On the Effects of Restricting Short-Term Investment

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We study the effects of policies proposed to address "short-termism" in financial markets. We examine a noisy rational expectations model in which investors' exposures and information about fundamentals endogenously vary across horizons. In this environment, taxing or outlawing short-term investment doesn't negatively affect the information in prices about long-term fundamentals. However, such a policy reduces short- and long-term investors' profits and utility. Changing policies about the release of short-term information can help long-term investors—an objective of some policy makers—at the expense of short-term investors. Doing so also makes prices less informative and increases costs of speculation. (*JEL* G11, G12, G14)

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For decades economists and policy makers have expressed concern about the potentially negative effects of "short-termism" among investors in financial markets. Research has argued that short-term investors may increase the volatility and reduce the informativeness of asset prices (Froot, Scharfstein, and Stein 1992), exacerbate fire sales and crashes (Cella, Ellul, and Giannetti 2013), inefficiently incentivize managers to focus on short-term projects (Shleifer and Vishny 1990), or reduce incentives of other investors to acquire information (Baldauf and Mollner 2017; Weller 2017), making prices less informative overall.

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Those who take the view that short-termism is bad for financial markets or the economy as a whole have proposed a broad array of policies to encourage long-term investment. One of the oldest is the tax on transactions of Tobin (1978).[1] Some policies directly depend on holding periods, such as the U.S. tax treatment of capital gains and dividends, the SEC's most recent proxy access rules, the proposed Long-Term Stock Exchange, linking corporate voting rights to tenure, and the proposal of Bolton and Samama (2013) for corporations to explicitly reward long-term investors.[2] Budish, Cramton, and Shim (2015) propose to eliminate trade at the very highest frequencies through frequent batch auctions, and proposals have been made to limit quarterly financial reports and earnings guidance in the United States, following similar changes in the United Kingdom, for example, by Dimon and Buffett (2018).[3] A number of these policies were endorsed in a letter from 2009 signed by leaders in business, finance, and law.[4]

This paper theoretically evaluates the effect of policies targeting short-termism on price informativeness and investor outcomes. Unlike the previous literature, we consider a simple and very general setting with investors who are ex ante identical and then may *endogenously* specialize into different horizons. Although some recent work delves into the consequences of various limits on information gathering ability and there have been empirical analyses of high-frequency traders, we are not aware of any other work that directly studies the effects of restrictions on short- and long-term strategies on price informativeness and investor profits in a general setting.[5]

The model is meant to be as simple and as general as possible. Two key features it must have are that investors choose among investment strategies at different horizons, and that they choose how much information to acquire about fundamentals across horizons. We study a version of the noisy rational expectations model developed in Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016). Whereas that paper studies investment in a cross-section of assets, we argue here that investment policies over time can be thought of as a choice of exposures on many different future dates. Each of those dates represents a different "asset", and the returns on those assets will be correlated

1 See also Stiglitz (1989), Summers and Summers (1989), and Habermeier and Kirilenko (2003).

2 See LTSE.com and Osipovich and Berman (2017).

3 See also Schacht et al. (2007), Pozen (2014), and Nallareddy, Pozen, and Rajgopal (2016).

4 See the Institute (2009) and Stiglitz (2015).

5 In much recent work, including Cartea and Penalva (2012), Baldauf and Mollner (2017), and Biais, Foucault, and Moinas (2015), high-frequency or short-term investors are somehow different from others, either in preferences or trading technologies. Those models are better suited to studying high-frequency trade specifically. For recent analyses of limits on information gathering ability, see Banerjee and Green (2015), Goldstein and Yang (2015), Dávila and Parlatore (2016), and Farboodi and Veldkamp (2017).

across dates.[6] The model in this paper is notable for allowing an arbitrarily long horizon (as opposed to two or three periods), with turnover at any frequency.

It is important to note that the model is not fully dynamic: all trade happens on date 0, so investors cannot rebalance in response to news or to the realization of fundamentals, even though they might desire to. The model takes a dynamic problem, with information flowing and investment choices being made over time, and compresses it into a single time period, along the lines of the classic Arrow-Debreu-type analysis, but without a complete set of state-contingent contracts. Dynamic market equilibria are difficult or impossible to solve, and we do not contribute to that area.[7] The paper's focus is instead on the choice of short- versus long-term investment strategies and information acquisition. Short-term investors arise naturally in the model as agents whose exposures to fundamentals rapidly fluctuate across dates because of the type of information they have acquired. The relevant concept of short- versus long-term here ranges between days and years; the model is not designed to analyze technical features of higher-frequency trading, like market microstructure effects or exchange fragmentation.

A policy maker might want to regulate investment strategies for at least three potential reasons. First, if price informativeness at long horizons is more important for economic decisions like physical investment, then long-term information acquisition might be encouraged. Second, policy makers might have a general bias toward long-term investors, perhaps because they are more likely to be people saving for retirement. Finally, policy makers might want to limit the losses of retail investors who make poor investment decisions. As is common in the literature on price efficiency, we do not explicitly model these regulatory motives.[8] Instead, we use the model to examine how restrictions on investment policies affect price informativeness and the profits and utility of the various investors in order to help inform the policy debate.

The paper examines a number of specific policies, including direct restrictions on investment strategies, taxes on transactions, and taxing or subsidizing information acquisition. We first show that when sophisticated agents are restricted from investing and trading at some frequency, prices

[6] The paper uses a frequency transformation that allows the model to be solved by hand. For other related work on frequency transformations, see Bandi and Tamoni (2014), Bernhardt, Seiler, and Taub (2010), Chinco and Ye (2017), Chaudhuri and Lo (2016), Dew-Becker and Giglio (2016), and Kasa, Walker, and Whiteman (2013).

[7] Work on the infinite regress problem typically assumes that investors have only single-period objectives and does not for a choice of information across horizons. See Makarov and Rytchkov (2012), Kasa, Walker, and Whiteman (2013), and Rondina and Walker (2017). Recent work also examines dynamic models with strategic trade (with similar restrictions regarding horizons) (see Vayanos 1999, 2001; Ostrovsky 2012; Banerjee and Breon-Drisch 2016; Foucault, Hombert, and Roşu 2016; Du and Zhu 2017; Dugast and Foucault 2018.

[8] Bond, Edmans, and Goldstein (2012) review the literature on the value of price efficiency and identify two spillovers. First, information that stock prices reveal guides real activity through investment decisions (Dow and Gorton 1997; Kurlat and Veldkamp 2015) and the decisions of outside investors and regulators to intervene in a firm's activities (Bond, Goldstein, and Prescott 2009; Bond and Goldstein 2015). Second, price informativeness allows shareholders to tie manager compensation to equity prices, thus improving the real efficiency of management activities (Fishman and Hagerty 1989; Holmström and Tirole 1993; Farboodi and Veldkamp 2017).

become uninformative at that frequency. So if a policy were implemented saying that investors could no longer maintain positions for less than a month, variation in prices at frequencies less than a month would become uninformative for fundamentals, and instead be driven purely by liquidity demand. High-frequency price volatility and mean reversion would also rise.

However, there is no spillover across horizons. A short-term restriction or transaction tax does not reduce price informativeness or increase return volatility at longer horizons, so prices would remain informative at frequencies lower than a month (in an extension of the model, informativeness can even rise). This separability across horizons follows from a statistical result showing that there is a robust independence across frequencies in stationary models, along with a separability in mean-variance (or constant absolute risk aversion [CARA]) preferences.

The next question is how investment restrictions affect investor outcomes. An increase in short-term investment (e.g., because of a change in technology that makes short-term information acquisition or trading cheaper) turns out to make long-term investors worse off, essentially taking away some of the long-term investors' trading opportunities. But restricting short-term investment does not transfer profits back to long-term investors; instead it simply eliminates those profits, making both short- and long-term investors worse off.

In the context of the model, the way to tilt markets in favor of long-term investors—if that is one's goal—is to make acquisition of short-term information more expensive for investors. There have been numerous recent proposals to do just that, for example by limiting quarterly earnings guidance. The model in this paper is well suited to analyze such policies, and we show that they can shift the equilibrium toward long-term investors, increasing their average profits and utility (though the direction of this result depends on how one models information releases).

Finally, the paper examines the impact of the various policies on the profits of noise traders. Intuitively, the noise traders are constantly making mistakes, potentially affecting prices. There are two ways to protect them from those mistakes: stop them from trading or reduce the losses they take on each trade. Stopping them from trading is, in principle, simple—just close asset markets—but then one loses the information contained in prices, along with any gains from trade. More interestingly, the paper shows that a better alternative is to subsidize or otherwise encourage information acquisition, which causes prices to become more informative and less responsive to noise trader (perhaps speculative) demand. Such a policy can, in the limit, drive noise trader losses to zero, while simultaneously making prices more useful for economic decisions and reducing the excess volatility caused by noise trader speculation. However, it is the opposite of the policy that we showed helps the long-term investors. Furthermore, it is important to temper the results on noise traders with the knowledge that there is no single canonical model of noise traders. The paper examines robustness to an alternative formulation driven by time-varying hedging demand and shows that welfare predictions are more difficult to make, though the predictions for price informativeness and return volatility are similar.

Overall, then, we obtain three main results about policies aimed at short-termism:

1. Restricting short-term investment affects short-run, but not long-run, price informativeness and return volatility.
2. Restricting short-term investment hurts both short- and long-term investors, but helps noise traders.
3. Taxing or restricting the availability of short-term information helps long-term investors, hurts short-term investors and noise traders, and reduces short-term price efficiency. Subsidizing information or mandating greater disclosure by firms does the opposite.

On net, then, we would argue that mandatory information releases or subsidizing information acquisition are the most natural policies to address short-termism, as they both reduce speculative effects on prices and improve price efficiency. They do, however, come with costs to long-term investors, and also run against recent proposals to reduce quarterly reporting.

The answer to the question of how restrictions on trade affect price informativeness and welfare is not obvious ex ante. One view is that there might be some sort of separation across frequencies, so that restrictions in one realm do not affect outcomes in another. On the other hand, investors obviously interact—they trade with each other—so it would be surprising if policies targeting a particular type of investor did not act to benefit others. What we find is a mix of the two: market characteristics at high frequencies can affect the profits and utility of long-term investors—the model is not entirely separable across frequencies in that sense—but they do not affect low-frequency price informativeness in our baseline case. Furthermore, there is a tension between helping long-term investors, helping noise traders, and maintaining price informativeness. No single policy helps all the groups at the same time, because of a zero-sum aspect of the model, and policies that may be attractive to certain investors can come with negative side effects for agents outside the model—for example, executives or policy makers like the FOMC—who might make decisions based on asset prices.

1. The Model

1.1 Market structure

Time is denoted by $t \in \{-1, 0, 1, \ldots, T\}$, with T even and large. There is a fundamentals process D_t, on which investors trade forward contracts, with realizations on all dates, except -1 and 0. The time series is stacked into a vector $D \equiv [D_1, D_2, \ldots, D_T]'$ (versions of variables without time subscripts denote vectors) and is unconditionally distributed as

$$D \sim N(0, \Sigma_D). \tag{1}$$

For our benchmark results, we focus on the case in which fundamentals are stationary. Stationarity implies that Σ_D is constant along its diagonals, and we further assume that the eigenvalues of Σ_D are finite and bounded away from zero (which is satisfied by standard ARMA processes).

There is a set of futures claims on realizations of the fundamental. When we say that the model features a choice of investment across dates or horizons, we mean that investors will choose portfolio allocations across the futures contracts, which then yield exposures to the realization of fundamentals on different dates in the future.

A concrete example of a process D_t is the price of crude oil: oil prices follow some stochastic process and investors trade futures on oil at many maturities. D_t also could be the dividend on a stock, in which case the futures would be claims on dividends on individual dates. The analysis of futures is an abstraction for the sake of the theory, though we note that dividend futures are in fact traded (Binsbergen and Koijen 2017). While the concept of a futures market on the fundamentals will be a useful analytic tool, we will also price portfolios of futures. Equity, for example, is a claim to the stream of fundamentals over time. Holding any given combination of futures claims on the fundamental is equivalent to holding futures contracts on equity claims.

The model does not allow for stochastic volatility, nonlinearity, or other changes in the higher moments of D_t over time.[9] That we study the level of fundamentals, rather than their log, is also a restriction shared by CARA-Normal specifications (e.g., Grossman and Stiglitz 1980). The restrictions, along with those implicit in the preferences below, mean that the model is primarily useful for qualitative analysis.

1.2 Information structure

There is a unit mass of "sophisticated" or rational investors, indexed by $i \in [0,1]$, who have rational expectations, conditioning on both prices and private signals. The signals an agent observes are a collection $\left\{Y_{i,t}\right\}_{t=1}^{T}$ *observed on date 0* with

$$Y_{i,t} = D_t + \varepsilon_{i,t},\ \varepsilon_i \sim N(0, \Sigma_i), \tag{2}$$

where Σ_i^{-1} is investor i's signal precision matrix (which will be chosen endogenously). Through $Y_{i,t}$, investors can learn about fundamentals on all dates between 1 and T. $\varepsilon_{i,t}$ is a stationary error process in the sense that $Cov\left(\varepsilon_{i,t}, \varepsilon_{i,t+j}\right)$ depends on j, but not on t. Because $Var\left(\varepsilon_{i,t}\right)$ is the same for all t, all dates are equally difficult to learn about and no particular date is given special prominence in the model. Investors must choose an information policy that treats all dates symmetrically.

[9] The model could accommodate predictable changes in volatility, such as intraday patterns and volatility around scheduled announcements, through time change methods, like in Ané and Geman (2000) and Geman, Madan, and Yor (2001).

The signal structure generates one of our desired model features, which is that investors can choose to learn about fundamentals across different dates in the future. When the errors are positively correlated across dates, the signals are relatively less useful for forecasting trends in fundamentals because the errors also have persistent trends. Conversely, when errors are negatively correlated across dates, the signals are less useful for forecasting transitory variation and provide more accurate information about moving averages. What types of fluctuations investors are informed about will determine their investment strategies.

1.3 Investment objective

On date 0, there is a market for forward claims on fundamentals on all dates in the future. Investor i's demand for a date-t forward conditional on the set of prices and signals is denoted $Q_{i,t}$. Investors have mean-variance utility over terminal wealth:[10]

$$U_{0,i} = \max_{\{Q_{i,t}\}} T^{-1} E_{0,i}\left[\sum_{t=1}^{T} \beta^t Q_{i,t}(D_t - P_t)\right] - \frac{1}{2}(\rho T)^{-1} Var_{0,i}\left[\sum_{t=1}^{T} \beta^t Q_{i,t}(D_t - P_t)\right], \quad (3)$$

where $0 < \beta \leq 1$ is the discount factor, $E_{0,i}$ and $Var_{0,i}$ are the expectation and variance operators conditional on agent i's date-0 information set, $\{P, Y_i\}$, and ρ is risk-bearing capacity per unit of time. Investors have identical preferences: they can follow different strategies with different rates of portfolio turnover, but they all want to earn the highest possible returns, with the least amount of risk, in the shortest time. The sense in which the model maps into the colloquial use of the term "short-termism" is that agents in the model may choose to follow investment strategies featuring very rapid changes in their positions across dates. Short- and long-term investors are distinguished by how long they maintain positions, not by their objective.[11]

The key restriction here is that signals are acquired and trade occurs on date 0. In general settings, there is no known closed-form solution to even the partial-equilibrium dynamic portfolio choice problem, let alone to the full market equilibrium. Therefore, we use a relatively minimal static model that eliminates those problems by assumption. Nevertheless, the model has the two characteristics that we stated we desire in the introduction: it allows for

10 To see why this is over terminal wealth, note that when the profits from each futures claim, $D_t - P_t$, are reinvested at the riskless rate β^{-1}, terminal wealth, $W_{T,i}$, is $\sum_{t=1}^{T} \beta^{t-T} Q_{i,t}(D_t - P_t)$, which is simply β^{-T} times the argument of the expectation and variance in the preferences. For motivation, see Dumas and Luciano (1991).

11 The model can only accommodate mean-variance or constant absolute risk aversion preferences and remain tractable.

investment strategies that place different weight on fundamentals on different dates in the future, and it allows investors to make a choice about how precise their signals are for different types of fluctuations in fundamentals.[12]

The time discounting in (3) has the effect of making dates farther in the future less important in the objective of the investors. Define

$$\tilde{Q}_{i,t} \equiv \beta^t Q_{i,t} \tag{4}$$

to be agent i's discounted demand. In what follows, the $\tilde{Q}_{i,t}$ will be stationary processes. That means that $Q_{i,t} = \beta^{-t} \tilde{Q}_{i,t}$ will generally grow in magnitude with maturity t, though only to a relatively small extent for typical values of β and horizons on the order of 10–20 years.

1.4 Noise trader demand

To keep prices from being fully revealing, we assume there is uninformed demand from a set of noise trader who have irrational expectations. Their beliefs depend on a signal, Z_t, that is in reality uncorrelated with fundamentals and represents a sentiment shock. Appendix A shows their demand, denoted N_t, is then

$$\tilde{N}_t = Z_t - k P_t, \tag{5}$$

$$\text{where } \tilde{N}_t \equiv \beta^t N_t. \tag{6}$$

k is a coefficient determining the sensitivity of noise trader demand to prices, which depends on their risk aversion and how precise they believe their signals to be. In the benchmark case in which D_t is stationary in levels, we assume that Z_t is also stationary in levels, which yields a useful symmetry between fundamentals, supply, and the signals, in that they are all stationary processes.

1.5 Asset market equilibrium in the time domain

We begin by solving for the market equilibrium on date 0 that takes the agents' signal precisions, Σ_i^{-1}, as given.

Definition 1. For any given set of individual precisions $\{\Sigma_i\}_{i \in [0,1]}$, a date-0 asset market equilibrium is a set of demand functions, $\{Q_i(P, Y_i)\}_{i \in [0,1]}$, and a price vector P, such that investors maximize utility and all markets clear: $\int_i Q_{i,t} di + N_t = 0$ for all $t \geq 1$.

[12] The two key differences from a fully dynamic model are that agents cannot condition on past realizations of fundamentals and that there is not a full set of state-contingent contracts. The former restriction will bind more weakly when agents make decisions primarily based on private signals rather than the realization of fundamentals. The latter restriction could potentially lead to time inconsistency, depending on how one assumes agents update information sets and preferences over time.

Investors submit demand curves for each futures contract and the equilibrium price vector, P, is the one that clears all markets. The structure of the time-0 equilibrium is mathematically that of Admati (1985):

$$P = A_1 D + A_2 Z, \tag{7}$$

$$\text{where } A_1 \equiv I - \begin{pmatrix} \rho^2 \Sigma_{avg}^{-1} \Sigma_Z^{-1} \Sigma_{avg}^{-1} + \\ \Sigma_{avg}^{-1} + \rho^{-1} k + \Sigma_D^{-1} \end{pmatrix}^{-1} \left(\rho^{-1} k + \Sigma_D^{-1}\right), \tag{8}$$

$$A_2 \equiv \rho^{-1} A_1 \Sigma_{avg}^{-1}, \tag{9}$$

$$\Sigma_{avg}^{-1} \equiv \int_i \Sigma_i^{-1} di. \tag{10}$$

As Admati (1985) discusses, this equilibrium is not particularly Illuminating, because standard intuitions, including the idea that increases in demand should raise prices, do not hold. Prices of futures maturing on any particular date depend on fundamentals and demand for all other maturities, because the matrices A_1 and A_2 are not diagonal, except in knife-edge cases. Interpreting the equilibrium requires interpreting complicated products of matrix inverses. The following section shows that the equilibrium can be solved by hand nearly exactly when it is rewritten in terms of frequencies.

2. Frequency Domain Interpretation

2.1 Frequency portfolios

The basic difficulty of the model is that fundamentals, noise trader demand, and signal errors are all correlated across dates. For any one of those three processes, we could use a standard orthogonal (eigen-) decomposition to yield a set of independent components. But, in general, three time series with different correlation properties across dates will not have the same orthogonal decomposition. A central result from time-series analysis, though, is that a particular frequency transform asymptotically orthogonalizes all standard stationary time-series processes.

Such a transformation represents simply analyzing the prices of particular portfolios of futures instead of the futures themselves. It must satisfy three requirements. First, the transformation should be full rank, so that the set of portfolios allows an investor to obtain the same payoffs as the futures themselves. Second, the transformed portfolios should be independent of each other. Third, we are studying trade at different frequencies, so it would be nice if the portfolios also had a frequency interpretation.

Fluctuations at different frequencies can be conceptualized many ways. One might imagine step functions switching between +1 and -1 at different rates. For reasons that will become clear, using sines and cosines is most natural in our setting.

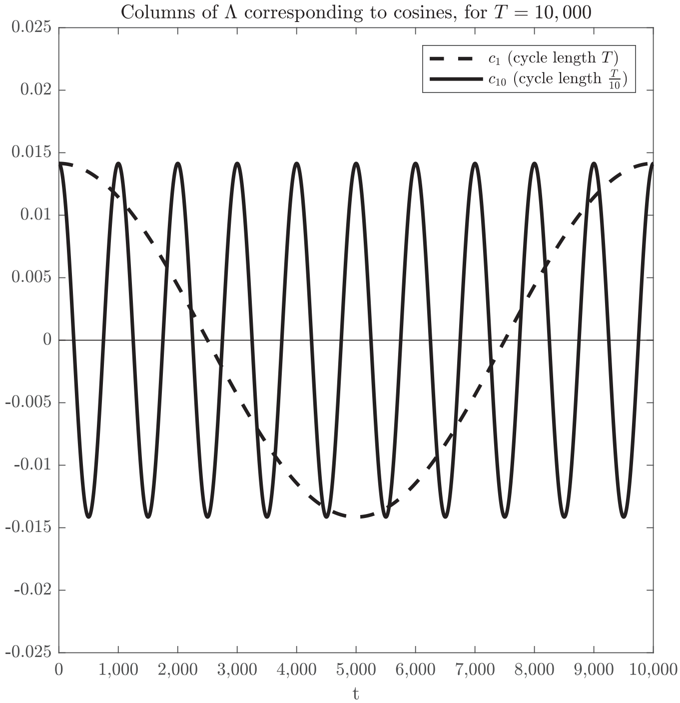

Figure 1
Portfolio weights.
Portfolio weights for the cosine frequency portfolios c1 and c0, as defined in the main text. The horizontal axis is time, or the maturity of the corresponding futures contract. The vertical axis is the weight each portfolio puts on that futures contract.

Formally, the portfolio weights are represented as vectors of the form

$$c_h \equiv \sqrt{\frac{2}{T}}\Big(\cos(\omega_h(t-1))\Big)_{t=1}^{T}, \tag{11}$$

$$s_h \equiv \sqrt{\frac{2}{T}}\Big(\sin(\omega_h(t-1))\Big)_{t=1}^{T}, \tag{12}$$

$$\text{where} \qquad \omega_h \equiv 2\pi h/T, \tag{13}$$

for different values of the integer $h \in \{0,1,\ldots,T/2\}$. c_0 is the lowest frequency portfolio, with the same weight on all dates, whereas $c_{\frac{T}{2}}$ is the highest frequency, with weights switching each period between ± 1.

Figure 1 plots the weights for a pair of those portfolios. The x-axis represents dates, and the y-axis is the weight of the portfolio on each date. The weights vary smoothly over time, with the rate at which they change signs depending on the frequency ω_h.

Economically, the idea is to think about the investment problem as being one of choosing exposure to different types of fluctuations in fundamentals. A long-term investor can be thought of as one whose exposure to fundamentals changes little over time, while a short-term investor holds a portfolio whose weights change more frequently and by larger amounts.

Our claim is that studying the frequency portfolios is more natural than studying individual futures claims. Investors do not typically acquire exposure to fundamentals on only a single date. Rather, they have exposures on multiple dates, and the portfolios we study are one way to express that. While investors will also obviously not hold a portfolio that takes the exact form of a cosine, any portfolio can be expressed as a sum of cyclical components. An investor whose portfolio loadings change frequently will have a portfolio whose weights are relatively larger on the high-frequency components, which Figure 1 shows generate rapid changes in loadings.

2.2 Properties of the frequency transformation

The portfolio weights can be combined into a matrix, Λ, which implements the frequency transformation,

$$\Lambda \equiv \left[\frac{1}{\sqrt{2}} c_0, c_1, s_1, c_2, s_2, \ldots, c_{\frac{T}{2}-1}, s_{\frac{T}{2}-1}, \frac{1}{\sqrt{2}} c_{\frac{T}{2}} \right]. \tag{14}$$

(s_0 and $s_{T/2}$ do not appear, because they are identically equal to zero; the $1/\sqrt{2}$ scaling for c_0 and $c_{T/2}$ gives them the same norms as the other vectors).

We use lowercase letters to denote frequency domain objects. So whereas $\tilde{Q}_i$ is investor i's vector of discounted allocations to the various futures, $\tilde{q}_i$ is their vector of discounted allocations to the frequency portfolios, with

$$\tilde{Q}_i = \Lambda \tilde{q}_i. \tag{15}$$

In what follows, the index $j = 1, \ldots, T$ identifies columns of Λ. The jth column of Λ is a vector that fluctuates at frequency $\omega_{\left\lfloor \frac{j}{2} \right\rfloor} = 2\pi \left\lfloor \frac{j}{2} \right\rfloor / T$, where $\lfloor \cdot \rfloor$ is the integer floor operator.[13] So there are two vectors, a sine and a cosine, for each characteristic frequency, with the exceptions of $j = 1$ (frequency 0, the lowest possible) and $j = T$ (frequency $\frac{T}{2}$, the highest possible).

Note also that Λ has the property that $\Lambda^{-1} = \Lambda'$, so that frequency domain vectors can be obtained through

$$\tilde{q}_i = \Lambda' \tilde{Q}_i. \tag{16}$$

In the same way that $\tilde{q}_i$ represents weights on frequency-specific portfolios, $d \equiv \Lambda' D$ is a representation of the realization of fundamentals written in terms of

13 $\lfloor x \rfloor$ is the largest integer that is less than or equal to x.

frequencies instead of time. The first element of d, for example, is proportional to the realized sample mean of D.

As a simple example, consider the case with $T=2$. The low-frequency or long-term component of dividends is then $d_0=(D_1+D_2)/\sqrt{2}$ and the high-frequency or transitory component is $d_1=(D_1-D_2)/\sqrt{2}$. Agents invest in the low-frequency component d_0 by buying an equal amount of the claims on D_1 and D_2 and they trade the high-frequency component d_1 by buying offsetting amounts of the claims on D_1 and D_2. A short-term investment in this case is one where the sign of the exposure to fundamentals changes, while the long-term investment has a fixed position.

The most important feature of the frequency transformation is that it approximately diagonalizes the variance matrices.

Definition 2. For an $n\times n$ matrix A with elements $a_{l,m}$, the **weak matrix norm** is

$$|A|\equiv\left(\frac{1}{n}\sum_{l=1}^{n}\sum_{m=1}^{n}a_{l,m}^2\right)^{1/2}. \tag{17}$$

If $|A-B|$ is small, then the elements of A and B are close in mean square.

The frequency transformation leads us to study the spectral densities of the various time series:

Definition 3. The **spectrum** at frequency ω of a stationary time series X_t is

$$f_X(\omega)\equiv\sigma_{X,0}+2\sum_{t=1}^{\infty}\cos(\omega t)\sigma_{X,t}, \tag{18}$$

$$\text{where } \sigma_{X,t}=cov(X_s,X_{s-t}). \tag{19}$$

The spectrum, or spectral density, is used widely in time-series analysis. The usual interpretation is that it represents a variance decomposition. $f_X(\omega)$ measures the part of the variance of X_t associated with fluctuations at frequency ω, which is formalized as follows.

Lemma 1. For any stationary time series $\{X_t\}_{t=1}^{T}$, with frequency representation $x\equiv\Lambda'X$, the elements of the vector x are approximately uncorrelated in the sense that the covariance matrix of x, $\Sigma_x\equiv\Lambda'\Sigma_X\Lambda$, is nearly diagonal,

$$|\Sigma_x-diag(f_X)|\leq bT^{-1/2}, \tag{20}$$

and x converges in distribution to

$$x\rightarrow_d N(0,diag(f_X)), \tag{21}$$

where b is a constant that depends on the autocorrelations of X,[14] and $diag(f_X)$ denotes a matrix with the vector $\left\{f_X\left(\omega_{\lfloor j/2\rfloor}\right)\right\}_{j=1}^{T}$ on the main diagonal and zeros elsewhere.[15]

Proof. These are textbook results (e.g., Brockwell and Davis 1991; Gray 2006). Online Appendix 1 provides a derivation of inequality (20) specific to our case. The convergence in distribution follows from Brillinger (1981), theorem 4.4.1. ■

Lemma 1 says that Λ approximately diagonalizes *all* stationary covariance matrices. So the frequency-specific components of fundamentals, prices, and noise trader demand are all (approximately) independent when analyzed in terms of frequencies. That is, $d=\Lambda' D$, $y_i=\Lambda' Y_i$, and $z=\Lambda' Z$ all have asymptotically diagonal variance matrices. That independence will substantially simplify our analysis, and it is a special property of the sines and cosines, as opposed to other conceptions of frequencies. The various primitive restrictions on the model, including mean-variance preferences and stationarity, are required in order to be able to take advantage of this diagonalization result.[16]

2.3 Market equilibrium in the frequency domain

Instead of solving jointly for the prices of all futures, the approximate diagonalization result allows us to solve a series of parallel scalar problems, one for each frequency. Intuitively, the frequency-specific portfolios have returns that are nearly uncorrelated with each other, so the investors' utility can be approximately written as a sum of mean-variance optimizations[17]

$$U_{0,i} \approx \max_{\{q_{i,j}\}} T^{-1} \sum_{j=1}^{T} \left\{ E_{0,i}\left[\tilde{q}_{i,j}\left(d_j - p_j\right)\right] - \frac{1}{2}\rho^{-1} Var_{0,i}\left[\tilde{q}_{i,j}\left(d_j - p_j\right)\right] \right\}. \tag{22}$$

14 Specifically, $b=4\left(\sum_{j=1}^{\infty}\left|j\sigma_{X,j}\right|\right)$.

15 A requirement of this lemma, which we impose for all the stationary processes studied in the paper, is that the autocovariances are summable in the sense that $\sum_{j=1}^{\infty}\left|j\sigma_{X,j}\right|$ is finite (which holds for finite-order stationary ARMA processes, for example). Trigonometric transforms of stationary time series converge in distribution under more general conditions, though. See Shumway and Stoffer (2011), Brillinger (1981), and Shao and Wu (2007).

16 Finally, note that infill asymptotics, where T grows by making the length of a time period shorter, are not sufficient for lemma 1 to hold. T must be large relative to the range of autocorrelation of the process X. So, for example, if fundamentals have nontrivial autocorrelations over a horizon of a year, T should be substantially larger than a year. If one shifts from annual to monthly data, then T should rise by a factor of 12 for the approximations to be equally accurate. Additionally, T should be as long as the investors' actual horizons. That is, if they are investing for retirement, it would be on the other of decades (if this is a model with geometric time discounting, then perhaps we should have $T\to+\infty$; we use a finite T to avoid the nontrivial technical challenges of dealing with infinitely large matrices). Overall, T should be long enough both for the approximations involved in the frequency transformation to be correct and to accurately represent the time horizon of individual investors.

17 This follows from lemma 1 combined with the fact that $\Lambda'\Lambda=I$, so that $Q_i' D=Q_i'\Lambda'\Lambda D=q_i' d$.

In what follows, we solve the model using the approximation for $U_{0,i}$, and then show that it converges to the true solution from Admati (1985). When utility is completely separable across frequencies, there is an equilibrium frequency by frequency:

Solution 1. Under the approximations $d \sim N(0, diag(f_D))$ and $z \sim N(0, diag(f_Z))$, the prices of the frequency-specific portfolios, p_j, satisfy, for all j

$$p_j = a_{1,j} d_j + a_{2,j} z_j \tag{23}$$

$$a_{1,j}, \equiv 1 - \frac{\rho^{-1} k + f_{D,j}^{-1}}{\left(\rho f_{avg,j}^{-1}\right)^2 f_{Z,j}^{-1} + f_{avg,j}^{-1} + f_{D,j}^{-1} + \rho^{-1} k}, \tag{24}$$

$$a_{2,j} \equiv \frac{a_{1,j}}{\rho f_{avg,j}^{-1}}, \tag{25}$$

where $f_{avg,j}^{-1} \equiv \int_i f_{i,j}^{-1} di$ is the average precision of the agents' signals at frequency j.

Proof. See Appendix B. ■

The price of the frequency j portfolio depends only on fundamentals and supply at that frequency due to the independence across frequencies. As usual, the informativeness of prices, $Var\left[d_j \mid p_j\right]$ can be shown to increase in the precision of the signals that investors obtain, while the impact of noise trader demand on prices is decreasing in signal precision and risk tolerance.

These solutions for the prices are standard results for scalar markets. What is different here is simply that the agents chose exposures across frequencies, rather than across dates; p_j is the price of a portfolio whose exposure to fundamentals fluctuates over time at frequency $2\pi \lfloor j/2 \rfloor / T$. Both prices and demands at frequency j only depend on signals and supply at frequency j. The problem is completely separable across frequencies.

While solution 1 is an approximation, Online Appendix 2.2 shows that the error for the coefficients a_1 and a_2 and the prices is of order $T^{-1/2}$. In other words, the standard time-domain solution for stationary time-series processes becomes arbitrarily close to a simple set of parallel scalar problems in the frequency domain for large T.

In what follows, we assume that $ka_{2,j} < 1$ for all j, which ensures that z represents a positive demand shock in equilibrium. The restriction is that noise trader demand not be too sensitive to prices; in the literature k is usually equal to zero.

2.4 Optimal information choice in the frequency domain

The analysis so far takes the precision of the signals as fixed. Following Van Nieuwerburgh and Veldkamp (2009) and Kacperczyk, Van Nieuwerburgh, and

Veldkamp (2016), we allow investors to choose their signal precisions, Σ_i^{-1} to maximize the expectation of their mean-variance objective (3) subject to an information cost,[18]

$$\max_{\{f_{i,j}\}} E_{-1}\left[U_{i,0} \mid \Sigma_i^{-1}\right] - \frac{\psi}{2T} tr\left(\Sigma_i^{-1}\right), \tag{26}$$

where E_{-1} is the expectation operator on date -1, that is, prior to the realization of signals and prices (as distinguished from $E_{i,0}$, which conditions on P and Y_i), and ψ is the per-period cost of information. Total information here is measured by the trace operator $tr\left(\Sigma_i^{-1}\right)$. Note that while Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) focus on the case in which investors have a fixed budget of precision, we are studying the dual problem in which information comes at a constant marginal cost. This can be thought of as a case in which an investment firm can choose how many analysts to hire at a fixed wage, with total precision scaling linearly with the number of analysts.[19]

Given the optimal demands, an agent's expected utility is linear in the precision they obtain at each frequency.

Lemma 2. Each informed investor's expected utility at time -1 may be written as a function of their own signal precisions, $f_{i,j}^{-1}$, and the average across other investors, $f_{avg,j}^{-1} \equiv \int_i f_{i,j}^{-1} di$, with

$$E_{-1}\left[U_{0,i} \mid \{f_{i,j}\}\right] = \frac{1}{2T} \sum_{j=1}^{T} \lambda_j \left(f_{avg,j}^{-1}\right) f_{i,j}^{-1} + \text{constant}, \tag{27}$$

where the constant does not depend on investor i's precision, and the functions λ_j satisfy $\lambda_j(x) > 0$ and $\lambda_j'(x) < 0$ for all $x \geq 0$ and all j.

Proof. See Online Appendix 2.3. ■

The terms $\lambda_j\left(f_{avg,j}^{-1}\right)$ represent the marginal utility of precision at each frequency, which the individual investor takes as given. Because expected utility and the information cost are both linear in the set of precisions that agent i chooses, $\left\{f_{i,j}^{-1}\right\}$, it immediately follows that agents purchase signals at whatever subset of frequencies has $\lambda_j\left(f_{avg,j}^{-1}\right) \geq \psi$.

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[18] The preferences can be equivalently written in terms of utility over terminal wealth, $W_{T,i}$. Specifically, maximization of $E_{-1}\left[-\rho^{-1}T^{-1}\log E_{0,i}\left[\exp\left(-\rho W_{T,i}\right)\right] \mid \Sigma_i^{-1}\right]$, where $E_{0,i}$ conditions on priors, agent i's signals, and prices, is equivalent to maximization of (26) because $U_{0,i} = \rho^{-1}T^{-1}\log E_{0,i}\left[\exp\left(-\rho W_{T,i}\right)\right]$.

[19] The constraint model corresponds to a world in which firms cannot expand the number of analysts that they employ, but rather shift them among tasks (frequencies). The cost model that we focus on represents a world in which firms are free to hire more analysts from an elastic supply. This is more relevant if the financial sector does not account for most of the employment of the people capable of doing research.

Solution 2. Information is allocated so that

$$f_{avg,j}^{-1}=\begin{cases}\lambda_j^{-1}(\psi) & \text{if } \lambda_j(0)\geq\psi,\\ 0 & \text{otherwise.}\end{cases} \tag{28}$$

Because attention cannot be negative, when $\lambda_j(0)\leq\psi$, no attention is allocated to frequency j. Otherwise, attention is allocated so that its marginal benefit and its marginal cost are equated. This result does not pin down precisely how any specific investor's attention is allocated; in this class of models, when information costs are not strictly convex, only the aggregate allocation of attention across frequencies is determinate. For the purposes of studying price informativeness, though, characterizing this aggregate allocation is all that is necessary. Solution 2 is the water-filling equilibrium of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016).

At this point, still no investors are explicitly "short-term" or "long-term."Investors can follow many different strategies, with different mixes of short- and long-term focuses. Even without any specialization to particular strategies, though, we now have sufficient structure to analyze the price effects of restrictions on the strategies that investors may follow.

3. Consequences of restricting investment for prices

This section focuses on the effects on prices of restrictions on the frequencies at which investment strategies can operate. It examines a particularly stark restriction that simply outlaws certain strategies. Specifically, we assume that investors are restricted to setting $\tilde{q}_{i,j}=0$ for j in some set $\mathcal{R}$. We leave the noise traders unconstrained, assuming that, like retail investors, they face different regulations from large and sophisticated institutions.

This restriction on exposures to the frequency portfolios reduces the number of degrees of freedom that an investor has in making choices. To see this intuitively, consider a model in which each time period is an hour, and T is a year, or 1,625 trading hours. A restriction that investors cannot invest at a frequency higher than a day (6.5 hours) would mean that they would go from a strategy with 1625 degrees of freedom to one with only 250. A pension that sets a portfolio once a quarter would have only four degrees of freedom. In that sense, then, a frequency restriction is similar to a shift from a continuous market to one with infrequent batch auctions, like in Budish, Cramton, and Shim (2015). Appendix C provides derivations of the results for the remainder of this section.

3.1 Results

3.1.1 Price informativeness across frequencies. In terms of frequencies, there is a complete separation in response to an investment restriction: prices become uninformative at restricted frequencies, while remaining unaffected at unrestricted frequencies.

Result 1. When investment by sophisticated investors is restricted at a set of frequencies $\mathcal{R}$, prices satisfy

$$p_j = \left\{ \begin{array}{c} k^{-1} z_j \text{ for } j \in \mathcal{R} \\ a_{1,j} d_j + a_{2,j} z_j \text{ otherwise} \end{array} \right\}, \tag{29}$$

where $a_{1,j}$ and $a_{2,j}$ are the same as those defined in solution 1.

Intuitively, when sophisticated investors are restricted at a particular frequency, prices only depend on sentiment, because the agents with information cannot express their opinions. Moreover, the market becomes illiquid, and it is cleared purely through prices rather than quantities. On the other hand, because the solution for information acquisition at a frequency j does not depend on anything about any other frequency, the information acquired at a frequency $j \notin \mathcal{R}$ is unaffected by the policy. As a result, we have the following corollary.

Corollary 1. When investors are restricted from holding portfolios with weights that fluctuate at some set of frequencies $j \in \mathcal{R}$, then prices at those frequencies, p_j, become completely uninformative about dividends. The informativeness of prices for $j \notin \mathcal{R}$ about dividends is unchanged. More formally, $Var\left[d_j \mid p_j\right]$ for $j \notin \mathcal{R}$ is unaffected by the restriction. For $j \in \mathcal{R}$, $Var\left[d_j \mid p_j\right] = Var\left[d_j\right]$.

So a policy that eliminates short-term investment, for example, by requiring holding periods of some minimum length, reduces the informativeness of prices for the short-term or transitory components of fundamentals, but has no effect on price informativeness in the long run.

3.1.2 Price informativeness across dates. The fact that prices remain equally informative at some frequencies does not mean that they remain equally informative for any particular *date*. Dates and frequencies are linked through a standard result:

$$Var(D_t \mid P) = \frac{1}{T} \sum_{j=1}^{T} Var\left[d_j \mid p_j\right]. \tag{30}$$

The variance of an estimate of fundamentals conditional on prices at a particular date is equal to the average of the variances across all frequencies. We then have the following:

Corollary 2. Investment restrictions reduce price informativeness for fundamentals on all dates by equal amounts, and by an amount that weakly increases with the number of frequencies that are restricted.

If a person is making decisions based on estimates of fundamentals from prices and they are worried that prices are contaminated by high-frequency noise due to a restriction on short-term investment, a natural response would be to examine an average of fundamentals and prices over time (across maturities of futures contracts).

Corollary 3. The informativeness of prices for the sum of fundamentals depends only on informativeness at the lowest frequency:

$$Var\left(T^{-1}\sum_{t=1}^{T} D_t \mid P\right) = Var\left[T^{-1/2} d_0 \mid p_0\right], \tag{31}$$

where d_0 is the lowest frequency portfolio—with equal weight each date—and p_0 is its price.

Corollary 3 immediately follows from the definition of d_0 and the independence across frequencies in the solution. It shows that the informativeness of prices for moving averages of fundamentals depends only on the very lowest frequency. So even if prices have little or no information at high frequencies—$Var\left[d_j \mid p_j\right]$ is high for large j—there need not be any degradation of information about averages of fundamentals over multiple periods, as they depend primarily on precision at lower frequencies (smaller values of j).

More concretely, going back to our example of oil futures, when investors are not allowed to choose exposure to the high-frequency portfolios, prices become noisier, making it more difficult to obtain an accurate forecast of the spot price of oil at some specific moment in the future. But if one is interested in the average of spot oil prices over a year, the model predicts that prices remain informative under restrictions on short-term strategies.

Thus, any restriction on investment reduces price informativeness for any particular date. But when short-term investment is restricted, there is little change in the behavior of moving averages of prices. So if a manager is making investment decisions based on fundamentals at a particular moment only, then those decisions will be hindered by the policy because prices now have more noise. But if decisions are made based on averages of fundamentals over longer periods, the model predicts that there need not be adverse consequences.

Finally, it is natural to examine the informativeness of *differences* in prices across dates. As an example, we can consider the variance of the first difference of fundamentals.

Corollary 4. The variance of an estimate of the change in fundamentals across dates conditional on observing the vector of prices is

$$Var\left[D_t - D_{t-1} \mid P\right] = \sum_{j=1}^{T} 2\left(1-\cos\left(\omega_{\lfloor j/2 \rfloor}\right)\right) Var\left[d_j \mid p_j\right]. \tag{32}$$

The function $2(1-\cos(\omega))$ is equal to 0 at $\omega=0$ and rises smoothly to 4 at the highest frequency, $\omega=\pi$. So period-by-period changes in fundamentals are driven primarily by high-frequency variation. As a result, policies restricting short-term investment will tend to have relatively large effects on the informativeness of prices for *changes* in fundamentals, as opposed to their limited effects on moving averages.[20]

3.1.3 Return volatility.

Corollary 5. Given an information policy $f_{avg,j}^{-1}$, the variance of returns at frequency j, $r_j \equiv d_j - p_j$, is

$$Var\left(r_j\right)=\begin{cases} f_{D,j}+\frac{f_{Z,j}}{k^2} \text{ for } j\in\mathcal{R} \\ \min\left(\psi,\lambda_j(0)\right) \text{ otherwise}\end{cases}. \tag{33}$$

Moreover, the variance of returns at restricted frequencies satisfies $Var(r_j) > f_{D,j}+\frac{f_{Z,j}}{(k+\rho f_{D,j}^{-1})^2}$, which is the variance that returns would have at the same frequency if investment were unrestricted but agents were uninformed.

The volatility of returns at a restricted frequency is higher than it would be if the sophisticated investors were allowed to trade, even if they gathered no information. When uninformed active investors have risk-bearing capacity ($\rho > 0$), they absorb some of the exogenous demand by simply trading against prices, buying when prices are below their means and selling when they are above. The greater is the risk-bearing capacity, the smaller is the effect of sentiment volatility on return volatility. Thus, the restriction affects return volatilities through its effects on both liquidity provision and information acquisition.

Restricting sophisticated investors from following short-term strategies in this model can thus substantially raise asset return volatility in the short run. Doing so can lead to, for example, large day-to-day fluctuations in prices (though those fluctuations in prices are, literally, variations in prices across maturities for different futures contracts on date 0). When unrestricted, sophisticated traders smooth prices across maturities, intermediating between excess demand on one day and excess supply in the next. When they are restricted from holding positions in futures that fluctuate from day to day, they can no longer provide that intermediation service, and short-term volatility increases. Restricting short-term investment increases transitory price volatility in this manner.

[20] Conversely, reductions in price informativeness at low frequencies have relatively large effects on moving averages and small effects on changes. This result can be relevant in situations in which long-term investment is restricted, like in the case of a trading desk that cannot have exposure to cycles lasting longer than a day (e.g., Brock and Kleidon 1992; Menkveld 2013).

3.2 Pricing equity

Equity is a claim on the entire future stream of fundamentals, so, in the model, we define it to be a claim that pays D_t on each date t. The payoff of an equity claim is simply the sum of fundamentals, so the date-1 equity claim has a payoff of exactly d_0. Corollary 3 then says that the absolute level of the price of equity remains equally informative under a short-run investment restriction, like in the unrestricted case. That result is natural: if only short-run investment is restricted, then long-run investors, who simply buy and hold equity, are unaffected and can continue to maintain price efficiency.

However, that does not mean that equity prices are unaffected by the restriction. In particular, while the level of equity prices on an individual date remains equally informative, *changes* in equity prices over time are not. In particular, note that

$$P_t^{equity} - P_{t+1}^{equity} = P_t, \tag{34}$$

where $P_t^{equity} \equiv \sum_{j=0}^{\infty} P_{t+j}$ is the price of equity on date t. The difference between equity prices between dates t and $t+1$ is exactly equal to the price of the single-period dividend claim on date t. That is because a strategy that holds equity on date t but then immediately sells it on date $t+1$ only actually has exposure to fundamentals on date t.

So when restrictions on short-term investment make the prices of the individual futures claims less informative, they also make changes in the value of equity over time less informative. The results above for the informativeness of individual futures claims map directly into informativeness of differences in equity prices across dates.

3.3 Numerical example

We now examine a numerical example to illustrate the predictions of the model for the behavior of investor positions, prices, and returns, both with and without restrictions on investment. Online Appendix 3 reports the details of the analysis and further results.

The length of a time period is set to a week.[21] The spectrum of fundamentals, f_D, is calibrated to match the features of dividend growth for the CRSP total market index. Dividends are nonstationary in the data, so the numerical calibration assumes that ΔD_t is stationary, so that the individual futures are claims to dividend growth.[22] Online Appendix 5 shows that the frequency analysis and theoretical results are essentially identical to those of the baseline

[21] As discussed above, the model is not intended to match subsecond scale features of financial markets, like limit order books and exchange fragmentation. It could be plausibly applied to a daily or perhaps hourly frequency. Here, we choose a week, which is the highest frequency at which aggregate economic indicators are released (specifically, initial claims for unemployment).

[22] Technically, the spectrum, f_D, is fit to the change in log dividends in calculating our calibration, but in the analysis that follows, we take f_D as applying to the first difference of the level of dividends.

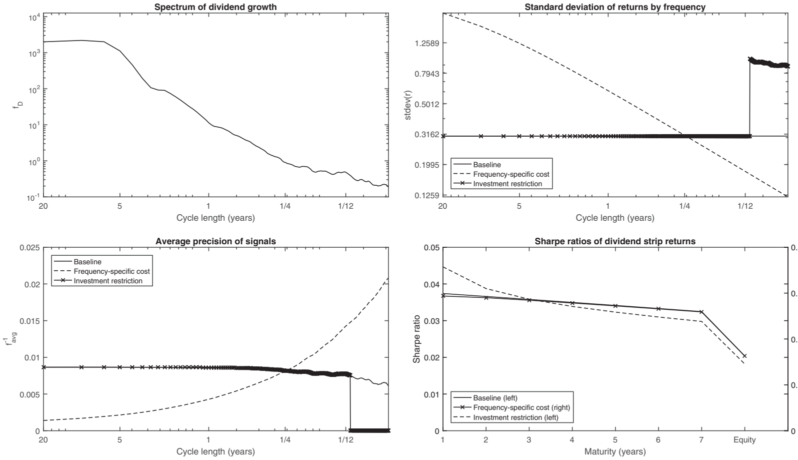

Figure 2
Numerical example.
The four panels plot results for the numerical example. The frequency-specific cost case is where the cost of information at frequency ω_j is proportional to $(\omega_j+\omega_1)^{-1}$. The investment restriction says that sophisticated investors cannot hold portfolios that uctuate at frequencies corresponding to cycles lasting 1 month or less. The Sharpe ratios for the dividend strip returns are calculated based on per-period returns as discussed in Online Appendix C and then annualized.

case.[23] The top-left panel of Figure 2 plots the calibrated spectrum for dividend growth, f_D. Empirically, there is substantial persistence in dividend growth, which causes f_D to peak at low frequencies.[24]

The top-right panel of Figure 2 plots the variance of returns on the dividend claims at each frequency, both with and without a restriction on investment at frequencies corresponding to cycles lasting less than one month ($\omega \geq 2\pi/4$), which could be thought of as similar to a tax on very short-term capital gains. Consistent with the results above, the variance of returns rises substantially at the restricted frequencies.

In addition to the benchmark case in which each frequency is equally difficult to learn about, we also consider an alternative specification for the information cost in which the cost of precision increases as the frequency falls. Formally, in the benchmark specification, the total cost of information is $\sum_j \psi f_{i,j}^{-1}$, and the alternative uses the generalization $\sum_j \psi_j f_{i,j}^{-1}$, with $\psi_j \propto \left(\omega_j+\omega_1\right)^{-1}$.[25] That specification has two uses. First, it illustrates what would happen if a regulator

23 See also Section 5 for a discussion of this case.

24 We set $\rho=57.8$, $k=0.2$, and f_Z to be one-eighth of the smallest value of f_D. The qualitative features of the model, as demonstrated in the results above, are not sensitive to those choices.

25 The average cost of information is set in this specification so that total information acquisition is equal to the baseline case, just shifted to higher frequencies.

taxed or subsidized information acquisition differentially across frequencies. Second, it will help match the empirical behavior of dividend strip variances.

The top-right panel shows that the consequence of that change is to cause the variance of returns to rise at low frequencies. The bottom-left panel of Figure 2 plots the average precision of the signals obtained by investors at each frequency. Under the investment restriction, the precision goes to zero, because the information becomes useless. When information costs vary across frequency, so does information acquisition, and approximately inversely to the cost.

Finally, the bottom-right panel of Figure 2 plots annualized Sharpe ratios of dividend strips at maturities of 1 to 7 years along with the equity claim (i.e., the claim to all dividend strips to maturity T) under the three different information policies.[26] The dividend strips are modeled as claims to the level of dividends on a given date in the future. Because ΔD is stationary here, a claim to D_t is equal to a claim to $\sum_{s=1}^{t} \Delta D_s$. We assume that there is a unit supply of equity, which induces positive average returns on claims to dividends. Because ΔD_1 affects the level of dividends on every date in the future, while ΔD_T affects only the level of dividends on date T, there is effectively greater supply of the shorter-maturity dividend claims, meaning that they earn higher returns in equilibrium, consistent with the findings of Binsbergen and Koijen (2017) and inducing downward-sloping Sharpe ratios.

In the benchmark case in which investors acquire information at all frequencies, returns have the same variance at all frequencies and horizons, which is inconsistent with the data in Binsbergen and Koijen (2017). The cost specification that increases at low frequencies causes the variance curve to slope upward strongly with frequency, generating more strongly downward-sloping Sharpe ratios, both of which are consistent with the results reported by Binsbergen and Koijen (2017). That result is obtained because the variances of the dividend strips depend on lower frequencies when their maturities are longer. Figures A.2 and A.3 in the Online Appendix report further results and compare the model to the data reported by Binsbergen and Koijen (2017).

The previous section argues that while a restriction on short-term investment does not affect the informativeness of the level of equity prices on date 1, it does affect the informativeness of differences across dates. Table 1 reports informativeness for both the level and the various changes in equity prices over time. For the level, informativeness is measured as the increase in precision from observing prices,

$$\log\left(\frac{\operatorname{var}\left[\sum_{t=1}^{T} D_t \mid P_1^{equity}\right]^{-1}}{\operatorname{var}\left[\sum_{t=1}^{T} D_t\right]^{-1}}\right). \tag{35}$$

[26] See van Binsbergen, Brandt, and Koijen (2012), Belo, Collin-Dufresne, and Goldstein (2015), **?**, and Hasler and Marfe (2016). Binsbergen and Koijen (2017) empirically study dividend strips with maturities of 1 to 7 years.

Table 1
Information losses relative to benchmark. Log difference in precision of functions of equity prices for fundamentals between the baseline model and the short-term restriction or alternative cost function that is high at low frequencies. The first row is for the level of the equity price on date 1. The second is for the difference in the price of equity in week 1 and week 2 (which isolates just the dividend in the first week). The third and fourth rows are for the difference between the price of equity on the first date and 1 month and 1 year later, respectively. Finally, the bottom row is the efficiency of $P_9^{equity} - 2P_5^{equity} + P_1^{equity}$, which measures the change in price growth across the first and second months.

	Short-term rest.	Low-freq. cost
Equity price	0.00	4.06
1-week difference	1.59	0.00
1-month difference	0.46	1.08
1-year difference	0.05	2.89
Monthly second difference	0.61	−0.11

Similarly, for the k-period change in equity prices, we report

$$\log\left(\frac{\operatorname{var}\left[\sum_{t=1}^{T} D_t - \sum_{t=k+1}^{T} D_t \mid P_{k+1}^{equity} - P_1^{equity}\right]^{-1}}{\operatorname{var}\left[\sum_{t=1}^{T} D_t - \sum_{t=k+1}^{T} D_t\right]^{-1}}\right). \qquad (36)$$

These measures of price informativeness map to the empirical measures of Bai, Philippon, and Savov (2016), who measure price informativeness across horizons based on the fraction of the variation in earnings explained by stock prices (see also Dávila and Parlatore 2018).

Table 1 shows that the level of equity prices is no less efficient under the short-term investment restriction while the difference in equity prices between the first and second weeks is substantially less efficient. As the length of the difference gets longer, so that it focuses on lower frequencies, the efficiency rises back toward the baseline. Finally, looking at a second difference, which measures the change in price growth across two periods, isolating higher frequencies, the short-term restriction again has measurable effects (see Corollary 4).[27]

4. Investor Outcomes

This section studies the consequences of investment restrictions on investor profits and utility. We obtain two main results, which initially appear to be in conflict:

1. A rise in short-term investment, due to a decline in the cost of high-frequency information or trading, reduces the profits and utility of long-term investors.

[27] The case in which low frequencies are more costly to learn about, for example, because of a tax on low-frequency information acquisition or a subsidy to high-frequency acquisition, leads to precisely the opposite effects.

2. However, restricting short-term investment in the way studied in section 3 only serves to *further* reduce the profits and utility of long-term investors.

Although long-term investors are worse off when short-term investment rises, cutting off short-term investment strategies—the ability to rapidly turn over portfolios—doesn't restore the old equilibrium or make the long-term investors better off. To achieve those goals, policies that change the cost of information acquisition are more suitable, as we will show.

The last part of the section examines the implications of the possible policy responses for noise traders, finding that noise traders are best off when prices are most informative.

4.1 Who are short- and long-term sophisticated investors?

We define a short-term investor as one whose portfolio is driven relatively more by high-frequency fluctuations, while a long-term investor holds a portfolio that is driven relatively more by low-frequency fluctuations. That definition can be formalized by a variance decomposition, using the facts

$$Var\left(\tilde{Q}_{i,t}\right)=\sum_{j=1}^{T}Var\left(\tilde{q}_{i,j}\right) \tag{37}$$

$$\text{and } \frac{d}{df_{i,j}^{-1}}\left[Var\left(\tilde{q}_{i,j}\right)\right]>0 \tag{38}$$

The component of the variance of $\tilde{Q}_{i,t}$ that is driven by fluctuations at frequency $j, Var\left(\tilde{q}_{i,j}\right)$, is increasing in the precision of the signals agent i acquires at frequency $j(f_{i,j}^{-1})$. So if two investors have the same total variance of their positions, $Var\left(\tilde{Q}_{1,t}\right)=Var\left(\tilde{Q}_{2,t}\right)$, but one of them has higher-precision signals at high frequencies, that is, $f_{1,j}^{-1} > f_{2,j}^{-1}$ for j above some cutoff, then variation in that investor's position is driven relatively more by high-frequency components.

For two investors with positions that have the same unconditional variance, the short-term investor—whose fluctuations happen relatively faster —is the one with relatively more precise signals about the transitory or high-frequency features of fundamentals. That is, *short-term investors have short-term/high-frequency information, and long-term investors have long-term/low-frequency information.* In the context of the example above, the short-term investor would be investor 1. As an extreme case—which is a simplification of the world for the sake of theoretical clarity—we assume short-term investors receive signals withpositive precision only for j above some cutoff j_{HF}, and long-term investors receive signals with positive precision only for j below some j_{LF} with $j_{HF} > j_{LF}$.

4.2 Investor profits and utility

Result 2. Let $R \equiv D - P$ be the vector of returns in the time domain. Investor i's average discounted profits are

$$E_{-1}\left[\tilde{Q}_i' R\right] = \sum_{j=1}^{T} (1 - ka_{2,j})\left(-E_{-1}\left[z_j r_j\right]\right) + ka_{1,j} E_{-1}\left[r_j d_j\right]$$

$$+\rho\left(f_{i,j}^{-1} - f_{avg,j}^{-1}\right) Var_{-1}\left[r_j\right], \tag{39}$$

and expected profits at each frequency are nonnegative,

$$E_{-1}\left[\tilde{q}_{i,j} r_j\right], \geq 0 \text{ for all } i, j \tag{40}$$

with equality only if $f_{i,j}^{-1} = 0$ and $f_{D,j}^{-1} = \rho f_{avg,j}^{-1} f_{Z,j}^{-1} k$ (i.e., in a knife-edge case). Finally, the average earnings of noise traders are

$$E\left[\sum_{t=1}^{T} \tilde{N}_t R_t\right] = \sum_j \underbrace{\left((1 - ka_{2,j}) z_j - ka_1 d_j\right)}_{\text{Noise trader demand}} \underbrace{\left((1 - a_{1,j}) d_j - a_{2,j} z_j\right)}_{\text{Returns}} \tag{41}$$

$$= -\sum_j \left[a_{2,j}\left(1 - ka_{2,j}\right) f_{Z,j} + ka_{1,j}\left(1 - a_{1,j}\right) f_{D,j}\right] \tag{42}$$

Each investor's expected discounted profits depend on three terms. The first represents the profits earned from noise traders. $E\left[z_j r_j\right] = -a_{2,j} f_{Z,j}^{-1} < 0$ because the sophisticated investors imperfectly accommodate their demand. When the noise traders have high demand (that is, when z is high), they drive prices up and expected returns down. The sophisticated investors earn profits from trading with that demand.[28]

The second term represents the profits that the informed investors earn by buying from the noise traders when they have positive signals on average. The coefficient $ka_{1,j}$ represents the slope of the supply curve that the informed investors face.

Finally, the third term in (39) represents a reallocation of profits from the less to the more informed sophisticated investors. An investor who has highly precise signals about fundamentals at frequency j can accurately distinguish periods when prices are high due to strong fundamentals from those when prices are high due to high sentiment. That allows them to earn relatively more profits on average than an uninformed investor.

[28] Note here that we are referring to flow profits, which do not include the cost of information acquisition that investors pay on date −1. We do this partly because flow profits are more readily measurable than the potential fixed costs of setting up information acquisition technologies, and also because flow profits are still relevant in the case in which investors face a constraint on information instead of a cost, or where the cost is in terms of utility units instead of money.

That said, an uninformed sophisticated investor does not earn negative expected profits at any frequency, even with $f_{i,j}^{-1}=0$. Profits always can be earned by trading with noise traders, except in the knife-edge case. Intuitively, this result follows from the separability of the problem across frequencies. The reason that an investor must always earn nonnegative expected profits is that if at some frequency j they did not, then they could simply set $q_{i,j}=0$, ensuring profits of zero and hence higher utility. An uninformed investor forecasts returns based only on prices, so that knife-edge case represents the condition under which prices alone have no forecasting power for returns, and they set $q_{i,j}=0$ in all states.

Result 2 yields two key insights. First, all investors, no matter their information, have the ability to earn profits at all frequencies through liquidity provision. Second, all else equal, investors who are informed about a particular frequency earn the most money from investing at these frequencies. Short-term investors—those with relatively more information about high-frequency fundamentals—earn relatively higher returns at high frequencies, whereas long-term investors earn relatively higher returns at low frequencies.

Finally, the decomposition of earnings for noise traders, equation (42), comes from the fact that the returns noise traders earn must be exactly the opposite of what the informed investors earn on average (i.e., equation (39) with $f_{i,j}^{-1}=f_{avg,j}^{-1}$). Average noise trader earnings are quadratic in the coefficients determining prices, $a_{1,j}$ and $a_{2,j}$. That is caused by the interaction of two effects. First, when expected returns are more responsive to their demand shocks ($a_{2,j}$ is large) or to fundamentals ($1-a_{1,j}$ is large), then expected returns vary more, giving more potential for losses. However, variation in prices inhibits their trading, because they have downward-sloping demand curves, with a slope of k

4.3 Effects of an increase in short-term investment

This section studies the consequences of a decline in the cost of acquiring information at high frequencies for short- and long-term investors, as well as potential policy responses.

4.3.1 Effects on short- and long-term investors. Under the specification of the model where the total cost of information is $\sum_j \psi_j f_{i,j}^{-1}$ (where the baseline is the special case of $\psi_j=\psi$ for all j), the equilibrium condition for information acquisition is

$$f_{avg,j}^{-1}=\begin{cases} \lambda_j^{-1}\left(\psi_j\right) & \text{if } \lambda_j(0)\geq\psi_j, \\ 0 & \text{otherwise.} \end{cases} \tag{43}$$

(See Online Appendix 8.1 for a derivation of the other theoretical results in this case.) We examine the effects of a marginal reduction in ψ_j for $j>j_{HF}$ starting from some the point $\psi_j=\lambda_j(0)$, that is, exactly where reducing information costs will lead to an initial increase in information acquisition. The investors who acquire information at those frequencies (setting $f_{i,j}^{-1} > 0$ for $j > j_{HF}$) are then the short-term investors, while those who do not, leaving $f_{i,j}^{-1} = 0$ for $j > j_{HF}$, are the long-term investors.[29]

Corollary 6. Starting from a ψ_j such that no investors acquire information at frequency j, a decline in ψ_j that leads to an increase in the equilibrium $f_{avg,j}^{-1}$ reduces profits and utility of an investor for whom $f_{i,j}^{-1}$ remains unchanged. Specifically,

$$\frac{d}{d\psi_j} E_{-1}\left[\tilde{q}_{LF,j} r_j\right]\Big|_{\psi_j=\lambda_j(0),-} > 0 \tag{44}$$

$$\frac{d}{d\psi_j} E_{-1}\left[\sum_t \tilde{Q}_{LF,t}(D_t - P_t)\right]\Big|_{\psi_j=\lambda_j(0),-} > 0 \tag{45}$$

$$\frac{d}{d\psi_j} E_{-1}\left[U_{LF,0}\right]\Big|_{\psi_j=\lambda_j(0)^-} > 0, \tag{46}$$

where the notation $\psi_j = \lambda_j(0)^-$ indicates the derivative is taken to the left and the LF subscripts denote positions and utility of a long-term investor who keeps $f_{i,j}^{-1} = 0$ at the affected frequency. Concretely, in an economy initially populated only by long-term investors who gather no short-term information, a decline in the cost of short-term information increases $f_{avg,j}^{-1}$ for $j > j_{HF}$ and therefore reduces the expected profits at those frequencies, total expected profits, and the utility of long-term investors.

The source of that result is the fact that investors with low-frequency information may still invest in the short run (i.e., have exposures that change from day to day). Suppose, for example, that not only does $f_{LF,j}^{-1} = 0$ for high j but also that $f_{avg,j}^{-1}$ does; in this case, nobody has short-term information . In that setting obviously any sophisticated investor will be happy to accommodate transitory fluctuations in noise trader demand. More concretely, an investor who has information that the long-term value of a stock is \$50 will be willing to provide liquidity in the short run, buying when the price is below \$50 and selling when the price is higher. That liquidity provision will have high-frequency components when liquidity demand (noise trader demand) has high-frequency components (i.e., $f_{z,j} > 0$ for $j > j_{HF}$). That is, if there are short-run variations in sentiment, then there will be short-run variation in the low-frequency investor's position.

29 Here, we assume that only a fraction of investors begin acquiring high-frequency information, but another valid equilibrium is one in which all investors acquire high-frequency information. The model does not pin down the cross-sectional distribution. However, generating specialization endogenously can be easily done, for example, through second-order differences in information acquisition skill across frequencies.

The increase in short-term investment hurts those with low-frequency information because those with high-frequency information are better at providing short-term liquidity. Result 2 and Corollary 6 formalize that idea and show how short-term investors hurt long-term investors: by crowding out their ability to provide liquidity. We highlight a critical point here: result 2 still shows that the increase in high-frequency investment never drives the profits earned by long-term investors below zero, even at high frequencies.

Furthermore, none of this is suboptimal from the perspective of the long-term investors. At the equilibrium, all investors are indifferent between acquiring information and not at any frequency where $\lambda_j(0) > \psi_j$, so their utility is not increased by acquiring more information at high frequencies. Moreover, that indifference also means that the short-term investors—those who actually create the increase in $f_{avg,j}^{-1}$—also experience declines in expected utility. The decline in ψ_j, by increasing information acquisition, makes prices more efficient, leaving less scope for investors to predict returns and earn profits.

The results also do not change the incentives of low-frequency investors to acquire information at low frequencies. While they lose money from a decrease in liquidity provision at high frequencies, their choices at low frequencies are unaffected. Therefore, a decline in the cost of high-frequency information has no effect on price informativeness at low frequencies. There is also nothing special about analyzing a shift in ψ_j at high frequencies; the economic results are the same if the cost of information changes at any frequency.

Nevertheless, there is something of an arms race here in that investor profits are decreasing in the information acquired by other investors. When the cost of high-frequency information falls, somebody will acquire more information, and the investor who does not will earn lower trading profits going forward. That said, making the arms race idea fully formal would require modeling a speed tournament or some sort of imperfect competition, so the link is only stylized.

Finally, we also note that the reduction in information costs and increase in short-term investment has positive effects on the overall market:

Corollary 7. A reduction in ψ_j that increases $f_{avg,j}^{-1}$ for $j > j_{HF}$, increases price informativeness and reduces return volatility at those frequencies. That is, for any frequency

$$\left.\frac{d}{d\psi_j} Var\left[d_j \mid p_j\right]\right|_{\psi_j=\lambda_j(0)^-} \geq 0, \tag{47}$$

$$\left.\frac{d}{d\psi_j} Var\left[r_j\right]\right|_{\psi_j=\lambda_j(0)^-} \geq 0. \tag{48}$$

Although long-term investors may be hurt by the reduction in information costs at high frequencies, to a regulator whose goal is simply to maximize price informativeness or minimize return volatility, short-term information acquisition and investment are beneficial.

4.3.2 Policy responses. If a decline in the cost of high-frequency information or trading hurts the incumbent long-term investors, a natural question to the incumbents might be how to restore the old equilibrium. We consider three responses that have been proposed: restricting or eliminating short-term investment, taxing transactions (or variation in positions), and limiting the availability of short-term information.

First, consider a restriction on short-term/high-frequency investment that says that no sophisticated investor may set $q_{i,j} \neq 0$ for j above some cutoff, like in the previous section (and like in a daily batch auction). Result 2 implies that such a restriction would, rather than restoring the profits and utility of the long-term investors, actually reduce them *further*. This follows from the fact that restricting investment eliminates the terms in the summation for j above the cutoff, which are all nonnegative. While short-term investors make liquidity provision at high frequencies more difficult, outlawing short-term investment simply makes it impossible. Eliminating short-term investment actually compounds the effect of the increase in short-term investment, rather than restoring the old equilibrium.

Corollary 8. Limiting short-term investment with a policy that restricts all sophisticated investors from holding $q_{i,j} \neq 0$ for $j > j_{HF}$ weakly reduces the profits and expected utility of all sophisticated investors.

Imposing a tax on changes in positions, specifically, a tax on $\left(Q_{i,t} - Q_{i,t-1}\right)^2$, will have similar effects to a restriction on short-term investment in that the tax is most costly for short-term strategies with high turnover. Online Appendix 6 formalizes that intuition.

The final policy response would be to somehow limit the acquisition of high-frequency information. In the context of the model, that would represent a restriction on the ability of investors to learn about period-to-period variation in fundamentals, for example by making it more costly to acquire high-frequency information. The most obvious response to a decline in the cost at frequency j is to directly impose a tax that exactly reverses the decline.

In the context of the model, a restriction on information acquisition could, in fact, exactly restore the equilibrium that exists in the absence of the short-term investors. Long-term investors do not acquire high-frequency information, so the restriction does not directly affect them. In terms of the results above, the reason that short-term investors harm long-term investors in the model is that they increase $f_{avg,j}^{-1}$ for high values of j. A policy that makes short-term information more expensive does the opposite, reducing $f_{avg,j}^{-1}$ and shifting the market back to the previous equilibrium.

A specific example of a policy that could make it more costly for investors to acquire high-frequency information might be a reduction in the information that firms freely release. For example, there have been suggestions to change financial reporting requirements so that less short-run information is revealed proposed by the CFA institute (Schacht et al. 2007) and Brookings Institution (Pozen 2014). In the United Kingdom, quarterly earnings reports are no longer mandatory (Gigler et al. 2014). When firms stop reporting quarterly earnings, or providing short-term earnings guidance, they are in a sense making information acquisition more expensive. Instead of simply reading and interpreting announcements, investors now must research short-term performance to try to measure it.

To be clear, the claim here is not that markets should be tilted in the direction of long-term investors. Restricting information can help long-term investors in some cases, but it would also have potential negative externalities from reduced price informativeness that have been studied in the literature, making investment decisions worse, making monitoring of firms more difficult, and limiting firms' ability to tie managerial pay to performance (Bond, Edmans, and Goldstein 2012). Furthermore, the next section shows that restricting information in the model, even though it helps one class of investors, will hurt others.

4.4 Outcomes for noise traders

The formalization of noise traders used here is that they are investors whose demand depends on an uninformative signal that they erroneously believe forecasts fundamentals. Under that interpretation, a natural objective of a policy maker might be to set policies to try to reduce the losses of these investors and to keep their speculative demand from affecting prices and creating volatility (that is the motivation of the transaction tax in Tobin [1978]).

The decomposition of average earnings of noise traders, Equation (42), shows that there are two ways to drive their losses to zero. One is for prices to be completely informative, with $a_{1,j}=1$ and $a_{2,j}=0$ (i.e., $p_j=d_j$) in Equation (42). That case is ideal in that noise traders have no losses and prices are maximally useful as signals for making decisions. Noise trader losses are zero in this case: informed investors have perfectly elastic demand curves and will trade any quantity, because they know the price is exactly equal to fundamentals. The second way to reduce noise trader losses to zero is to drive $a_{1,j}$ to zero and $a_{2,j}=k^{-1}$. In that case, prices are completely uninformative, and they move in such a way that there is no trade. This achieves the goal of minimizing noise trader losses, but at the cost of eliminating all information from asset markets.[30]

The two policies examined above—restricting trade and restricting information—drive in the direction of the second way, to reduce noise trader losses. However, they differ in their effects on price informativeness. Next, we consider each policy separately.

[30] Note, though, that noise trader profits are nonmonotonic in both a_1 and a_2, so although we can draw conclusions about the extreme cases of a_1 and a_2 equal to 0 or 1, the exact response of profits to interior values of those coefficients is parameter dependent.

4.4.1 Consequences of restricting investment strategies. Restricting all investment by the informed investors at a given frequency eliminates all information from prices, but it also means that the noise traders have nobody to trade with, so their losses are identically zero. Similarly, restricting information, by reducing $f_{avg,j}^{-1}$ to zero, sets $a_{1,j}$ to zero, so that the noise traders have no losses because sophisticated investors have no information (the second part of Equation (42)). We also have

$$f_{avg,j}^{-1}=0 \Rightarrow 1-ka_{2,j}=\frac{f_D^{-1}}{\rho^{-1}k+f_D^{-1}}. \tag{49}$$

Noise traders will still lose money to informed investors, in general, through the first term in Equation (42). As the amount of fundamental uncertainty grows, though–f_D^{-1} shrinks—the losses eventually fall to zero.

So, unlike above, for the purpose of protecting noise traders, instead of long-horizon investors, the trading restriction is more effective than the information restriction. The information restriction does not in general reduce the losses of the noise traders to zero, while the trade restriction does. Either policy is only second best, though, in the sense that they help noise traders by reducing the informativeness of prices and increasing price volatility.

The policy of restricting investment is most natural at frequencies where $f_{Z,j}$ is large and $f_{D,j}$ is small. At such a frequency, the information loss from restricting investment is relatively small—in fact, it could be zero if $\lambda_j(0)$ is sufficiently small (because in the absence of the restriction information acquisition would not occur)—and the benefit, which increases in f_Z (Equation (42)), is relatively large. So restrictions on investment make the most sense at frequencies with little variation in fundamentals but substantial variation in sentiment or noise trader demand.

Corollary 9. At any frequency where $f_{Z,j}$ is sufficiently large or $f_{D,j}$ sufficiently small that $\lambda_j(0) \leq \psi$ (recall that $\lambda_j(0)$ represents the marginal value of acquiring information when $f_{avg,j}^{-1}=0$), there is no information acquisition in equilibrium and prices are completely uninformative. At those frequencies, restricting trade by mandating that $q_{i,j}=0$ reduces the losses of noise traders to zero and has no effect on price efficiency, because prices are already uninformative.

A common view among economists and policy makers is that there is relatively little important economic news at high frequencies because economic decisions, such as physical investment, depend on relatively long-term expectations. In such a case, one would think that $f_{D,j}$ is small at high frequencies. The results here then show that it would be natural to restrict high-frequency investment because there is no information loss and the effects of noise trader demand or speculation are eliminated. The model here formalizes that common intuition.

4.4.2 Consequences of subsidizing information. On the other hand, if the goal was to reduce noise trader losses without any reduction in price informativeness, then the ideal policy would be one that *increases* $f_{avg,j}^{-1}$. Specifically, as the quantity of information that investors acquire becomes infinite ($f_{avg,j}^{-1} \to \infty$), prices become completely informative, in that $a_{1,j} \to 1$ and $a_{2,j} \to 0$, and noise traders have zero average losses,

$$\lim_{f_{avg,j}^{-1} \to \infty} E\left[n_j r_j\right] = 0. \tag{50}$$

Increases in $f_{avg,j}^{-1}$ could be encouraged by subsidizing or otherwise encouraging information production by investors (e.g., a tax credit for research). In the context of the discussion in the previous section, this corresponds to actively trying to reduce the ψ_j that investors face (ideally to zero, if the goal is to send $f_{avg,j}^{-1}$ to infinity). Certainly an information subsidy would not be costless to implement, and whether its benefits outweigh the costs is theoretically ambiguous.[31]

In the model, there is tension among the outcomes of short-term investors, long-term investors, and noise traders. Long-term investors benefit from reductions in $f_{avg,j}^{-1}$ at high frequencies, but that comes at the cost of reducing price informativeness and hurting noise traders and short-term investors. Noise traders benefit from increasing $f_{avg,j}^{-1}$ (when $f_{avg,j}^{-1}$ is sufficiently large, at least), or mandating greater disclosure about fundamentals, but that hurts the informed investors in general, because their trading opportunities shrink. These results follow from the zero-sum property of the payoffs in the game that the investors are playing. For policy makers wishing to promote price efficiency, encouraging greater information acquisition and higher $f_{avg,j}^{-1}$ is ideal.

5. Robustness and alternative specifications

The appendix reports results for a number of perturbations to the assumptions in the analysis discussed so far. We summarize them here.

First, the baseline analysis assumes that fundamentals are stationary in levels. Online Appendix 5 shows that the results are nearly identical when fundamentals and noise trader demand are instead stationary in first differences. In particular, all the frequency portfolios take precisely the same form as in the level-stationary case, except for the very lowest frequency. That means that the analysis above also goes through identically, except for a small change at that single frequency.

Second, there are alternative specifications for the information acquisition problem that investors face. In the first, investors have a fixed budget of

[31] A closely related policy—and one that might be cheaper than a subsidy for research—is to mandate greater information production by firms, such as more frequent or thorough earnings announcements. Online Appendix 4 examines a version of the model in which a public signal is revealed on date 0, and the next section discusses the results.

information instead of access to unlimited amounts of information at a constant cost (see Online Appendix 8.2). In that case, because the total supply of information is constant, restrictions on trade cause reallocations across frequencies; for example, while restricting high-frequency investment always reduces high-frequency informativeness, in this case it can also increase the informativeness of prices at low frequencies. This leads to ambiguity about the effects of such restrictions on price informativess in the time domain and on investor profits and utility.

The second alternative information specification is to assume that the cost of information depends on the entropy of the signals, rather than their precision. Online Appendix 8.3 shows that the main results continue to hold in that case and that the entropy specification also helps match the behavior of dividend strip returns.

The third potential modification of the model is to treat the noise trader demand as being driven by liquidity shocks rather than sentiment shocks. While the basic equilibrium is identical in this case to the baseline, some of the results on investor utility change. With rational liquidity demand, there are gains from trade that do not exist in the zero-sum baseline model. Online Appendix 9 reports results for this case.

Finally, in the analysis of information restrictions, we suggested that a decrease in mandatory reporting might constitute an increase in the cost of information. An alternative interpretation is that it would represent a decline in the precision of a public signal that all investors can freely observe. Online Appendix 4 examines that case and shows that an increase in public disclosures can reverse the declines in utility and profits following a decline in the cost of information. The effects of disclosure therefore depend on modeling choices, which need to be evaluated empirically. The model is about costly information acquisition and processing, so it is somewhat inconsistent with the general approach to assume that it is costless for investors to interpret, for example, financial statements. Nevertheless, the basic pattern of the effects is the same, in that both a tax on information acquisition, which raises ψ_j, and an increase in the precision of public signals, raise the profits of low-frequency investors and noise traders and reduce the profits of high-frequency investors. A public signal has the added advantage of increasing price informativeness.

Online Appendix 6 examines an extension of the model with quadratic trading costs. Those costs can be viewed as either technological or representing a transaction tax implemented by a regulator. It shows that an increase in trading costs affects high frequencies more strongly than low frequencies, causing investors to acquire less information at high frequencies and reduce high-frequency investment.

6. Conclusion

The aim of this paper is to understand the effects of policies aimed at reducing "short-termism" in financial markets. It develops results on the effects on price informativeness and investor welfare of restrictions on investment and information acquisition at different frequencies. To study those questions, we develop a model in which investors can make meaningful decisions about the horizon of their investment strategies and in which they face endogenous information choices.

We obtain three main results in the baseline specification of the model:

1. Restricting short-term investment affects short-run, but not long-run, price informativeness and return volatility.
2. Restricting short-term investment hurts both short- and long-term investors, but helps noise traders.
3. Taxing or restricting the availability of short-term information helps long-term investors, hurts short-term investors and noise traders, and reduces short-term price efficiency.

The first result is a natural consequence of the statistical independence of the model across frequencies. The second result shows that, while lower costs of acquiring high-frequency information reduce the utility and profits of long-term investors, restricting short-term *investment* in response does not make long-term investors better off. A buy-and-hold investor is able to provide the market short-term liquidity. A person with a price target of $50 should be willing to accommodate transitory demand shocks that drive the price above their target. But when high frequency information is cheaper, more investors chose to focus on short-term strategies. These short-term investors are also better at providing short-term liquidity than long-term investors; this is what makes long-term investors worse off. However, eliminating all short-term investment does not solve the problem. In fact, it makes it worse by eliminating the earnings from liquidity provision for all investors. However, the results for noise traders are reversed: they benefit from restrictions on investment and are hurt by limits on information.

Finally, the third result shows that information policies have distributional effects. If one's goal is to both maximize price informativeness and limit the impact of speculation by noise traders, subsidizing information acquisition can potentially (if the subsidy is sufficiently strong) solve both of those problems. However, because there is not a single accepted model of noise trading, the third result is relatively more delicate. We also examine an alternative specification in which noise traders are replaced by investors with time-varying hedging demand. In that case, it is more difficult to obtain clear predictions for welfare, but the first two main results continue to hold. Furthermore, price efficiency may have positive externalities that are not modeled here, as discussed in Bond, Edmans, and Goldstein (2012).

We do not make normative claims about what the right objective is. Many externalities are not considered here. For example, price informativeness is important to many agents in the economy who are not represented in our model. We also have a specific model of noise traders as irrational agents, but the role of noise trader demand in facilitating trade also can be played by agents who simply have exogenous liquidity needs, in which case the optimal policy response would more clearly tilt toward information subsidies. It is also not obvious whether short- or long-term investors should necessarily be supported. The goal of the paper is not to resolve the question of which policy is best, but rather simply to provide a general analysis of the effects of the various policies.

A. Noise Trader Demand

We assume that noise traders have preferences similar to those of sophisticates, but they have different information. They receive signals about fundamentals, and believe that the signals are informative, although the signals are actually random. The signals are also perfectly correlated across the noise traders, so that they do not wash out in the aggregate. They can be therefore thought of as common sentiment shocks among noise traders. Furthermore, the noise traders assume that prices contain no information about fundamentals.

The noise traders optimize

$$\max_{\{N_t\}_{t=1}^T} T^{-1}\sum_{t=1}^{T}\beta^t N_t E_{0,N}[D_t-P_t]-\frac{1}{2}(\rho T)^{-1}Var_{0,N}\left[\sum_{t=1}^{T}\beta^t N_t(D_t-P_t)\right], \tag{A1}$$

where N_t is the demand of the noise traders and $E_{0,N}$ and $Var_{0,N}$ are their expectation and variance operators conditional on their signals.

We model the noise traders as being Bayesians who simply misunderstand the informativeness of their signals, and ignore prices. Their prior belief, before receiving signals, is that

$$D\sim N\left(0,\Sigma_N^{prior}\right). \tag{A2}$$

They then receive signals that they believe (incorrectly) are of the form

$$S\sim N\left(D,\Sigma_N^{signal}\right). \tag{A3}$$

The usual Bayesian update then yields the distribution of D conditional on S,

$$D\,|\,S\sim N\left(\Sigma_N\left(\Sigma_N^{signal}\right)^{-1}S,\Sigma_N\right), \tag{A4}$$

$$\text{where }\Sigma_N\equiv\left(\left(\Sigma_N^{signal}\right)^{-1}+\left(\Sigma_N^{prior}\right)^{-1}\right)^{-1}. \tag{A5}$$

So we have

$$E_{0,N}[D]=\Sigma_N\left(\Sigma_N^{signal}\right)^{-1}S, \tag{A6}$$

$$Var_{0,N}[D]=\Sigma_N. \tag{A7}$$

Define $\tilde{N}_t\equiv\beta^t N_t$ and $\tilde{N}=[N_1,\ldots,N_T]'$. The optimization problem then becomes

$$\max_{\tilde{N}} T^{-1}\tilde{N}'\left(\Sigma_N\left(\Sigma_N^{signal}\right)^{-1}S-P\right)-\frac{1}{2}(\rho T)^{-1}\tilde{N}'\Sigma_N\tilde{N}. \tag{A8}$$

This has the solution:

$$\tilde{N}=\rho^{-1}\Sigma_N^{-1}\left(\Sigma_N\left(\Sigma_N^{signal}\right)^{-1}S-P\right), \tag{A9}$$

$$=\rho^{-1}\left(\left(\Sigma_N^{signal}\right)^{-1}S-\Sigma_N^{-1}P\right). \tag{A10}$$

For the sake of simplicity, we assume that $\Sigma_N=k^{-1}I$, where I is the identity matrix and k is a parameter. (This can be obtained, for instance, by assuming that $\Sigma_N^{signal}=\Sigma_N^{prior}=2kI$.) We then have

$$\tilde{N}=\rho^{-1}\left(\Sigma_N^{signal}\right)^{-1}S-kP, \tag{A11}$$

so that the vector $Z=(Z_1,\ldots,Z_T)'$ from the main text is

$$Z\equiv\rho^{-1}\left(\Sigma_N^{signal}\right)^{-1}S, \tag{A12}$$

and the true variance of S, Σ_S, always can be chosen to yield any particular $\Sigma_Z\equiv Var(Z)$ by setting

$$\Sigma_S=\rho^2\Sigma_N^{signal}\Sigma_Z\Sigma_N^{signal}. \tag{A13}$$

B. Derivation of Solution 1

To save notation, we suppress the j subscripts indicating frequencies in this section when they are not necessary for clarity. So in this section f_D, for example, is a scalar representing the spectral density of fundamentals at some arbitrary frequency (rather than vectors, which is what the unsubscripted variables represent in the main text).

In this section we solve a general version of the model that allows for a constant component of the supply, denoted s. This can be thought of as the mean aggregate supply of the underlying. The main results implicitly set $s=0$, but the analysis of equity returns uses nonzero s. We assume that the noise traders' demand curve depends on prices relative to their mean, so that supply does not enter. This is without loss of generality as it is simply a normalization.

B.1 Statistical inference

We guess that prices take the form

$$p=a_1d+a_2z+a_3s, \tag{B1}$$

where s is nonstochastic. Standard analysis then yields that

$$E[d\,|\,y_i,p]=\tau_i^{-1}\left(f_i^{-1}y_i+\frac{a_1}{a_2^2}f_Z^{-1}(p-a_3s)\right) \tag{B2}$$

The Online Appendix reports a full derivation.

B.2 Demand and equilibrium

The agent's utility function is (where variables without subscripts here indicate vectors)

$$U_i = \max_{\{Q_{i,t}\}} \rho^{-1} E_{0,i}\left[T^{-1}\tilde{Q}_i'(D-P)\right] - \frac{1}{2}\rho^{-2} Var_{0,i}\left[T^{-1/2}\tilde{Q}_i'(D-P)\right] \tag{B3}$$

$$= \max_{\{Q_{i,t}\}} \rho^{-1} E_{0,i}\left[T^{-1}\tilde{q}_i'(d-p)\right] - \frac{1}{2}\rho^{-2} Var_{0,i}\left[T^{-1/2}\tilde{q}_i'(d-p)\right] \tag{B4}$$

$$= \max_{\{Q_{i,t}\}} \rho^{-1} T^{-1} \sum_j \tilde{q}_{i,j} E_{0,i}\left[\left(d_j - p_j\right)\right]$$

$$-\frac{1}{2}\rho^{-2} T^{-1} \sum_j \tilde{q}_{i,j}^2 Var_{0,i}\left[d_j - p_j\right], \tag{B5}$$

where the last line follows by imposing the asymptotic independence of d across frequencies (the Online Appendix analyzes the error induced by that approximation). The utility function is thus entirely separable across frequencies, with the optimization problem for each $\tilde{q}_{i,j}$ independent from all others.

The remainder of the equilibrium computation then follows standard steps, which the Online Appendix fully reports.

C. Results on Price Informativeness with Restricted Frequencies

C.1 Result 1 and Corollaries 1 and 5

When there are no active investors and just exogenous supply, we have that $0 = z_j + kp_j$ and so

$$p_j = k^{-1} z_j, \tag{C1}$$

$$r_j = d_j - k^{-1} z_j. \tag{C2}$$

Because of the separability of information choices across frequencies, the coefficients $a_{1,j}$ and $a_{2,j}$ are unchanged at all other frequencies. Moreover, it is clear that $Var(d_j|p_j) = Var(d_j)$ at the restricted frequencies, because prices now only carry information about supply, which is uncorrelated with dividends.

Note that for any $j \in \mathcal{R}$,

$$Var(r_j) = f_{D,j} + \frac{f_{Z,j}}{k^2}. \tag{C3}$$

Additionally, if investors were allowed to hold *exposure* at those frequencies, but the endogenously chose not to allocate any *attention* to the frequency, the return volatility would be

$$Var_{unrestr.}(r_j) = \lambda_j(0) = f_{D,j} + \frac{f_{Z,j}}{\left(k + \rho f_{D,j}^{-1}\right)^2} < Var(r_j). \tag{C4}$$

C.2 Corollary 2 and result 4

Under the diagonal approximation, we have

$$D \mid P \sim N\left(\bar{D}, \Lambda diag\left(\tau_0^{-1}\right)\Lambda'\right) \tag{C5}$$

where τ_0 is a vector of frequency-specific precisions conditional on prices, as of time 0. Given the independence of prices across frequencies, the jth element of τ_0 is

$$\tau_{0,j}^{-1} = Var(d_j \mid p_j). \tag{C6}$$

Using this expression, we can compute:

$$Var(D_t \mid P) = 1_t' \Lambda diag\left(\tau_0^{-1}\right) \Lambda' 1_t \tag{C7}$$

$$= \left(\Lambda' 1_t\right)' diag\left(\tau_0^{-1}\right) \left(\Lambda' 1_t\right) \tag{C8}$$

$$= \sum_j \lambda_{t,j}^2 Var(d_j \mid p_j) \tag{C9}$$

$$= \lambda_{t,0}^2 Var(d_0 | p_0) + \lambda_{t,T}^2 Var(d_{\frac{T}{2}} \mid p_{\frac{T}{2}})$$

$$+ \sum_{k=1}^{T/2-1} \left(\lambda_{t,2k}^2 + \lambda_{t,2k+1}^2\right) Var(d_k \mid p_k), \tag{C10}$$

where 1_t is a vector equal to 1 in its tth element and zero elsewhere, and $\lambda_{t,j}$ is the t,j element of Λ. The last line follows from the fact that the spectrum has $f_{X,2k} = f_{X,2k+1}$ for $0 < k < T/2-1$. Furthermore, note that for $0 < k < T/2-1$,

$$\lambda_{t,2k}^2 + \lambda_{t,2k+1}^2 = \frac{2}{T}\cos(\omega_k(t-1))^2 + \frac{2}{T}\sin(\omega_k(t-1))^2 \tag{C11}$$

$$= \frac{2}{T}, \tag{C12}$$

which yields Equation (30). Result 3 immediately follows from this expression and the fact that $\lambda_{t,0}^2 = \lambda_{t,T}^2 = \frac{1}{T}$.

Result 4 uses the fact that

$$Var(D_t - D_{t-1} \mid P) = \left(\lambda_{t,1} - \lambda_{t-1,1}\right)^2 \tau_{0,1}^{-1} + \left(\lambda_{t,T} - \lambda_{t-1,T}\right)^2 \tau_{0,\frac{T}{2}}^{-1} \tag{C13}$$

$$+ \sum_{k=1}^{T/2-1} \left[\begin{array}{c} \left(\lambda_{t,2k} - \lambda_{t-1,2k}\right)^2 \\ + \left(\lambda_{t,2k+1} - \lambda_{t-1,2k+1}\right)^2 \end{array} \right] \tau_{0,k}^{-1}, \tag{C14}$$

and the fact that $(\cos(x) - \cos(y))^2 + (\sin(x) - \sin(y))^2 = 4\sin\left(\frac{1}{2}(x-y)\right)^2 = 2(1 - \cos(x-y))$.

D. Result 2

Expression (37) in the main text follows from the steps used in Appendix C.2. The Online Appendix shows that, omitting the j notation,

$$\tilde{q}_i = \rho\left(f_i^{-1} y_i + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) p\right) \tag{D1}$$

$$= \rho f_i^{-1} \varepsilon_i + \rho\left(f_i^{-1} + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) a_1\right) d + \rho\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) a_2 z. \tag{D2}$$

Recall also that

$$\tau_i = \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} + f_D^{-1} + f_i^{-1}, \tag{D3}$$

so that

$$\tilde{q}_i = \rho\left(\tau_i - \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} - f_D^{-1}\right)\varepsilon_i + \rho\left(f_i^{-1} + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right)a_1\right)d$$

$$+\rho\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right)a_2 z. \tag{D4}$$

Moreover,

$$f_i^{-1} - a_1\tau_i + \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} = \tau_i - \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} - f_D^{-1} - a_1\tau_i + \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} \tag{D5}$$

$$= (1-a_1)\tau_i - f_D^{-1}. \tag{D6}$$

Therefore,

$$\rho^{-1}\tilde{q}_i = \left(\tau_i - \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} - f_D^{-1}\right)\varepsilon_i + \left((1-a_1)\tau_i - f_D^{-1}\right)d + \left(\frac{a_1}{a_2} f_Z^{-1} - a_2\tau_i\right)z, \tag{D7}$$

so that

$$\rho^{-2}Var(\tilde{q}_i) = \left(\tau_i - \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} - f_D^{-1}\right) + \left((1-a_1)\tau_i - f_D^{-1}\right)^2 f_D + \left(\frac{a_1}{a_2} f_Z^{-1} - a_2\tau_i\right)^2 f_Z. \tag{D8}$$

(where the first term uses the fact that $Var\left(f_i^{-1}\varepsilon_i\right) = f_i^{-1}$). The derivative of this expression with respect to τ_i is

$$\begin{aligned}
\rho^{-2}\frac{\partial Var(\tilde{q}_i)}{\partial \tau_i} &= 2\tau_i\left((1-a_1)^2 f_D + a_2^2 f_Z\right) - 1 \\
&\geq 2\left(f_D^{-1} + \left(\tfrac{a_1}{a_2}\right)^2 f_Z^{-1}\right)\left((1-a_1)^2 f_D + a_2^2 f_Z\right) - 1 \\
&= 2\left((1-a_1)^2 + a_2^2 f_Z f_D^{-1} + (1-a_1)^2\left(\tfrac{a_1}{a_2}\right)^2 f_Z^{-1} f_D + a_1^2\right) - 1 \\
&= 2\left(1 - 2a_1(1-a_1) + a_2^2 f_Z f_D^{-1} + (1-a_1)^2\left(\tfrac{a_1}{a_2}\right)^2 f_Z^{-1} f_D\right) - 1 \\
&= 2\left(-2a_1(1-a_1) + a_2^2 f_Z f_D^{-1} + (1-a_1)^2\left(\tfrac{a_1}{a_2}\right)^2 f_Z^{-1} f_D\right) + 1 \\
&= 2\left((1-a_1)\left(\tfrac{a_1}{a_2}\right)(f_Z^{-1} f_D)^{\frac{1}{2}} - a_2(f_Z^{-1} f_D)^{-\frac{1}{2}}\right)^2 + 1 \\
&> 0,
\end{aligned} \tag{D9}$$

where to go from the first to the second line, we used the fact that $\tau_i \geq \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} + f_D^{-1}$, and where we also used the fact that $a_1 \leq 1$. Because τ_i is a monotonic transformation of f_i^{-1}, this establishes Equation (38) from the main text.

For result 2, first note that $E_{-1}\left[\tilde{Q}_i' R\right] = E_{-1}\left[\tilde{q}_i' \Lambda' \Lambda r\right] = E_{-1}\left[\tilde{q}_i r\right] = \sum_j E_{-1}\left[\tilde{q}_{i,j} r_j\right]$, where the last equality follows from the diagonal approximation. Moreover, straightforward but tedious algebra shows that

$$f_i^{-1} + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right)a_1 = \rho(f_i^{-1} - f_{avg}^{-1})(1-a_1) + ka_1, \tag{D10}$$

$$\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right)a_2 = -\rho(f_i^{-1} - f_{avg}^{-1})a_2 + (ka_2 - 1). \tag{D11}$$

We can use these expressions, and the fact that $r=(1-a_1)d-a_2 z$ to rewrite $\tilde{q}_i$ as

$$\tilde{q}_i=\rho f_i^{-1}\varepsilon_i+\rho\left(f_i^{-1}-f_{avg}^{-1}\right)r+ka_1 d+(ka_2-1)z. \tag{D12}$$

Therefore,

$$E_{-1}[\tilde{q}_i r]=\rho\left(f_i^{-1}-f_{avg}^{-1}\right)Var(r)+ka_1 E_{-1}[rd]+(ka_2-1)E_{-1}[rz], \tag{D13}$$

which is the decomposition from result 2.

The result that expected profits are nonnegative is a simple consequence of the investors' objective:

$$\max_{\{\tilde{q}_{i,j}\}}\rho^{-1}T^{-1}\sum_j E_{0,i}\left[\tilde{q}_{i,j}\left(d_j-p_j\right)\right]-\frac{1}{2}\rho^{-2}T^{-1}\sum_j Var_{0,i}\left[\tilde{q}_{i,j}\left(d_j-p_j\right)\right] \tag{D14}$$

Because the variance is linear in $\tilde{q}_{i,j}^2$, if $E_{0,i}\left[\tilde{q}_{i,j}r_j\right]<0$, utility can always be increased by setting $\tilde{q}_{i,j}=0$ (or, even more, by reversing the sign of $\tilde{q}_{i,j}$). For $E_{-1}\left[\tilde{q}_{i,j}r_j\right]=0$, it must be the case that $Var_{-1,i}\left[E_{0,i}\left[d_j-p_j\right]\right]=0$, because any deviation of $E_{0,i}\left[d_j-p_j\right]$ will cause the investor to optimally take a nonzero position. We have, from above,

$$a_1=\frac{\tau_{avg}-f_D^{-1}}{\tau_{avg}+\rho^{-1}k}=\frac{\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1}+f_{avg}^{-1}}{\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1}+f_{avg}^{-1}+f_D^{-1}+\rho^{-1}k} \tag{D15}$$

$$a_2=\frac{a_1}{\rho f_{avg}^{-1}} \tag{D16}$$

$$\tau_{avg}\equiv\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1}+f_{avg}^{-1}+f_D^{-1} \tag{D17}$$

The expression for a_2 is invalid in the case when $f_{avg}^{-1}=0$. In that case, we have

$$E[d\,|\,y_i,p]=\tau_i^{-1}\left(f_i^{-1}y_i+\frac{a_1}{a_2^2}f_Z^{-1}p\right) \tag{D18}$$

$$E[d-p\,|\,y_i,p]=\tau_i^{-1}f_i^{-1}y_i+\left(\tau_i^{-1}\frac{a_1}{a_2^2}f_Z^{-1}-1\right)(a_1 d+a_2 z) \tag{D19}$$

$$Var[E[d-p\,|\,y_i,p]]=\left(\tau_i^{-1}f_i^{-1}+\left(\tau_i^{-1}\frac{a_1}{a_2^2}f_Z^{-1}-1\right)a_1\right)^2 f_D$$
$$+\left(\tau_i^{-1}\frac{a_1}{a_2^2}f_Z^{-1}-1\right)^2 a_2^2 f_Z \tag{D20}$$

Now, first, we must have $\tau_i^{-1}\frac{a_1}{a_2^2}f_Z^{-1}-1=0$ for the third term to be zero. But if that is true, then, for the first term to be zero, we must have $f_i^{-1}=0$ (because τ_i^{-1} is always positive). Combining $f_i^{-1}=0$ with $\tau_i^{-1}\frac{a_1}{a_2^2}f_Z^{-1}-1=0$, we obtain

$$f_D^{-1}=\rho f_{avg}^{-1}f_Z^{-1}k. \tag{D21}$$

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