Abstract

We document that the rise of factors such as software, intellectual property, brand, and innovative business processes, collectively known as “intangible capital” can explain much of the weakness in physical capital investment since 2000. Moreover, intangibles have distinct economic features compared to physical capital. For example, they are scalable (e.g., software) though some also have legal protections (e.g., patents or copyrights). These characteristics may have enabled the rise in industry concentration over the last two decades. Indeed, we show that the rise in intangibles is driven by industry leaders and coincides with increases in their market share and hence, rising industry concentration. Moreover, intangibles are associated with at least two drivers of rising concentration: market power and productivity gains. Productivity gains derived from intangibles are strongest in the Consumer sector, while market power derived from intangibles is strongest in the Healthcare sector. These shifts have important policy implications, since intangible capital is less interest-sensitive and less collateralizable than physical capital, potentially weakening traditional transmission mechanisms. However, these shifts also create opportunities for policy innovation around new market mechanisms for intangible capital.
I Introduction

During the global financial crisis, governments and monetary institutions around the world intervened to keep credit and financial markets functioning, and as markets stabilized, to keep interest rates low across the yield curve. Yet despite sustained historically low interest rates, business investment recovered slowly from its collapse during the crisis, and did not return to pre-crisis levels. Even as the recovery continued, investment remained sluggish (Hall, 2015; Gutiérrez and Philippon, 2017).

Of course, interest rates and more generally, the cost of capital, are not the only fundamental determinant of investment. Expected cash flows, as indicators of the rate of return on capital, are also crucial, as is the availability of financing, either through retained earnings or through the financial sector. Yet in the recovery, corporate profitability was strong, and importantly, the corporate sector was a net source of savings to the rest of the economy (Alexander and Eberly, 2018). And while overall output growth remained modest (Fernald et al., 2017), valuations and hence Tobin’s Q, as a measure of the expected return to capital, boomed along with profitability. These observations are hard to square with explanations of weak investment based solely on weak expected growth or lack of financing. In general, purely crisis-based explanation of weak investment are likely to be incomplete, since the data suggest that investment started to weaken earlier – closer to 2000 – before the financial crisis and the Great Recession (Gutiérrez and Philippon, 2017; Alexander and Eberly, 2018).

The pattern of investment across industries instead contains some clues to the reason for low aggregate investment. As shown by Alexander and Eberly (2018), investment remained high in structures and related sectors, such as Oil and Gas or Telecoms, which put in place platforms, pipelines, or towers. Investment in these “spatially grounded” sectors, where physical capital is hard to relocate or replace with other inputs, showed little sign of weakness, instead responding strongly to positive shocks (hydraulic fracturing, for instance). Moreover, in the Manufacturing and production sectors, historically the engines of aggregate capital accumulation, investment underwent a long relative decline since the mid-1990’s.

However, these sectors’ share of value has also been in decline, and they do not account for the growth in profitability and valuations discussed above. Not surprisingly, much of that growth comes from the High-tech sector. But as its share of sales, income, and valuation climbed, the High-tech sector’s share of investment stagnated. Likewise, the Retail sector has been growing as a share of value added, but investment there has been weak. This discrepancy left a gap in aggregate capital accumulation, as companies with the highest growth and valuations failed to fuel investment demand.¹

¹Simple back-of-the-envelope computations using industry-level data, reported in Figure 1 of the Online Appendix, indicate that had the High-tech and Retail sectors kept investing at the same rate as in 2000, the aggregate investment rate in 2015 would have been approximately 20% higher, and its overall decline would largely have been averted.
The low investment puzzle is thus concentrated among some of the most successful sectors and firms in the economy. Given the growth in sales and profitability of these sectors and firms, it is difficult to argue that “low investment” results from a binding constraint limiting capital accumulation. Instead, firms may have chosen a lower level of capital investment than historic norms would indicate – but why? Resolving this question is important for public policy decisions, as investment is often a leading target of public policy interventions, either through monetary policy and low interest rates, or fiscal policy through accelerated depreciation or tax credits. If the explanation for weak investment is a temporary suspension of the transmission mechanism, due, for example, to credit rationing, the policy implications are quite different than a change in the capital allocation choices made by firms. Such changes could have a more fundamental impact on policy transmission and decisions.

We examine the possibility that investment weakened because the composition of the capital stock used by firms has changed over time. The weak investment puzzle states that the level of physical investment — investment in PPE (property, plant, and equipment) — has been low relative to valuations and expected growth. But firms increasingly use other fixed factors than PPE for production and sales, and in particular factors known as “intangible capital” (Corrado et al., 2005). For example, they rely more heavily on software to produce and sell goods and services than did their predecessors. The platform developed by an online retailer is just as crucial to producing revenue as an oil platform is to an energy firm. Aside from own-account software, “intangible capital” includes intellectual property (including those related to R&D), brand, and innovative business processes.

Some of these factors are now measured and included as “capital” for purposes of National Income Accounting. The BEA defines capital, in principle, as resources set aside today to produce output in the future. Hence purchases of software and development of intellectual property are indeed investments in capital. Expenditure-based measures for these two types of intangible factors are now available at the industry level. At the firm level, both the flow and stock of intangibles are more difficult to measure. Firm level accounting data conventions mean that intangible investment is generally not capitalized, though some of it may eventually be booked as “intangible” capital, especially (though not only) following acquisitions or mergers. For instance, while the oil platforms of an energy firm will systematically be reflected in its PPE stock, an online retailer’s platform will not, and may not be easy to identify using balance sheet data. As a result, measures of profitability, such as Tobin’s Q, are typically restricted to PPE, and omit intangibles altogether.²

Our first step is to discuss how the omission of intangibles may affect estimates of the relationship between

²See Corrado et al. (2009) and Nakamura (2010) for an application to the aggregate data and the measurement of US economic growth. See Lev (2000) for intangibles at the firm level, and the difficulties inherent in measuring them.
PPE investment and valuations. We analyze omitted factors in a conventional production setting, and show what their exclusion does to empirical estimates that ignore them. In general, the omission of a fixed factor generates an “investment gap” between expected PPE investment and that actually observed. The higher the share of omitted capital, the higher the gap. We then show that, consistent with the omitted factors hypothesis, measures of the PPE investment gap are correlated with measures of the share of intangible capital, both in the cross-section and over time. We obtain industry-level measures of the composition of the capital from the BEA, and construct firm-level measures from Compustat, using balance sheet proxies for intangible capital, as well as expenditure-based measures that map to the BEA definitions of capital. Investment gap measures are reduced by one quarter (in firm-level data) to three quarters (in industry data), by adjusting for the presence (and importance) of intangible capital.

This analysis demonstrates that intangible capital, when treated as an omitted factor in production, can fill a substantial part of the apparent gap left by weak physical capital investment. But the growth of this factor may have other implications, as well. Research documenting the rising role of intangible capital, in the US and internationally (Corrado et al., 2009; Haskel and Westlake, 2017; Bhandari and McGrattan, 2018), points out that properly accounting for intangible capital may affect growth accounting exercises and measures of productivity, the net effect on productivity estimates in these studies is generally small. However, intangible capital has different economic characteristics than physical capital. Corrado et al. (2005) identify three main categories of business intangibles: computerized information, innovative property, and economic competencies. Examples of items in these three categories include software, scientific R&D, and brand values, respectively. These types of capital are quite distinct from PPE capital. For example, intangible capital is more readily scalable and less excludable than physical capital: a piece of software can be more easily replicated than a piece of equipment. This implies that ownership may be less palpable and more contractual, requiring patent and copyright protection, as we see in intellectual property and software. Similarly, investments in branding and business processes, such as online platforms and order systems, may be readily scalable, but then protected by trademarks. These distinct economic characteristics potentially allow intangible capital to play a different role in generating revenue and profit than traditional capital inputs. In particular, these properties may promote economies of scale, while the protections afforded intangibles (patents, copyrights) may exclude competitors and generate market power.

These qualities are of particular interest given the growing evidence of rising concentration in US industries. Recent work (Autor et al., 2017) has emphasized rising measures of industry concentration across a range of US business sectors — coincident with the rise in intangible capital. We explore what role intangible capital may play in rising concentration.

We first document that the increase in concentration is correlated with the rise in intangibles across
industries. Specifically, we show that both across and within firms, market shares are positively related to firms’ intangible intensity, defined as their ratio of intangible to total capital. This suggests that the accumulation of intangible capital has occurred hand-in-hand with the increase in the market share of industry leaders and the increasing concentration of US industries.

The consequence of rising concentration, however, depends on its source. Two potential causes for the rise in concentration have been put forward in the literature. One is market power. Empirical work by De Loecker and Eeckhout (2017), Gutiérrez and Philippon (2018) and Hall (2018) all suggest that the rise in concentration has been accompanied by rising markups, though the estimated degree of the increase varies. Rising markups provide stronger evidence of market power, over and above increasing concentration. Alternatively, rising concentration may result from adoption of technologies that favor large firms, as suggested by the “superstar firm” literature and emphasized by Autor et al. (2017). These hypotheses have vastly different implications for welfare and for policy. If concentration is rising because of the expansion of the most productive firms, it may be efficient. If, on the other hand, rising concentration reflects greater market power, it may imply inefficiencies and resource misallocation. The source of rising concentration is thus important for understanding the extent to which rising concentration is efficient or not, and possible policy implications.

While both hypotheses are consistent with rising concentration, they can be identified separately from measures of markups and productivity. That is, if concentration is due to market power, we should see rising markups, whereas if it is due to productivity, we should see advancing productivity among market leaders. These explanations are not mutually exclusive and need not play the same role in every industry, as we found in earlier work focusing on the Retail sector (Crouzet and Eberly, 2018). Hence, we break the data into industry groups and examine them separately.

We provide evidence that there are pervasive links between intangible investment, market power, and the productivity gap in the sectors we examine, though the nature of the link varies across sectors. In the Consumer sector, productivity growth appears to be the primary cause of growing concentration. Moreover, it is closely associated with intangible investment, both across and within firms. This result is intuitive, since the Consumer sector has been transformed by process innovation, from inventory and distribution methods to online platforms, which are embodied in intangible capital. By contrast, in the Healthcare sector, productivity is stable but markups have risen consistently. Again, the increase in markups is associated with intangible investment, both across and within firms. These results likely reflect innovation in Healthcare that is also embodied in intangibles, but more likely to be patentable product innovations. In the High-tech sector, both factors appear to be at work. Markups rise considerably, and productivity measures also increase. Both trends — in markups and productivity — are closely correlated with intangible investment,
even within firms. Finally, the Manufacturing sector exhibits none of these trends, with more stable markups and productivity, and modest growth in intangible capital.

A potential issue with the interpretation of these correlations is that our main firm-level measure of intangible capital is balance sheet intangibles. As we discuss in sections II and IV, balance sheet intangibles capture the value of acquired intangible assets; thus, they are potentially measure with error, because upon acquisition, firms may over- or under-value intangibles relative to fundamentals. These valuation errors could themselves be correlated with current or subsequent firm sales, potentially creating bias in our estimates. We address this issue by instrumenting for balance sheet intangibles in two ways. At the industry level, we use the BEA’s measure of intangible capital, which includes accumulated spending on software, R&D, and intellectual property related to arts and entertainment. At the firm level, we use a proxy for internally developed intangible assets, from the work of Peters and Taylor (2017), which takes a similar approach to the BEA by capitalizing firm-level expenditures on intangibles. This proxy is the capitalized value of past expenditures on sales, general and administrative expenses and, where available, on R&D. This instrumentation approach satisfies the exclusion restriction if the measurement error in Peters and Taylor (2017) intangibles (likely due to the inclusion of expenditures not directly related to intangible investment) are uncorrelated with the measurement error in balance sheet intangibles (which, as mentioned above, is likely driven by over- or under-valuation of acquired intangible assets.) Using this approach, we find results that are consistent, both in terms of sign and statistical significance, with the baseline correlations obtained with simple OLS.

Together, these results suggest that intangible capital is important to understanding weak investment in physical capital, as well as changes in market structure. In the latter, intellectual property, software, and other forms of “intangible capital” can generate scale economies and enhance productivity, creating “superstar firms”. However, intangible capital can also differentiate products and exclude competitors (branding, patent protection), which can confer pricing power.

We conclude by discussing the policy implications of the rise in intangible capital. Investment in intangible assets differs from traditional investment in many ways, but two are particularly relevant to policymakers. First, the user cost of intangible assets tends to be less interest-sensitive than that of physical assets, because depreciation rates of intangibles are substantially higher. Second, markets for intangible assets are generally illiquid (if they even exist), and as a result, intangible capital is more difficult to use as collateral in obtaining external financing. Both of these remarks imply that monetary policy is unlikely to influence intangible investment as strongly as it does traditional investment. Thus, to the extent that shifts in market structure and market power are indeed attributable to intangible investment, policy should focus on other levers than interest rates, such as strengthening competition regulation and intellectual property rights enforcement,
and encouraging the development of markets for intangible assets.

The paper is organized as follows. Section 2 discusses the facts on the decline in physical investment and the rise in intangibles. Section 3 connects the two phenomena both theoretically and empirically. Section 4 then documents the empirical relationship between intangible investment and market concentration, and studies two potential economic mechanisms linking the two — rising market power of industry leaders, and an increasing gap between leaders’ and followers’ productivities. Section 5 draws the policy implications of our results, and section 6 concludes.

II Background facts on the level and composition of investment

Recent research on investment in the US has emphasized that while investment has been weak following the global financial crisis, this weakness predated the crisis itself. Similar to the dynamics of employment, the crisis punctuated, and perhaps exacerbated, longer run, underlying dynamics and structural changes. This weakness is evident in the raw data, reported in Figure 1a. This figure shows investment rates using both firm-level data for publicly traded firms and industry-level data from the BEA’s fixed asset tables; the level and trends coincide across data sources.

The weakness in investment may simply be due to weak fundamentals (Fernald et al., 2017), such as slow output growth, as the economy recovered from the financial crisis. However, strength in corporate profitability and high valuations suggest that weak investment is not so easily explained. Gutiérrez and Philippon (2017) and Alexander and Eberly (2018) show that controlling for standard determinants of investment, an “investment gap” opens up starting around the year 2000, rising throughout the decade. Both these papers use average Tobin’s Q as a measure of the incentive to invest. Average Q is defined as the ratio of the market value of the firm to the replacement cost of its (physical) assets. The “investment gap” can be measured by the time effects in a standard regression of investment on average Q. Figure 2a shows that these time effects become negative and significant around the year 2000, and remain so for the rest of the sample period. This is a sign that, since the 2000’s, actual investment has fallen consistently short of the level consistent with observed average Q. (We have also controlled for cash flow in these investment regressions, in order to allow for liquidity constraints or measurement error in Q, since these considerations are not central to our analysis and are often included in standard investment regressions.)

The decline in physical investment, and the associated “investment gap”, are also visible at the industry level. Figures 1b and 2b report investment rates, and investment gaps, for four particular groups of industries.

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3Throughout the paper, the sample of publicly traded firms we use consists only of firms incorporated in the US. The data is drawn from Compustat; see the Online Appendix for a complete description of data construction and sample selection.
Consumer, High-tech, Healthcare, and Manufacturing. These four sectors together accounted for 54% of total value added and 60% of total investment in 2001; all have suffered a decline in their investment rates since the mid-2000’s.

As emphasized by Alexander and Eberly (2018), not all industries follow this pattern (although the four highlighted above do). As a result, the distribution of investment across industries has changed, following a pattern of “hollowing out” reminiscent of similar trends in labor markets (Autor, 2010). Figure 4 illustrates this evolution. Prominent in the “missing middle” is the Manufacturing sector, where offshoring and outsourcing might have replaced domestic investment. A growing “left tail” of sectors with reasonably high investment rates still exists; these sectors are those which require a local presence, an in particular, the energy sector and the transportation and warehousing sectors. The “right tail” of the distribution is made up of sectors such as Healthcare and High-tech, which are growing in value-added terms, but with no commensurate rise in their share of investment.

While physical investment waned over the 1995-2015 period, the accumulation of other forms of capital did not. Figures 3a and 3b report trends in the importance of intangible capital as a share of total capital. In this figure, we use two measures of the share of intangible assets, as a fraction of total capital. The first one, derived from firm-level data, is the ratio of balance sheet intangibles to total capital (where total capital is defined as the sum of balance sheet intangibles, plus physical capital). Balance sheet intangibles are at best an imperfect measure of intangible capital. First, because they reflect the overall value of acquired intangibles, they conflate different types of intangible capital: software, intellectual property, brand, and business processes. The total book value of acquired intangibles may itself be a biased measure of their fundamental value, because of potential over- or under-valuation at the time of acquisitions. Second, balance sheet intangibles will in general miss any own-account intangible investment, which may be an important source of intangible capital accumulation. To address some of these concerns, we also report a second measure of the composition of the capital stock, constructed using industry-level data from the BEA’s fixed asset tables. There, the intangible share is defined as the ratio of the estimated replacement cost of software, R&D, and intellectual property rights (in entertainment and arts), to the replacement cost cost of software, R&D, and intellectual property rights (in entertainment and arts), to the replacement cost

4See the Online Appendix for details on the industrial classification used throughout the paper.
5Balance sheet intangibles are in general created upon acquisition of another firm. The only cases in which intangibles may appear on the balance sheet of a firm through internal accumulation is for the development of software intended for sale or leasing, and exploratory costs in the Oil & Gas sectors; see FASB ASC 370. FASB standards require firms to classify acquired intangibles into two categories: identifiable intangibles, and goodwill. FAS 141 describes in detail the types of intangibles that are classified as “identifiable”; they include, among others, customer-related intangibles (such as customer lists) and technology-based intangibles (such as patents). Goodwill contains all other intangible assets not classifiable under FAS 141 categories. We include goodwill as part of our main measure of balance sheet intangibles because, while it includes other forms of value booked in an acquisition, it also includes intangible capital acquired by firms. The 2017 10K of Amazon, for instance, describes goodwill as follows (p.54): “The goodwill of the acquired companies is primarily related to expected improvements in technology performance and functionality, as well as sales growth from future product and service offerings and new customers, together with certain intangible assets that do not qualify for separate recognition.”
6In section IV, we use capitalized measures of spending on intangibles to adress some of the potential measurement error issues regarding balance sheet intangibles.
of the total capital stock. This latter measure is constructed using industry-level estimates of spending on these three types of intangibles.

At the aggregate level, the rise in both types of intangibles over the sample period, and particularly since 1995, is striking. However, just as for the decline in physical investment, the rise in intangibles is not uniform across industries. Manufacturing, for instance, stands out as having experienced a relatively mild increase in intangible intensity. Crouzet and Eberly (2018) show that intangible capital plays an important role in the Retail sector, where the reliance on physical capital has historically been low, and where the investment gap has been particularly pronounced since the mid-1990’s.\textsuperscript{7} We come back to this variation across industries in section IV.

III Weak physical investment and omitted factors

The previous section has shown that for the past two decades, measures of physical investment have been weak, in particular relative to average $Q$. At the same time, other forms of capital seem to have gained importance. In this section, we first show, theoretically, that if the firm utilizes other, non-physical forms of capital in production, then average $Q$ is not a sufficient statistic for physical investment. This is true even when there are constant returns to scale to capital. We show that average $Q$ in fact always overstates the incentive to invest in physical capital, when other forms of capital contribute to the production process. We then ask whether investment-$Q$ regressions, once properly adjusted for non-physical capital, still suggest that physical investment (and investment overall) has been weak relative to $Q$.

III.A Theory

In order to understand how omitted capital might change the interpretation of investment-$Q$ regressions, we start from a standard real model of investment with adjustment costs. Specifically, we take a discrete-time version of the Hayashi (1982) model, and modify the revenue function. We allow for two types of capital, where both face costs of adjustment. In particular, the revenue function takes the form:

$$\Pi_t = A_t \left( \alpha K^\rho_{1,t} + (1 - \alpha) K^\rho_{2,t} \right)^{\frac{1}{\rho}}.$$  \hspace{1cm} (1)

Here, $A_t$ is a stochastic process characterizing business conditions (both productivity as well as, potentially, demand or labor and intermediate input costs), and $K_{1,t}$ and $K_{2,t}$ are the two capital inputs used by the firm. Throughout, we will think of $K_{1,t}$ as physical capital, and $K_{2,t}$ as non-physical capital. The parameter

\textsuperscript{7}Retail — NAICS subsector 44 — makes up the majority of the Consumer sector.
\( \rho \in [-\infty; 1] \) controls the elasticity of substitution between the two capital inputs. When \( \rho > 0 \), the two types of capital are substitutes, while when \( \rho < 0 \), they are complements.\(^8\)

In this model, the equation determining the physical investment rate, \( i_{1,t} = \frac{I_{1,t}}{K_{1,t}} \), is the standard marginal \( q \) condition:

\[
i_{1,t} = \frac{1}{\gamma_1} \left( \frac{1}{1 + \rho E_t} [q_{1,t+1} - P_{1,t}] \right), \tag{2}
\]

where \( P_{1,t} \) is the price of physical assets, \( \gamma_1 \) is a parameter governing the curvature of the physical capital adjustment costs, and:

\[
q_{1,t} = \frac{\partial V}{\partial K_1}(K_{1,t}, K_{2,t}; X_t)
\]

is the marginal value of one unit of physical capital. (\( X_t \) collects exogenous processes.)

However, even though there are constant returns to capital overall, average (physical) \( Q \) is not a good measure of the marginal value of one unit of physical capital. To see why, note that under constant returns, the value of the firm, \( V(K_{1,t}, K_{2,t}; X_t) \), can be decomposed as:

\[
V(K_{1,t}, K_{2,t}; X_t) = q_{1,t} K_{1,t} + q_{2,t} K_{2,t} = q_1 (\nu_t; X_t) K_{1,t} + q_2 (\nu_t; X_t) K_{2,t}, \tag{3}
\]

where \( q_{2,t} \) denotes the marginal value of a unit of non-physical capital, and:

\[
\nu_t = \frac{K_{2,t}}{K_{1,t}}
\]

is the ratio of non-physical to physical capital. The definition of average physical \( Q \) is:

\[
Q_{1,t} \equiv \frac{V(K_{1,t}, K_{2,t}; X_t)}{K_{1,t}},
\]

which can be rewritten, using equation 3, as:

\[
Q_{1,t} = q_1 (\nu_t; X_t) + \nu_t q_2 (\nu_t; X_t). \tag{4}
\]

Equations 2 to 4 lead to two important remarks on the relationship between investment and average physical \( Q \).

\(^8\)Appendix A contains a detailed statement of the model and a derivation of the key results below.
Remark 1: Average $Q$ systematically overstates the incentive to invest in physical capital.

Combining equation 4 with equation 2, we obtain:

$$i_{1,t} = \frac{1}{\gamma_1} \left( \frac{1}{1 + r} E_t [Q_{1,t+1}] - P_{1,t} \right) - \frac{1}{\gamma_1} \left( \frac{1}{1 + r} E_t [Q_{2,t+1}] \right) \nu_{t+1},$$

where $\nu_{t+1} = \frac{K_{2,t+1}}{K_{1,t+1}}$ is determined at time $t$. This equation indicates that there is wedge $w_t$ between physical investment, and average physical $Q$ (net of the price of capital goods):

$$w_t = \frac{1}{\gamma_1} \left( \frac{1}{1 + r} E_t [Q_{2,t+1}] \right) \nu_{t+1}.$$

Note that, so long as the the marginal revenue product of non-physical capital is positive, then $q_{2,t} > 0$, so that $w_t > 0$. Hence, average $Q$ (net of the price of capital goods) is an over-estimate of the optimal investment rate. (The fact that the wedge between average $Q$, $Q_{1,t}$, and marginal $q$, $q_{1,t}$ is always positive is also directly visible in equation 4, given that $q_{2,t} > 0$.) Note that this wedge is not a mechanical result of adding another capital good in the computation of the physical investment rate; the left-hand side of equation 5 is $I_{1,t} K_{1,t}$, not $I_{1,t} K_{1,t} + K_{2,t}$.

It may be surprising that the sign of this wedge does not depend on whether capital goods are complements or substitutes. The reason is that the value of the firm, $V(K_{1,t}, K_{2,t}; X_t)$, captures the value of both types of capital; so it overstates the value of investing in either one. In order to obtain a correct measure of the incentive to invest in physical assets, we need to subtract the value of non-physical capital from $\frac{V}{K_{1,t}}$; this value is given by $\nu q_{2,t}$. So long as this value is positive (which is the case whenever the marginal revenue product of non-physical capital is positive), this adjustment will be negative.

Note, additionally, that all other things equal, the magnitude of the wedge is increasing in $\nu_{t+1}$, the ratio of non-physical to physical capital. Thus, the adjustment will be larger, the larger the share of non-physical capital.

Remark 2: Using a measure of “total” $Q$ in investment regressions will still overstate the incentive to invest in physical assets. Additionally, one may think that simple adjustments in the computation of investment rates and $Q$ may suffice to address the omitted variable bias. For instance, given a measure of $K_{2,t}$, consider estimating a regression of $i_{1,t}$ on $Q_{t}^{(tot)}$, where:

$$Q_{t}^{(tot)} = \frac{V(K_{1,t}, K_{2,t}; X_t)}{K_{1,t} + K_{2,t}}$$

\(^9\)Appendix A contains a formal proof of this statement.
is the ratio of the value of the firm to the sum total of the two capital stocks. Using equation 3, we have that:

\[ Q_t^{(\text{tot})} = \frac{1}{1 + \nu_t} q_{1,t} + \frac{\nu_t}{1 + \nu_t} q_{2,t}, \]  

that is, \( Q_t^{(\text{tot})} \) equals a time-varying weighted sum of the two marginal \( q \) terms. Using equation 4, we have the investment equation for physical capital:

\[ i_{1,t} = \frac{1}{\gamma_1} \left( \frac{1}{1 + r} (1 + \nu_{t+1}) \mathbb{E}_t \left[ Q_{t+1}^{(\text{tot})} \right] - P_{1,t} \right) - \frac{1}{\gamma_1} \left( \frac{1}{1 + r} \mathbb{E}_t \left[ q_{2,t+1} \right] - P_{2,t} \right), \]  

Thus, in general, even using \( Q_t^{(\text{tot})} \) overstates the incentive to invest in physical assets.

Moreover, even regressions of total investment on total \( Q \) may, in general, remain biased. Using equations 2 to 4, the total investment rate \( i_t^{(\text{tot})} = \frac{i_{1,t} + i_{2,t}}{K_{1,t} + K_{2,t}} \) is given by:

\[ i_t^{(\text{tot})} = \frac{1}{\gamma_1} \left( \frac{1}{1 + r} \mathbb{E}_t \left[ Q_{t+1}^{(\text{tot})} \right] - P_t \right) + \left( \frac{1}{\gamma_2} - \frac{1}{\gamma_1} \right) \frac{\nu_t}{1 + \nu_t} \left( \frac{1}{1 + r} \mathbb{E}_t \left[ q_{2,t+1} \right] - P_{2,t} \right) \]  

where the firm-specific investment price \( \tilde{P}_t \) is given by:

\[ \tilde{P}_t = \frac{1}{1 + \nu_t} P_{1,t} + \frac{\nu_t}{1 + \nu_t} P_{2,t}. \]  

Again, in general, total \( Q \) still provides an incorrect estimate of the overall incentive to invest. However, whether this is an over- or under-estimate now depends on the relative magnitude of adjustment costs, as well as the sign of the optimal intangible investment rate \( i_{2,t} = \frac{1}{\gamma_2} \left( \frac{1}{1 + r} \mathbb{E}_t \left[ q_{2,t+1} \right] - P_{2,t} \right) \). Even under the assumption of identical convexity of adjustment costs, \( \gamma_1 = \gamma_2 \), the standard regression remains mis-specified, because the total investment price \( \tilde{P}_t \) is a firm-specific weighted average of capital prices, as opposed to an (industry) index. For instance, the intercept of a cross-sectional regression of total investment on total \( Q \) will not identify the price of capital anymore; one therefore requires direct measures of the price of capital to estimate the regression.

The requirements for regressions using total \( Q \) to be correctly specified are very specific; in particular, it must be the case that:

\[ q_{1,t} = q_{2,t} = Q_t^{(\text{tot})}. \]  

When the marginal \( q \)'s of the two types of capital are equal (and therefore, using equation 6, equal to total \( Q \)), it is clear that the physical investment-total \( Q \) regression is correctly specified. (If, additionally, \( \gamma_1 = \gamma_2 \), then the total \( Q \)-total investment regression 8 is also correctly specified). Appendix A discusses
the specific assumptions under which the relationship \( q_{1,t} = q_{2,t} \) holds.\(^{10}\) Two conditions, in particular, are necessary: identical depreciation rates across types of capital, and perfect substitutability \((\rho = 1)\). We discuss differences in rates of depreciation in section V.A; available evidence suggests that intangible assets have shorter economic lives than physical assets. Overall, we do not find the requirements to use “total \( Q \)” very plausible in practice, and if implemented to match investment trends, they would require price trends that seem at odds with the data.

Thus, even in generalized \( Q \) regressions, the fundamental problem remains: firm value captures the joint value of both types of capital, but investment is determined by their respective marginal \( q \)’s. Since the marginal \( q \) of one type of capital need not be proportional to the other, linear combinations do not address the problem.

III.B Evidence: can missing factors account for weak physical investment?

In this section, we use the theoretical predictions of the model described above to test whether weak physical investment, relative to \( Q \), can plausibly be explained by investment in an omitted capital input. The omitted factor(s) we consider are intangible capital, the rising importance of which was described in section II.

III.B.i Industry-level evidence

In section II, we defined the “investment gap” of a particular industry as the time effects \( \gamma_t \) obtained from a panel regression of the form:

\[
i_{j,t} = \alpha_j + \gamma_t + \delta Q_{j,t} + \beta' X_{j,t-1} + \epsilon_{j,t},
\]

(11)

where \( j \) indexes a firm, \( t \) indexes a year, \( Q_{j,t} \) is average physical \( Q \), and \( X_{j,t-1} \) are lagged firm-level controls (including, but not necessarily limited to, the ratio of EBITDA to physical assets). The time effects \( \gamma_t \), which in a number of industries display a sharp downward trend, are an estimate of the average discrepancy between physical investment and average physical \( Q \) in a particular industry.

Equation 5 states that, in a world with an omitted factor, the firm-level gap between physical investment and average physical \( Q \) should be given by:

\[
b_{j,t} = \nu_{j,t+1} q^{(e)}_{2,j,t+1},
\]

(12)

where \( q^{(e)}_{2,j,t+1} = \frac{1}{1+r} E_{j,t} [q_{2,j,t+1}] \). Taking cross-section averages, the average gap in the industry should

\(^{10}\)The appendix also discusses how the model outlined in this section relates to Peters and Taylor (2017).
then be given by:

\[ b_t = \mathbb{E}_t [\nu_{j,t+1}] \mathbb{E}_t [q_{2,j,t+1}^{(e)}] + \text{cov}_t (\nu_{j,t+1}, q_{2,j,t+1}^{(e)}) \, . \] (13)

In particular, the average gap should be increasing with the cross-sectional average intangible ratio \( \bar{\nu}_{t+1} = \mathbb{E}_t [\nu_{j,t+1}] \) in the industry. To the extent that the empirical investment gaps \( \gamma_t \) are indeed due to an omitted factor, and that this omitted factor is in fact intangible capital, one should therefore expect them to be negatively correlated with industry/year averages of the intangible ratio \( \bar{\nu}_{t+1} \).

Figure 5 plots the industry-level estimates of \( \gamma_t \) for KLEMS industries, aggregated up to the 12 sectors, against the cross-sectional averages of the intangible asset share, \( s_{j,t+1} = \frac{K_{2,j,t+1}}{K_{1,j,t+1} + K_{2,j,t+1}} = \frac{\nu_{j,t+1}}{1 + \nu_{j,t+1}} \), also aggregated up to the 12-sector level.\(^{11}\) The figure indicates that there is indeed a negative correlation between the physical investment gap at the industry level, and the industry-wide average of the intangible share.

Table 1 reports regressions of the investment gap \( \gamma_t \) on the intangible share \( s_t \); an observation in these regressions is a KLEMS industry/year. The first two lines of the table report the simple OLS coefficients associated with the intangible share, either with no fixed effects (first column), KLEMS industry effects (second column), and KLEMS industry/time effects (third column). In order to address potential endogeneity arising from measurement error, in the third to fifth lines of table 1, we instrument the Compustat balance intangibles with the BEA’s industry-level estimate of the ratio of software and intellectual capital to total capital.\(^{12}\) The IV estimates are consistent in sign, and, for the first two specifications, in magnitude with the simple OLS. They are also somewhat more significant than the simple OLS estimate, in particular in the saturated specification containing industry and year effects. Overall, both the simple OLS and IV results suggest that the relationship between the investment gap and the intangible share is robust, statistically significant, and hold both within and across industries.

The second column of table 1, in particular, indicates that within an industry, a one percentage point increase in the intangible share is associated with an increase of the investment gap of 0.21 percentage points. This number is significant at the 1% level. The magnitude of the effect is economically large: the economy-wide increase in the (weighted) intangible share of assets in about 30 percentage points from the mid-1990’s to 2010, as discussed in section II. This would translate into an incremental investment gap of roughly 6 percentage points, or two thirds of the aggregate investment gap documented in Figure 2a. In a similar spirit, figures 6a and 6b plot, along with the estimates of the investment gap, the residuals from

\(^{11}\)We use the share \( s_{j,t+1} \) instead of the ratio \( \nu_{j,t+1} \) because the former is bounded between 0 and 1, so that cross-sectional averages are less sensitive to outliers; however, the two are increasing transformations of one another, so from a theoretical standpoint, the prediction that the investment gap should be increasing in the intangible share if it is due to omitted intangible capital still holds. The sectors are defined in the Online Appendix to the paper.

\(^{12}\)See section IV.B.1 for a more detailed discussion of the motivation for and validity of instrumenting the Compustat balance sheet intangibles using the BEA’s measure of intangible capital.
the OLS projection of the investment gap on the intangible share, controlling for industry effects — that is, the residuals from model (2) in table 1.\(^1\) These "residual" investment gaps are between two thirds and three quarters smaller, in magnitude, than the actual investment gap. Overall, these industry-level results suggest that the rising importance of intangibles may well account for a substantial portion of the observed discrepancy between physical investment and average physical \(Q\).

III.B.ii Firm-level evidence

We next turn to firm-level evidence on the relationship between the investment gap, as measured in equations of the type \(11\), and the potential presence of an omitted factor. An important feature of the data on public firms is that not all of them carry substantial amounts of intangibles; in fact, it is not infrequent for firms to have zero balance sheet intangibles altogether. For firms whose capital inputs are purely or primarily physical assets, equation 5 indicates that the investment gap should be smaller (in magnitude) than the investment gap of other firms.

Figure 7 reports a simple test of this prediction. In each KLEMS industry and year, firms are sorted by their intangible share. We define intangible intensity as the ratio of the intangible capital stock to total assets of the firm. High intangible-intensity firms are defined as those in the top quartile by intangible intensity, while low intangible-intensity firms are defined as those in the bottom quartile. In all groups, the bottom quartile by intangible intensity corresponds to firms whose intangible share is 0-5% of total assets. The threshold for the top quartile rises over time, from roughly 40% to 60% for all industries except manufacturing, where it rises from 35% to 50%. The figure suggests that the investment gap is indeed larger among more intangible-intensive firms (and conversely, lower for firms whose capital is primarily physical).

Finally, figure 8 reports estimates of the time effects \(\zeta_t\) in a regression of the form:

\[
i_{j,t} = \alpha_j + \zeta_t + \xi \nu_{j,t+1} + \delta Q_{j,t} + \beta' X_{j,t-1} + \epsilon_{j,t},
\]

where \(\nu_{j,t+1} = \frac{K_{2,t+1}}{K_{1,t+1}}\) is the end-of-period ratio of intangible to physical capital. This control mimics the expression of the wedge apparent in expression 5, \(w_{j,t} = \nu_{j,t+1} \frac{1}{1+r_t} E_t [q_{2,t+1}^{(c)}]\). There is no good empirical proxy for the marginal value of intangible assets, so that the wedge is measured with error, and the estimates of the coefficient \(\xi\) are likely to suffer from attenuation bias.\(^1\)

---

\(^1\)The graphs are constructed in two steps: first, the residuals are estimated from the OLS regression of model (2); then, these residuals are averaged at the sector- or economy-wide levels using the share of nominal value added of the sector in 2001 as weights.

\(^2\)Further analysis of the model indicates that \(q_{2,t} = \frac{\partial Q_t}{\partial \eta_t}\), so that one could in principle try to approximate \(q_2\) by the ratio of the change in \(Q\) to the change in \(\nu\). However, this leads to estimates of \(q_2\) that display a high degree of volatility, as the change in \(\nu\) is often small.
estimates of $\xi$ are negative and significant. Figure 8 suggests that despite the likely attenuation bias, the investment gap in the Consumer, High-tech, and Healthcare sectors falls by about a quarter relative to the baseline model where $\xi = 0$. In the manufacturing sector, consistent with figure 7, the firm-level evidence indicates that the investment gap seems less closely related to the increase in the intangible share. Recall, however, that the trend of a rising intangible share is weakest in the manufacturing sector.

The evidence in this section thus points to the fact that the gap in physical investment, relative to $Q$, can largely be traced back to the increase in the intangible intensity of sectors and firms. This is consistent with the simple model laid out in the earlier part of the section, which suggests that in the presence of an omitted factor — in this case, intangible capital — we should expect $Q$ to overstate the true marginal value of physical assets, and investment-$Q$ regressions to be significantly biased.

IV The link with market structure

In this section, we go beyond weak physical investment and ask whether the rise in intangible capital has other implications. In particular, we examine another important macroeconomic trend: the apparent shift in market structure toward more concentration in a number of US industries. We first review work on concentration and proposed explanations, and then show that there is a strong empirical relationship between concentration and intangibles, both at the industry and firm levels. This relationship could reflect either intangibles’ effect on productivity or their effect on market power, so we then examine each channel separately.

IV.A Market concentration and intangible capital

The previous section documented the increasing role of intangible capital in the US business sector. At the same time, recent research emphasizes a coincident increase in concentration in many industries. The properties of intangible capital suggest that this timing may not be only coincidental. Haskel and Westlake (2017) emphasize that intangible capital tends to be scalable, such as software or intellectual property, facilitating the growth of large, intangible-intensive firms. If this approach were correct, one would expect that intangible capital would be more prevalent in the leading firms of an industry, leading these more productive firms to dominate, increasing their own market share and industry concentration. In addition, other forms of intangible capital, such as brand value, may increase market power by reducing the price elasticity of demand for the firm’s products. Indeed, a number of papers have documented the potential for
“customer capital” to explain a number of patterns of firm pricing and investment decisions. In addition, even if intangibles such as intellectual property enhance firms’ productivity, patent and other protections may deter competition and increase concentration.

To date, the literature that documenting the rise in concentration has proposed several explanations. Autor et al. (2017) show that the rising concentration in many US industries coincides with a falling labor share in those industries. They argue that this may result from technological change, in which industry leaders adopt new technologies, increase efficiency and advance their market share. This necessarily raises concentration and reduces their labor share of compensation.

Other researchers point out rising markups along with concentration, and argue that concentration may be associated with rising market power and weakening competition (Gutiérrez and Philippon, 2018, 2017; De Loecker and Eeckhout, 2017). Gutiérrez and Philippon (2018) argue that rising market power may also lead to weak investment, as less competitive firms have less incentive to invest.

While these two hypotheses, technology and market power, are not mutually exclusive, they have sharply different implications for economic efficiency and welfare. Technological improvement is associated with rising productivity and investment, and higher welfare as a result. On the contrary, increasing market power lowers investment and reduces welfare. Hence, the source of rising concentration is crucial to understanding whether rising concentration is efficient.

We first document the empirical link between rising concentration and intangible capital intensity, and then use this link to differentiate between market power and efficiency explanations for the rise in concentration. Table 2 shows that there is a positive correlation between intangibles and market share at the firm level in our data; that is, industry leaders tend to be intangible-intensive. This result holds both in the cross section and in the time series: within industries, firms with higher market share tend to be more intangible-intensive; additionally, when a firm’s intangible share rises, its market share increases. This result is robust to controlling for firm effects (which subsume industry effects) and time effects. This result complements the observations of section III, which emphasized that, in many industries, the investment gap is driven by intangible-intensive firms. Here, we find that high, and rising, intangibles intensity, is associated with higher market share, and rising market share, respectively.

Finding that industry leaders tend to be intangible-intensive naturally leads to the question of whether this increase in firms’ market shares translates into greater industry concentration. That is, are more intangibles intensive industries also more concentrated — and hence potentially responsible for the increase in concentration noted in the literature? Because there is so much heterogeneity across industries in both intangibles and concentration, we also break the data into industry clusters, as we did for the investment

See, for instance, Gourio and Rudanko (2014).
data. Table 3 shows that in each of the four industry clusters, the Herfindahl index is on average higher when the firms in those industries are more intangible-intensive. This effect is statistically significant in all four industry clusters. The comovement between concentration and intangible intensity is evident in Figure 9, which charts the industry clusters of Herfindahl indexes and firm-level intangible shares.\(^\text{16}\)

Overall, intangibles appear to be more important in industries in which concentration is higher, and the rise in concentration seems to have been connected to an increasing intangible intensity in a number of sectors. However, what is the mechanism connecting intangibles to concentration? In the next section, we explore the two hypotheses suggested earlier to explain rising concentration — productivity and market power — and their links to intangible capital.

## IV.B  Market power and productivity explanations for rising concentration

In this section, we examine the mechanism behind rising concentration. Specifically, are intangibles associated with concentration because they confer market power, or are they instead a source of competitive advantage? Either is certainly plausible, as patents, for example, may represent technological improvements that enhance productivity, yet they may also exclude competition. We conduct the analysis both at the industry level — where more precise, expenditure-based measures of intangible capital from the BEA are available — and at the firm level, where sample sizes are larger.

### IV.B.i  Intangibles and market power

Our main measures of market power are industry- and firm-level markups. We first discuss their measurement, and then the link to intangible investment.

**Markup measurement**  We combine two approaches in our measurement of markups: the industry-level approach of Hall (1988) and Hall (2018), and the firm-level approach of De Loecker and Eeckhout (2017). Specifically, using industry-level data, we estimate the overall level of markups, between 1988 and 2015, as the elasticity of the change in output to the change in total inputs, including quasi-fixed inputs such as capital.\(^\text{17}\) We then adjust the (firm-level) ratios

\[
\frac{\text{sales}_{j,t}}{\text{cogs}_{j,t}}
\]

to match, within a KLEMS industry, the average markup estimated using the industry-level data. As discussed by De Loecker and Eeckhout (2017),

\(^{16}\)Two features of these graphs are notable. First, in the High-tech sector, concentration is falling during the mid- to late 1990’s. This is entirely due to entry into the Compustat sector for one particular industry, NAICS 518 (Data processing, internet publishing, and other information services); it captures the effect of the dotcom bubble on the decision to go public. The second trend is the reversal in concentration in the transportation industry, NAICS 336, but concentration in other manufacturing industries, in the Compustat sample, also seems to have stabilized — if not declined — around that period.

\(^{17}\)See Hall (2018) for details on the methodology. In particular, in estimating the elasticity, we instrument changes in output at the industry level using proxies for aggregate demand, specifically, government purchases of different categories of final goods and services, as well as changes in oil prices.
within a particular industry, the ratio of revenue to cost of goods sold is a correct measure of firm-level markups, up to an industry-specific constant.\footnote{This constant is the elasticity of output to cost of goods sold; it must be assumed to be constant over time and across firms in an industry for the prior statement to hold.} Our procedure amounts to choosing the industry-specific constant in such a way that markups obtained from the firm-level \cite{DeLoecker2017} approach match the average levels documented by Hall (2018). Note that while this affects the levels of markups, it does not change the trends, which are only driven by changes in the ratio of sales to cost of goods sold at the firm-level.\footnote{The Online Appendix reports results using the inverse Lerner index (which includes other operating costs than those captured in \textit{cogs}) in order to proxy for markups. The results are largely consistent with those reported here. We focus on the ratio of sales to cost of goods in the main body of the paper because operating margins may also reflect fixed or quasi-fixed costs.}

\textbf{Industry-level evidence} Figures 10a and 10b report averages of these estimates at the aggregate level, and at the level of 4 broad groups of sectors. In constructing these figures, one average markup time series is first calculated for each KLEMS industry. This average is value-weighted using firm sales, so that it primarily reflects the markups charged by the largest firms in an industry. (We come back below to within-industry variation in markups.) The average markup across industries is then computed using industries’ shares of total nominal value added in 2001.

Consistent with \cite{DeLoecker2017} and Hall (2018), we find that markups have increased over the span of the sample, though the increase is more moderate than documented in \cite{DeLoecker2017}, with the average markup rising from approximately 1.2 to 1.4. The trends, however, differ sharply across the four major sector groups. In particular, in the Consumer sector, which is primarily made up of the Retail and Wholesale Trade industries, markups are stable, consistent with the evidence reported in Crouzet and Eberly (2018). They also appear to be stable in the Manufacturing group. By contrast, in both High-Tech and Healthcare, markups have been rising sharply, particularly so after 1995.

Table 4 provides further evidence of this pattern in the relationship between markups and intangibles, at a more disaggregated level. It reports OLS and IV estimates of the elasticity of markups with respect to the Compustat intangible share, at the level of the KLEMS industry, splitting the sample between the four major sector groups.\footnote{Industries not in one of the four major sector groups are omitted; see the Online Appendix for details on the industry classification used in this paper.} The instrument used for the Compustat intangible share is the BEA intangible share. We instrument for the Compustat intangible share in order to address the concern that balance sheet intangibles are likely to measure true underlying intangible capital with substantial error. The measurement error could arise either because firms do not apply consistent standards in capitalizing past expenditures on intangibles, or because goodwill deviates from the true underlying value of acquired intangible capital (for instance, because of over- or under-valuation at the time of acquisition). Both sources of measurement error
could plausibly be correlated to subsequent outcomes, either at the firm or at the industry level. The BEA intangible capital stock, while only capturing a small portion of the overall intangible stock (primarily that related to R&D), is immune to both measurement problems, since it is derived only from expenditures on inputs (not from acquisition values), and since those measures are consistently capitalized. Any measurement error in the BEA stock is thus plausibly uncorrelated with the measurement error in balance sheet intangibles.

Results of this analysis are broadly consistent with the message of Figures 10a and 10b. For the Consumer sector group, the relationship between the intangible share and markups is negative, and economically small. In the Manufacturing, High-tech and Healthcare groups, the relationship is positive and significant (though, in the Manufacturing sector, the low Cragg-Donald F statistic indicates potentially weak instruments.)

**Firm-level evidence** We next document whether, within an industry, more intangible-intensive firms tend to charge higher markups. Table 5 summarizes the firm-level relationship between markups and the intangible share. Panel A reports results from a specification containing industry-year fixed effects. The first row of the table reports OLS results, and the second row reports instrumental variables results. The motivation for using IV is the same as in the industry regressions, namely measurement error. The instruments in these regressions are the estimates of R&D capital and organizational capital proposed by Peters and Taylor (2017). While these stocks are also imprecise estimates of underlying intangible capital, their measurement error is likely to be independent to that affecting balance sheet intangibles, since the Peters and Taylor (2017) are based only on expenditures (as opposed to also incorporating acquisition values, in balance sheet intangibles), that are furthermore consistently capitalized across firms.\(^{21}\)

The results of panel A indicate that within an industry/year, firms with a higher intangible share also tend to have higher markups, even after controlling for size, age, profitability, and leverage. The effect is strongest in the Healthcare and High-tech groups. The results of panel B in table 5, however, suggest that this may be partially a firm specific effect. Panel B adds firm fixed effects to both the OLS and IV specifications.\(^{22}\) The results in Panel B confirm a very strong effect of intangibles on markups in the Healthcare sector. In High-tech, removing the firm effect reduces the magnitude of the effect substantially, but it is still large and

\(^{21}\)Another concern, specific to the regressions with markups as the dependent variable, is that the dependent variable is also measured with error, and that this error may be correlated with the instrument itself. The measurement error in the dependent variable may arise because some of the variable costs of the firm may not be included in the denominator of the markup measure, \textit{cogs}. Indeed, SG&A may contain in part operating costs; for instance, some firms in the Consumer sector use SG&A to book a part of labor costs, as discussed in Crouzet and Eberly (2018). Thus firms with high SG&A will tend to have high (measured) markups; this may then create a mechanical correlation between measured markups and measures of the intangible stock that contain capitalized SG&A, such as the organizational capital stock measure of Peters and Taylor (2017). However, the exclusion restriction would only be threatened if (past) SG&A spending were correlated with the measurement error in balance sheet intangibles. It is not obvious why this would be the case, given that the measurement error in balance sheet intangibles is primarily due to either heterogeneous report conventions, or under-/over-valuation of intangibles. For industry regressions, the problem is less pressing because the BEA measures are unlikely to contain expenditures on inputs contributing toward current sales.

\(^{22}\)We also explored a first-difference specification; however, the results were generally insignificant, likely due to measurement error in both the dependent and independent variables.
statistically significant, while in Manufacturing, the effect remains close to unity and significant. The effect of intangibles on markups in the Consumer group remains precisely estimated, but quantitatively small.

Given the strength of the result in Healthcare, it is worth noting that the correlation is significant both across firms, and within firms, meaning that firms with more intangibles charge higher markups, and markups rise as firms increase their intangible share. This sector group includes both pharmaceuticals and device makers, as well as hospitals and health care providers. However, the largest firms in the sector are the leading drug manufacturers, and belong to NAICS subsector 325, chemicals manufacturing. Results using weighted regressions suggest that high markups are primarily driven by those firms, as opposed to either smaller firms, or firms in service-related Healthcare sectors.\footnote{These results are available from the authors upon request.}

In the next section, we explore what else may be driving the link between intangibles and concentration, in addition to market power.

\section*{IV.B.\textit{i}} \textbf{Intangibles and productivity}

We next turn to the evidence on the connection between productivity and intangible investment. We focus on measures of average labor productivity — as opposed to total factor productivity — because they can be constructed from firm-level data without specific assumptions on the production function. The most natural measure of average labor productivity at the firm level is sales per worker, \( lp_{j,t} = \frac{s_{j,t}}{l_{j,t}} \). Since sales are measures at market prices, not at cost, markups are embedded in this measure; to quantify this, note that one can decompose the ratio as:

\begin{equation}
lp_{j,t} = \frac{\mu_{j,t} c_{j,t}}{\theta l_{j,t}}, \quad \mu_{j,t} \equiv \theta \frac{s_{j,t}}{c_{j,t}},
\end{equation}

where \( c_{j,t} \) denotes the cost of goods sold, \( \theta \) denotes the elasticity of revenue to inputs, and \( \mu_{j,t} \) denotes the firm’s markup. In order to isolate the part of the ratio of sales to worker related to productivity from that related to the markup, we therefore report results using the ratio:

\begin{equation}
lpc_{j,t} = \frac{c_{j,t}}{l_{j,t}},
\end{equation}

which we refer to as “sales per worker, at cost”.

Figure 11a reports the economy-wide average of the measures \( lp_{j,t} \) and \( lpc_{j,t} \), and compares them with output per hour, obtained from the BLS. The three time series display consistent trends at the aggregate level, roughly doubling over the duration of the sample, with a slowdown after 2009. At the more disaggregated
sectoral level, the measures derived from firm-level data again line up with the BLS output per hour measure, as reported in Figure 11b. However, trends differ markedly across sectors. Labor productivity growth was weak overall in the Manufacturing and Healthcare sectors; in the Consumer sector, it was close to the aggregate trend; and in the High-tech sector, labor productivity boomed, rising almost five-fold over the decade.

Industry-level evidence  Table 6 reports results from industry-level regressions for the four major sector groups of Figure 11a. These results reinforce the view that the experience of Healthcare and Manufacturing — both sectors where productivity grew slowly — differed sharply from those of the Consumer and High-tech sectors. The first panel of 11a reports the results of a simple OLS regression of our measure of labor productivity derived from firm-level data (sales per worker, at cost) on the Compustat industry-level intangible share. These regressions contain (KLEMS-level) industry effects, so that they ask whether trends in the Compustat intangible share coincided with the trends in labor productivity. The results are consistently positive across sectors — productivity tends to comove positively with intangible intensity. However, the second panel of the table, in which the Compustat intangible share is instrumented using the BEA fixed asset tables’ measure of the intangible share, suggests that this result is not robust for the Manufacturing and Healthcare sectors; in particular, the coefficient falls by about two-thirds in the Healthcare sector, and becomes insignificant in the Manufacturing sector. However, in the Consumer and High-tech sectors, the correlation remains significant (and in fact, increases in magnitude) once potential measurement error is accounted for. Thus, the industry-level connection between intangibles and labor productivity seems strongest precisely in those sectors where labor productivity grew most, the Consumer and High-tech sectors.

Firm-level evidence  Table 7 describes the relationship between intangibles and our main measure of labor productivity at the firm level. Panel A compares firms within the same (KLEMS) industry and year. The first row reports OLS results, and the second row reports instrumental variables results, as for the earlier analysis of markups. The Consumer and Healthcare sectors are essentially mirror images of their results for markups, as we saw in the industry data. Intangibles are associated with positive productivity effects in the Consumer group, but a negative effect in Healthcare. In Manufacturing and High-tech, these effects are positive, but weak. Moreover, this appears to be largely the firm-fixed effect. Panel B controls for firm fixed effects, and the results show a significant positive effect of intangibles on productivity in the Manufacturing and High-tech sectors, as we saw in the industry data. However, the within-firm effect is

24 The exception is the healthcare sector, where the trends derived from aggregating firm-level data appear to be different from the trends based on BLS data. However, the discrepancy is only before 1993; after 1993, there is virtually no labor productivity growth in the sector, according to either source. The rise in productivity in the healthcare sector over the 1990-1993 period in the firm-level data is entirely attributable to NAICS sector 621, ambulatory health services.
stronger in Healthcare (negatively) and Consumer (positively), in particular with the IV estimates.\textsuperscript{25}

Across both industries and firms, the analysis of productivity provides a consistent message. In the two sectors that experienced sharp increases in labor productivity, Consumer and High-tech, there was a correlated rise in intangible intensity. These estimates are also quantitatively important, as the increase in intangibles in the Consumer group would imply roughly doubling of productivity (compared to the actual increase of 150 percent). In High-tech, the share is smaller, with rising intangibles explaining about a 50 percent increase in productivity. The fact that these more intangible-intensive firms also tend to be leaders in market share, as emphasized above, suggests a potential explanation for the rise in concentration in those sectors. As leading firms invested in intangible assets, these firms increased their productivity and market share lead over competitors, opening a wider “productivity gap” relative to the rest of the industry. By contrast, in the Healthcare and Manufacturing sectors, which did not experience large increases in labor productivity, the relationship between intangibles and productivity is insignificant or tends to even be negative.

\textbf{IV.C Summary: intangibles, markups, and productivity}

Intangible capital has become more important at the same time that concentration has risen in many industries. We show that these trends are more than coincident, as the industries and firms with greater intangibles intensity also tend to have greater concentration, and as firms increase their intangibles intensity, their market share also tends to rise. This comovement suggests a potential mechanism for rising concentration, as firms invest more heavily in intangible capital, facilitating economies of scale or the exercise of market power. In particular, rising concentration may result from changes in technology in an otherwise competitive environment, and thus be largely efficient. Or it may arise from market power, leading to bigger wedges between price and marginal cost, and potentially inefficient allocations. Intangible investment could play a role in both scenarios, given that intangible assets can serve both to establish market power and to advance productivity. Patents, for instance, confer exclusivity and thus pricing power, but could also be indicative of productivity-enhancing research conducted by a firm.

We examined these possibilities by looking at productivity and markups by industry and by firm. We confirm that in general, both markups and productivity have risen along with concentration, so both hypotheses...
play a role. However, there are sharp differences across broad sector groups.

In the Consumer sector group, the rise in concentration was primarily driven by productivity, not markups. Moreover, the increase in productivity was highly correlated with intangible investment, both between and within firms. The Consumer sector contains retail giants like Walmart and Amazon, who have transformed distribution through innovative processes, which generate intangible capital and drive productivity growth. Interestingly, these processes, such as inventory management methods, are not generally patentable. However, to the extent that they require large scale (such as big box distribution), they may nonetheless deter replication or entry.

The Healthcare group is the mirror image of the Consumer sector, where the primary phenomenon behind the rise in concentration was markups, yet the increase in markups was also highly correlated with intangible investment. While there has clearly been extensive innovation in the Healthcare sector, many goods (pharmaceuticals, devices) are subject to patent protection, allowing for market power. This may explain why intangibles are more associated with markups than productivity increases in Healthcare.

In the High-tech sector, both forces are at work. High-tech is where one might have most expected to see intangible investment leading to productivity gains, but the data show that the large increase in intangible capital also led to rising markups. Thus, in High-tech, intangible investment may contribute to product differentiation as much as it contributes to productivity growth. Both are plausible sources of the large increase in concentration in High-tech in recent years.

Finally, concentration rose mildly in Manufacturing; but there, productivity and markups were stable, and intangible investment, which rose much more slowly than in other sectors, seems related to rising markups, but the effect is economically small. Given the long decline in manufacturing employment, consolidation may be a reasonable explanation for the remaining rise concentration, though our evidence does not speak to this or other alternative hypotheses directly.

V Policy implications

So far, we have shown that the rising share of intangible capital can help to explain the weak performance of investment in the 2000s and beyond. Moreover, investment in intangible capital contributed to rising industry concentration, though for different reasons depending on the industry. Does the increasing role of intangible capital change how policymakers should think about their impact on investment, and on the economy more broadly? We have focused the fact that intangible capital has unique properties compared to physical capital, and here we argue that these properties also affect the policy transmission mechanism.
V.A  User cost

Investment is one of the main channels through which interest rates affect the real economy. This is most evident in the user cost of capital, which measures the effective rental rate of capital. The interest rate plays a central role, since it measures the opportunity cost of investing in capital rather than in other assets. The user cost also depends on taxes, since tax considerations affect the rate of return to capital. The classic formulation of the user cost was presented by Hall and Jorgenson in 1967 as

\[ u = P (r + \delta) - \dot{P}, \tag{17} \]

where \( u \) is the user cost of capital, \( p \) is the price of capital, \( r \) is the relevant discount or interest rate, \( \delta \) is the economic depreciation rate, and \( \dot{P} \) is the growth rate of the price of capital, or the capital gains on holding physical capital. Hence, the cost of holding physical capital is the opportunity cost \( r \) plus depreciation, less any capital gain, or price increase, on the capital. Setting the capital gain term to zero for simplicity, they show that the tax adjustment generates a user cost of the form

\[ u = P (r + \delta) \frac{(1 - t_c)(1 - \tau z)}{(1 - \tau)}, \tag{18} \]

where \( t_c \) is the investment tax credit, \( \tau \) is the firm’s tax rate, and \( z \) is the present value of depreciation allowances on a dollar of investment. The power of monetary policy traditionally enters through the interest rate \( r \), since the response of the user cost to changes in \( r \) is given by the elasticity

\[ \frac{r}{u} \frac{\partial u}{\partial r} = \frac{r}{r + \delta}. \tag{19} \]

When the depreciation rate is low, this elasticity is nearly unity, and the user cost moves one-for-one with changes in the interest rate.

However, with intangible capital, that narrative is lost; in fact, the elasticity goes to zero as the depreciation rate becomes high relative to the interest rate. The depreciation rate of intangible capital as calculated and implemented by the BEA ranges from 20 percent for some innovative property to almost 70 percent for some forms of economic competences such as advertising. Other estimates are higher; Li and Hall (2016), for example estimate that R&D capital has a 30 percent annual depreciation rate, while Computer System Design has an almost 50 percent depreciation rate.

In the elasticity of the user cost with respect to the interest rate above, the higher is the depreciation rate, the lower is the elasticity. For an interest rate of 5 percent, the elasticity falls from 33 percent to around 10%.
percent as the depreciation rises from 10 percent to the range of estimates presented by Hall and Li (30 to 50 percent). Hence, the higher depreciation rates embodied in intangible capital necessarily mean the user cost is less sensitive to interest rates by a factor of 3 or more — and hence less responsive to monetary policy. Thus, if intangible capital represents a large share of firms’ investment, investment becomes less interest sensitive. This may have the benefit of reducing overall investment volatility, but it does pose a potential challenge to policy makers by dampening monetary policy transmission.

It is worth noting that this does not necessarily imply a larger role for fiscal policy. Going back to Hall and Jorgenson’s formula for the tax adjusted user cost, the elasticity of the user cost with respect to the tax rate is

$$\frac{\tau}{\partial u} = \tau \frac{1 - z}{1 - \tau (1 - \tau z)}.$$ (20)

When a capital good is expensed, $z = 1$, the above elasticity is zero, and the user cost is unaffected by the corporate tax rate. This is the case for most intangibles. This means that tax policy is also mostly ineffective in moving the user cost of intangible capital. When a capital good is amortized, as has been implemented for R&D spending, then a cut in the corporate tax rate decreases the user cost (the first term), but this is effect is muted by depreciation allowances (the second term).

Other forms of fiscal policy can have a more direct effect on intangible investment. The investment tax credit has a direct impact on any form of capital investment. Similarly, for any amortized capital investment, accelerated depreciation increases the present value of the depreciation allowances. Either of these policies would be effective with respect to intangible capital.

V.B Liquidity

A feature common to most forms of intangible assets is that there are limited, and sometimes no markets on which they can be readily sold to other potential users. Blair and Wallman (2000) suggest classifying intangibles into three categories: those that can be owned and sold, such as patents, copyrights and brands; those that can be controlled by the firm, but not separated from it, such as proprietary databases or software, or in-process R&D; and those that cannot be fully controlled by the firm, such as human capital or customer/supplier relationships. Intangibles in the second and third category are either too firm-specific, or not sufficiently under the firms’ direct control, to be easily transferred to other users; as a result, no markets for them exist.

The scarcity of markets for intangibles has an important implication: business lending against intangible assets is difficult, if at all possible. A substantial portion of business lending — and in particular, bank-based business lending — is asset-backed. The backing can be explicit, in the form of a lien on specific physical
assets, such as machinery, or liquid financial assets, such as account receivables. Or it can take the form of a protection of the creditor’s claim on overall firm assets in case of liquidation (by committing not to pledge assets to future creditors, a practice known as “negative lien”). Either way, this requires that, in case of a default event, firm assets can be sold and transferred to another user. Unsurprisingly, examples of lending purely backed by intangibles thus tend to be limited, mostly to intellectual property.26

An implication of the limited liquidity of intangible assets is that business lending in intangible-intensive industries may be constrained, relative to industries where capital inputs are primarily tangible. There are some signs of this in the industry-level data; figure 12 shows that there is a negative (and significant, after controlling for industry effects) relationship between total leverage and total intangible intensity across US industries.

This has at least two implications for the transmission of monetary policy to corporate investment. The first, straightforward implication is that a declining portion of overall investment may be financed through debt, and in particular through bank debt. Attempts to leverage the bank lending channel of monetary policy transmission may therefore face diminishing returns, as more economic activity moves toward intangible-intensive sectors.27 The second implication is that even conditional on begin financed by debt, the sensitivity of investment to monetary policy shocks may still decline. First, as noted above, the high depreciation rates of intangibles make them less interest-sensitive; as a result, the response of (intangible) asset prices to declining interest rates should fall, which in turn should mute the response of firm net worth. Thus, balance sheet effects may be weaker in an intangible-intensive economy. Second, the same change in asset prices might do less relax credit constraints if assets are more difficult to pledge. This latter channel is an important source of amplification in collateral constraints models, such as Kiyotaki and Moore (1997).

One direction for policy to consider is then to relax requirements that might affect banks’ incentive to lend against intangibles, or short of this, directly underwrite and potentially guarantee intangible-based lending. The Singaporean government, offered, between 2016 and 2018, an intellectual property financing scheme, in which it employed its intellectual property (IP) office to conduct summary valuation exercises of the IP offered by the borrower as collateral. In addition to those efforts, the general promotion of "IP marketplaces" could help generate more precise estimates of collateral values; it has indeed been under consideration in a number of countries, including the UK.28

However, these solutions are still relevant only to the types of intangibles that can plausibly be transferred to other users; this is only a fraction of the total stock of intangible assets which must be financed. For

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26See Kamiyama et al. (2006) and Loumioti (2012) for a discussion of the use of intangibles, and intellectual property in particular, as collateral in bank financing.
28See OECD (2015) for a discussion of these efforts.
non-transferrable intangibles, lending may simply not be the optimal financial arrangement. Collateral is, after all, the typical solution to the moral hazard problems associated with lending, but it may just not be available in these situations. The structural change toward a more intangible-intensive economy might instead require a broader shift in incentives away from debt financing and toward equity financing. There are no shortage of arguments and policy proposals to equalize the treatment of equity and debt in the corporate tax system; the difficulty inherent financing intangibles through debt adds to that list.29

VI Conclusion

While physical investment has been low relative to corporate valuations since the mid-2000’s, we document that a substantial portion of this gap is associated with a shift toward more investment in intangible capital. This intangible capital, which includes patents, brands and business processes, accounts for an increasingly large share of corporations’ overall capital stock.

Importantly, the rise in intangible capital is concentrated among industry leaders, and thus closely related to the rise in industry concentration which took place over the same period. The evidence furthermore suggests that intangible investment by leaders has been associated with productivity gains in some industries, in particular the Consumer and High-tech sectors. In those industries, intangible investment may have helped leaders further their competitive advantage and gain market share. By contrast, in the Healthcare sector (and to a less extent in High-tech), intangible investment is closely associated with rising markups at leading firms. In that sector, intangible investment may instead have enabled industry leaders to exert market power. These results emphasize that a “one size fits all” approach to concentration may be misguided, and at least premature. The sources — and hence the implications — of concentration need not be the same in all firms nor in all industries.

More work is needed to understand why intangible capital may confer market power in certain circumstances (the Healthcare sector) but not in others (the Consumer sector), where it instead enhances productivity. This distinction is particularly important in policy settings where there is scope for unintended consequences of well-intentioned policies. Policies intended to promote intangible capital (such as intellectual property protections) may instead do more to create market power. Or on the contrary, policies intended to reduce concentration might instead discourage productivity-enhancing intangible investment.

One approach to this distinction is to identify certain types of intangible investment — such as R&D-related intellectual property or business processes — that promote productivity gains, while other types — such as patentable R&D, trademarks or brands — that confer market power. Features common to all forms of

29See, for instance, Mirrlees et al. (2012) for the UK. Belgium has recently moved in this direction, with the introduction of a notional interest deduction on the basis of shareholder equity.
intangible capital, such as scalability or non-transferability, may also have different implications depending on the type of goods or services which the industry produces. Pursuing these hypotheses requires more refined data on intangible capital, which is not generally available at the firm level, though data collection is improving and may change with updates to accounting and measurement of intangible capital. In addition, modeling of industry equilibrium will provide more structure for the analysis of industry concentration.

As we work to refine and understand the role of intangible capital, however, the fact is that software, intellectual property, business practices, and other intangibles are already a substantial presence on firms’ balance sheets and in the national accounts.

References


Figure 1: Aggregate and sector-level investment. In Compustat data, investment is defined as $\frac{capx}{ppge}$. The two Compustat investment rates reported are the industry-wide average (blue line) and the ratio of industry total capital expenditures to industry total property, plant and equipment (purple line). In the BEA data, investment is defined as the ratio of investment in physical assets to their replacement cost. Investment rates are computed first at the KLEMS industry level, then weighted across industries using the value added of the industry in 2001. The top panel reports the economy-wide average, and the bottom panel reports the average for four major sector groups. Details on the data sources and the industrial classification used to define the four sectors are reported in the Online Appendix.
Figure 2: The economy-wide and sectoral investment gaps. The investment gap is measured first at the KLEMS industry level, as the time effect $\gamma_t$ in the regression $i_{j,t} = \alpha_j + \gamma_t + \delta Q_{j,t-1} + 3CF_{j,t-1} + \epsilon_{j,t}$. In this regression, $j$ indexes a firm and $t$ indexes a year. The regression controls for $Q$, as well as for the ratio of cash flow to assets, $CF$; see the Online Appendix for a definition of these in terms of Compustat variables. The top panel shows economy-wide averages, while the second panel shows average for four major sector groups. Averages of the time effects $\gamma_t$ across industries are weighted using the industry’s nominal value added in 2001. Details on the data sources and the industrial classification used to define the four sectors are reported in the Online Appendix.
**Figure 3:** Intangible capital as a fraction of total firm capital. The graphs report measures of the intangible share using Compustat data (solid lines) and BEA data (dashed line). The Compustat measures use the ratio \( \frac{\text{intangible assets}}{\text{total capital}} \). The two Compustat ratios reported are industry-level averages of firm-level intangible shares, and industry-level total intangible capital, as a fraction of total capital. In the BEA data, the intangible share is defined as the ratio of the replacement cost of non-physical capital (own-account software, R&D, and other intellectual property), to the replacement cost of total capital. Averages across industries are computed using the industry’s share of nominal value added in 2001. Details on the data sources and the industrial classification are reported in the Online Appendix.
Figure 4: Changes in the composition of investment. This graph reports the change in the share of aggregate investment of seven major sectors between 1988 and 2015. See the Online Appendix for details on the industrial classification; the Utilities, Finance, Construction, and Others sectors are omitted. The data are from the BEA fixed asset tables.

Figure 5: Scatterplot of the industry-level investment gap, $\gamma_t$, against the average intangible share. The investment gaps are estimated at the level of the 52 KLEMS industries, then averaged up to the 12 sector level, using the value added share of the industry in 2001 as weights. The intangible share is defined at the firm level as the ratio $\frac{\text{intang}}{\text{total \_assets}}$. The average intangible share is computed for each KLEMS industry and year separately; it is then averaged to the 12 sector and year level, using the same weights as for the measures of the investment gap.
Figure 6: The investment gap, before and after projecting onto average firm-level intangible shares. The top panel shows economy-wide averages, while the second panel shows average for four major sector groups. The blue line reports averages of the industry-level estimates of the investment gap, with a 2 standard error confidence band. The dashed green line plots the residual from the regression of the industry-level investment gap on the industry-level Compustat intangible share, including industry fixed effects. This is the same specification as in the first line, second column of table 1. Averaging across industries, to the sector- or economy-wide levels is done using the industry’s share of nominal value added in 2001.
Figure 7: The investment gap among high and low intangible intensity firms. The orange line plots the estimated investment gap among low intangible intensity firms, and the shaded area reports the +/- 2 standard error bands. The blue line reports the investment gap among high intangible intensity firms. Low intangible intensity firms are defined as the bottom quartile of the intangible share in a particular KLEMS industry/year; high intangible intensity firms are defined at the top quartile. The results are computed at the KLEMS industry level, and averaged across industries weighting by industries’ share of nominal value added. We use Compustat balance sheet intangibles \textit{intan} to total assets \textit{intan+ppegt} as our measure of intangible intensity.
Figure 8: The investment gap after controlling for intangibles. The red line plots the estimated investment gap among all firms, and the +/- 2 standard error bands. The green line reports the investment gap after controlling for the end-of-period ratio $\nu_{t+1} = \frac{K_{2,t+1}}{K_{1,t+1}}$ at the firm level, where $K_{1,t+1}$ is physical capital and $K_{2,t+1}$ is intangible capital. We use Compustat balance sheet intangibles intan to total assets intan+ppeg as our proxy for $\nu_{t+1}$.
Figure 9: The graphs plot the average concentration of sales in four broad sector groups, jointly with the average share of intangibles. The share of intangibles is measured using Compustat balance sheet intangibles. Averages across industries (within a sector) are weighted using the share of value added of the industry in 2001. Details on the data sources and the industrial classification are reported in the Online Appendix.
Figure 10: Trends in markups. The top panel shows economy-wide averages, while the second panel shows average for four major sector groups. The purple line shows the raw average of sales/cogs; the long dashed grey line shows the average of sales/cogs, adjusted to match the Hall (2018) estimates of average industry markups over the period; the short dashed blue line shows estimates of the markups obtained using the methodology of De Loecker and Eeckhout (2017). For the last line, we estimate output elasticities θ at the KLEMS industry level, but do not adjust for the residuals in that estimation; see the Online Appendix for details. All estimates are constructed at the KLEMS industry level first, then averaged across industries using their share of nominal value added in 2001. At the industry level, the markup ratios are averaged using firm-level sales in that year as weights. Markups are winsorized at the 1st and 99th percentiles, by year. Finally, the agricultural and mining sectors are dropped, as markup measures obtained using the KLEMS data are negative in both cases.
Figure 11: Trends in labor productivity across industries. The top panel shows economy-wide averages, while the second panel shows averages for four major sector groups. The two solid lines are derived from Compustat data. The purple line is the ratio of industry/year total sales (Compustat variable `sale`) to industry/year total workers (Compustat variable `emp`). The blue line is the ratio of industry/year total cost of goods (Compustat variable `cogs`) to industry/year total workers. Variables are deflated by the output price index for the corresponding KLEMS industry, obtained from the KLEMS database. The dashed line is output per hour from the KLEMS database. All estimates are constructed at the KLEMS industry level first, then averaged across industries using their share of nominal value added in 2001. All estimates are normalized to 100 in 1988.
Figure 12: Relationship between intangible intensity and leverage at the industry level. Leverage is defined at the ratio of total debt (Compustat items dlc and dltt) to total book assets (Compustat item at). Intangibles are defined as Compustat balance sheet intangibles, but the results are robust to different definitions of the intangible share. The data are aggregated to the twelve-sector KLEMS level, weighting underlying KLEMS industries by their share of value added in 2001. Both leverage and the intangible share are plotted net of industry effects. The slope is $-0.16$, with a t-stat of $-1.64$ with industry-clustered standard errors.
### Table 1: Regressions of the investment gap on average industry-wide intangible shares. An observation is a KLEMS industry/year. Model (1) contains no fixed effects; model (2) contains KLEMS industry effects; model (3) contains KLEMS industry and year effects. In all models, standard errors are clustered at the industry and time level; t-statistics are reported in parentheses. The first and second line contains estimates of a simple OLS regression of the investment gap on the Compustat intangible share, while the third to fifth lines contain estimates where the Compustat intangible share has been instrumented using the BEA’s estimate of the intangible capital share in the KLEMS industry. See main text for a definition of the dependent and independent variables. *: \( p < 0.10 \), **: \( p < 0.05 \), ***: \( p < 0.01 \).

<table>
<thead>
<tr>
<th>Dependent variable: investment gap</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compustat intangible share  ( s_t ) (OLS)</td>
<td>(-0.137^{***})</td>
<td>(-0.210^{***})</td>
<td>(-0.065^{**})</td>
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<tr>
<td></td>
<td>((-6.84))</td>
<td>((-10.44))</td>
<td>((-2.38))</td>
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<tr>
<td>Compustat intangible share  ( s_t ) (IV)</td>
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<td>(-0.303^{***})</td>
<td>(-0.269^{***})</td>
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<td></td>
<td>((-2.82))</td>
<td>((-7.68))</td>
<td>((-3.02))</td>
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<tr>
<td><strong>First-stage F-stat</strong></td>
<td>47.94</td>
<td>33.72</td>
<td>13.54</td>
</tr>
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</table>

| Observations       | 1456    | 1456    | 1456    |
| Industry f.e.      | No      | Yes     | Yes     |
| Year f.e.          | No      | No      | Yes     |

### Table 2: Firm-level relationship between market share and intangible share. The dependent variable is the firm’s share of total sales in its KLEMS industry, and the independent variable is its intangible share. Model (A) contains year-industry fixed effects; models (B) and (C) contain firm effects (and, in the case of model (C), year effects). In model (A), standard errors are clustered at the industry and year level; in models (B) and (C), they are clustered at the firm and year level. t-statistics in parentheses. ***: \( p < 0.10 \), **: \( p < 0.05 \), ***: \( p < 0.01 \).

<table>
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<tr>
<th>Dependent variable: market share</th>
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<th>(B)</th>
<th>(C)</th>
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<tbody>
<tr>
<td>Compustat intangible share</td>
<td>0.0131^{***}</td>
<td>0.0096^{***}</td>
<td>0.0073^{***}</td>
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<tr>
<td></td>
<td>(17.69)</td>
<td>(5.40)</td>
<td>(4.91)</td>
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<p>| Observations       | 98520    | 97245    | 97245    |
| Industry ( \times ) year f.e. | Yes      | No       | No       |
| Firm f.e.          | No       | Yes      | Yes      |
| Year f.e.          | No       | No       | Yes      |</p>
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<th>Sector group</th>
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<th>Av. OLS coefficient</th>
<th>t-stat of av. &gt; 2?</th>
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<td>0.78</td>
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<td>0.09</td>
<td>0.11</td>
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<tr>
<td>Healthcare</td>
<td>0.09</td>
<td>0.19</td>
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<tr>
<td>Manufacturing</td>
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<td>0.19</td>
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</tr>
<tr>
<td>Other</td>
<td>0.46</td>
<td>-0.20</td>
<td>no</td>
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</tbody>
</table>

Table 3: Industry-level average correlation between intangible intensity and concentration. For each KLEMS industry, a regression of the Herfindahl index of sales on the industry-level intangible intensity is estimated. The coefficients are then averaged across industries in a particular sector group. The standard errors are also averaged across industries; the t-stat of the average is the ratio of the two.

<table>
<thead>
<tr>
<th>Dependent variable : markup (log)</th>
<th>Consumer</th>
<th>Manufacturing</th>
<th>High-tech</th>
<th>Healthcare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compustat intangible share $s_t$ (OLS)</td>
<td>-0.132***</td>
<td>0.044*</td>
<td>0.452***</td>
<td>0.709***</td>
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<tr>
<td></td>
<td>(-6.32)</td>
<td>(1.62)</td>
<td>(5.90)</td>
<td>(6.01)</td>
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<td>Compustat intangible share $s_t$ (IV)</td>
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<td>0.879***</td>
<td>0.498***</td>
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<td>(-8.75)</td>
<td>(2.98)</td>
<td>(2.81)</td>
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<td>802.12</td>
<td>10.47</td>
<td>89.31</td>
<td>617.89</td>
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<td>112</td>
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<td>Industry f.e.</td>
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<td>Yes</td>
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Table 4: Industry-level relationship between markups and the share of intangible assets. The dependent variable is the log of the industry-wide average markup, defined as the ratio $sale/cogs$, adjusted to match the industry-level average markups estimated using the Hall (2018) method. Results are reported separately for 4 broad group of sectors. All regressions contain industry effects. The first panel reports the simple OLS coefficient, while the second panel report coefficients when the Compustat intangible share is instrumented using the BEA measure of intangibles. The t-statistics reported in parentheses are computed using heteroskedasticity-robust standard errors. Cragg-Donald F statistic reported for the first stage. *: $p < 0.10$, **: $p < 0.05$, ***: $p < 0.01$. 

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Table 5: Firm-level relationship between intangibles and markups. The depend variable is \( \log(\text{sale/cogs}) \). Panel A reports results from specifications without firm fixed effects, while panel B reports results from specifications with firm fixed effects. The Compustat intangible share is \( \frac{\text{intan}}{(\text{ppegt} + \text{intan})} \). The instruments in the IV specifications are either the ratio of capitalized R&D expenditures to capital (excluding balance sheet intangibles), \( \frac{k_{\text{know}}}{k_{\text{know}} + k_{\text{org}} + \text{ppegt}} \), or the ratio of (a fraction of) capitalized SG&A expenditures to capital, \( \frac{k_{\text{org}}}{k_{\text{know}} + k_{\text{org}} + \text{ppegt}} \), or both. The variables \( k_{\text{know}} \) and \( k_{\text{org}} \) are obtained from Peters and Taylor (2017). All dependent variables are measured at the beginning of the observation year. Firm controls are: size (\( \log(\text{ppegt}) \)), age (years since first appearance in CRSP), leverage ((\( \text{dlc}+\text{dltt} \))/\( \text{at} \)), and cash flow to assets (\( \frac{\text{ebitda}}{\text{at}} \)). Kleibergen-Paap (KP) Wald F statistics are reported for the IV specifications. The excluded instruments (the Peters-Taylor intangible shares) are selected according to the following criterion: if the KP statistic is higher than the Stock-Yogo critical values for 15% maximal IV size, keep both; otherwise, keep the one with the highest KP statistics. This criterion selects both the SG&A and R&D share for the High-tech and Manufacturing sector, and only the SG&A share for Consumer and Healthcare sectors. * : \( p < 0.10 \), ** : \( p < 0.05 \), *** : \( p < 0.01 \).
<table>
<thead>
<tr>
<th></th>
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<th>Manufacturing</th>
<th>High-tech</th>
<th>Healthcare</th>
</tr>
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<tbody>
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<td><strong>Dependent variable</strong>: labor productivity (log)</td>
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<td>Compustat intangible share $s_t$ (OLS)</td>
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<td>2.153***</td>
<td>1.713***</td>
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<td>(16.49)</td>
<td>(17.67)</td>
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<td>Compustat intangible share $s_t$ (IV)</td>
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<td>(4.76)</td>
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<td><strong>First-stage F-stat</strong></td>
<td>214.98</td>
<td>10.01</td>
<td>116.16</td>
<td>1547.90</td>
</tr>
<tr>
<td>Observations</td>
<td>56</td>
<td>504</td>
<td>168</td>
<td>112</td>
</tr>
<tr>
<td>Industry f.e.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Table 6:** Industry-level relationship between intangibles and labor productivity. The dependent variable is the log of the industry-wide ratio of cost of goods sold (cogs) to industry-wide employment (emp), after deflating cogs using the KLEMS price indices. Results are reported separately for 4 broad group of sectors. All regressions contain industry effects. The first panel reports the simple OLS coefficient, while the second panel reports coefficients when the Compustat intangible share is instrumented using the BEA measure of intangibles. The t-statistics reported in parentheses are computed using heteroskedasticity-robust standard errors. * : $p < 0.10$, ** : $p < 0.05$, *** : $p < 0.01$. 
### Panel A: Cross-sectional (between) regressions

<table>
<thead>
<tr>
<th></th>
<th>Consumer</th>
<th>Manufacturing</th>
<th>High-tech</th>
<th>Healthcare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compustat intangible share $s_{j,t}$ (OLS)</td>
<td>0.561***</td>
<td>0.123</td>
<td>0.082</td>
<td>-0.316***</td>
</tr>
<tr>
<td>Compustat intangible share $s_{j,t}$ (IV)</td>
<td>0.593</td>
<td>0.428</td>
<td>0.840**</td>
<td>-1.610***</td>
</tr>
<tr>
<td>First-stage F stat</td>
<td>71.3</td>
<td>200.7</td>
<td>41.5</td>
<td>79.7</td>
</tr>
<tr>
<td>Observations</td>
<td>8027</td>
<td>24436</td>
<td>19730</td>
<td>10296</td>
</tr>
<tr>
<td>Firms</td>
<td>646</td>
<td>1726</td>
<td>1718</td>
<td>878</td>
</tr>
<tr>
<td>Firm-level controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Standard error clustering</td>
<td>Industry-year and firm</td>
<td>Industry-year and firm</td>
<td>Industry-year and firm</td>
<td>Industry-year and firm</td>
</tr>
<tr>
<td>Industry-year f.e.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm f.e.</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

### Panel B: Panel (within) regressions

<table>
<thead>
<tr>
<th></th>
<th>Consumer</th>
<th>Manufacturing</th>
<th>High-tech</th>
<th>Healthcare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compustat intangible share $s_{j,t}$ (OLS)</td>
<td>0.247***</td>
<td>-0.016</td>
<td>0.044</td>
<td>-0.033</td>
</tr>
<tr>
<td>Compustat intangible share $s_{j,t}$ (IV)</td>
<td>2.736***</td>
<td>0.854</td>
<td>0.834***</td>
<td>-3.988**</td>
</tr>
<tr>
<td>First-stage F stat</td>
<td>11.3</td>
<td>20.3</td>
<td>30.9</td>
<td>9.1</td>
</tr>
<tr>
<td>Observations</td>
<td>8027</td>
<td>24436</td>
<td>19730</td>
<td>10296</td>
</tr>
<tr>
<td>Firms</td>
<td>646</td>
<td>1726</td>
<td>1718</td>
<td>878</td>
</tr>
<tr>
<td>Firm-level controls</td>
<td>Yes (excl. age)</td>
<td>Yes (excl. age)</td>
<td>Yes (excl. age)</td>
<td>Yes (excl. age)</td>
</tr>
<tr>
<td>Standard error clustering</td>
<td>Industry-year and firm</td>
<td>Industry-year and firm</td>
<td>Industry-year and firm</td>
<td>Industry-year and firm</td>
</tr>
<tr>
<td>Industry-year f.e.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm f.e.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 7: Firm-level relationship between intangibles and labor productivity. The depend variable is log (cogs/emp), where cogs has been deflated at the KLEMS industry level using the KLEMS output price deflators. Panel A reports results from specifications without firm fixed effects, while panel B reports results from specifications with firm fixed effects. The Compustat intangible share is $\frac{intan}{pegt + intan}$. The instruments in the IV specifications are either the ratio of capitalized R&D expenditures to capital (excluding balance sheet intangibles), $\frac{k_{know}}{k_{know} + k_{org} + pegt}$, or the ratio of (a fraction of) capitalized SG&A expenditures to capital, $\frac{k_{org}}{k_{know} + k_{org} + pegt}$, or both. The variables $k_{know}$ and $k_{org}$ are obtained from Peters and Taylor (2017). All dependent variables are measured at the beginning of the observation year. Firm controls are: size (log(pegt)), age (years since first appearance in CRSP), leverage ((dlc+dltt)/at), and cash flow to assets (ebida/at). t-statistics in parentheses. Kleibergen-Paap (KP) Wald F statistics are reported for the IV specifications. The excluded instruments (the Peters-Taylor intangible shares) are selected according to the following criterion: if the KP statistic is higher than the Stock-Yogo critical values for 15% maximal IV size, keep both; otherwise, keep the one with the highest KP statistics. This criterion selects both the SG&A and R&D share for the High-tech and Manufacturing sector, and only the SG&A share for Consumer and Healthcare sectors. * : $p < 0.10$, ** : $p < 0.05$, *** : $p < 0.01$. 

A Theory

This appendix describes in detail the model used in section III to discuss how omitted factors may affect the interpretation of empirical investment-\(Q\) regressions.

I.A Firm model

Time is discrete. The firm solves:

\[
V(K_{1,t}, K_{2,t}, X_t) = \max A_t (\alpha K_{1,t}^\rho + (1 - \alpha) K_{2,t}^\rho)^\frac{1}{\rho} - C(1)(K_{1,t}, I_{1,t}, P_{1,t}) - C(2)(K_{2,t}, I_{2,t}, P_{2,t})
\]

\[
+ \frac{1}{1 + r} \mathbb{E}_t [V(K_{1,t+1}, K_{2,t+1}; X_{t+1})]
\]

\[s.t.\]

\[
K_{1,t+1} = I_{1,t+1} + (1 - \delta_1) K_{1,t}
\]

\[
K_{2,t+1} = I_{2,t+1} + (1 - \delta_2) K_{2,t}
\]

where \(K_{1,t}\) and \(K_{2,t}\) are two types of capital, and \(X_t = (A_t, P_{1,t}, P_{2,t})\) collects the exogenous processes entering the firm’s problem. \(A_t\) captures both firm-specific and aggregate business conditions, and \(P_{n,t}\) denotes the price of capital of type \(n\). We assume that all three follow stationary first-order Markov processes. Note that the two types of capital are also allowed to depreciate at different rates. Furthermore, following Hayashi (1982), we assume that adjustment costs for each type of capital are quadratic:

\[
C(n)(I_{n,t}, K_{n,t}, P_{n,t}) = \left( P_{n,t} \frac{I_{n,t}}{K_{n,t}} + \frac{\gamma_n}{2} \left( \frac{I_{n,t}}{K_{n,t}} \right)^2 \right) K_{n,t},
\]

where \(\gamma_n\) captures the curvature of the adjustment costs for capital of type \(i\).

The revenue function of the firm has constant returns to scale with respect to the two capital inputs \((K_{1,t}, K_{2,t})\). The parameter \(\rho \in [-\infty, 1]\) controls the elasticity of substitution between the two types of capital. They are substitutes if \(0 < \rho < 1\), with \(\rho = 1\) the perfect substitutes (additive) case. They are complements if \(\rho < 0\), with \(\rho = -\infty\), the perfect complements (Leontief) case. When \(\rho = 0\), the revenue function is Cobb-Douglas in the two forms of capital, with shares \((\alpha, 1 - \alpha)\). 30

I.B Optimal investment and firm value

It is straightforward to show that the first-order conditions for optimal investment are:

\[
i_{n,t} = \frac{1}{\gamma_n} \left( \frac{1}{1 + r} \mathbb{E}_t [q_{n,t+1}] - P_{n,t} \right), \quad n = 1, 2
\]

where:

\[
i_{n,t} \equiv \frac{I_{n,t}}{K_{n,t}}, \quad q_{n,t} \equiv \frac{\partial V}{\partial K_n}(K_{1,t}, K_{2,t}; X_t).
\]

30Here, we use the terms “complements” and “substitutes” to refer to the responsiveness of the ratio of the two capital types to the ratio of their prices. To see why the cases \(\rho < 0\) and \(\rho > 0\) map to complementarity and substitutability in this model, note that, when \(\gamma_1 = \gamma_2 = 0\), and when all stochastic processes are assumed to be constant, the optimal investment policy involves choosing:

\[
\frac{K_2}{K_1} = \left( \frac{1 - \alpha \delta_1 + r P_1}{\alpha \delta_2 + r P_2} \right)^\frac{1}{\rho}.
\]

A 1% fall in the relative user cost of type-2 capital therefore leads to a \(\frac{1}{\rho} \%\) increase in the ratio of type-2 to type-1 capital. This elasticity is \(+\infty\) in the perfect substitutes case, 0 in the perfect complements case, and 1 in the Cobb-Douglas case. Note that, when \(\rho > 0\), the marginal revenue product of one type of capital is increasing in the other capital’s input, while when \(\rho < 0\), it is decreasing.
Moreover, the constant returns to scale assumption implies the following decomposition of firm value.

**Lemma 1.** The value function of the firm can be written as:

\[ V(K_{1,t}, K_{2,t}; X_t) = q_1(\nu_t; X_t) K_{1,t} + q_2(\nu_t; X_t) K_{2,t}, \]  

(26)

where:

\[ \nu_t \equiv \frac{K_{2,t}}{K_{1,t}} \]  

(27)

**Proof.** We first establish the homogeneity of degree 1 of \( V(K_{1,t}, K_{2,t}; X_t) \) in \((K_{1,t}, K_{2,t})\) following the method of Abel and Eberly (1994). Fix \( K_{1,t}, K_{2,t} \). Let \( a > 0 \). We have:

\[ V(aK_{1,t}, aK_{2,t}; X_t) \geq aV(K_{1,t}, K_{2,t}; X_t), \]  

(28)

Indeed, the investment plan \((aI_{1,t+k}, aI_{2,t+k})\) is implementable from \((aK_{1,t}, aK_{1,t})\) and yields \( aV(K_{1,t}, K_{2,t}) \), because of the linear homogeneity of the objective and constraints. Since we have established this for arbitrary \((K_{1,t}, K_{2,t}, a)\), we also have that:

\[ V\left(\frac{1}{a}(aK_{1,t}), \frac{1}{a}(aK_{2,t}); X_t\right) \geq \frac{1}{a} V(aK_{1,t}, aK_{2,t}; X_t). \]  

(29)

So,

\[ V(aK_{1,t}, aK_{2,t}; X_t) = aV(K_{1,t}, K_{2,t}; X_t), \forall a > 0, \]  

(30)

that is, \( V(K_{1,t}, K_{2,t}; X_t) \) is homogeneous of degree one in \((K_{1,t}, K_{2,t})\). Using Euler’s theorem, the homogeneity of degree one implies that:

\[ V(K_{1,t}, K_{2,t}; X_t) \equiv \frac{\partial V}{\partial K_1}(K_{1,t}, K_{2,t}; X_t) K_{1,t} + \frac{\partial V}{\partial K_2}(K_{1,t}, K_{2,t}; X_t) K_{2,t}. \]  

(31)

Additionally, the partial derivatives of \( V(K_{1,t}, K_{2,t}; X_t) \) with respect to \( K_{1,t} \) and \( K_{2,t} \) are homogeneous of degree 0 with respect to \((K_{1,t}, K_{2,t})\). This implies that we can write:

\[ q_{1,t} = \frac{\partial V}{\partial K_1}(K_{1,t}, K_{2,t}; X_t) = \frac{\partial V}{\partial K_1} \left(1, \frac{K_{2,t}}{K_{1,t}}; X_t\right) \equiv q_1(\nu_t; X_t), \quad \frac{\partial V}{\partial K_2}(K_{1,t}, K_{2,t}; X_t) = q_2(\nu_t; X_t). \]  

(32)

Moreover, the envelope theorem implies that the marginal \( q_t \) functions satisfy the following two dynamic equations:

\[
q_1(\nu_t; X_t) = \alpha A_t (\alpha + (1 - \alpha) \nu_t^\rho)^{\frac{1}{\rho} - 1} + \frac{\gamma_1}{2} (i_{1,t})^2 + \frac{1 - \delta_1}{1 + \rho} E_t[q_1(\nu_{t+1}; X_{t+1})].
\]

\[
q_2(\nu_t; X_t) = (1 - \alpha) A_t (\alpha + (1 - \alpha) \nu_t^\rho)^{\frac{1}{\rho} - 1} \nu_t^{-1} + \frac{\gamma_2}{2} (i_{2,t})^2 + \frac{1 - \delta_2}{1 + \rho} E_t[q_2(\nu_{t+1}, X_{t+1})],
\]

(33)

where \( i_{1,t}, i_{2,t} \) and \( \nu_{t+1} = i_{2,t} + (1 - \delta_2) \nu_t \) denotes the optimal investment policies. Both marginal \( q \) functions can thus be expressed as the expected discounted present value of positive terms; therefore, \( q_{1,t}(\nu_t; X_t) \geq 0, \ i = 1, 2. \)

**I.C Generalization to other cost functions, and the \( q_1 = q_2 \) case**

Note that assuming perfect substitutability \((\rho = 1)\) in the previous model is in general not sufficient to eliminate biases in \( Q \) regressions, even when replacing average \( Q \) by ”total” average \( Q, \frac{V}{K_1 + K_2} \). Indeed,
in the perfect substitutes case, the dynamic equations for marginal \( q \) become:

\[
q_1(\nu_t; X_t) = \alpha A_t + \frac{\gamma_1}{2} \hat{q}_{1,t} + \frac{1 - \delta_1}{1 + r} E_t [q_1(\nu_{t+1}; X_{t+1})].
\]

\[
q_2(\nu_t; X_t) = (1 - \alpha) A_t + \frac{\gamma_2}{2} \hat{q}_{2,t} + \frac{1 - \delta_2}{1 + r} E_t [q_2(\nu_{t+1}, X_{t+1})].
\]

(34)

In this case, the marginal \( q \) functions are independent of \( \nu_t \) (\( q_1(\nu_t; X_t) = q_1(X_t) \)). But even so, they are not necessarily equal — at least not so long as \( \alpha \neq \frac{1}{2}, \delta_1 \neq \delta_2, \gamma_1 = \gamma_2 \) and \( P_{1,t} \neq P_{2,t} \). As a result, total \( Q \) is still different from either of the marginal \( q \) functions.

More assumptions are needed to obtain equal marginal \( q \) functions. Assume that the adjustment cost functions take a general form:

\[
C_i(I_{1,t}, I_{2,t}, K_{1,t}, K_{2,t}; X_t),
\]

(35)

while remaining homogeneous of degree 1 in \((I_{1,t}, I_{2,t}, K_{1,t}, K_{2,t})\) and convex. With these general cost functions we still obtain the firm value decomposition:

\[
V(K_{1,t}, K_{2,t}; X_t) = q_1(\nu_t; X_t) K_{1,t} + q_2(\nu_t; X_t) K_{2,t},
\]

(36)

where \( q_1 \) and \( q_2 \) are again marginal \( q \) for each capital type, which are now governed by the following dynamic equations:

\[
q_1(\nu_t; X_t) = \alpha A_t + (1 - \alpha) \nu_t^{\rho - 1} - \frac{\partial C_1, t}{\partial K_{1,t}} - \frac{\partial C_2, t}{\partial K_{2,t}} + \frac{1 - \delta_1}{1 + r} E_t [q_1(\nu_{t+1}; X_{t+1})]
\]

\[
q_2(\nu_t; X_t) = (1 - \alpha) A_t + (1 - \alpha) \nu_t^{\rho - 1} + \frac{\partial C_1, t}{\partial K_{1,t}} + \frac{\partial C_2, t}{\partial K_{2,t}} + \frac{1 - \delta_2}{1 + r} E_t [q_2(\nu_{t+1}; X_{t+1})]
\]

(37)

Because of their homogeneity of degree 0, the partial derivatives of the cost functions can always be written as a function only of \( \nu_t \) and investment rates.

Using this generalized cost function, we can pinpoint more specifically the cases in which \( q_1 = q_2 \). Assume that the cost functions have the property that:

\[
\frac{\partial C_1, t}{\partial K_{1,t}} + \frac{\partial C_2, t}{\partial K_{2,t}} = \frac{\partial C_1, t}{\partial K_{1,t}} + \frac{\partial C_2, t}{\partial K_{2,t}} = \frac{1}{2} \gamma_1 I_{1,t} + \gamma_2 I_{2,t} - \frac{1}{1 + r} E_t [q_1(\nu_{t+1}; X_{t+1})] = -\frac{1}{2} (\gamma_1 \hat{q}_{1,t} + \gamma_2 \hat{q}_{2,t}),
\]

(38)

where the investment rates \( \hat{I}_{i,t} = \frac{I_{i,t}}{K_{1,t} + K_{2,t}} \). This is the assumption made in Peters and Taylor (2017), who use the following cost function:

\[
C_{i, t}^{PT} = \left( P_{i,t} - \frac{I_{i,t}}{K_{1,t} + K_{2,t}} + \frac{\gamma_i}{2} \left( \frac{I_{i,t}}{K_{1,t} + K_{2,t}} \right) \right)^2 (K_{1,t} + K_{2,t})
\]

(39)

When adjustment cost functions satisfy the condition above, we can write the dynamic \( q \) equations as:

\[
q_1(\nu_t; X_t) = \alpha A_t + (1 - \alpha) \nu_t^{\rho - 1} + \frac{1}{2} (\gamma_1 \hat{q}_{1,t} + \gamma_2 \hat{q}_{2,t}) + \frac{1 - \delta_1}{1 + r} E_t [q_1(\nu_{t+1}; X_{t+1})]
\]

\[
q_2(\nu_t; X_t) = (1 - \alpha) A_t + (1 - \alpha) \nu_t^{\rho - 1} + \frac{1}{2} (\gamma_1 \hat{q}_{1,t} + \gamma_2 \hat{q}_{2,t}) + \frac{1 - \delta_2}{1 + r} E_t [q_2(\nu_t; X_t)]
\]

(40)

Next, assume that inputs are perfect substitutes and have the same rates of depreciation:

\[
\rho = 1, \quad \alpha = (1 - \alpha) = \frac{1}{2}, \quad \delta_1 = \delta_2.
\]

(41)
Under these assumptions the dynamic equations simplify to:

\[ q_1(\nu_t; X_t) = \frac{1}{2} A_t + \frac{1}{2} (\gamma_1 \tilde{i}_{1,t} + \gamma_2 \tilde{i}_{2,t}) + \frac{1 - \delta}{1 + r} E_t [q_1(\nu_{t+1}; X_{t+1})] \]

\[ q_2(\nu_t; X_t) = \frac{1}{2} A_t + \frac{1}{2} (\gamma_1 \tilde{i}_{1,t} + \gamma_2 \tilde{i}_{2,t}) + \frac{1 - \delta}{1 + r} E_t [q_2(\nu_{t+1}; X_{t+1})] \]

and so we have:

\[ q_1(\nu_t; X_t) = q_2(\nu_t; X_t) = q(X_t). \]  

(43)

In that case,

\[ Q_t = \frac{V_t}{K_{1,t} + K_{2,t}} = \frac{1}{1 + \nu_t} q_1(X_t) + \frac{\nu_t}{1 + \nu_t} q_2(X_t) = q_1(X_t) = q_2(X_t), \]

(44)

so that average \(Q\) regressions are unbiased. Note that the model is still not fully symmetric, as it allows for different prices of investment and different adjustment costs. But because inputs are perfect substitutes, both in the production and cost functions, only (weighted) averages of the capital prices and adjustment costs matter.