Online appendix

A.1 Theory

This appendix provides details for the theoretical results reported in Section 2..

A.1.1 Main model

We start by a describing and analyzing in more detail the balanced growth model described in Section 2. and used in Section 3. for empirical analysis. This model uses a value added production function.

A.1.1.1 Description

Firm The representative firm solves:

$$TC_t = \min_{\mathbf{K}_t, L_t} \quad \sum_{n=1}^{2} R_{n,t} K_{n,t} + W_t L_t \quad \text{s.t.} \quad Z_t \left(K_{1,t}^{1-\eta} K_{2,t}^{\eta} \right)^{\alpha} L_t^{1-\alpha} \geq Y_t \quad [MC_t]$$

where TC_t denotes total costs of production, $\mathbf{K}_t = \{K_{n,t}\}_{n=1}^{2}$ is a vector of capital inputs, with $K_{1,t}$ the measured capital input, and $K_{2,t}$ the omitted intangible input, L_t is labor input, $\{R_{n,t}\}_{n=1}^{2}$ is a vector of user costs, W_t is the wage rate, Z_t is total factor productivity, $1-\alpha$ is the elasticity of output with respect to labor, and η is the elasticity of the total capital input $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. Total factor productivity evolves exogenously, following:

$$dZ_t = g_Z Z_t dt.$$

The solution to this problem is:

$$TC_t = MC_t Y_t \tag{19}$$

$$
\begin{aligned}
MC_t &= \frac{1}{Z_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha} \\
L_t &= MC_t\frac{(1-\alpha)Y_t}{W_t} \\
K_t &= MC_t\frac{\alpha Y_t}{R_t} \\
K_t &= K_{1,t}^{1-\eta}K_{2,t}^{\eta} \\
R_t &= \left(\frac{R_{1,t}}{1-\eta}\right)^{1-\eta}\left(\frac{R_{2,t}}{\eta}\right)^{\eta} \\
K_{1,t} &= (1-\eta)\frac{R_t}{R_{1,t}}K_t \\
K_{2,t} &= \eta\frac{R_t}{R_{2,t}}K_t
\end{aligned} \tag{20}
$$

The firm's revenue is $S_t = P_tY_t$ and its profits are $\Pi_t = P_tY_t - TC_t = (P_t - MC_t)Y_t$, where P_t is the price of consumption goods. The labor share is:

$$
s_{L,t} \equiv \frac{W_tL_t}{P_tY_t} = (1-\alpha)\frac{MC_t}{P_t}.
$$

Household The representative household solves:

$$
\begin{aligned}
U(\mathbf{K}_t;\mathbf{X}_t) = \max_{\{C_{t+h},\mathbf{I}_{t+h}\}_{h\geq 0}} & \int_0^{+\infty} e^{-\rho h}\frac{C_{t+h}^{1-\sigma}}{1-\sigma}dh \\
\text{s.t.} \quad & dK_{n,t} = (I_{n,t} - \delta_n K_{n,t})\,dt, \quad n = 1,2 \\
& \sum_{n=1}^{2} R_{n,t}K_{n,t} + W_tL_t + \Pi_t = \sum_{n=1}^{2} Q_{n,t}I_{n,t} + C_t
\end{aligned}
$$

Here, ρ is the household's discount factor, $\sigma \geq 1$ is the intertemporal elasticity of substitution in consumption, $\{\delta_n\}_{n=1}^2$ is the rate of depreciation of capital, and the vector $\mathbf{X}_t$ collects all variables that are either exogenous or taken as given by the household when making consumption plans: $\mathbf{X}_t = \{W_t, L_t, \Pi_t, R_{1,t}, R_{2,t}, Q_{1,t}, Q_{2,t}\}$. In particular, the prices of investment goods, $\{Q_{n,t}\}_{n=1}^{2}$, and labor supply, L_t, all evolve exogenously, following:

$$\begin{aligned} dQ_{n,t} &= g_{Q_n} Q_{n,t} dt, \quad n = 1, 2 \\ dL_t &= g_L L_t dt. \end{aligned}$$

Equilibrium An equilibrium is a set of deterministic sequences for all endogenous variables such that (1) given the exogenous processes for labor, productivity, and the prices of capital goods, the endogenous variables satisfy the solution to the firm's problem and solve the representative consumer's problem; and (2) the price of consumption goods and their marginal cost of production are related through:

$$P_t = \mu MC_t,$$

where $\mu > 1$ is the exogenous price-cost markup. Finally, in equilibrium, we normalize the price level to $P_t = 1$, so that all other prices are expressed relative to consumption goods.

A.1.1.2 Balanced growth path

We next derive the unique stationary, or balanced growth, equilibrium of the model. Define an aggregate price index for capital goods Q_t as:

$$Q_t = Q_{1,t}^{1-\eta} Q_{2,t}^{\eta}.$$

Next, define the trend growth factor X_t as:

$$T_{X,t} = L_t Z_t^{\frac{1}{1-\alpha}} Q_t^{-\frac{\alpha}{1-\alpha}},$$

and define the detrended variables:

$$c_t \equiv \frac{C_t}{T_{X,t}}$$

$$w_t \equiv \frac{W_t L_t}{T_{X,t}}$$

$$\pi_t \equiv \frac{\Pi_t}{T_{X,t}}$$

$$i_{n,t} \equiv \frac{Q_{n,t} I_{n,t}}{T_{X,t}}, \quad n = 1, 2$$

$$k_{n,t} \equiv \frac{Q_{n,t} K_{n,t}}{T_{X,t}}, \quad n = 1, 2$$

$$R_{d,n,t} \equiv \frac{R_{n,t}}{Q_{n,t}}, \quad n = 1, 2$$

$$R_{d,t} \equiv \frac{R_t}{Q_t}$$

$$y_t \equiv \frac{Y_t}{T_{X,t}}$$

$$k_t \equiv \frac{Q_t K_t}{T_t}$$

Moreover, define the trend growth rate (the growth rate of $T_{X,t}$), the capital price growth rate (the growth rate of Q_t), and the discount rate r as:

$$\begin{aligned} g &\equiv g_L + \frac{1}{1-\alpha} g_Z - \frac{\alpha}{1-\alpha} g_Q, \\ g_Q &\equiv (1-\eta) g_{Q_1} + \eta g_{Q_2}, \\ r &\equiv \rho + \sigma g. \end{aligned}$$

Using these detrended variables, the Hamilton-Jacobi-Bellman equation corresponding to the household's problem can be written as:

$$
\begin{aligned}
(r-g)\,u(\mathbf{k}_t,\mathbf{x}_t) &= \max_{c_t,\mathbf{i}_t} \quad \frac{c_t^{1-\sigma}}{1-\sigma} \quad + \quad \sum_{n=1}^{2}\left(i_{n,t}-\left(g+\delta_n-g_{Q_n}\right)k_{n,t}\right)u_{k_n,t}(\mathbf{k}_t,\mathbf{x}_t) \\
&\qquad\qquad + \sum_j \dot{x}_{j,t}u_{x_j,t}(\mathbf{k}_t,\mathbf{x}_t) \\
&\quad \text{s.t.} \quad \sum_{n=1}^{2} R_{d,n,t}k_{n,t}+w_t+\pi_t=\sum_{n=1}^{2} i_{n,t}+c_t
\end{aligned}
$$

where $\mathbf{i}_t=\{i_{n,t}\}_{n=1}^2$, $\mathbf{k}_t=\{k_{n,t}\}_{n=1}^2$, $\mathbf{x}_t=\{w_t,\pi_t,R_{d,1,t},R_{d,2,t}\}$, $u(\mathbf{k}_t,\mathbf{x}_t)=U(\mathbf{K}_t,\mathbf{X}_t)/T_{X,t}^{1-\sigma}$, and $\dot{x}_{j,t}\equiv\frac{dx_{j,t}}{dt}$. After simplifications, the two Euler equations associated with this problem can be written as:

$$
\begin{aligned}
\left(\sum_{n=1}^{2} R_{d,n,t}k_{n,t}+w_t+\pi_t-c_t-\sum_{n=1}^{2}(\delta_n+g-g_{Q_n})k_{n,t}\right)\frac{c_{k_n,t}}{c_t} &= \\
\frac{R_{d,n,t}-(r+\delta_n-g_{Q_n})+\sum_j \dot{x}_{j,t}u_{x_jk_n,t}}{\sigma}, &\qquad n=1,2.
\end{aligned}
$$

A balanced growth path is defined as an equilibrium where detrended variables are constant: $\dot{x}_{j,t}=0$ for all j, $\dot{k}_{n,t}=0$, $n=1,2$, and so on. Detrended variables without a time index indicate these constant values.

Since $\dot{k}_{n,t}=i_{n,t}-(g+\delta_n-g_{Q_n})k_{n,t}, n=1,2$, on the balanced growth path,

$$
i_{n,t}=i_n=(g+\delta_n-g_{Q_n})k_n, \quad n=1,2.
$$

Plugging this into the first-order condition, and using the budget constraint of the household and the fact that $\dot{x}_{j,t}=0$ for all j, we obtain that along the balanced growth path, detrended user costs must satisfy:

$$
R_{d,n,t}=R_{d,n}=r+\delta_n-g_{Q_n}, \quad n=1,2,
$$

the standard Hall-Jorgenson formula. rest of the balanced growth path is then given by:

$$
\begin{aligned}
MC &= \frac{1}{\mu} \\
R_d &= \left(\frac{R_{d,1}}{1-\eta}\right)^{1-\eta}\left(\frac{R_{d,2}}{\eta}\right)^{\eta} \\
y &= k^{\alpha} \\
k &= \left(\frac{\alpha}{\mu R_d}\right)^{\frac{1}{1-\alpha}} \\
w &= \frac{1-\alpha}{\mu}k^{\alpha} \\
\pi &= \frac{\mu-1}{\mu}k^{\alpha} \\
k_1 &= (1-\eta)\frac{R_d}{R_{d,1}}k \\
k_2 &= \eta\frac{R_d}{R_{d,2}}k \\
i_n &= (g+\delta_n-g_{Q_n})k_n = (R_{d,n}-(r-g))k_n, \quad n=1,2.
\end{aligned}
$$

A.1.1.3 Chained GDP growth vs. growth of output in consumption units

Finally, we discuss the relationship between growth of output in consumption units, $\frac{dY_t}{Y_t}$, and chained GDP growth, the usual empirical measures of real output. This discussion follows Oulton (2007).

First, note that we assume that measured nominal output is the sum of consumption expenditures, plus measured investment expenditures:

$$P_t\hat{Y}_t = P_tC_t + Q_{1,t}I_{1,t},$$

where $P_t\hat{Y}_t$ is measured nominal output (with P_t referring to the price of consumption goods, and $\hat{Y}_t$ measured output in consumption units); C_t is consumption; and $Q_{1,t}I_{1,t}$ are measured investment expenditures. In our model, measured chained GDP growth, $\frac{d\hat{Y}_t^{\text{ch}}}{\hat{Y}_t^{\text{ch}}}$, is *defined* as the share-weighted growth rate of real consumption and measured real investment:

$$\frac{d\hat{Y}_t^{\text{ch}}}{\hat{Y}_t^{\text{ch}}} \equiv (1 - s_{I_1,t})\frac{dC_t}{C_t} + s_{I_1,t}\frac{dI_{1,t}}{I_{1,t}},$$

where the share of investment in measured nominal GDP is:

$$s_{I,t} \equiv \frac{Q_{1,t}I_{1,t}}{P_t\hat{Y}_t},$$

where variables are defined as above. By contrast, since we have:

$$\hat{Y}_t = C_t + (Q_{1,t}/P_t)I_{1,t},$$

growth of measured output in consumption units is given by:

$$\frac{d\hat{Y}_t}{\hat{Y}_t} = (1 - s_{I_1,t})\frac{dC_t}{C_t} + s_{I_1,t}\left(\frac{dI_{1,t}}{I_{1,t}} + \frac{dQ_{1,t}}{Q_{1,t}} - \frac{dP_t}{P_t}\right).$$

Therefore, chained GDP growth is not equal to the growth rate of measured output in consumption units. Instead:

$$\frac{d\hat{Y}_t^{\text{ch}}}{\hat{Y}_t^{\text{ch}}} - \frac{d\hat{Y}_t}{\hat{Y}_t} = -s_{I_1,t}\left(\frac{dQ_{1,t}}{Q_{1,t}} - \frac{dP_t}{P_t}\right).$$

It is straightforward to see that this bias remains nonzero even in the balanced growth path. (Note that in the balanced growth path, since we normalize $P_t = 1$, the expression boils down to $-s_{I_1,t}\frac{dQ_{1,t}}{Q_{1,t}}$; however, $\frac{dQ_{1,t}}{Q_{1,t}}$ should then be interpreted as the change of the price of measured investment goods relative the price of consumption goods.)

Next, assume that, instead of measuring output growth using output in consumption units (as we do in our baseline approach), we were to measure it using chained GDP growth.

Then, denoting by $\dfrac{d\hat{Z}_t^{ch}}{\hat{Z}_t^{ch}}$ the Solow residual obtained using chained GDP growth, since our other measures of input growth and input shares are unchanged, we have:

$$\frac{d\hat{Z}_t^{ch}}{\hat{Z}_t^{ch}} = \frac{d\hat{Z}_t}{\hat{Z}_t} - s_{I_1,t}\left(\frac{dQ_{1,t}}{Q_{1,t}} - \frac{dP_t}{P_t}\right).$$

Note that, consistent with $\frac{dQ_{1,t}}{Q_{1,t}} < \frac{dP_t}{P_t}$, Table A2 shows that the Solow residual obtained using chained GDP growth is higher than the one obtained using growth of output in consumption units.

The rest of the derivations regarding the bias between the true rate of growth of neutral technology, $\frac{dZ_t}{Z_t}$, and the Solow residual $\dfrac{d\hat{Z}_t}{\hat{Z}_t}$, is unchanged. Therefore, we can express the bias between the chained GDP Solow residual, and the true growth rate of neutral technology, as:

$$\frac{d\hat{Z}_t^{ch}}{\hat{Z}_t^{ch}} - \frac{dZ_t}{Z_t} = \Delta_t^{(1)} + \Delta_t^{(2)} + \Delta_t^{(3)} + \Delta_t^{(4)},$$

where the terms $\Delta_t^{(1)}$, $\Delta_t^{(2)}$, and $\Delta_t^{(3)}$ are defined as in the baseline model, and:

$$\Delta_t^{(4)} \equiv -s_{I_1,t}\left(\frac{dQ_{1,t}}{Q_{1,t}} - \frac{dP_t}{P_t}\right).$$

In other words, with the chained GDP Solow residual, the analysis of the baseline text is unchanged, except that there is a fourth bias term. This term reflects the fact that with investment-specific technical change of the form assumed in our baseline model, the chained GDP Solow residual does not appropriately measure the growth rate of neutral technology. In our baseline approach, rather than adding the fourth bias terms $\Delta_t^{(4)}$, we instead measure the Solow residual using the growth rate of output in consumption units; as the previous discussion shows, the two approaches are equivalent, up to the additional bias term, $\Delta_t^{(4)}$.

A.1.1.4 Additional results and proofs

We next state the following result about the case of no markups ($\mu = 1$), and provide a proof below.

Result 5. *Assume that $\mu = 1$, $\sigma = 1$, and that the stock of intangibles is growing ($dK_{2,t}/K_{2,t} > 0$, or $g > g_{Q_2}$). Then, along the balanced growth path, TFP growth is biased downward ($\Delta < 0$), if and only if: $g_{Q_2} \geq \bar{\mathbf{g}}_2(\eta)$, where:*

$$\bar{\mathbf{g}}_2(\eta) = \frac{g_L + \delta_2}{\rho + \delta_2} g_Z + \frac{\rho}{\rho + g_L + \delta_2} g_{Q_1} + O(\eta). \tag{21}$$

In particular, if $\delta_2 \gg \rho$, measured TFP growth is biased downward, if and only if: $g_{Q_2} > g_Z$.[30]

Proof. [Result 5] Along the balanced growth path, using the fact that $\sigma = 1$, we have:

$$\begin{aligned} \Delta &= \alpha\eta \frac{-(r-g)\left(g_{Q_2} - g_{Q_1}\right) + \left(g + \delta_2 - g_{Q_2}\right)\left(g_Z - g_{Q_1} - (1+\alpha\eta)\left(g_{Q_2} - g_{Q_1}\right)\right)}{r - g + (1-\alpha\eta)\left(g + \delta_2 - g_{Q_2}\right)} \\ &= \alpha\eta \frac{-\rho\left(g_{Q_2} - g_{Q_1}\right) + \left(g + \delta_2 - g_{Q_2}\right)\left(g_Z - g_{Q_1} - (1+\alpha\eta)\left(g_{Q_2} - g_{Q_1}\right)\right)}{\rho + (1-\alpha\eta)\left(g + \delta_2 - g_{Q_2}\right)} \qquad \text{when } \sigma = 1 \end{aligned}$$

The stock of intangibles is growing if and only:

$$\begin{aligned} \frac{dK_{2,t}}{K_{2,t}} > 0 \quad &\Longleftrightarrow \quad g - g_{Q_2} > 0 \\ &\Longleftrightarrow \quad g_L + \frac{1}{1-\alpha}\left(g_Z - g_{Q_1}\right) - \frac{1-\alpha+\alpha\eta}{1-\alpha}\left(g_{Q_2} - g_{Q_1}\right) > 0 \end{aligned}$$

Define $\quad x \equiv g_{Q_2} - g_{Q_1}$. Then:

$$\begin{aligned} g - g_{Q_2} > 0 \quad &\Longleftrightarrow \quad g_L + \frac{1}{1-\alpha}\left(g_Z - g_{Q_1}\right) - \frac{1-\alpha+\alpha\eta}{1-\alpha} x > 0 \\ &\Longleftrightarrow \quad x \leq \bar{\bar{x}}(\eta) \equiv \frac{1-\alpha}{1-\alpha+\alpha\eta}\left[g_L + \frac{1}{1-\alpha}\left(g_Z - g_{Q_1}\right)\right] \end{aligned}$$

[30] The proof is reported in Appendix A.1.1.4, and illustrated in Appendix Figure A1 illustrates the result.

Thus intangible capital is growing so long as $x \leq \bar{\bar{x}}(\eta)$. In that case, note that the sign of the total bias only depends on the sign of its numerator. The sign of the numerator in Δ is the same as the sign of:

$$\begin{aligned}
\Delta(x,\eta) &= (x - B(\eta))\,(x - C(\eta)) - A(\eta)x \\
A(\eta) &\equiv \frac{(1-\alpha)(1+\alpha\eta)}{1-\alpha+\alpha\eta}\rho \\
B(\eta) &\equiv \frac{g_Z - g_{Q_1}}{1+\alpha\eta} \\
C(\eta) &\equiv \frac{1-\alpha}{1-\alpha+\alpha\eta}\left(g_L + \frac{1}{1-\alpha}\left(g_Z - g_{Q_1}\right) + \delta_2\right)
\end{aligned}$$

The minimum of $\Delta(.,\eta)$ is attained at $\hat{x}(\eta) \equiv \frac{1}{2}\left(A(\eta) + B(\eta) + C(\eta)\right)$. Moreover, a sufficient condition for $\hat{x}(\eta) > \bar{\bar{x}}(\eta)$ for all η is:

$$\delta_2 + \rho > \frac{\alpha}{(1-\alpha)(1+\alpha\eta)}\left(g_Z - g_{Q_1}\right) + g_L. \quad (1)$$

The discriminant of the polynomial $\Delta(.,\eta)$ can be rewritten as:

$$D(\eta) \;= (A(\eta) + B(\eta) + C(\eta))^2 - 4BC(\eta) = (C(\eta) - B(\eta))^2 + A(\eta)(A(\eta) + 2(B(\eta) + C(\eta)))$$

A sufficient condition for $D(\eta) > 0$ for all η is:

$$g_Z - g_{Q1} > 0. \quad (2)$$

Assume conditions that (1) and (2) hold. Then, given that $D(.,\eta)$ is a convex function with global minimum $\hat{x}(\eta)$,

$$D(x,\eta) < 0 \quad \text{and} \quad g > g_{Q_2} \quad \Longleftrightarrow \quad x_1(\eta) \leq x \leq \bar{\bar{x}}(\eta),$$

where x_1 is the smallest root of the polynominal $\Delta(x)$:

$$x_1(\eta) = \frac{1}{2}\left(A(\eta) + B(\eta) + C(\eta)\right)\left(1 - \sqrt{1 - \frac{4B(\eta)C(\eta)}{(A(\eta) + B(\eta) + C(\eta))^2}}\right)$$

The threshold reported in the Result is therefore given by:

$$\bar{\mathbf{g}}_2(\eta) \equiv g_{Q_1} + x_1(\eta).$$

For the expansion, assume that $C(\eta), A(\eta) \gg B(\eta)$. Then:

$$\begin{aligned}
x_1(\eta) &= \frac{1}{2}\left(A(\eta) + B(\eta) + C(\eta)\right)\left(1 - \sqrt{1 - \frac{4B(\eta)C(\eta)}{(A(\eta) + B(\eta) + C(\eta))^2}}\right) \\
&= \frac{1}{2}\left(A(\eta) + B(\eta) + C(\eta)\right) 2 \frac{B(\eta)C(\eta)}{(A(\eta) + B(\eta) + C(\eta))^2} + O(\eta) \\
&= \frac{B(\eta)C(\eta)}{A(\eta) + B(\eta) + C(\eta)} + O(\eta) \\
&= \frac{g_L + \delta_2}{\rho + g_L + \delta_2}\left(g_Z - g_{Q_1}\right) + O(\eta).
\end{aligned}$$

□

Appendix Figure A1 illustrates this result. In a first parameter region (highlighted in blue), the price of omitted capital is growing too slowly to generate negative measurement error in TFP growth, while in the second one (highlighted in green), the price of omitted capital is growing sufficiently fast so as to generate negative measurement bias. The frontier between the two regions — corresponding to the threshold $\bar{\mathbf{g}}_2(\eta)$ in Result 5 — depends on η, the Cobb-Douglas intangible share, but, as indicated by Result 5, its slope is small.

A.1.2 Gross output model

This appendix provides more details on the results relating to gross-output production functions. The main difference is that gross output is not mismeasured, but its components are, which will contribute to mismeasured productivity growth as in the value-added approach.

A.1.2.1 General results

Assume that gross output is given by $X_t = Z_{X,t}G(M_t, L_t, K_t)$, where G is homogeneous of degree 1, M_t are intermediate inputs, and $Z_{X,t}$ is "gross output" total factor productivity.[31] Define the "gross output" Solow residual as:

$$\frac{d\widehat{Z}_{X,t}}{\widehat{Z}_{X,t}} = \frac{d\hat{X}_t}{\hat{X}_t} - \widehat{s}_{X,M,t}\frac{d\widehat{M}_t}{\widehat{M}_t} - \widehat{s}_{X,L,t}\frac{d\widehat{L}_t}{\widehat{L}_t} - (1 - \widehat{s}_{X,M,t} - \widehat{s}_{X,L,t})\frac{d\widehat{K}_t}{\widehat{K}_t}. \tag{22}$$

Here, $\widehat{s}_{X,M,t}$ and $\widehat{s}_{X,L,t}$ are the shares of intermediate input and labor in *gross output*:

$$\widehat{s}_{X,M,t} \equiv \frac{\widehat{P}_{M,t}\widehat{M}_t}{P_t X_t}, \quad \widehat{s}_{X,L,t} \equiv \frac{W_t L_t}{P_t X_t}, \tag{23}$$

and $\widehat{P}_{M,t}\widehat{M}_t$ is nominal expenditure on intermediate inputs.

Analogous to the value added case, we are interested in whether the gross-output Solow residual is a biased measure of gross-output TFP growth when there are markups and omitted intangibles. Markups are defined as the wedge between the price of consumption goods, P_t, and the marginal cost of gross output, $\mu_{X,t} \equiv \frac{P_t}{MC_{X,t}}$.[32] With omitted intangibles, gross output is always correctly measured. Omitting intangibles only affects its distribution between purchases of intermediates and purchases of investment goods.

As before, denoting by B_t misclassified purchases of intangibles, we have: $\widehat{P}_{M,t}\widehat{M}_t = P_{M,t}M_t - B_t$, where $P_{M,t}M_t$ are actual purchases of intermediate inputs. Similar to the ratio b_t in the value added case, the ratio $c_t = \dfrac{P_{M,t}M_t}{\widehat{P}_{M,t}\widehat{M}_t} \leq 1$ captures the amount of mismeasurement due to omitted intangibles; in particular, when $c_t = 1$, there is no mismeasurement.

[31] The terminology "multi-factor productivity" is sometimes used to refer to $Z_{X,t}$, but we use "gross output" TFP in order to distinguish it from the notion of productivity in our value-added approach.

[32] Note that $\mu_{X,t}$ is a gross-output, or a sales, markup. Under cost minimization and constant returns, we have $\mu_t = (\mu_{X,t} - \epsilon_{X,M,t})/(1 - \epsilon_{X,M,t}) \geq \mu_{X,t}$, where $\epsilon_{X,M,t}$ is the elasticity of gross output with respect to intermediate inputs. This can also be written as $\mu_t = \mu_{X,t}(1 - c_t\hat{s}_{X,M,t})/(1 - c_t\mu_{X,t}\hat{s}_{X,M,t})$. Additionally, as in the case of the value-added approach, we assume that X_t is expressed in units of consumption goods.

Result 6. *Assume that labor and intermediate inputs are chosen to minimize total variable cost $P_{M,t}M_t + W_tL_t$. Then, the bias in the gross-output Solow residual, relative to gross-output TFP growth, can be written as:*

$$\frac{d\hat{Z}_{X,t}}{\hat{Z}_{X,t}} - \frac{dZ_{X,t}}{Z_{X,t}} \equiv \Delta_{X,t} = \Delta^{(1)}_{X,t} + \Delta^{(2,L)}_{X,t} + \Delta^{(2,M)}_{X,t} + \Delta^{(3)}_{X,t}. \quad (24)$$

The components of the bias in the gross-output Solow residual, relative to gross-output TFP growth, are given by:

$$\begin{aligned}
\Delta^{(1)}_{X,t} &= \hat{s}_{X,M,t}\left(\frac{dM_t}{M_t} - \frac{d\hat{M}_t}{\hat{M}_t}\right) && \textit{(intermediate growth bias)} \\
\Delta^{(2,L)}_{X,t} &= -\hat{s}_{X,L,t}\left(\mu_{X,t} - 1\right)\left(\frac{d\hat{K}_t}{\hat{K}_t} - \frac{dL_t}{L_t}\right) && \textit{(labor share bias)} \\
\Delta^{(2,M)}_{X,t} &= -\hat{s}_{X,M,t}\left(\mu_{X,t}c_t - 1\right)\left(\frac{d\hat{K}_t}{\hat{K}_t} - \frac{dM_t}{M_t}\right) && \textit{(intermediate share bias)} \\
\Delta^{(3)}_{X,t} &= \left(1 - \left(\epsilon_{X,L,t} + \epsilon_{X,M,t}\right)\right)\left(\frac{dK_t}{K_t} - \frac{d\hat{K}_t}{\hat{K}_t}\right) && \textit{(capital growth bias)}
\end{aligned} \quad (25)$$

and $\epsilon_{X,L,t}$ and $\epsilon_{X,M,t}$ are the elasticities of gross output with respect to labor and intermediate input, respectively.

The similarities and differences with respect to the value added case are the following.

First, there is no mis-measurement in gross output growth (whereas, in the value-added approach, output growth is potentially mismeasured). The term $\Delta^{(1)}_{X,t}$ instead reflects mis-measurement in the growth rate of intermediate inputs.

Second, the labor share of gross output $\hat{s}_{X,L,t}$ is not affected by the omission of intangibles, because gross output and the wage bill are correctly measured (whereas, in the value-added approach, the omission of intangibles can affect the measurement of the labor share). Thus, the labor share bias $\Delta^{(2,L)}_{X,t}$ only reflects markups.

Third, the intermediate share of gross output $\hat{s}_{X,M,t}$ is affected by the omission of intangibles. This creates an "intermediate share" bias, $\Delta_{X,t}^{(2,M)}$, the expression of which is closely analogous to the "labor share bias" in Result 3.

Finally, the mismeasurement of capital growth rates also creates a bias, $\Delta_{X,t}^{(3)}$, with the same intuition as in the value added case.

A.1.2.2 Model

Next, we describe a version of our model in which firms use a gross output production function. We then derive results on measurement bias in this model along the balanced growth path.

Firm The representative firm solves:

$$TC_{X,t} = \min_{\mathbf{K}_t, L_t} \quad \sum_{n=1}^{2} R_{n,t}K_{n,t} + W_t L_t + P_{M,t}M_t \quad \text{s.t.} \quad Z_{X,t}\left(\left(K_{1,t}^{1-\eta}K_{2,t}^{\eta}\right)^{\alpha} L_t^{1-\alpha}\right)^{1-\beta} M_t^{\beta} \geq X_t$$

where $TC_{X,t}$ denotes total costs of production, $\mathbf{K}_t = \{K_{n,t}\}_{n=1}^{2}$ is a vector of capital inputs, with $K_{1,t}$ the measured capital input, and $K_{2,t}$ the omitted intangible input, L_t is labor input, $\{R_{n,t}\}_{n=1}^{2}$ is a vector of user costs, W_t is the wage rate, M_t are intermediate inputs, $Z_{X,t}$ is total factor productivity (over all factors), β is the elasticity of output with respect to intermediate inputs, $(1-\alpha)(1-\beta)$ is the elasticity of output with respect to labor, η is the elasticity of the total capital input $K_t = K_{1,t}^{1-\eta}K_{2,t}^{\eta}$ with respect to intangibles. Total factor productivity (over all factors, including gross output) evolves exogenously, following:

$$dZ_{X,t} = g_{Z_X} Z_{X,t} dt.$$

The price of intermediate goods also evolves exogenously, following:

$$dP_{M,t} \quad = \quad g_M P_{M,t} dt.$$

Define $MC_{X,t}$, the marginal cost of capital, labor, and intermediates , to be the Lagrange multiplier on the constraint. The solution to this problem is:

$$TC_t = MC_{X,t} Y_t \tag{26}$$

$$MC_{X,t} = \frac{1}{Z_{X,t}} \left(\frac{P_{M,t}}{\beta}\right)^{\beta} \left(\frac{W_t}{(1-\beta)(1-\alpha)}\right)^{(1-\beta)(1-\alpha)} \left(\frac{R_t}{(1-\beta)\alpha}\right)^{(1-\beta)\alpha}$$

$$M_t = MC_{X,t} \frac{\beta X_t}{P_{M,t}}$$

$$L_t = MC_{X,t} \frac{(1-\beta)(1-\alpha) X_t}{W_t}$$

$$K_t = MC_{X,t} \frac{(1-\beta)\alpha X_t}{R_t}$$

$$K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$$

$$R_t = \left(\frac{R_{1,t}}{1-\eta}\right)^{1-\eta} \left(\frac{R_{2,t}}{\eta}\right)^{\eta}$$

$$K_{1,t} = (1-\eta) \frac{R_t}{R_{1,t}} K_t$$

$$K_{2,t} = \eta \frac{R_t}{R_{2,t}} K_t \tag{27}$$

The firm's revenue is $S_t = P_t X_t$ and its profits are $\Pi_t = P_t X_t - TC_{X,t} = (P_t - MC_{X,t}) X_t$, where P_t is the price of consumption goods.

Household The representative household solves the same problem as in the model with a value added production function, so we do not re-state it here.

Equilibrium An equilibrium is a set of deterministic sequences for all endogenous variables such that (1) given the exogenous processes for labor, productivity, the price of intermediate goods, and the prices of capital goods, the endogenous variables satisfy the solution to the firm's problem and solve the representative consumer's problem; and (2) the price of consumption goods and their marginal cost of production are related through:

$$P_t = \mu_X MC_{X,t},$$

where $\mu_X > 1$ is the exogenous price-cost markup of price over the marginal cost of labor, capital, and intermediate inputs — the gross output markup, for short. Finally, in equilibrium, we normalize the price level to $P_t = 1$, so that all other prices are expressed relative to consumption goods.

A.1.2.3 Equivalence with the value added model

Aggregate accounting Intermediate output was introduced above assuming a "roundabout" production function, where the representative firm both produces consumption goods, and uses consumption goods as intermediate input (converting them to intermediate output at rate $P_t/P_{M,t}$) within the same period, while still behaving as though it were purchasing consumption goods from a perfectly competitive market.

Using the normalization $P_t = 1$, gross output is given by:

$$\begin{aligned} X_t &= Z_{X,t}\left(\left(K_{1,t}^{1-\eta}K_{2,t}^{\eta}\right)^{\alpha} L_t^{1-\alpha}\right)^{1-\beta} M_t^{\beta} \\ &= W_t L_t + R_{1,t}K_{1,t} + R_{2,t}K_{2,t} + P_{M,t}M_t + \Pi_t \\ &= C_t + Q_{1,t}I_{1,t} + Q_{2,t}I_{2,t} + P_{M,t}M_t \end{aligned}$$

The first relationship uses the definition of the production function (the output approach), the second uses the definition firm profits (the income approach), and the third relationship uses the budget constraint of the household (the expenditure approach). Value added is defined as:

$$\begin{aligned} Y_t &\equiv X_t - P_{M,t}M_t \\ &= W_tL_t + R_{1,t}K_{1,t} + R_{2,t}K_{2,t} \\ &= C_t + Q_{1,t}I_{1,t} + Q_{2,t}I_{2,t}. \end{aligned}$$

The second line is the income approach definition of GDP, and the third line is the expenditure approach to GDP.

Value added representation The following result describes the equivalence between the value added and gross output models.

Result 7. *Define:*

$$\begin{aligned} Z_t &= \frac{\mu_X - \beta}{1-\beta}\left(\frac{Z_{X,t}}{\mu_X}\left(\frac{P_{M,t}}{\beta}\right)^{-\beta}\right)^{\frac{1}{1-\beta}} \\ \mu &= \frac{\mu_X - \beta}{1-\beta} \end{aligned} \tag{28}$$

Then, all quantities and prices in the gross output model are the same as in a value added model where total factor productivity Z_t and markups μ are given by Equation (28).[33]

This equivalence result says that one can think of the value-added model as being derived from an underlying gross output model. The expressions in (28) then highlights two points. First, the link between the (value-added) markup μ in the value added model and the (gross output) markup μ_X in the gross output model depends on the intermediate share β. Second, value-added TFP growth g_Z in the value-added model is related to gross-output TFP growth g_{Z_X} through: $g_Z = \frac{1}{1-\beta}\left(g_{\tilde{Z}} - \beta g_{P_M}\right)$. Value-added TFP in the value-added model should therefore be thought of as reflecting a combination of technical change and change in the price of intermediate products. This equivalence result implies that all the result results regarding how the simple value-added Solow residual $d\hat{Z}/\hat{Z}_t$ potentially mis-measures value-added TFP, g_Z, follow through in the gross output model.

[33]This is with the exception of intermediate inputs M_t, gross output marginal cost $MC_{X,t}$, and gross output X_t, which are undefined in the value-added model.

Proof. [Result 7] Since the household's problem is the same in both models, we only need to show (1) that the first-order conditions of the firm's problem are the same as in the value-added model, under the definitions of value-added TFP Z_t and markups μ given above; and (2) that Y_t and MC_t defined as:

$$Y_t = Z_t L_t^{1-\alpha} \left(K_{1,t}^{1-\eta} K_{2,t}^{\eta} \right)^{\alpha}, \tag{29}$$

$$MC_t = \frac{1}{Z_t} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{R_t}{\alpha} \right)^{\alpha}. \tag{30}$$

indeed measure value added and its marginal cost in the gross output model.

Combining the first-order conditions for capital and labor from the firm's problem in the gross output model, (26), we obtain (for any Z_t):

$$Z_t K_t^{\alpha} L_t^{1-\alpha} = \frac{MC_{X,t}}{MC_t}(1-\beta)X_t, \tag{31}$$

where we defined MC_t as in Equation (30). Plugging this back into the first-order conditions for capital and labor, this implies that they are the same as in the value added model:

$$W_t = \frac{(1-\alpha)MC_t Y_t}{L_t}$$

$$R_t = \frac{\alpha MC_t Y_t}{L_t}$$

where Y_t is defined as in equation (29). Note, additionally, that equation (31) implies:

$$MC_t Y_t = (1-\beta) MC_{X,t} X_t. \tag{32}$$

In the equilibrium of the gross output model, $MC_{X,t} = \mu_X^{-1}$. Therefore:

$$\left(\frac{L_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{K_t}{\alpha} \right)^{\alpha} = \left(\frac{Z_{X,t}}{\mu_X} \left(\frac{P_{M,t}}{\beta} \right)^{-\beta} \right)^{\frac{1}{1-\beta}}.$$

Therefore, using the definitions of Z_t and μ

$$MC_t = \frac{1}{Z_t}\left(\frac{L_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{K_t}{\alpha}\right)^{\alpha}$$
$$= \frac{1-\beta}{\mu_X-\beta} = \frac{1}{\mu}.$$

This proves that $1 = P_t = \mu MC_t$, as in the value-added model. Additionally, it implies that:

$$\frac{MC_{X,t}}{MC_t} = \frac{\mu_X-\beta}{1-\beta}\frac{1}{\mu_X}.$$

Therefore:

$$X_t - P_{M,t}M_t = \left(1-\frac{\beta}{\mu_X}\right)X_t$$
$$= \frac{\mu_X-\beta}{\mu_X}\frac{MC_t}{MC_{X,t}}(1-\beta)Y_t$$
$$= \frac{\mu_X-\beta}{\mu_X}\frac{\mu_X(1-\beta)}{\mu_X-\beta}(1-\beta)Y_t$$
$$= Y_t,$$

where to go from the first to the second line, we used equation (32). So Y_t indeed measures value added. Moreover:

$$MC_tY_t = MC_{X,t}X_t - \beta MC_{X,t}X_t = TC_{X,t} - P_{M,t}M_t = TC_t$$

where TC_t is the total cost of production of output minus intermediate costs. So MC_t measures the marginal cost of value added. □

A.1.2.4 Balanced growth

For completeness, we next report the balanced growth expressions for the solution of the gross output model. The steps are the same as for the value added model, so we do not detail them. Define the aggregate price index for capital goods Q_t as:

$$Q_t = Q_{1,t}^{1-\eta} Q_{2,t}^{\eta}.$$

Next, define the trend growth factor $T_{X,t}$ as:

$$T_{X,t} = L_t Z_{X,t}^{\frac{1}{(1-\beta)(1-\alpha)}} Q_t^{-\frac{\alpha}{1-\alpha}} P_{M,t}^{-\frac{\beta}{(1-\beta)(1-\alpha)}},$$

and define the detrended variables:

$$c_t \equiv \frac{C_t}{T_{X,t}} \tag{33}$$

$$w_t \equiv \frac{W_t L_t}{T_{X,t}} \tag{34}$$

$$m_t \equiv \frac{P_{M,t} M_t}{T_{X,t}} \tag{35}$$

$$\pi_t \equiv \frac{\Pi_t}{T_{X,t}} \tag{36}$$

$$i_{n,t} \equiv \frac{Q_{n,t} I_{n,t}}{T_{X,t}}, \quad n = 1, 2 \tag{37}$$

$$k_{n,t} \equiv \frac{Q_{n,t} K_{n,t}}{T_{X,t}}, \quad n = 1, 2 \tag{38}$$

$$R_{d,n,t} \equiv \frac{R_{n,t}}{Q_{n,t}}, \quad n = 1, 2 \tag{39}$$

$$R_{d,t} \equiv \frac{R_t}{Q_t} \tag{40}$$

$$x_t \equiv \frac{X_t}{T_{X,t}} \tag{41}$$

$$k_t \equiv \frac{Q_t K_t}{T_{X,t}} \tag{42}$$

Moreover, define the trend growth rate (the growth rate of $T_{X,t}$), the capital price growth rate (the growth rate of Q_t), and the discount rate r as:

$$\begin{aligned}
g &\equiv g_L + \frac{1}{1-\alpha}\left(g_{Z_X} - \frac{\beta}{1-\beta} g_{P_M}\right) - \frac{\alpha}{1-\alpha} g_Q, \\
g_Q &\equiv (1-\eta) g_{Q_1} + \eta g_{Q_2}, \\
r &\equiv \rho + \sigma g.
\end{aligned}$$

Note that $g_Z = g_{Z_X} - \frac{\beta}{1-\beta} g_{P_M}$, where Z_t is defined in Equation (28). Detrended user costs must satisfy:

$$R_{d,n,t} = R_{d,n} = r + \delta_n - g_{Q_n}, \quad n = 1, 2,$$

the standard Hall-Jorgenson formula. The balanced growth path is then given by:

$$\begin{aligned}
MC &= \frac{1}{\mu_X} \\
R_d &= \left(\frac{R_{d,1}}{1-\eta}\right)^{1-\eta} \left(\frac{R_{d,2}}{\eta}\right)^{\eta} \\
x &= \left(\frac{\beta}{\mu_X}\right)^{\frac{\beta}{(1-\beta)(1-\alpha)}} \left(\frac{(1-\beta)\alpha}{\mu_X R_d}\right)^{\frac{\alpha}{1-\alpha}} \\
k &= \frac{(1-\beta)\alpha}{\mu_X R_d} x \\
m &= \frac{\beta}{\mu_X} x \\
w &= \frac{(1-\alpha)(1-\beta)}{\mu_X} x \\
\pi &= \frac{\mu_X - 1}{\mu_X} x
\end{aligned}$$

$$k_1 = (1-\eta)\frac{R_d}{R_{d,1}}k$$

$$k_2 = \eta\frac{R_d}{R_{d,2}}k$$

$$i_n = (g + \delta_n - g_{Q_n})k_n = (R_{d,n} - (r-g))k_n, \quad n = 1, 2.$$

A.1.2.5 TFP measurement on the balanced growth path

Assumptions We make the same assumptions about (mis)measurement in aggregate accounts as we do in the analysis of Section 2.. First, gross output X_t is correctly measured. Second, measured value added excludes $Q_{2,t}I_{2,t}$. We have:

$$\begin{aligned} \widehat{Y}_t &= Y_t - Q_{2,t}I_{2,t} \\ &= X_t - \widehat{P_{M,t}M_t} \\ \widehat{P_{M,t}M_t} &= P_{M,t}M_t + Q_{2,t}I_{2,t} \end{aligned}$$

Here, $\widehat{P_{M,t}M_t}$ is measured nominal purchases of intermediates (which are too large, because $Q_{2,t}I_{2,t}$ is misclassified). In what follows, we use the following two ratios (the first of which is also the one we use in the analysis of the value-added model):

$$\begin{aligned} b_t &= \frac{\widehat{Y}_t}{Y_t} \\ c_t &= \frac{P_{M,t}M_t}{P_{M,t}M_t + Q_{2,t}I_{2,t}} \end{aligned}$$

The case of no omitted intangibles corresponds to $\eta = 0$. Using the expressions from Section A.1.2.4, we obtain that along the balanced growth path:

$$\begin{aligned} b_t = b &= 1 - \frac{1-\beta}{\mu_X - \beta}\frac{g + \delta_2 - g_{Q_2}}{\rho + g + \delta_2 - g_{Q_2}}\alpha\eta \\ c_t = c &= 1 - \frac{1-\beta}{\beta}\frac{g + \delta_2 - g_{Q_2}}{\rho + g + \delta_2 - g_{Q_2}}\alpha\eta \end{aligned}$$

The expression for b_t is the same as Equation (14), for the value-added model, when $\mu = \frac{\mu_X - \beta}{1-\beta}$. These expressions indicate that there are no omitted intangibles ($\eta = 0$), if and only if, $b = 1$ and $c = 0$.

Mis-measurement of value-added TFP growth (g_Z) Recall that in the gross output model, value-added TFP (in levels) is defined as:

$$Z_t = \frac{\mu_X - \beta}{1 - \beta} \left(\frac{Z_{X,t}}{\mu_X} \left(\frac{P_{M,t}}{\beta} \right)^{-\beta} \right)^{\frac{1}{1-\beta}},$$

so that, in growth rates.

$$g_Z = \frac{1}{1-\beta} \left(g_{Z_X} - \beta g_{P_M} \right).$$

Given the equivalence between the gross output and value added approaches developed in Result (7), all the results of Section 2. on the mis-measurement of value-added TFP growth go through. Define the (value-added) Solow residual as:

$$\frac{d\hat{Z}_t}{\hat{Z}_t} = \frac{d\hat{Y}_t}{\hat{Y}_t} - \hat{s}_{L,t} \frac{d\hat{L}_t}{\hat{L}_t} - (1 - \hat{s}_{L,t}) \frac{d\hat{K}_t}{\hat{K}_t},$$

where $\hat{s}_{L,t}$ is the labor share of value added, which, on the balanced growth path, is given by:

$$\hat{s}_{L,t} = \hat{s}_L = \frac{W_t L_t}{\hat{Y}_t} = \frac{(1-\beta)(1/\mu_X)}{b(1-\beta/\mu_X)}(1-\alpha) = \frac{1-\alpha}{b\mu}.$$

Then $d\hat{Z}_t/\hat{Z}_t$ is a biased measure of g_Z, and the bias can be decomposed into a capital growth bias (which is zero whenever there are no omitted intangibles), and a labor share bias (which is driven by markups, but can amplify the omitted capital bias), and their expressions are given as in (4).

Mis-measurement of gross output TFP growth (g_{Z_X}) This model also has predictions for the bias between the *gross-output* Solow residual $d\hat{Z}_{X,t}/\hat{Z}_{X,t}$, and gross-output TFP growth g_{Z_X}, in the presence of markups and omitted intangibles, analogous to Result 6. These predictions are summarized in the follow result.

Result 8. *Assume that the growth rate of intermediate goods prices is correctly measured. Then, along the balanced growth path:*

$$\Delta_{X,t} \;=\; \Delta_X \;=\; \Delta_X^{(1)} + \Delta_X^{(2,M)} + \Delta_X^{(2,L)} + \Delta_X^{(3)} \tag{43}$$

where $\Delta_X^{(1)} = 0$*, and:*

$$\Delta_X^{(2,L)} \;=\; -(1-\beta)\frac{\mu_X - 1}{\mu_X}\left(g_Z - g_{Q_1} - \alpha\eta(g_{Q_2} - g_{Q_1})\right) \qquad \textit{(labor share bias)}$$

$$\Delta_X^{(2,M)} \;=\; -\beta\frac{\mu_X c - 1}{\mu_X c}\left(g_{P_M} - g_{Q_1}\right) \qquad \textit{(intermediate share bias)}$$

$$\Delta_X^{(3)} \;=\; -(1-\beta)\alpha\eta\left(g_{Q_2} - g_{Q_1}\right) \qquad \textit{(capital growth bias),}$$

where along the balanced growth path, $c = 1 - \dfrac{1-\beta}{\beta}\dfrac{g + \delta_2 - g_{Q_2}}{r + \delta_2 - g_{Q_2}}\alpha\eta.$

Result (8) reports expressions for the components of the bias between the gross output Solow residual and gross output TFP growth, derived from applying to balanced growth solution to Result (6). In this result, we have assumed that the real growth rate of actual intermediate inputs, which is equal to $g - g_{P_M}$ in the balanced growth path, is the same as the real growth rate of measured intermediate inputs. The latter growth rates depends on the measured growth rate for intermediate inputs, $g_{\hat{P}_M}$. If this growth rate is correctly measured, the contribution of mismeasurement of intermediate input growth along the balanced growth path (the term $\Delta_{X,1,t}$) is zero; otherwise, the contribution of this term is equal to $-\frac{\beta}{\mu_X}(g_{P_M} - g_{\hat{P}_M})$.

An important difference with the value-added case is that, so long as $d\hat{K}_t/\hat{K}_t > dL_t/L_t$ (the empirically relevant case), the labor share bias will be (weakly) negative. Thus, a sufficient condition for the overall bias to be negative is $g_{Q_2} > g_{P_M} > g_{Q_1}$. As discussed in Section 3., this condition is empirically plausible, as the types of intangible investments most likely to be misclassified as omitted intangibles are also among the intermediate goods with highest relative price growth.

In the empirical applications, we focus on quantifying mis-measurement of value-added TFP growth g_Z by the value-added Solow residual $d\hat{Z}_t/\hat{Z}_t$, and not on mismeasurement of g_{Z_X} using the gross-output Solow residual $d\hat{Z}_{X,t}/\hat{Z}_{X,t}$. We make this choice because we are interested in understanding trends in value-added TFP growth which can be compared with the relevant literature, but, in principle, the analysis could be extended to gross-output TFP growth.

A.2 Empirics

This section of the appendix provides more details on the empirical analysis.

A.2.1 The decline in measured TFP growth

In order to document the decline in measured TFP growth, we use the time series constructed by Fernald (2014). This data covers the period 1947q1-2020q1, and provides measures of the growth rate of real output, labor input, capital input, and the labor share, for the business sector. This comprises all corporate and non-corporate for-profit businesses, as well as other business entities, such as non-profits and certain government agencies; see Bureau of Economic Analysis (2017).

We make one main modification to the data of Fernald (2014): in Solow residual computations, we use the growth rate of GDP in *consumption units*. In computing the Solow residual, Fernald (2014) use the quarterly growth rate of real value added by businesses in chained dollars (NIPA table 1.3.6; FRED series A195RX1Q020SBEA). Instead, we use the quarterly growth rate in the ratio of nominal value added by businesses (NIPA Table 1.3.5;

FRED series A195RC1Q027SBEA) to the implicit price deflator for personal consumption expenditures (NIPA Table 1.1.9; FRED series DPCERD3Q086SBEA).

We choose to do this because, in our balanced growth model, the notion of output we consider, Y_t, is directly defined in consumption units, and is not necessarily equal to chained GDP growth. We explain this point, which is explained more generally in Oulton 2007, in Appendix A.1.1.3. We compare below the results of the simple growth accounting decomposition when chained GDP growth is used instead of the growth of output in consumption units.

Other than this difference, three points about these data are worth noting. First, the data on capital input growth are constructed from estimated stocks for nine types of capital, including specific estimated stocks for R&D capital and software. These stocks are themselves derived from NIPA series on investment capitalized using perpetual inventory methods. The nine types of capital are: land; business inventory; business residential real estate; information processing equipment; other equipment; structures; software; R&D; artistic originals. Investment in different capital goods is deflated using capital-specific price indices, so that the resulting growth rates in stocks are real. Aggregate capital growth is obtained by weighting these series by their estimated user cost shares. Second, the labor share is measured as the ratio of total labor payments to total value added; the capital share is obtained as the residual (one minus the labor share), as opposed to being directly imputed from estimates of the user cost of capital. Proprietor's income, in particular, is allocated so as to ensure that the aggregate labor share is equal to the labor share of non-financial corporations. Third, the data also contain an adjustment for variable capacity utilization; we compare trends with and without this adjustment below.

Figure 1 reports the time-series for TFP growth without adjustments for capacity utilization, defined as the simple Solow residual:

$$\frac{d\hat{Z}_t}{\hat{Z}_t} = \frac{d\hat{Y}_t}{\hat{Y}_t} - \hat{s}_{L,t}\frac{d\hat{L}_t}{\hat{L}_t} - (1 - \hat{s}_{L,t})\frac{d\hat{K}_t}{\hat{K}_t} \tag{44}$$

where $d\hat{Y}_t/\hat{Y}_t$ denotes the growth rate of output in consumption units,

$$\hat{s}_{L,t} = \frac{W_t L_t}{N_t} \tag{45}$$

denotes the labor share in nominal business value added, N_t, $d\hat{L}_t/\hat{L}_t$ denotes the growth rate of labor, and $d\hat{K}_t/\hat{K}_t$ denotes the measured growth rate of capital. The series show that, after a period of rapid increase in the early to mid-1990's, TFP growth reach a plateau, and then declined. This decline lasted until late 2007, but was not followed by a persistent rebound; instead, productivity growth has remained subdued since 2010.

Table 1 reports simple averages on the decline in TFP growth, comparing the 1947-1996 period, to the 1997-2018 period. Before 1997, TFP growth in the US had been, on average, 1.11% per year; after 1995, it fell to 0.62% per year, a 0.49% decline. By contrast, between the two periods, growth of output in consumption units fell by 0.92%; 0.43% of that decline is therefore attributable to a decline in input growth, and the rest to the TFP growth decline. Additionally, the labor share of income fell by 4 p.p. over the period. Finally, the last line in the table highlights the fact that the utilization adjustment constructed by Fernald (2014) using the methodology of Basu et al. (2013) only leads to a very small difference in the decline of measured TFP growth.

Table A2 compares output growth and the Solow residuals obtained using output in consumption units (our baseline approach), to the values obtained using chained GDP (the data provided in Fernald (2014)). The table shows that the growth rate of GDP in consumption units is lower than the growth rate of chained GDP by approximately 0.25% in both the 1947-1996 and 1997-2018 periods. As a result the Solow residual obtained using chained GDP is higher than in our baseline approach (by 0.25%) in both periods. However, the *change* in both GDP growth and the Solow residual is the almost identical under the two approaches. This indicates that the bias created by the fact that Y_t, in the model, does not correspond to chained GDP in the data is stable across periods and does not affect our measurement of the *decline* in the the Solow residual.

A.2.2 Methodology using only expenditure data

The value of the growth rate g_{Q_2} such that *all* of the gap between true TFP growth and the Solow residual $\hat{g}_Z$ is due to mismeasurement is given by:

$$
\begin{aligned}
g_{Q_2} &= \frac{1}{2}\left(\hat{r}+\delta_2+\hat{g}-\hat{g}_K+\hat{\xi}-\sqrt{\left(\hat{\xi}+(\hat{r}-\hat{g}-(\hat{g}_K+\delta_2))\right)^2+4(\hat{r}-\hat{g})(\hat{g}_K+\delta_2)}\right), \\
\hat{\xi} &= \frac{\hat{s}_L\hat{b}}{(1-\hat{b})(1-\alpha)}\left[\tilde{g}_Z-(\hat{g}-(1-\alpha)\hat{g}_L-\alpha\hat{g}_K)\right],
\end{aligned}
$$

where $\hat{r}=\rho+\sigma\hat{g}$. This result is derived as follows.

Replacing g_{Q_2} with x, and omitting the hat notation for measured variables, the conditions from the balanced growth model are:

$$
\begin{aligned}
g_Z &= g-(1-\alpha)g_L-\alpha g_K+\alpha\eta\left(x-(g-g_K)\right) \\
\mu &= \frac{1-\alpha}{sb} \\
\eta &= \mu\frac{1-b}{\alpha}\frac{r+\delta_2-x}{g+\delta_2-x}
\end{aligned}
$$

Substituting the expression for the markup,

$$
\begin{aligned}
g_Z &= g-(1-\alpha)g_L-\alpha g_K+\alpha\eta\left(x-(g-g_K)\right) \\
\eta &= \frac{(1-b)(1-\alpha)}{\alpha sb}\frac{r+\delta_2-x}{g+\delta_2-x}
\end{aligned}
$$

Substituting the expression for η into the expression for the production function,

$$
g_Z = g-(1-\alpha)g_L-\alpha g_K+\frac{(1-b)(1-\alpha)}{sb}\frac{r+\delta_2-x}{g+\delta_2-x}\left(x-(g-g_K)\right)
$$

Let:

$$\xi \equiv \frac{sb}{(1-b)(1-\alpha)}\left[g_Z - (g - (1-\alpha)g_L - \alpha g_K)\right],$$

then we can write this as:

$$((r+\delta_2) - x)\,(x - (g - g_K)) - \xi(g + \delta_2 - x) \;=\; 0$$

Let:

$$a \;\equiv\; r + \delta_2$$

$$b \;\equiv\; g - g_K$$

$$c \;\equiv\; g + \delta_2 < a$$

The solution must satisfy:

$$b < x < c < a.$$

Indeed, the condition $b < x$ ensures that the implied growth rate of prices of omitted intangible capital is higher than than the growth rate of prices of measured capital. The condition $x < c$ ensures that the detrended user cost of omitted intangible capital is strictly positive. The equation for x can be rewritten as:

$$(a - x)\,(x - b) - \xi(c - x) = 0,$$

or:

$$\frac{(a-x)(x-b)}{c-x} = \xi.$$

Using the fact that $b < c < a$, it can be shown that the left-hand side in this equation is a strictly increasing mapping from $]b, c[$ to $]0, +\infty[$, so there is always a unique solution to this equation in $]b, c[$. The unique solution in this interval is given by:

$$x \;=\; \frac{1}{2}\left(a + b + \xi - \sqrt{(\xi + (a + b - 2c))^2 + 4(a-c)(c-b)}\right).$$

In terms of the original variables, the solution can be written as:

$$x = \frac{1}{2}\left(r+\delta_2+g-g_K+\xi-\sqrt{(\xi+(r-g-(g_K+\delta_2)))^2+4(r-g)(g_K+\delta_2)}\right),$$

$$\xi = \frac{sb}{(1-b)(1-\alpha)}\left[g_Z-(g-(1-\alpha)g_L-\alpha g_K)\right].$$

A.2.3 Other data sources

BLS price indices In Section 3.4, as an alternative empirical proxy for g_{Q_2}, we use the BLS' Producer Price Indices for commodities.[34] There are a number of challenges in mapping these data to the Input-Output tables. The main one is that the level of aggregation differs from that of the IO tables. Information on the producer prices for commodities are substantially more granular than in the Input-Output tables; but it tends to be less granular for service prices. We focus on BLS price indices reported at the 3- and 4-NAICS levels, and match them, based on names, to the IO table classification. This matching is available from the authors on request. Not all IO commodity and service groups are matched (for instance, Data processing, in the IO tables, does not have a clear match to the BLS commodity groups), and for the IO groups with several more granular matches in the BLS PPI tables, we take the simple average of prices across matches.

Table A5 reports results from a simple regression using the matched BEA-BLS sample. In all specifications, the dependent variable is $g_{Q_2}^{(BEA)}$, the empirical proxy for g_{Q_2} constructed using the BEA GDP-by-industry data and described in Section 3.2, and the independent variable is the equivalent empirical proxy constructed using the BLS price deflators. The results of the table indicate that there is a robust correlation between the two variables, even within industry and year, though there remains independent variation between the two sets of price indices, with R^2s in the order of 65% across specifications.

[34] The PPI commodity tables are available at https://download.bls.gov/pub/time.series/wp/.

Non-financial public firms We obtain data on spending on organization capital from the sample of Compustat non-financial firms, for the 1997-2018 period. We use standard selection criteria in order to obtain the sample of domestically incorporated, publicly traded-firms not in the utility or financial sector.[35] The sample we obtain covers approximately 70% of aggregate investment and gross operating surplus in the corporate non-financial sector, as documented in Crouzet and Eberly (ming).

Our objective is to use this sample to construct an alternative measure of adjusted to unadjusted GDP, after reclassifying expenditures on organization capital, M as investment:

$$\hat{b}_{CS} = \frac{\hat{Y}_{CS}}{\hat{Y}_{CS} + \hat{M}_{CS}},$$

where $\hat{Y}_{CS}$ is total value added in the Compustat sample, and $\hat{M}_{CS}$ are expenditures on organization capital. As discussed in the main text, intermediate expenditures on the three key service groups closely relate to the notion of organization capital developed in the macro and finance literature on intangible capital (Atkeson and Kehoe, 2005; Eisfeldt and Papanikolaou, 2013). As an empirical proxy for $\hat{M}_{CS}$, we use the measure developed by Eisfeldt and Papanikolaou (2013), who propose to measure organization capital spending as $0.3 \times (\texttt{xsga} - \texttt{xrd})$, where `xsga` denotes spending on sales and general and administrative expenses, and `xrd` denotes $R\&D$ spending.

Measuring value added, $\hat{Y}_{CS}$, is more challenging, because Compustat firms do not report separate line items for wage payments. In order to address this issue, we map the Compustat data to the 61 sectors of Make tables of the Input-Output accounts. This match uses the NAICS-3 and NAICS-4 classification of firms in Compustat, and is available from the authors on request. For each sector s, we then impute Compustat wages using:

$$W_s^{(CS)} = \frac{S_s^{(IO)}}{S_s^{(CS)}} W_s^{(IO)},$$

[35] We use the same sample selection criteria as Crouzet and Eberly (ming); see the appendix of that paper for more details.

where $S_s^{(IO)}$ is sector gross output at producer prices from the IO tables, $S_s^{(CS)}$ is total revenue for the sector from Compsutat, and $W_s^{(IO)}$ are total wage payments for the sector from the IO tables. Given imputed wages for the sector, we then compute:

$$\hat{Y}_s^{(CS)} = \Pi_s^{(CS)} + W_s^{(CS)} + RD_s^{(CS)},$$

where $\Pi_s^{(CS)}$ is total EBITDA in the sector, and $RD_s^{(CS)}$ are total $R\&D$ expenditures in the sector. The former is the closest firm accounting counterpart to gross operating surplus, so that adding back wages provides an estimate of value added. The main difference with national accounting definitions of gross operating surplus is that $R\&D$ expenditures as treated as intermediate expenditures (operating costs) in firm accounting data, so that they need to be added back to EBITDA in order to obtain a measure of value added consistent with the national accounts definition. Finally, we define the Compustat proxy for the ratio of unadjusted to adjusted value added as:

$$\hat{b}^{(CS)} = \frac{\sum_s \hat{Y}_s^{(CS)}}{\sum_s \hat{Y}_s^{(CS)} + \hat{M}_s^{(CS)}}.$$

Figure A4 reports the resulting time series for $\hat{b}^{(CS)}$, along with the ratio of nominal investment to value added, with and without adjustment for investment in organization capital.

A.2.4 Robustness

Other measures of relative price growth We use the Producer Price Indices for commodities from the Bureau of Labor Statistics as an alternative empirical proxy for g_{Q_2}. Appendix A.2.3 discusses the differences between BLS and BEA data, and shows that there is independent variation between the two sets of price measures, though they are highly correlated. Appendix Table A6 reports results obtained using this alternative empirical proxy for g_{Q_2}. For two of the three key service groups, g_{Q_2} is lower than in our baseline analysis.[36]

[36]Price information in the BLS data is missing for the third key service group, Management.

As a result, the implied adjustments for TFP growth are lower than in the baseline; the total adjustment is approximately 21bps, instead of 32bps in the baseline. However, the adjustment remains positive, because even the BLS proxies for g_{Q_2} are higher than our estimate of g_{Q_1}, which is negative throughout the 1997-2018 period.

Estimating organization capital spending from firm data We use firm accounting data in order to construct an alternative proxy for $\hat{b}$. Our adjustment builds on the empirical measures of investment in organization capital proposed by Eisfeldt and Papanikolaou (2013). Conceptually, this form of intangible investment corresponds most closely to what might be misclassified as intermediate expenditures on the three key service groups highlighted in our baseline analysis. Appendix A.2.3 explains in detail how the empirical proxy for $\hat{b}$ is constructed in Compustat data, and Appendix Figure A4 reports the resulting time-series.

The most important point to note about this empirical proxy for $\hat{b}$ is that it contains both *externally purchased* investments in organization capital (which is also what our baseline approach estimates from the Use tables), and, potentially, *own-account* intangibles. Own-account intangibles could include, for instance, worker training, in-house investments in logistics, or expenditures on product management and branding, so long as they are not externally contracted or purchased. Because these expenditures would not correspond to service or commodity purchase in the Use tables, our baseline approach would not capture them.[37]

The inclusion of own-account intangibles in this alternative measure of $\hat{b}$ suggests that its resulting values could be lower (i.e. the intangible adjustment larger) than those obtained from the Use tables. On the other hand, the estimates of $\hat{b}$ measure organization capital investment as a constant fraction $\gamma = 30\%$ of sales, general and administrative expenses (SG&A), but there is evidence that this fraction may vary across industries, and could be as high as 50% in industries such as Healthcare and High-tech (Ewens et al., 2019). This

[37]The exception to this is managerial time spent on organization capital, as this may be as use of intermediate inputs produced by the Management of Companies and Enterprises sector in the Use tables.

could lead the values of $\hat{b}$ estimated from Compustat data to be lower than in the Use tables. Appendix Figure A4 (top panel) reports the time series for the ratio of $\hat{b}$ obtained from Compustat data; it is generally close to our most extensive adjustments from the Use tables (using Professional Services, Management, and Administrative Services), suggesting both of the effects described (the higher estimates due to own-account spending on organization capital, and the lower estimates due to the value of γ used) potentially affect estimates of $\hat{b}$.

The magnitude of the adjustment is similar to what we obtained in our baseline analysis when reclassifying expenditures on PSTS and Management services in the Use tables. Table 4 then reports the implied TFP growth rates when using estimates for $\hat{b}$ from Compustat data.[38] Our mechanisms explain 29bps of the 49bps TFP growth decline in that case.

Alternative breakpoints Our baseline analysis uses 1997 as the breakpoint relative to which we analyze the decline of the Solow residual compared to its historical values. We use this breakpoint as our baseline for two main reasons. First, after 1997, the ratio of unadjusted GDP to GDP adjusted for misclassified investment stabilizes, after a long period of decline that starts in the 1980s, as indicated by Figure 3. In other words, the size of potentially misclassified investment, relative to GDP is closer to being constant after 1997, consistent with the assumptions of our balanced growth in Section 2.2. Since our goal is to understand the effects of misclassification of intangibles on TFP growth measurement, it is natural to date our breakpoint using this change in the trend of the ratio of unadjusted to adjusted GDP. Second, papers focusing on the slowdown in productivity growth have noted that this slowdown in productivity growth in the US started some time between the late 1990s and the mid-2000s (Cette et al., 2016; Byrne et al., 2013; Fernald, 2015).[39]

However, as emphasized in other papers, the breaks in the data is not sharp, so we also consider results using alternative breakpoints. Following the literature, we look at

[38] Appendix Figure A3 reports year-by-year results from this exercise for the 1997-2018 period.

[39] Cette et al. (2016) dates the start of the slowdown in TFP growth relative to the US, in a sample of advanced economies, in 1997. Fernald (2015) dates the slowdown in productivity growth in most US industries to 2004.

breakpoints in 2000 and 2004. Additionally, we consider an earlier breakpoint, 1993, as further robustness check. In Appendix Table A3, we report key data moments (the growth rate of inputs, output, and the resulting Solow residuals) for these three breakpoints. Using the later breakpoints, the implied decline in TFP growth is higher, with the drop in measured TFP growth rising to 0.68% for the 2004 breakpoint (compared to 0.49% in our baseline), reflecting the brief acceleration of TFP growth in the late 1990s, also noted in Byrne et al. (2013) and Fernald (2015).

Table A4 then reports results analogous to those of Table 4 (the effect of adjusting for markups and misclassified intangible investment on measured TFP growth) for these alternative breakpoints. The earlier breakpoint (1994) makes no notable difference to the results. However, the results for the later breakpoints are more muted than in our baseline. For the 2000 breakpoint, markups and intangibles together account for half of the decline in TFP growth (or 0.29% out of the 0.58%), while after 2004, they account for one-third of the decline in TFP growth (or 0.22% out of 0.68%). By contrast, in our baseline, they account for two-thirds (or 0.33% out of 0.49%) of the decline in TFP growth. The key reason for this difference is that the growth rate of the relative price of potentially misclassified intangibles – g_{Q_2} – fell somewhat during the 2004-207 period, though it remains larger than the growth rate of the price of measured capital (and positive overall, as indicated in Appendix Table A3). Thus, to the extent that the growth in the relative price of misclassified intangibles slowed down over time, the source of mismeasurement we highlight will also decline.

Values of δ_2 Appendix Figure A5 reports comparative statics for the adjusted Solow residual obtained in Section 3.3.2, when changing the value of the rate of depreciation of omitted intangibles. Our estimates are relatively insensitive to this parameter: compared to our baseline estimate of 0.95% when adjusting for two of the three key service inputs, implied TFP growth (the adjusted Solow residual) declines from 0.96% to 0.89% as δ_2 increases from 0.05 to 0.40. The intuition for the sign of the effect is that with lower depreciation, the stock of omitted intangibles, and therefore its user cost share, is larger, magnifying the effect of the capital growth mismeasurement on TFP growth.[40]

[40] A potential alternative to calibrating the value of δ_2 is to try to estimate it directly. This could in principle be done with data on the income share of omitted intangible capital and on the required rate of return to capital r_t. However, contrary to the measures of omitted intangible investment explored in this section, an empirical proxy for the intangible capital income share is more challenging to construct.

	1947-1996	1997-2018	Change
Growth rate of K/L ratio	2.27	2.34	0.06
Measured labor share	0.68	0.64	-0.04
Bias ($\epsilon_L = 1.00$)	-0.73	-0.84	-0.11
Bias ($\epsilon_L = 0.68$)	0.00	-0.09	-0.09

Table A1: Potential size of the bias in measured TFP growth induced by markups. The first two lines report sample averages of the measured rate of change of the capital to labor ratio and of the labor income share). The last two lines report estimates of the bias in measured TFP growth; the third line is the absolute upper bound, when all measured capital income is pure profits; and the fourth line is the estimate obtained when setting the output elasticity of labor equal to the 1947-1996 sample average of the measured labor income share, $\bar{\hat{s}}_L = 0.68$.

	1947-1996		1997-2018		Change	
	GDP in cons. units	Chained GDP	GDP in cons. units	Chained GDP	GDP in cons. units	Chained GDP
GDP growth (p.p.)	3.36	3.62	2.44	2.68	-0.92	-0.93
TFP growth (p.p.)	1.11	1.36	0.62	0.86	-0.49	-0.50
TFP growth (util.-adj.; p.p.)	1.13	1.39	0.66	0.91	-0.47	-0.49

Table A2: Differences in output growth and Solow residual using GDP in consumption units and chained GDP growth. The data are the same as in Table 1, except that in the columns marked "Chained GDP", the measure of GDP growth is the growth of business value added in chained dollars; see Appendix A.2.1 for more details on data sources. TFP growth (the Solow residual) is constructed as $\hat{g}_Z = \hat{g} - \hat{s}_L \hat{g}_L - (1 - \hat{s}_L)\hat{g}_K$, where $\hat{g}$ is either the growth rate of output in consumption units (as defined in Appendix A.2.1), or chained output growth; $\hat{s}_L$ is the average measured labor income share; $\hat{g}_L$ is the average growth rate of labor input; and $\hat{g}_K$ is the average growth rate of capital. Utilization-adjusted TFP growth is constructed as $\hat{g}_Z = \hat{g} - \hat{s}_L \hat{g}_L - (1 - \hat{s}_L)\hat{g}_K - \hat{g}_u$, where $\hat{g}_u$ is the average growth rate of utilization.

	Average change (after minus before breakpoint)			
Breakpoint	**1997**	**2000**	**2004**	**1994**
GDP growth (p.p.)	-0.92	-1.34	-1.22	-0.75
Labor growth (p.p.)	-0.54	-0.80	-0.31	-0.15
Capital growth (p.p.)	-0.48	-0.96	-1.30	-0.39
Labor share of income	-0.04	-0.04	-0.05	-0.04
TFP growth (p.p.)	-0.49	-0.58	-0.68	-0.59
TFP growth (util.-adj.; p.p.)	-0.47	-0.57	-0.87	-0.58

Table A3: Data moments with alternative breakpoints. This table reports the change average output growth (with output measured in consumption units), labor growth, capital growth, the labor share of income, TFP growth, and utilization-adjusted TFP growth, for alternative breakpoints between the two samples we consider: 1997 (our baseline breakpoint); 2001; 2005; and 1993. The data are the same as in Tables 1 and A2.

	$\hat{b}$	$\hat{g}_{Q_2}$ (%)	g_Z (%)	μ	η
1947-1996	0	0	1.11	1.00	0
1997-2018					
No adj., no markups	0	0	0.62	1.00	0
No adj., markups	0	0	0.71	1.06	0
Intan. adj., markups	0.89	0.65	0.95	1.19	0.50

(a) Breakpoint: 1997

	$\hat{b}$	$\hat{g}_{Q_2}$ (%)	g_Z (%)	μ	η
1947-2000	0	0	1.11	1.00	0
2001-2018					
No adj., no markups	0	0	0.53	1.00	0
No adj., markups	0	0	0.63	1.07	0
Intan. adj., markups	0.89	0.36	0.82	1.20	0.51

(b) Breakpoint: 2000

	$\hat{b}$	$\hat{g}_{Q_2}$ (%)	g_Z (%)	μ	η
1947-2003	0	0	1.11	1.00	0
2004-2018					
No adj., no markups	0	0	0.43	1.00	0
No adj., markups	0	0	0.51	1.09	0
Intan. adj., markups	0.88	0.28	0.65	1.23	0.53

(c) Breakpoint: 2004

	$\hat{b}$	$\hat{g}_{Q_2}$ (%)	g_Z (%)	μ	η
1947-1993	0	0	1.17	1.00	0
1994-2018					
No adj., no markups	0	0	0.62	1.00	0
No adj., markups	0	0	0.71	1.06	0
Intan. adj., markups	0.89	0.65	0.95	1.19	0.50

(d) Breakpoint: 1994

Table A4: Results with alternative breakpoints. Each panel reports the effects of adjusting for markups and for intangibles when the breakpoints used are 1997 (our baseline); 2001; 2005; and 1993. The intangible adjustment used is for Professional Services, Management, and Administrative services (corresponding to the penultimate line of Table 4). The adjustments are made following the second of the two approaches described in Section 3.1, which uses data on both expenditures and prices of intangibles.

	(1)	(2)	(3)	(4)
$g_{Q_2}^{(BLS)}$	0.97***	0.97***	1.04***	1.05***
	(0.18)	(0.18)	(0.17)	(0.18)
Commodity/service FE	no	yes	no	yes
Year FE	no	no	yes	yes
Clustering of s.e.	commodity + year	commodity + year	commodity + year	commodity + year
R^2	0.603	0.633	0.643	0.673
N	829	829	829	829

Table A5: Simple correlations in proxies for g_{Q_2}, for BEA and PPI price indices. The sample is the set of year and commodity or service gorups for which the BEA GDP-by-industry and the BLS PPI commodity price indices can be matched. In all specification, the dependent variable is $g_{Q_2}^{(BEA)}$, the empirical proxy for g_{Q_2} derived from the BEA's GDP-by-industry tables and described in Section 3.2.

	$\hat{b}$	g_{Q_2} (%, BEA)	g_{Q_2} (%, BLS)
Services			
Professional, scientific, and technical services	0.940	0.59	0.25
Other real estate	0.952	-1.75	n.a.
Administrative and support services	0.964	0.20	0.33
Insurance carriers and related activities	0.972	-0.21	0.28
Credit intermediation and related activities	0.973	1.06	1.59
Management of companies and enterprises	0.974	1.54 w	n.a.
Commodities			
Chemical products	0.962	1.31	1.26
Oil and gas extraction	0.972	2.09	1.38
Petroleum and coal products	0.973	3.78	3.36
Food and beverage and tobacco products	0.976	1.12	0.37

(a) Individual commodity and service groups

	BEA			BLS		
	g_Z (%)	μ	η	g_Z (%)	μ	η
1947-1996	1.11	1.00	0	1.11	1.00	0
1997-2018						
No adjustment, no markups	0.62	1.00	0	0.62	1.00	0
No adjustment, markups	0.71	1.06	0	0.71	1.06	0
Adjusted for Prof. services	0.83	1.13	0.25	0.80	1.13	0.25
Adjusted for Prof. services + Admin.	0.88	1.17	0.40	0.83	1.17	0.40
Adjusted for Org. capital (Compustat)	0.88	1.16	0.38	0.83	1.16	0.38

(b) Aggregated service groups

Table A6: Comparison of results using BEA and BLS price indices for mismeasured investment goods. The top panel reports the 10 commodity or service groups with the smallest value of unadjusted GDP to adjusted GDP, as in Table 2. The average is computed over the 1997-2018 period, for each commodity or service group. The second column reports average values for the relative price growth of omitted capital, computed using price deflators from the BEA GDP-by-industry tables, as described in Section 3.2. The third column reports price indices obtained from the BLS, as described in Section 3.4. The bottom panel reports results from adjusting TFP growth measures for intangibles and markups, as in Table 4.

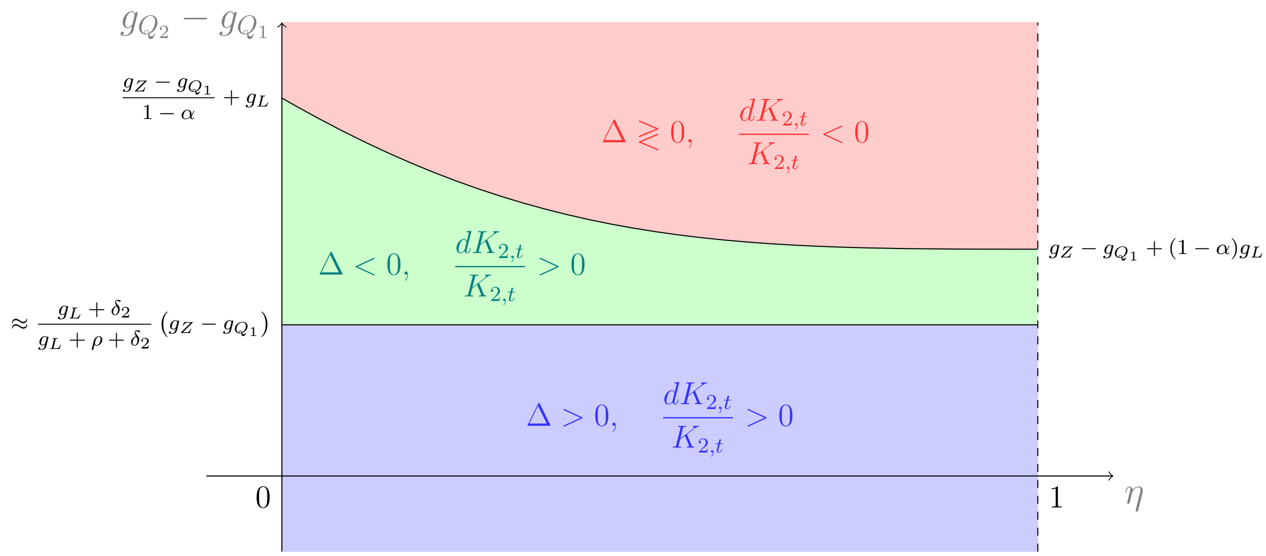

Figure A1: Sign of the total bias in measured TFP growth on the balanced growth path, depending on model parameters. The total bias is $\Delta = d\hat{Z}_t/Z_t - dZ_t/Z_t = d\hat{Z}_t/Z_t - g_Z$, where $d\hat{Z}_t/\hat{Z}_t$ is measured TFP on the balanced growth path, and $dZ_t/Z_t = g_Z$ is actual TFP growth. The horizontal axis corresponds to different values of η, the Cobb-Douglas share of omitted capital in production, and the vertical axis corresponds to different values of $g_{Q_2} - g_{Q_1}$, the difference between the growth rate of prices of omitted and measured capital.

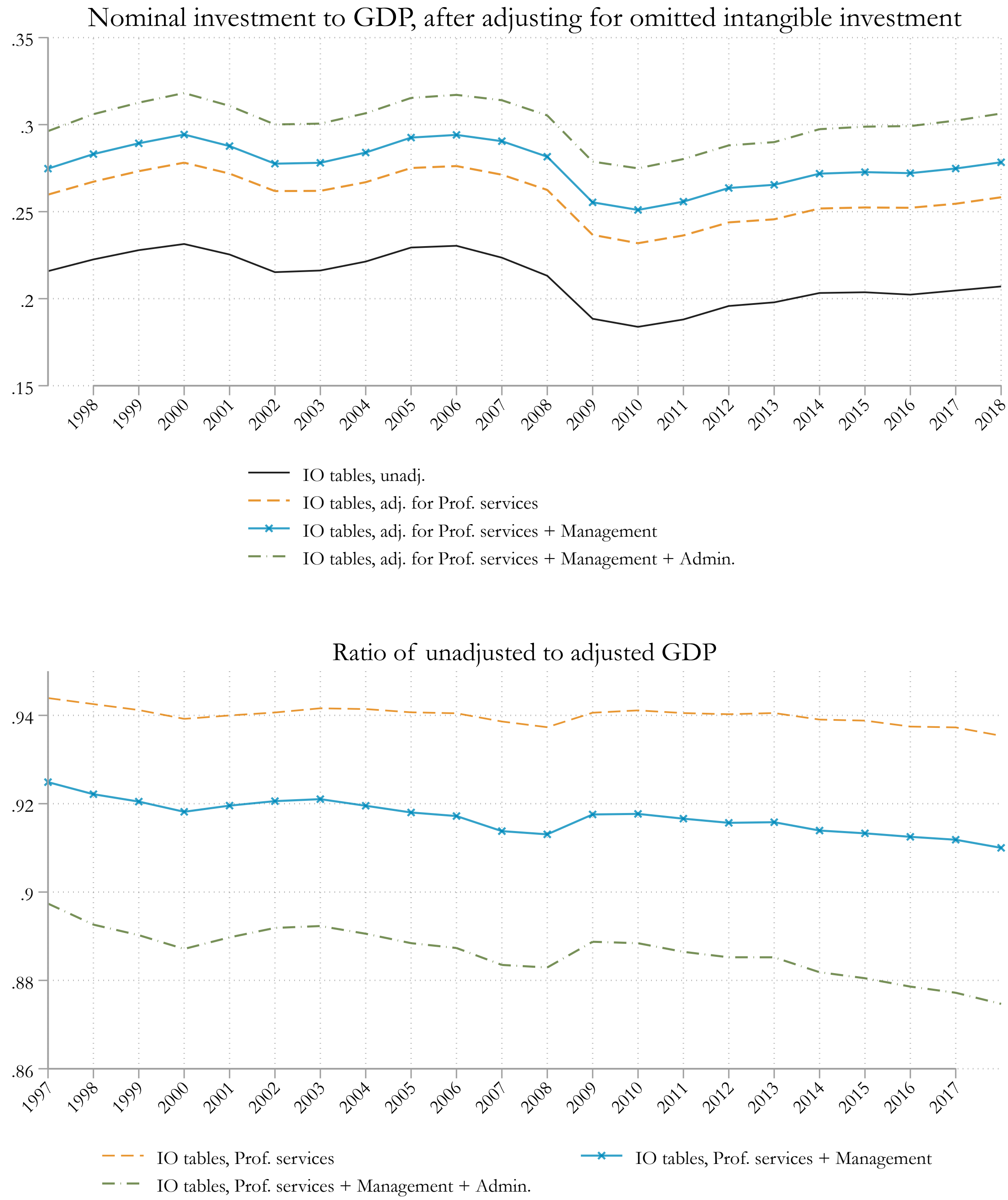

Figure A2: Time series for the ratio of unadjusted GDP to GDP adjusted for omitted intangibles (top panel), and for the ratio of investment to GDP without and with adjustments for omitted intangibles (bottom panel). The top panel reports the time series for $\hat{b}_t = P_tY_t/(P_tY_t + M_t)$, where P_tY_t is total GDP at producer prices, and M_t is the nominal value of intermediate input use of a group of services, where the latter is obtained from the Use tables of the benchmark Input-Output accounts. Each line corresponds to the ratio obtained when treating a different group of services as misclassified intangible investment. The bottom panel reports the time series $\iota_t = (Q_tI_t + M_t)/(P_tY_t + M_t)$, where Q_tI_t is measured aggregate spending on investment goods, also obtained from the Input-Output accounts.

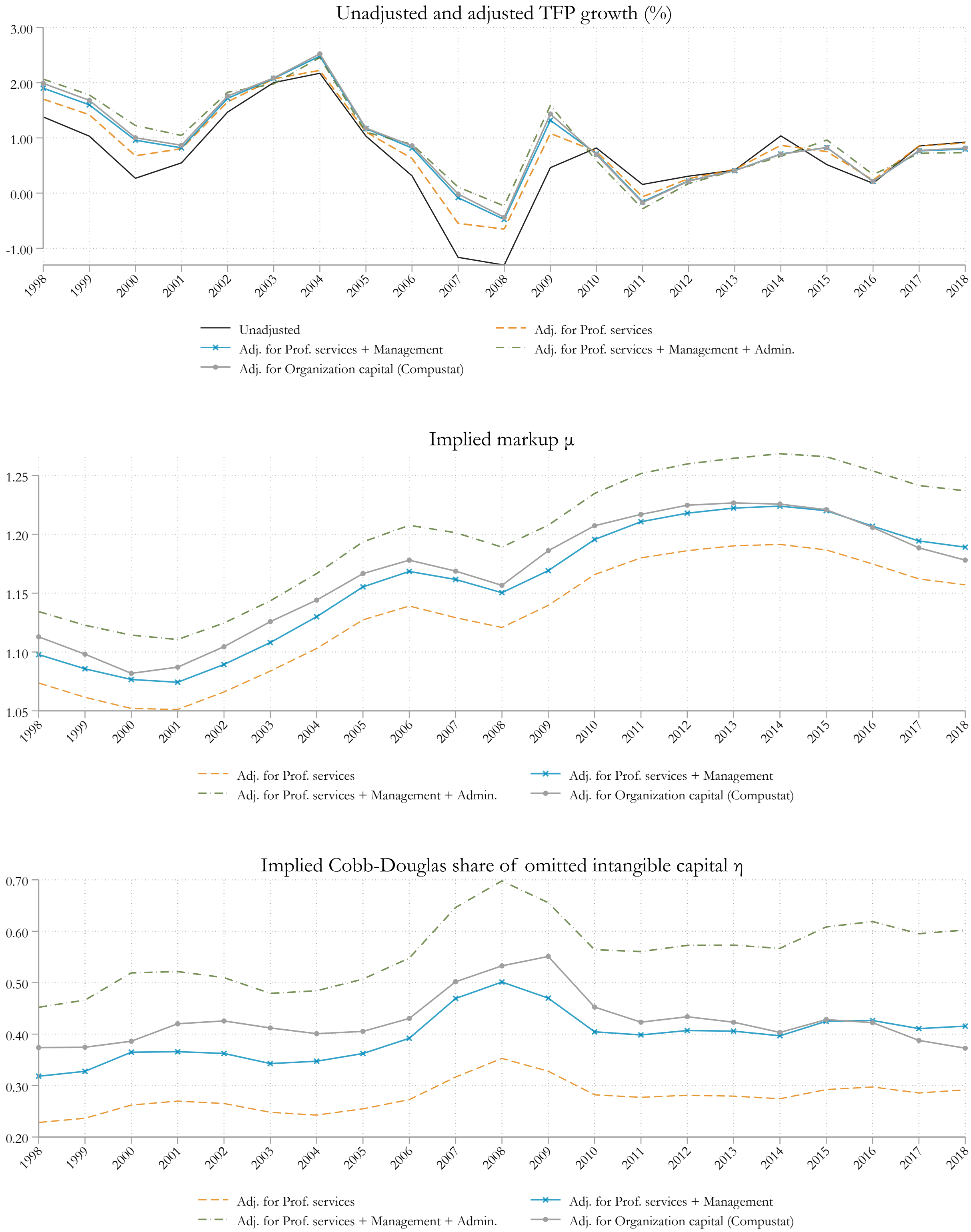

Figure A3: Time series for implied moments when adjusting for three key service groups. Adjusted TFP growth, markups, and the Cobb-Douglas share of omitted intangibles in the production function are computed following the second of the two approaches described in Section 3.1, which uses data on both expenditures and prices. The implied moments are constructed for each year separately. The series marked "unadjusted TFP growth" is the simple Solow residual.

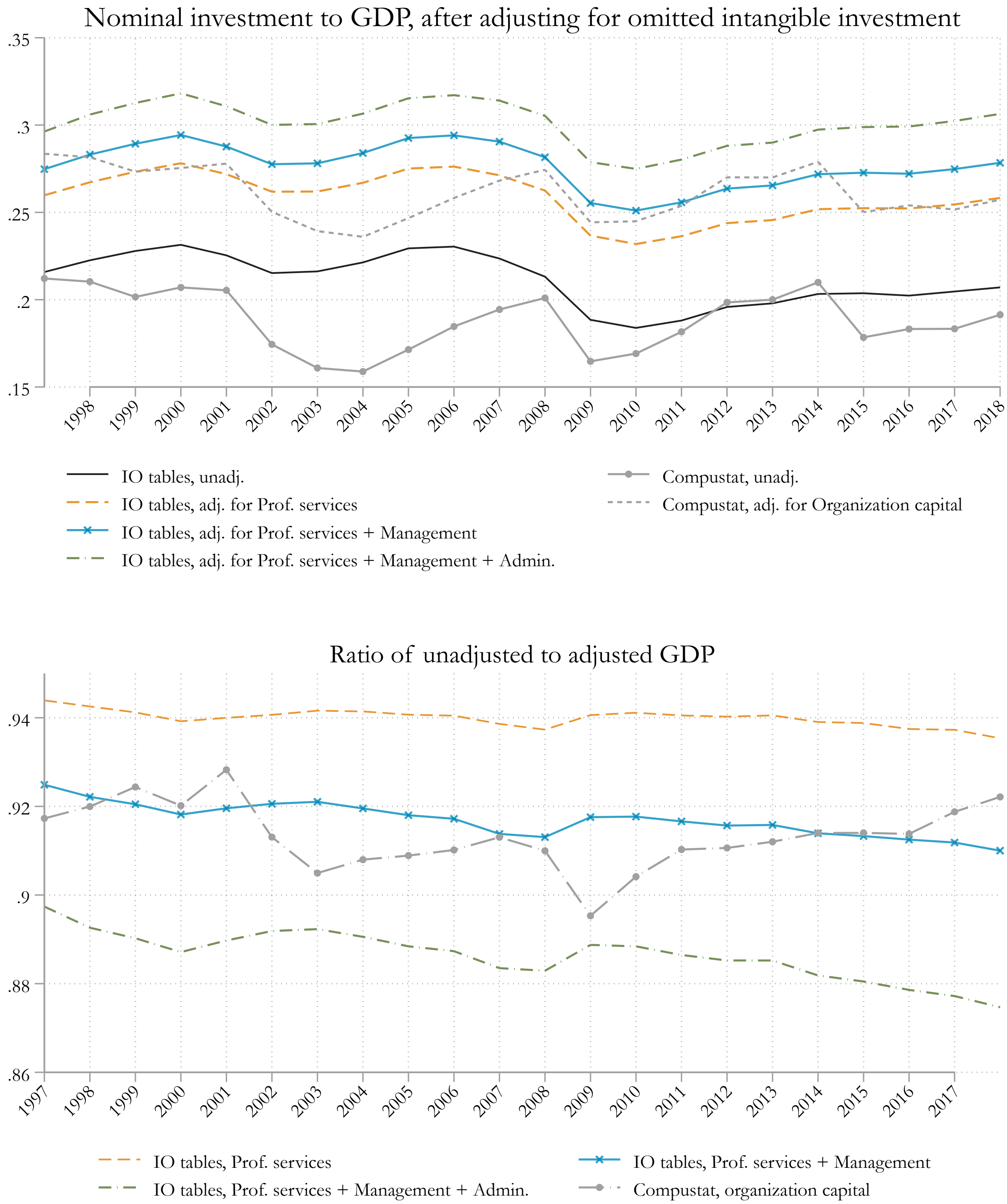

Figure A4: Compustat vs. IO tables: time series for unnadjusted GDP to GDP adjusted for ommited intangibles (top panel), and for the ratio of investment to GDP without and with adjustments for omitted intangibles (bottom panel). Relative to Figure A2, the only difference is the addition of the Compustat time series. The top panel reports the time series for $\hat{b}_t = P_tY_t/(P_tY_t + M_t)$, where P_tY_t is total GDP at producer prices, and M_t is the nominal value of intermediate input use of a group of services, where the latter is obtained from the Use tables of the benchmark Input-Output accounts. The bottom panel reports the time series $\iota_t = (Q_tI_t + M_t)/(P_tY_t + M_t)$, where Q_tI_t is measured aggregate spending on investment goods. See Section 3.2 for details on time series constructed from the IO tables, and A.2.3 for the time series constructed from Compustat.

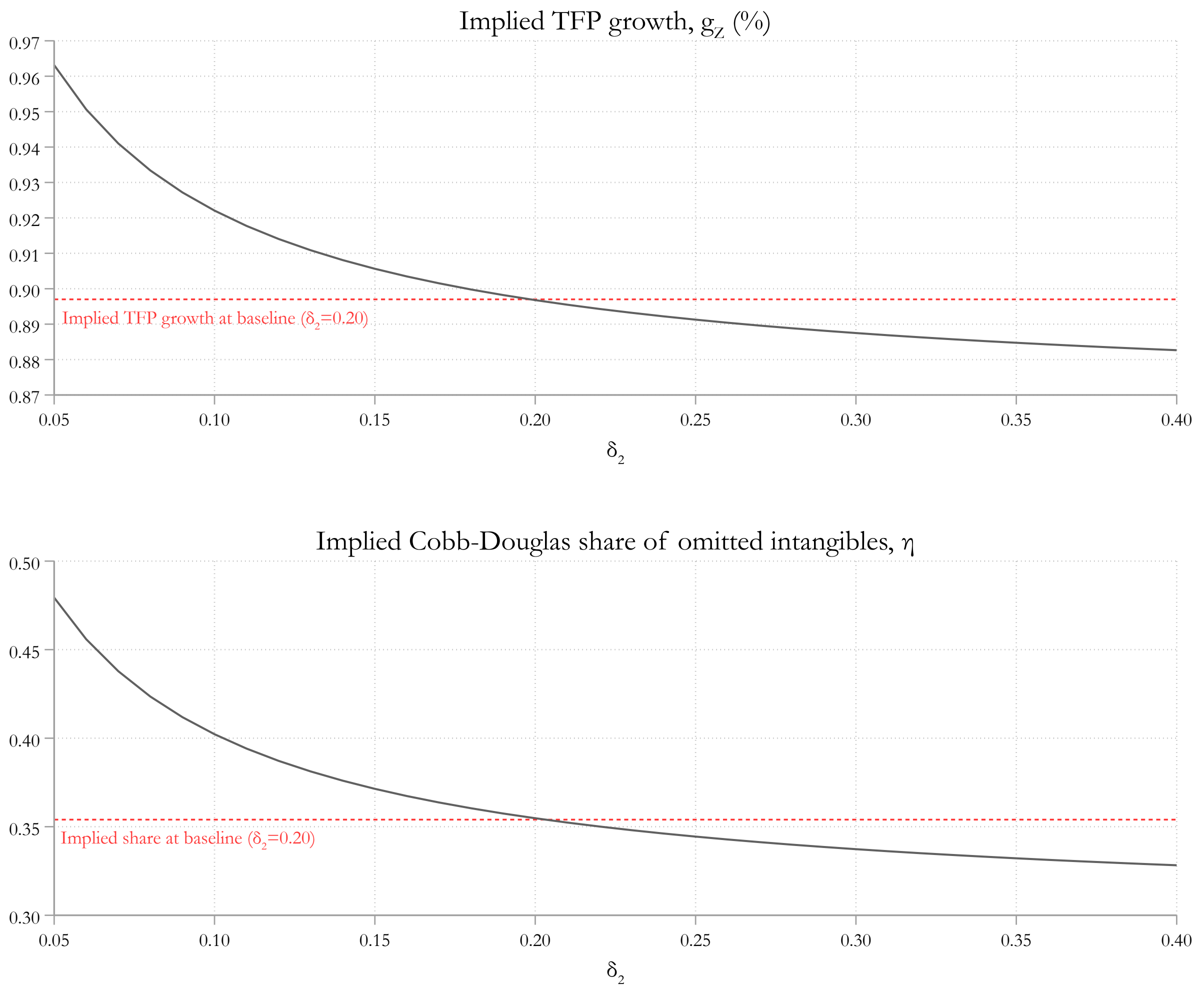

Figure A5: Implied moments for alternative values of the depreciation rate of omitted capital, δ_2. The bottom graph reports implied productivity growth g_Z, and the bottom graph reports the implied value of the Cobb-Douglas share of intangible capital, η, obtained using the second of the two approaches describes in Section 3.1. For the values of $\hat{b}$ and $\hat{g}_{Q_2}$, we use those corresponding to the case when only intermediate expenditures on Professional, Technical and Scientific services (PSTS) and Management services are reclassified as intangibles. This corresponds to the fifth line in Table 4.

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