Online appendix to "What do inventories tell us about news-driven business cycles?"

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Abstract

This appendix reports the following:

A. the conditions characterizing the equilibrium of the baseline model of section 2;

B. a description of a real-business-cycle version of the stock-elastic demand model of Bils and Kahn (2000), a characterization of the elasticity of intertemporal substitution in production (EISP) in this model, and a derivation of analytical restrictions on structural parameters that guarantee positive comovement of inventories and sales in response to news;

C. a similar treatment of the stockout-avoidance model of Kryvtsov and Midrigan (2013);

D. verification of our empirical SVAR strategy using model simulated data;

E. sources for the data used in section 4, and additional results for alternative specifications of the empirical exercise of that section.

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A Detailed equilibrium conditions of the baseline model

The Lagrangian associated with the problem of the representative agent is:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t\geq 0} \beta^t \Bigg[& U(c_t, n_t; \psi_t) + \mu_t \left\{ y_t - inv_t + (1-\delta_i) inv_{t-1} - s_t \left(1 + \chi\left(\frac{inv_t}{s_t}\right)\right)\right\} \\ & + \lambda_t \left\{ s_t - (c_t + i_t) \right\} \\ & + mc_t \left\{ z_t (u_t k_t)^{1-\alpha} n_t^{\alpha} - y_t \right\} \\ & + q_t \left\{ i_t \left[1 - \phi\left(\frac{i_t}{i_{t-1}}\right)\right] + (1-\delta_k(u_t))k_t - k_{t+1} \right\} \Bigg]\end{aligned} \tag{1}$$

Eliminating $\mu_t = mc_t$, the set of equations characterizing a solution of (1) is:

$$\left(c_t - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}}\right)^{-\sigma} = \lambda_t \tag{2}$$

$$\left(c_t - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}}\right)^{-\sigma} \psi_t n_t^{\xi^{-1}} = mc_t \alpha z_t (u_t k_t)^{1-\alpha} n_t^{\alpha-1} \tag{3}$$

$$c_t + i_t = s_t \tag{4}$$

$$s_t + inv_t + s_t \chi\left(\frac{inv_t}{s_t}\right) = (1-\delta_i) inv_{t-1} + y_t \tag{5}$$

$$1 + \chi'\left(\frac{inv_t}{s_t}\right) = \mathbb{E}_t \left[\frac{\beta(1-\delta_i) mc_{t+1}}{mc_t}\right] \tag{6}$$

$$mc_t \left(1 + \chi\left(\frac{inv_t}{s_t}\right) - \frac{inv_t}{s_t}\chi'\left(\frac{inv_t}{s_t}\right)\right) = \lambda_t \tag{7}$$

$$\begin{aligned} q_t & \left(1 - \phi\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right)\phi'\left(\frac{i_t}{i_{t-1}}\right)\right) \\ & + \beta \mathbb{E}_t \left[q_{t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 \phi'\left(\frac{i_{t+1}}{i_t}\right)\right] = \lambda_t \end{aligned} \tag{8}$$

$$\beta \mathbb{E}_t \left[(1-\alpha) mc_{t+1} z_{t+1} u_{t+1}^{1-\alpha} \left(\frac{k_{t+1}}{n_{t+1}}\right)^{-\alpha} + (1-\delta_k(u_{t+1})) q_{t+1}\right] = q_t \tag{9}$$

$$mc_t (1-\alpha) z_t u_t^{-\alpha} k_t^{1-\alpha} n_t^{\alpha} = q_t \delta_k'(u_t) k_t \tag{10}$$

$$i_t \left(1 - \phi\left(\frac{i_t}{i_{t-1}}\right)\right) + (1-\delta_k(u_t)) k_t = k_{t+1} \tag{11}$$

$$z_t (u_t k_t)^{1-\alpha} n_t^{\alpha} = y_t \tag{12}$$

$$mc_t = \mu_t \tag{13}$$

B News shocks in the stock-elastic-demand inventory model

In this section, we describe and analyze a general equilibrium model of inventory dynamics based on the work of Pindyck (1994), Bils and Kahn (2000), and Jung and Yun (2006).

The key feature of the so-called "stock-elastic" demand model is the assumption that sales of a firm are elastic to the amount of goods available for sale, which we term "on-shelf goods." The positive elasticity of sales to on-shelf goods captures the idea that with more on-shelf goods, customers are more likely to find a good match and purchase the product.

B.1 Description of the stock-elastic demand model

The economy consists of a representative household and monopolistically competitive firms. The output of the firms are storable goods, of which they keep a positive inventory. We start with the household problem.

Household problem A representative household maximizes the following expected sum of discounted utility,

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t, n_t; \psi_t)\right], \tag{14}$$

where c_t is the consumption of the final good, n_t denotes the supply of labor services, and ψ_t is an exogenous variable that introduces a wedge between consumption and leisure, which we call the "labor wedge." We assume the following GHH period utility function:

$$U(c, n; \psi) = \frac{1}{1-\sigma}\left(c - \psi\frac{n^{1+\xi^{-1}}}{1+\xi^{-1}}\right)^{1-\sigma},$$

where ξ is the Frisch elasticity of labor supply and σ denotes the inverse of the elasticity of the household's intertemporal substitution. The household's maximization problem is subject to the following constraints:

$$\int_0^1 p_t(j)s_t(j)dj + \mathbb{E}_t\left[Q_{t,t+1}B_{t+1}\right] \leq W_t n_t + R_t k_t + \int_0^1 \pi_t(j)dj + B_t, \tag{15}$$

$$k_{t+1} = i_t\left[1 - \phi\left(\frac{i_t}{i_{t-1}}\right)\right] + (1-\delta_k)k_t, \tag{16}$$

$$c_t + i_t \leq x_t, \tag{17}$$

$$x_t = \left(\int_0^1 v_t(j)^{\frac{1}{\theta}} s_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}. \tag{18}$$

Equation (15) is the household budget constraint. The household earns income each period by providing labor n_t at a given nominal wage W_t, lending capital k_t at a rate R_t, claiming the nominal profit $\pi_t(j)$ from each firm $j \in [0,1]$, and receiving nominal bond payments B_t. It spends its income in purchases of each variety in the amount $s_t(j)$ at a price $p_t(j)$, and in purchases of the state-contingent one-period bonds B_{t+1}. The probability-adjusted price of each of these nominal bonds is $Q_{t,t+1}$, for each state in period $t+1$.

Equation (16) is the law of motion of capital with adjustment costs to investment. The adjustment cost function $\phi(\cdot)$ is twice-differentiable, with $\phi(1) = \phi'(1) = 0$ and $\phi''(1) > 0$. When firms' desired future level of capital is high, this type of adjustment cost forces them to smooth out the desired increase over time and start investing today.

Equation (17) states that the household's consumption and investment cannot exceed its total absorption of final goods, x_t, which is constructed by aggregating their purchase of intermediate goods $\{s_t(j)\}_{j\in[0,1]}$. The aggregation of the intermediate goods $\{s_t(j)\}_{j\in[0,1]}$ into x_t is given by a Dixit-Stiglitz type aggregator (18) where $v_t(j)$ is the taste-shifter for each product j and θ is the elasticity of substitution across intermediate goods. It follows from expenditure minimization that the demand function for each good and the aggregate price level take the following forms:

$$s_t(j) = v_t(j) \left(\frac{p_t(j)}{P_t} \right)^{-\theta} x_t, \qquad P_t = \left(\int_0^1 v_t(j) p_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}.$$

In the stock-elastic demand model, the taste shifter for variety j is assumed to depend on the amount of goods on shelf proposed by the firm producing variety j, $a_t(j)$, in the following fashion:

$$v_t(j) = \left(\frac{a_t(j)}{a_t} \right)^{\zeta}, \tag{19}$$

where the normalization by a_t, defined as the the economy-wide average of on-shelf goods, ensures that the mean of $\nu_t(j)$ across goods is equal to 1. The parameter $\zeta > 0$ controls the degree of the shift in taste due to the relative amount of goods on shelf.

Finally, the household is given an initial level of capital k_0 and bonds B_0, and its optimization problem is subject to no-Ponzi conditions for both capital and stage-contingent bond holdings.

Firm problem Each monopolistically competitive firm $j \in [0,1]$ maximizes the expected discounted sum of profits

$$E_0 \left[\sum_{t=0}^{\infty} Q_{0,t} \pi_t(j) \right], \tag{20}$$

where

$$\pi_t(j) = p_t(j) s_t(j) - W_t n_t(j) - R_t k_t(j). \tag{21}$$

Note that the profit in each period is the revenue from sales net of the cost from hiring labor $n_t(j)$ and renting capital $k_t(j)$ at their respective prices W_t and R_t. The term $Q_{0,t}$ is the discount factor of between period 0 and t, so that $Q_{0,t} = \prod_{T=0}^{t-1} Q_{T,T+1}$. This discount factor is consistent with households being the final owners of firms. The firm faces the following constraints:

$$a_t(j) = (1-\delta_i) inv_{t-1}(j) + y_t(j), \tag{22}$$

$$inv_t(j) = a_t(j) - s_t(j), \tag{23}$$

$$y_t(j) = z_t k_t^{1-\alpha}(j) n_t^{\alpha}(j), \tag{24}$$

$$s_t(j) = \left(\frac{a_t(j)}{a_t} \right)^{\zeta} \left(\frac{p_t(j)}{P_t} \right)^{-\theta} x_t. \tag{25}$$

Equation (22) is the inventory stock accumulation equation. The stock (on-shelf goods) of the firm, $a_t(j)$, consists of the undepreciated stock of inventories from the previous period $(1-\delta_i) inv_{t-1}(j)$ and of current production $y_t(j)$. The parameter δ_i denotes the depreciation rate of inventories. Equation (23) states that on-shelf goods that are unsold are accounted as inventories.[1] Equation (24) is the production function. Firms use a constant returns to scale production function, with capital and labor as inputs. The variable z_t represents total factor productivity and is exogenous. Finally, monopolistically competitive firms face the demand function (25) stemming from the household problem.

[1] In the data, this is recorded as the end-of-period inventory stock in each period.

Market clearing Labor and capital markets clear, and net bond holdings is zero:

$$n_t = \int_0^1 n_t(j)dj, \tag{26}$$

$$k_t = \int_0^1 k_t(j)dj, \tag{27}$$

$$B_t = 0. \tag{28}$$

Sales of goods for each variety j also clear, as is implicit in the expression of the demand function (25). The average level of on-shelf goods in the economy a_t is defined by:

$$a_t = \int_0^1 a_t(j)dj. \tag{29}$$

Since the price of the consumption good P_t is a numeraire, in what follows we will use the lower-case variables $w_t = W_t/P_t$ for real wage, $r_t = R_t/P_t$ for real rental rate of capital, $b_t = B_t/P_t$ for real bond holdings, $q_{t,t+1} = Q_{t,t+1}P_{t+1}/P_t$ for the real stochastic discount factor, and $\tilde{p}_t(j) = p_t(j)/P_t$ for the relative price of good j.

B.2 Equilibrium

A market equilibrium of this economy is a set of stochastic processes for aggregate variables

$$c_t, n_t, k_{t+1}, i_t, b_{t+1}, x_t, a_t, w_t, r_t, q_{t,t+1},$$

and firm-level variables

$$\{a_t(j)\}, \{n_t(j)\}, \{k_t(j)\}, \{v_t(j)\}, \{s_t(j)\}, \{y_t(j)\}, \{inv_t(j)\}, \{\tilde{p}_t(j)\},$$

such that, given the exogenous stochastic processes z_t, ψ_t, as well as initial conditions k_0, b_0 and $\{inv_{-1}(j)\}$:

- households maximize (14) subject to (15) - (19) and two no-Ponzi conditions,
- each firm $j \in [0, 1]$ maximizes (20) subject to (21) - (25),
- markets clear according to (26) - (29).

A market equilibrium of the stock-elastic demand model is always symmetric: $a_t(j) = a_t$, $s_t(j) = s_t$, $inv_t(j) = inv_t$, $y_t(j) = y_t$, and $p_t(j) = p_t$ for all j. Along with the law of motion for exogenous variable, the market equilibrium is characterized by the following set of equations:

$$\left(c_t - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}}\right)^{-\sigma} = \lambda_t \tag{30}$$

$$w_t = \psi_t n_t^{\xi^{-1}} \tag{31}$$

$$\xi_t \left(1 - \phi\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right)\phi'\left(\frac{i_t}{i_{t-1}}\right)\right) + \beta \mathbb{E}_t \left[\xi_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 \phi'\left(\frac{i_{t+1}}{i_t}\right)\right] = \lambda_t \tag{32}$$

$$i_t \left(1 - \phi\left(\frac{i_t}{i_{t-1}}\right)\right) + (1-\delta_k)k_t = k_{t+1} \tag{33}$$

$$\beta \mathbb{E}_t \left[(1-\delta_k)\xi_{t+1} + \lambda_{t+1} r_{t+1}\right] = \xi_t \tag{34}$$

$$c_t + i_t = x_t \tag{35}$$

$$z_t k_t^{1-\alpha} n_t^{\alpha} = y_t \tag{36}$$

$$mc_t \alpha \frac{y_t}{n_t} = w_t \tag{37}$$

$$mc_t (1-\alpha) \frac{y_t}{k_t} = r_t \tag{38}$$

$$(1-\delta_i) inv_{t-1} + y_t = s_t + inv_t \tag{39}$$

$$s_t + inv_t = a_t \tag{40}$$

$$\mathbb{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t}(1-\delta_i)\frac{mc_{t+1}}{mc_t}\right] = \gamma_t \tag{41}$$

$$\frac{1}{\mathbb{E}_t \left[(1-\delta_i) q_{t,t+1} mc_{t+1}\right]} = \mu_t \tag{42}$$

$$\zeta \frac{1}{1 + \frac{inv_t}{s_t}} = \frac{\frac{1}{\gamma_t} - 1}{\mu_t - 1} \tag{43}$$

$$\frac{\theta}{\theta - 1} = \mu_t \tag{44}$$

$$s_t = x_t \tag{45}$$

Conditions (30)-(35) characterize the optimum of the household's problem, conditions (36)-(43) characterize that of the firm, and condition (45) reflects market clearing for goods. Condition (43) characterizes its optimal choice of inventory holdings, while conditions (42) and (44) characterize optimal pricing by monopolistic firms in this environment. Conditions (39) and (40) are the law of motion for inventories, and the definition of goods on shelf, respectively.

The two exogenous processes in our economy are total factor productivity z_t and the labor wedge ψ_t. As in the main paper, we assume that these processes are driven by both surprise innovations and news shocks.

B.3 The optimal choice of inventories

The optimal stock choice of firms is governed by the equation:

$$mc_t = \frac{\partial s_t}{\partial a_t} + \left(1 - \frac{\partial s_t}{\partial a_t}\right) \mathbb{E}_t[q_{t,t+1}(1-\delta_i)mc_{t+1}]. \tag{46}$$

The left hand side of this equation represents the cost of adding an extra unit of goods to the stock of goods on sale, a_t, which equals the current marginal cost of production. The right hand side represents the two benefits of adding this extra unit. First, by producing and stocking an extra unit, the firm is able generate an additional fraction $(\partial s_t/\partial a_t)$ of sales. Second, since some of the extra goods stocked will not be sold and will be stored as inventories for the next period, future production costs are reduced.

It is important to notice that at the nonstochastic steady state of the economy, the stock of inventories is positive. Since the real interest rate and the inventory depreciation rate are both positive at the steady state, holding inventories is costly. However, consistent with the first term on the right hand side of (46), there is a convenience yield associated with holding a positive amount of inventories in each period. In the model, the convenience yield is the additional sales created by holding a positive level of stock. Therefore, there will be a positive amount of inventories in steady state, despite the intertemporal costs that holding inventories implies.

Rearranging, (46) can be expressed as:

$$\frac{\partial s_t}{\partial a_t} = \frac{\gamma_t^{-1} - 1}{\mu_t - 1}, \tag{47}$$

where:

$$\mu_t \equiv \frac{1}{(1-\delta_i)\mathbb{E}_t[q_{t,t+1}mc_{t+1}]}, \quad \gamma_t \equiv (1-\delta_i)\mathbb{E}_t\left[\frac{q_{t,t+1}mc_{t+1}}{mc_t}\right].$$

The variable μ_t is the markup of price over expected discounted future marginal cost. This is the relevant markup concept in an economy where firms produce to stock: indeed, the true cost of sales is not current but future marginal cost, since selling an extra unit reduces tomorrow's stock of goods. The variable γ_t is the expected discounted growth rate of marginal cost, which summarizes the firm's opportunity cost of producing today. The optimal stocking behavior of a firm balances these 3 margins: markup, discounted growth rate of marginal cost, and additional sales generated by extra inventory holdings.

In equilibrium, the optimal choice of inventories can be approximated up to first order as:

$$\widehat{inv}_t = \hat{s}_t + \eta^{SE} \hat{\gamma}_t,$$

where hatted variables represent log-deviations from its steady-state.[2] As in the main paper, this condition states that two forces determine the dynamics of inventories: a demand channel, according to which firms in this economy build up their inventories when sales are high; and an intertemporal substitution channel. The reduced-form parameter η^{SE} is the elasticity of intertemporal substitution in production (EISP) in this model; its relationship to structural parameters in derived in proposition 1 below.

B.4 The propagation of news

We now turn to studying the effect of news shocks in this model economy. Like in the main paper, we derive analytical conditions under which news shocks result in positive comovement on impact between sales and inventories, assess whether those conditions are likely to hold in reasonable calibrations of the model, and inspect the mechanisms underpinning the result.

B.4.1 Impact comovement

We analyze a first-order log-linear approximation of the model around its steady-state. The following framework summarizes the equilibrium conditions needed for the purpose of our subsequent analysis.

[2] This equation is derived by combining (23), (47) and the optimal pricing condition $\hat{\mu}_t = 0$.

Proposition 1 *On impact and without surprise shocks, so that $\hat{z}_t = 0$ and $\hat{\psi}_t = 0$, the market equilibrium can be approximated by:*

$$
\begin{aligned}
\widehat{mc}_t &= \omega \hat{y}_t, && (48)\\
\kappa \hat{y}_t &= \hat{s}_t + \frac{\kappa - 1}{\delta_i}[\widehat{inv}_t - (1-\delta_i)\widehat{inv}_{t-1}], && (49)\\
\widehat{inv}_t &= \hat{s}_t + \tau^{SE}\hat{\mu}_t + \eta^{SE}\hat{\gamma}_t, && (50)\\
\hat{\mu}_t &= 0, && (51)\\
\hat{\mu}_t + \hat{\gamma}_t + \widehat{mc}_t &= 0. && (52)
\end{aligned}
$$

The mapping from the structural model parameters to the parameters of the reduced-form equations is given by:

$$
\begin{aligned}
\omega &= \frac{1 + (1-\alpha)\xi}{\alpha\xi}, && (53)\\
\kappa &= 1 + \delta_i IS, && (54)\\
\eta^{SE} &= \frac{1 + IS}{IS}\frac{1}{1 - \beta(1-\delta_i)}, && (55)\\
\tau^{SE} &= \frac{1 + IS}{IS}\theta,
\end{aligned}
$$

where IS is the steady-state inventory-sales ratio, given by

$$
IS = \frac{(\theta - 1)(1 - \beta(1-\delta_i))}{\zeta\beta(1-\delta_i) - (\theta-1)(1-\beta(1-\delta_i))}.
$$

Equation (48) relates marginal cost to output, which is derived by combining the labor supply and demand conditions, and the production function. Importantly, this equation is not connected to the introduction of inventories in our model. The parameter ω is the elasticity of marginal cost with respect to output, keeping constant total factor productivity; it expression in terms of structural parameters is identical to the main paper.

Equation (49) is the law of motion for the stock of inventories, obtained from combining equations (22) and (23). This law of motion states that output should equal sales plus inventory investment. The parameter κ in (49) denotes the steady-state output to sales ratio.

Equations (50) and (51) are the optimal stocking and pricing conditions, respectively. Combining these two equations, we see that inventories are determined by the demand channel ($\hat{s}_t$) and the intertemporal substitution channel ($\eta^{SE}\hat{\gamma}_t$). Equation (55) indicates that a lower bound for η^{SE} is:

$$\eta^{SE} \geq \underline{\eta}^{SE} = \frac{1}{1-\beta(1-\delta_i))}.$$

The lower bound depends on two parameters, β and δ_i. First, the household discount factor β governs the opportunity cost of holding inventories. In the limiting case where $\beta = 1$, there is no opportunity cost of holding inventories since the real interest rate $1/\beta - 1$ is 0. Second, the depreciation rate of inventories δ_i represent the physical cost of holding inventories. Therefore, the value $1 - \beta(1 - \delta_i)$ represents the overall intertemporal cost of adjusting inventories.

Lastly, equation (52) follows from the definition of μ_t and γ_t. Using this first-order approximation, it is straightforward to establish the following analytical result.

Proposition 2 (The impact response of inventories to a good news about the future) *With news shocks ($\hat{z}_t = 0$ and $\hat{\psi}_t = 0$ but $\mathbb{E}_t\hat{z}_{t+k} \neq 0$ or $\mathbb{E}_t\hat{\psi}_{t+k} \neq 0$ for some $k > 0$) , inventories and sales positively comove on impact if and only if:*

$$\eta^{SE}\omega < \kappa.$$

This proposition indicates that the positive comovement between inventories and sales only depends on the three parameters discussed above, κ, ω and η. Note that κ is, in general, very close to 1; in NIPA data, it is approximately equalt to 1.005. Thus, following our discussion in the main paper, a conservative upper bound on κ/ω is $\frac{1.005}{0.33} \approx 3.3$. For the values of $\beta = 0.99$ and $\delta_i = 0.025$ used in the main paper, the lower bound on η is:

$$\underline{\eta}^{SE} = 28.8.$$

Thus, the condition for positive comovement between inventories and sales in this model is not met, and in fact, fails by an order of magnitude.

Parameter	Value	Description / Target
β	0.99	Subjective discount factor
σ	1	Household elasticity of intertemporal substitution
ξ	2.5	Frisch elasticity of labor supply
α	0.67	Labor share of income
$\bar{\delta}_k$	0.025	Capital depreciation rate
$\phi_I''(1)$	9.11	Investment adjustment cost
n	0.2	Steady state hours worked
ρ_z	0.99	Persistence of the productivity process
ρ_ψ	0.95	Persistence of the labor wedge process
δ_i	0.025	Inventory depreciation rate
ζ	0.25	$IS = 0.75$ in steady-state
θ	1.25	$\mu = \frac{\theta}{\theta-1} = 1.25$ in steady-state

Table 1: Calibration of the SE model used in the main paper.

B.4.2 Dynamic comovement

In the main paper, we report the impulse responses of the stock-elastic demand model, and compares them to that of our baseline model. The calibration of the stock-elastic demand model used in to construct these impulse responses is reported in table 1. This calibration matches the same target for the IS ratio as in the baseline calibration of the main paper, and all non-inventory parameters are identical. Note that the implied EISP in this model, η^{SE}, is larger than the lower bound $\bar{\eta}^{SE}$, since η is finite. As a result, intertemporal substitution is stronger in this model than in the baseline, and the fall in inventories is deeper and more protracted after the shock.

C News shocks in the stockout-avoidance inventory model

In this section, we describe a Real Business Cycle version of the stockout-avoidance models of Kahn (1987) and Kryvtsov and Midrigan (2013), and analyze its impact response to news shocks.

C.1 Model description

The economy consists of a representative household and monopolistically competitive firms, where again firms produce storable goods. Since many aspects of the model are similar to the stock-elastic demand model, we refer directly to these equations.

Household problem A representative household maximizes (14), subject to the household budget constraint (15), capital accumulation rule (16), and the resource constraint (17). The aggregation of goods $\{s_t(j)\}_{j\in[0,1]}$ into x_t is given by (18), where $v_t(j)$ is the taste shifter for product j in period t.

In stockout-avoidance models, in contrast to the stock-elastic demand models, this taste shifter is assumed to be exogenous. In particular, we assume it is identically distributed across firms and over time according to a cumulative distribution function $F(\cdot)$ with a support $\Omega(\cdot)$:

$$v_t(j) \sim F, \qquad v_t(j) \in \Omega. \tag{56}$$

For each product j, households cannot buy more than the goods on shelf $a_t(j)$, which is chosen by firms:

$$s_t(j) \leq a_t(j), \qquad \forall j \in [0,1]. \tag{57}$$

Although (57) also holds for the stock-elastic demand model, it has not been mentioned since it was never binding. Households observe these shocks, and the amount of goods on shelf $a_t(j)$, before making their purchase decisions. Firms, however, do not observe the shock $v_t(j)$ when deciding upon the amount $a_t(j)$ of goods that are placed on shelf, so that (57) occasionally binds, resulting in a stockout.

Again, a demand function and a price aggregator can be obtained from the expenditure minimization problem of the household. The demand function for product j becomes

$$s_t(j) = \min\left\{ v_t(j)\left(\frac{p_t(j)}{P_t}\right)^{-\theta} x_t,\ a_t(j) \right\}, \tag{58}$$

which states that when $v_t(j)$ is high enough so that demand is higher than the amount of on-shelf goods, a stockout occurs and demand is truncated at $a_t(j)$. The price aggregator P_t is given by:

$$P_t = \left(\int_0^1 v_t(j)\tilde{p}_t(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}. \tag{59}$$

The variable $\tilde{p}_t(j)$ is the Lagrange multiplier on constraint (57). It reflects the household's shadow valuation of goods of variety j. For varieties that do not stock out, $\tilde{p}_t(j) = p_t(j)$, whereas for varieties that do stock out, $\tilde{p}_t(j) > p_t(j)$.

Firm problem Each monopolistically competitive firm $j \in [0,1]$ maximizes (20) with $\pi_t(j)$ defined as

$$\pi_t(j) = p_t(j)\tilde{s}_t(j) - W_t n_t(j) - R_t k_t(j). \tag{60}$$

As explained before, firms do not observe the exogenous taste shifter $v_t(j)$ and hence their demand $s_t(j)$ when making their price and quantity decisions in period t. Therefore, they will have to form conditional expectations on sales $s_t(j)$. This conditional expectation is denoted by $\tilde{s}_t(j)$.

The constraints on the firm are (22), (23), (24) and the demand function (58) with a known distribution for the taste shifter $v_t(j)$ in (56). Notice that this distribution is identical across all firms and invariant to aggregate conditions. By law of large numbers, firms observe P_t and x_t in their demand function. Therefore, $\tilde{s}_t(j)$ in (60) is given by:

$$\tilde{s}_t(j) = \int_{v \in \Omega(v)} \min \left\{ v \left(\frac{p_t(j)}{P_t} \right)^{-\theta} x_t,\ a_t(j) \right\} dF(v). \tag{61}$$

Market clearing The market clearing conditions for labor, capital, and bond markets are identical to the stock-elastic model and are given by (26), (27) and (28). Sales of goods also clear by the demand function for each variety.

C.2 Equilibrium

A market equilibrium of the stockout-avoidance model is defined as follows.

Definition 1 (Market equilibrium of the stockout-avoidance model) *A market equilibrium in the stockout-avoidance model is a set of stochastic processes:*

$$c_t, n_t, k_{t+1}, i_t, B_{t+1}, x_t, \{a_t(j)\}, \{v_t(j)\}, \{s_t(j)\}, \{\tilde{s}_t(j)\}, \{y_t(j)\}, \{inv_t(j)\}, \{p_t(j)\}, W_t, R_t, P_t, Q_{t,t+1}$$

such that, given the exogenous stochastic process z_t and initial conditions k_0, B_0, and $\{inv_{-1}(j)\}$:

- *households maximize (14) subject to (15) - (18), (56) - (57), and a no-Ponzi condition,*

- *each firm $j \in [0,1]$ maximizes profits subject to (22) - (24), (60) - (61),*
- *markets clear according to (26) - (28).*

In what follows, we use the following notation for aggregate output, sales, and inventories:

$$y_t = \int_0^1 y_t(j)dj, \qquad s_t = \int_0^1 s_t(j)dj, \qquad inv_t = \int_0^1 inv_t(j)dj. \tag{62}$$

C.2.1 Equilibrium symmetry and the stock-out wedge

In stockout-avoidance models, a market equilibrium is not symmetric across firms. Indeed, because of the idiosyncratic taste shifters $\{\nu_t(j)\}$, realized sales $\{s_t(j)\}$ and end-of-period inventories $\{inv_t(j)\}$ differ across firms.

However, it can be shown that all firms make identical *ex-ante* choices. To see this, note first that for the same reason mentioned for the stock-elastic demand model, marginal cost is constant across firms. Second, the first-order conditions for optimal pricing and optimal choice of stock are given, respectively, by:

$$mc_t = \frac{\partial \tilde{s}_t(j)}{\partial a_t(j)} \frac{p_t(j)}{P_t} + \left(1 - \frac{\partial \tilde{s}_t(j)}{\partial a_t(j)}\right)(1-\delta_i)\mathbb{E}_t\left[q_{t,t+1} mc_{t+1}\right],$$

$$\frac{p_t(j)/P_t}{(1-\delta_i)\mathbb{E}_t\left[q_{t,t+1} mc_{t+1}\right]} = \frac{\theta}{\theta - 1 - \dfrac{\tilde{s}_t(j)}{p_t(j)} \dfrac{\partial \tilde{s}_t(j)}{\partial p_t(j)}},$$

where mc_t denotes nominal marginal cost deflated by P_t. Here, $\tilde{s}_t(j)$ denotes firm j's expected sales. Following equation (61), expected sales of firm j depend only on price $p_t(j)$ and on-shelf goods $a_t(j)$, and aggregate variables. In turn, the above optimality conditions can be solved to obtain a decision rule for $a_t(j)$ and $p_t(j)$ as a function of current and expected values of aggregate variables, so that the choices of individual firms for these variables are symmetric. This implies that there is a unique threshold of the taste shifter, common across firms, above which firms stock out. From (58), this threshold is given by:

$$\nu_t^*(j) = \nu_t^* = \left(\frac{p_t}{P_t}\right)^\theta \frac{a_t}{x_t}.$$

The fact that those firms with a taste shifter $\nu_t(j) \geq \nu_t^*$ run out of goods to sell implies that

$p_t \neq P_t$. Indeed, as emphasized in (59), the aggregate price level P_t depends on the household's marginal value of good j, $\tilde{p}_t(j)$. This marginal value equals the (symmetric) sales price p_t for all varieties that do not stockout. However, for varieties that run out of stock, households would like to purchase more of that good than what is on sale. Therefore, the household's marginal value of the good is higher than their market price: $\tilde{p}_t(j) > p_t$. Thus, the standard aggregation relation $P_t = p_t$ fails to hold, and instead, $P_t > p_t$. In what follows, we denote:

$$d_t = \frac{p_t}{P_t}.$$

This relative price term can be thought of as a stockout wedge. It is smaller when the household's valuation of the aggregate bundle of goods is large relative to the market price of varieties, that is, when stockouts are more likely. Formally, it can be shown that the wedge d_t is a strictly increasing function of ν_t^*, and therefore a decreasing function of the probability of stocking out, $1 - F(\nu_t^*)$.

With the stockout wedge, the firm-level markup μ_t differs from the definition of the aggregate markup used above (the inverse of the aggregate expected marginal cost). In what follows, we denote the aggregate by μ_t^A. Since $\mu_t = \frac{p_t}{P_t}\mu_t^A$, we have:

$$\mu_t = d_t \mu_t^A. \tag{63}$$

C.2.2 Equilibrium conditions

Using the results mentioned above, the equilibrium of the stockout-avoidance model is then described by (30)-(41) (where μ is replaced by μ_t^A), along with the following equations:

$$1 - F(\nu_t^*) = \frac{\frac{1}{\gamma_t} - 1}{\mu_t - 1}, \tag{64}$$

$$\frac{\theta}{\theta - 1 - \frac{1 - F(\nu_t^*)}{\int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu)}} = \mu_t, \tag{65}$$

$$\frac{\int_{\nu \leq \nu_t^*} \left(1 - \frac{\nu}{\nu_t^*}\right) dF(\nu)}{\int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu) + 1 - F(\nu_t^*)} = \frac{inv_t}{s_t}, \tag{66}$$

$$\mu_t = d_t \mu_t^A \tag{67}$$

$$\left(\int_{\nu\leq\nu_t^*}\nu dF(\nu)+\nu_t^*\int_{\nu>\nu_t^*}\left(\frac{\nu}{\nu_t^*}\right)^{\frac{1}{\theta}}dF(\nu)\right)^{\frac{1}{\theta-1}}=d_t, \tag{68}$$

$$\frac{\left((\nu_t^*)^{\frac{1}{\theta}}\int_{\nu\leq\nu_t^*}\frac{\nu}{\nu_t^*}dF(\nu)+\int_{\nu>\nu_t^*}\nu^{\frac{1}{\theta}}dF(\nu)\right)^{\frac{\theta}{\theta-1}}}{\int_{\nu\leq\nu_t^*}\frac{\nu}{\nu_t^*}dF(\nu)+1-F(\nu_t^*)}s_t=x_t. \tag{69}$$

Condition (64) determines the optimal choice of stock in the stockout avoidance model. Here, ν_t^* is related to the aggregate IS ratio through (66). Condition (65) is the optimal markup choice in the stockout-avoidance model which also depends on the IS ratio through (66), reflecting the dependence of the price elasticity of demand on the stock of goods on sale in this (not iso-elastic) model. The firm markup μ_t and the aggregate markup μ_t^A are linked by the stockout wedge d_t in equation (67). The stockout wedge itself is given by (68). Finally, condition (69) reflects market clearing when some varieties are out of stock.

C.3 An alternative log-linearized framework

There are two important differences between stockout-avoidance models and the stock-elastic demand model. The first difference is the occurrence of stockouts, which implies the existence of the stockout wedge and hence the difference between firm-level and aggregate markups as described above. The second difference is that, even in our flexible-price environment, firm-level markups are not set at a constant rate over future marginal cost, as they did in the stock-elastic demand model. These two differences mean that unlike stock-elastic demand models, we cannot exactly map this class of models into the same log-linearized framework. We need an alternative framework, which we provide in the following lemma.

Lemma 2 (The log-linearized framework for the stockout-avoidance model) *In an equilibrium of the stockout-avoidance model, if productivity z_t is at its steady-state value, on impact, up to a first order approximation around the steady-state, equations (48) and (49) hold (with μ_t replaced by μ_t^A), along with:*

$$\widehat{inv}_t=\hat{s}_t+\tau^{SA}\hat{\mu}_t+\eta^{SA}\hat{\gamma}_t, \tag{70}$$

$$\hat{\mu}_t=\hat{d}_t+\hat{\mu}_t^A, \tag{71}$$

$$\hat{d}_t = \epsilon_d \left(\widehat{inv}_t - \hat{s}_t \right), \tag{72}$$

$$\hat{\mu}_t = \epsilon_\mu \left(\widehat{inv}_t - \hat{s}_t \right). \tag{73}$$

In this approximation, the parameters ω and κ are given by the same expressions as in the stock-elastic demand model, (53) and (54), while the EISP $\eta^{SA} > 0$, and the reduced-form parameters $\tau^{SA} > 0$, $\epsilon_d > 0$, and ϵ_μ are given in section C.5.

Several points are worth mentioning.

First, note that the effect of intertemporal substitution in production on inventories is again summarized by a reduced-form parameter, the EISP η^{SA}. The EISP in the stockout-avoidance model can be written as:

$$\eta^{SA} = \Xi(IS, \mu) \underbrace{\frac{1 + IS}{IS} \frac{1}{1 - \beta(1 - \delta_i)}}_{= \eta^{SE}},$$

where:

$$\Xi(IS, \mu) = \frac{1 - F(\nu^*(IS, \mu))}{\nu^*(IS, \mu) f(\nu^*(IS, \mu))} (1 - (1 - F(\nu^*(IS, \mu))(1 + IS)).$$

The value $\nu^*(IS, \mu)$ is the steady state of the cutoff for stocking out, as functions of the steady-state IS ration and the firm-level markup, so that $(1 - F(\nu^*(IS, \mu))$ is the steady-state stockout probability. The value $\nu^*(IS, \mu)$ does not have a closed-form expression; section C.5 discusses the equations that implicitly define this cutoff. Note that the expression for η^{SA} is similar to the relative marginal cost elasticity in the stock-elastic demand model η^{SE}, save for the term $\Xi(IS, \mu)$. This term is related to the generalized hazard rate characterizing the cumulative distribution function of taste shifters. For the type of distributions considered in the literature, $\Xi(IS, \mu)$ is typically smaller than 1. Thus in general, $\eta^{SA} \leq \eta^{SE}$. That is, the intertemporal substitution channel is weaker in these models than in the stock-elastic demand model. The fact that some firms stock out of their varieties prevents them altogether from smoothing production over time by storing goods or depleting inventories. However, setting the targets at $IS = 0.75$ and $\mu = 1.25$, and assuming that the taste shifter follows a log-normal distribution, η^{SA} is approximately two thirds of η^{SA}. Therefore, given the large magnitude of the EISP in the stock-elastic demand model, the intertemporal substitution motive remains large even in the stockout-avoidance model.

Second, the optimal choice of inventories (70) depends on the firm-level markup $\hat{\mu}_t$, which is not equal to the aggregate markup $\hat{\mu}_t^A$.

Third, in equation (71), aggregate markups and firm-level markups are linked by the stockout wedge $\hat{d}_t$. This follows from the definition of firm-level markup and stockout wedge given in (63).

Fourth, note that the framework of lemma 2 now includes (72), an equation linking the stockout wedge to the aggregate IS ratio. As we argued previously, the stockout wedge is negatively related to the probability of stocking out. In turn, one can show that there is a strictly decreasing mapping between the stockout probability, or equivalently a strictly increasing mapping between ν_t^*, and the ratio of the average end-of-period inventory to sales:

$$IS_t = \frac{inv_t}{s_t} = \frac{\int_0^1 inv_t(j)dj}{\int_0^1 s_t(j)dj}.$$

A lower probability of stocking out (i.e. a higher ν_t^*) implies that firms will, on average, be left with a higher stock of inventories relative to the amount of goods sold. Combining these two mappings, we obtain that the stockout wedge is increasing in the aggregate IS ratio, so that $\epsilon_d > 0$.

Lastly, the framework of lemma 2 includes variable firm-level markups, as described in equation (73). This is because in stockout-avoidance models, the desired firm-level markup is not constant. Instead, it depends on the ratio of goods on-shelf to expected demand, which itself is linked to the probability of stocking out. One can show that for log-normal and pareto-distributed idiosyncratic demand shocks, μ_t is a strictly decreasing function of ν_t^*, and therefore an increasing function of the probability of stocking out. Thus, the elasticity ϵ_μ is typically negative. Intuitively, this is because when firms are likely to stock out, the price-elasticity of demand is lower, and therefore markups are higher. Indeed, with a high stockout probability, demand is mostly constrained by the amount of goods available for sale, and does not vary much with price changes. The converse intuition holds when the stockout probability is low.

Before moving on, note that this framework reduces to the stock-elastic demand model when the stockout wedge is absent and firm-level markups are constant, so that $\hat{d}_t = \hat{\mu}_t = \hat{\mu}_t^A = 0$. Hence the framework could also be viewed as a generalized version the stock-elastic demand model.

C.4 The response to news shocks

C.4.1 The impact response to news shocks

We now turn to discussing the effects of a news shock using our new log-linearized framework. We again maintain the assumption that the shock has the effect of increasing sales, $\hat{s}_t > 0$, while leaving current productivity unchanged, $\hat{z}_t = 0$, so that we can indeed used the log-linearized framework of lemma 2. Combining the equations of lemma 2, it is straightforward to rewrite the optimality condition for inventory choice as:

$$\widehat{inv}_t = -\tilde{\eta}^{SA}\omega\widehat{mc}_t + \hat{s}_t.$$

In this expression, the elasticity of inventories to relative marginal cost, $\tilde{\eta}^{SA}$ is given by:

$$\tilde{\eta}^{SA} = \frac{1}{1 - \eta^{SA}\epsilon_d + (\eta^{SA} - \tau^{SA})\epsilon_\mu}\eta^{SA} \tag{74}$$

In contrast to the stock-elastic demand model, $\tilde{\eta}^{SA}$ does not purely reflect the EISP. The EISP is now compensated for markup movements (the terms τ^{SA} and ϵ_μ) and for movements in the stockout wedge (the term ϵ_d).

Unlike in the stock-elastic demand model, the sign of $\tilde{\eta}^{SA}$ cannot in general be established. This is because its sign depends on the distribution of the idiosyncratic taste shock. However, for a very wide range of calibrations and for the pareto and log-normal distributions, $\tilde{\eta}^{SA}$ is negative. We document this in table 2. There, we compute different values of $\tilde{\eta}^{SA}$, for different pairs of values of σ_d, the standard deviation of the shock, and different values of the steady-state markup. In all cases, we force the shock to have a mean equal to 1. The standard deviations we consider range from 0.1 to 1, and the markups range from 1.05 to 1.75. In all cases, $\tilde{\eta}^{SA}$ is negative. In table 3, we perform the same exercise for pareto-distributed shocks, and results are similar.

These results can be understood using (74). First, as discussed before, since $\epsilon_\mu < 0$ for standard distributions, markups fall when the IS ratio increases. With a higher IS ratio, a stockout is less likely for a firm, so that its price elasticity of demand is high, and its charges low markups. Second, because $(\eta^{SA} - \tau)\epsilon_\mu > 0$, markup movements tend to attenuate the intertemporal substitution channel; that is, if we were to set $\epsilon_d = 0$, then $\tilde{\eta}^{SA} < \eta^{SA}$. Lower markups signal a higher future

Value of $\tilde{\eta}^{SA}$					
$\sigma_d \downarrow \|\|\mu \rightarrow$	**1.05**	**1.1**	**1.25**	**1.5**	**1.75**
0.1	-729.12	-278.08	-121.98	-77.39	-61.87
0.25	-307.22	-116.94	-51.42	-32.71	-26.20
0.5	-167.04	-63.17	-27.66	-17.57	-14.06
0.75	-120.68	-45.25	-19.59	-12.35	-9.85
1	-97.75	-36.33	-15.51	-9.66	-7.66

Implied IS ratio					
$\sigma_d \downarrow \|\|\mu \rightarrow$	**1.05**	**1.1**	**1.25**	**1.5**	**1.75**
0.1	0.05	0.09	0.15	0.18	0.21
0.25	0.12	0.23	0.39	0.50	0.57
0.5	0.23	0.47	0.83	1.13	1.32
0.75	0.32	0.69	1.31	1.88	2.26
1	0.41	0.90	1.81	2.73	3.36

Table 2: Value of $\tilde{\eta}^{SA}$ when idiosyncratic demand shocks follow a log-normal distribution with mean 1. Different lines correspond to different standard deviations of the associated normal distribution, and different columns to different steady-state markups. Values are for $\beta = 0.99$ and $\delta_i = 0.011$.

Value of $\tilde{\eta}^{SA}$					
$\sigma_d \downarrow \|\|\mu \rightarrow$	**1.05**	**1.1**	**1.25**	**1.5**	**1.75**
0.1	-1959.13	-297.78	-62.89	-27.02	-18.16
0.25	-926.82	-142.18	-30.44	-13.30	-9.06
0.5	-598.66	-92.85	-20.20	-8.98	-6.20
0.75	-499.86	-78.04	-17.14	-7.69	-5.35
1	-456.12	-71.51	-15.80	-7.13	-4.97

Implied IS ratio					
$\sigma_d \downarrow \|\|\mu \rightarrow$	**1.05**	**1.1**	**1.25**	**1.5**	**1.75**
0.1	0.03	0.07	0.15	0.22	0.26
0.25	0.05	0.15	0.34	0.51	0.63
0.5	0.09	0.25	0.57	0.90	1.13
0.75	0.10	0.30	0.71	1.15	1.48
1	0.11	0.33	0.80	1.31	1.70

Table 3: Value of $\tilde{\eta}^{SA}$ when shock follow a Pareto distribution with mean 1. Different lines correspond to different standard deviations for the Pareto distribution, and different columns to different steady-state markups. Values are for $\beta = 0.99$ and $\delta_i = 0.011$.

marginal cost to the firm, thereby leading it to increase inventories (for fixed current marginal cost). At the same time, higher markups lead the firm to increase its sales relative to available goods, leaving it with fewer inventories at the end of the period. On net, the first effect dominates, leading to higher inventories at the end of the period, and reducing thus the inventory-depleting effects of the shock. Finally, $\eta^{SA}\epsilon_d - (\eta^{SA} - \tau)\epsilon_\mu > 1$, so that $\tilde{\eta}^{SA} < 0$. Therefore, movements in the stockout wedge change the sign of the elasticity of inventories to marginal cost.

With $\tilde{\eta}^{SA} < 0$, the following results hold for the impact response of news shocks in the stockout-avoidance model.

Proposition 3 (The impact response to news shocks in the stockout-avoidance model) *In the stockout-avoidance model with $\tilde{\eta}^{SA} < 0$, when news arrive:*

1. *the inventory-to-sales ratio comoves positively with sales;*

2. *inventories and sales comove positively, if and only if:*

$$(-\tilde{\eta}^{SA})\omega < \kappa IS.$$

The first part of this proposition is by itself daunting to news shocks, since the IS ratio tends to be countercyclical (Kryvtsov and Midrigan, 2013).

The second part of proposition 3 provides a condition under which inventories comove positively with sales. Much as in the case of the stock-elastic demand model and the simple model of the main paper, this condition relates the degree of real rigidities, and a parameter depending on the IESP, $(-\tilde{\eta}^{SA})$. If the model is calibrated to match the same IS ratio as the baseline ($IS = 0.75$), along with identical intertemporal inventory holding costs ($\beta = 0.99$ and $\delta_i = 0.025$) and the same degree of real rigidities, then:

$$\eta^{SA} = 19.3 \quad , \quad -\tilde{\eta}^{SA} = 22.4 \quad \text{and} \quad \frac{\kappa IS}{\omega} = 0.7.$$

Again, the analytical restriction fails, this time by over an order of magnitude, and inventories comove negatively with sales on impact. As discussed in the main paper, this remains true across calibrations, for example, as one varies the steady-state IS ratio.

Parameter	Value	Description / Target
β	0.99	Subjective discount factor
σ	1	Household elasticity of intertemporal substitution
ξ	2.5	Frisch elasticity of labor supply
α	0.67	Labor share of income
$\bar{\delta}_k$	0.025	Capital depreciation rate
$\phi_I''(1)$	9.11	Investment adjustment cost
n	0.2	Steady state hours worked
ρ_z	0.99	Persistence of the productivity process
ρ_ψ	0.95	Persistence of the labor wedge process
δ_i	0.025	Inventory depreciation rate
σ_ν	0.604	SD of demand shock, chosen to match $IS = 0.75$
θ	6.7	chosen to match $\mu = 1.25$ in steady-state

Table 4: Calibration of the SA model used in the main paper.

C.4.2 Dynamic response

The main paper reports the impulse responses of the stockout-avoidance model, and compares them to that of our baseline model. The calibration is reported in table 4, and uses a log-normal distribution for idiosyncratic taste shocks. This calibration again matches the same target for the IS ratio as in the baseline calibration, and again all non-inventory parameters are identical.

C.5 Expressions for the reduced-form coefficients of lemma 2

In what follows, we denote the steady-state stockout probability by:

$$\Gamma = 1 - F(\nu^*).$$

First, note that the log-linear approximation of equation (66) is:

$$\widehat{inv}_t - \hat{s}_t = (1 - \Gamma(1 + IS))\frac{1 + IS}{IS}\hat{\nu}_t^*.$$

This implies that the IS ratio and the stockout threshold move in the same direction. Indeed, the restriction:

$$1 > \Gamma(1 + IS)$$

follows from the fact that in the steady state,

$$IS = \frac{\int_{\nu \le \nu^*} \left(1 - \frac{\nu}{\nu^*}\right) dF(\nu)}{\int_{\nu \le \nu^*} \frac{\nu}{\nu^*} dF(\nu) + \Gamma} \Leftrightarrow \frac{1}{1+IS} - \Gamma = \int_{\nu \le \nu^*} \frac{\nu}{\nu^*} dF(\nu) > 0.$$

Second, it can be shown that the log-linear approximations to equations (64), (65) and (68) are respectively given by:

$$\frac{\nu^* f(\nu^*)}{\Gamma} \hat{\nu}_t^* = \frac{\mu}{\mu - 1} \hat{\mu}_t^F + \frac{1}{1 - \beta(1-\delta_i)} \hat{\gamma}_t,$$

$$\hat{\mu}_t^F = (\mu - 1)\Gamma(1+IS) \left(1 - \frac{\nu^* f(\nu^*)}{\Gamma} \frac{1}{1 - \Gamma(1+IS)}\right) \nu_t^*,$$

$$\hat{d}_t = \frac{\mu - 1}{\mu} (1 - \Gamma(1+IS)) \Delta \hat{\nu}_t^*.$$

Here, the coefficient $\Delta \in (0,1]$ is defined as:

$$\Delta \equiv \frac{\int_{\nu > \nu^*} \left(\frac{\nu}{\nu^*}\right)^{\frac{1}{\theta}} dF(\nu)}{\int_{\nu \le \nu^*} \frac{\nu}{\nu^*} dF(\nu) + \int_{\nu > \nu^*} \left(\frac{\nu}{\nu^*}\right)^{\frac{1}{\theta}} dF(\nu)},$$

where the relationship between the parameter θ and the steady-state markup is given by:

$$\theta = \frac{\mu}{\mu - 1} \frac{1}{1 - \Gamma(1+IS)}.$$

Combining these equations, one arrives at the following expressions for the different reduced-form parameters defining the log-linear framework of lemma 2:

$$\tau^{SA} = \frac{\Gamma}{\nu^* f(\nu^*)} (1 - \Gamma(1+IS)) \frac{1+IS}{IS} \frac{\mu}{\mu - 1} > 0, \tag{75}$$

$$\eta^{SA} = \frac{\Gamma}{\nu^* f(\nu^*)} (1 - \Gamma(1+IS)) \frac{1+IS}{IS} \frac{1}{1 - \gamma} > 0, \tag{76}$$

$$\epsilon_d = \frac{IS}{1+IS} \frac{1}{1 - \Gamma(1+IS)} \frac{\mu - 1}{\mu} (1 - \Gamma(1+IS)) \Delta > 0, \tag{77}$$

$$\epsilon_\mu = \frac{IS}{1+IS} \frac{1}{1 - \Gamma(1+IS)} (\mu - 1)\Gamma(1+IS) \left(1 - \frac{\nu^* f(\nu^*)}{\Gamma} \frac{1}{1 - \Gamma(1+IS)}\right). \tag{78}$$

The implicit equations determining ν^*, along with the variance of F, σ_ν, as functions of IS and μ, are:

$$IS = \frac{\int_{\nu \le \nu^*} \left(1 - \frac{\nu}{\nu^*}\right) dF(\nu)}{\int_{\nu \le \nu^*} \frac{\nu}{\nu^*} dF(\nu) + (1 - F(\nu^*))},$$

$$1 - F(\nu^*) = \frac{\frac{1}{\beta(1-\delta_i)} - 1}{\mu - 1}.$$

D Monte-Carlo simulation

In this section, we illustrate the performance of our SVAR with 2-period dynamic restrictions (i.e. 2-period negative comovement between inventory investment and consumption as well as investment), in terms of recovering the "correct" impulse-response functions and forecast-error variance of output with regards to news shocks in our model-simulated data. Data are simulated from the calibrated model in the main paper. For clarity of exposition, TFP is assumed to be the sole exogenous process, and labor wedge movements are suppressed in the simulation. The TFP process has two innovations: surprise and news. The standard deviation of surprise innovation is set at 1. The experiment is done with 3 different numbers for the standard deviation of news innovations: 1/2, 1, 2.

The same VAR as in section 4 in the main paper is assumed, in particular, with 4 variables in logs (consumption, investment, inventories and output) and with 4 lags and a constant. Since 2 shocks are used in generating the data, small i.i.d. measurement errors are included (with standard deviations of 0.001) for each variable to avoid stochastic singularity in the estimation.

Figure 1 plots both the theoretical impulse response based on the model (blue dashed line) and the 80% credible set of the identified impulse response using simulated data, with different standard deviations for news innovations assumed.[3] Our 2-period dynamic sign restriction captures well both the average size and the longer run model dynamics of our key macro variables.

Figure 2 plots the posterior probability density of FEV for output. As expected, the posterior median FEV is close to the theoretical FEV. One thing to note from this figure is that as news shocks are assumed to explain a larger share of output volatility in the model, our 2-period dynamic restriction may potentially underestimate the importance of news shocks. In particular, when news shocks are assumed to account for around 77 percent of output volatility in 20 quarters, the posterior density estimates the FEV of output to be 68 percent. This may be due to the prior density of the VAR parameters implying a smaller FEV than the model. If more information is used to identify

[3]The scale of the theoretical impulse response is the standard deviation of the news shock times $\sqrt{2/\pi}$, which is the mean of the folded standard normal distribution.

news shocks consistent with the model, the posterior density may move closer to the true FEV.

To explore this channel, figure 3 plots the posterior probability density of FEV for output, under the case where news shocks are assumed to account for a large variation of output dynamics (standard deviation of news innovations = 2). Panel (a) plots the case when only impact restrictions are imposed. We find that the posterior median FEV (black solid line) is significantly smaller than the model FEV (blue dashed line). However, when more information is used, we find that the posterior median FEV aligns well with the model FEV. Assuming 2-period restrictions in panel (b) and 3-period restrictions in panel (c), the posterior median FEV becomes closer to the model FEV. Hence, given that proper sign restrictions are imposed, we verify in our simulated example that using more information on the comovement of inventories and demand improves on recovering the true FEV of output.

E Empirical appendix

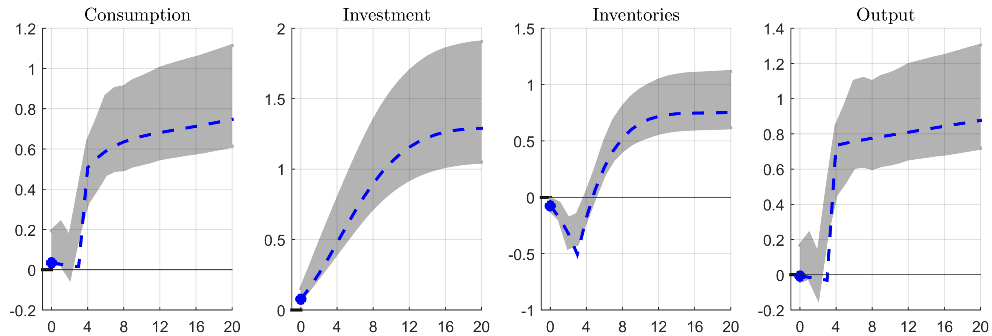
E.1 Data sources

Consumption is the sum of nominal nondurables and services (NIPA table 1.1.5), divided by the GDP deflator (NIPA table 1.1.9) and the civilian noninstitutional population, obtained from the BLS. Investment is the sum of nominal durable consumption and fixed investment (NIPA table 1.1.5), divided by the GDP deflator (NIPA table 1.1.9) and civilian noninstitutional population, obtained from the BLS. The inventory data (nonfarm private and retail) are obtained from NIPA tables 5.8.6A and 5.8.6B, divided by the civilian noninstitutional population, obtained from the BLS.

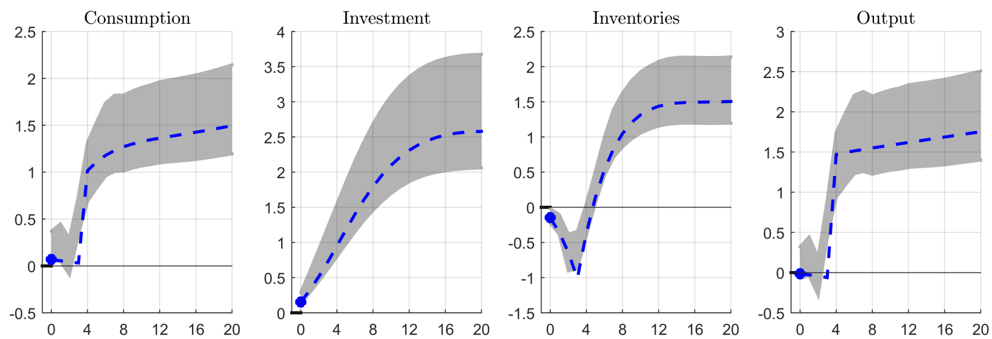
E.2 Additional results

Two additional robustness checks of our main result are reported. We first check whether it is sensitive to alternative detrending of the data by HP-filtering each time series. Figure 4 reports the results obtained using the impact sign restriction. Figure 5 reports the posterior distribution of the forecast error variance of output. Similar to our benchmark specification, the identified shock accounts for 10 percent of output variation in the short run and 25 percent in the long run.

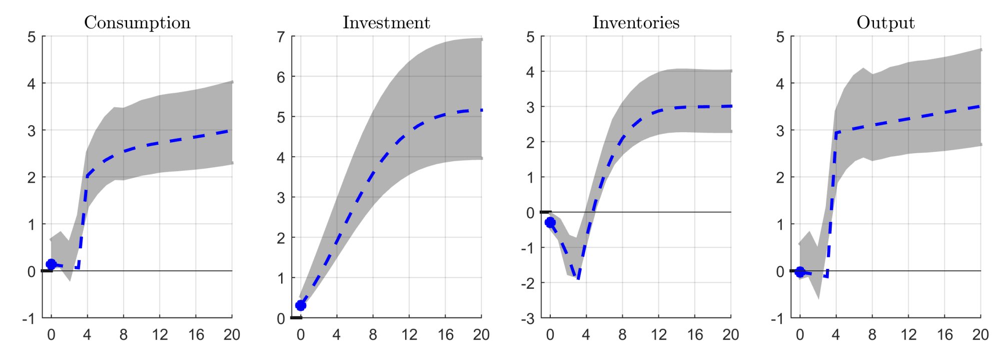
Second, in our benchmark estimation, we used real GDP as a measure of output. To be consistent with our model definition of output, we also constructed an alternative output series which subtracts government spending and net exports from the GDP series. That is, the alternative output measure is nominal GDP net of government spending and net exports, deflated by the GDP deflator, expressed in per capita terms. Figures 6 and 7 indicate that the results obtained using the impact restriction are not sensitive to this alternative definition of output.



(a) Standard deviation of TFP news innovation: 1/2.

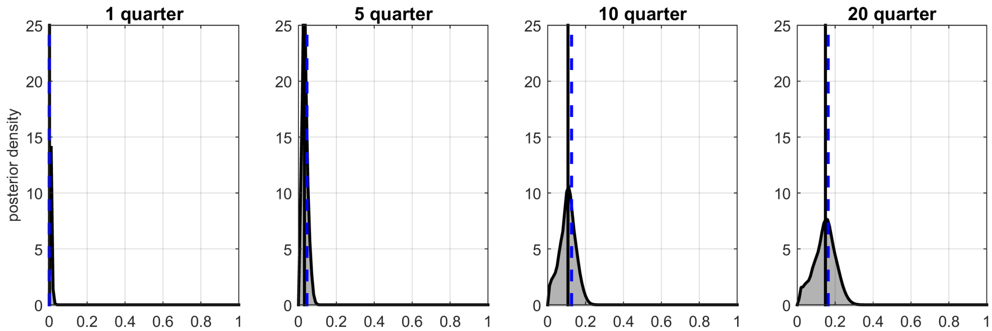


(b) Standard deviation of TFP news innovation: 1.

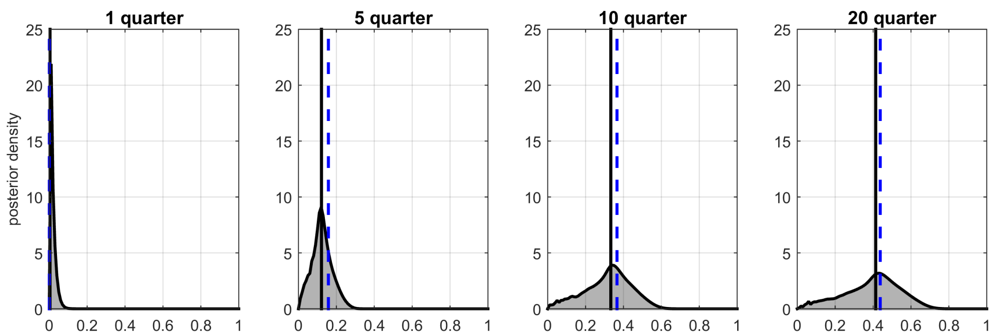


(c) Standard deviation of TFP news innovation: 2.

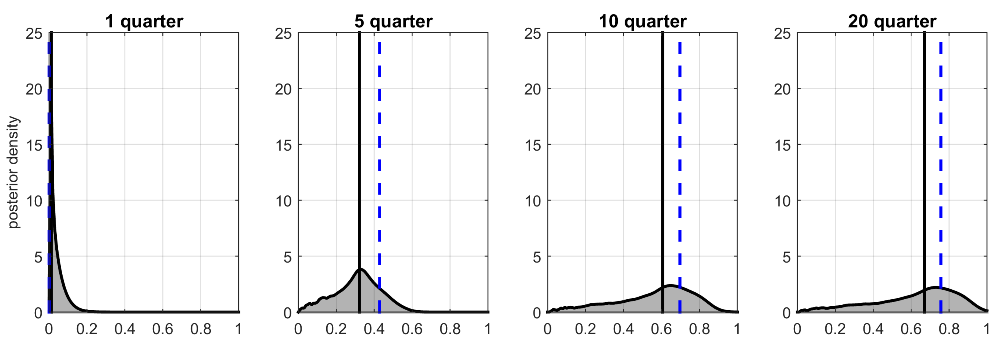
Figure 1: TFP news impulse responses. Blue dashed line: Model average impulse response. Shaded area: 80% credible set of SVAR using simulated data. News shocks in SVAR are identified using 2-period sign restrictions on inventories, consumption, and investment.



(a) Standard deviation of TFP news innovation: 1/2.

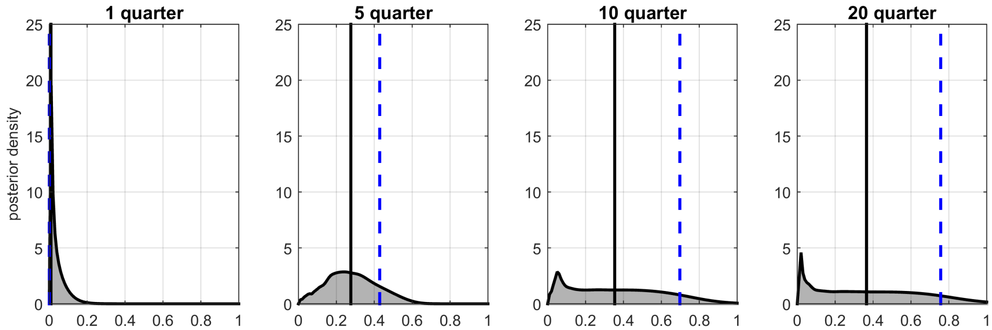


(b) Standard deviation of TFP news innovation: 1.

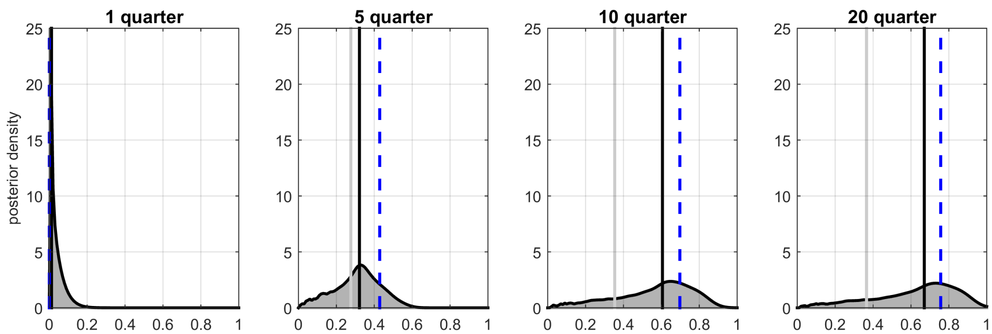


(c) Standard deviation of TFP news innovation: 2.

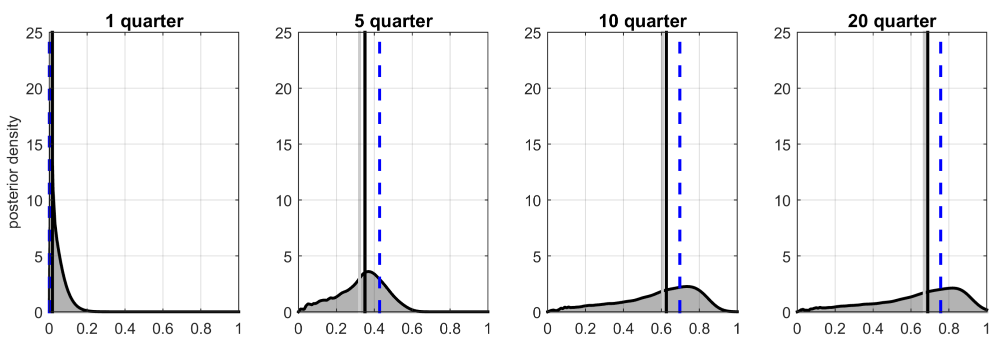
Figure 2: Share of FEV of output at horizons 1, 5, 10, 20, attributable to news shocks. News shocks are identified using 2-period sign restrictions on inventories, consumption, and investment. Blue dashed line: Model FEV. Shaded area: Posterior probability density of FEV based on SVAR. Black solid line: Median FEV based on SVAR.



(a) Impact restriction (Standard deviation of TFP news innovation: 2).



(b) Two period restriction (Standard deviation of TFP news innovation: 2).



(c) Three period restriction (Standard deviation of TFP news innovation: 2).

Figure 3: Share of FEV of output at horizons 1, 5, 10, 20, attributable to news shocks. News shocks are identified using 1-, 2-, and 3-period sign restrictions on inventories, consumption, and investment. Blue dashed line: Model FEV. Shaded area: Posterior probability density of FEV based on SVAR. Black solid line: Median FEV based on SVAR. Grey solid line in (b): Median FEV based on (a). Grey solid line in (c): Median FEV based on (b).

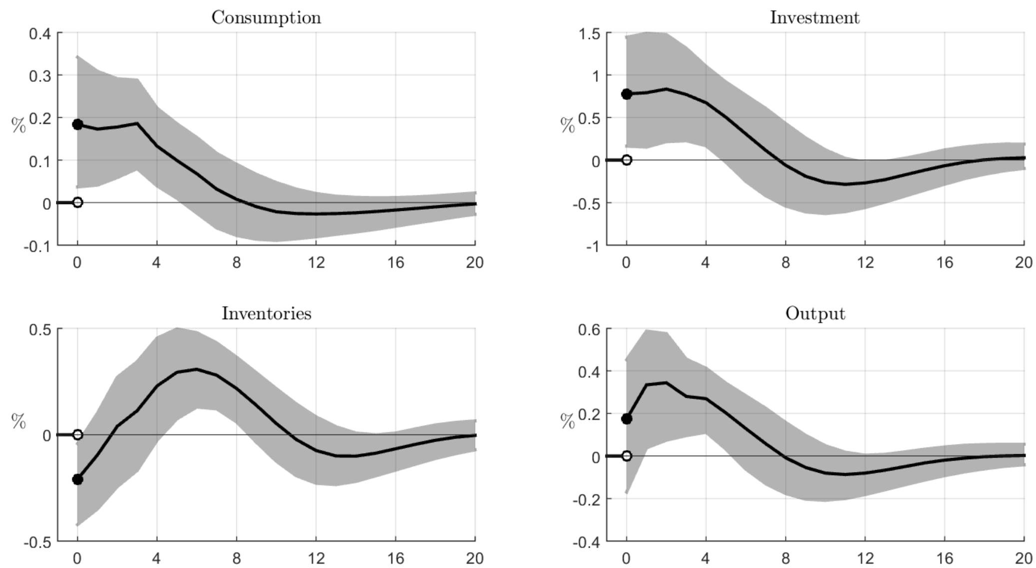


Figure 4: Median (solid line) and 80% credit set (shaded area) for the impulse responses to news shocks for HP filtered series. News shocks are identified using sign restrictions on inventories, consumption, and investment.

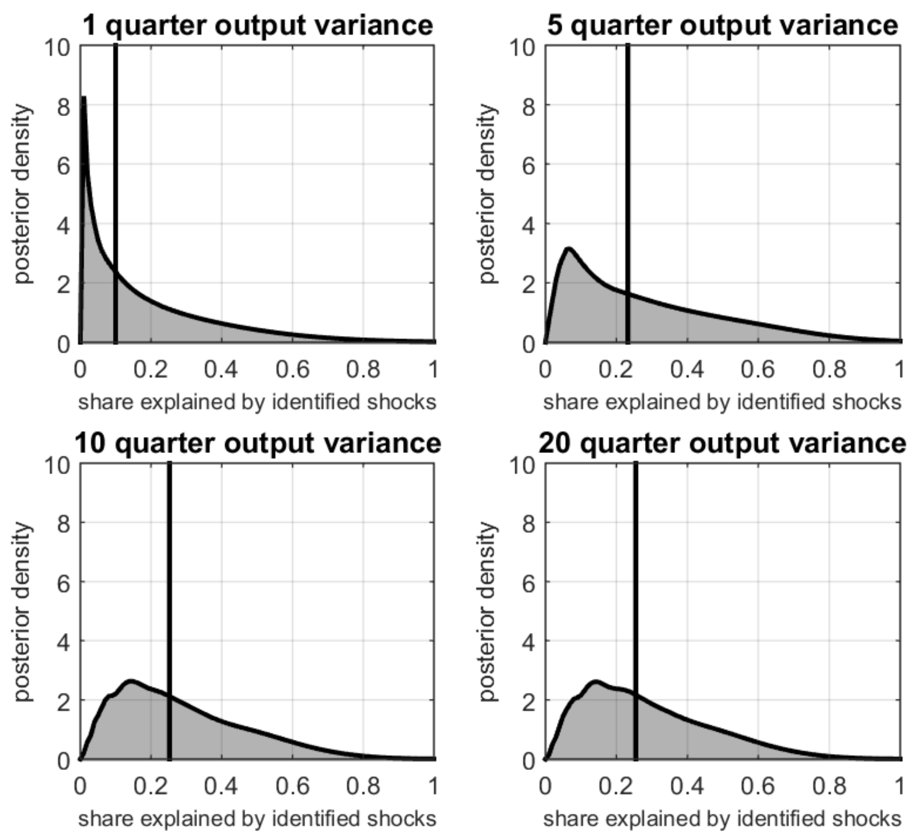


Figure 5: Posterior probability density and median (vertical line) FEV of output attributable to news shocks for HP filtered series. News shocks are identified using sign restrictions on inventories, consumption, and investment.

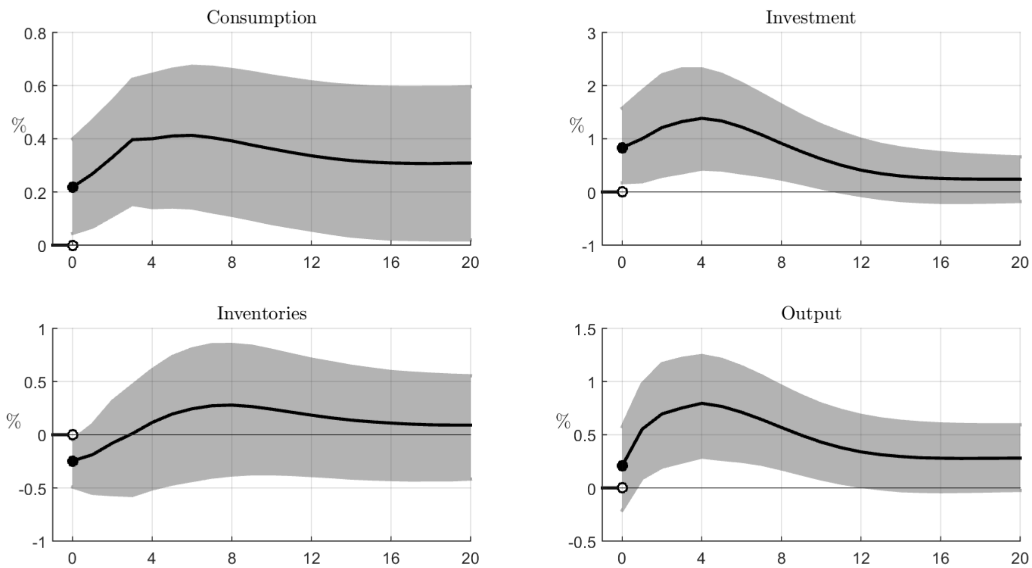


Figure 6: Median (solid line) and 80% credit set (shaded area) for the impulse responses to news shocks for domestic output series. News shocks are identified using sign restrictions on inventories, consumption, and investment.

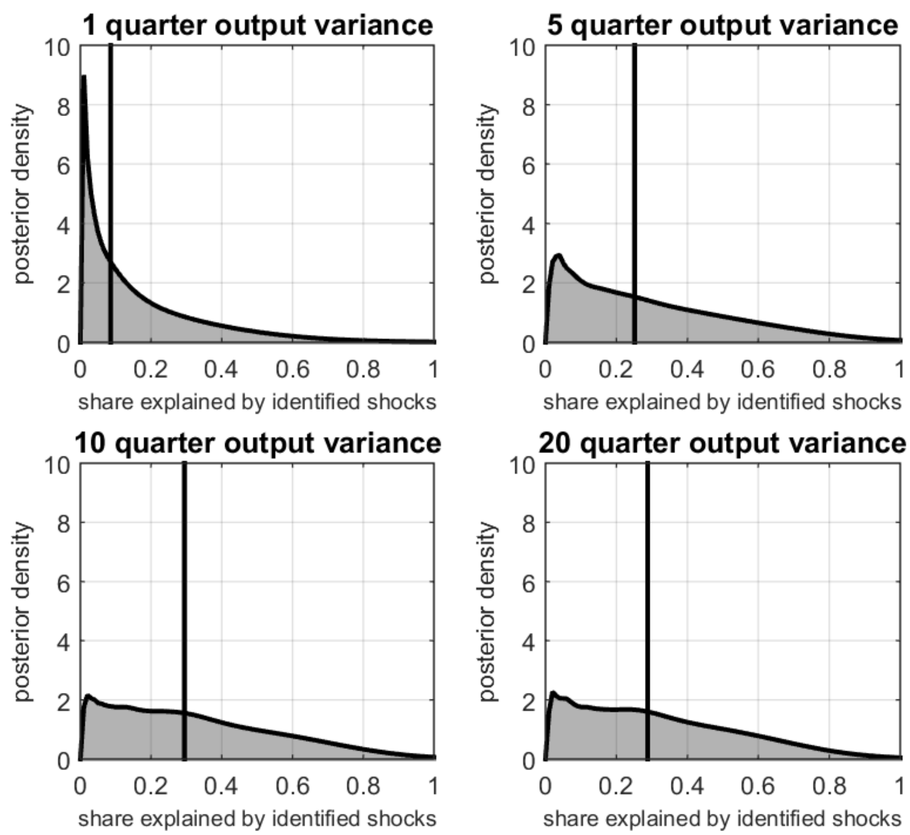


Figure 7: Posterior probability density and median (vertical line) FEV of output attributable to news shocks for domestic output series. News shocks are identified using sign restrictions on inventories, consumption, and investment.

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