flexible-price environment, firm-level markups are not set at a constant rate over future marginal cost, as they did in the stock-elastic demand model. These two differences mean that unlike stock-elastic demand models, we cannot exactly map this class of models into the log-linearized framework of section 3. We need a more general framework, which we provide in the following lemma.

Lemma 2 (The log-linearized framework for the stockout-avoidance model) *In an equilibrium of the stockout-avoidance model, if productivity z_t is at its steady-state value, on impact, up to a first order approximation around the steady-state, equations (M.20) and (M.21) hold, along with:*

$$\hat{inv}_t = \hat{s}_t + \tau \hat{\mu}_t^F + \eta \hat{\gamma}_t, \tag{13}$$

$$\hat{\mu}_t^F = \hat{d}_t + \hat{\mu}_t, \tag{14}$$

$$\hat{d}_t = \epsilon_d \left(\hat{inv}_t - \hat{s}_t \right), \tag{15}$$

$$\hat{\mu}_t^F = \epsilon_\mu \left(\hat{inv}_t - \hat{s}_t \right). \tag{16}$$

In this approximation, the parameters ω and κ are given by (M.25) and (M.26), while the parameters $\eta > 0$, $\tau > 0$, $\epsilon_d > 0$, and ϵ_μ differ and are given in section B.8.1.

We discuss (13)-(16), which are new to this framework. First, the optimal choice of inventories (13) depends on the firm-level markup $\hat{\mu}_t^F$ that is not equal to the aggregate markup $\hat{\mu}_t$. The parameters expressed as τ and γ also have a different expression that will be discussed later.

Second, in equation (14), aggregate markups and firm-level markups are linked with the stockout wedge $\hat{d}_t$. This follows from the definition of firm-level markup and stockout wedge given in (12).

Third, note that the framework of lemma 2 now includes (15), an equation linking the stockout wedge to the aggregate IS ratio. As we argued previously, the stockout wedge is negatively related to the probability of stocking out. In turn, one can show that there is a strictly decreasing mapping between the stockout probability, or equivalently a strictly increasing mapping between ν_t^*, and the ratio of average end of period inventory to average sales:

$$IS_t = \frac{inv_t}{s_t} = \frac{\int_0^1 inv_t(j)dj}{\int_0^1 s_t(j)dj}.$$

A lower probability of stocking out (a higher ν_t^*) implies that firms will, on average, be left with a higher stock of inventories relative to the amount of goods sold. Combining these two mappings, we obtain that the stockout wedge is increasing in the aggregate IS ratio, so that $\epsilon_d > 0$.

Lastly, the framework of lemma 2 includes variable firm-level markups, as described in equation (16). This is because in stockout-avoidance models, the desired firm-level markup is not constant. Instead, it depends on the ratio of goods on-shelf to expected demand, which itself is linked to the probability of stocking out. One can show that for log-normal and pareto-distributed idiosyncratic demand shocks, μ_t^F is a strictly decreasing function of ν_t^*, and therefore an increasing function of the probability of stocking out. Thus, the elasticity ϵ_μ is typically negative. Intuitively, this is because when firms are likely to stock out, the price-elasticity of demand is lower, and therefore markups are higher. Indeed, with a high stockout probability, demand is mostly constrained by the amount of goods available for sale, and does not vary much with price changes. The converse intuition holds when the stockout probability is low.

Before moving on, note that this framework reduces to the framework of section 3 when the stockout wedge is absent and firm-level markups are constant, so that $\hat{d}_t = \hat{\mu}_t^F = \hat{\mu}_t = 0$. Hence the framework is a generalized version of the basic framework of section 3, nesting it as a particular case with $\epsilon_d = \epsilon_\mu = 0$.

B.5 The impact response to news shocks

We now turn to discussing the effects of a news shock using our new log-linearized framework. We again maintain the assumption that the shock has the effect of increasing sales, $\hat{s}_t > 0$, while leaving current productivity unchanged, $\hat{z}_t = 0$, so that we can indeed used the log-linearized framework of lemma 2. Combining the equations of lemma 2, it is straightforward to rewrite the

Value of $\tilde{\eta}$					
$\sigma_d \downarrow \|\|\mu \rightarrow$	**1.05**	**1.1**	**1.25**	**1.5**	**1.75**
0.1	-729.12	-278.08	-121.98	-77.39	-61.87
0.25	-307.22	-116.94	-51.42	-32.71	-26.20
0.5	-167.04	-63.17	-27.66	-17.57	-14.06
0.75	-120.68	-45.25	-19.59	-12.35	-9.85
1	-97.75	-36.33	-15.51	-9.66	-7.66

Implied IS ratio					
$\sigma_d \downarrow \|\|\mu \rightarrow$	**1.05**	**1.1**	**1.25**	**1.5**	**1.75**
0.1	0.05	0.09	0.15	0.18	0.21
0.25	0.12	0.23	0.39	0.50	0.57
0.5	0.23	0.47	0.83	1.13	1.32
0.75	0.32	0.69	1.31	1.88	2.26
1	0.41	0.90	1.81	2.73	3.36

Table 1: Value of $\tilde{\eta}$ when idiosyncratic demand shocks follow a log-normal distribution with mean 1. Different lines correspond to different standard deviations of the associated normal distribution, and different columns to different steady-state markups. Values are for $\beta = 0.99$ and $\delta_i = 0.011$.

optimality condition for inventory choice as:

$$\hat{inv}_t = -\tilde{\eta}\omega\widehat{mc}_t + \hat{s}_t.$$

In this expression, the elasticity of inventories to relative marginal cost, $\tilde{\eta}$ is given by:

$$\tilde{\eta} = \frac{1}{1 - \eta\epsilon_d + (\eta - \tau)\epsilon_\mu}\eta \tag{17}$$

In contrast to the stock-elastic demand model, $\tilde{\eta}$ does not purely reflect the intertemporal substitution of production anymore. The relative marginal cost elasticity η is now compensated for markup movements (the terms τ and ϵ_μ)and for movements in the stockout wedge (the term ϵ_d).

Unlike in the stock-elastic demand model, the sign of $\tilde{\eta}$ cannot in general be established.[1] This is because its sign depends on the distribution of the idiosyncratic taste shock. However, for a very wide range of calibrations and for the Pareto and Log-normal distributions, $\tilde{\eta}$ is negative. We document this in Table 1. There, we compute different values of $\tilde{\eta}$, for different pairs of values of

[1]In the variant of this model considered by Wen (2011), it can however be proved that the analogous reduced-form parameter $\tilde{\eta}$ is strictly negative regardless of the shock distribution. The proof is available from the authors upon request.

Value of $\tilde{\eta}$					
$\sigma_d \downarrow \|\| \mu \rightarrow$	**1.05**	**1.1**	**1.25**	**1.5**	**1.75**
0.1	-1959.13	-297.78	-62.89	-27.02	-18.16
0.25	-926.82	-142.18	-30.44	-13.30	-9.06
0.5	-598.66	-92.85	-20.20	-8.98	-6.20
0.75	-499.86	-78.04	-17.14	-7.69	-5.35
1	-456.12	-71.51	-15.80	-7.13	-4.97

Implied IS ratio					
$\sigma_d \downarrow \|\| \mu \rightarrow$	**1.05**	**1.1**	**1.25**	**1.5**	**1.75**
0.1	0.03	0.07	0.15	0.22	0.26
0.25	0.05	0.15	0.34	0.51	0.63
0.5	0.09	0.25	0.57	0.90	1.13
0.75	0.10	0.30	0.71	1.15	1.48
1	0.11	0.33	0.80	1.31	1.70

Table 2: Value of $\tilde{\eta}$ when shock follow a Pareto distribution with mean 1. Different lines correspond to different standard deviations for the Pareto distribution, and different columns to different steady-state markups. Values are for $\beta = 0.99$ and $\delta_i = 0.011$.

σ_d, the standard deviation of the shock, and different values of the steady-state markup. In all cases, we constraint the shock to have a mean equal to 1. The standard deviations we consider range from 0.1 to 1, and the markups range from 1.05 to 1.75. In all cases, $\tilde{\eta}$ is negative. In table 2, we perform the same exercise for Pareto-distributed shocks, and results are similar.

These results can be understood using (17). First, as discussed before, since $\epsilon_\mu < 0$ for standard distributions, markups fall when the IS ratio increases. With a higher IS ratio, a stockout is less likely for a firm, so that its price elasticity of demand is high, and its charges low markups. Second, because $(\eta - \tau)\epsilon_\mu > 0$, markup movements tend to attenuate the intertemporal substitution channel; that is, if we were to set $\epsilon_d = 0$, then $\tilde{\eta} < \eta$. Lower markups signal a higher future marginal cost to the firm, thereby leading it to increase inventories (for fixed current marginal cost). At the same time, higher markups lead the firm to increase its sales relative to available goods, leaving it with fewer inventories at the end of the period. On net, the first effect dominates, leading to higher inventories at the end of the period, and reducing thus the inventory-depleting effects of the shock. Finally, $\eta\epsilon_d - (\eta - \tau)\epsilon_\mu > 1$, so that $\tilde{\eta} < 0$. Therefore, movements in the stockout wedge change the sign of the elasticity of inventories to marginal cost.

With $\tilde{\eta} < 0$, the following results hold for the impact response of news shocks in the stockout avoidance model.

Proposition 1 (The impact response to news shocks in the stockout-avoidance model) *In the stockout-avoidance model with $\tilde{\eta} < 0$, after a news shock:*

1. *inventory-sales ratio and inventories move in the same direction;*

2. *inventories increase, if and only if:*

$$-\tilde{\eta} < \frac{\kappa}{\omega}\frac{\delta_i}{\kappa - 1}.$$

The first part of this proposition is by itself daunting to news shocks, since it implies a counterfactual positive comovement between the IS ratio and inventories in response to a news shock. The second part states the condition under which inventories could be procyclical. This condition is similar to that of proposition 1 in the main text, with $-\tilde{\eta}$ taking place instead of η on the left hand side, and κ/ω multiplied by $\delta_i/(\kappa - 1)$ on the right hand side. Again, inventories are procyclical if the degree of real rigidities represented by the inverse of ω is high compared to the absolute value of the elasticity of inventories to relative marginal cost $-\tilde{\eta}$. We turn to a discussion of the numerical values of the parameters for this condition to hold.

B.6 When do inventories respond positively to news shocks?

The second part of proposition 1 provides a condition under which inventories are procyclical. Much as in the case of the stock-elastic demand model, this condition for procyclicality of inventories implies a lower bound for the degree real rigidities (alternatively, an upper bound for ω). We now provide a numerical illustration of this bound, by setting $\beta = 0.99$ and considering the same range of steady-state IS ratios, 0.25, 0.5 and 0.75, as in section 3. Given these values and a depreciation rate of inventories δ_i, the value $\overline{\omega}$ was uniquely pinned down in section 3. However, in the stockout-avoidance model considered above, the three variables are not sufficient to determine $\overline{\omega}$. Hence we also target the steady-state gross markup μ at 1.25, which is within the range of estimates considered in the literature.[2]

[2] It should be noted that with given values of the steady-state markup, the steady-state IS ratio, and the rate of depreciation of inventories, a unique steady-state stockout probability is implied. Indeed, in this model, a higher IS ratio implies a lower stockout probability, while at the same time, it is linked to a higher markup. The IS ratio and the markup thus cannot be targeted independently of the stockout probability.

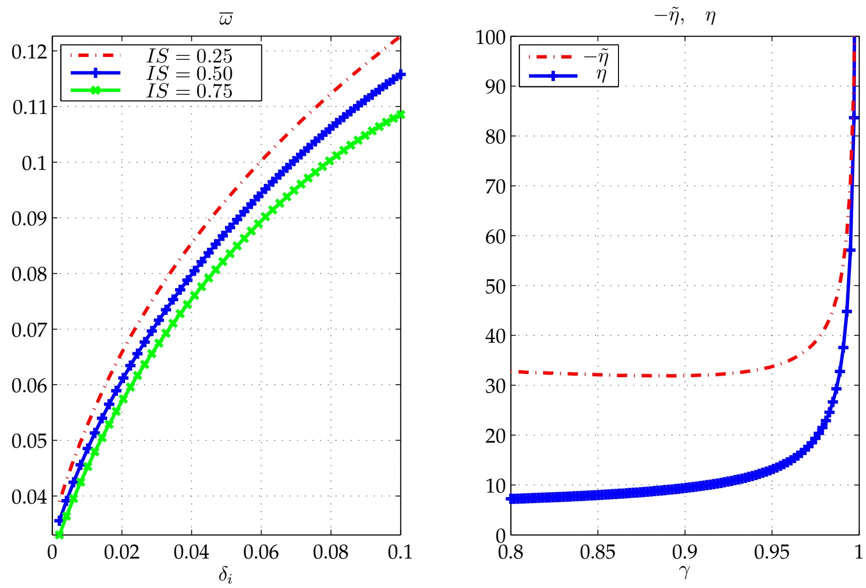

Figure 3: Implied parameter values for the stockout avoidance model. The left panel provides the upper bound on ω for procyclical inventories, derived from targeting the steady-state IS ratio and $\mu = 1.25$. The right panel provides the value of $-\tilde{\eta}$ and η as a function of $\gamma(= \beta(1-\delta_i))$, holding fixed all the other structural parameters.

In figure 3, we plot the upper bound of ω for inventories to be procyclical, assuming a log-normal distribution for the taste-shifter. We observe that inventories are procyclical only with low levels of ω. For a quarterly depreciation of 2 percent, the upper bound of ω is below 0.07, much lower than the existing measures. Hence with reasonable numerical values, the model still implies that inventories fall with regards to news shocks.

B.7 Is the response of inventories dominated by intertemporal substitution?

The inequality condition in proposition 1 does not hold because $-\tilde{\eta}$ is large. An immediate question is whether this large value is due to the high intertemporal substitution, as was the case in section 3. Since the reduced-form parameter η summarizes the intensity of the intertemporal substitution motive, we need to verify whether η is large and positively related to $-\tilde{\eta}$.

First, the value η in the stockout-avoidance model is determined by the following:

$$\eta = \underbrace{\frac{1}{1-\beta(1-\delta_i)} \frac{1+IS}{IS}}_{=\eta^{SE}} \left(1 - \Gamma(1+IS)\right) \frac{1}{H(\Gamma)}.$$

Here, Γ denotes the steady-state stockout probability. Note that this expression is similar to the relative marginal cost elasticity in the stock-elastic demand model, save for the two terms that depend on the stockout probability Γ. The function $H(\Gamma)$ is related to the hazard rate characterizing the cumulative distribution function of taste shocks F. For the type of distributions considered in the literature, $H(\Gamma)$ is typically larger than 1. Thus in general, $\eta \leq \eta^{SE}$, where η^{SE} is the expression for η in the stock-elastic demand model. That is, the intertemporal substitution channel is weaker in these models than in the stock-elastic demand model. The fact that some firms stock out of their varieties prevents them altogether from smoothing production over time by storing goods or depleting inventories.

However, setting the targets at $IS = 0.5$ and $\mu = 1.25$, and assuming that the taste-shifter follows a log-normal distribution, η is computed to be two thirds of the value in the stock-elastic demand model. Given that the lower bound for η^{SE} was above 30, η in the stockout-avoidance model is above 20, implying that a 1 percent increase in the present value of future marginal cost leads firms to adjust more than 20 percent of inventories relative to sales. Hence the intertemporal substitution motive remains large in the stockout-avoidance model.

Second, we need to verify whether a large η implies a large $-\tilde{\eta}$. However, both parameters are in reduced form, and therefore the link between the two cannot be directly measured. Instead, we show whether the two values are positively correlated with $\gamma = \beta(1 - \delta_i)$. Setting the benchmark targets at $IS = 0.5$ and $\mu = 1.25$, we fix the structural parameters, assuming that the taste-shifter follows a log-normal distribution. Given the structural parameters, we vary γ and plot the implied value of η and $-\tilde{\eta}$ on the right panel of figure 3. Note that both values are increasing in γ as γ approaches 1. This suggests that the value of $-\tilde{\eta}$ is again dominated by the value of η in (17), especially when γ is close to 1.[3] In this sense, the strong intertemporal substitution channel again dominates the overall response of inventories to news shocks.

B.8 Additional results for the stockout avoidance model

The following equations are consitute an equilibrium of the stockout avoidance model:

[3]The same result holds for a wide range of distributions for the taste-shifter.

$$1 - F(\nu_t^*) = \frac{\frac{1}{\gamma_t} - 1}{\mu_t^F - 1}, \tag{18}$$

$$\frac{\theta}{\theta - 1 - \frac{1 - F(\nu_t^*)}{\int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu)}} = \mu_t^F, \tag{19}$$

$$\frac{\int_{\nu \leq \nu_t^*} \left(1 - \frac{\nu}{\nu_t^*}\right) dF(\nu)}{\int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu) + 1 - F(\nu_t^*)} = \frac{inv_t}{s_t}, \tag{20}$$

$$\mu_t^F = d_t \mu_t \tag{21}$$

$$\left(\int_{\nu \leq \nu_t^*} \nu dF(\nu) + \nu_t^* \int_{\nu > \nu_t^*} \left(\frac{\nu}{\nu_t^*}\right)^{\frac{1}{\theta}} dF(\nu) \right)^{\frac{1}{\theta - 1}} = d_t, \tag{22}$$

$$\frac{\left((\nu_t^*)^{\frac{1}{\theta}} \int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu) + \int_{\nu > \nu_t^*} \nu^{\frac{1}{\theta}} dF(\nu) \right)^{\frac{\theta}{\theta - 1}}}{\int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu) + 1 - F(\nu_t^*)} s_t = x_t. \tag{23}$$

Condition (18) determines the optimal choice of stock in the stockout avoidance model. Here, ν_t^* is related to the aggregate IS ratio through (20). Condition (19) is the optimal markup choice in the stockout avoidance model which also depends on the IS ratio through (20), reflecting the dependence of the price elasticity of demand on the stock of goods on sale in this (not iso-elastic) model. The firm markup μ_t^F and the aggregate markup μ_t are linked by the stockout wedge d_t in equation (21). The stockout wedge itself is given by (22). Finally, condition (23) reflects market clearing when some varieties stock out.

Because some firms stock out while others do not, the equilibrium of the stockout avoidance model is not symmetric across firms. We define the aggregate variables s_t and inv_t as the aggregate sales and inventories, respectively:

$$inv_t \equiv \int_{j \in [0,1]} inv_t(j) dj \quad , \quad s_t =\equiv \int_{j \in [0,1]} s_t(j) dj.$$

However, the choices of price $p_t(j)$ and goods on shelf $a_t(j)$ are identical across firms. To see this, note first that for the same reason mentioned for the stock-elastic demand model, marginal cost is constant across firms. Second, the first-order conditions for optimal pricing and optimal choice of stock are given, respectively, by:

$$mc_t = \frac{\partial \tilde{s}_t(j)}{\partial a_t(j)} \frac{p_t(j)}{P_t} + \left(1 - \frac{\partial \tilde{s}_t(j)}{\partial a_t(j)}\right)(1-\delta_i)\mathbb{E}_t\left[q_{t,t+1} mc_{t+1}\right],$$

$$\frac{p_t(j)/P_t}{(1-\delta_i)\mathbb{E}_t\left[q_{t,t+1} mc_{t+1}\right]} = \frac{\theta}{\theta - 1 - \frac{\tilde{s}_t(j)}{p_t(j)} \frac{\partial \tilde{s}_t(j)}{\partial p_t(j)}}$$

where mc_t denotes nominal marginal cost deflated by the CPI, P_t. Here, $\tilde{s}_t(j)$ denotes firm j's expected sales. Following equation (10), expected sales of firm j depend only on price $p_t(j)$ and on-shelf goods $a_t(j)$, and aggregate variables. In turn, the above optimality conditions can be solved to obtain a decision rule for $a_t(j)$ and $p_t(j)$ as a function of current and expected values of aggregate values, so that the choices of individual firms for these variables are symmetric. This implies in turn that the stockout cutoff,

$$\nu_t^*(j) = \left(\frac{p_t(j)}{P_t}\right)^{\theta} \frac{a_t(j)}{x_t},$$

is also symmetric across firms.

B.8.1 Expressions for the reduced-form coefficients of lemma 2

In what follows, we denote the steady-state stockout probability by:

$$\Gamma = 1 - F(\nu^*).$$

First, note that the log-linear approximation of equation (20) is:

$$\widehat{inv}_t - \hat{s}_t = (1 - \Gamma(1+IS))\frac{1+IS}{IS}\hat{\nu}_t^*$$

This implies that the IS ratio and the stockout threshold move in the same direction. Indeed, the restriction:

$$1 > \Gamma(1+IS)$$

follows from the fact that in the steady state,

$$IS = \frac{\int_{\nu \le \nu^*} \left(1 - \frac{\nu}{\nu^*}\right) dF(\nu)}{\int_{\nu \le \nu^*} \frac{\nu}{\nu^*} dF(\nu) + \Gamma} \Leftrightarrow \frac{1}{1+IS} - \Gamma = \int_{\nu \le \nu^*} \frac{\nu}{\nu^*} dF(\nu) > 0.$$

Second, it can be shown that the log-linear approximations to equations (18), (19) and (22) are respectively given by:

$$\frac{\nu^* f(\nu^*)}{\Gamma} \hat{\nu}_t^* = \frac{\mu^F}{\mu^F - 1} \hat{\mu}_t^F + \frac{1}{1-\gamma} \hat{\gamma}_t,$$

$$\hat{\mu}_t^F = (\mu^F - 1)\Gamma(1+IS) \left(1 - \frac{\nu^* f(\nu^*)}{\Gamma} \frac{1}{1 - \Gamma(1+IS)}\right) \nu_t^*,$$

$$\hat{d}_t = \frac{\mu^F - 1}{\mu^F} (1 - \Gamma(1+IS)) \Delta \hat{\nu}_t^*.$$

Here, the coefficient $\Delta \in (0, 1]$ is defined as:

$$\Delta \equiv \frac{\int_{\nu > \nu^*} \left(\frac{\nu}{\nu^*}\right)^{\frac{1}{\theta}} dF(\nu)}{\int_{\nu \le \nu^*} \frac{\nu}{\nu^*} dF(\nu) + \int_{\nu > \nu^*} \left(\frac{\nu}{\nu^*}\right)^{\frac{1}{\theta}} dF(\nu)},$$

where the relationship between the parameter θ and the steady-state markup is given by:

$$\theta = \frac{\mu^F}{\mu^F - 1} \frac{1}{1 - \Gamma(1+IS)}.$$

Combining these equations, one arrives at the following expressions for the different reduced-form parameters defining the log-linear framework of lemma 2:

$$\tau = \frac{\Gamma}{\nu^* f(\nu^*)} (1 - \Gamma(1+IS)) \frac{1+IS}{IS} \frac{\mu^F}{\mu^F - 1} > 0, \tag{24}$$

$$\eta = \frac{\Gamma}{\nu^* f(\nu^*)} (1 - \Gamma(1+IS)) \frac{1+IS}{IS} \frac{1}{1-\gamma} > 0, \tag{25}$$

$$\epsilon_d = \frac{IS}{1+IS} \frac{1}{1 - \Gamma(1+IS)} \frac{\mu^F - 1}{\mu^F} (1 - \Gamma(1+IS)) \Delta > 0, \tag{26}$$

$$\epsilon_\mu = \frac{IS}{1+IS} \frac{1}{1 - \Gamma(1+IS)} (\mu^F - 1)\Gamma(1+IS) \left(1 - \frac{\nu^* f(\nu^*)}{\Gamma} \frac{1}{1 - \Gamma(1+IS)}\right). \tag{27}$$

C The stock-elastic demand model for estimation

We describe the stock-elastic inventory model, allowing for trends and both stationary and non-stationary shocks as in Schmitt-Grohé and Uribe (2012). We start by defining the trend components of the model.

C.1 Trends in the model

The two sources of nonstationarity in the model of Schmitt-Grohé and Uribe (2012) are neutral and investment-specific productivity. Aggregate sales S_t can be written as

$$S_t = C_t + Z_t^I I_t + G_t,$$

where Z_t^I is the nonstationary investment-specific productivity. From this equation and balanced growth path, we observe that $Z_t^I I_t / S_t$ is stationary. Letting the trend of aggregate sales to be X_t^Y and the trend of I_t to be X_t^I, the balanced-growth condition tells us that

$$X_t^Y = Z_t^I X_t^I. \tag{28}$$

Moreover, from the capital accumulation function, capital and investment should follow the same trend. Writing X_t^K as the trend of capital, the second condition is

$$X_t^K = X_t^I. \tag{29}$$

Lastly, the production function is

$$Y_t = z_t (u_t K_t)^{\alpha_K} (X_t n_t)^{\alpha_N} (X_t L)^{1-\alpha_K-\alpha_N}.$$

Since the trend must also be consistent, we have the following equation

$$X_t^Y = (X_t^K)^{\alpha_K} X_t^{1-\alpha_K}. \tag{30}$$

From the three conditions (28), (29) and (30), we can solve for the trends X_t^Y, X_t^I, X_t^K as

$$X_t^Y = X_t (Z_t^I)^{\frac{\alpha_K}{\alpha_K - 1}}, \qquad X_t^K = X_t^I = X_t (Z_t^I)^{\frac{1}{\alpha_K - 1}}.$$

We are now ready to write the stationary problem. It will be useful to write the stationary variables in lower cases as follows:

$$y_t = \frac{Y_t}{X_t^Y}, \quad c_t = \frac{C_t}{X_t^Y}, \quad i_t = \frac{I_t}{X_t^I}, \quad k_{t+1} = \frac{K_{t+1}}{X_t^K}, \quad g_t = \frac{G_t}{X_t^G}.$$

Note that the trend on government spending X_t^G is defined as a smoothed version of X_t^Y:

$$X_t^G = (X_{t-1}^G)^{\rho_{xg}} (X_{t-1}^Y)^{1-\rho_{xg}}.$$

We can also express the two exogenous trends in stationary variables:

$$\mu_t^X = \frac{X_t}{X_{t-1}}, \quad \mu_t^A = \frac{Z_t^I}{Z_{t-1}^I}.$$

Using this, we get an expression for the endogenous trends:

$$\mu_t^Y = \mu_t^X (\mu_t^A)^{\frac{\alpha_K}{\alpha_K - 1}}, \qquad \mu_t^I = \mu_t^K = \frac{\mu_t^Y}{\mu_t^A}.$$

We also define x_t^G as the relative trend of government spending:

$$x_t^G \equiv \frac{X_t^G}{X_t^Y} = \frac{(X_{t-1}^G)^{\rho_{xg}} (X_{t-1}^Y)^{1-\rho_{xg}}}{X_t^Y} = \frac{(x_{t-1}^G)^{\rho_{xg}}}{\mu_t^Y}.$$

With these stationary variables, we can express the problem in terms of stationary variables. We start with the household problem.

C.2 Household problem

To write down all the equilibrium conditions, the household utility is defined as follows:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_{h,t} \frac{M_t^{1-\sigma} - 1}{1-\sigma},$$

$$M_t = C_t - bC_{t-1} - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} H_t,$$

$$H_t = (C_t - bC_{t-1})^{\gamma_h} H_{t-1}^{1-\gamma_h}.$$

The household constraints are the following:

$$\int_0^1 \frac{p_t(j)}{P_t} S_t(j) dj + \mathbb{E}_t q_{t,t+1} B_{t+1} = W_t n_t + R_t u_t K_t + B_t + \Pi_t,$$

$$S_t = \left(\int_0^1 \nu_t(j)^{\frac{1}{\theta}} S_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}},$$

$$C_t + Z_t^I I_t + G_t = S_t,$$

$$K_{t+1} = z_t^k I_t \left(1 - \phi \left(\frac{I_t}{I_{t-1}} \right) \right) + (1 - \delta(u_t)) K_t.$$

Notice that given the symmetry of the firm behavior, $\nu_t(j) = 1$ and $\int_0^1 \frac{p_t(j)}{P_t} S_t(j) dj = S_t$. Hence the household problem can be written as

$$\begin{aligned}
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \zeta_{h,t} \frac{M_t^{1-\sigma} - 1}{1-\sigma} + \Lambda_{m,t} \left[C_t - bC_{t-1} - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} H_t - M_t \right] \\
& + \Lambda_{h,t} [H_t - (C_t - bC_{t-1})^{\gamma_h} H_{t-1}^{1-\gamma_h}] \\
& + \Lambda_t [W_t n_t + R_t u_t K_t + \Pi_t + B_t - C_t - Z_t^I I_t - G_t - \mathbb{E}_t q_{t,t+1} B_{t+1}] \\
& + \Lambda_{k,t} \left[z_t^k I_t \left(1 - \phi \left(\frac{I_t}{I_{t-1}} \right) \right) + (1 - \delta(u_t)) K_t - K_{t+1} \right] \Bigg\}.
\end{aligned}$$

Hence the household first order conditions are characterized by the following:

$$[M_t] : \zeta_{h,t} M_t^{-\sigma} = \Lambda_{m,t}, \tag{31}$$

$$[H_t] : \Lambda_{h,t} - \Lambda_{m,t}\psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} = \beta \mathbb{E}_t \Lambda_{h,t+1}(C_{t+1} - bC_t)^{\gamma_h} H_t^{-\gamma_h}(1-\gamma_h), \tag{32}$$

$$[C_t] : \Lambda_{m,t} - \Lambda_{h,t}\gamma_h (C_t - bC_{t-1})^{\gamma_h - 1} H_{t-1}^{1-\gamma_h} - \beta b \mathbb{E}_t[\Lambda_{m,t+1} - \Lambda_{h,t+1}\gamma_h (C_{t+1} - bC_t)^{\gamma_h-1} H_t^{1-\gamma_h}] = \Lambda_t, \tag{33}$$

$$[n_t] : \Lambda_t W_t = \Lambda_{m,t}\psi_t n_t^{\xi^{-1}} H_t, \tag{34}$$

$$[I_t] : Z_t^I \Lambda_t - \Lambda_{k,t} z_t^k \left(1 - \phi\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right)\phi'\left(\frac{I_t}{I_{t-1}}\right)\right) = \beta \mathbb{E}_t \Lambda_{k,t+1} z_{t+1}^k \left(\frac{I_{t+1}}{I_t}\right)^2 \phi'\left(\frac{I_{t+1}}{I_t}\right), \tag{35}$$

$$[K_{t+1}] : \Lambda_{k,t} = \beta \mathbb{E}_t[\Lambda_{t+1} R_{t+1} u_{t+1} + \Lambda_{k,t+1}(1 - \delta(u_{t+1}))], \tag{36}$$

$$[u_t] : \Lambda_t R_t = \Lambda_{k,t}\delta'(u_t), \qquad [u_t = 1 \text{ if not allowed to vary}], \tag{37}$$

$$[B_{t+1}] : q_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}, \tag{38}$$

$$[\Lambda_{m,t}] : M_t = C_t - bC_{t-1} - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} H_t, \tag{39}$$

$$[\Lambda_{h,t}] : H_t = (C_t - bC_{t-1})^{\gamma_h} H_{t-1}^{1-\gamma_h}, \tag{40}$$

$$[\Lambda_{k,t}] : K_{t+1} = z_t^k I_t \left(1 - \phi\left(\frac{I_t}{I_{t-1}}\right)\right) + (1 - \delta(u_t))K_t. \tag{41}$$

and the household budget constraint. We also want private spending S_t^p and total absorption S_t as

$$S_t^p = C_t + Z_t^I I_t, \tag{42}$$

$$S_t = C_t + Z_t^I I_t + G_t. \tag{43}$$

We define the following stationary variables:

$$\lambda_{m,t} = \frac{\Lambda_{m,t}}{(X_t^Y)^{-\sigma}}, \quad \lambda_{h,t} = \frac{\Lambda_{h,t}}{(X_t^Y)^{-\sigma}}, \quad \lambda_t = \frac{\Lambda_t}{(X_t^Y)^{-\sigma}}, \quad \lambda_{k,t} = \frac{\Lambda_{k,t}}{(X_t^Y)^{-\sigma} Z_t^I}, \quad w_t = \frac{W_t}{X_t^Y}, \quad r_t = \frac{R_t}{Z_t^I}.$$

Using these expressions as well as the ones defined in the previous section, we rewrite the household first order condition in terms of stationary variables:

$$[m_t] : \zeta_{h,t} m_t^{-\sigma} = \lambda_{m,t}, \tag{44}$$

$$[h_t] : \lambda_{h,t} - \lambda_{m,t} \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} = \beta \mathbb{E}_t \lambda_{h,t+1} (\mu_{t+1}^Y)^{-\sigma} (c_{t+1}\mu_{t+1}^Y - bc_t)^{\gamma_h} h_t^{-\gamma_h} (1-\gamma_h), \tag{45}$$

$$[c_t] : \lambda_{m,t} - \lambda_{h,t}\gamma_h \left(\frac{c_t \mu_t^Y - bc_{t-1}}{h_{t-1}}\right)^{\gamma_h - 1} - \beta b \mathbb{E}_t (\mu_{t+1}^Y)^{-\sigma} \left[\lambda_{m,t+1} - \lambda_{h,t+1}\gamma_h \left(\frac{c_{t+1}\mu_{t+1}^Y - bc_t}{h_t}\right)^{\gamma_h - 1}\right] = \lambda_t, \tag{46}$$

$$[n_t] : \lambda_t w_t = \lambda_{m,t} \psi_t n_t^{\xi^{-1}} h_t, \tag{47}$$

$$[i_t] : \lambda_t - \lambda_{k,t} z_t^k \left(1 - \phi\left(\frac{i_t}{i_{t-1}}\mu_t^I\right) - \left(\frac{i_t}{i_{t-1}}\mu_t^I\right)\phi'\left(\frac{i_t}{i_{t-1}}\mu_t^I\right)\right)$$

$$= \beta \mathbb{E}_t \lambda_{k,t+1} \frac{\mu_{t+1}^A}{(\mu_{t+1}^Y)^\sigma} z_{t+1}^k \left(\frac{i_{t+1}}{i_t}\mu_{t+1}^I\right)^2 \phi'\left(\frac{i_{t+1}}{i_t}\mu_{t+1}^I\right), \tag{48}$$

$$[k_{t+1}] : \lambda_{k,t} = \beta \mathbb{E}_t (\mu_{t+1}^Y)^{-\sigma} \mu_{t+1}^A [\lambda_{t+1} r_{t+1} u_{t+1} + \lambda_{k,t+1}(1 - \delta(u_{t+1}))], \tag{49}$$

$$[u_t] : \lambda_t r_t = \lambda_{k,t} \delta'(u_t), \qquad [u_t = 1 \text{ if not allowed to vary}], \tag{50}$$

$$[b_{t+1}] : q_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} (\mu_{t+1}^Y)^{-\sigma}, \tag{51}$$

$$[\lambda_{m,t}] : m_t = c_t - b\frac{c_{t-1}}{\mu_t^Y} - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} h_t, \tag{52}$$

$$[\lambda_{h,t}] : h_t = \left(c_t - b\frac{c_{t-1}}{\mu_t^Y}\right)^{\gamma_h} \left(\frac{h_{t-1}}{\mu_t^Y}\right)^{1-\gamma_h}, \tag{53}$$

$$[\lambda_{k,t}] : k_{t+1} = z_t^k i_t \left(1 - \phi\left(\frac{i_t}{i_{t-1}}\mu_t^I\right)\right) + (1 - \delta(u_t))\frac{k_t}{\mu_t^I}, \tag{54}$$

$$[s_t^p] : s_t^p = c_t + i_t, \tag{55}$$

$$[s_t] : s_t = c_t + i_t + g_t x_t^G. \tag{56}$$

Now, in log-linearized form:

$$[m_t]: \hat{\zeta}_{h,t} - \sigma \hat{m}_t = \hat{\lambda}_{m,t}, \tag{57}$$

$$[h_t]: \hat{\lambda}_{h,t} - \left[1 - \beta(1-\gamma_h)(\mu^Y)^{1-\sigma}\right]\left[\hat{\lambda}_{m,t} + \hat{\psi}_t + (1+\xi^{-1})\hat{n}_t\right]$$
$$= \beta(1-\gamma_h)(\mu^Y)^{1-\sigma}\left[\mathbb{E}_t\hat{\lambda}_{h,t+1} + (1-\sigma)\mathbb{E}_t\hat{\mu}^Y_{t+1} + \mathbb{E}_t\hat{h}_{t+1} - \hat{h}_t\right], \tag{58}$$

$$[c_t]: \lambda\hat{\lambda}_t = \lambda_m\hat{\lambda}_{m,t} - \lambda_h\gamma_h(\mu^Y)^{1-\frac{1}{\gamma_h}}\left[\hat{\lambda}_{h,t} + \hat{h}_t - \frac{\mu^Y}{\mu^Y - b}\hat{c}_t + \frac{b}{\mu^Y-b}\hat{c}_{t-1} - \frac{b}{\mu^Y-b}\hat{\mu}^Y_t\right]$$
$$+ \sigma\beta b(\mu^Y)^{-\sigma}\left[\lambda_m - \lambda_h\gamma_h(\mu^Y)^{1-\frac{1}{\gamma_h}}\right]\mathbb{E}_t\hat{\mu}^Y_{t+1} - \beta b(\mu^Y)^{-\sigma}\lambda_m\mathbb{E}_t\hat{\lambda}_{m,t+1}$$
$$+ \beta b(\mu^Y)^{-\sigma}\lambda_h\gamma_h(\mu^Y)^{1-\frac{1}{\gamma_h}}\left[\mathbb{E}_t\hat{\lambda}_{h,t+1} + \mathbb{E}_t\hat{h}_{t+1} - \frac{\mu^Y}{\mu^Y-b}\mathbb{E}_t\hat{c}_{t+1} + \frac{b}{\mu^Y-b}\hat{c}_t - \frac{b}{\mu^Y-b}\mathbb{E}_t\hat{\mu}^Y_{t+1}\right], \tag{59}$$

$$[n_t]: \hat{\lambda}_t + \hat{w}_t = \hat{\lambda}_{m,t} + \hat{\psi}_t + \frac{1}{\xi}\hat{n}_t + \hat{h}_t, \tag{60}$$

$$[i_t]: \hat{\lambda}_{k,t} = \hat{\lambda}_t - \hat{z}^k_t + \mu^I\phi_I''(\hat{i}_t - \hat{i}_{t-1} + \hat{\mu}^I_t) - \beta\frac{\mu^A}{(\mu^Y)^\sigma}(\mu^I)^3\phi_I''(\mathbb{E}_t\hat{i}_{t+1} - \hat{i}_t + \mathbb{E}_t\hat{\mu}^I_{t+1}), \tag{61}$$

$$[k_{t+1}]: \hat{\lambda}_{k,t} = \mathbb{E}_t\hat{\mu}^A_{t+1} - \sigma\mathbb{E}_t\hat{\mu}^Y_{t+1} + \beta(\mu^Y)^{-\sigma}\mu^A(1-\delta_k)\mathbb{E}_t\hat{\lambda}_{k,t+1}$$
$$+ \left[1 - \beta(\mu^Y)^{-\sigma}\mu^A(1-\delta_k)\right](\mathbb{E}_t\hat{\lambda}_{t+1} + \mathbb{E}_t\hat{r}_{t+1} + \mathbb{E}_t\hat{u}_{t+1}) - \beta(\mu^Y)^{-\sigma}\mu^A\delta_k'\mathbb{E}_t\hat{u}_{t+1}, \tag{62}$$

$$[u_t]: \hat{\lambda}_t + \hat{r}_t = \hat{\lambda}_{k,t} + \frac{\delta_k''}{\delta_k'}\hat{u}_t, \qquad [\hat{u}_t = 0 \text{ if not allowed to vary}], \tag{63}$$

$$[b_{t+1}]: \mathbb{E}_t\hat{q}_{t,t+1} = \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \sigma\mathbb{E}_t\hat{\mu}^Y_{t+1}, \tag{64}$$

$$[\lambda_{m,t}]: m\hat{m}_t = c\hat{c}_t - b\frac{c}{\mu^Y}\hat{c}_{t-1} + b\frac{c}{\mu^Y}\hat{\mu}^Y_t - \psi\frac{n^{1+\xi^{-1}}}{1+\xi^{-1}}h\left[\hat{\psi}_t + \hat{h}_t + (1+\xi^{-1})\hat{n}_t\right], \tag{65}$$

$$[\lambda_{h,t}]: \hat{h}_t = \frac{\gamma_h\mu^Y}{\mu^Y-b}\hat{c}_t - b\frac{\gamma_h}{\mu^Y-b}\hat{c}_{t-1} + b\frac{\gamma_h}{\mu^Y-b}\hat{\mu}^Y_t + (1-\gamma_h)\hat{h}_{t-1} - (1-\gamma_h)\hat{\mu}^Y_t, \tag{66}$$

$$[\lambda_{k,t}]: \hat{k}_{t+1} = \left(1 - \frac{1-\delta_k}{\mu^I}\right)\hat{z}^k_t + \left(1 - \frac{1-\delta_k}{\mu^I}\right)\hat{i}_t + \frac{1-\delta_k}{\mu^I}\hat{k}_t - \frac{1-\delta_k}{\mu^I}\hat{\mu}^I_t - \frac{\delta_k'}{\mu^I}\hat{u}_t, \tag{67}$$

$$[s^p_t]: \hat{s}^p_t = \frac{c}{c+i}\hat{c}_t + \frac{i}{c+i}\hat{i}_t, \tag{68}$$

$$[s_t]: \hat{s}_t = \frac{c}{s}\hat{c}_t + \frac{i}{s}\hat{i}_t + \frac{gx^G}{s}\hat{g}_t + \frac{gx^G}{s}\hat{x}^G_t, \tag{69}$$

$$[\mu^Y_t]: \hat{\mu}^Y_t = \hat{\mu}^X_t + \frac{\alpha_K}{\alpha_K - 1}\hat{\mu}^A_t, \tag{70}$$

$$[\mu^I_t]: \hat{\mu}^I_t = \hat{\mu}^Y_t - \hat{\mu}^A_t, \tag{71}$$

$$[x^G_t]: \hat{x}^G_t = \rho_{xg}\hat{x}^G_{t-1} - \hat{\mu}^Y_t. \tag{72}$$

C.3 Firm problem without inventories

This section is only for completeness. The readers should skip this section and read the firm problem with stock-elastic inventories. The firm side is subject to monopolistic competition. As you will see, this aspect itself will introduce no changes in the dynamics relative to the real model since no price rigidity is assumed. Firm $j \in [0,1]$ solves the following problem:

$$\max \ \mathbb{E}_0 q_{0,t} \left[\frac{p_t(j)}{P_t} S_t(j) - W_t n_t(j) - R_t u_t(j) K_t(j) \right],$$

subject to

$$\begin{aligned} S_t(j) &= \left(\frac{p_t(j)}{P_t} \right)^{-\theta} S_t, \\ Y_t(j) &= z_t (u_t(j) K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K}, \\ Y_t(j) &= S_t(j). \end{aligned}$$

As is well known, the last constraint is the demand constraint when no inventory adjustment is allowed. Letting the multiplier on this constraint to be the marginal cost, we can state the firm problem as the following:

$$\begin{aligned} \max \mathbb{E}_0 q_{0,t} \Bigg[& \frac{p_t(j)^{1-\theta}}{P_t^{1-\theta}} S_t - W_t n_t(j) - R_t u_t(j) K_t(j) \\ & + mc_t(j) \left\{ z_t (u_t(j) K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K} - \left(\frac{p_t(j)}{P_t} \right)^{-\theta} S_t \right\} \Bigg]. \end{aligned}$$

Hence the first order conditions are:

$$\begin{aligned} [p_t(j)] &: \frac{p_t(j)/P_t}{mc_t(j)} = \frac{\theta}{\theta - 1}, \\ [n_t(j)] &: \alpha_N mc_t(j) \frac{Y_t(j)}{n_t(j)} = W_t, \\ [u_t(j) K_t(j)] &: \alpha_K mc_t(j) \frac{Y_t(j)}{u_t(j) K_t(j)} = R_t, \\ [mc_t(j)] &: Y_t(j) = S_t(j), \end{aligned}$$

and a technology constraint: $Y_t(j) = z_t (u_t(j) K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K}$.

In a symmetric equilibrium the following conditions hold:

$$[p_t] : \frac{1}{mc_t} = \frac{\theta}{\theta - 1},$$
$$[n_t] : \alpha_N mc_t \frac{Y_t}{n_t} = W_t,$$
$$[u_t k_t] : \alpha_K mc_t \frac{Y_t}{u_t K_t} = R_t,$$
$$[mc_t] : Y_t = S_t,$$
$$[tech] : Y_t = z_t (u_t K_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K}.$$

Writing in terms of stationary variables, we have:

$$[p_t] : \frac{1}{mc_t} = \frac{\theta}{\theta - 1},$$
$$[n_t] : \alpha_N mc_t \frac{y_t}{n_t} = w_t,$$
$$[u_t k_t] : \alpha_K mc_t \frac{y_t}{u_t k_t} = \frac{r_t}{\mu_t^I},$$
$$[mc_t] : y_t = s_t,$$
$$[tech] : y_t = z_t (u_t k_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N} (\mu_t^I)^{-\alpha_K}.$$

In a log-linear setup, we can rewrite these conditions as

$$[p_t] : \widehat{mc}_t = 0, \tag{73}$$
$$[n_t] : \widehat{mc}_t + \hat{y}_t - \hat{n}_t = \hat{w}_t, \tag{74}$$
$$[u_t k_t] : \widehat{mc}_t + \hat{y}_t - \hat{u}_t - \hat{k}_t = \hat{r}_t - \hat{\mu}_t^I, \tag{75}$$
$$[mc_t] : \hat{y}_t = \hat{s}_t, \tag{76}$$
$$[tech] : \hat{y}_t = \hat{z}_t + \alpha_K \hat{u}_t + \alpha_K \hat{k}_t + \alpha_N \hat{n}_t - \alpha_K \hat{\mu}_t^I. \tag{77}$$

C.4 Computing the steady state in the no-inventory model

First of all, by targeting the markup μ, we get $\theta = \mu/(\mu - 1)$. Also, $mc = 1/\mu$. The other targets we want to force are labor supply n, steady-state output growth rate μ^Y, and steady-state investment growth rate μ^I.

Now from the capital investment condition, we get that $\lambda = \lambda_k$. Hence the capital stock condition tells us that $r = (\mu^Y)^\sigma(\mu^A\beta)^{-1} - 1 + \delta_k$. With $u = 1$, the utilization condition forces the depreciation acceleration due to utilization to be $\delta_k' = r$. Using the capital rental condition at the firm side, we get the steady-state capital:

$$k = \mu^I \left[\alpha_K \frac{mc}{r} n^{\alpha_N} l^{1-\alpha_K-\alpha_N}\right]^{\frac{1}{1-\alpha_K}}.$$

Therefore, output is $y = k^{\alpha_K} n^{\alpha_N} l^{1-\alpha_K-\alpha_N}(\mu^I)^{-\alpha_K}$ and investment is $i = (1-(1-\delta_k)/\mu^I)k$. Real wage is $w = \alpha_N mcy/n$ and consumption is therefore $c = y - i - x^G g$.

With these pillars, we also get the household utility aspects. The stock of habit is $h = c(\mu^Y - b)(\mu^Y)^{-1/\gamma_h}$. We have the following steady-state conditions:

$$\begin{aligned}
m^{-\sigma} &= \lambda_m, \\
\lambda_h(1-\beta(\mu^Y)^{1-\sigma}(1-\gamma_h)) &= \lambda_m \psi \frac{n^{1+\xi^{-1}}}{1+\xi^{-1}}, \\
\frac{\lambda}{\lambda_m} &= \left(1 - \frac{\beta b}{(\mu^Y)^\sigma}\right)\left[1 - \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \frac{\lambda_h}{\lambda_m}\right], \\
\frac{\lambda}{\lambda_m} &= \frac{\psi n^{\xi^{-1}} h}{w}, \\
m &= \left(1 - \frac{b}{\mu^Y}\right) c - \psi \frac{n^{1+\xi^{-1}}}{1+\xi^{-1}} h.
\end{aligned}$$

The first thing to pin down is ψ. Using the second to fourth conditions above, we can obtain ψ:

$$\psi = (1-\beta b(\mu^Y)^{-\sigma}) / \left[\frac{n^{\xi^{-1}} h}{w} + \frac{(1-\beta b(\mu^Y)^{-\sigma})\gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} n^{1+\xi^{-1}}}{(1+\xi^{-1})(1-\beta(\mu^Y)^{1-\sigma}(1-\gamma_h))}\right].$$

Once you pin down ψ, you can also obtain m as above. Then, from the first condition, you also get λ_m. Therefore λ_h and λ are also obtained and we are done.

C.5 Writing down all the equilibrium conditions for the no-inventory model

The 21 endogenous variables are

$$m_t, \lambda_{m,t}, \lambda_{h,t}, n_t, c_t, h_t, \lambda_t, w_t, \lambda_{k,t}, i_t, r_t, u_t, r_t^f, k_{t+1}, s_t^p, s_t, mc_t, y_t, x_t^G, \mu_t^Y, \mu_t^I,$$

and the 7 exogenous processes are $\zeta_{h,t}, \psi_t, z_t, z_t^k, g_t, \mu_t^X, \mu_t^A$. The 21 endogenous equations are:

$$[m_t] : \hat{\zeta}_{h,t} - \sigma \hat{m}_t = \hat{\lambda}_{m,t}, \tag{78}$$

$$\begin{aligned}[h_t] : &\hat{\lambda}_{h,t} - \left[1 - \beta(1-\gamma_h)(\mu^Y)^{1-\sigma}\right] \left[\hat{\lambda}_{m,t} + \hat{\psi}_t + (1+\xi^{-1})\hat{n}_t\right] \\ &= \beta(1-\gamma_h)(\mu^Y)^{1-\sigma} \left[\mathbb{E}_t \hat{\lambda}_{h,t+1} + (1-\sigma)\mathbb{E}_t \hat{\mu}^Y_{t+1} + \mathbb{E}_t \hat{h}_{t+1} - \hat{h}_t\right],\end{aligned} \tag{79}$$

$$\begin{aligned}[c_t] : &\lambda \hat{\lambda}_t = \lambda_m \hat{\lambda}_{m,t} - \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \left[\hat{\lambda}_{h,t} + \hat{h}_t - \frac{\mu^Y}{\mu^Y - b}\hat{c}_t + \frac{b}{\mu^Y - b}\hat{c}_{t-1} - \frac{b}{\mu^Y - b}\hat{\mu}^Y_t\right] \\ &+ \sigma\beta b (\mu^Y)^{-\sigma} \left[\lambda_m - \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}}\right] \mathbb{E}_t \hat{\mu}^Y_{t+1} - \beta b (\mu^Y)^{-\sigma} \lambda_m \mathbb{E}_t \hat{\lambda}_{m,t+1} \\ &+ \beta b (\mu^Y)^{-\sigma} \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \left[\mathbb{E}_t \hat{\lambda}_{h,t+1} + \mathbb{E}_t \hat{h}_{t+1} - \frac{\mu^Y}{\mu^Y - b}\mathbb{E}_t \hat{c}_{t+1} + \frac{b}{\mu^Y - b}\hat{c}_t - \frac{b}{\mu^Y - b}\mathbb{E}_t \hat{\mu}^Y_{t+1}\right],\end{aligned} \tag{80}$$

$$[n_t] : \hat{\lambda}_t + \hat{w}_t = \hat{\lambda}_{m,t} + \hat{\psi}_t + \frac{1}{\xi}\hat{n}_t + \hat{h}_t, \tag{81}$$

$$[i_t] : \hat{\lambda}_{k,t} = \hat{\lambda}_t - \hat{z}^k_t + \mu^I \phi''(\mu^I)(\hat{i}_t - \hat{i}_{t-1} + \hat{\mu}^I_t) - \beta \frac{\mu^A}{(\mu^Y)^\sigma}(\mu^I)^3 \phi''(\mu^I)(\mathbb{E}_t \hat{i}_{t+1} - \hat{i}_t + \mathbb{E}_t \hat{\mu}^I_{t+1}), \tag{82}$$

$$\begin{aligned}[k_{t+1}] : &\hat{\lambda}_{k,t} = \mathbb{E}_t \hat{\mu}^A_{t+1} - \sigma \mathbb{E}_t \hat{\mu}^Y_{t+1} + \beta(\mu^Y)^{-\sigma}\mu^A(1-\delta_k)\mathbb{E}_t \hat{\lambda}_{k,t+1} \\ &+ [1 - \beta(\mu^Y)^{-\sigma}\mu^A(1-\delta_k)](\mathbb{E}_t \hat{\lambda}_{t+1} + \mathbb{E}_t \hat{r}_{t+1} + \mathbb{E}_t \hat{u}_{t+1}) - \beta(\mu^Y)^{-\sigma}\mu^A \delta'_k \mathbb{E}_t \hat{u}_{t+1},\end{aligned} \tag{83}$$

$$[u_t] : \hat{\lambda}_t + \hat{r}_t = \hat{\lambda}_{k,t} + \frac{\delta''_k}{\delta'_k}\hat{u}_t, \qquad [\hat{u}_t = 0 \text{ if not allowed to vary}], \tag{84}$$

$$[b_{t+1}] : -\hat{r}^f_t = \mathbb{E}_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - \sigma \mathbb{E}_t \hat{\mu}^Y_{t+1}, \qquad [\text{written in terms of the real interest rate}], \tag{85}$$

$$[\lambda_{m,t}] : m\hat{m}_t = c\hat{c}_t - b\frac{c}{\mu^Y}\hat{c}_{t-1} + b\frac{c}{\mu^Y}\hat{\mu}^Y_t - \psi \frac{n^{1+\xi^{-1}}}{1+\xi^{-1}} h \left[\hat{\psi}_t + \hat{h}_t + (1+\xi^{-1})\hat{n}_t\right], \tag{86}$$

$$[\lambda_{h,t}] : \hat{h}_t = \frac{\gamma_h \mu^Y}{\mu^Y - b}\hat{c}_t - b\frac{\gamma_h}{\mu^Y - b}\hat{c}_{t-1} + b\frac{\gamma_h}{\mu^Y - b}\hat{\mu}^Y_t + (1-\gamma_h)\hat{h}_{t-1} - (1-\gamma_h)\hat{\mu}^Y_t, \tag{87}$$

$$[\lambda_{k,t}] : \hat{k}_{t+1} = \left(1 - \frac{1-\delta_k}{\mu^I}\right)\hat{z}^k_t + \left(1 - \frac{1-\delta_k}{\mu^I}\right)\hat{i}_t + \frac{1-\delta_k}{\mu^I}\hat{k}_t - \frac{1-\delta_k}{\mu^I}\hat{\mu}^I_t - \frac{\delta'_k}{\mu^I}\hat{u}_t, \tag{88}$$

$$[s^p_t] : \hat{s}^p_t = \frac{c}{c+i}\hat{c}_t + \frac{i}{c+i}\hat{i}_t, \tag{89}$$

$$[s_t] : \hat{s}_t = \frac{c}{s}\hat{c}_t + \frac{i}{s}\hat{i}_t + \frac{gx^G}{s}\hat{g}_t + \frac{gx^G}{s}\hat{x}^G_t, \tag{90}$$

$$[\mu_t^Y] : \hat{\mu}_t^Y = \hat{\mu}_t^X + \frac{\alpha_K}{\alpha_K - 1}\hat{\mu}_t^A, \tag{91}$$

$$[\mu_t^I] : \hat{\mu}_t^I = \hat{\mu}_t^Y - \hat{\mu}_t^A, \tag{92}$$

$$[x_t^G] : \hat{x}_t^G = \rho_{xg}\hat{x}_{t-1}^G - \hat{\mu}_t^Y, \tag{93}$$

$$[p_t] : \widehat{mc}_t = 0, \tag{94}$$

$$[n_t] : \widehat{mc}_t + \hat{y}_t - \hat{n}_t = \hat{w}_t, \tag{95}$$

$$[u_t k_t] : \widehat{mc}_t + \hat{y}_t - \hat{u}_t - \hat{k}_t = \hat{r}_t - \hat{\mu}_t^I, \tag{96}$$

$$[mc_t] : \hat{y}_t = \hat{s}_t, \tag{97}$$

$$[tech] : \hat{y}_t = \hat{z}_t + \alpha_K\hat{u}_t + \alpha_K\hat{k}_t + \alpha_N\hat{n}_t - \alpha_K\hat{\mu}_t^I. \tag{98}$$

C.6 Firm problem with stock-elastic inventories

Again, the firm side is subject to monopolistic competition. Firm $j \in [0, 1]$ solves the following problem:

$$\max \mathbb{E}_0 q_{0,t}\left[\frac{p_t(j)}{P_t}S_t(j) - W_t n_t(j) - R_t u_t(j)K_t(j)\right],$$

subject to

$$\begin{aligned}
S_t(j) &= \left(\frac{A_t(j)}{A_t}\right)^{\zeta_t}\left(\frac{p_t(j)}{P_t}\right)^{-\theta_t} S_t, \\
Y_t(j) &= z_t(u_t(j)K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K}, \\
A_t(j) &= (1-\delta_i)(A_{t-1}(j) - S_{t-1}(j)) + Y_t(j) \\
&\quad - \phi_y\left(\frac{Y_t(j)}{Y_{t-1}(j)}\right)Y_t(j) - \phi_{inv}\left(\frac{INV_t(j)}{INV_{t-1}(j)}\right)INV_t(j) - \phi_a\left(\frac{A_t(j)}{A_{t-1}(j)}\right)A_t(j), \\
INV_t(j) &= A_t(j) - S_t(j).
\end{aligned}$$

The firm problem now has an active dynamic margin by storing more goods and selling in the future, at the same time by being able to create more demand by producing more goods.[4] We can

[4] For quantitative issues on matching the smoothness of the aggregate stock of inventories, we also allow for adjustment costs for inventories. As we noted in the main paper, the smoothness of the stock of inventories relative to sales remains a challenge on inventory models. We leave this as future research and approximate that aspect by allowing for adjustment costs. However, we believe that the moment we focus on (which is the comovement property between inventories and components of sales) is not sensitive to the smoothness of the inventory series that we observe in the data.

state the firm problem as the following:

$$
\begin{aligned}
&\max \mathbb{E}_0 q_{0,t}\Bigg[\frac{p_t(j)}{P_t}S_t(j) - W_t n_t(j) - R_t u_t(j)K_t(j) + \tau_t(j)\{z_t(u_t(j)K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_N-\alpha_K} X_t^{1-\alpha_K} - Y_t(j)\} \\
&\quad + mc_t(j)\Bigg\{Y_t(j) + (1-\delta_i)(A_{t-1}(j) - S_{t-1}(j)) - A_t(j) - \phi_y\left(\frac{Y_t(j)}{Y_{t-1}(j)}\right)Y_t(j) \\
&\quad -\phi_{inv}\left(\frac{INV_t(j)}{INV_{t-1}(j)}\right)INV_t(j) - \phi_a\left(\frac{A_t(j)}{A_{t-1}(j)}\right)A_t(j)\Bigg\} + \varsigma_t(j)\left\{\left(\frac{A_t(j)}{A_t}\right)^{\zeta_t}\left(\frac{p_t(j)}{P_t}\right)^{-\theta_t}S_t - S_t(j)\right\}\Bigg],
\end{aligned}
$$

The first order conditions turn out to be the following:

$$
\begin{aligned}
[p_t(j)] &: S_t(j) = \theta_t\varsigma_t(j)\left(\frac{A_t(j)}{A_t}\right)^{\zeta_t}\left(\frac{p_t(j)}{P_t}\right)^{-\theta_t-1}S_t, \\
[S_t(j)] &: \frac{p_t(j)}{P_t} + mc_t(j)\left(\phi_{inv}\left(\frac{INV_t(j)}{INV_{t-1}(j)}\right) + \frac{INV_t(j)}{INV_{t-1}(j)}\phi'_{inv}\left(\frac{INV_t(j)}{INV_{t-1}(j)}\right)\right) \\
&\quad = \varsigma_t(j) + \mathbb{E}_t q_{t,t+1} mc_{t+1}(j)(1-\delta_i) + \mathbb{E}_t q_{t,t+1} mc_{t+1}(j)\left(\frac{INV_{t+1}(j)}{INV_t(j)}\right)^2\phi'_{inv}\left(\frac{INV_{t+1}(j)}{INV_t(j)}\right), \\
[Y_t(j)] &: \tau_t(j) = mc_t(j)\left(1 - \phi_y\left(\frac{Y_t(j)}{Y_{t-1}(j)}\right) - \phi'_y\left(\frac{Y_t(j)}{Y_{t-1}(j)}\right)\right) \\
&\quad + \mathbb{E}_t q_{t,t+1} mc_{t+1}(j)\left(\frac{Y_{t+1}(j)}{Y_t(j)}\right)^2\phi'_y\left(\frac{Y_{t+1}(j)}{Y_t(j)}\right), \\
[n_t(j)] &: \alpha_N\tau_t(j)\frac{Y_t(j)}{N_t(j)} = W_t, \\
[u_t(j)K_t(j)] &: \alpha_k\tau_t(j)\frac{Y_t(j)}{u_t(j)K_t(j)} = R_t,
\end{aligned}
$$

$$
\begin{aligned}
[A_t(j)] &: mc_t(j)\Bigg(1 + \phi_{inv}\left(\frac{INV_t(j)}{INV_{t-1}(j)}\right) + \frac{INV_t(j)}{INV_{t-1}(j)}\phi'_{inv}\left(\frac{INV_t(j)}{INV_{t-1}(j)}\right) \\
&\quad +\phi_a\left(\frac{A_t(j)}{A_{t-1}(j)}\right) + \frac{A_t(j)}{A_{t-1}(j)}\phi'_a\left(\frac{A_t(j)}{A_{t-1}(j)}\right)\Bigg) = \varsigma_t(j)\zeta_t\left(\frac{A_t(j)}{A_t}\right)^{\zeta_t}\left(\frac{p_t(j)}{P_t}\right)^{-\theta_t}\frac{S_t}{A_t(j)} \\
&\quad + \mathbb{E}_t q_{t,t+1} mc_{t+1}(j)\left[(1-\delta_i) + \left(\frac{INV_{t+1}(j)}{INV_t(j)}\right)^2\phi'_{inv}\left(\frac{INV_{t+1}(j)}{INV_t(j)}\right) + \left(\frac{A_{t+1}(j)}{A_t(j)}\right)^2\phi'_a\left(\frac{A_{t+1}(j)}{A_t(j)}\right)\right], \\
[INV_t(j)] &: INV_t(j) = A_t(j) - S_t(j).
\end{aligned}
$$

In a symmetric equilibrium, the following conditions hold:

$$
\begin{aligned}
[\tau_t] &: Y_t = z_t(u_tK_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K},\\
[mc_t] &: A_t = (1-\delta_i)(A_{t-1}-S_{t-1}) + Y_t - Y_t\phi_y\left(\frac{Y_t}{Y_{t-1}}\right) - INV_t\phi_{inv}\left(\frac{INV_t}{INV_{t-1}}\right) - A_t\phi_a\left(\frac{A_t}{A_{t-1}}\right),\\
[p_t] &: 1 = \theta_t\varsigma_t,\\
[S_t] &: 1 + mc_t\left(\phi_{inv}\left(\frac{INV_t}{INV_{t-1}}\right) + \frac{INV_t}{INV_{t-1}}\phi'_{inv}\left(\frac{INV_t}{INV_{t-1}}\right)\right)\\
&= \varsigma_t + \mathbb{E}_t q_{t,t+1} mc_{t+1}\left[1-\delta_i + \left(\frac{INV_{t+1}}{INV_t}\right)^2 \phi'_{inv}\left(\frac{INV_{t+1}}{INV_t}\right)\right],\\
[Y_t] &: \tau_t = mc_t\left(1-\phi_y\left(\frac{Y_t}{Y_{t-1}}\right) - \phi'_y\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \mathbb{E}_t q_{t,t+1} mc_{t+1}\left(\frac{Y_{t+1}}{Y_t}\right)^2 \phi'_y\left(\frac{Y_{t+1}}{Y_t}\right),\\
[n_t] &: \alpha_N\tau_t\frac{Y_t}{n_t} = W_t,\\
[u_tK_t] &: \alpha_K\tau_t\frac{Y_t}{u_tK_t} = R_t,\\
[A_t] &: mc_t\left(1+\phi_{inv}\left(\frac{INV_t}{INV_{t-1}}\right) + \frac{INV_t}{INV_{t-1}}\phi'_{inv}\left(\frac{INV_t}{INV_{t-1}}\right) + \phi_a\left(\frac{A_t}{A_{t-1}}\right) + \frac{A_t}{A_{t-1}}\phi'_a\left(\frac{A_t}{A_{t-1}}\right)\right)\\
&= \varsigma_t\zeta_t\frac{S_t}{A_t} + \mathbb{E}_t q_{t,t+1} mc_{t+1}\left[(1-\delta_i) + \left(\frac{INV_{t+1}}{INV_t}\right)^2\phi'_{inv}\left(\frac{INV_{t+1}}{INV_t}\right) + \left(\frac{A_{t+1}}{A_t}\right)^2\phi'_a\left(\frac{A_{t+1}}{A_t}\right)\right],\\
[INV_t] &: INV_t = A_t - S_t.
\end{aligned}
$$

Note that $\varsigma_t = 1/\theta_t$. Hence simplifying the above notation we get the following 8 conditions:

$$[\tau_t] : Y_t = z_t(u_tK_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N},$$

$$[mc_t]: A_t = (1-\delta_i)(A_{t-1} - S_{t-1}) + Y_t - Y_t\phi_y\left(\frac{Y_t}{Y_{t-1}}\right) - INV_t\phi_{inv}\left(\frac{INV_t}{INV_{t-1}}\right) - A_t\phi_a\left(\frac{A_t}{A_{t-1}}\right),$$

$$[S_t]: \frac{\theta_t - 1}{\theta_t} + mc_t\left(\phi_{inv}\left(\frac{INV_t}{INV_{t-1}}\right) + \frac{INV_t}{INV_{t-1}}\phi'_{inv}\left(\frac{INV_t}{INV_{t-1}}\right)\right)$$

$$= \mathbb{E}_t q_{t,t+1} mc_{t+1}\left[(1-\delta_i) + \left(\frac{INV_{t+1}}{INV_t}\right)^2 \phi'_{inv}\left(\frac{INV_{t+1}}{INV_t}\right)\right],$$

$$[Y_t]: \tau_t = mc_t\left(1 - \phi_y\left(\frac{Y_t}{Y_{t-1}}\right) - \phi'_y\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \mathbb{E}_t q_{t,t+1} mc_{t+1}\left(\frac{Y_{t+1}}{Y_t}\right)^2 \phi'_y\left(\frac{Y_{t+1}}{Y_t}\right),$$

$$[n_t]: \alpha_N \tau_t \frac{Y_t}{n_t} = W_t,$$

$$[u_t K_t]: \alpha_K \tau_t \frac{Y_t}{u_t K_t} = R_t,$$

$$[A_t]: mc_t\left(1 + \phi_{inv}\left(\frac{INV_t}{INV_{t-1}}\right) + \frac{INV_t}{INV_{t-1}}\phi'_{inv}\left(\frac{INV_t}{INV_{t-1}}\right) + \phi_a\left(\frac{A_t}{A_{t-1}}\right) + \frac{A_t}{A_{t-1}}\phi'_a\left(\frac{A_t}{A_{t-1}}\right)\right)$$

$$= \frac{\zeta_t}{\theta_t}\frac{S_t}{A_t} + \mathbb{E}_t q_{t,t+1} mc_{t+1}\left[(1-\delta_i) + \left(\frac{INV_{t+1}}{INV_t}\right)^2 \phi'_{inv}\left(\frac{INV_{t+1}}{INV_t}\right) + \left(\frac{A_{t+1}}{A_t}\right)^2 \phi'_a\left(\frac{A_{t+1}}{A_t}\right)\right],$$

$$[INV_t]: INV_t = A_t - S_t.$$

Expressing these into stationary variables (with $A_t = a_t X_t^Y$ and $INV_t = inv_t X_t^Y$):

$$[\tau_t]: y_t = z_t (u_t k_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N} (\mu_t^I)^{-\alpha_K},$$

$$[mc_t]: a_t\mu_t^Y = (1-\delta_i)(a_{t-1} - s_{t-1}) + y_t\mu_t^Y - y_t\mu_t^Y\phi_y\left(\frac{y_t}{y_{t-1}}\mu_t^Y\right)$$

$$- inv_t\mu_t^Y\phi_{inv}\left(\frac{inv_t}{inv_{t-1}}\mu_t^Y\right) - a_t\mu_t^Y\phi_a\left(\frac{a_t}{a_{t-1}}\mu_t^Y\right),$$

$$[s_t]: \frac{\theta_t - 1}{\theta_t} + mc_t\left(\phi_{inv}\left(\frac{inv_t}{inv_{t-1}}\mu_t^Y\right) + \frac{inv_t}{inv_{t-1}}\mu_t^Y\phi'_{inv}\left(\frac{inv_t}{inv_{t-1}}\mu_t^Y\right)\right)$$

$$= \mathbb{E}_t q_{t,t+1} mc_{t+1}\left[(1-\delta_i) + \left(\frac{inv_{t+1}}{inv_t}\mu_{t+1}^Y\right)^2 \phi'_{inv}\left(\frac{inv_{t+1}}{inv_t}\mu_{t+1}^Y\right)\right],$$

$$[y_t]: \tau_t = mc_t\left(1 - \phi_y\left(\frac{y_t}{y_{t-1}}\mu_t^Y\right) - \phi'_y\left(\frac{y_t}{y_{t-1}}\mu_t^Y\right)\right) + \mathbb{E}_t q_{t,t+1} mc_{t+1}\left(\frac{y_{t+1}}{y_t}\mu_{t+1}^Y\right)^2 \phi'_y\left(\frac{y_{t+1}}{y_t}\mu_{t+1}^Y\right),$$

$$[n_t]: \alpha_N \tau_t \frac{y_t}{n_t} = w_t,$$

$$[u_t k_t]: \alpha_K \tau_t \frac{y_t}{u_t k_t} = \frac{r_t}{\mu_t^I},$$

$$[A_t] : mc_t \left[1 + \phi_{inv}\left(\frac{inv_t}{inv_{t-1}}\mu_t^Y\right) + \left(\frac{inv_t}{inv_{t-1}}\mu_t^Y\right)\phi'_{inv}\left(\frac{inv_t}{inv_{t-1}}\mu_t^Y\right) + \phi_a\left(\frac{a_t}{a_{t-1}}\mu_t^Y\right) + \left(\frac{a_t}{a_{t-1}}\mu_t^Y\right)\phi'_a\left(\frac{a_t}{a_{t-1}}\mu_t^Y\right)\right] = \frac{\zeta_t}{\theta_t}\frac{s_t}{a_t}$$
$$+ \mathbb{E}_t q_{t,t+1} mc_{t+1}\left[(1-\delta_i) + \left(\frac{inv_{t+1}}{inv_t}\mu_{t+1}^Y\right)^2 \phi'_{inv}\left(\frac{inv_{t+1}}{inv_t}\mu_{t+1}^Y\right) + \left(\frac{a_{t+1}}{a_t}\mu_{t+1}^Y\right)^2 \phi'_a\left(\frac{a_{t+1}}{a_t}\mu_{t+1}^Y\right)\right],$$

$$[INV_t] : inv_t = a_t - s_t.$$

Writing $\mu_t = \theta_t/(\theta_t - 1)$, the 8 log-linearized conditions are the following:

$$[\tau_t] : \hat{y}_t = \hat{z}_t + \alpha_K \hat{u}_t + \alpha_K \hat{k}_t + \alpha_N \hat{n}_t - \alpha_K \hat{\mu}_t^I, \quad (99)$$

$$[mc_t] : a\mu^Y \hat{a}_t + a\mu^Y \hat{\mu}_t^Y = (1-\delta_i)a\hat{a}_{t-1} - (1-\delta_i)s\hat{s}_{t-1} + y\mu^Y \hat{y}_t + y\mu^Y \hat{\mu}_t^Y, \quad (100)$$

$$[s_t] : (\mu^Y)^2 \phi''_{inv}(\widehat{inv}_t - \widehat{inv}_{t-1} + \hat{\mu}_t^Y)$$
$$= \beta(\mu^Y)^{-\sigma}(1-\delta_i)[\hat{\mu}_t - \hat{r}_t^f + \mathbb{E}_t\widehat{mc}_{t+1}] + \beta(\mu^Y)^{3-\sigma}\phi''_{inv}[\mathbb{E}_t\widehat{inv}_{t+1} - \widehat{inv}_t + \mathbb{E}_t\hat{\mu}_{t+1}^Y], \quad (101)$$

$$[y_t] : \hat{\tau}_t = \widehat{mc}_t + \beta(\mu^Y)^{3-\sigma}\phi''_y \mathbb{E}_t\hat{y}_{t+1} - (\mu^Y + \beta(\mu^Y)^{3-\sigma})\phi''_y \hat{y}_t + \mu^Y\phi''_y\hat{y}_{t-1}$$
$$+ \beta(\mu^Y)^{3-\sigma}\phi''_y \mathbb{E}_t\hat{\mu}_{t+1}^Y - \mu^Y\phi''_y\hat{\mu}_t^Y, \quad (102)$$

$$[n_t] : \hat{\tau}_t + \hat{y}_t - \hat{n}_t = \hat{w}_t, \quad (103)$$

$$[u_t k_t] : \hat{\tau}_t + \hat{y}_t - \hat{u}_t - \hat{k}_t = \hat{r}_t - \hat{\mu}_t^I, \quad (104)$$

$$[a_t] : \widehat{mc}_t + (\mu^Y)^2\phi''_{inv}[\widehat{inv}_t - \widehat{inv}_{t-1} + \hat{\mu}_t^Y] + (\mu^Y)^2\phi''_a[\hat{a}_t - \hat{a}_{t-1} + \hat{\mu}_t^Y]$$
$$= (1 - \beta(\mu^Y)^{-\sigma}(1-\delta_i))\left(\hat{\zeta}_t + \hat{s}_t - \hat{a}_t + \frac{1}{\mu - 1}\hat{\mu}_t\right) + \beta(\mu^Y)^{-\sigma}(1-\delta_i)(-\hat{r}_t^f + \mathbb{E}_t\widehat{mc}_{t+1})$$
$$+ \beta(\mu^Y)^{3-\sigma}\phi''_{inv}[\mathbb{E}_t\widehat{inv}_{t+1} - \widehat{inv}_t + \mathbb{E}_t\hat{\mu}_{t+1}^Y] + \beta(\mu^Y)^{3-\sigma}\phi''_a[\mathbb{E}_t\hat{a}_{t+1} - \hat{a}_t + \mathbb{E}_t\hat{\mu}_{t+1}^Y], \quad (105)$$

$$[inv_t] : inv\widehat{inv}_t = a\hat{a}_t - s\hat{s}_t. \quad (106)$$

C.7 Computing the steady state in the stock-elastic inventory model

Again, we target directly the markup μ and in the inventory model, note that $mc = [\mu\beta(\mu^Y)^{-\sigma}(1-\delta_i)]^{-1}$. The values for n, μ^Y, μ^I, r, u, δ'_k, k, y, i, and w are all obtained in the same manner as in the no-inventory model.

The new parameters and steady-state values we compute are ζ, δ_i, a, inv, τ. First, δ_i is calibrated directly and $\tau = mc$. To obtain ζ, we target the steady-state stock-sales ratio a/s in the data. Using the two inventory conditions, we get

$$\zeta = \frac{1}{\mu - 1}\left(\frac{1 - \beta(\mu^Y)^{-\sigma}(1 - \delta_i)}{\beta(\mu^Y)^{-\sigma}(1 - \delta_i)}\right)\frac{a}{s}.$$

From this, we also get

$$s = \frac{\mu^Y y}{\mu^Y - 1 + \delta_i} / \left(\frac{a}{s} + \frac{1 - \delta_i}{\mu^Y - 1 + \delta_i}\right),$$

$$a = \frac{a}{s}s.$$

Therefore, $c = s - i - x^G g$. The same procedure follows in getting the values for h, ψ, m, λ_m, λ_h, λ, λ_k.

C.8 Writing down all the equilibrium conditions for the stock-elastic inventory model

The 24 endogenous variables are

$$m_t, \lambda_{m,t}, \lambda_{h,t}, n_t, c_t, h_t, \lambda_t, w_t, \lambda_{k,t}, i_t, r_t, u_t, r_t^f, k_{t+1}, s_t^p, s_t, mc_t, y_t, \tau_t, a_t, inv_t, x_t^G, \mu_t^Y, \mu_t^I.$$

The 3 endogenous variables τ_t, a_t, inv_t are newly added in the inventory model. The 9 exogenous processes are $\zeta_{h,t}, \psi_t, z_t, z_t^k, g_t, \mu_t^X, \mu_t^A, \zeta_t, \mu_t$. The 24 endogenous equations are:

$$[m_t] : \hat{\zeta}_{h,t} - \sigma \hat{m}_t = \hat{\lambda}_{m,t}, \tag{107}$$

$$\begin{aligned} [h_t] : &\hat{\lambda}_{h,t} - [1 - \beta(\mu^Y)^{1-\sigma}(1-\gamma_h)] \left[\hat{\lambda}_{m,t} + \hat{\psi}_t + (1+\xi^{-1})\hat{n}_t\right] \\ &= \beta(\mu^Y)^{1-\sigma}(1-\gamma_h)[\mathbb{E}_t\lambda_{h,t+1} + (1-\sigma)\mathbb{E}_t\hat{\mu}_{t+1}^Y + \mathbb{E}_t\hat{h}_{t+1} - \hat{h}_t], \end{aligned} \tag{108}$$

$$\begin{aligned} [c_t] : &\lambda\hat{\lambda}_t = \lambda_m\hat{\lambda}_{m,t} - \lambda_h\gamma_h(\mu^Y)^{1-\frac{1}{\gamma_h}} \left[\hat{\lambda}_{h,t} + \hat{h}_t - \frac{\mu^Y}{\mu^Y - b}\hat{c}_t + \frac{b}{\mu^Y - b}\hat{c}_{t-1} - \frac{b}{\mu^Y - b}\hat{\mu}_t^Y\right] \\ &+ \sigma\beta b(\mu^Y)^{-\sigma}\left[\lambda_m - \lambda_h\gamma_h(\mu^Y)^{1-\frac{1}{\gamma_h}}\right]\mathbb{E}_t\hat{\mu}_{t+1}^Y - \beta b(\mu^Y)^{-\sigma}\lambda_m\mathbb{E}_t\hat{\lambda}_{m,t+1} \\ &+ \beta b(\mu^Y)^{-\sigma}\lambda_h\gamma_h(\mu^Y)^{1-\frac{1}{\gamma_h}}\left[\mathbb{E}_t\hat{\lambda}_{h,t+1} + \mathbb{E}_t\hat{h}_{t+1} - \frac{\mu^Y}{\mu^Y - b}\mathbb{E}_t\hat{c}_{t+1} + \frac{b}{\mu^Y - b}\hat{c}_t - \frac{b}{\mu^Y - b}\mathbb{E}_t\hat{\mu}_{t+1}^Y\right], \end{aligned} \tag{109}$$

$$[n_t] : \hat{\lambda}_t + \hat{w}_t = \hat{\lambda}_{m,t} + \hat{\psi}_t + \frac{1}{\xi}\hat{n}_t + \hat{h}_t, \tag{110}$$

$$[i_t] : \hat{\lambda}_{k,t} = \hat{\lambda}_t - \hat{z}_t^k + \mu^I\phi''(\mu^I)(\hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^I) - \beta\frac{\mu^A}{(\mu^Y)^\sigma}(\mu^I)^3\phi''(\mu^I)(\mathbb{E}_t\hat{i}_{t+1} - \hat{i}_t + \mathbb{E}_t\hat{\mu}_{t+1}^I), \tag{111}$$

$$\begin{aligned} [k_{t+1}] : &\hat{\lambda}_{k,t} = \mathbb{E}_t\hat{\mu}_{t+1}^A - \sigma\mathbb{E}_t\hat{\mu}_{t+1}^Y + \beta(\mu^Y)^{-\sigma}\mu^A(1-\delta_k)\mathbb{E}_t\hat{\lambda}_{k,t+1} \\ &+ [1 - \beta(\mu^Y)^{-\sigma}\mu^A(1-\delta_k)](\mathbb{E}_t\hat{\lambda}_{t+1} + \mathbb{E}_t\hat{r}_{t+1} + \mathbb{E}_t\hat{u}_{t+1}) - \beta(\mu^Y)^{-\sigma}\mu^A\delta_k'\mathbb{E}_t\hat{u}_{t+1}, \end{aligned} \tag{112}$$

$$[u_t] : \hat{\lambda}_t + \hat{r}_t = \hat{\lambda}_{k,t} + \frac{\delta_k''}{\delta_k'}\hat{u}_t, \qquad [\hat{u}_t = 0 \text{ if not allowed to vary}], \tag{113}$$

$$[b_{t+1}] : -\hat{r}_t^f = \mathbb{E}_t\hat{\lambda}_{t+1} - \hat{\lambda}_t - \sigma\mathbb{E}_t\hat{\mu}_{t+1}^Y, \qquad [\text{written in terms of the real interest rate}], \tag{114}$$

$$[\lambda_{m,t}] : m\hat{m}_t = c\hat{c}_t - b\frac{c}{\mu^Y}\hat{c}_{t-1} + b\frac{c}{\mu^Y}\hat{\mu}_t^Y - \psi\frac{n^{1+\xi^{-1}}}{1+\xi^{-1}}h\left[\hat{\psi}_t + \hat{h}_t + (1+\xi^{-1})\hat{n}_t\right], \tag{115}$$

$$[\lambda_{h,t}] : \hat{h}_t = \frac{\gamma_h\mu^Y}{\mu^Y - b}\hat{c}_t - b\frac{\gamma_h}{\mu^Y - b}\hat{c}_{t-1} + b\frac{\gamma_h}{\mu^Y - b}\hat{\mu}_t^Y + (1-\gamma_h)\hat{h}_{t-1} - (1-\gamma_h)\hat{\mu}_t^Y, \tag{116}$$

$$[\lambda_{k,t}] : \hat{k}_{t+1} = \left(1 - \frac{1-\delta_k}{\mu^I}\right)\hat{z}_t^k + \left(1 - \frac{1-\delta_k}{\mu^I}\right)\hat{i}_t + \frac{1-\delta_k}{\mu^I}\hat{k}_t - \frac{1-\delta_k}{\mu^I}\hat{\mu}_t^I - \frac{\delta_k'}{\mu^I}\hat{u}_t, \tag{117}$$

$$[s_t^p] : \hat{s}_t^p = \frac{c}{c+i}\hat{c}_t + \frac{i}{c+i}\hat{i}_t, \tag{118}$$

$$[s_t] : \hat{s}_t = \frac{c}{s}\hat{c}_t + \frac{i}{s}\hat{i}_t + \frac{gx^G}{s}\hat{g}_t + \frac{gx^G}{s}\hat{x}_t^G, \tag{119}$$

$$[\mu_t^Y]: \hat{\mu}_t^Y = \hat{\mu}_t^X + \frac{\alpha_K}{\alpha_K - 1}\hat{\mu}_t^A, \tag{120}$$

$$[\mu_t^I]: \hat{\mu}_t^I = \hat{\mu}_t^Y - \hat{\mu}_t^A, \tag{121}$$

$$[x_t^G]: \hat{x}_t^G = \rho_{xg}\hat{x}_{t-1}^G - \hat{\mu}_t^Y, \tag{122}$$

$$[\tau_t]: \hat{y}_t = \hat{z}_t + \alpha_K\hat{u}_t + \alpha_K\hat{k}_t + \alpha_N\hat{n}_t - \alpha_K\hat{\mu}_t^I, \tag{123}$$

$$[mc_t]: a\mu^Y\hat{a}_t + a\mu^Y\hat{\mu}_t^Y = (1-\delta_i)a\hat{a}_{t-1} - (1-\delta_i)s\hat{s}_{t-1} + y\mu^Y\hat{y}_t + y\mu^Y\hat{\mu}_t^Y, \tag{124}$$

$$\begin{aligned}[s_t]: &(\mu^Y)^2\phi''_{inv}(\widehat{inv}_t - \widehat{inv}_{t-1} + \hat{\mu}_t^Y)\\ &= \beta(\mu^Y)^{-\sigma}(1-\delta_i)[\hat{\mu}_t - \hat{r}_t^f + \mathbb{E}_t\widehat{mc}_{t+1}] + \beta(\mu^Y)^{3-\sigma}\phi''_{inv}[\mathbb{E}_t\widehat{inv}_{t+1} - \widehat{inv}_t + \mathbb{E}_t\hat{\mu}_{t+1}^Y],\end{aligned} \tag{125}$$

$$\begin{aligned}[y_t]: \hat{\tau}_t &= \widehat{mc}_t + \beta(\mu^Y)^{3-\sigma}\phi''_y\mathbb{E}_t\hat{y}_{t+1} - (\mu^Y + \beta(\mu^Y)^{3-\sigma})\phi''_y\hat{y}_t + \mu^Y\phi''_y\hat{y}_{t-1}\\ &+ \beta(\mu^Y)^{3-\sigma}\phi''_y\mathbb{E}_t\hat{\mu}_{t+1}^Y - \mu^Y\phi''_y\hat{\mu}_t^Y,\end{aligned} \tag{126}$$

$$[n_t]: \hat{\tau}_t + \hat{y}_t - \hat{n}_t = \hat{w}_t, \tag{127}$$

$$[u_tk_t]: \hat{\tau}_t + \hat{y}_t - \hat{u}_t - \hat{k}_t = \hat{r}_t - \hat{\mu}_t^I, \tag{128}$$

$$\begin{aligned}[a_t]: &\widehat{mc}_t + (\mu^Y)^2\phi''_{inv}[\widehat{inv}_t - \widehat{inv}_{t-1} + \hat{\mu}_t^Y] + (\mu^Y)^2\phi''_a[\hat{a}_t - \hat{a}_{t-1} + \hat{\mu}_t^Y]\\ &= (1-\beta(\mu^Y)^{-\sigma}(1-\delta_i))\left(\hat{\zeta}_t + \hat{s}_t - \hat{a}_t + \frac{1}{\mu-1}\hat{\mu}_t\right) + \beta(\mu^Y)^{-\sigma}(1-\delta_i)(-\hat{r}_t^f + \mathbb{E}_t\widehat{mc}_{t+1})\\ &+ \beta(\mu^Y)^{3-\sigma}\phi''_{inv}[\mathbb{E}_t\widehat{inv}_{t+1} - \widehat{inv}_t + \mathbb{E}_t\hat{\mu}_{t+1}^Y] + \beta(\mu^Y)^{3-\sigma}\phi''_a[\mathbb{E}_t\hat{a}_{t+1} - \hat{a}_t + \mathbb{E}_t\hat{\mu}_{t+1}^Y],\end{aligned} \tag{129}$$

$$[inv_t]: inv\widehat{inv}_t = a\hat{a}_t - s\hat{s}_t. \tag{130}$$

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