On the effects of restricting short-term investment*

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Abstract

We study the effects of policies proposed for addressing "short-termism" in financial markets. We examine a noisy rational expectations model in which investors' exposures and information about fundamentals endogenously vary across horizons. In this environment, taxing or outlawing short-term investment has no negative effect on the information in prices about long-term fundamentals. However, such a policy reduces the profits and utility of short- and long-term investors. Changing policies on the release of short-term information can help long-term investors – an objective of some policymakers – at the expense of short-term investors, but it also makes prices less informative and increases costs of speculation.

For decades economists and policymakers have expressed concern about the potentially negative effects of "short-termism" among investors in financial markets. Research has argued that short-term investors may increase the volatility and reduce the informativeness of asset prices (Froot, Scharfstein, and Stein (1992)), exacerbate fire sales and crashes (Cella, Ellul, and Giannetti (2013)), inefficiently incentivize managers to focus on short-term projects (Shleifer and Vishny (1990)), or reduce incentives of other investors to acquire information (Baldauf and Mollner (2017); Weller (2017)), making prices less informative overall.

Those who take the view that short-termism is bad for financial markets or the economy as a whole have proposed a broad array of policies to encourage long-term investment. One of the

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oldest proposals is the tax on transactions of Tobin (1978).[1] Some policies directly depend on holding periods, such as US tax treatment of capital gains and dividends, the SEC's most recent proxy access rules, the proposed Long-Term Stock Exchange, linking corporate voting rights to tenure, and the proposal of Bolton and Samama (2013) for corporations to explicitly reward long-term investors.[2] Budish, Cramton, and Shim (2015) propose to eliminate trade at the very highest frequencies by shifting markets from continuous operation to frequent batch auctions, and there have also been proposals to limit or eliminate quarterly financial reports and earnings guidance in the US, following similar changes in the UK, e.g. by Dimon and Buffett (2018).[3] A number of these policies were endorsed in a letter from 2009 signed by leaders in business, finance, and law.[4]

This paper theoretically evaluates the effect of policies targeting short-termism on price informativeness and investor outcomes. Unlike the previous literature, we consider a simple and very general setting with investors who are ex ante identical and then may *endogenously* specialize into different horizons. While there is some recent work on the consequences of various limits on information gathering ability and there have been empirical analyses of high-frequency traders, we are not aware of any other work that directly studies the effects of restrictions on short- and long-term strategies on price informativeness and investor profits in a general setting.[5]

The model is designed to be as simple and general as possible. Two key features that it must have are that investors choose among investment strategies at different horizons, and that they choose how much information to acquire about fundamentals across horizons. We study a version of the noisy rational expectations model developed in Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016). Whereas that paper studies investment in a cross-section of assets, we argue here that investment policies over time can be thought of as a choice of exposures on many different future dates. Each of those dates represents a different "asset", and the returns on those assets

[1] See also Stiglitz (1989), Summers and Summers (1989), and Habermeier and Kirilenko (2003)

[2] See LTSE.org and Osipovich and Berman (2017).

[3] See also Nallareddy, Pozen, and Rajgopal (2016).

[4] "Overcoming Short-termism: A Call for a More Responsible Approach to Investment and Business Management", available at https://assets.aspeninstitute.org/content/uploads/files/content/docs/pubs/overcome_short_state0909_0.pdf. See also Stiglitz (2015).

[5] In much recent work, including Cartea and Penalva (2012), Baldauf and Mollner (2017) and Biais, Foucault, and Moinas (2015), high-frequency or short-term investors are somehow different from others, either in preferences or trading technologies. Those models are better suited to studying high-frequency trade specifically.

For recent analyses of limits on information gathering ability, see Banerjee and Green (2015), Goldstein and Yang (2015), Dávila and Parlatore (2016), and Farboodi and Veldkamp (2017).

will be correlated across dates.[6] The model in this paper is notable for allowing an arbitrarily long horizon (as opposed to two or three periods), with turnover at any frequency.

It is important to note that the model is not fully dynamic – all trade happens on date 0, so investors cannot rebalance in response to news or the realization of fundamentals, even though they might desire to. The model takes a dynamic problem, with information flowing and investment choices being made over time, and compresses it into a single time period, along the lines of the classic Arrow–Debreu type analysis, but without a complete set of state-contingent contracts. Dynamic market equilibria are difficult or impossible to solve, and we do not contribute to that area.[7] The paper's focus is instead on the choice of short- versus long-term investment strategies and information acquisition. Short-term investors arise naturally in the model as agents whose exposures to fundamentals fluctuate rapidly across dates due to the type of information they have acquired. The relevant concept of short- versus long-term here ranges between days and years – the model is not designed to analyze technical features of higher frequency trading, like market microstructure effects or exchange fragmentation.

There are a number of potential reasons why a policymaker might want to regulate investment strategies. As is common in the literature, those reasons are somewhat outside the model. For example, research often examines how policies affect price efficiency, even though the models studied do not generally imply that price efficiency raises welfare (see Bond, Edmans, and Goldstein (2012) for a review of the literature on the value of price efficiency).[8] There are at least three potential motivations for regulation. First, if price informativeness at long horizons is more important

[6] The paper uses a frequency transformation that allows the model to be solved by hand. For other related work on frequency transformations, see Bandi and Tamoni (2014), Bernhardt, Seiler, and Taub (2010), Chinco and Ye (2017), Chaudhuri and Lo (2016), Dew-Becker and Giglio (2016), and Kasa, Walker, and Whiteman (2013).

[7] The lack of dynamics means that the model is not suited for studying the relationship between investor horizon and bubbles, such as those of Blanchard (1979). There is work that has made substantial progress in solving the infinite regress problem, but those models assume that investors have only single-period objectives and they do not allow for a choice of information across horizons. See Makarov and Rytchkov (2012), Kasa, Walker, and Whiteman (2013), and Rondina and Walker (2017). Recent work also examines dynamic models with strategic trade (with similar restrictions regarding horizons), whereas here we study a fully competitive setting in which all investors are price takers – see Vayanos (1999, 2001), Ostrovsky (2012), Banerjee and Breon-Drisch (2016), Foucault, Hombert, and Rosu (2016), Du and Zhu (2017), and Dugast and Foucault (2017).

[8] Bond, Edmans, and Goldstein (2012) identify two channels for such spillovers. First, information that stock prices reveal may guide real activity through investment decisions (Dow and Gorton (1997); Kurlat and Veldkamp (2015)) and the decisions of outside investors and regulators to intervene in a firm's activities (Bond, Goldstein, and Prescott (2009); Bond and Goldstein (2015)). Second, price informativeness allows shareholders to tie manager compensation to equity prices, thus improving the real efficiency of management activities (Fishman and Hagerty (1989); Holmström and Tirole (1993); Farboodi and Veldkamp (2017)).

for economic decisions like physical investment, then long-term information acquisition might be encouraged. Second, policymakers might have a general bias toward long-term investors, perhaps because they are more likely to be people saving for retirement. Finally, one might think of the noise traders in the model as retail investors who make poor investment decisions driven by sentiment, or perhaps as uninformed speculators.[9] We use the model to examine how restrictions on investment policies affect price informativeness and the profits and utility of the various investors in order to help inform the policy debate. If the goal is to reduce mistakes or uninformed speculation, then one would ask how to reduce the losses borne by noise traders and their effects on prices.

The paper examines a number of specific policies, including direct restrictions on investment strategies (which map to the batch auction mechanism of Budish, Cramton, and Shim (2015)), taxes on transactions, and taxing or subsidizing information acquisition. As to transaction taxes and investment restrictions, we show that when sophisticated agents are restricted from investing and trading at some frequency, prices become uninformative at that frequency. So if a policy were implemented saying that investors could no longer maintain positions for less than a month, variation in prices within the month would become uninformative for fundamentals, and instead be driven purely by liquidity demand. Intra-month price volatility and mean reversion would also rise.

However, there is no spillover across horizons. A short-term restriction or transaction tax does not reduce price informativeness or increase return volatility at longer horizons, so prices would remain informative at frequencies lower than a month (in an extension of the model, informativeness can even rise). This separability across horizons follows from a statistical result showing that there is a robust independence across frequencies in stationary models, along with a separability in mean-variance (or constant absolute risk aversion) preferences.

The next question is how investment restrictions affect investor outcomes. While it seems inevitable that a restriction on short-term investment would reduce the welfare of short-term in-

[9] One view is that policies aimed at short-termism are trying to reduce speculation, but that term is somewhat ill-defined. Sometimes speculators are simply investors with no fundamental hedging demand, in which case we would say that all the sophisticated investors in our model are speculators. Alternatively, speculators might be agents who invest based on signals about the demand of others, rather than about fundamentals. In the present setting, a signal about demand, after conditioning on prices, is directly informative about fundamentals, so there is little economic difference between the two here. We thus focus on motivations for addressing short-termism that have direct counterparts in the model.

vestors, it is less obvious what would happen to long-term investors or noise traders. An increase in short-term investment (e.g. due to a change in technology that makes short-term information acquisition or trading cheaper) turns out to make long-term investors worse off, essentially taking away some of the long-term investors' trading opportunities. But restricting short-term investment does not transfer profits back to long-term investors; instead it simply eliminates those profits, making both short- and long-term investors worse off.

In the context of the model, the way to tilt markets in favor of long-term investors – if that is one's goal – is to make acquisition of short-term information more expensive for investors. There have been numerous recent proposals to do just that, for example by limiting quarterly earnings guidance (e.g. Schacht et al. (2007), Pozen (2014), Dimon and Buffett (2018), and the Aspen Institute's report). In the UK, in fact, such reports are no longer mandatory for publicly traded companies. The model in this paper is well suited to analyze such policies, and we show that they can shift the equilibrium toward long-term investors, increasing their average profits and utility (though the direction of this result depends on how one models information releases).

Finally, the paper examines the impact of the various policies on the profits of noise traders. Intuitively, the noise traders are constantly making mistakes, potentially affecting prices. There are two ways to protect them from those mistakes: stop them from trading, or reduce the losses they take on each trade. Stopping them from trading is in principle simple – just close asset markets – but then one loses the information contained in prices, along with any gains from trade.

More interestingly, the paper shows that a better alternative is to subsidize or otherwise encourage information acquisition, which causes prices to become more informative and less responsive to noise trader (perhaps speculative) demand. Such a policy can, in the limit, drive noise trader losses to zero, while simultaneously making prices more useful for economic decisions and reducing the excess volatility caused by noise trader speculation. However, and interestingly, it is the opposite of the policy that we showed helps the long-term investors. Furthermore, it is important to temper the results on noise traders with the knowledge that there is no single canonical model of noise traders. The paper examines robustness to an alternative formulation driven by time-varying hedging demand and shows that welfare predictions are more difficult to make, though the predictions for price informativeness and return volatility are similar.

Overall, then, we obtain three basic results about policies aimed at short-termism:

1. Restricting short-term investment affects short-run but not long-run price informativeness and return volatility.

2. Restricting short-term investment hurts both short- and long-term investors, but helps noise traders.

3. Taxing or restricting the availability of short-term information helps long-term investors, hurts short-term investors and noise traders, and reduces short-term price efficiency. Subsidizing information or mandating greater disclosure by firms does the opposite.

On net, then, we would argue that mandatory information releases or subsidizing information acquisition are the most natural policies to address short-termism, as they both reduce speculative effects on prices and improve price efficiency. They do, however, come with costs to long-term investors, and also run against recent proposals to reduce quarterly reporting.

The answers to the questions of how restrictions on trade affect price informativeness and welfare are not obvious ex ante. One view is that there might be some sort of separation across frequencies, so that restrictions in one realm do not affect outcomes in another. On the other hand, investors obviously interact – they trade with each other – so it would be surprising if policies targeting a particular type of investor did not act to benefit others. What we find is a mix of the two: market characteristics at high frequencies can affect the profits and utility of long-term investors – the model is not entirely separable across frequencies in that sense – but they do not affect low-frequency price informativeness in our baseline case. Furthermore, there is a tension between helping long-term investors, helping noise traders, and maintaining price informativeness. No single policy helps all the groups at the same time due to a zero-sum aspect of the model, and policies that may be attractive to certain investors can come with negative side effects for agents outside the model – e.g. executives, or policymakers like the FOMC – who might make decisions based on asset prices.

The remainder of the paper is organized as follows. Sections 1 and 2 lay out the model and its solution. Section 3 examines the effects for price volatility and informativeness of restrictions on investment at different horizons, while section 4 examines the impacts of such policies on the profits and welfare of different investors. Section 4 also examines the impact of restrictions on information releases such as earnings announcements, and section 5 concludes.

1 The model

1.1 Market structure

Time is denoted by $t \in \{-1, 0, 1, ..., T\}$, with T even, and we will focus on cases in which T may be treated as large. There is a fundamentals process D_t, on which investors trade forward contracts, with realizations on all dates except -1 and 0. The time series is stacked into a vector $D \equiv [D_1, D_2, ..., D_T]'$ (versions of variables without time subscripts denote vectors) and is unconditionally distributed as

$$D \sim N(0, \Sigma_D). \tag{1}$$

For our benchmark results, we focus on the case where fundamentals are stationary. Appendix H shows that the results extend naturally to a case in which fundamentals are stationary in their growth rate, rather than their level. We discuss that case further below. Stationarity implies that Σ_D is constant along its diagonals, and we further assume that the eigenvalues of Σ_D are finite and bounded away from zero (which is satisfied by standard ARMA processes).

The biggest restriction imposed by the stationarity assumption (whether in levels or differences) is that we are assuming that the distribution of fundamentals is determined entirely by the matrix Σ_D. The model thus does not allow for stochastic volatility or more general changes in the higher moments of D_t over time (though it could handle deterministic changes)[10], nor does it allow for nonlinearities in the time series dependence of D. The fact that we study the level (or change) in fundamentals, rather than their log, is also a restriction, though one that is generally shared by CARA–Normal specifications (e.g. Grossman and Stiglitz (1980)).

Those restrictions, along with those implicit in the preferences below, mean that the model is useful primarily for qualitative analysis – it does admit the functional forms required for a realistic quantitative analysis. In exchange, though, the assumptions yield tractability and closed-form solutions.

[10] All the variables in the model are heteroskedastic. The model could accommodate predictable cahgnes in volatility, such as intra-day patterns and volatility around scheduled announcements, through time-change methods as in Ané and Geman (2000), and Geman, Madan, and Yor (2001).

There is a set of futures claims on realizations of the fundamental. When we say that the model features a choice of investment across dates or horizons, we mean that investors will choose portfolio allocations across the futures contracts, which then yield exposures to the realization of fundamentals on different dates in the future.

A concrete example of a process D_t is the price of crude oil: oil prices follow some stochastic process and investors trade futures on oil at many maturities. D_t could also be the dividend on a stock, in which case the futures would be claims on dividends on individual dates. The analysis of futures is an abstraction for the sake of the theory, though we note that dividend futures are in fact traded (Binsbergen and Koijen (2017)). While the concept of a futures market on the fundamentals will be a useful analytic tool, we will also price portfolios of futures. Equity, for example, is a claim to the stream of fundamentals over time. Holding any given combination of futures claims on the fundamental is equivalent to holding futures contracts on equity claims.

1.2 Information structure

There is a unit mass of "sophisticated" or rational investors, indexed by $i \in [0, 1]$, who have rational expectations, conditioning on both prices and private signals. The realization of the time series of fundamentals, $\{D_t\}_{t=1}^{T}$, can be thought of as a single draw from a multivariate normal distribution. The signals an agent observes are a collection $\{Y_{i,t}\}_{t=1}^{T}$ *observed on date 0* with

$$Y_{i,t} = D_t + \varepsilon_{i,t},\ \varepsilon_i \sim N\left(0, \Sigma_i\right), \tag{2}$$

where Σ_i^{-1} is investor i's signal precision matrix (which will be chosen endogenously below). Through $Y_{i,t}$, investors can learn about fundamentals on all dates between 1 and T. $\varepsilon_{i,t}$ is a stationary error process in the sense that $Cov\left(\varepsilon_{i,t}, \varepsilon_{i,t+j}\right)$ depends on j but not t. That also implies that $Var\left(\varepsilon_{i,t}\right)$ is the same for all t, so all dates are equally difficult to learn about. The stationarity assumption is imposed so that no particular date is given special prominence in the model. Investors must choose an information policy that treats all dates symmetrically, and they are not allowed to choose to learn about a single date.

The signal structure generates one of our desired model features, which is that investors can choose to learn about fundamentals across different dates in the future. When the errors are positively correlated across dates, the signals are relatively less useful for forecasting trends in fundamentals since the errors also have persistent trends. Conversely, when errors are negatively correlated across dates, the signals are less useful for forecasting transitory variation and provide more accurate information about moving averages. What types of fluctuations investors are informed about will determine their investment strategies.

1.3 Investment objective

On date 0, there is a market for forward claims on fundamentals on all dates in the future. Investor i's demand for a date-t forward conditional on the set of prices and signals is denoted $Q_{i,t}$. Investors have mean-variance utility over terminal wealth:[11]

$$U_{0,i} = \max_{\{Q_{i,t}\}} T^{-1} E_{0,i} \left[\sum_{t=1}^{T} \beta^t Q_{i,t} \left(D_t - P_t\right) \right] - \frac{1}{2} \left(\rho T\right)^{-1} Var_{0,i} \left[\sum_{t=1}^{T} \beta^t Q_{i,t} \left(D_t - P_t\right) \right], \tag{3}$$

where $0 < \beta \leq 1$ is the discount factor, $E_{0,i}$ and $Var_{0,i}$ are the expectation and variance operators conditional on agent i's date-0 information set, $\{P, Y_i\}$, and ρ is risk-bearing capacity per unit of time. We treat all investors as having identical horizons, T – they can follow different strategies, and may have different rates of portfolio turnover, but they all want to earn the highest possible returns, with the least amount of risk, in the shortest time. The sense in which the model maps into the colloquial use of the term "short-termism" is that agents in the model may choose to follow investment strategies featuring very rapid changes in their positions across dates. Appendix B shows that the horizon does not affect information choices in the model. Short- and long-term investors are distinguished by how long they maintain positions, not by their objective.

The key restriction here (beyond those implicit in the mean-variance assumption) is that signals are acquired and trade occurs on date 0. In general settings there is no known closed-form solution to even the partial-equilibrium dynamic portfolio choice problem, let alone to the full market equilibrium.[12] The dynamic portfolio choice problem is difficult due to the presence of dynamic

[11] To see why this is over terminal wealth, note that when the profits from each futures claim, $D_t - P_t$, are reinvested at the riskless rate β^{-1}, terminal wealth, $W_{T,i}$, is $\sum_{t=1}^{T} \beta^{t-T} Q_{i,t} \left(D_t - P_t\right)$, which is simply β^{-T} times the argument of the expectation and variance in the preferences. For motivation, see Dumas and Luciano (1991). Other papers using similar preferences include Carpenter (2000), Cox and Leland (2000), Li and Ng (2000).

[12] Frequency-domain solutions to the infinite regress problem, such as Kasa, Walker, and Whiteman (2013) and Makarov and Rytchkov (2012), restrict preferences to depend on wealth one period ahead in order to avoid the dynamic portfolio problem.

hedging motives and rebalancing in response to news. Moreover, allowing agents to obtain signals repeatedly yields a highly nontrivial statistical updating task. We therefore use a relatively minimal static model which eliminates those problems by assumption. The model nevertheless has the two characteristics that we stated we desire in the introduction: it allows for investment strategies that place different weight on fundamentals on different dates in the future, and it allows investors to make a choice about how precise their signals are for different types of fluctuations in fundamentals.[13]

It should also be noted that the model can only accommodate mean-variance (or constant absolute risk aversion) preferences and remain tractable. The specification used here does not allow for generalized recursive utility, for example.

The time discounting in (3) has the effect of making dates farther in the future less important in the objective of the investors. We therefore define

$$\tilde{Q}_{i,t} \equiv \beta^t Q_{i,t} \tag{4}$$

to be agent i's discounted demand. In what follows, the $\tilde{Q}_{i,t}$ will be stationary processes. That means that $Q_{i,t} = \beta^{-t}\tilde{Q}_{i,t}$ will generally grow in magnitude with maturity t, though only to a relatively small extent for typical values of β and horizons on the order of 10–20 years.

1.4 Noise trader demand

In order to keep prices from being fully revealing, we assume there is uninformed demand from a set of noise traders. The noise traders are investors with the same objective as the rational agents, but whose expectations are formed differently. Specifically, their expectations of fundamentals depend on a signal, Z_t, that is in reality uncorrelated with fundamentals, so it can be viewed as a type of sentiment shock. The noise traders can also be viewed as uninformed speculators. Appendix L examines all of our results in an alternative model in which exogenous demand is due to hedging.

[13] In a dynamic model, signals are revealed and investment decisions are made in each period. here, information flows and investment decisions are compressed into a single period. The two key differences from a fully dynamic model are that agents cannot condition on the realization of fundamentals and that there is not a full set of state-contingent contracts. The former restriction will bind more weakly when agents make decisions primarily based on private signals rather than the realization of fundamentals. The latter restriction could potentially lead to a form of time inconsistency here, depending on how one assumes agents update information sets and preferences over time.

Appendix A shows that when the noise traders maximize an objective of the form of (3) but with their incorrect expectations, then their demand, denoted N_t, can be written as

$$\tilde{N}_t = Z_t - kP_t, \tag{5}$$

$$\text{where } \tilde{N}_t \equiv \beta^t N_t. \tag{6}$$

Z_t depends on the signals the noise traders receive (which are assumed to be common across them) and k is a coefficient determining the sensitivity of noise trader demand to prices, which depends on their risk aversion and how precise they believe their signals to be. In principle, N_t can depend on prices on all dates (depending on the structure of priors and signals), but we restrict attention to the case where N_t depends only on P_t for the sake of simplicity.

In the benchmark case where D_t is stationary in levels, we assume that Z_t is also stationary in levels – the noise traders have a signal technology with the same stationarity properties as that of the sophisticates – which yields a useful symmetry between fundamentals, supply, and the signals, in that they are all assumed to be stationary processes.

1.5 Asset market equilibrium in the time domain

We begin by solving for the market equilibrium on date 0 that takes the agents' signal precisions, Σ_i^{-1}, as given. The Σ_i^{-1} are chosen on date -1, and that optimization is discussed below.

Definition 1 *For any given set of individual precisions $\{\Sigma_i\}_{i\in[0,1]}$, a date-0 asset market equilibrium is a set of demand functions, $\{Q_i(P,Y_i)\}_{i\in[0,1]}$, and a price vector P, such that investors maximize utility and all markets clear: $\int_i Q_{i,t}di + N_t = 0$ for all $t \geq 1$.*

Investors submit demand curves for each futures contract and the equilibrium price vector, P, is the one that clears all markets. The structure of the time-0 equilibrium is mathematically that of Admati (1985), who studies investment in a cross-section of assets, and the solution from that paper applies directly here (with the minor difference that supply is also a function of prices):

$$\begin{aligned} P &= A_1 D + A_2 Z, && (7) \\ A_1 &\equiv I - \left(\rho^2 \Sigma_{avg}^{-1} \Sigma_Z^{-1} \Sigma_{avg}^{-1} + \Sigma_{avg}^{-1} + \Sigma_D^{-1} + \rho^{-1} k\right)^{-1} \left(\rho^{-1} k + \Sigma_D^{-1}\right), && (8) \\ A_2 &\equiv \rho^{-1} A_1 \Sigma_{avg}^{-1}, && (9) \\ \text{where } \Sigma_{avg}^{-1} &\equiv \int_i \Sigma_i^{-1} di. && (10) \end{aligned}$$

As Admati (1985) discusses, this equilibrium is not particularly illuminating since standard intuitions, including the idea that increases in demand should raise prices, do not hold. Prices of futures maturing on any particular date depend on fundamentals and demand for all other maturities except in knife-edge cases. Interpreting the equilibrium requires interpreting complicated products of matrix inverses. The following section shows that the equilibrium can be solved by hand nearly exactly when it is rewritten in terms of frequencies.

2 Frequency domain interpretation

2.1 Frequency portfolios

The basic difficulty of the model is that fundamentals, noise trader demand, and signal errors are all correlated across dates. For any one of those three processes, we could use a standard orthogonal (eigen-) decomposition to yield a set of independent components. But in general three time series with different correlation properties across dates will not have the same orthogonal decomposition. A central result from time series analysis, though, is that a particular frequency transform asymptotically orthogonalizes all standard stationary time series processes.

Such a transformation represents simply analyzing the prices of particular portfolios of futures instead of the futures themselves. It must satisfy three requirements. First, the transformation should be full rank, so that the set of portfolios allows an investor to obtain the same payoffs as the futures themselves. Second, the transformed portfolios should be independent of each other. And third, since we are studying trade at different frequencies, it would be nice if the portfolios also had a frequency interpretation.

There are many different conceptions of fluctuations at different frequencies. One might imagine step functions switching between +1 and -1 at different rates. For reasons we will see below, using sines and cosines will be most natural in our setting. The portfolios that we study – representing investor exposures – vary smoothly over time in the form $\cos(\omega t)$ and $\sin(\omega t)$.

Formally, the portfolio weights are represented as vectors of the form

$$c_h \equiv \sqrt{\frac{2}{T}}\Big(\cos\left(\omega_h\left(t-1\right)\right)\Big)_{t=1}^{T}, \tag{11}$$

$$s_h \equiv \sqrt{\frac{2}{T}}\Big(\sin\left(\omega_h\left(t-1\right)\right)\Big)_{t=1}^{T}, \tag{12}$$

$$\text{where} \quad \omega_h \equiv 2\pi h/T, \tag{13}$$

for different values of the integer $h \in \{0, 1, ..., T/2\}$. c_0 is the lowest frequency portfolio, with the same weight on all dates, while $c_{\frac{T}{2}}$ is the highest frequency, with weights switching each period between ± 1.

Figure 1 plots the weights for a pair of those portfolios. The x-axis represents dates and the y-axis is the weight of the portfolio on each date. The weights vary smoothly over time, with the rate at which they change signs depending on the frequency ω.

Economically, the idea is to think about the investment problem as being one of choosing exposure to different types of fluctuations in fundamentals. A long-term investor can be thought of as one whose exposure to fundamentals changes little over time, while a short-term investor holds a portfolio whose weights change more frequently and by larger amounts.

Our claim is that studying the frequency portfolios is more natural than studying individual futures claims. Investors do not typically acquire exposure to fundamentals on only a single date. Rather, they have exposures on multiple dates, and the portfolios we study are one way to express that. While investors will also obviously not hold a portfolio that takes the exact form of a cosine, any portfolio can be expressed as a sum of cyclical components. An investor whose portfolio loadings change frequently will have a portfolio whose weights are relatively larger on the high-frequency components, which figure 1 shows generate rapid changes in loadings.

2.2 Properties of the frequency transformation

The portfolio weights can be combined into a matrix, Λ, which implements the frequency transformation.

$$\Lambda \equiv \left[\frac{1}{\sqrt{2}}c_0, c_1, s_1, c_2, s_2, ..., c_{\frac{T}{2}-1}, s_{\frac{T}{2}-1}, \frac{1}{\sqrt{2}}c_{\frac{T}{2}}\right] \tag{14}$$

(s_0 and $s_{T/2}$ do not appear since they are identically equal to zero; the $1/\sqrt{2}$ scaling for c_0 and $c_{T/2}$ gives them the same norms as the other vectors).

We use lower-case letters to denote frequency-domain objects. So whereas $\tilde{Q}_t$ is investor i's vector of discounted allocations to the various futures, $\tilde{q}_i$ is their vector of discounted allocations to the frequency portfolios, with

$$\tilde{Q}_i = \Lambda \tilde{q}_i. \tag{15}$$

In what follows, the index $j = 1, ..., T$ identifies columns of Λ. The jth column of Λ is a vector that fluctuates at frequency $\omega_{\left\lfloor \frac{j}{2} \right\rfloor} = 2\pi \left\lfloor \frac{j}{2} \right\rfloor /T$, where $\lfloor \cdot \rfloor$ is the integer floor operator.[14] So there are two vectors, a sine and a cosine, for each characteristic frequency, with the exceptions of $j = 1$ (frequency 0, the lowest possible) and $j = T$ (frequency $\frac{T}{2}$, the highest possible).

Note also that Λ has the property that $\Lambda^{-1} = \Lambda'$, so that frequency-domain vectors can be obtained through

$$\tilde{q}_i = \Lambda' \tilde{Q}_i. \tag{16}$$

In the same way that $\tilde{q}_i$ represents weights on frequency-specific portfolios, $d \equiv \Lambda' D$ is a representation of the realization of fundamentals written in terms of frequencies instead of time. The first element of d, for example, is proportional to the realized sample mean of D. Equivalently, d is the set of regression coefficients of D on the columns of Λ (which generate an R^2 of 1).

As a simple example, consider the case with $T = 2$. The low-frequency or long-term component of dividends is then $d_0 = (D_1 + D_2)/\sqrt{2}$ and the high-frequency or transitory component is $d_1 = (D_1 - D_2)/\sqrt{2}$. Agents invest in the low-frequency component d_0 by buying an equal amount of the claims on D_1 and D_2 and they trade the high-frequency component d_1 by buying offsetting amounts of the claims on D_1 and D_2. A short-term investment in this case is one where the sign of the exposure to fundamentals changes, while the long-term investment has a fixed position.

[14] $\lfloor x \rfloor$ is the largest integer that is less than or equal to x.

The most important feature of the frequency transformation is that it approximately diagonalizes the variance matrices.

Definition 2 *For an $n \times n$ matrix A with elements $a_{l,m}$, the **weak matrix norm** is*

$$|A| \equiv \left(\frac{1}{n} \sum_{l=1}^{n} \sum_{m=1}^{n} a_{l,m}^2 \right)^{1/2}. \tag{17}$$

If $|A - B|$ is small, then the elements of A and B are close in mean square.

The frequency transform will lead us to study the spectral densities of the various time series:

Definition 3 *The **spectrum** at frequency ω of a stationary time series X_t is*

$$f_X(\omega) \equiv \sigma_{X,0} + 2 \sum_{t=1}^{\infty} \cos(\omega t)\, \sigma_{X,t}, \tag{18}$$

$$\textit{where } \sigma_{X,t} = cov\left(X_s, X_{s-t}\right). \tag{19}$$

The spectrum, or spectral density, is used widely in time series analysis. The usual interpretation is that it represents a variance decomposition. $f_X(\omega)$ measures the part of the variance of X_t associated with fluctuations at frequency ω, which is formalized as follows.

Lemma 1 *For any stationary time series $\{X_t\}_{t=1}^{T}$, with frequency representation $x \equiv \Lambda' X$, the elements of the vector x are approximately uncorrelated in the sense that the covariance matrix of x, $\Sigma_x \equiv \Lambda' \Sigma_X \Lambda$, is nearly diagonal,*

$$|\Sigma_x - diag(f_X)| \leq b T^{-1/2}, \tag{20}$$

and x converges in distribution to

$$x \rightarrow_d N(0, diag(f_X)), \tag{21}$$

where b is a constant that depends on the autocorrelations of X,[15] *and $diag(f_X)$ denotes a matrix with the vector $\left\{ f_X\left(\omega_{\lfloor j/2 \rfloor}\right)\right\}_{j=1}^{T}$ on the main diagonal and zeros elsewhere.*[16]

[15] Specifically, $b = 4\left(\sum_{j=1}^{\infty} |j \sigma_{X,j}|\right)$.

[16] A requirement of this lemma, which we impose for all the stationary processes studied in the paper, is that

Proof. These are textbook results (e.g. Brockwell and Davis (1991) and Gray (2006)). Appendix C.1 provides a derivation of the inequality (20) specific to our case. The convergence in distribution follows from Brillinger (1981), theorem 4.4.1. ■

Lemma 1 says that Λ approximately diagonalizes *all* stationary covariance matrices. So the frequency-specific components of fundamentals, prices, and noise trader demand are all (approximately) independent when analyzed in terms of frequencies. That is, $d = \Lambda' D$, $y_i = \Lambda' Y_i$, and $z = \Lambda' Z$ all have asymptotically diagonal variance matrices. That independence will substantially simplify our analysis, and it is a special property of the sines and cosines, as opposed to other conceptions of frequencies.[17] The various primitive restrictions on the model, including mean-variance preferences, stationarity, and homoskedasticity, are required in order to be able to take advantage of this diagonalization result.

2.3 Market equilibrium in the frequency domain

2.3.1 Approximate diagonalization

Instead of solving jointly for the prices of all futures, the approximate diagonalization result allows us to solve a series of parallel scalar problems, one for each frequency. Intuitively, since the frequency-specific portfolios have returns that are nearly uncorrelated with each other, the investors' utility can be written approximately as a sum of mean-variance optimizations[18]

$$U_{0,i} \approx \max_{\{q_{i,j}\}} T^{-1} \sum_{j=1}^{T} \left\{ E_{0,i}\left[\tilde{q}_{i,j}\left(d_j - p_j\right)\right] - \frac{1}{2}\rho^{-1} Var_{0,i}\left[\tilde{q}_{i,j}\left(d_j - p_j\right)\right] \right\}. \tag{22}$$

In what follows, we solve the model using the approximation for $U_{0,i}$, and then show that it converges to the true solution from Admati (1985). When utility is completely separable across frequencies,

the autocovariances are summable in the sense that $\sum_{j=1}^{\infty} |j \sigma_{X,j}|$ is finite (which holds for finite-order stationary ARMA processes, for example). Trigonometric transforms of stationary time series converge in distribution under more general conditions, though. See Shumway and Stoffer (2011), Brillinger (1981), and Shao and Wu (2007).

[17] Finally, it is should be noted that infill asymptotics, where T grows by making the length of a time period shorter, are not sufficient for lemma 1 to hold. What is important is that T is large relative to the range of autocorrelation of the process X. So, for example, if fundamentals have nontrivial autocorrelations over a horizon of a year, then it is important that T be substantially larger than a year. Van Binsbergen and Koijen (2017), for example, examine data on dividend futures with maturities as long as 16 years. This also means that the correct numerical value for T depends on the length of a time period. If one shifts from annual to monthly data, then T should rise by a factor of 12 for the approximations to be equally accurate. T should thus be both long enough for the frequency approximation to be accurate and also to give a reasonable representation of investor horizons.

[18] This follows from lemma 1 combined with the fact that $\Lambda' \Lambda = I$, so that $Q_i' D = Q_i' \Lambda' \Lambda D = q_i' d$.

there is an equilibrium frequency by frequency:

Solution 1 *Under the approximations $d \sim N(0, diag(f_D))$ and $z \sim N(0, diag(f_Z))$, the prices of the frequency-specific portfolios, p_j, satisfy, for all j*

$$p_j = a_{1,j} d_j + a_{2,j} z_j \tag{23}$$

$$a_{1,j} \equiv 1 - \frac{\rho^{-1} k + f_{D,j}^{-1}}{\left(\rho f_{avg,j}^{-1}\right)^2 f_{Z,j}^{-1} + f_{avg,j}^{-1} + f_{D,j}^{-1} + \rho^{-1} k} \tag{24}$$

$$a_{2,j} \equiv \frac{a_{1,j}}{\rho f_{avg,j}^{-1}} \tag{25}$$

where $f_{avg,j}^{-1} \equiv \int_i f_{i,j}^{-1} di$ is the average precision of the agents' signals at frequency j.

Proof. See appendix C.2. ■

The price of the frequency-j portfolio depends only on fundamentals and supply at that frequency due to the independence across frequencies. As usual, the informativeness of prices, $Var[d_j \mid p_j]$ can be shown to increase in the precision of the signals that investors obtain, while the impact of noise trader demand on prices is decreasing in signal precision and risk tolerance.

These solutions for the prices are standard results for scalar markets. What is different here is simply that the agents chose exposures across frequencies, rather than across dates; p_j is the price of a portfolio whose exposure to fundamentals fluctuates over time at frequency $2\pi \lfloor j/2 \rfloor / T$. Both prices and demands at frequency j depend only on signals and supply at frequency j – the problem is completely separable across frequencies.

In what follows, we assume that k is sufficiently small that $ka_{2,j} < 1$ for all j, which ensures that z represents a positive demand shock in equilibrium (though most of the results hold without that assumption). The restriction is that noise trader demand not be too sensitive to prices; in the literature k is usually equal to zero.

2.3.2 Quality of the approximation

While solution 1 is an approximation, its error can be bounded. The time domain solution is obtained from the frequency domain solution by premultiplying by Λ (from equation (15)), and we have,

Proposition 1 *The difference between solution 1 and the exact Admati (1985) solution is small in the sense that*

$$\left|A_1 - \Lambda diag\left(a_1\right)\Lambda'\right| \leq c_1 T^{-1/2} \tag{26}$$

$$\left|A_2 - \Lambda diag\left(a_2\right)\Lambda'\right| \leq c_2 T^{-1/2} \tag{27}$$

for constants c_1 and c_2. Furthermore, the variances of the approximation error for prices and quantities are bounded by:

$$\left|Var\left(\Lambda p - P\right)\right| \leq c_P T^{-1/2} \tag{28}$$

$$\left|Var\left(\Lambda \tilde{q}_i - \tilde{Q}_i\right)\right| \leq c_Q T^{-1/2} \tag{29}$$

for constants c_P and c_Q.

Proof. See appendix C.3. ■

Proposition 1 shows that the frequency domain solution to the market equilibrium provides a close approximation to the true solution in the sense that the solution in (23), once it is rotated back to the time domain, converges to equations (7–9). Moreover, Λp is stochastically close to P in the sense that the variance of the pricing errors is of order $T^{-1/2}$. So the standard time-domain solution for stationary time series processes becomes arbitrarily close to a simple set of parallel scalar problems in the frequency domain for large T.

2.4 Optimal information choice in the frequency domain

The analysis so far takes the precision of the signals as fixed. Following Van Nieuwerburgh and Veldkamp (2009) and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), we allow investors to choose their signal precisions, Σ_i^{-1} to maximize the expectation of their mean-variance objective

(3) subject to an information cost,[19]

$$\max_{\{f_{i,j}\}} E_{-1}\left[U_{i,0} \mid \Sigma_i^{-1}\right] - \frac{\psi}{2T} tr\left(\Sigma_i^{-1}\right), \tag{30}$$

where E_{-1} is the expectation operator on date -1, i.e. prior to the realization of signals and prices (as distinguished from $E_{i,0}$, which conditions on P and Y_i), and ψ is the per-period cost of information. Total information here is measured by the trace operator $tr\left(\Sigma_i^{-1}\right)$. Note that while Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016) focus on the case where investors have a fixed budget of precision, we are studying the dual problem in which information comes at a constant marginal cost. This can be thought of as a case where an investment firm can choose how many analysts to hire at a fixed wage, with total precision scaling linearly with the number of analysts. We discuss below how the choice of a constraint versus cost affects the main results.[20] Appendix K.3 and section 3.4 discuss another alternative specification where information flows are measured by entropy.

Given the optimal demands, an agent's expected utility is linear in the precision they obtain at each frequency.

Lemma 2 *Each informed investor's expected utility at time -1 may be written as a function of their own signal precisions, $f_{i,j}^{-1}$, and the average across other investors, $f_{avg,j}^{-1} \equiv \int_i f_{i,j}^{-1} di$, with*

$$E_{-1}\left[U_{0,i} \mid \{f_{i,j}\}\right] = \frac{1}{2T}\sum_{j=1}^{T} \lambda_j\left(f_{avg,j}^{-1}\right) f_{i,j}^{-1} + constant, \tag{31}$$

where the constant does not depend on investor i's precision and $\lambda_j(x) > 0$ and $\lambda_j'(x) < 0$ for all $x \geq 0$.

Proof. See appendix C.4. ■

Since expected utility and the information cost are both linear in the set of precisions that

[19] The preferences can equivalently be written in terms of utility over terminal wealth, $W_{T,i}$. Specifically, maximization of $E_{-1}\left[-\rho^{-1}T^{-1}\log E_{0,i}\left[\exp\left(-\rho W_{T,i}\right)\right] \mid \Sigma_i^{-1}\right]$, where $E_{0,i}$ conditions on priors, agent i's signals, and prices, is equivalent to maximization of (30) since $U_{0,i} = \rho^{-1}T^{-1}\log E_{0,i}\left[\exp\left(-\rho W_{T,i}\right)\right]$.

[20] The constraint model corresponds to a world where firms cannot expand the number of analysts that they employ, just shift them among tasks (frequencies). The cost model that we focus on represents a world where firms are free to hire more analysts from an elastic supply. This is more relevant if the financial sector does not account for most of the employment of the people capable of doing research.

agent i chooses, $\left\{f_{i,j}^{-1}\right\}$, it immediately follows that agents purchase signals at whatever subset of frequencies has $\lambda_j\left(f_{avg,j}^{-1}\right) \geq \psi$.

Solution 2 *Information is allocated so that*

$$f_{avg,j}^{-1} = \begin{cases} \lambda_j^{-1}(\psi) & \text{if } \lambda_j(0) \geq \psi, \\ 0 & \text{otherwise.} \end{cases} \tag{32}$$

Because attention cannot be negative, when $\lambda_j(0) \leq \psi$, no attention is allocated to frequency j. Otherwise, attention is allocated so that its marginal benefit and its marginal cost are equated. This result does not pin down precisely how any specific investor's attention is allocated; this class of models, with a non-convex information cost, only determines the aggregate allocation of attention across frequencies. For the purposes of studying price informativeness, though, characterizing this aggregate allocation is all that is necessary.

Solution 2 is the water-filling equilibrium of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016). In their case it applied to the variances of principal components of a cross-section of assets, where here it applies to variances of frequency portfolios – the spectrum.

At this point there are still no investors who are explicitly "short-term" or "long-term". Investors can follow many different strategies, with different mixes of short- and long-term focus. Even without any specialization to particular strategies, though, we now have sufficient structure to analyze the effects of restrictions on the strategies that investors may follow. Later on, we explicitly discuss how to think about short- and long-term investors.

3 The consequences of restricting investment frequencies for prices

This section focuses on the effects on prices of restrictions on the frequencies at which investment strategies can operate. It examines a particularly stark restriction that simply outlaws certain strategies. Section 4 studies information restrictions, while appendix I shows that the results here are similar to those obtained by imposing a quadratic tax on trading.

3.1 Restricting investment frequencies

The assumption in this section is that investors are restricted to setting $\tilde{q}_{i,j} = 0$ for j in some set $\mathcal{R}$. We leave the noise traders unconstrained, assuming that, like retail investors, they face different regulations from large and sophisticated institutions.

Intuitively, if an investor is restricted from exposures at frequencies shorter than a day (i.e. $\mathcal{R}$ is the set of frequencies corresponding to cycles lasting less than one day), then they can effectively only choose exposures once per day. Rather than forcing the investor to literally only trade once a day, though, the restriction in our case corresponds to a portfolio that varies smoothly between days. So (approximately) if the investor can choose daily exposures, then their actual exposures, minute-by-minute, might be represented by a spline that smooths between the daily exposures.

More formally, a restriction on exposures to the frequency portfolios reduces the number of degrees of freedom that an investor has in making choices. Consider a model in which each time period is an hour, and T is a year, or 1625 trading hours. A restriction that investors cannot invest at a frequency higher than a day (6.5 hours) would mean that they would go from a strategy with 1625 degrees of freedom to one with only 250. A pension that sets a portfolio once a quarter would have only four degrees of freedom. In that sense, then, a frequency restriction is similar to a shift from a continuous market to one with infrequent batch auctions, as in Budish, Cramton, and Shim (2015). While that paper proposes holding the auctions still very frequently (i.e. more than once per second), a more aggressive restriction could have auctions only once per day, or once per hour.

Appendix H examines the version of the model in which fundamentals are stationary in differences instead of levels (i.e. they have a unit root). In that case, the analysis goes through nearly identically – frequency restrictions still represent decreases in the degrees of freedom available to investors – but with a single small change: the lowest frequency portfolio, rather than being one that puts equal weight on fundamentals on all dates, puts weight on fundamentals only on the final date, T. Intuitively, an investor who wants to take a position in long-run growth rates does that by buying a claim just to the level on date T. On the other hand, an investor who holds a portfolio that loads on rapid changes in the growth rate of fundamentals will have a portfolio with weights on the level of fundamentals that also change quickly. So in that case, the example of restricting investment in portfolios with frequencies higher than a day continues to impose the same limit on the set of strategies investors can choose from.

Derivations of the results in the remainder of this section are in appendix D.

3.2 Results

We begin by describing price informativeness at different frequencies to demonstrate our key separation result. We then show what happens to prices of standard claims in the time domain.

3.2.1 Price informativeness across frequencies

In terms of frequencies, there is a complete separation: prices become uninformative at restricted frequencies, while remaining unaffected at unrestricted frequencies.

Result 1 *When investment by sophisticated investors is restricted at a set of frequencies $\mathcal{R}$, prices satisfy*

$$p_j = \left\{ \begin{array}{c} k^{-1} z_j \textit{ for } j \in \mathcal{R} \\ a_{1,j} d_j + a_{2,j} z_j \textit{ otherwise} \end{array} \right\}, \tag{33}$$

where $a_{1,j}$ and $a_{2,j}$ are the same as those defined in solution 1.

Intuitively, when sophisticated investors are restricted, prices depend only on sentiment, since the agents with information cannot express their opinions. Moreover, the market becomes illiquid, and it is cleared purely through prices rather than quantities.

Since the solution for information acquisition at a frequency j does not depend on anything about any other frequency, the information acquired at a frequency $j \notin \mathcal{R}$ is also unaffected by the policy. We then have the result that:

Corollary 1.1 *When investors are restricted from holding portfolios with weights that fluctuate at some set of frequencies $j \in \mathcal{R}$, then prices at those frequencies, p_j, become completely uninformative about dividends. The informativeness of prices for $j \notin \mathcal{R}$ about dividends is unchanged. More formally, $Var[d_j \mid p_j]$ for $j \notin \mathcal{R}$ is unaffected by the restriction. For $j \in \mathcal{R}$, $Var[d_j \mid p_j] = Var[d_j]$.*

So a policy that eliminates short-term investment, e.g. by requiring holding periods of some minimum length, reduces the informativeness of prices for the short-term or transitory components of fundamentals, but has no effect on price informativeness in the long-run.

3.2.2 Price informativeness across dates

The fact that prices remain equally informative at some frequencies does not mean that they remain equally informative for any particular *date*. Dates and frequencies are linked through a standard Fourier result

$$Var\left(D_t \mid P\right) = \frac{1}{T}\sum_{j=1}^{T} Var\left[d_j \mid p_j\right]. \tag{34}$$

The variance of an estimate of fundamentals conditional on prices at a particular date is equal to the average of the variances across all frequencies. So when uncertainty rises at some set of frequencies, the informativeness of prices for fundamentals on every date falls by an equal amount.

Corollary 1.2 *Investment restrictions reduce price informativeness for fundamentals on all dates by equal amounts, and by an amount that weakly increases with the number of frequencies that are restricted.*

If a person is making decisions based on estimates of fundamentals from prices and they are worried that prices are contaminated by high-frequency noise due to a restriction on short-term investment, a natural response would be to examine an average of fundamentals and prices over time (across maturities of futures contracts).

Corollary 1.3 *The informativeness of prices for the sum of fundamentals depends only on informativeness at the lowest frequency:*

$$Var\left(T^{-1}\sum_{t=1}^{T} D_t \mid P\right) = Var\left[T^{-1/2} d_0 \mid p_0\right], \tag{35}$$

where d_0 is the lowest frequency portfolio – with equal weight each date – and p_0 is its price.

Result 1.3 follows immediately from the definition of d_0 and the independence across frequencies in the solution. It shows that the informativeness of prices for moving averages of fundamentals depends only on the very lowest frequency. So even if prices have little or no information at high frequencies – $Var\left[d_j \mid p_j\right]$ is high for large j – there need not be any degradation of information about averages of fundamentals over multiple periods, as they depend primarily on precision at lower frequencies (smaller values of j).

More concretely, going back to our example of oil futures, when investors are not allowed to choose exposure to the high-frequency portfolios, prices become noisier, making it more difficult to obtain an accurate forecast of the spot price of oil at some specific moment in the future. But if one is interested in the average of spot oil prices over a year, the model predicts that prices remain informative under restrictions on short-term strategies. It is possible to derive a similar result for moving averages shorter than T; in that case the weights on the frequencies are given by the Fejér kernel.

In the case where fundamentals are stationary in terms of growth rates instead of levels, the results in this section also hold, but replacing D_t by its first difference. In particular, result 1.3 then states that $Var(D_T \mid P)$ is equal to the variance of the lowest frequency portfolio. This is unsurprising since, as we had previously noted, in the difference-stationary case, the lowest frequency portfolio is the one that places weight only on D_T. In that case, the prediction of the model is that $Var\,(D_T \mid P)$ is unaffected by restrictions on short-term investment.

When long-run investment strategies are restricted, on the other hand, as in the case of a trading desk that cannot have exposure to cycles lasting longer than a day (e.g. Brock and Kleidon (1992) and Menkveld (2013)), then it is natural to examine the informativeness of *differences* in prices across dates. As an example, we can consider the variance of the first difference of fundamentals.

Corollary 1.4 *The variance of an estimate of the change in fundamentals across dates conditional on observing the vector of prices is*

$$Var\left[D_t - D_{t-1} \mid P\right] = \sum_{j=1}^{T} 2\left(1 - \cos\left(\omega_{\lfloor j/2 \rfloor}\right)\right) Var\left[d_j \mid p_j\right]. \tag{36}$$

The function $2 - 2\cos(\omega)$ is equal to 0 at $\omega = 0$ and rises smoothly to 4 at the highest frequency, $\omega = \pi$. So period-by-period changes in fundamentals are driven primarily by high-frequency variation. Reductions in price informativeness at low frequencies have relatively large effects on moving averages and small effects on changes, while the reverse is true for reductions in informativeness at high frequencies.

To summarize, any restriction on investment reduces price informativeness for any particular date. But when short-term investment is restricted, there is little change in the behavior of moving averages of prices. So if a manager is making investment decisions based on fundamentals only at a particular moment, then that decision will be hindered by the policy since prices now have more noise. But if decisions are made based on averages of fundamentals over longer periods, the model predicts that there need not be adverse consequences.

Similar results appear if investors face a constraint on the total precision of their signals, rather than a fixed cost. At targeted frequencies, informativeness still falls to zero. In addition, though, in the constraint specification a decline in information acquisition at the restricted frequencies mechanically leads to an increase in acquisition at unrestricted frequencies. For result 1, then, the a_1 and a_2 coefficients can change at the unrestricted frequencies, with a_1 increasing. For corollary 1.1, price informativeness at unrestricted frequencies actually increases. So in either the constraint or cost case, a restriction on investment at some set of frequencies does no damage to informativeness at the unrestricted frequencies, and in the constraint case it will actually increase informativeness. Corollary 1.2 becomes ambiguous in the constraint case because informativeness falls at some frequencies and rises at others. Since $Var\left(D_t \mid P\right)$ depends on all frequencies, it is natural that in the constraint case the effects would be ambiguous, since then the total amount of precision is held fixed.

3.2.3 Return volatility

Corollary 1.5 *Given an information policy $f_{avg,j}^{-1}$, the variance of returns at frequency j, $r_j \equiv d_j - p_j$ is*

$$Var\left(r_j\right) = \begin{cases} f_{D,j} + \frac{f_{Z,j}}{k^2} \text{ for } j \in \mathcal{R} \\ \min\left(\psi, \lambda_j\left(0\right)\right) \text{ otherwise} \end{cases} . \tag{37}$$

Moreover, the variance of returns at restricted frequencies satisfies $Var(r_j) > f_{D,j} + \frac{f_{Z,j}}{(k+\rho f_{D,j}^{-1})^2}$, which is the variance that returns would have at the same frequency if investment were unrestricted but agents were uninformed.

The volatility of returns at a restricted frequency is higher than it would be if the sophisticated investors were allowed to trade, even if they gathered no information. When uninformed active investors have risk-bearing capacity ($\rho > 0$), they absorb some of the exogenous demand by simply trading against prices, buying when prices are below their means and selling when they are above. The greater is the risk-bearing capacity, the smaller is the effect of sentiment volatility on return volatility. Thus, the restriction affects return volatilities through its effects on both liquidity provision and information acquisition.

Restricting sophisticated investors from following short-term strategies in this model can thus substantially raise asset return volatility in the short-run – it can lead to, for example, large day-to-day fluctuations in prices (though those fluctuations in prices are, literally, variations in prices across maturities for different futures contracts on date 0). Sophisticated traders typically play a role of smoothing prices across maturities, intermediating between excess demand on one day and excess supply in the next. When they are restricted from holding positions in futures that fluctuate from day to day, they can no longer provide that intermediation service, and short-term volatility increases. So while there might be other reasons why one might want to restrict short-term investment (e.g. due to incentive effects on managers, as in Shleifer and Vishny (1990), or reducing losses of noise traders, as we discuss below), a consequence will be that transitory and inefficient price volatility will increase.

Finally, we note that the results in this section could be extended fairly easily to account for more general types of restrictions, such as placing restrictions only on the trade of a subset of agents, or perhaps bounding the size of the positions of some agents at certain frequencies.[21]

3.3 The pricing of equity

Equity is a claim on the entire future stream of fundamentals, so in the model we define it to be a claim that pays D_t on each date t. Since the payoff of an equity claim is simply the sum of fundamentals, in the case where fundamentals are stationary in levels the date-1 equity claim has a payoff of exactly d_0. Corollary 1.3 then says that the absolute level of the price of equity remains equally informative under a short-run investment restriction as in the unrestricted case (though this does not hold for non-stationary fundamentals). That result is natural: if only short-run investment is restricted, then long-run investors, who simply buy and hold equity, are unaffected and can continue to maintain price efficiency.

However, that does not mean that equity prices are unaffected by the restriction. In particular,

[21] See Dávila and Parlatore (2018) for an extensive analysis of the relationship between informativeness and volatility.

while the level of equity prices on an individual date remains equally informative, *changes* in equity prices over time are not. In particular, note that

$$P_t^{equity} - P_{t+1}^{equity} = P_t \tag{38}$$

where $P_t^{equity} \equiv \sum_{j=0}^{\infty} P_{t+j}$ is the price of equity on date t. The difference between equity prices between dates t and $t+1$ is exactly equal to the price of the single-period dividend claim on date t. That is because a strategy that holds equity on date t but then immediately sells it on date $t+1$ only actually has exposure to fundamentals on date t.

So when restrictions on short-term investment make the prices of the individual futures claims less informative, they also make changes in the value of equity over time less informative. The results above for the informativeness of individual futures claims map directly into informativeness of differences in equity prices across dates.

3.4 Numerical example

We now examine a numerical example to illustrate the predictions of the model for the behavior of investor positions, prices, and returns, both with and without restrictions on investment.

The length of a time period is set to a week.[22] The spectrum of fundamentals, f_D, is calibrated to match the features of dividend growth for the CRSP total market index. Since dividends are nonstationary in the data, the numerical calibration assumes that ΔD_t is stationary, so that the individual futures are claims to dividend growth (see appendix H).[23] Appendix F provides full details of the estimation. The top-left panel of figure 2 plots the calibrated spectrum for dividend growth, f_D. Empirically, there is substantial persistence in dividend growth, which causes f_D to peak at low frequencies.[24]

The top-right panel of figure 2 plots the variance of returns on the dividend claims at each

[22] As discussed above, the model is not intended to match sub-second scale features of financial markets, like limit order books and exchange fragmentation. It could be plausibly applied to a daily or perhaps hourly frequency. Here we choose a week because that is the highest frequency at which aggregate economic indicators are released (specifically, initial claims for unemployment).

[23] Technically, the spectrum, f_D, is fit to the change in log dividends in calculating our calibration, but in the analysis that follows, we take f_D as applying to the first difference of the level of dividends.

[24] We set $\rho = 57.8$, $k = 0.2$, and f_Z to be 1/8th of the smallest value of f_D. The qualitative features of the model, as demonstrated in the results above, are not sensitive to those choices.

frequency, both with and without a restriction on investment at frequencies corresponding to cycles lasting less than one month ($\omega \geq 2\pi/4$), which could be thought of as similar to a tax on very short-term capital gains. Consistent with the results above, the variance of returns rises substantially at the restricted frequencies.

In addition to the benchmark case where each frequency is equally difficult to learn about, we also consider an alternative specification for the information cost in which the cost of precision that increases as the frequency falls. Formally, in the benchmark specification, the total cost of information is $\sum_j \psi f_{i,j}^{-1}$, and the alternative uses the generalization $\sum_j \psi_j f_{i,j}^{-1}$, with $\psi_j \propto (\omega_j + \omega_1)^{-1}$.[25]

That specification has two uses. First, it illustrates what would happen if a regulator taxed or subsidized information acquisition differentially across frequencies. Second, it will help match the empirical behavior of dividend strip variances. The top-right panel shows that the consequence of that change is to cause the variance of returns to rise at low frequencies. The bottom-left panel of figure 2 plots the average precision of the signals obtained by investors at each frequency. Under the investment restriction, the precision goes to zero, since the information becomes useless. When information costs vary across frequency, so does information acquisition, and approximately inversely to the cost.

Finally, the bottom-right panel of figure 2 plots annualized Sharpe ratios of dividend strips at maturities of 1 to 7 years along with the equity claim (i.e. the claim to all dividend strips to maturity T) under the three different information policies.[26] The dividend strips are modeled as claims to the level of dividends on a given date in the future. Since it is ΔD that is stationary here, a claim to D_t is equal to a claim to $\sum_{s=1}^{t} \Delta D_s$.

We assume that there is a unit supply of equity, which induces positive average returns on claims to dividends (see appendices C.2 and F.1). Because ΔD_1 affects the level of dividends on every date in the future, while ΔD_T affects only the level of dividends on date T, there is effectively greater supply of the shorter-maturity dividend claims, meaning that they earn higher returns in equilibrium, consistent with the findings of Binsbergen and Koijen (2017) and inducing

[25] The average cost of information is set in this specification so that total information acquisition is equal to the baseline case, just shifted to higher frequencies.

[26] See Binsbergen, Brandt, and Koijen (2012), Collin-Dufresne, and Goldstein (2015), Binsbergen and Koijen (2017), and Hasler and Marfe (2016). Binsbergen and Koijen (2017) empirically study dividend strips with maturities of one to seven years.

downward-sloping Sharpe ratios.

In the benchmark case where investors acquire information at all frequencies, returns have the same variance at all frequencies and horizons, which is inconsistent with the data in Binsbergen and Koijen (2017). The cost specification that increases at low frequencies causes the variance curve to slope upward strongly with frequency, generating more strongly downward-sloping Sharpe ratios, both of which are consistent with the results reported by Binsbergen and Koijen (2017). That result is obtained because the variances of the dividend strips depend on lower frequencies when their maturities are longer (see appendix F.2). Figures A.2 and A.3 report further results and compare the model to the data reported by Binsbergen and Koijen (2017); see appendix F.3.

As an alternative to the frequency-specific cost function, appendix K.3 shows that similar results are obtained when information flows are measured by entropy. The entropy case also is able to match the increasing variance of dividend strip returns across maturities (see figure A.1). In other words, the model is able to match the slope of dividend strip variances without necessarily assuming high low-frequency information costs.

The previous section argues that while a restriction on short-term investment does not affect the informativeness of the level of equity prices on date 1, it does affect the informativeness of differences across dates. Table 1 reports informativeness for both the level and various changes in equity prices over time. For the level, informativeness is measured as the increase in precision from observing prices,

$$\log\left(\frac{\operatorname{var}\left[\sum_{t=1}^{T} D_t \mid P_1^{equity}\right]^{-1}}{\operatorname{var}\left[\sum_{t=1}^{T} D_t\right]^{-1}}\right) \tag{39}$$

Similarly, for the k-period change in equity prices, we report

$$\log\left(\frac{\operatorname{var}\left[\sum_{t=1}^{T} D_t - \sum_{t=k+1}^{T} D_t \mid P_{k+1}^{equity} - P_t^{equity}\right]^{-1}}{\operatorname{var}\left[\sum_{t=1}^{T} D_t - \sum_{t=k+1}^{T} D_t\right]^{-1}}\right) \tag{40}$$

These measures of price informativeness map to the empirical measures of Bai, Philippon, and Savov (2016), who measure price informativeness across horizons based on the fraction of the variation in earnings explained by stock prices (see also Dávila and Parlatore (2018) for a related analysis).

Table 1 shows that the level of equity prices is no less efficient under the short-term investment restriction while the difference in equity prices between the first and second weeks is substantially less efficient. As the length of the difference gets longer, so that it focuses on lower frequencies, the efficiency rises back toward the baseline. Finally, looking at a second difference, which measures the change in price growth across two periods, isolating higher frequencies, the short-term restriction again has measurable effects (see corollary 1.4). The case where low frequencies are more costly to learn about, e.g. because of a tax on low-frequency information acquisition or a subsidy to high-frequency acquisition, leads to precisely the opposite effects.

4 Investor outcomes

This section studies the impacts of the various policies studied above on investor profits and utility. The particular scenario it examines is a decline in the cost of acquiring high-frequency information, which then leads to an increase in high-frequency investment. High-frequency investment and the related policy responses have been an area of active interest, but the results in this section also apply to shifts at other frequencies.

We obtain two main results for outcomes for the sophisticated investors, which initially appear to be in conflict:

1. A rise in short-term investment reduces the profits and utility of long-term investors.

2. Restricting short-term investment further reduces the profits and utility of long-term investors.

So while long-term investors are worse off when short-term investment rises, cutting off short-term investment strategies (the ability to rapidly turn over portfolios) neither restores the old equilibrium, nor does it make the long-term investors better off. Instead, policies that change the cost of information acquisition are better targeted.

The last part of the section examines the implications of the possible policy responses for noise traders, finding that noise traders are best off when prices are most informative.

4.1 Who are short- and long-term sophisticated investors?

We define a short-term investor as one whose portfolio is driven relatively more by high-frequency fluctuations, while a long-term investor holds a portfolio that is driven relatively more by low-frequency fluctuations. That definition can be formalized by a variance decomposition, using the facts

$$Var\left(\tilde{Q}_{i,t}\right) = \sum_{j=1}^{T} Var\left(\tilde{q}_{i,j}\right) \tag{41}$$

$$\text{and } \frac{d}{df_{i,j}^{-1}}\left[Var\left(\tilde{q}_{i,j}\right)\right] > 0 \tag{42}$$

The component of the variance of $\tilde{Q}_{i,t}$ that is driven by fluctuations at frequency j, $Var\left(\tilde{q}_{i,j}\right)$, is increasing in the precision of the signals agent i acquires at frequency j ($f_{i,j}^{-1}$). So if two investors have the same total variance of their positions, $Var\left(\tilde{Q}_{1,t}\right) = Var\left(\tilde{Q}_{2,t}\right)$, but one of them has higher-precision signals at high frequencies, i.e. $f_{1,j}^{-1} > f_{2,j}^{-1}$ for j above some cutoff, then variation in that investor's position is driven relatively more by high-frequency components.

(42) shows that $Var\left(\tilde{q}_{i,j}\right)$ is increasing in the precision of the signals that agent i receives. When an investor has more precise signals at a given frequency, they trade more aggressively for two reasons. First, since their signals are more precise, their demand is more sensitive to their own signals. Second, the quality of their signals also means that they can worry less about adverse selection, so they trade more strongly to accommodate demand shocks from noise traders.

For two investors with positions that have the same unconditional variance, the short–term investor – whose fluctuations happen relatively faster – is the one with relatively more precise signals about the transitory or high-frequency features of fundamentals. That is, *short-term investors have short-term/high-frequency information, and long-term investors have long-term/low-frequency information.* As an extreme case – which is a simplification of the world for the sake of theoretical clarity – we will take short-term investors as people whose signals have positive precision only for j above some cutoff j_{HF}, and long-term investors have signals with positive precision only for j below some j_{LF} with $j_{HF} > j_{LF}$.

4.2 Investor profits and utility

Result 2 *Let $R = D - P$ be the vector of returns in the time domain. Investor i's average discounted profits are*

$$E_{-1}\left[\tilde{Q}_i' R\right] = \sum_{j=1}^{T} (1 - ka_2)\left(-E_{-1}\left[z_j r_j\right]\right) + ka_1 E_{-1}\left[r_j d_j\right] + \rho\left(f_{i,j}^{-1} - f_{avg,j}^{-1}\right) Var_{-1}\left[r_j\right] \tag{43}$$

and expected profits at each frequency are nonnegative,

$$E_{-1}\left[\tilde{q}_{i,j} r_j\right] \geq 0 \text{ for all } i, j \tag{44}$$

with equality only if $f_{i,j}^{-1} = 0$ and $f_{D,j}^{-1} = \rho f_{avg,j}^{-1} f_{Z,j}^{-1} k$ (i.e. in a knife-edge case).

Each investor's expected discounted profits depend on three terms. The first represents the profits earned from noise traders. $E\left[z_j r_j\right] = -a_2 f_Z^{-1} < 0$ since the sophisticated investors imperfectly accommodate their demand. When the noise traders have high demand (that is, when z is high), they drive prices up and expected returns down. The sophisticated investors earn profits from trading with that demand.[27]

The second term represents the profits that the informed investors earn by buying from the noise traders when they have positive signals on average. The coefficient $ka_{1,j}$ represents the slope of the supply curve that the informed investors face.

Finally, the third term in (43) represents a reallocation of profits from the less to the more informed sophisticated investors. An investor who has highly precise signals about fundamentals at frequency j can accurately distinguish periods when prices are high due to strong fundamentals from those when prices are high due to high sentiment. That allows them to earn relatively more profits on average than an uninformed investor.

That said, an uninformed sophisticated investor does not earn negative expected profits at any frequency, even with $f_{i,j}^{-1} = 0$. There are always, except in a knife-edge case, profits to be earned

[27] Note here that we are referring to flow profits, which do not include the cost of information acquisition that investors pay on date -1. We do this partly because flow profits are more readily measurable than the potential fixed costs of setting up information acquisition technologies, and also because flow profits are still relevant in the case where investors face a constraint on information instead of a cost, or where the cost is in terms of utility units instead of money.

by trading with noise traders. Intuitively, this result follows from the separability of the problem across frequencies. The reason that an investor must always earn nonnegative expected profits is that if at some frequency j they did not, then they could simply set $q_{i,j} = 0$, ensuring profits of zero and hence higher utility.[28] An uninformed investor forecasts returns based only on prices, so that knife-edge case represents the condition under which prices alone have no forecasting power for returns, and they set $q_{i,j} = 0$ in all states.

Result 2 yields two key insights. First, all investors, no matter their information, have the ability to earn profits at all frequencies through liquidity provision. Second, all else equal, investors who are informed about a particular frequency earn the most money from investing at these frequencies. Short-term investors – those with relatively more information about high-frequency fundamentals – earn relatively higher returns at high frequencies, while long-term investors earn relatively higher returns at low frequencies.

4.3 The effects of an increase in short-term investment

This section studies the consequences of a decline in the cost of acquiring information at high frequencies for short- and long-term investors, as well as potential policy responses.

4.3.1 Effects on short- and long-term investors

Formally, under the specification of the model where the total cost of information is $\sum_j \psi_j f_{i,j}^{-1}$ (where the baseline is the special case of $\psi_j = \psi$ for all j), the equilibrium condition for information acquisition is

$$f_{avg,j}^{-1} = \begin{cases} \lambda_j^{-1}\left(\psi_j\right) & \text{if } \lambda_j\left(0\right) \geq \psi_j, \\ 0 & \text{otherwise.} \end{cases} \tag{45}$$

(see appendix K.1 for a derivation of the other theoretical results in this case). We examine the effects of a marginal reduction in ψ_j for $j > j_{HF}$ from some point $\psi_j > \lambda_j(0)$ to the point $\psi_j = \lambda_j(0)$ – i.e. exactly where reducing information costs will lead to an initial increase in information acquisition. The investors who acquire information at those frequencies (setting $f_{i,j}^{-1} > 0$ for

[28] Even better, if some $q_{i,j}$ yields negative expected returns, then $-q_{i,j}$ must yield positive expected returns with the same variance.

$j > j_{HF}$) are then the short-term investors, while those who do not, leaving $f_{i,j}^{-1} = 0$ for $j > j_{HF}$, are the long-term investors.

Corollary 2.1 *Starting from a ψ_j such that no investors acquire information at frequency j, a decline in ψ_j that leads to an increase in the equilibrium $f_{avg,j}^{-1}$ reduces profits and utility of an investor for whom $f_{i,j}^{-1}$ remains unchanged. Specifically,*

$$\left.\frac{d}{d\psi_j} E_{-1}\left[\tilde{q}_{LF,j} r_j\right]\right|_{\psi_j = \lambda_j(0)^-} > 0 \tag{46}$$

$$\left.\frac{d}{d\psi_j} E_{-1}\left[\sum_t \tilde{Q}_{LF,t}\left(D_t - P_t\right)\right]\right|_{\psi_j = \lambda_j(0)^-} > 0 \tag{47}$$

$$\left.\frac{d}{d\psi_j} E_{-1}\left[U_{LF,0}\right]\right|_{\psi_j = \lambda_j(0)^-} > 0 \tag{48}$$

where the notation $\psi_j = \lambda_j(0)^-$ indicates the derivative is taken to the left (i.e. comparing $\psi_j = \lambda_j(0)$ to $\psi_j = \lambda_j(0) - \varepsilon$ for a small ε) and the LF subscripts denote positions and utility of a long-term investor who keeps $f_{i,j}^{-1} = 0$ at the affected frequency. Concretely, in an economy populated only by long-term investors who gather no short-term information, an increase in short-term investment – caused by a decline in the cost of short-term information – increases $f_{avg,j}^{-1}$ for $j > j_{HF}$ and therefore reduces the expected profits at those frequencies, total expected profits, and the utility of long-term investors.

The source of that result is the fact that investors with low-frequency information may still invest in the short-run (i.e. have exposures that change from day to day). Suppose, for example, that not only does $f_{LF,j}^{-1} = 0$ for high j, but also that $f_{avg,j}^{-1}$ does – nobody has short-term information. In that setting obviously any sophisticated investor will be happy to accommodate transitory fluctuations in noise trader demand. More concretely, an investor who has information that the long-term value of a stock is \$50 will be willing to provide liquidity in the short-run, buying when the price is below \$50 and selling when the price is higher. That liquidity provision will have high-frequency components when liquidity demand (noise trader demand) has high-frequency components (i.e. $f_{z,j} > 0$ for $j > j_{HF}$). That is, if there are short-run variations in sentiment, then there will be short-run variation in the low-frequency investor's position.

The increase in short-term investment hurts those with low-frequency information because those with high-frequency information are better at providing short-term liquidity. Result 2 and corollary 2.1 formalize that idea and shows that how short-term investors hurt long-term investors – by crowding out their ability to provide liquidity. It is critical to note, though, that result 2 still shows that the increase in high-frequency investment never drives the profits earned by long-term investors below zero, even at high frequencies.

Furthermore, none of this is suboptimal from the perspective of the long-term investors – at the equilibrium, all investors are indifferent between acquiring information and not at any frequency where $\lambda_j(0) > \psi_j$, so their utility is not increased by acquiring more information at high frequencies. Moreover, that indifference also means that the short-term investors – those who actually create the increase in $f_{avg,j}^{-1}$ – also see declines in expected utility. The decline in ψ_j, by increasing information acquisition, makes prices more efficient, leaving less scope for investors to predict returns and earn profits.

The results also do not change the incentives of low-frequency investors to acquire information at low frequencies. While they lose money from a decrease in liquidity provision at high frequencies, their choices at low frequencies are unaffected, so if one's primary concern is price informativeness at low frequencies, the entry of short-term investors will have no effect. There is also nothing special about analyzing a shift in ψ_j at high frequencies – the economic results are the same if the cost of information changes at any frequency.

Nevertheless, there is something of an arms race here in that investor profits are decreasing in the information acquired by other investors. When the cost of high-frequency information falls, somebody will acquire more information, and the investor who does not will earn lower trading profits going forward. Appendix I shows how a reduction in trading costs, which could also be due to a decline in the cost of high-speed communication, similarly leads to an increase in average precision and a sort of arms race. That said, making the arms race idea fully formal would require modeling a speed tournament or some sort of imperfect competition, so the link is at best stylized, but could be an extension of the present framework.

Finally, we also note that the entrance of short-term investors has positive effects on the overall market:

Corollary 2.2 *An increase in high-frequency investment, due to a reduction in ψ_j that increases $f_{avg,j}^{-1}$ for $j > j_{HF}$, increases price informativeness and reduces return volatility at those frequencies. That is, for any frequency*

$$\left.\frac{d}{d\psi_j} Var\left[d_j \mid p_j\right]\right|_{\psi_j=\lambda_j(0)^-} \geq 0 \tag{49}$$

$$\left.\frac{d}{d\psi_j} Var\left[r_j\right]\right|_{\psi_j=\lambda_j(0)^-} \geq 0 \tag{50}$$

So while long-term investors may be hurt by the entry of the short-term investors, to a regulator whose goal is simply to maximize price informativeness or minimize return volatility, the short-term investors are beneficial.[29]

Appendix K.2 discusses the robustness of these results to the constraint versus cost specification for information acquisition. Under the constraint model, profits and utility still fall at the frequencies where ψ_j falls. However, they also weakly rise at the other frequencies, since attention is reallocated away from them. The results for total profits and total utility integrated across frequencies then become ambiguous.

4.3.2 Policy responses

If a decline in the cost of high-frequency information or trading hurts the incumbent long-term investors, a natural question to the incumbents might be how to restore the old equilibrium. We consider three responses that have been proposed: restricting or eliminating short-term investment, taxing transactions (or variation in positions), and limiting the availability of short-term information.

First, consider a restriction on short-term/high-frequency investment that says that no sophisticated investor may set $q_{i,j} \neq 0$ for j above some cutoff, as in the previous section. A concrete example of such a policy is an infrequent batch auction mechanism, similar to Budish, Cramton, and Shim (2015). Restricting investment above the daily frequency would approximately corre-

[29] Under a constraint specification for information, investors would be essentially constrained in their number of analysts, so a reduction in ψ_j would lead to a shift in analysts away from low frequencies. The analysis is further complicated by the fact that in the constraint case there is both that substitution effect and also an income effect, since a reduction in any ψ_j relaxes the constraint that sophisticated investors face. While the first derivative in corollary 2.1 retains its sign in the constraint case, the others in this section become ambiguous.

spond to having an auction once per day. Result 2 shows that such a restriction would, rather than restoring the profits and utility of the long-term investors, actually reduce them *further.* The result follows from the fact that restricting investment eliminates the terms in the summation for j above the cutoff, which are all nonnegative. While short-term investors make liquidity provision at high frequencies more difficult, outlawing short-term investment simply makes it impossible. So eliminating short-term investment does not restore the old equilibrium – it actually compounds the effect of the entrance of short-term investors.

Corollary 2.3 *Limiting short-term investment with a policy that restricts sophisticated investors from holding* $q_{i,j} \neq 0$ *for* $j > j_{HF}$ *weakly reduces the profits and expected utility of all sophisticated investors.*

Imposing a tax on changes in positions, specifically, a tax on $(Q_{i,t} - Q_{i,t-1})^2$, will have similar effects to a restriction on short-term investment in that the tax is most costly for short-term strategies with high turnover. Appendix J formalizes that intuition.

The final policy response would be to somehow limit the acquisition of high-frequency information. In the context of the model, that would represent a restriction on the ability of investors to learn about period-to-period variation in fundamentals, for example by making it more costly to acquire high-frequency information. The most obvious response to a decline in the cost at frequency j is to directly impose a tax that exactly reverses the decline.

In the context of the model, a restriction on information acquisition could in fact exactly restore the equilibrium that exists in the absence of the short-term investors. Since the long-term investors do not acquire high-frequency information, the restriction has no direct effect on them. In terms of the results above, the reason that short-term investors harm long-term investors in the model is that they increase $f_{avg,j}^{-1}$ for high values of j. A policy that makes short-term information more expensive does the opposite, reducing $f_{avg,j}^{-1}$ and shifting the market back to the previous equilibrium.

A specific example of a policy that could make it more costly for investors to acquire high-frequency information might be a reduction in the information that firms freely release. For example, there have been suggestions to change financial reporting requirements so that less short-run information is revealed proposed by the CFA institute (Schacht et al. (2007)) and Brookings Institution (Pozen (2014)). In the UK quarterly earnings reports are no longer mandatory, and Gigler et al. (2014) argue that reducing reporting frequency can reduce managerial biases toward short-term projects. When firms stop reporting quarterly earnings, or providing short-term earnings guidance, they are in a sense making information acquisition more expensive – instead of simply reading and interpreting announcements, investors now must do research to try to measure short-term performance.

To be clear, the claim here is not that markets should be tilted in the direction of long-term investors. Restricting information can help long-term investors in some cases, but it would also have potential negative externalities from reduced price informativeness that have been studied in the literature, making investment decisions worse, making monitoring of firms more difficult, and limiting firms' ability to tie managerial pay to performance (see Bond, Edmans, and Goldstein (2012) for a review). Furthermore, the next section shows that restricting information in the model, even though it helps one class of investors, will hurt others.

That analysis here takes the view that interpreting financial reports is not free – an investment firm must pay an analyst to say how the report affects the conditional expectation of future dividends. An alternative interpretation of a change in mandatory financial reporting is that such reports might represent noisy public signals about fundamentals that are freely available to all investors – that is, interpreting the report is costless. In that case, a decrease in disclosures, rather than corresponding to an increase in ψ_j, would represent decline in the precision of public signals. Appendix G shows that public disclosures reverse the declines in utility and profits following a decline in ψ_j. The effects of disclosure therefore depend on modeling choices, which need to be evaluated empirically. The model is about costly information acquisition and processing, so it is somewhat inconsistent with the general approach to assume that it costless for investors to interpret, for example, financial statements. Nevertheless, the basic pattern of the effects is the same, in that both a tax on information acquisition, raising ψ_j, and an increase in the precision of public signals raises the profits of low-frequency investors and noise traders (as we show in the next section) and reduces the profits of high-frequency investors. A public signal has the added advantage of increasing price informativeness.

4.4 Outcomes for noise traders

The formalization of noise traders used here is that they are investors whose demand depends on an uninformative signal that they erroneously believe forecasts fundamentals. Under that interpretation, a natural objective of a policymaker might be to set policies to try to reduce the losses of these investors. That is, if one thinks that retail investors trade based on sentiment, then one might want to try to limit their losses and keep speculative demand from affecting prices and creating volatility (that is the motivation of the transaction tax in Tobin (1978)).

4.4.1 Noise trader profits

The policies examined in the previous section have direct implications for the losses of noise traders. First, note that the average returns that noise traders earn must be exactly the opposite of what the informed investors earn on average (i.e. equation (43) with $f_{i,j}^{-1} = f_{avg,j}^{-1}$):

Corollary 2.4 *The average earnings of noise traders are*

$$E\left[\sum_{t=1}^{T} \tilde{N}_t R_t\right] = \sum_j \underbrace{\left(\left(1 - k a_{2,j}\right) z_j - k a_1 d_j\right)}_{\textit{Noise trader demand}} \underbrace{\left(\left(1 - a_{1,j}\right) d_j - a_{2,j} z_j\right)}_{\textit{Returns}} \tag{51}$$

$$= -\sum_j \left[a_{2,j}\left(1 - k a_{2,j}\right) f_{Z,j} + k a_{1,j}\left(1 - a_{1,j}\right) f_{D,j}\right] \tag{52}$$

Average noise trader earnings are quadratic in the coefficients determining prices, $a_{1,j}$ and $a_{2,j}$. That is caused by the interaction of two effects. First, when expected returns are more responsive to their demand shocks ($a_{2,j}$ is large) or to fundamentals ($1 - a_{1,j}$ is large), then expected returns vary more, giving more potential for losses. However, variation in prices inhibits their trading since they have downward sloping demand curves, with slope k. So when $1 - k a_{2,j}$ is small or $k a_{1,j}$ is small, losses are smaller.

There are thus two ways to drive the losses of noise traders to zero. One is for prices to be completely informative, with $a_{1,j} = 1$ and $a_{2,j} = 0$ (i.e. $p_j = d_j$). That case is obviously ideal in that noise traders have no losses and prices are also most useful as signals for making decisions. Noise trader losses are zero in this case because informed investors have perfectly elastic demand curves, and will trade any quantity since they know the price is exactly equal to fundamentals.

The second way to reduce noise trader losses to zero is to drive $a_{1,j}$ to zero and $a_{2,j} = k^{-1}$. In that case, prices are completely uninformative, and they move in such a way that there is no trade. This achieves the goal of minimizing noise trader losses, but at the cost of eliminating all information from asset markets. Note, though, that noise trader profits are non-monotonic in both a_1 and a_2, so while we can draw conclusions about the extreme cases of a_1 and a_2 equal to 0 or 1, the exact response of profits to interior values of those coefficients is parameter dependent.

4.4.2 The consequences of restricting investment strategies

The two policies examined above – restricting trade and restricting information – both drive in the direction of the second way to reduce noise trader losses. Restricting all investment by the informed investors at a given frequency eliminates all information from prices, but it also means that the noise traders have nobody to trade with, so their losses are identically zero. Similarly, restricting information, by reducing $f_{avg,j}^{-1}$ to zero, sets $a_{1,j}$ to zero, so that the noise traders have no losses due to the information held by the sophisticated investors (the second part of equation (52)). We also have

$$f_{avg,j} = 0 \Rightarrow 1 - ka_{2,j} = \frac{f_D^{-1}}{\rho^{-1}k + f_D^{-1}} \tag{53}$$

The noise traders will still lose money to the informed investors in general, through the first term in equation (52). As the amount of fundamental uncertainty grows, though – f_D^{-1} shrinks – the losses eventually fall to zero.

So unlike above, for the purpose of protecting noise traders, instead of long-horizon investors, the trading restriction is more effective than the information restriction. The information restriction does not in general reduce the losses of the noise traders to zero, while the trade restriction does. Either policy is only second-best, though, in the sense that they help noise traders by reducing the informativeness of prices and increasing price volatility.

The policy of restricting investment would be most natural if there were some frequencies at which f_Z was particularly large and f_D particularly small. At such a frequency, the information loss from restricting investment is relatively small – in fact it could potentially even be zero if $\lambda_j(0)$ is sufficiently small (since there would also be no information acquisition in the absence of the restriction) – and the benefit is relatively large, since it increases in f_Z (equation (52)). So restrictions on investment make the most sense at frequencies with little variation in fundamentals but substantial variation in sentiment or noise trader demand.

Corollary 2.5 *At any frequency where $f_{Z,j}$ is sufficiently large or $f_{D,j}$ sufficiently small that $\lambda_j(0) \leq \psi$ (recall that $\lambda_j(0)$ represents the marginal value of acquiring information when $f_{avg,j}^{-1} = 0$), there is no information acquisition in equilibrium and prices are completely uninformative. At those frequencies, restricting trade by mandating that $q_{i,j} = 0$ reduces the losses of noise traders to zero and has no effect on price efficiency, since prices are already uninformative.*[30]

A common view is that there is relatively little important economic news at high frequencies since economic decisions such as physical investment depend on relatively long-term expectations. In such a case, one would think that $f_{D,j}$ is small at high frequencies. The results here then show that it would be natural to restrict high-frequency investment since there is no information loss and the effects of noise trader demand or speculation are eliminated. The model here formalizes that common intuition.

4.4.3 The consequences of subsidizing information

On the other hand, if the goal was to reduce noise trader losses without any reduction in price informativeness, then the ideal policy would be one that *increases* $f_{avg,j}^{-1}$. Specifically, as the quantity of information that investors acquire becomes infinite, prices become completely informative:

$$\lim_{f_{avg,j}^{-1} \to \infty} a_{1,j} = 1 \tag{54}$$

$$\lim_{f_{avg,j}^{-1} \to \infty} a_{2,j} = 0 \tag{55}$$

and noise traders have zero average losses:

$$\lim_{f_{avg,j}^{-1} \to \infty} E[n_j r_j] = 0 \tag{56}$$

[30] An alternative model of exogenous demand (which breaks the no-trade theorem) is that instead of having sentiment shocks, agents could simply have exogenous liquidity or hedging needs (see Dávila and Parlatore (2018), for example). In that case, restricting investment at any frequency would be very bad for them, since trading is fundamentally valuable. The optimal policy for agents of that type would be to subsidize information in order to reduce $a_{2,j}$ towards zero, since that would mean that their liquidity needs did not affect prices (e.g. when forced to buy they would not drive prices up). See section 4.4.4.

Increases in $f_{avg,j}^{-1}$ could be encouraged by subsidizing or otherwise encouraging information production by investors (e.g. a tax credit for research). In the context of the discussion in the previous section, this corresponds to actively trying to reduce the ψ_j that investors face (ideally to zero, if the goal is to send $f_{avg,j}^{-1}$ to infinity). More generally, if the noise traders are thought of as speculators, these results say that the effect of speculators is eventually reduced when the sophisticated investors have sufficient information. Certainly an information subsidy would not be costless to implement, and whether its benefits outweigh the costs is theoretically ambiguous.

A closely related policy, and one that might be cheaper than a subsidy for research, is to mandate greater information production by firms, such as more frequent or thorough earnings announcements. Appendix G examines a version of the model in which a public signal is revealed on date 0. It shows that such a revelation reduces noise trader losses (and in this case that result holds across the parameter space, not just in the limit; appendix G illustrates this result with a numerical example). So both methods of increasing price informativeness have the effect of reducing noise trader losses.

Note, interestingly, that the optimal policy for helping noise traders and simultaneously increasing price informativeness is the opposite of what we found above would help the long-term investors. The simple reason is that the profits of the long-term investors are earned at the expense of the noise traders.

In the end, therefore, there is a clear tension in the model among short-term investors, long-term investors, and noise traders. Long-term investors benefit from reductions in f_{avg}^{-1} at high frequencies, but that comes at the cost of reducing price informativeness and hurting noise traders and short-term investors. Noise traders benefit from increasing f_{avg}^{-1} (when f_{avg}^{-1} is sufficiently large, at least), or mandating greater disclosure about fundamentals, but that hurts the informed investors in general, since their trading opportunities shrink. These results follow from the simple fact that these investors are playing a game with zero-sum payoffs. To those who sit outside the financial market, if what matters most is price efficiency, then obviously a policy encouraging greater information acquisition and higher $f_{avg,j}^{-1}$ will be ideal, all else equal.

4.4.4 Noise traders as hedgers

As discussed above, an alternative model of the exogenous demand, Z, is that there is a set of rational investors with an outside investment opportunity that is correlated with the fundamental, D. Their demand, Z, is driven by hedging. Appendix L, using an extension of the model in Wang (1994), shows that all the results up to this point go through essentially unchanged.[31] When the exogenous demand is from hedging, though, instead of sentiment shocks, the welfare implications of the model change. While the model with noise traders is zero-sum, with hedgers it is positive sum since there are gains from trade. The immediate consequence of that fact is that, unlike noise traders, hedgers are not made better off by restrictions on trade – instead, their utility is weakly reduced, just like the speculators (again, because not investing is always an option). In fact, the hedgers are in at least some cases made better off by a greater presence of sophisticated investors, since they then have greater hedging opportunities.

However, like the noise traders, it is possible to show that the hedgers are helped by an information subsidy in at least one sense:

$$\lim_{f_{avg}^{-1}\to\infty} EU_H\left(f_{avg}^{-1}\right) > EU_H\left(0\right) \tag{57}$$

where EU_H is the expected utility of the hedgers. Hedgers are better off when prices are fully informative (where $\lim_{f_{avg}^{-1}\to\infty} a_1 = 1$ and $\lim_{f_{avg}^{-1}\to\infty} a_2 = 0$) compared to when they are uninformative ($a_1 = 0$). Between those two cases, though, the effect of information subsidies is ambiguous. The appendix also shows, though, that the sign of $dEU_H\left(f_{avg}^{-1}\right)/df_{avg}^{-1}$ is positive as $f_{avg}^{-1} \to \infty$, indicating that more information eventually makes hedgers better off, just like the noise traders.

5 Conclusion

The aim of this paper is to understand the effects of policies aimed at reducing "short-termism" in financial markets. It develops results on the effects on price informativeness and investor welfare of restrictions on investment and information acquisition at different frequencies. In order to study those questions, we develop a model in which investors can make meaningful decisions about the

[31] See also Savov (2014) for a related model in which households trade in order to hedge outside income.

horizon of their investment strategies, and in which they face endogenous information choices.

We obtain three main results:

1. Restricting short-term investment affects short-run but not long-run price informativeness and return volatility.

2. Restricting short-term investment hurts both short- and long-term investors, but helps noise traders.

3. Taxing or restricting the availability of short-term information helps long-term investors, hurts short-term investors and noise traders, and reduces short-term price efficiency.

The first result is a natural consequence of the statistical independence of the model across frequencies. The second result shows that while the entry of short-term *investors* reduces the utility and profits of long-term investors, restricting short-term *investment* in response to that entry does not make long-term investors better off. A buy-and-hold investor is able to provide the market short-term liquidity – a person with a price target of $50 should be willing to accommodate transitory demand shocks that drive the price above their target. Short-term investors are better at such liquidity provision; that is why their entry makes long-term investors worse off. But eliminating all short-term investment does not solve the problem. In fact, it makes it worse by eliminating the earnings from liquidity provision for all investors. However, the results for noise traders are reversed – they benefit from restrictions on investment and are hurt by limits on information.

Finally, the third result shows that information policies have distributional effects. If one's goal is to both maximize price informativeness and limit the impact of speculation by noise traders, subsidizing information acquisition can potentially (if the subsidy is sufficiently strong) solve both of those problems. However, since there is not a single accepted model of noise trading, the third result is relatively more delicate. We also examine an alternative specification in which noise traders are replaced by investors with time-varying hedging demand. In that case, it is more difficult to obtain clear predictions for welfare, but the first two main results continue to hold. Furthermore, price efficiency may have positive externalities that are not modeled here, as discussed in Bond, Edmans, and Goldstein (2012).

We do not make normative claims about what the right objective is. There are many externalities not considered here. For example, price informativeness is important to many agents in the economy who are not represented in our model. We also have a specific model of noise traders as irrational agents, but the role of noise trader demand in facilitating trade can also be played by agents who simply have exogenous liquidity needs, in which case the optimal policy response would more clearly tilt towards information subsidies. It is also not obvious whether short- or long-term investors should necessarily be supported. The goal of the paper is not to resolve the question of which policy is best, but rather simply to provide a general analysis of the effects of the various policies.

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A Noise trader demand

We assume that noise traders have preferences similar to those of sophisticates, but they have different information. They receive signals about fundamentals, and believe that the signals are informative, although the signals are actually random. The signals are also perfectly correlated across the noise traders, so that they do not wash out in the aggregate. They can be therefore thought of as common sentiment shocks among noise traders. Furthermore, the noise traders assume that prices contain no information about fundamentals.

The noise traders optimize

$$\max_{\{N_t\}_{t=1}^{T}} T^{-1} \sum_{t=1}^{T} \beta^t N_t E_{0,N} \left[D_t - P_t \right] - \frac{1}{2} (\rho T)^{-1} Var_{0,N} \left[\sum_{t=1}^{T} \beta^t N_t \left(D_t - P_t \right) \right] \tag{58}$$

where N_t is the demand of the noise traders and $E_{0,N}$ and $Var_{0,N}$ are their expectation and variance operators conditional on their signals.

We model the noise traders as being Bayesians who simply misunderstand the informativeness of their signals, and ignore prices. Their prior belief, before receiving signals, is that

$$D \sim N\left(0, \Sigma_N^{prior}\right). \tag{59}$$

They then receive signals that they believe (incorrectly) are of the form

$$S \sim N\left(D, \Sigma_N^{signal}\right). \tag{60}$$

The usual Bayesian update then yields the distribution of D conditional on S,

$$D \quad | \quad S \sim N\left(\Sigma_N \left(\Sigma_N^{signal}\right)^{-1} S, \Sigma_N\right) \tag{61}$$

$$\text{where } \Sigma_N \quad \equiv \quad \left(\left(\Sigma_N^{signal}\right)^{-1} + \left(\Sigma_N^{prior}\right)^{-1}\right)^{-1}. \tag{62}$$

So we have

$$E_{0,N}[D] = \Sigma_N \left(\Sigma_N^{signal}\right)^{-1} S \tag{63}$$

$$Var_{0,N}[D] = \Sigma_N \tag{64}$$

Define $\tilde{N}_t \equiv \beta^t N_t$ and $\tilde{N} = [N_1, ..., N_T]'$. The optimization problem then becomes

$$\max_{\tilde{N}} T^{-1} \tilde{N}' \left(\Sigma_N \left(\Sigma_N^{signal}\right)^{-1} S - P\right) - \frac{1}{2} (\rho T)^{-1} \tilde{N}' \Sigma_N \tilde{N}. \tag{65}$$

This has the solution:

$$\begin{aligned} \tilde{N} &= \rho^{-1} \Sigma_N^{-1} \left(\Sigma_N \left(\Sigma_N^{signal}\right)^{-1} S - P\right) && (66) \\ &= \rho^{-1} \left(\left(\Sigma_N^{signal}\right)^{-1} S - \Sigma_N^{-1} P\right). && (67) \end{aligned}$$

For the sake of simplicity, we assume that $\Sigma_N = k^{-1} I$, where I is the identity matrix and k is a parameter. (This can be obtained, for instance, by assuming that $\Sigma_N^{signal} = \Sigma_N^{prior} = 2kI$). We then have

$$\tilde{N} = \rho^{-1} \left(\Sigma_N^{signal}\right)^{-1} S - kP, \tag{68}$$

so that the vector $Z = (Z_1, ..., Z_T)'$ from the main text is:

$$Z \equiv \rho^{-1} \left(\Sigma_N^{signal}\right)^{-1} S, \tag{69}$$

and the true variance of S, Σ_S, can always be chosen to yield any particular $\Sigma_Z \equiv Var(Z)$ by setting

$$\Sigma_S = \rho^2 \Sigma_N^{signal} \Sigma_Z \Sigma_N^{signal}. \tag{70}$$

B Time horizon and investment

At first glance, the assumption of mean-variance utility over cumulative returns over a long period of time ($T \to \infty$) may appear to give investors an incentive to primarily worry about long-horizon performance, whereas a small value of T would make investors more concerned about short-term performance. In the present setting, that intuition is not correct – the $T \to \infty$ limit determines how detailed investment strategies may be, rather than incentivizing certain types of strategies.

The easiest way to see why the time horizon controls only the detail of the investment strategies is to consider settings in which T is a power of 2 . If $T = 2^k$, then the set of fundamental frequencies is

$$\left\{2\pi j/2^k\right\}_{j=0}^{2^{k-1}} \tag{71}$$

For $T = 2^{k-1}$, the set of frequencies is

$$\left\{2\pi j/2^{k-1}\right\}_{j=0}^{2^{k-2}} = \left\{2\pi\left(2j\right)/2^k\right\}_{j=0}^{2^{k-2}} \tag{72}$$

That is, when T falls from 2^k to 2^{k-1}, the effect is to simply eliminate alternate frequencies. Reducing T does not change the lowest or highest available frequencies (which are always 0 and π, respectively). It just discretizes the $[0, \pi]$ interval more coarsely; or, equivalently, it means that the matrix Λ is constructed from a smaller set of basis vectors.

When T is smaller – there are fewer available basis functions – Q and its frequency domain analog $q \equiv \Lambda' Q$ have fewer degrees of freedom and hence must be less detailed. So the effect of a small value of T is to make it more difficult for an investor to *isolate* particularly short or long run fluctuations in fundamentals (or any other narrow frequency range). But in no way does T cause the investor's portfolio to depend more on one set of frequencies than another.

C Results on the frequency solution

C.1 Proof of lemma 1

The proof here follows "Time Series Analysis" lecture notes of Suhasini Subba Rao. The broad idea of the proof is as follows. Let Σ be any matrix of the form:

$$\Sigma = \begin{pmatrix} \sigma_0 & \sigma_1 & ... & ... & \sigma_{T-1} \\ \\ \sigma_1 & \sigma_0 & \sigma_1 & ... & \sigma_{T-2} \\ \\ ... & ... & ... & ... & ... \\ \\ \sigma_{T-1} & ... & ... & ... & \sigma_0 \end{pmatrix} \tag{73}$$

where $x_0 > 0$. Matrices of this type contain all the variance-covariance matrices of order T of arbitrary weakly stationary processes. The lemma follows from "approximating" Σ by the circulant matrix:

$$\Sigma_{circ} = circ(\sigma_{circ}) \quad , \quad \sigma \equiv (\sigma_0, \sigma_1 + \sigma_{T-1}, \sigma_2 + \sigma_{T-2}, ..., \sigma_{T-2} + \sigma_2, \sigma_{T-1} + \sigma_1)', \tag{74}$$

where, for any real vector $\{x_i\}_{i=0}^{T-1}$,

$$circ(x) \equiv \begin{pmatrix} x_0 & \cdot & \cdot & x_{T-1} \\ x_{T-1} & x_0 & \cdot & x_{T-2} \\ \cdot & & & \\ x_1 & \cdot & \cdot & x_0 \end{pmatrix}. \tag{75}$$

In order to obtain this approximation, we first need the following result.

Appendix lemma 4 *For any matrix Σ of the form given above, and associated circulant matrix Σ_{circ}, the family of vectors Λ defined in the main text exactly diagonalizes Σ_{circ}:*

$$\Sigma_{circ}\Lambda = \Lambda diag\left(\left\{f_\Sigma\left(\omega_{\lfloor j/2 \rfloor}\right)\right\}_{j=1}^{T}\right), \tag{76}$$

where each distinct eigenvalue in $\left\{f_\Sigma\left(\omega_{\lfloor j/2\rfloor}\right)\right\}_{j=1}^{T}$ *is given by:*

$$f_\Sigma(\omega_h) = \sigma_0 + 2\sum_{t=1}^{T-1}\sigma_t cos(\omega_h t), \quad \omega_h \equiv 2\pi h/T, \tag{77}$$

for some $h = 0, ..., \frac{T}{2}$.

Given that Λ is orthonormal,

$$\Lambda'\Sigma_{circ}\Lambda = diag\left(f_\Sigma\right). \tag{78}$$

The approximate diagonalization of the matrix Σ consists in writing:

$$\Lambda'\Sigma\Lambda = diag\left(f_\Sigma\right) + R_\Sigma, \tag{79}$$

where the $T \times T$ matrix R_Σ is given by:

$$R_\Sigma \equiv \Lambda'\left(\Sigma - \Sigma_{circ}\right)\Lambda. \tag{80}$$

This is an approximation in the sense that R_Σ is generically small. Specifically, it is of order T^{-1} element-wise. The following lemma proves the first result stated in lemma 1 of the main text.

Appendix lemma 5 *For any* $T \geq 2$*, we have:*

$$|R_\Sigma| \leq \frac{4}{\sqrt{T}}\sum_{j=1}^{T-1}|j\sigma_j|, \tag{81}$$

where $|M|$ *denotes the weak matrix norm, as in the main text.*

Proof. Define $\Delta\Sigma = \Sigma_{circ} - \Sigma$. First note that since:

$$\Sigma^{(i,j)} = \begin{cases} \sigma_0 & \text{if } i = j \\ \sigma_{|i-j|} & \text{otherwise} \end{cases}, \tag{82}$$

$$\Sigma_{circ}^{(i,j)} = \begin{cases} \sigma_0 & \text{if } i = j \\ \sigma_{|i-j|} + \sigma_{T-|i-j|} & \text{otherwise} \end{cases}, \tag{83}$$

we have:

$$\Delta\Sigma^{(i,j)} = \begin{cases} 0 & \text{if } i = j \\ \sigma_{T-|i-j|} & \text{otherwise} \end{cases} \tag{84}$$

where $\Sigma^{(i,j)}$ is the (i,j) element of Σ. This means that the matrix $\Delta\Sigma$ has constant and symmetric diagonals. Moreover, the first subdiagonals both contain σ_{T-1}, the second contain σ_{T-2}, and so on. That is,

$$\Delta\Sigma = \begin{pmatrix} 0 & \sigma_{T-1} & \sigma_{T-2} & & \sigma_2 & \sigma_1 \\ \sigma_{T-1} & \ddots & \ddots & \ddots & & \sigma_2 \\ \sigma_{T-2} & \ddots & \ddots & \ddots & \ddots & \\ & \ddots & \ddots & \ddots & \ddots & \sigma_{T-2} \\ \sigma_2 & & \ddots & \ddots & \ddots & \sigma_{T-1} \\ \sigma_1 & \sigma_2 & & \sigma_{T-2} & \sigma_{T-1} & 0 \end{pmatrix} \tag{85}$$

Therefore,

$$\sum_{i=1}^{T}\sum_{j=1}^{T} |\Delta\sigma_{i,j}| = 2\sum_{j=1}^{T-1} |j\sigma_j|. \tag{86}$$

Let λ_k denote the k-th column of the matrix Λ. For any $(l,m) \in [1,T]^2$, we have:

$$\begin{aligned} \left|R_\Sigma^{(l,m)}\right| &= \left|\lambda_l' \Delta\Sigma \lambda_m\right| \\ &= \left|\sum_{i=1}^{T}\sum_{j=1}^{T} \lambda_{i,l}\lambda_{j,m}\Delta\sigma_{i,j}\right| \\ &\leq \sum_{i=1}^{T}\sum_{j=1}^{T} |\lambda_{i,l}|\,|\lambda_{j,m}|\,|\Delta\sigma_{i,j}| \\ &\leq \sum_{i=1}^{T}\sum_{j=1}^{T} \frac{\sqrt{2}}{\sqrt{T}}\frac{\sqrt{2}}{\sqrt{T}}|\Delta\sigma_{i,j}| \\ &= \tfrac{4}{T}\sum_{j=1}^{T-1} |j\sigma_j|. \end{aligned} \tag{87}$$

This implies that:

$$||R_\Sigma||_\infty \leq \frac{4}{T}\sum_{j=1}^{T-1} |j\sigma_j|, \tag{88}$$

where $||.||_\infty$ is the element-wise max norm. The inequality for the weak norm follows from the fact that the weak norm and the element-wise max norm satisfy $|.| \leq \sqrt{T}\,||.||_\infty$. $\blacksquare$

C.2 Derivation of solution 1

To save notation, we suppress the j subscripts indicating frequencies in this section when they are not necessary for clarity. So in this section f_D, for example, is a scalar representing the spectral density of fundamentals at some arbitrary frequency (rather than vectors, which is what the unsubscripted variables represent in the main text).

In this section we solve a general version of the model that allows for a constant component of the supply, denoted s. This can be thought of as the mean aggregate supply of the underlying. The main results implicitly set $s = 0$, but the analysis of equity returns uses nonzero s. We assume that the noise traders' demand curve depends on prices relative to their mean, so that supply does not enter. This is without loss of generality as it is simply a normalization.

C.2.1 Statistical inference

We guess that prices take the form

$$p = a_1 d + a_2 z + a_3 s \tag{89}$$

where s is nonstochastic. The joint distribution of fundamentals, signals, and prices is then

$$\begin{bmatrix} d \\ y_i \\ p - a_3 s \end{bmatrix} \sim N\left(0, \begin{bmatrix} f_D & f_D & a_1 f_D \\ f_D & f_D + f_i & a_1 f_D \\ a_1 f_D & a_1 f_D & a_1^2 f_D + a_2^2 f_Z \end{bmatrix}\right) \tag{90}$$

The expectation of fundamentals conditional on the signal and price is

$$E[d \mid y_i, p] = \begin{bmatrix} f_D & a_1 f_D \end{bmatrix} \begin{bmatrix} f_D + f_i & a_1 f_D \\ a_1 f_D & a_1^2 f_D + a_2^2 f_Z \end{bmatrix}^{-1} \begin{bmatrix} y_i \\ p - a_3 s \end{bmatrix} \tag{91}$$

$$= [1, a_1] \begin{bmatrix} 1 + f_i f_D^{-1} & a_1 \\ a_1 & a_1^2 + a_2^2 f_Z f_D^{-1} \end{bmatrix}^{-1} \begin{bmatrix} y_i \\ p - a_3 s \end{bmatrix} \tag{92}$$

and the variance satisfies

$$\tau_i \equiv Var[d \mid y_i, p]^{-1} = f_D^{-1} \left(1 - \begin{bmatrix} 1 & a_1 \end{bmatrix} \begin{bmatrix} 1 + f_i f_D^{-1} & a_1 \\ a_1 & a_1^2 + a_2^2 f_Z f_D^{-1} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ a_1 \end{bmatrix} \right)^{-1} \tag{93}$$

$$= \frac{a_1^2}{a_2^2} f_Z^{-1} + f_i^{-1} + f_D^{-1} \tag{94}$$

We use the notation τ to denote a posterior precision, while f^{-1} denotes a prior precision of one of the basic variables of the model. The above then implies that

$$E[d \mid y_i, p] = \tau_i^{-1} \left(f_i^{-1} y_i + \frac{a_1}{a_2^2} f_Z^{-1} (p - a_3 s) \right) \tag{95}$$

C.2.2 Demand and equilibrium

The agent's utility function is (where variables without subscripts here indicate vectors),

$$U_i = \max_{\{Q_{i,t}\}} \rho^{-1} E_{0,i} \left[T^{-1} \tilde{Q}_i' (D - P) \right] - \frac{1}{2} \rho^{-2} Var_{0,i} \left[T^{-1/2} \tilde{Q}_i' (D - P) \right] \tag{96}$$

$$= \max_{\{Q_{i,t}\}} \rho^{-1} E_{0,i} \left[T^{-1} \tilde{q}_i' (d - p) \right] - \frac{1}{2} \rho^{-2} Var_{0,i} \left[T^{-1/2} \tilde{q}_i' (d - p) \right] \tag{97}$$

$$= \max_{\{Q_{i,t}\}} \rho^{-1} T^{-1} \sum_j \tilde{q}_{i,j} E_{0,i} [(d_j - p_j)] - \frac{1}{2} \rho^{-2} T^{-1} \sum_j \tilde{q}_{i,j}^2 Var_{0,i} [d_j - p_j], \tag{98}$$

where the last line follows by imposing the asymptotic independence of d across frequencies (we analyze the error induced by that approximation below). The utility function is thus entirely separable across frequencies, with the optimization problem for each $\tilde{q}_{i,j}$ independent from all others.

Taking the first-order condition associated with the last line above for a single frequency (with $\tilde{q}_i$, d, etc. again representing scalars, for any j), we obtain

$$\begin{aligned}
\tilde{q}_i &= \rho\tau_i E\left[d - p \mid y_i, p\right] && (99)\\
&= \rho\left(f_i^{-1} y_i + \frac{a_1}{a_2^2} f_Z^{-1}\left(p - a_3 s\right) - \tau_i p\right) && (100)\\
&= \rho\left(f_i^{-1} y_i + \frac{a_1}{a_2^2} f_Z^{-1}\left(a_1 d + a_2 z\right) - \tau_i\left(a_1 d + a_2 z + a_3 s\right)\right) && (101)
\end{aligned}$$

Summing up all demands and inserting the guess for the price yields

$$\begin{aligned}
-z + k\left(a_1 d + a_2 z\right) + s &= \int_i \rho\left(f_i^{-1} y_i + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right)\left(a_1 d + a_2 z\right) - \tau_i a_3 s\right) di && (102)\\
&= \int_i \rho\left(f_i^{-1} d + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right)\left(a_1 d + a_2 z\right) - \tau_i a_3 s\right) di, && (103)
\end{aligned}$$

where the second line uses the law of large numbers. Matching coefficients on d, z, and s then yields

$$\begin{aligned}
\int_i \rho\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) di &= -a_2^{-1}\left(1 - k a_2\right) && (104)\\
\int_i \rho f_i^{-1} a_1^{-1} + \rho\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) di &= k && (105)
\end{aligned}$$

$$a_3 = \frac{-1}{\rho \int_i \tau_i di} \qquad (106)$$

and therefore

$$\begin{aligned}
k - \int_i \rho f_i^{-1} a_1^{-1} &= a_2^{-1}\left(k a_2 - 1\right) && (107)\\
\int_i \rho f_i^{-1} &= \frac{a_1}{a_2} && (108)
\end{aligned}$$

Now define aggregate precision to be

$$f_{avg}^{-1} \equiv \int_i f_i^{-1} di \qquad (109)$$

We then have

$$\tau_i = \frac{a_1^2}{a_2^2} f_Z^{-1} + f_i^{-1} + f_D^{-1} \tag{110}$$

$$\tau_{avg} \equiv \int \tau_i di = \left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_{avg}^{-1} + f_D^{-1} \tag{111}$$

Inserting the expression for τ_i into (104) yields

$$a_1 = \frac{\tau_{avg} - f_D^{-1}}{\tau_{avg} + \rho^{-1} k} \tag{112}$$

$$a_2 = \frac{a_1}{\rho f_{avg}^{-1}} \tag{113}$$

$$a_3 = \frac{-1}{\rho \tau_{avg}} \tag{114}$$

The expression for a_1 can be written more explicitly as:

$$a_1 = \frac{\tau_{avg} - f_D^{-1}}{\tau_{avg} + \rho^{-1} k} = \frac{\frac{a_1^2}{a_2^2} f_Z^{-1} + f_{avg}^{-1} + f_D^{-1} + \rho^{-1} k - \rho^{-1} k - f_D^{-1}}{\frac{a_1^2}{a_2^2} f_Z^{-1} + f_{avg}^{-1} + f_D^{-1} + \rho^{-1} k} \tag{115}$$

$$= 1 - \frac{\rho^{-1} k + f_D^{-1}}{\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_{avg}^{-1} + \rho^{-1} k + f_D^{-1}}. \tag{116}$$

The expression for a_2 is invalid in the case when $f_{avg}^{-1} = 0$. In that case, we have

$$a_2 = \frac{1}{\rho f_D^{-1} + k}. \tag{117}$$

C.3 Proof of Proposition 1

This section considers the case where supply is set to zero, so that $s = 0$.

We use the notation $\bar{O}$ to mean that, for any matrices A and B,

$$|A - B| = \bar{O}\left(T^{-1/2}\right) \Longleftrightarrow |A - B| \leq bT^{-1/2} \tag{118}$$

for some constant b and for all T. This is a stronger statement than typical big-O notation in that it holds for all T, as opposed to holding only for some sufficiently large T. Standard properties of norms yield the following result. If $|A-B|=\bar{O}\left(T^{-1/2}\right)$ and $|C-D|=\bar{O}\left(T^{-1/2}\right)$, then

$$
\begin{aligned}
|cA-cB| &= \bar{O}\left(T^{-1/2}\right) && (119)\\
\left|A^{-1}-B^{-1}\right| &= \bar{O}\left(T^{-1/2}\right) && (120)\\
|(A+C)-(B+D)| &= \bar{O}\left(T^{-1/2}\right) && (121)\\
|AC-BD| &= \bar{O}\left(T^{-1/2}\right) && (122)
\end{aligned}
$$

In other words, convergence in weak norm carries through under addition, multiplication, and inversion. Following the time domain solution (8), A_1 and A_2 can be expressed as a function of the Toeplitz matrices Σ_D, Σ_Z and Σ_{avg} using those operations. it follows that $|A_1-\Lambda diag\left(a_1\right)\Lambda'|\leq c_1T^{-\frac{1}{2}}$ for some constant c_1, and the same holds for A_2 for some constant c_2.

For the variance of prices, we define

$$
\begin{aligned}
R_1 &\equiv A_1-\Lambda diag\left(a_1\right)\Lambda', && (123)\\
R_2 &\equiv A_2-\Lambda diag\left(a_2\right)\Lambda'. && (124)
\end{aligned}
$$

In what follows, we use the strong norm $\|\cdot\|$, defined as:

$$
\|A\|=\max_{x'x=a}\left(x'A'Ax\right)^{\frac{1}{2}}. \tag{125}
$$

Finally, we use the following property of the weak norm: for any two square matrices A, B of size $T\times T$,

$$
|AB|\leq\sqrt{T}\,|A|\,|B|. \tag{126}
$$

The proof for this inequality is standard and reported at the end of this section. We then have the following:

$$
\begin{aligned}
|Var\left[P-\Lambda p\right]| &= \left|Var\left[(A_1-\Lambda a_1\Lambda')D+(A_2-\Lambda a_2\Lambda')Z\right]\right| && (127)\\
&\leq \left|R_1\Sigma_D R_1'\right|+\left|R_2\Sigma_Z R_2'\right| && (128)\\
&\leq \sqrt{T}\left(\left|R_1\Sigma_D\right|\left|R_1\right|+\left|R_2\Sigma_Z\right|\left|R_2\right|\right) && (129)\\
&\leq \sqrt{T}\left(\left\|\Sigma_D\right\|\left|R_1\right|^2+\left\|\Sigma_Z\right\|\left|R_2\right|^2\right) && (130)\\
&\leq \sqrt{T}K\left(\left|R_1\right|^2+\left|R_2\right|^2\right). && (131)
\end{aligned}
$$

The second line follows from the triangle inequality. The third line comes from property (126). The fourth line uses the fact that for any two square matrices G, H, $||GH|| \leq \|G\|\,|H|$; for a proof, see Gray (2006), lemma 2.3. The last line follows from the assumption that the eigenvalues of Σ_D and Σ_Z are bounded. Indeed, since Σ_D and Σ_Z are symmetric and real, they are Hermitian; following Gray (2006), eq. (2.16), we then have $\|\Sigma_Z\| = \max_t |\alpha_{Z,t}|$ and $\|\Sigma_D\| = \max_t |\alpha_{D,t}|$, where $\alpha_{X,t}$ denotes the eigenvalues of the matrix X.

Given that $|R_1| \leq c_1 T^{-\frac{1}{2}}$ and $|R_2| \leq c_2 T^{-\frac{1}{2}}$, this implies:

$$
\begin{aligned}
|Var\left[P-\Lambda p\right]| &\leq K\sqrt{T}\left(c_1^2+c_2^2\right)T^{-1} && (132)\\
&= c_P T^{-\frac{1}{2}}. && (133)
\end{aligned}
$$

A similar proof establishes the result for $\left|Var\left[\tilde{Q}-\Lambda\tilde{q}\right]\right|$.

To prove inequality (126), note that:

$$
\begin{aligned}
|AB|^2 &= \tfrac{1}{T}\sum_{m,n}\left(\sum_{t=1}^{T} a_{mt}b_{tn}\right)^2\\
&\leq \tfrac{1}{T}\sum_{m,n}\left(\sum_{t=1}^{T} a_{mt}^2\right)\left(\sum_{t=1}^{T} b_{tn}^2\right)\\
&= \tfrac{1}{T}\left(\sum_{m,t} a_{mt}^2\right)\left(\sum_{n,t} b_{nt}^2\right)\\
&= T\left(\tfrac{1}{T}\left(\sum_{m,t} a_{mt}^2\right)\right)\left(\tfrac{1}{T}\left(\sum_{n,t} b_{nt}^2\right)\right)\\
&= T\,|A|^2\,|B|^2,
\end{aligned}
\qquad (134)
$$

so that $|AB| \leq \sqrt{T}\,|A|\,|B|$. In this sequence of inequalities, going from the second to the third line uses the Cauchy-Schwarz inequality.

C.4 Proof of lemma 2

First, since the trace operator is invariant under rotations,

$$tr\left(\Sigma_i^{-1}\right) = \sum_j f_{i,j}^{-1}. \tag{135}$$

The information constraint is linear in the frequency-specific precisions. Investors also face a technical constraint that the elements of $f_{i,j}$ corresponding to paired sines and cosines must have the same value. That is, if $\lfloor j/2 \rfloor = \lfloor k/2 \rfloor$, then $f_{i,j} = f_{i,k}$; this condition is necessary for $\varepsilon_{i,t}$ to be stationary.

Inserting the optimal value of $q_{i,j}$ into the utility function, we obtain

$$E_{-1}\left[U_{i,0}\right] \equiv \frac{1}{2} E\left[T^{-1} \sum_j \tau_{i,j} E\left[d_j - p_j \mid y_{i,j}, p_j\right]^2\right] \tag{136}$$

$U_{i,0}$ is utility conditional on an observed set of signals and prices. $E_{-1}\left[U_{i,0}\right]$ is then the expectation taken over the distributions of prices and signals.

$Var\left[E\left[d_j - p_j \mid y_{i,j}, p_j\right]\right]$ is the variance of the part of the return on portfolio j explained by $y_{i,j}$ and p_j, while $\tau_{i,j}^{-1}$ is the residual variance. The law of total variance says

$$Var\left[d_j - p_j\right] = Var\left[E\left[d_j - p_j \mid y_{i,j}, p_j\right]\right] + E\left[Var\left[d_j - p_j \mid y_{i,j}, p_j\right]\right] \tag{137}$$

where the second term on the right-hand side is just $\tau_{i,j}^{-1}$ and the first term is $E\left[E\left[d_j - p_j \mid y_{i,j}, p_j\right]^2\right]$ since everything has zero mean. The unconditional variance of returns is

$$Var(r_j) = Var\left[d_j - p_j\right] = \left(1 - a_{1,j}\right)^2 f_{D,j} + \frac{a_{1,j}^2}{\left(\rho f_{avg,j}^{-1}\right)^2} f_{Z,j}. \tag{138}$$

So then

$$E_{-1}\left[U_{i,0}\right]=\frac{1}{2}T^{-1}\sum_{j}\left[\left(\left(1-a_{1,j}\right)^{2}f_{D,j}+\frac{a_{1,j}^{2}}{\left(\rho f_{avg}^{-1}\right)^{2}}f_{Z,j}\right)\tau_{i,j}-1\right]. \tag{139}$$

We thus obtain the result that agent i's expected utility is linear in the precision of the signals that they receive (since $\tau_{i,j}$ is linear in $f_{i,j}^{-1}$; see equation 110). Now define

$$\lambda_{j}\left(f_{avg,j}^{-1}\right)\equiv(1-a_{1,j})^{2}f_{D,j}+\left(\frac{a_{1,j}}{\rho f_{avg,j}^{-1}}\right)^{2}f_{Z,j}=Var(r_{j}). \tag{140}$$

From equations (111)-(112), when $f_{avg,j}^{-1}>0$, λ_j can be re-written as:

$$\lambda_{j}\left(f_{avg,j}^{-1}\right)=\frac{f_{D,j}\left(f_{D,j}^{-1}+\rho^{-1}k\right)^{2}+(\rho f_{avg,j}^{-1})^{2}f_{Z,j}^{-1}+f_{Z,j}\rho^{-2}}{\left((\rho f_{avg,j}^{-1})^{2}f_{Z,j}^{-1}+f_{D,j}^{-1}+\rho^{-1}k+f_{avg,j}^{-1}\right)^{2}}, \tag{141}$$

which can be further decomposed as:

$$\begin{aligned}\lambda_{j}\left(f_{avg,j}^{-1}\right) &= \frac{1}{\left((\rho f_{avg,j}^{-1})^{2}f_{Z,j}^{-1}+f_{D,j}^{-1}+\rho^{-1}k+f_{avg,j}^{-1}\right)^{2}}\\ &+ \frac{f_{Z,j}-\frac{f_{avg,j}^{-1}}{\rho}}{\left((\rho f_{avg,j}^{-1})^{2}f_{Z,j}^{-1}+f_{D,j}^{-1}+\rho^{-1}k+f_{avg,j}^{-1}\right)^{2}}\\ &+ \frac{\rho^{-1}k(1+f_{D,j}^{-1}\rho^{-1}k)}{\left((\rho f_{avg,j}^{-1})^{2}f_{Z,j}^{-1}+f_{D,j}^{-1}+\rho^{-1}k+f_{avg,j}^{-1}\right)^{2}}\end{aligned} \tag{142}$$

Each of these three terms is decreasing in $f_{avg,j}^{-1}$, so that the function $\lambda_j(\cdot)$ is decreasing.

$$E_{-1}\left[U_{i,0}\right]=\frac{1}{2}T^{-1}\sum_{j}\left[\frac{a_{1}^{2}}{a_{2}^{2}}f_{Z}^{-1}+f_{D}^{-1}\right] \tag{143}$$

D Results on price informativeness with restricted frequencies

D.1 Result 1 and corollaries 1.1 and 1.5

When there are no active investors and just exogenous supply, we have that $0=z_j+kp_j$ and so:

$$p_{j} = k^{-1}z_{j}, \tag{144}$$

$$r_{j} = d_{j}-k^{-1}z_{j}. \tag{145}$$

Because of the separability of information choices across frequencies, the coefficients $a_{1,j}$ and $a_{2,j}$ are unchanged at all other frequencies. Moreover, it is clear that $Var(d_j|p_j) = Var(d_j)$ at the restricted frequencies, since prices now only carry information about supply, which is uncorrelated with dividends.

Note that for any $j \in \mathcal{R}$,

$$Var(r_j) = f_{D,j} + \frac{f_{Z,j}}{k^2}. \tag{146}$$

Additionally, if investors were allowed to hold *exposure* at those frequencies, but the endogenously chose not to allocate any *attention* to the frequency, the return volatility would be:

$$Var_{unrestr.}(r_j) = \lambda_j(0) = f_{D,j} + \frac{f_{Z,j}}{\left(k + \rho f_{D,j}^{-1}\right)^2} < Var(r_j). \tag{147}$$

D.2 Corollary 1.2 and result 1.4

Under the diagonal approximation, we have:

$$D \mid P \sim N\left(\bar{D}, \Lambda diag\left(\tau_0^{-1}\right)\Lambda'\right) \tag{148}$$

where τ_0 is a vector of frequency-specific precisions conditional on prices, as of time 0. Given the independence of prices across frequencies, the j-th element of τ_0 is:

$$\tau_{0,j}^{-1} = Var(d_j \mid p_j). \tag{149}$$

Using this expression, we can compute:

$$\begin{aligned}
Var\left(D_t \mid P\right) &= 1_t'\Lambda diag\left(\tau_0^{-1}\right)\Lambda' 1_t && (150)\\
&= \left(\Lambda' 1_t\right)' diag\left(\tau_0^{-1}\right)\left(\Lambda' 1_t\right) && (151)\\
&= \sum_j \lambda_{t,j}^2 Var(d_j \mid p_j) && (152)\\
&= \lambda_{t,0}^2 Var(d_0|p_0) + \lambda_{t,T}^2 Var(d_{\frac{T}{2}} \mid p_{\frac{T}{2}}) + \sum_{k=1}^{T/2-1}\left(\lambda_{t,2k}^2 + \lambda_{t,2k+1}^2\right) Var(d_k \mid p_k) && (153)
\end{aligned}$$

where 1_t is a vector equal to 1 in its t-th element and zero elsewhere, and $\lambda_{t,j}$ is the t,j element of Λ. The last line follows from the fact that the spectrum has $f_{X,2k} = f_{X,2k+1}$ for $0 < k < T/2 - 1$. Furthermore, note that for $0 < k < T/2 - 1$,

$$\lambda_{t,2k}^2 + \lambda_{t,2k+1}^2 = \frac{2}{T}\cos\left(\omega_k\left(t-1\right)\right)^2 + \frac{2}{T}\sin\left(\omega_k\left(t-1\right)\right)^2 \tag{154}$$

$$= \frac{2}{T} \tag{155}$$

which yields equation (34). Result 3 immediately follows from this expression and the fact that $\lambda_{t,0}^2 = \lambda_{t,T}^2 = \frac{1}{T}$.

Result 1.4 uses the fact that

$$Var\left(D_t - D_{t-1} \mid P\right) = \left(\lambda_{t,1} - \lambda_{t-1,1}\right)^2 \tau_{0,1}^{-1} + \left(\lambda_{t,T} - \lambda_{t-1,T}\right)^2 \tau_{0,\frac{T}{2}}^{-1} \tag{156}$$

$$+ \sum_{k=1}^{T/2-1} \left[\left(\lambda_{t,2k} - \lambda_{t-1,2k}\right)^2 + \left(\lambda_{t,2k+1} - \lambda_{t-1,2k+1}\right)^2\right] \tau_{0,k}^{-1}, \tag{157}$$

and the fact that $(\cos(x) - \cos(y))^2 + (\sin(x) - \sin(y))^2 = 4\sin\left(\frac{1}{2}(x-y)\right)^2 = 2\left(1 - \cos(x-y)\right)$.

E Results on investor outcomes

E.1 Result 2

Expression (41) in the main text follows from the steps used in appendix D.2. Recall from (100) that, omitting the j notation,

$$\tilde{q}_i = \rho\left(f_i^{-1} y_i + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) p\right) \tag{158}$$

$$= \rho f_i^{-1} \varepsilon_i + \rho\left(f_i^{-1} + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) a_1\right) d + \rho\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) a_2 z \tag{159}$$

Recall also that:

$$\tau_i = \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} + f_D^{-1} + f_i^{-1}, \tag{160}$$

so that:

$$\tilde{q}_i = \rho \left(\tau_i - \left(\frac{a_1}{a_2} \right)^2 f_Z^{-1} - f_D^{-1} \right) \varepsilon_i + \rho \left(f_i^{-1} + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i \right) a_1 \right) d + \rho \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i \right) a_2 z \quad (161)$$

Moreover,

$$\begin{aligned} f_i^{-1} - a_1 \tau_i + \left(\frac{a_1}{a_2} \right)^2 f_Z^{-1} &= \tau_i - \left(\frac{a_1}{a_2} \right)^2 f_Z^{-1} - f_D^{-1} - a_1 \tau_i + \left(\frac{a_1}{a_2} \right)^2 f_Z^{-1} && (162) \\ &= (1 - a_1) \tau_i - f_D^{-1}. && (163) \end{aligned}$$

Therefore

$$\rho^{-1} \tilde{q}_i = \left(\tau_i - \left(\frac{a_1}{a_2} \right)^2 f_Z^{-1} - f_D^{-1} \right) \varepsilon_i + \left((1 - a_1) \tau_i - f_D^{-1} \right) d + \left(\frac{a_1}{a_2} f_Z^{-1} - a_2 \tau_i \right) z, \quad (164)$$

so that

$$\rho^{-2} Var\left(\tilde{q}_i\right) = \left(\tau_i - \left(\frac{a_1}{a_2} \right)^2 f_Z^{-1} - f_D^{-1} \right) + \left((1 - a_1) \tau_i - f_D^{-1} \right)^2 f_D + \left(\frac{a_1}{a_2} f_Z^{-1} - a_2 \tau_i \right)^2 f_Z. \quad (165)$$

(where the first term uses the fact that $Var\left(f_i^{-1} \varepsilon_i\right) = f_i^{-1}$). The derivative of this expression with respect to τ_i is:

$$\begin{aligned} \rho^{-2} \frac{\partial Var\left(\tilde{q}_i\right)}{\partial \tau_i} &= 2 \tau_i \left((1 - a_1)^2 f_D + a_2^2 f_Z \right) - 1 \\ &\geq 2 \left(f_D^{-1} + \left(\tfrac{a_1}{a_2} \right)^2 f_Z^{-1} \right) \left((1 - a_1)^2 f_D + a_2^2 f_Z \right) - 1 \\ &= 2 \left((1 - a_1)^2 + a_2^2 f_Z f_D^{-1} + (1 - a_1)^2 \left(\tfrac{a_1}{a_2} \right)^2 f_Z^{-1} f_D + a_1^2 \right) - 1 \\ &= 2 \left(1 - 2 a_1 (1 - a_1) + a_2^2 f_Z f_D^{-1} + (1 - a_1)^2 \left(\tfrac{a_1}{a_2} \right)^2 f_Z^{-1} f_D \right) - 1 && (166) \\ &= 2 \left(-2 a_1 (1 - a_1) + a_2^2 f_Z f_D^{-1} + (1 - a_1)^2 \left(\tfrac{a_1}{a_2} \right)^2 f_Z^{-1} f_D \right) + 1 \\ &= 2 \left((1 - a_1) \left(\tfrac{a_1}{a_2} \right) (f_Z^{-1} f_D)^{\frac{1}{2}} - a_2 (f_Z^{-1} f_D)^{-\frac{1}{2}} \right)^2 + 1 \\ &> 0, \end{aligned}$$

where to go from the first to the second line, we used the fact that $\tau_i \geq \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} + f_D^{-1}$, and where we also used the fact that $a_1 \leq 1$. Since τ_i is a monotonic transformation of f_i^{-1}, this establishes equation (42) from the main text.

For result 2, first note that $E_{-1}\left[\tilde{Q}_i' R\right] = E_{-1}\left[\tilde{q}_i' \Lambda' \Lambda r\right] = E_{-1}\left[\tilde{q}_i r\right] = \sum_j E_{-1}\left[\tilde{q}_{i,j} r_j\right]$, where the last equality follows from the diagonal approximation. Moreover, straightforward but tedious algebra shows that:

$$f_i^{-1} + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) a_1 = \rho(f_i^{-1} - f_{avg}^{-1})(1 - a_1) + k a_1, \tag{167}$$

$$\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) a_2 = -\rho(f_i^{-1} - f_{avg}^{-1}) a_2 + (k a_2 - 1). \tag{168}$$

We can use these expressions, and the fact that $r = (1 - a_1)d - a_2 z$ to re-write $\tilde{q}_i$ as:

$$\tilde{q}_i = \rho f_i^{-1} \varepsilon_i + \rho\left(f_i^{-1} - f_{avg}^{-1}\right) r + k a_1 d + (k a_2 - 1) z. \tag{169}$$

Therefore,

$$E_{-1}\left[\tilde{q}_i r\right] = \rho\left(f_i^{-1} - f_{avg}^{-1}\right) Var(r) + k a_1 E_{-1}[rd] + (k a_2 - 1) E_{-1}[rz], \tag{170}$$

which is the decomposition from result 2.

The result that expected profits are nonnegative is a simple consequence of the investors' objective:

$$\max_{\{\tilde{q}_{i,j}\}} \rho^{-1} T^{-1} \sum_j E_{0,i}\left[\tilde{q}_{i,j}(d_j - p_j)\right] - \frac{1}{2}\rho^{-2} T^{-1} \sum_j Var_{0,i}\left[\tilde{q}_{i,j}(d_j - p_j)\right] \tag{171}$$

Since the variance is linear in $\tilde{q}_{i,j}^2$, if $E_{0,i}\left[\tilde{q}_{i,j} r_j\right] < 0$, utility can always be increased by setting $\tilde{q}_{i,j} = 0$ (or, even more, by reversing the sign of $\tilde{q}_{i,j}$). In order for $E_{-1}\left[\tilde{q}_{i,j} r_j\right] = 0$, it must be the case that $Var_{-1,i}\left[E_{0,i}\left[d_j - p_j\right]\right] = 0$, since any deviation of $E_{0,i}\left[d_j - p_j\right]$ will cause the investor to optimally take a nonzero position. We have, from above,

$$a_1 = \frac{\tau_{avg} - f_D^{-1}}{\tau_{avg} + \rho^{-1}k} = \frac{\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_{avg}^{-1}}{\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_{avg}^{-1} + f_D^{-1} + \rho^{-1}k} \tag{172}$$

$$a_2 = \frac{a_1}{\rho f_{avg}^{-1}} \tag{173}$$

$$\tau_{avg} \equiv \left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_{avg}^{-1} + f_D^{-1} \tag{174}$$

The expression for a_2 is invalid in the case when $f_{avg}^{-1} = 0$. In that case, we have

$$E\left[d \mid y_i, p\right] = \tau_i^{-1}\left(f_i^{-1} y_i + \frac{a_1}{a_2^2} f_Z^{-1} p\right) \tag{175}$$

$$E\left[d - p \mid y_i, p\right] = \tau_i^{-1} f_i^{-1} y_i + \left(\tau_i^{-1}\frac{a_1}{a_2^2} f_Z^{-1} - 1\right)\left(a_1 d + a_2 z\right) \tag{176}$$

$$Var\left[E\left[d - p \mid y_i, p\right]\right] = \left(\tau_i^{-1} f_i^{-1} + \left(\tau_i^{-1}\frac{a_1}{a_2^2} f_Z^{-1} - 1\right) a_1\right)^2 f_D + \left(\tau_i^{-1}\frac{a_1}{a_2^2} f_Z^{-1} - 1\right)^2 a_2^2 f_Z \tag{177}$$

Now first we must have $\tau_i^{-1}\frac{a_1}{a_2^2} f_Z^{-1} - 1 = 0$ in order for the third term to be zero. But if that is true, then for the first term to be zero we must have $f_i^{-1} = 0$ (since τ_i^{-1} is always positive). Combining $f_i^{-1} = 0$ with $\tau_i^{-1}\frac{a_1}{a_2^2} f_Z^{-1} - 1 = 0$, we obtain

$$f_D^{-1} = \rho f_{avg}^{-1} f_Z^{-1} k. \tag{178}$$

E.2 Corollary 2.1

Assume that long-term investors are initially uninformed about the frequency; then $f_i^{-1} = 0$, for all i so:

$$\tau_i = \left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} + f_D^{-1}. \tag{179}$$

Using expression (164), we then have

$$\rho^{-1}\tilde{q}_{LF,i} = \left(\left(1 - a_1\right)\left(\frac{a_1}{a_2}\right)^2 f_Z^{-1} - a_1 f_D^{-1}\right) d + \left(\frac{a_1(1 - a_1)}{a_2} f_Z^{-1} - a_2 f_D^{-1}\right) z. \tag{180}$$

Given that $r = (1 - a_1)d - a_2 z$ and that z and d are independent,

$$\begin{aligned}
\rho^{-1} E_{-1}\left[\tilde{q}_{LF,i} r\right] &= \left((1-a_1)\left(\tfrac{a_1}{a_2}\right)^2 f_Z^{-1} - a_1 f_D^{-1}\right)(1-a_1) f_D - \left(\tfrac{a_1(1-a_1)}{a_2} f_Z^{-1} - a_2 f_D^{-1}\right) a_2 f_Z \\
&= (1-a_1)^2 \left(\tfrac{a_1}{a_2}\right)^2 f_Z^{-1} f_D - 2a_1(1-a_1) + a_2^2 f_Z f_D^{-1} \\
&= \left((1-a_1)\left(\tfrac{a_1}{a_2}\right)(f_Z^{-1} f_D)^{\frac{1}{2}} - a_2 (f_Z f_D^{-1})^{\frac{1}{2}}\right)^2
\end{aligned} \tag{181}$$

$$(f_Z^{-1} f_D)\left((1-a_1)\left(\frac{a_1}{a_2}\right) - a_2 f_Z f_D^{-1}\right)^2 \tag{182}$$

From the equilibrium condition for $f_{avg,j}^{-1}$ stated in the text, a marginal reduction in α_j at $\alpha_j = \lambda_j(0)/\psi$ leads to a marginal increase in $f_{avg,j}^{-1}$, so the signs of the derivatives with respect to α_j are simply the reverse of the signs of the derivatives with respect to $f_{avg,j}^{-1}$. We now calculate derivatives with respect to $f_{avg,j}^{-1}$.

For any $f_{avg}^{-1} > 0$, where $a_1/a_2 = \rho f_{avg}^{-1}$, the derivative of this expression with respect to f_{avg}^{-1} is

$$\begin{aligned}
\rho^{-1} \frac{dE_{-1}\left[\tilde{q}_{LF,i} r\right]}{df_{avg}^{-1}} &= 2\left((1-a_1)\left(\tfrac{a_1}{a_2}\right)(f_Z^{-1} f_D)^{\frac{1}{2}} - a_2 (f_Z f_D^{-1})^{\frac{1}{2}}\right) \\
&\times \left\{\rho\left[(1-a_1)(f_Z^{-1} f_D)^{\frac{1}{2}} - a_1 (f_Z f_D^{-1})^{\frac{1}{2}}\right] - \left[(f_Z^{-1} f_D)^{\frac{1}{2}} + (f_Z f_D^{-1})^{\frac{1}{2}}\right]\rho \tfrac{\partial a_1}{\partial f_{avg}^{-1}} f_{avg}^{-1}\right\}
\end{aligned} \tag{183}$$

Moreover, when $f_{avg}^{-1} > 0$,

$$\frac{\partial a_1}{\partial f_{avg}^{-1}} f_{avg}^{-1} = a_1(1-a_1) + (1-a_1)\frac{(\rho f_{avg}^{-1})^2 f_Z^{-1}}{(\rho f_{avg}^{-1})^2 f_Z^{-1} + f_{avg}^{-1} + f_D^{-1} + \rho^{-1} k}. \tag{184}$$

The following limits follow from the discussion in Appendix C.2.2:

$$\lim_{f_{avg}^{-1} \to 0^+} a_1 = 0, \qquad \lim_{f_{avg}^{-1} \to 0^+} a_2 = \frac{1}{\rho f_D^{-1} + k}. \tag{185}$$

Using these limits and the expressions just derived, we arrive at

$$\lim_{f_{avg}^{-1} \to 0^+} \frac{\partial E_{-1}\left[\tilde{q}_{LF,i} r\right]}{\partial f_{avg}^{-1}} = -2\rho \frac{(f_Z f_D^{-1})^{\frac{1}{2}} (f_Z^{-1} f_D)^{\frac{1}{2}}}{f_D^{-1} + \rho^{-1} k} < 0. \tag{186}$$

Re-introducing the notation j, for the frequency at which entry takes place, we then have

$$\frac{d}{df_{avg,j}^{-1}}E_{-1}\left[\sum_t \tilde{Q}_{LF,t}\left(D_t-P_t\right)\right]=\frac{d}{df_{avg,j}^{-1}}\sum_k E_{-1}\left[\tilde{q}_{LF,k}r_k\right]=\frac{d}{df_{avg,j}^{-1}}E_{-1}\left[\tilde{q}_{LF,j}r_j\right]<0; \tag{187}$$

that is, all the effect of entry on total profits is concentrated on frequency j, where entry reduces profits, as just established.

For the last result, we again use the frequency separability,

$$\frac{d}{df_{avg,j}^{-1}}E_{-1}\left[U_{LF,0}\right]=\frac{d}{df_{avg,j}^{-1}}E_{-1}\left[u_{LF,0,j}\right], \tag{188}$$

where

$$E_{-1}[u_{LF,0,j}]\equiv\frac{1}{2}T^{-1}\left[\left((1-a_{1,j})^2 f_{D,j}+a_{2,j}^2 f_{Z,j}\right)\tau_{i,j}-1\right] \tag{189}$$

is the component of utility from fluctuations at at frequency j. This latter definition uses expression (139), derived in Appendix C.4. Omitting the j notation for clarity, the derivative of this expression with respect to f_{avg} assuming that $f_i^{-1}=0$ is:

$$\begin{aligned}2T\tfrac{dE_{-1}[u_{LF,0}]}{df_{avg}^{-1}} &= \left((1-a_1)^2 f_D+a_1^2(\rho f_{avg}^{-1})^2 f_Z\right)2\rho^2 f_Z^{-1}f_{avg}^{-1}\\ &+\left(-2(1-a_1)\tfrac{\partial a_1}{\partial f_{avg}^{-1}}f_D+2a_1\tfrac{\partial a_1}{\partial f_{avg}^{-1}}\left(\rho f_{avg}^{-1}\right)^2 f_Z+2a_1^2\rho^2 f_Z f_{avg}^{-1}\right)\left(\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1}+f_D^{-1}\right)\end{aligned} \tag{190}$$

Given that:

$$\lim_{f_{avg}^{-1}\to 0^+} a_1=0, \tag{191}$$

the only term in this expression for which the limit may not be 0 as $f_{avg}^{-1}\to 0^+$ is:

$$-2(1-a_1)\frac{\partial a_1}{\partial f_{avg}^{-1}}f_D+2a_1\frac{\partial a_1}{\partial f_{avg}^{-1}}\rho f_{avg}^{-1}f_Z. \tag{192}$$

However, given equation (184), we have that:

$$\lim_{f_{avg}^{-1}\to 0^+}\frac{\partial a_1}{\partial f_{avg}^{-1}}f_{avg}^{-1}=0, \tag{193}$$

and so the second term in (192) goes to 0 as $f_{avg}^{-1}\to 0^+$. For the second term, note that, using

(184) we have that:

$$\frac{\partial a_1}{\partial f_{avg}^{-1}} = \frac{a_1}{f_{avg}^{-1}} + o(1) = \frac{1 + (\rho f_{avg}^{-1}) f_Z^{-1}}{(\rho f_{avg}^{-1})^2 f_Z^{-1} + f_D^{-1} + f_{avg}^{-1} + \rho^{-1} k} + o(1). \tag{194}$$

Therefore,

$$\lim_{f_{avg}^{-1} \to 0^+} 2T \frac{dE_{-1}[u_{LF,0}]}{df_{avg}^{-1}} = -2 \frac{f_D}{f_D^{-1} + \rho^{-1} k} = -2 f_D a_2 < 0, \tag{195}$$

which proves the last statement of corollary 2.1.

E.3 Corollary 2.2

The second inequality follows immediately from the facts proved above that $\frac{d}{df_{avg,j}^{-1}} \lambda_j \left(f_{avg,j}^{-1} \right) < 0$ and $\lambda_j \left(f_{avg,j}^{-1} \right) = Var\left(r_j\right)$. The first inequality follows from the fact proved above that

$$Var\left[d_j \mid p_j\right] = \left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_D^{-1} \tag{196}$$

F Explanation of numerical calibration

Our goal is to calibrate the model to be consistent with the behavior of aggregate stock market dividends at the annual frequency, and also use information about other major economic time series to provide reasonable values for the spectrum at higher frequencies. The reason that we use the annual frequency for dividends is that there are seasonal effects within the year, in that most dividends are paid quarterly, but in different months.

We first calculate the spectrum of annual dividend growth by calculating the periodogram – the squared Fourier transform – of annual data obtained from CRSP.

To obtain information about high frequencies, we use weekly initial unemployment claims. That series is obviously somewhat removed from dividends, but has the advantage of being perhaps the only economic indicator that is available at such high frequencies. It is used, for example, by the Federal Reserve Bank of Philadelphia's real-time business conditions index. It is strongly cyclical, and closely related to the unemployment rate, so we use it is as general measure of economic activity. It also has the advantage that its sample periodogram has a highly similar shape to that of dividend growth at the frequencies where they overlap.

To estimate the spectrum we first shift the level of the periodogram for initial claims so that it has the same mean as that of dividend growth at the frequencies where they overlap.

We estimate the true spectrum as a latent variable. We model it with a Gaussian prior for its log such that the covariance between any pair of frequencies is proportional to $\exp\left(-\phi\left|\omega_1-\omega_2\right|\right)$, where ϕ is a parameter determining the smoothness of the estimated spectrum. The factor of proportionality, denoted σ_P^2, is the prior variance for the level of the log spectrum (we use the log because the log periodogram is homoskedastic).

The two periodograms yield a pair of samples, $\{X_1, F_1\}$ and $\{X_2, F_2\}$, where the X vectors are the sample Fourier frequencies and the F vectors are the values of the log periodogram. Those two samples are stacked into a pair of large vectors, $\bar{X}$ and $\bar{F}$. The prior covariance matrix is then Σ, where the i, j entry is $\sigma_P^2 \exp\left(-\phi\left|\bar{X}_i-\bar{X}_j\right|\right)$. Denote the estimate of the true spectrum as $\hat{F}+b$, where b is a constant.

Technically, the log periodogram is not normal – it is distributed as the log of a $\chi_2^2/2$. We treat it as normal for simplicity, following a quasi-maximum likelihood approach. Denote the estimated spectrum with the vector $\hat{F}$. Then the quasi-log-likelihood, taking into account the prior and the data likelihood, is

$$-\hat{F}\Sigma^{-1}\hat{F}-\left(\hat{F}+b-\bar{F}\right)\Sigma_{samp}^{-1}\left(\hat{F}+b1-\bar{F}\right) \tag{197}$$

where 1 is a vector of 1's and Σ_{samp} is the variance matrix of the log spectrum. This is, given basic properties of the periodogram, the variance of a $\chi_2^2/2$ (see, e.g., Brillinger (1981)).

The first-order condition for b is

$$0 = 1'\Sigma_{samp}^{-1}\left(\hat{F}+b1-\bar{F}\right) \tag{198}$$

$$b = \left(1'\Sigma_{samp}^{-1}1\right)^{-1}1'\Sigma_{samp}^{-1}\left(\hat{F}-\bar{F}\right) \tag{199}$$

Inserting that into the optimization, the first-order condition for $\hat{F}$ is

$$\max_{\hat{F}} -\hat{F}\Sigma^{-1}\hat{F}-\left(\hat{F}+1\left(1'\Sigma_{samp}^{-1}1\right)^{-1}1'\Sigma_{samp}^{-1}\left(\hat{F}-\bar{F}\right)-\bar{F}\right)\Sigma_{samp}^{-1}\left(\hat{F}+1\left(1'\Sigma_{samp}^{-1}1\right)^{-1}1'\Sigma_{samp}^{-1}\left(\hat{F}-\bar{F}\right)-\bar{F}\right) \tag{200}$$

Yielding

$$\hat{F} = \left(\Sigma^{-1} + V\right)^{-1} V\bar{F} \tag{201}$$

$$\text{where } V \equiv \left(I + 1\left(1'\Sigma_{samp}^{-1}1\right)^{-1} 1'\Sigma_{samp}^{-1}\right) \Sigma_{samp}^{-1} \left(I + 1\left(1'\Sigma_{samp}^{-1}1\right)^{-1} 1'\Sigma_{samp}^{-1}\right) \tag{202}$$

Because the set of frequencies, $\bar{X}$, at which we have data is not the same as the set of frequencies in the numerical example, we linearly interpolate from $\hat{F} + b$ to obtain f_D.

F.1 Calculating returns on dividend strips and equity

As noted in the text, the numerical example uses the case where fundamentals (dividends, in this case) are stationary in first differences (see section H for the derivations in that case). Since the model is calibrated to the weekly frequency, a claim on the level of dividends at the end of the first year is a claim to $\sum_{t=1}^{52} \Delta D_t$, where Δ is the first-difference operator. The futures claims are in this case claims to ΔD_t, with prices P_t. The price of the 1-year dividend strip is then $\sum_{t=1}^{52} P_t$. An n-year dividend future, giving claims to the level of dividends at the end of year n is then a claim to $\sum_{t=1}^{52n} \Delta D_t$ with price $\sum_{t=1}^{52n} P_t$. Equity is a claim to dividends in each period. It is straightforward to show that its final payoff is $\sum_{t=1}^{T} (T + 1 - t) \Delta D_t$.

A difficulty with interpreting the returns on these contracts is that they all have different maturities. Note that an individual futures return is a single-period return. The per-period return on a dividend strip is then just the average return on the individual futures,

$$R_n^{period} = \frac{\sum_{t=1}^{52n} (\Delta D_t - P_t)}{52n} \tag{203}$$

The analogous calculation for equity is

$$R_{Equity}^{period} = \frac{\sum_{t=1}^{T} (T + 1 - t) (\Delta D_t - P_t)}{\sum_{t=1}^{T} (T + 1 - t)} \tag{204}$$

We calculate per-period return variances similarly. Specifically,

$$\sigma^2_{period,n} \equiv \frac{\text{var}\left(\sum_{t=1}^{52n}(\Delta D_t - P_t)\right)}{52n} \tag{205}$$

$$\sigma^2_{period,Equity} \equiv \frac{\text{var}\left(\sum_{t=1}^{T}(T+1-t)(\Delta D_t - P_t)\right)}{\sum_{t=1}^{T}(T+1-t)} \tag{206}$$

These values are all multiplied by 52 to put them into annual terms.

To account for the positive average returns on dividend strips and equity, we give them a positive supply in the model (since the sophisticated investors bear that supply, they drive the price down and returns up). We assume that there is a unit supply of equity. Since equity has a payoff of $\sum_{t=1}^{T}(T+1-t)\Delta D_t$, that means that the supply of the claim to date-t dividend growth is $T+1-t$.

F.2 Variance of dividend strip returns

In the nonstationary model, the variance of dividend growth in a single period is

$$Var(\Delta D_t - P_t) = 1_t' \Lambda diag(f_R) \Lambda' 1_t \tag{207}$$

$$= (\Lambda' 1_t)' diag(f_R)(\Lambda' 1_t) \tag{208}$$

$$= \sum_j \lambda_{t,j}^2 Var(d_j - p_j) \tag{209}$$

$$= \lambda_{t,0}^2 Var(d_0 - p_0) + \lambda_{t,T/2}^2 Var(d_{\frac{T}{2}} - p_{\frac{T}{2}}) + \sum_{j=1}^{T/2-1}\left(\lambda_{t,2j}^2 + \lambda_{t,2j+1}^2\right) Var(d_j - p_j) \tag{210}$$

where f_R is the spectrum of returns, $f_R \equiv Var(d_j - p_j)$, 1_t is a vector equal to 1 in its tth element and zero elsewhere, and $\lambda_{t,j}$ is the jth trigonometric transform evaluated at t. This takes advantage of the fact that the variance at the cosine and sine associated with a given frequency must be the same. From here on, we write $r_j \equiv d_j - p_j$.

More generally, then

$$
\begin{aligned}
Var\left(\frac{1}{s}\sum_{m=0}^{s-1}R_{t+m}\right) &= \frac{1}{s^2}\left(\sum_{m=0}^{s-1}1_{t+m}\right)'\Lambda diag\left(f_R\right)\Lambda'\left(\sum_{m=0}^{s-1}1_{t+m}\right) && (211)\\
&= \frac{1}{s^2}\left(\sum_{m=0}^{s-1}\lambda_{t+m,0}\right)^2 f_{R,0}+\frac{1}{s^2}\left(\sum_{m=0}^{s-1}\lambda_{t+m,T/2}\right)^2 f_{R,T/2} && (212)\\
&\quad +\frac{1}{s^2}\sum_{j=1}^{T/2-1}\left[\left(\sum_{m=0}^{s-1}\lambda_{t+m,2j}\right)^2+\left(\sum_{m=0}^{s-1}\lambda_{t+m,2j+1}\right)^2\right]f_{R,j} && (213)
\end{aligned}
$$

For $0<j<T/2$

$$
\left(\sum_{m=0}^{s-1}\lambda_{t+m,j}\right)^2+\left(\sum_{m=0}^{s-1}\lambda_{t+m,j}\right)^2=\sum_{m=0}^{s-1}\sum_{k=0}^{s-1}\frac{2}{T}\left[\begin{array}{c}\cos\left(2\pi j\left(t+m-1\right)/T\right)\cos\left(2\pi j\left(t+k-1\right)/T\right)\\ +\sin\left(2\pi j\left(t+m-1\right)/T\right)\sin\left(2\pi j\left(t+k-1\right)/T\right)\end{array}\right] \tag{214}
$$

Now note that

$$
2\cos\left(x\right)\cos\left(y\right)+2\sin\left(x\right)\sin\left(y\right)=2\cos\left(x-y\right) \tag{215}
$$

So we have

$$
\begin{aligned}
\left(\sum_{m=0}^{s-1}\lambda_{t+m,j}\right)^2+\left(\sum_{m=0}^{s-1}\lambda_{t+m,j}\right)^2 &= \frac{2}{T}\sum_{m=0}^{s-1}\sum_{k=0}^{s-1}\cos\left(\frac{2\pi j}{T}\left(m-k\right)\right) && (216)\\
&= 2\frac{s}{T}\sum_{m=-(s-1)}^{s-1}\frac{s-|m|}{s}\cos\left(\frac{2\pi j}{T}m\right) && (217)
\end{aligned}
$$

$$
=2\frac{s}{T}F_s\left(\frac{2\pi j}{T}\right) \tag{218}
$$

where F_s denotes the sth-order Fejér kernel. Note that when $s=T$, the above immediately reduces to zero, since $\cos\left(2\pi j\right)=0$. That is the desired result, as an average over all dates should be unaffected by fluctuations at any frequency except zero. For $j = 0$,

$$\left(\sum_{m=0}^{s-1} f_{t+m,0}\right)^2 = \left(\sum_{m=0}^{s-1} \sqrt{1/T}\right)^2 \tag{219}$$

$$= \left(s\frac{1}{T^{1/2}}\right)^2 \tag{220}$$

$$= \frac{s}{T}F_s(0), \tag{221}$$

since $F_s(0) = s$ (technically, this holds as a limit: $\lim_{x\to 0} F_s(x) = s$). For $j = T/2$,

$$\left(\sum_{m=0}^{s-1} f_{t+m,T/2}\right)^2 = \frac{1}{T}\left(\sum_{m=1}^{s}(-1)^m\right)^2 = \begin{cases} \frac{1}{T} \text{ for odd } s \\ 0 \text{ otherwise} \end{cases} \tag{222}$$

$$= \frac{s}{T}\frac{1}{s}\left(\frac{\sin(s\pi/2)}{\sin(\pi/2)}\right)^2 = \frac{s}{T}F_s(\pi) \tag{223}$$

So we finally have that

$$Var\left(\frac{1}{s}\sum_{m=0}^{s-1} R_{t+m}\right) = \frac{1}{sT}\left(F_s(0)\, f_{R,0} + \sum_{j=1}^{T/2-1} F_s(\omega_j)\, f_{R,2j} + F_s(\pi)\, f_{R,T/2}\right) \tag{224}$$

In the case where fundamentals are difference-stationary, the return on a claim to the level of fundamentals on date s is exactly $\sum_{t=1}^{s} R_t$.

F.3 Further numerical results

Figures A.1, A.2, and A.3 give further detail in addition to the results reported in the main text. Figure A.1 replicates figure 2, but replacing the case with frequency-specific information costs with a case where information flows are measured by their entropy rather than precision. Appendix K.3 described the analysis for that case. Figures A.2 and A.3 report the mean, standard deviation, and Sharpe ratio of the dividend strips and equity in the model with frequency-specific information costs and entropy costs. They also report the average of the values reported for dividend strips across four markets in Binsbergen and Koijen (2017).

G Public release of information

This section considers a simple extension of the model in which there is a public signal that is revealed on date 0. It has the same structure as the other signals in that it takes the form, at each frequency,

$$\zeta = d + \varepsilon_\zeta \tag{225}$$

$$Var\left(\varepsilon_\zeta\right) = f_\zeta \tag{226}$$

This section examines the effects of varying the precision of that signal, f_ζ^{-1}.

G.1 Statistical inference

We guess that prices take the form

$$p = a_1 d + a_2 z + a_\zeta \zeta \tag{227}$$

$\left(p - a_\zeta \zeta\right)/a_1$ is a signal about the dividend with noise equal to $\left(a_2/a_1\right)z$, which has variance $\left(a_2/a_1\right)^2 f_Z$. The posterior variance of dividends is then

$$\tau_i = \frac{a_1^2}{a_2^2} f_Z^{-1} + f_i^{-1} + f_\zeta^{-1} + f_D^{-1} \tag{228}$$

and the posterior mean is

$$E\left[d - p \mid y_i, p - a_\zeta \zeta\right] = \tau_i^{-1} \frac{a_1^2}{a_2^2} f_Z^{-1} \left(p - a_\zeta \zeta\right) a_1^{-1} + \tau_i^{-1} f_i^{-1} y_i + \tau_i^{-1} f_\zeta^{-1} \zeta - p \tag{229}$$

It will be useful later to calculate the variance of fundamentals conditional on just observing prices, which is

$$Var\left(d \mid p\right) = \frac{a_2^2}{\left(a_1 + a_\zeta\right)^2} f_Z + \frac{a_\zeta^2}{\left(a_1 + a_\zeta\right)^2} f_\zeta \tag{230}$$

G.2 Demand and equilibrium

Agent i's demand is

$$\begin{aligned}
\tilde{q}_i &= \rho\tau_i E\left[d - p \mid y_i, p\right] && (231)\\
&= \rho\left(\frac{a_1^2}{a_2^2} f_Z^{-1}\left(p - a_\zeta\zeta\right) a_1^{-1} + f_i^{-1} y_i + f_\zeta^{-1}\zeta - \tau_i p\right) && (232)\\
&= \rho\left(\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) p + f_i^{-1} y_i + \left(f_\zeta^{-1} - \frac{a_1}{a_2^2} a_\zeta f_Z^{-1}\right)\zeta\right) && (233)
\end{aligned}$$

Summing up all demands and inserting the guess for the price yields

$$\begin{aligned}
-z + k\left(a_1 d + a_2 z + a_\zeta\zeta\right) &= \rho\int_i\left(\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) p + f_i^{-1} y_i + \left(f_\zeta^{-1} - \frac{a_1}{a_2^2} a_\zeta f_Z^{-1}\right)\zeta\right) di && (234)\\
&= \rho\left(\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_{avg}\right)\left(a_1 d + a_2 z + a_\zeta\zeta\right) + f_{avg}^{-1} d + \left(f_\zeta^{-1} - \frac{a_1}{a_2^2} a_\zeta f_Z^{-1}\right)\zeta\right) && (235)
\end{aligned}$$

where the second line uses the law of large numbers. Matching coefficients on d, z, and ζ then yields

$$\begin{aligned}
k &= \rho\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_{avg}\right) + \rho f_{avg}^{-1} a_1^{-1} && (236)\\
-a_2^{-1}\left(1 - k a_2\right) &= \rho\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_{avg}\right) && (237)\\
k &= \rho\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_{avg}\right) + \rho\left(a_\zeta^{-1} f_\zeta^{-1} - \frac{a_1}{a_2^2} f_Z^{-1}\right) && (238)
\end{aligned}$$

$$a_\zeta = f_\zeta^{-1} a_1\left(f_{avg}^{-1} + \frac{a_1^2}{a_2^2} f_Z^{-1}\right)^{-1} \tag{239}$$

$$\frac{a_1}{a_2} = \rho f_{avg}^{-1} \tag{240}$$

$$a_1 = \frac{\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_{avg}^{-1}}{\tau_{avg} + \rho^{-1} k} \tag{241}$$

which implies

$$a_\zeta = \frac{f_\zeta^{-1}}{\tau_{avg} + \rho^{-1} k} \tag{242}$$

G.3 Utility and profits

Utility, as before, is equal to the variance of returns multiplied by precision,

$$E_{-1}\left[u_{i,0}\right] = \lambda_j\left(f_{avg}^{-1}, f_\zeta^{-1}\right)\left(\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_i^{-1} + f_D^{-1} + f_\zeta^{-1}\right) - 1 \tag{243}$$

where $\lambda_j\left(f_{avg}^{-1}, f_\zeta^{-1}\right)$ is the variance of returns, and we write it as a function of f_ζ^{-1} since that is a choice variable of a regulator in this case.

It is straightforward to show that average profits are also linear in $\lambda_j\left(f_{avg}^{-1}, f_\zeta^{-1}\right) \Big(\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_i^{-1} + f_D^{-1} + f_\zeta$ so results on utility will map directly into results on profits (with appropriate adjustments for the cost of information).

G.4 Results

G.4.1 Limits and noise trader profits

The main text considers a limit as the information of investors becomes infinite. Here, that would correspond to setting $f_\zeta^{-1} \to \infty$. That immediately implies $\tau_{avg} \to \infty$, $a_1 \to 0$, $a_2 \to 0$, and $a_\zeta \to 1$. The analog to the first two limits from section 4.4.3 in this case is that prices are perfectly informative in the sense that they depend just on fundamentals, since when $f_\zeta^{-1} \to \infty$, $\zeta = d$, and hence $p = d$.

Noise trader profits are,

$$E\left[\left(z - k\left(\left(a_1 + a_\zeta\right)d + a_2 z + a_\zeta \varepsilon_\zeta\right)\right)\left(\left(1 - a_1 - a_\zeta\right)d - a_2 z - a_\zeta \varepsilon_\zeta\right)\right] \tag{244}$$

$$= -a_2 f_Z - k\left(a_1 + a_\zeta\right)\left(1 - a_1 - a_\zeta\right) f_D + k a_2^2 f_Z + k a_\zeta^2 f_\zeta \tag{245}$$

So when $a_1 = 0$, $a_2 = 0$, and $a_\zeta = 1$, noise trader losses are zero, yielding the third limit from section 4.4.3. Note, again, that this is the opposite of average profits of informed investors, so when $f_\zeta^{-1} \to \infty$, the average profits of informed investors also go to zero.

G.4.2 Information acquisition, profits, and utility

The profits and utility of uninformed investors – the long-term investors in the example in the text, are linear in

$$\lambda_j\left(f_{avg}^{-1}, f_\zeta^{-1}\right)\left(\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_D^{-1} + f_\zeta^{-1}\right) \tag{246}$$

We are interested in changes in f_ζ, which will affect f_{avg}^{-1} in equilibrium. When there is any positive amount of information acquisition, we have $\lambda_j\left(f_{avg}^{-1}, f_\zeta^{-1}\right) = \psi_j$. Taking a total derivative with respect to f_ζ^{-1} (or just invoking the implicit function theorem) yields

$$\frac{df_{avg}^{-1}}{df_\zeta^{-1}} = -\frac{\lambda_{j,2}\left(f_{avg}^{-1}, f_\zeta^{-1}\right)}{\lambda_{j,1}\left(f_{avg}^{-1}, f_\zeta^{-1}\right)} \tag{247}$$

where $\lambda_{j,k}$ denotes the derivative of λ_j with respect to its kth argument.

The derivative of profits when information is being acquired ($\lambda_j\left(f_{avg}^{-1}, f_\zeta^{-1}\right) = \psi_j$) is then

$$\frac{d}{df_\zeta^{-1}}\left[\lambda_j\left(f_{avg}^{-1}, f_\zeta^{-1}\right)\left(\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1} + f_D^{-1} + f_\zeta^{-1}\right)\right] = \psi_j\left(2\rho^2 f_{avg}^{-1} f_Z^{-1}\frac{df_{avg}^{-1}}{df_\zeta^{-1}} + 1\right) \tag{248}$$

When information is not being acquired, $f_{avg}^{-1} = 0$, the derivative becomes

$$\lambda\left(0, f_\zeta^{-1}\right) + \left(f_D^{-1} + f_\zeta^{-1}\right)\lambda_{j,2}\left(0, f_\zeta^{-1}\right) \tag{249}$$

We have the following results

1. Information acquisition is weakly decreasing in f_ζ^{-1}.

– This result is obtained by simply showing that $\lambda_{j,2}\left(f_{avg}^{-1}, f_\zeta^{-1}\right) < 0$.

2. The profits and utility of passive investors increase in disclosure when $f_{avg}^{-1} > 0$.

– This involves simply confirming that (248) is positive.

3. The utility of all sophisticated investors increase in f_ζ^{-1} when $f_{avg}^{-1} > 0$.

– This follows directly from the second result.

4. The average profits of sophisticated investors, and the losses of noise traders, decrease in f_ζ^{-1} when $f_{avg}^{-1} > 0$.

– The derivative of the profits of the average investor, who has signal precision f_{avg}^{-1}, is

$$\psi_j\left(\left(2\rho^2 f_{avg}^{-1} f_Z^{-1}+1\right)\frac{df_{avg}^{-1}}{df_\zeta^{-1}}+1\right) \tag{250}$$

which can be shown to be negative

5. The losses of noise traders and the utility of sophisticated investors converge to zero as $f_\zeta^{-1}\to\infty$.

– See the previous section.

6. The total precision for fundamentals in public information – prices and the public signal – increases in f_ζ^{-1}.

– Public precision is $\left(\rho f_{avg}^{-1}\right)^2 f_Z^{-1}+f_D^{-1}+f_\zeta^{-1}$. The derivative with respect to f_ζ^{-1} is

$$2\rho^2 f_{avg}^{-1} f_Z^{-1}\frac{df_{avg}^{-1}}{df_\zeta^{-1}}+1 \tag{251}$$

which is the same as the derivative used for the second result. When $f_{avg}^{-1}=0$, the result holds trivially.

G.5 Numerical example

We consider a simple numerical example with $\rho=k=f_D^{-1}=f_Z^{-1}=1$ and $\psi=0.4757$ and examine how profits, utility, and price informativeness vary with f_ζ^{-1}. The four panels of figure A.4 plot results from a numerical solution, with f_ζ^{-1} varying along the x-axis. Note that the scales are generally in logs.

The top-left panel plots the profits of the various agents. The dotted line is the expected profits for uninformed sophisticated investors. They initially benefit as information is released publicly since it reduces their informational disadvantage compared to more highly informed agents (at $f_\zeta^{-1}=0$, their profits are not zero, just numerically very small). Eventually f_ζ^{-1} rises sufficiently high that $f_{avg}^{-1}=0$. At that point, more precision for public signals just makes prices more informative and reduces the profits of all sophisticated investors.

The solid line in the top-left panel plots the average profits of a sophisticated investor with the average level of precision, f_{avg}^{-1}. Their profits fall as f_ζ^{-1} rises because they acquire less information

– f_{avg}^{-1} falls. Since profits are zero sum, as their average profits fall, the (negative) average profits of the noise traders rise – they lose less money.

The bottom-left panel of figure A.4 plots f_{avg}^{-1}. It shows that increases in the precision of the public signal reduce incentives for agents to acquire information.

The top-right panel shows that utility initially increases with the public signal – agents are able to trade with the noise traders facing less risk (since they are better informed about fundamentals) without having to pay for private signals. Eventually, though, when there is sufficient information, prices become so efficient that profits and hence utility fall, eventually to the point where there are no profits to be earned.

Finally, the bottom-right panel of figure A.4 reports the information available to investors, either purely from prices or from combining prices and the public signal. In both cases, we see that they rise as the public signal becomes more precise.

H Results when fundamentals are difference-stationary

In the main text, we assume that the level of fundamentals is stationary. Here we examine an extension in which fundamentals are stationary in terms of first differences and show that the results go through nearly identically, with the primary difference being in how the long-term portfolio is defined.

H.1 Informed investors under difference stationarity

We assume that D_0 is known to investors when making decisions, and without loss of generality normalize $D_0 = 0$. Define Δ to be the first difference operator so that

$$\Delta D_t = D_t - D_{t-1} \tag{252}$$

and define the vector $\Delta D \equiv [\Delta D_1, \Delta D_2, ...\Delta D_T]'$. We assume that

$$\Delta D \sim N(0, \Sigma_D). \tag{253}$$

For any given allocation to the futures contracts, there is an allocation to claims on ΔD that gives an identical payoff. Specifically, an allocation $Q_i' D$ can be exactly replicated by

$$\begin{aligned} Q_i' D &= Q_i' L_1 \Delta D && (254) \\ &= \left(L_1' Q_i\right)' \Delta D && (255) \end{aligned}$$

where L_1 is a matrix that creates partial sums,

$$L_1 \equiv \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 1 & 1 & 0 & \\ 1 & 1 & 1 & \\ \vdots & & & \ddots \end{bmatrix} \tag{256}$$

So an allocation of Q_i to the futures is equivalent to an allocation of $L_1' Q_i$ to claims on the first differences of fundamentals, which we will call the growth rate futures. Define the notation

$$Q_{\Delta,i} \equiv L_1' Q_i \tag{257}$$

Furthermore, the prices of the growth rate futures are simply the vector ΔP (by the law of one price). We can therefore rewrite the optimization problem equivalently as

$$\max T^{-1} \sum_{t=1}^{T} \beta^t Q_{\Delta,i,t} E_{0,i} \left[\Delta D_t - \Delta P_t\right] - \frac{1}{2} \left(\rho T^{-1}\right) Var_{0,i} \left[\sum_{t=1}^{T} \beta^t Q_{\Delta,i,t} \left(\Delta D_t - \Delta P_t\right)\right] \tag{258}$$

Now suppose for the moment that we are able to solve the entire model in terms of first differences (that is not obvious as we will need to ensure that noise trader demand is also difference stationary). So we have an allocation $Q_{\Delta D,i}$. An allocation to the first differences is then equivalent to an allocation of $\left(L_1'\right)^{-1} Q_{\Delta,i}$ to the levels (which follows trivially from the definition of $Q_{\Delta,i}$ in (257)).

Since our maintained assumption is that we will solve the model in first differences in the same way we did in the main text for levels, that means that we will continue to use the rotation Λ, but now in first differences. So the frequency domain allocations in terms of first differences will be

$$\tilde{Q}_{\Delta D,i} = \Lambda \tilde{q}_{\Delta,i} \tag{259}$$

where $\tilde{Q}_{\Delta D,i,t} \equiv Q_{\Delta D,i,t}\beta^t$. $\tilde{q}_{\Delta,i}$ now represents the allocations to different frequencies of growth in fundamentals. The key question, then, is what that implies for the behavior of portfolios in terms of levels. We have

$$\begin{aligned} \tilde{Q}_i &= \left(L_1'\right)^{-1} \tilde{Q}_{\Delta,i} && (260) \\ &= \left(L_1'\right)^{-1} \Lambda \tilde{q}_{\Delta,i} && (261) \end{aligned}$$

So in terms of levels, the basis vectors, instead of being Λ, are $\left(L_1'\right)^{-1}\Lambda$.

For $\left(L_1'\right)^{-1}$ we have

$$\left(L_1'\right)^{-1} \equiv \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & & \vdots \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & & & \ddots & -1 \\ 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \tag{262}$$

So the way that $\left(L_1'\right)^{-1}$ transforms a matrix is to take a forward difference of each column, and then retaining the value of the final row. A way to see the implications of that transformation is to approximate the finite differences of the sines and cosines as derivatives. The columns of $\left(L_1'\right)^{-1}\Lambda$ are equal to $\left(L_1'\right)^{-1} c_j$ and $\left(L_1'\right)^{-1} s_j$, which can be written using standard trigonometric formulas as:

$$\begin{aligned} \left(L_1'\right)^{-1} c_j &\approx \begin{bmatrix} 2\sin\left(\frac{1}{2}\omega_j\right)\sqrt{\frac{2}{T}}\left\{\sin\left(\omega_j\left(t-\frac{1}{2}\right)\right)\right\}_{t=2}^{T} \\ \sqrt{\frac{2}{T}}\cos\left(\omega_j(T-1)\right) \end{bmatrix} && (263) \\ \left(L_1'\right)^{-1} s_j &\approx \begin{bmatrix} -2\sin\left(\frac{1}{2}\omega_j\right)\sqrt{\frac{2}{T}}\left\{\cos\left(\omega_j\left(t-\frac{1}{2}\right)\right)\right\}_{t=2}^{T} \\ \sqrt{\frac{2}{T}}\sin\left(\omega_j(T-1)\right) \end{bmatrix} && (264) \end{aligned}$$

The column c_j represents a portfolio in terms of the first differences of fundamentals with weights equal to a cosine fluctuating at frequency ω_j. $\left(L_1'\right)^{-1} c_j$ measures the loadings of that portfolio on claims to the level of fundamentals. These loadings also fluctuate at frequency ω_j, with the only difference being the replacement of the cosine with a sine function. (Intuitive, the loadings are approximately equal to the derivative of the columns of Λ with respect to time; taking derivatives does not affect the characteristic frequency of fluctuations.)

So consider a relatively short-term investor, whose portfolio weights are all close to zero except for a large value in the vector $q_{\Delta,i}$ at some large value of j. By assumption, that investor holds a portfolio whose loadings on the first differences of fundamentals fluctuate at frequency ω_j. What the approximations in (263–264) show, though, is that that investor's positions measured in terms of the *level* of fundamentals (i.e. $\tilde{Q}_i$) has loadings that also fluctuate at frequency ω_j.

One subtlety is in the lowest-frequency portfolio, $(L_1'^{-1}\left(\frac{1}{\sqrt{2}}c_0\right)$. That portfolio puts equal weight on growth in fundamentals on all dates – it is a bet on the sample mean growth rate. In terms of levels, note that $(L_1')^{-1}\left(\frac{1}{\sqrt{2}}c_0\right) = \left[0,0,0,...,\sqrt{2/T}\right]$. A person who wants to bet on the mean growth rate between dates 1 and T can do that by buying a claim to fundamentals only on date T.[32]

H.2 Noise traders under difference stationarity

Last, we need to show that noise trader demand will also take a form such that the entire model can be solved in terms of first differences (and then shifted back into levels for interpretation). First, as above, since the model expressed in first differences is just a linear transformation of the levels, the noise traders' optimization problem can be written in terms of first differences,

$$\max T^{-1}\sum_{t=1}^{T}\beta^t N_{\Delta,t}E_{0,N}\left[\Delta D_t-\Delta P_t\right]-\frac{1}{2}\left(\rho T^{-1}\right)Var_{0,N}\left[\sum_{t=1}^{T}\beta^t N_{\Delta,t}\left(\Delta D_t-\Delta P_t\right)\right] \tag{265}$$

where $N_{\Delta,t}$ is the demand of the noise traders for the claims on first differences.

We assume that the noise traders understand that fundamentals have a unit root and that they therefore have priors and signals that refer to the change in fundamentals. The analogs to (59) and

[32] The highest frequency portfolio, $(L_1'^{-1}\left(\frac{1}{\sqrt{2}}c_{\frac{T}{2}}\right)$, is given by $1/\sqrt{T}\,(2,-2,...,2,1)'$, and therefore fluctuates at the highest sample frequency.

(60) are then

$$\Delta D \sim N\left(0, \Sigma_{N\Delta}^{prior}\right) \tag{266}$$

$$S \sim N\left(\Delta D, \Sigma_{N\Delta}^{signal}\right) \tag{267}$$

and the Bayesian update is

$$\Delta D \quad | \quad S \sim N\left(\Sigma_{N\Delta}\left(\Sigma_{N\Delta}^{signal}\right)^{-1} S, \Sigma_{N\Delta}\right) \tag{268}$$

$$\text{where } \Sigma_{N\Delta} \equiv \left(\left(\Sigma_{N\Delta}^{signal}\right)^{-1} + \left(\Sigma_{N\Delta}^{prior}\right)^{-1}\right)^{-1} \tag{269}$$

I Quadratic trading costs

The restriction that investors have *exactly* zero exposure at certain frequencies is a natural one to study in the model. But there are other ways of imposing limits on investors' exposures across frequencies. This appendix examines the equilibrium when there are quadratic costs of trading. Relative to the frictionless benchmark, introducing these costs has analogous effects to the more abstract restriction $q_{i,j} = 0$ for $j \in \mathcal{R}$. Changes in trading costs could be caused either by the imposition of a quadratic tax on shares traded (i.e. a particular form of a Tobin tax), or by changes in the trading technology. The proofs for this section follow in appendix J

The model does not literally have trade over time. However, the exposures that investors choose in the futures market can be replicated through a commitment to trade (at a fixed price) the fundamental on future dates. That is, define a date-t equity claim to be an asset that pays dividends equal to the fundamental on each date from $t+1$ to T. Since the futures contracts involve exchanging money only at maturity, the date-t cost of an equity claim is $P_t^{equity} = \sum_{j=1}^{T-t} \beta^{-j} P_{t+j}$. An investor's exposure to fundamentals on date t, $Q_{i,t}$ can be acquired either by buying $Q_{i,t}$ units of forwards on date 0 or by holding $Q_{i,t}^{EQ}$ units of equity entering date t. In the latter case, the volume of trade by investor i would be equal to the change in $Q_{i,t}$ over time. That is, $\Delta Q_{i,t}^{EQ} = \Delta Q_{i,t}$.

We assume that investors now maximize the following objective:

$$U_{0,i} = \max_{\{Q_{i,t}\}} E_{0,i}\left[T^{-1}\sum_{t=1}^{T} Q_{i,t}\left(D_t - P_t\right)\right] - \frac{1}{2}cT^{-2}E_{0,i}\left[QV\left\{Q_i\right\}\right] - \frac{1}{2}bT^{-2}E_{0,i}\left[\sum_{t=1}^{T} Q_{i,t}^2\right], \quad (270)$$

where $b > 0$ is a cost of holding large positions in the assets, $c \geq 0$ is a cost incurred from quadratic variation in positions, with quadratic variation defined as:

$$QV\left\{Q_i\right\} \equiv \left[\sum_{t=2}^{T}\left(Q_{i,t} - Q_{i,t-1}\right)^2 + \left(Q_{i,1} - Q_{i,T}\right)^2\right]. \quad (271)$$

The term involving b in (270) replaces the aversion to variance in the benchmark setting. That change is made for the sake of tractability, but its economic consequences are minimal (see, e.g., Kasa, Walker, and Whiteman (2013)). We also set discount rates to zero here to maintain tractability.

Appendix J shows that:

$$T^{-1}QV\left\{Q_i\right\} = 2\sum_{j=1}^{T}\sin^2\left(\omega_{\lfloor j/2\rfloor}/2\right)q_{i,j}^2. \quad (272)$$

Note that we have defined quadratic variation as the sum of the squared changes in $Q_{i,t}$ between $t = 2$ and T plus $\left(Q_{i,1} - Q_{i,T}\right)^2$. Without the final term, there would be no cost to investors of entering and exiting very large positions at the beginning and end of the investment period. This term helps account for that, and has the added benefit of yielding the simple closed-form expression in the frequency domain reported above. The right-hand side shows that the quadratic variation in the volume induced by an investor depends on their squared exposures at each frequency scaled by $\sin^2\left(\omega_{\lfloor j/2\rfloor}/2\right)$, which rises from 0 to 1 as j rises. Intuitively, when $c > 0$, holding exposure to higher frequency fluctuations in fundamentals is more costly because it requires more frequent portfolio rebalancing.

The equilibrium of the model is described in detail in Appendix J. Here, we highlight key results and explain how they relate to the previous results on restricting trade frequencies.

Result 3 *When $c > 0$, all else equal, investors' equilibrium signal precision is higher at lower frequencies.*

With the assumption of fixed quadratic trading costs, the marginal benefit of increasing precision at frequency j is given by:

$$\frac{1}{2}\left(c\sin^2\left(\omega_{\lfloor j/2\rfloor}/2\right)+b\right)^{-1}Var\left[d_j \mid p_j, y_{i,j}\right]^2. \tag{273}$$

In particular, it is declining with both the signal precision and the frequency of exposure. Given that the marginal cost of information is the same across frequencies, investors choose higher signal precisions at lower frequencies, all else equal.

The main result regarding the effect of the quadratic trading cost is the following.

Result 4 *A small increase in trading costs, when starting from zero, reduces information acquisition at all frequencies except frequency* 0*. The effect is larger at higher frequencies. As a corollary, the effect of an increase in trading costs on price informativeness is weaker at longer horizons.*

The first part of this result suggests that if the goal is to reduce short-term investment, then a quadratic tax is a more blunt instrument than placing an explicit restriction on investment at targeted frequencies. A tax on volume affects all investors, regardless of the strategy that they follow. However, the second part of the result says that trading costs affect short-term strategies most strongly. The quadratic cost thus leads, endogenously, to the same changes in information acquisition studied in the main model; namely, the variance of dividends conditional on prices, $Var(d_j|p_j)$, rises more at higher frequencies. The corollary regarding price informativeness refers to the fact that the variance of moving averages of the form:

$$Var\left(\frac{1}{n}\sum_{m=0}^{n-1}D_{t+m} \mid P\right) \tag{274}$$

increases less as a result of the increase in trading costs for longer horizons n. In the extreme case of $n = T$, which corresponds to the frequency 0 component of the signals, the increase in trading costs has in fact *no* effect on equilibrium signal precision and thus price informativeness. This can be seen from the expression for the marginal benefit of signal precision above, which is independent of c when $j = 0$.

Finally, to examine the effects of trading costs on noise trader profits, we have

Result 5 *Prices continue to take the form*

$$p_j = a_{1,j} d_j + a_{2,j} z_j \tag{275}$$

At all frequencies, increases in trading costs weakly reduce $a_{1,j}$ *and strictly increase* $a_{2,j}$ *(except at frequency zero, where they have no effect).*

Again, an increase in trading costs is broadly similar to a restriction on investment in the sense that it makes markets less liquid and prices less informative. By liquid what we mean is that an exogenous demand shock – an increase in z_j – has a larger effect on prices when trading costs are larger. This policy can therefore reduce the losses of noise traders by reducing their overall trade with the informed investors, but again at the cost of less informative prices. As above, if one has evidence that f_Z is large relative to f_D at high frequencies, then this trade-off may be favorable. There is not much to learn about, so losing information has relatively low costs, and since the sentiment shocks are large, inhibiting them is particularly valuable.

Thus, overall, the message of the model with quadratic costs is consistent with the previous analysis. Increasing trading costs leads to less informed trading and the effect is tilted toward high frequencies; at lower frequencies, information acquisition decisions are less impacted. As a result, the effect of the increase on the informativeness of prices for fundamentals in the long run is limited.

J Quadratic costs proofs

J.1 Frequency domain expressions for trading costs

Using $Q_i = \Lambda q_i$, each agent's position at time t can be written as

$$Q_{i,t} = \sum_j \left[\begin{array}{c} q_j \cos(2\pi j t/T) \\ +q_{j'} \sin(2\pi j t/T) \end{array} \right]. \tag{276}$$

Trading costs are then written in terms of $(Q_{i,t} - Q_{i,t-1})^2$ as:

$$QV\{Q_i\} \equiv \sum_{t=2}^{T} (Q_{i,t} - Q_{i,t-1})^2 + (Q_{i,1} - Q_{i,T})^2 . \tag{277}$$

We can write that as

$$QV\left\{Q_i\right\} = (DQ)'(DQ) \tag{278}$$

where D is a matrix that generates first differences,

$$D \equiv \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 1 \\ 1 & 0 & \cdots & 0 & -1 \end{bmatrix}. \tag{279}$$

Using again the fact that $Q_i = \Lambda q_i$,

$$QV\left\{Q_i\right\} = q'\Lambda'D'D\Lambda q \tag{280}$$

In what follows, we will need to evaluate the matrix $\Lambda'D'D\Lambda$. The m, n element of that matrix is the inner product of the m and n columns of $D\Lambda$. Each column of $D\Lambda$ contains the first difference of the corresponding column of Λ, with the exception of the last element, $(D\Lambda)_{m,T}$, which is equal to $\Lambda_{m,t} - \Lambda_{n,T}$. We have the following standard trigonometric results: for $m \neq n$:

$$\begin{aligned}
\sum_{t=1}^{T} \left(\cos\left(\omega_m t\right) - \cos\left(\omega_m\left(t-1\right)\right)\right)\left(\cos\left(\omega_n t\right) - \cos\left(\omega_n\left(t-1\right)\right)\right) &= 0, && (281) \\
\sum_{t=1}^{T} \left(\cos\left(\omega_m t\right) - \cos\left(\omega_m\left(t-1\right)\right)\right)\left(\sin\left(\omega_n t\right) - \sin\left(\omega_n\left(t-1\right)\right)\right) &= 0, && (282) \\
\sum_{t=1}^{T} \left(\sin\left(\omega_m t\right) - \sin\left(\omega_m\left(t-1\right)\right)\right)\left(\sin\left(\omega_n t\right) - \sin\left(\omega_n\left(t-1\right)\right)\right) &= 0, && (283)
\end{aligned}$$

where recall that $\omega_m = \frac{2\pi m}{T}$, and:

$$\sum_{t=1}^{T} \left(\cos\left(\omega_m t\right) - \cos\left(\omega_m \left(t-1\right)\right)\right)^2 = 2T\sin^2\left(\omega_m/2\right), \quad (284)$$

$$\sum_{t=1}^{T} \left(\sin\left(\omega_m t\right) - \sin\left(\omega_m \left(t-1\right)\right)\right)^2 = 2T\sin^2\left(\omega_m/2\right), \quad (285)$$

$$\sum_{t=1}^{T} \left(\cos\left(\omega_m t\right) - \cos\left(\omega_m \left(t-1\right)\right)\right)\left(\sin\left(\omega_m t\right) - \sin\left(\omega_m \left(t-1\right)\right)\right) = 0. \quad (286)$$

These results immediately imply that the off-diagonal elements of $\Lambda' D' D \Lambda$ are equal to zero and the jth element of the main diagonal is $2T\sin^2\left(\omega_{\lfloor j/2 \rfloor}/2\right)$.

We then have

$$QV\left\{Q_i\right\} = q\Lambda' D' D\Lambda q \quad (287)$$

$$= \sum_{j=1}^{T} 2T\sin^2\left(\omega_{\lfloor j/2 \rfloor}/2\right) q_{i,j}^2 \quad (288)$$

Total holding costs can be written as:

$$\sum_{t=1}^{T} Q_t^2 = \sum_{j=1}^{T} q_j^2, \quad (289)$$

which is just Parseval's theorem.

J.2 Equilibrium of the trading cost model

Throughout the analysis, unless it is necessary, we omit the index j of the particular frequency in order to simplify notation.

J.2.1 Investment and equilibrium

The first-order condition for frequency j is

$$
\begin{aligned}
0 &= E\left[d_j - p_j \mid y_{i,j}, p_j\right] - 2c\sin^2\left(\omega_{\lfloor j/2\rfloor}/2\right) q_j - bq_j && (290)\\
q &= \frac{E\left[d_j - p_j \mid y_{i,j}, p_j\right]}{\gamma_j} && (291)\\
&= \gamma_j^{-1}\tau_i^{-1}\left(f_i^{-1} y_i + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) p\right) && (292)
\end{aligned}
$$

where

$$
\gamma_j \equiv 2c\sin^2\left(\omega_{\lfloor j/2\rfloor}/2\right) + b \tag{293}
$$

is the marginal cost of q_j. We can then solve for the coefficients a_1 and a_2 as before.

Inserting the formula for the conditional expectation and integrating across investors yields

$$
\begin{aligned}
\int_i \gamma_j^{-1}\tau_i^{-1}\left(f_i^{-1} y_i + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right)(a_1 d - a_2 z)\right) di &= z_j && (294)\\
\int_i \gamma_j^{-1}\tau_i^{-1}\left(f_i^{-1} d + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right)(a_1 d - a_2 z)\right) di &= z_j && (295)
\end{aligned}
$$

Matching coefficients then yields

$$
\begin{aligned}
\int_i \gamma_j^{-1}\tau_i^{-1}\left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) di &= -a_2^{-1} && (296)\\
\int_i \gamma_j^{-1}\tau_i^{-1}\left(f_i^{-1} + \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_i\right) a_1\right) di &= 0 && (297)
\end{aligned}
$$

Combining those two equations, we obtain

$$
\int_i \gamma_j^{-1}\tau_i^{-1} f_i^{-1} di = \frac{a_1}{a_2} \tag{298}
$$

Now put the definition of τ_i into that equation for f_i^{-1}

$$\int_i \gamma_j^{-1} \tau_i^{-1} \left(\tau_i - \frac{a_1^2}{a_2^2} f_Z^{-1} - f_D^{-1} \right) di = \frac{a_1}{a_2} \tag{299}$$

$$\gamma_j^{-1} \int_i 1 - \left(\frac{a_1^2}{a_2^2} f_Z^{-1} - f_D^{-1} \right) \tau_i^{-1} di = \frac{a_1}{a_2} \tag{300}$$

J.2.2 Expected utility

At any particular frequency,

$$U_{i,j} = q_{i,j} E_{0,i} \left[d_j - p_j \right] - \frac{1}{2} q_{i,j}^2 2c \sin^2 \left(\omega_{\lfloor j/2 \rfloor} / 2 \right) - \frac{1}{2} b q_{i,j}^2 \tag{301}$$

$$= \frac{1}{2} \frac{E \left[d_j - p_j \mid y_{i,j}, p_j \right]^2}{\gamma_j} \tag{302}$$

Expected utility prior to observing signals is then

$$EU_{i,j} \equiv \frac{1}{2} E \left[\frac{E \left[d_j - p_j \mid y_{i,j}, p_j \right]^2}{\gamma_j} \right] \tag{303}$$

$E \left[E \left[d_j - p_j \mid y_{i,j}, p_j \right]^2 \right]$ is the variance of the part of the return on portfolio j explained by $y_{i,j}$ and p_j, while $\tau_{i,j}$ is the residual variance. We know from the law of total variance that

$$Var \left[d_j - p_j \right] = Var \left[E \left[d_j - p_j \mid y_{i,j}, p_j \right] \right] + E \left[Var \left[d_j - p_j \mid y_{i,j}, p_j \right] \right] \tag{304}$$

where the second term on the right-hand side is just $\tau_{i,j}^{-1}$ and the first term is $E \left[E \left[d_j - p_j \mid y_{i,j}, p_j \right]^2 \right]$ since everything has zero mean. The unconditional variance of returns is simply

$$Var \left[d_j - p_j \right] = Var \left[\left(1 - a_1 \right) d_j + a_2 z_j \right] \tag{305}$$

$$= \left(1 - a_{1,j} \right)^2 f_{D,j} + a_2^2 f_{Z,j} \tag{306}$$

So then

$$EU_{i,j} = \frac{1}{2} \frac{Var \left[d_j - p_j \right] - \tau_{i,j}^{-1}}{\gamma_j} \tag{307}$$

What we end up with is that utility is decreasing in $\tau_{i,j}^{-1}$. That is,

$$EU_{i,j} = -\frac{1}{2}\frac{\tau_{i,j}^{-1}}{\gamma_j} + \text{constants.} \tag{308}$$

J.2.3 Information choice

With the linear cost on precision, agents maximize

$$\begin{aligned} & -\frac{1}{2}\frac{\tau_{i,j}^{-1}}{\gamma_j} - \psi f_{i,j}^{-1} && (309) \\ = & -\frac{1}{2}\left(\frac{a_1^2}{a_2^2} f_{Z,j}^{-1} + f_{i,j}^{-1} + f_{D,j}^{-1}\right)^{-1} \gamma_j^{-1} - \psi f_{i,j}^{-1} && (310) \end{aligned}$$

The FOC for $f_{i,j}^{-1}$ is

$$\begin{aligned} \psi &= \frac{1}{2}\tau_{i,j}^{-2}\gamma_j^{-1} && (311) \\ \tau_{i,j} &= \frac{1}{\sqrt{2}}\psi^{-1/2}\gamma_j^{-1/2} && (312) \end{aligned}$$

But τ has a lower bound of $\frac{a_1^2}{a_2^2} f_Z^{-1} + f_D^{-1}$, so it's possible that this has no solution. That would be a state where agents do no learning. Formally,

$$\tau_{i,j} = \max\left(\frac{a_1^2}{a_2^2} f_Z^{-1} + f_D^{-1}, \frac{1}{\sqrt{2}}\psi^{-1/2}\gamma_j^{-1/2}\right) \tag{313}$$

Note that, unlike in the other model, the equilibrium is unique here – all agents individually face a concave problem with an interior solution.

Frequencies with no learning Now using the result for a_1/a_2 from above, at the frequencies where nobody learns, $f_i^{-1} = 0$, we have

$$\begin{aligned} \frac{a_1}{a_2} &= \int_i \gamma_j^{-1}\tau_i^{-1} f_i^{-1} di && (314) \\ &= 0 && (315) \end{aligned}$$

which then implies

$$\tau_{i,j} = \max\left(f_D^{-1}, \frac{1}{\sqrt{2}}\psi^{-1/2}\gamma_j^{-1/2}\right) \tag{316}$$

To get a_2, we have

$$\begin{aligned} \int_i \left(cj^2 + b\right)\tau_i^{-1}\left(\frac{a_1}{a_2^2}f_Z^{-1} - \tau_i\right)di &= -a_2^{-1} && (317) \\ \gamma_j &= a_2 && (318) \end{aligned}$$

So the sensitivity of the price to supply shocks is increasing in the cost of holding inventory, b, and the trading costs, c. It is also higher at higher frequencies – it is harder to temporarily push through supply than to do it persistently.

Frequencies with learning At the frequencies at which there is learning, where

$$f_D^{-1} < \frac{1}{\sqrt{2}}\psi^{-1/2}\gamma_j^{-1/2} \tag{319}$$

we have, just by rewriting the τ equation,

$$f_i^{-1} = \tau_i - \frac{a_1^2}{a_2^2}f_Z^{-1} - f_D^{-1} \tag{320}$$

Using the second equation from above,

$$\begin{aligned} \int_i \gamma_j^{-1}\tau_i^{-1}\left(\frac{a_1}{a_2^2}f_Z^{-1} - \tau_i\right)di &= -a_2^{-1} && (321) \\ \int_i \gamma_j^{-1}\tau_i^{-1}\left(\frac{a_1}{a_2}f_Z^{-1} - a_2\tau_i\right)di &= -1 && (322) \\ \int_i \gamma_j^{-1}\left(\tau_i^{-1}\frac{a_1}{a_2}f_Z^{-1} - a_2\right)di &= -1 && (323) \end{aligned}$$

Under the assumption of a symmetric strategy, this is

$$\begin{aligned} \tau^{-1}\frac{a_1}{a_2}f_Z^{-1} - a_2 &= -\gamma_j && (324) \\ \frac{a_1}{a_2} &= \tau f_Z\left(-\gamma_j + a_2\right) && (325) \end{aligned}$$

Using the other equilibrium condition, we have

$$\int_i \gamma_j^{-1}\tau_i^{-1}\left(\tau_i - \frac{a_1^2}{a_2^2} f_Z^{-1} - f_D^{-1}\right) di = \frac{a_1}{a_2} \tag{326}$$

$$\int_i \gamma_j^{-1}\left(1 - \tau_i^{-1}\frac{a_1}{a_2} f_Z^{-1}\frac{a_1}{a_2} - \tau_i^{-1} f_D^{-1}\right) di = \frac{a_1}{a_2} \tag{327}$$

$$1 - \left(-\gamma_j + a_2\right)\frac{a_1}{a_2} - \tau_i^{-1} f_D^{-1} = \left(cj^2 + b\right)\frac{a_1}{a_2} \tag{328}$$

$$1 - \tau_i^{-1} f_D^{-1} = a_1 \tag{329}$$

Plugging in the formula for τ_i when there is learning,

$$1 - \sqrt{2}\psi^{1/2}\gamma_j^{1/2} f_D^{-1} = a_1. \tag{330}$$

The expression for a_2 can be obtained from:

$$\frac{a_1}{\tau f_Z} = \left(-\gamma_j + a_2\right) a_2. \tag{331}$$

Since $a_1/\tau f_Z > 0$, we know that there is only one solution to this equation for $a_2 > 0$. The positive root is

$$a_2 = \frac{\gamma_j + \sqrt{\gamma_j^2 + 4\frac{a_1}{\tau f_Z}}}{2} \tag{332}$$

K Alternative information cost specifications

This section considers alternative specifications for information costs. In each case, we examine the robustness of all of the paper's theoretical results. The following results hold regardless of the information cost structure:

- Corollaries 1.3 and 1.4 depend only on the properties of the frequency transformation
- Result 2, corollaries 2.3 and 2.4, and the limits for $a_{1,j}$ and $a_{2,j}$ under information subsidies depend only on the properties of the date-0 rational expectations equilibrium (REE).

K.1 Frequency-specific linear costs

This section reports results for the case where the total cost of information is $\sum_{j=1}^{T} \psi_j f_{i,j}^{-1}$ instead of $\sum_{j=1}^{T} \psi f_{i,j}^{-1}$. Expected utility is the same, so it is simple to show that the equilibrium information choices are

$$f_{avg,j}^{-1} = \begin{cases} 0 & \text{if } \psi_j > \lambda_j(0) \\ \\ \lambda_j^{-1}\left(\psi_j\right) & \text{if } \psi_j \leq \lambda_j(0) \end{cases} \tag{333}$$

where, as before $\lambda_j(0) = f_{D,j} + \frac{f_{Z,j}}{\left(\rho f_{D,j}^{-1} + k\right)^2}$.

K.1.1 Result 1 and corollaries 1.1 and 1.2

These results rely on the fact that the equilibrium information choices are independent across frequencies. Since that holds in this case, corollaries 1.1 and 1.2 are unchanged.

K.1.2 Corollary 1.5

This result depends primarily on the date-0 REE. The only change is that at the unrestricted frequencies, the variance of returns is $\min\left(\psi_j, \lambda_j\left(0\right)\right)$ – the cost now has a frequency index.

K.1.3 Corollary 2.1 and 2.2

These results are derived in appendix E.

K.1.4 Corollary 2.5

This result again follows from the separability of the information choice across frequencies and continues to hold with ψ replaced by ψ_j.

K.2 Information capacity constraint

This section examines the case where investors are constrained in the total information they can acquire, rather than facing a linear cost of adding more precision. These problems mathematically are duals of each other, meaning that they coincide holding the parameters fixed. The comparative statics, however, are different in some cases.

K.2.1 Information cost structure

The constraint specification is

$$\max_{\{f_{i,j}^{-1}\}} \sum_{j=1}^{T} \lambda_j \left(f_{avg,j}^{-1}\right) f_{i,j}^{-1} \quad \text{s.t.} \quad \sum_{j=1}^{T} \psi_j f_{i,j}^{-1} \leq C \tag{334}$$

for some C. Denoting the Lagrange multiplier on the information constraint by μ, the equilibrium information choices are

$$f_{avg,j}^{-1} = \begin{cases} 0 & \text{if } \mu > \frac{\lambda_j(0)}{\psi_j} \\ \lambda_j^{-1}\left(\mu\psi_j\right) & \text{if } \mu \leq \frac{\lambda_j(0)}{\psi_j} \end{cases}, \tag{335}$$

where μ is the solution to

$$\sum_{j \text{ s.t. } \mu \leq \frac{\lambda_j(0)}{\psi_j}} \lambda_j^{-1}\left(\mu\psi_j\right) = C. \tag{336}$$

K.2.2 Result 1

The equation for prices at restricted frequencies continues to hold since it depends only on the date-0 REE. The values of $a_{1,j}$ and $a_{2,j}$ at unrestricted frequencies shift in response to the restriction, unlike in the baseline case, due to the lack of complete separability. $a_{1,j}$ weakly rises, while the effect on $a_{2,j}$ is ambiguous.

K.2.3 Corollary 1.1

It remains the case that prices at the restricted frequencies become completely uninformative. At the unrestricted frequencies price informativeness weakly increases, depending on whether attention is reallocated to those frequencies. Specifically, we have the following result.

Lemma 3 (Corollary 1.1, modified) *When investors are restricted from holding portfolios with weights that fluctuate at some set of frequencies $j \in \mathcal{R}$, the prices at those frequencies, p_j, become completely uninformative about dividends. The informativeness of prices for $j \in \mathcal{R}$ about dividends* *weakly increases. More formally, $Var[d_j|p_j]$ for $j \notin \mathcal{R}$ weakly increases following the restriction. For $j \in \mathcal{R}$, $Var[d_j|p_j] = Var[d_j]$.*

Proof. First we have:

$$Var[d_j|p_j] = \frac{1}{f_{D,j}^{-1} + (\rho f_{avg,j}^{-1}) f_{Z,j}^{-1}},$$

so price informativeness is strictly increasing in $f_{avg,j}^{-1}$. Moreover, at any restricted frequency, $f_{avg,j}^{-1} = 0$ so $Var[d_j|p_j] = f_D = Var[d_j]$.

Let μ_{unr} be the marginal value of capacity in the unrestricted case. If $\{j \text{ s.t. } \mu_{unr}\psi_j \leq \lambda_j(0)\} \cup \mathcal{R} = \emptyset$, then the restriction has no effect on information choices, and $\mu_{res} = \mu_{unr}$, where μ_{res} is the marginal value of capacity under the restriction.

Consider the case where $\{j \text{ s.t. } \mu_{unr}\psi_j \leq \lambda_j(0)\} \cap \mathcal{R} \neq \emptyset$. We next show that in that case, $\mu_{res} < \mu_{unr}$.

Assume otherwise, i.e. $\mu_{res} \geq \mu_{unr}$. Then $\forall j$, $\lambda_j^{-1}(\mu_{res}\psi_j) \leq \lambda_j^{-1}(\mu_{unr}\psi_j)$. Moreover, if $\lambda_j(0) \geq \mu_{res}\psi_j$, then $\lambda_j(0) \geq \mu_{unr}\psi_j$. So:

$$\begin{aligned}
\sum_{j \text{ s.t. } \lambda_j(0) \geq \mu_{res}\psi_j, j \notin R} \lambda_j^{-1}\left(\mu_{res}\alpha_j\right) &\leq \sum_{j \text{ s.t. } \lambda_j(0) \geq \mu_{res}\psi_j, j \notin R} \lambda_j^{-1}\left(\mu_{unr}\psi_j\right) \\
&\leq \sum_{j \text{ s.t. } \lambda_j(0) \geq \mu_{unr}\psi_j, j \notin R} \lambda_j^{-1}\left(\mu_{unr}\psi_j\right) \\
&< \sum_{j \text{ s.t. } \lambda_j(0) \geq \mu_{unr}\psi_j} \lambda_j^{-1}\left(\mu_{unr}\psi_j\right) \\
&= C.
\end{aligned} \tag{337}$$

This contradicts optimality in the restricted case (the investors are not exhausting their information budget). Therefore $\mu_{res} < \mu_{unr}$.

So the restriction implies that $\mu_{res} \leq \mu_{unr}$, with equality if and only if no restricted frequencies where being learned about before the restriction. In turn, $\mu_{res} \leq \mu_{unr}$ implies that learning at all unrestricted frequencies weakly increases, using the first-order condition (346). ■

So by contrast with the fixed marginal cost case, where learning is unchanged at unrestricted frequencies, here it goes up weakly, as attention is reallocated toward unrestricted frequencies.

K.2.4 Corollary 1.2

This result changes under the constraint. The properties of the frequency transformation yield

$$Var(D_t|P) = \frac{1}{T}\sum_{j=1}^{T} Var[d_j|p_j] \tag{338}$$

the effect of the restriction is now to increase $Var[d_j|p_j]$ but to weakly reduce it at other frequencies. The net effect on the informativeness of the vector of prices then becomes ambiguous. It remains the case, though, that informativeness on all dates is affected equally.

K.2.5 Corollary 1.5

This result continues to hold but with the modification at the unrestricted frequencies of $\min(\mu_{res}\psi_j, \lambda_j(0))$, where μ_{res} is the Lagrange multiplier in the constrained case. Note that since $f_{avg,j}^{-1}$ weakly increases at the unrestricted frequencies, return variance weakly decreases at those frequencies.

K.2.6 Corollary 2.1

In the case of a constraint, a change in the cost of information acquisition at a particular has both an income and a substitution effect. The substitution effect will cause agents to shift attention from the frequencies whose costs have not fallen to those that fall. The income effect causes agents to (weakly) increase attention on all frequencies, since the constraint relaxes. The consequence is that the first part of the result,

$$\frac{d}{d\psi_j} E_{-1}\left[\tilde{q}_{LF,j} r_j\right]\Big|_{\psi_j=\lambda_j(0)^-} > 0 \tag{339}$$

continues to hold, since it does not depend on anything about the other frequencies. However, the two other inequalities no longer hold.

This corollary gives the clearest motivation for the use of the cost specification instead of the constraint. The constraint specification means that a decline in information acquisition costs does not lead investors to acquire information (other than mechanically) since, by assumption, they cannot.

K.2.7 Corollary 2.5

This result is also unchanged except that "$\lambda_j(0) \leq \psi$" is replaced by "$\lambda_j(0) \leq \mu_{res}\psi_j$".

K.3 Entropy cost for information

This section examines a specification where instead of the cost of information being measured in terms of total precision, it is measured in terms of the joint entropy of the prior and posterior, as in Sims (2003). Specifically, the information flow contained in the signals can be measured by the difference between the prior entropy, which is equal to $\frac{1}{2}\log|\Sigma_D|$ plus a constant, and the posterior entropy, $\frac{1}{2}\log\left|\left(\Sigma_D^{-1}+\Sigma_i^{-1}\right)^{-1}\right|$. As in Kacperczyk, van Nieuwerburgh, and Veldkamp (2016), we exponentiate the entropy. Using the frequency transformation (and ignoring approximation error) and ignoring constants, total information flow is then measured by

$$\prod_{j=1}\left(f_{i,j}^{-1}+f_{D,j}^{-1}\right) \tag{340}$$

K.3.1 Information cost structure and equilibrium

The attention allocation problem with an entropy cost function can be written as:

$$\max_{\left\{f_{i,j}^{-1}\right\}}\sum_{j=1}^{T}\lambda_j\left(f_{avg,j}^{-1}\right)f_{i,j}^{-1}-\kappa\prod_{j=1}^{T}\left(f_{i,j}^{-1}+f_{D,j}^{-1}\right) \tag{341}$$

Assume that there exists j such that:

$$\lambda_j > \kappa\prod_{k\neq j}f_{D,k}^{-1}. \tag{342}$$

Then the problem is unbounded (a value of $+\infty$ can be reached by setting $f_{i,k}^{-1}=0$ for $k\neq j$ and $f_{i,j}^{-1}=+\infty$). Therefore, it must be the case in equilibrium that

$$\lambda_j \leq \kappa\prod_{k\neq j}f_{D,k}^{-1}\quad\forall j. \tag{343}$$

It is straightforward to confirm that optimization requires that investors only allocate attention to a single frequency that achieves the maximum value across all frequencies of

$$\lambda_j - \kappa \prod_{k \neq j} f_{D,k}^{-1} \tag{344}$$

It is possible that there are multiple frequencies with this property. Regardless, each investor only allocates attention to a single frequency. Define

$$\xi \equiv \kappa \prod_{k} f_{D,k}^{-1} \tag{345}$$

The equilibrium information choices are then given by:

$$f_{avg,j}^{-1} = \begin{cases} 0 & \text{if } \frac{\lambda_j(0)}{f_{D,j}} < \xi \\ \lambda_j^{-1}\left(f_{D,j}\xi\right) & \text{if } \frac{\lambda_j(0)}{f_{D,j}} \geq \xi \end{cases}. \tag{346}$$

As in the baseline case, the allocation of precision across investors is indeterminate, up to the fact that investors must learn about at most one frequency.

This version of the model is very similar to the linear cost with heterogeneous frequency-specific cost case, with $f_{D,j}$ playing the role of ψ_j. The model retains the linearity of utility with respect to precision, and the information decisions remain completely separable across frequencies. Those facts mean that all the results go through without any changes except corollary 2.1.

For corollary 2.1, we modify the entropy constraint to make the cost of precision frequency dependent, as

$$\max_{\{f_{i,j}^{-1}\}} \sum_{j=1}^{T} \lambda_j \left(f_{avg,j}^{-1}\right) f_{i,j}^{-1} - \kappa \prod_{j=1}^{T} \left(\psi_j f_{i,j}^{-1} + f_{D,j}^{-1}\right) \tag{347}$$

Then the equilibrium information allocation is

$$f_{avg,j}^{-1} = \begin{cases} 0 & \text{if } \xi > \frac{\lambda_j(0)}{f_{D,j}\psi_j} \\ \lambda_j^{-1}\left(f_{D,j}\xi\psi_j\right) & \text{if } \xi \leq \frac{\lambda_j(0)}{f_{D,j}\psi_j} \end{cases}, \tag{348}$$

At that point, the analysis from the baseline version applies. The only changes are that $\min\left(\psi, \lambda_j(0)\right)$ must be replaced by $\min\left(f_{D,j}\xi, \lambda_j(0)\right)$, where $\xi \equiv \kappa \prod_k f_{D,k}^{-1}$ in corollary 1.5 and $\lambda_j(0) \leq \psi$ should be replaced by $\lambda_j(0) \leq f_{D,j}\xi$, with $\xi \equiv \kappa \prod_k f_{D,k}^{-1}$ in corollary 2.5.

L Hedging model

This section provides the full derivation for an alternative model of "noise traders". The key feature that the model needs in order for there to be trade – i.e. for prices to not be fully revealing – is that there must be shocks to demand for the fundamental that are uncorrelated with the realization of the fundamental. In the main text, those shocks are driven by uninformative signals that a subset of investors erroneously treat as informative. Here, we study an alternative case in which the demand shocks represent hedging demands from a subset of investors. The analysis is similar to that of Wang (1994), and extended to account for the information structure in this paper.

The analysis in this section takes place entirely in the frequency domain and applies to a representative frequency, so we drop the j subscripts.

Suppose there is a set of investors who have a private technology that they can invest in. It has payoffs that are correlated with the fundamental, so that trading the fundamental is useful for hedging purposes. For simplicity, we assume that these investors do not have any other signals about fundamentals. We call these investors the hedgers.

The hedgers have investment opportunities that are imperfectly correlated. Each individual hedger, indexed by h, has an investment opportunity z_h, where the distribution of z_h across the hedgers is $N\left(z, \sigma_z^2\right)$, with $z \sim N\left(0, f_Z\right)$. z and z_h are both random variables drawn on date 0, z is not directly observed by any investor, while z_h is observed by hedger h, but not by any other investors.

Investing a quantity k_h in the project yields a random payoff of $k_h x_h$, where $x_h = z_h + d + \varepsilon_{x,h}$, with $\varepsilon_{x,h} \sim N\left(0, \sigma_x^2\right)$. The inclusion of d as part of the payoff means that the agent can hedge the project by trading the fundamental. z_h is the expected payoff in the absence of any other information, while $\varepsilon_{x,h}$ represents uninsurable risk that investor h faces.

We guess that prices follow

$$p = a_1 d - a_2 z \tag{349}$$

The hedgers' optimization is then over both their investment in the private opportunity, k_h, and their investment in the fundamental, q_h.

$$\max_{k_h,q_h} E\left[k_h x_h + q_h(d-p) \mid p, z_h\right] - \frac{\rho_H^{-1}}{2}\operatorname{var}\left[k_h x_h + q_h(d-p) \mid p, z_h\right] \tag{350}$$

$$= \max_{k_h,q_h} E\left[k_h x_h + q_h\left(E\left[d \mid p, z_h\right] - p\right)\right] - \frac{\rho_H^{-1}}{2}\left(\left(k_h + q_h\right)^2 \operatorname{var}\left[d \mid p, z_h\right] + k_h^2\sigma_x^2\right) \tag{351}$$

L.1 Beliefs

The optimization involves means and variances conditional on z_h and p. It is possible to obtain them in general, but a useful simplification is to assume that hedgers only forecast d using p, not their private investment opportunity, z_h. That corresponds to the limiting case where $\sigma_z \to \infty$, since then each hedger's investment opportunity is minimally informative about the average investment opportunity. So in what follows all expectations and variances condition only on p. We have

$$\operatorname{var}(d \mid p) = f_D\left(1 - \frac{a_1^2 f_D}{a_1^2 f_D + a_2^2 f_Z}\right) = f_D\left(\frac{f_Z}{\frac{a_1^2}{a_2^2} f_D + f_Z}\right) \tag{352}$$

$$E\left[d \mid p\right] = \frac{a_1 f_D}{a_1^2 f_D + a_2^2 f_Z} p \tag{353}$$

L.2 Optimization

Note that

$$k_h x_h + q_h(d-p) = k_h z_h + \left(k_h + q_h\right)(d-p) + k_h \varepsilon_{x,h} + k_h p \tag{354}$$

The optimization problem is then

$$\max_{k_h,q_h} E\left[k_h x_h + q_h(d-p) \mid p\right] - \frac{\rho_H^{-1}}{2}\left(\left(k_h + q_h\right)^2 \operatorname{var}\left[d \mid p\right] + k_h^2\sigma_x^2\right) \tag{355}$$

$$= \max_{k_h,q_h} k_h z_h + k_h p + E\left[\left(k_h + q_h\right)(d-p) \mid p\right] - \frac{\rho_H^{-1}}{2}\left(k_h + q_h\right)^2 \operatorname{var}\left[(d-p) \mid p\right] - \frac{\rho_H^{-1}}{2} k_h^2\sigma_x^2 \tag{356}$$

The two first-order conditions (FOCs) are

$$\rho_H\left(z_h + E\left[d \mid p, z_h\right]\right) = \left(k_h + q_h\right)\operatorname{var}\left(d - p \mid p, z_h\right) + k_h\sigma_x^2 \tag{357}$$

$$\rho_H E\left[d - p \mid p, z_h\right] = \left(k_h + q_h\right)\operatorname{var}\left(d - p \mid p, z_h\right) \tag{358}$$

Subtracting the second equation from the first yields

$$k_h = \frac{\rho_H (z_h + p)}{\sigma_x^2} \tag{359}$$

So, naturally, agents invest more in their private project when its expected return is higher or its risk is lower. $z_h + p$ is the expected return on an investment that is long one unit of the private investment and short one unit of the fundamental, and σ_x^2 is its variance, so this is the standard mean-variance optimal quantity invested.

Combining that result with the FOC for q_h, we have

$$\begin{aligned} q_h &= \frac{\rho_H E[d - p \mid p, z_h]}{\operatorname{var}(d - p \mid p, z_h)} - \frac{\rho_H (z_h + p)}{\sigma_x^2} && (360) \\ &= \rho_H \frac{\left(a_1 f_D - a_1^2 f_D - a_2^2 f_Z\right) p}{f_D a_2^2 f_Z} - \frac{\rho_H (z_h + p)}{\sigma_x^2} && (361) \\ &= \rho_H \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_H - \sigma_x^{-2} \right) p - \rho_H \sigma_x^{-2} z_h && (362) \end{aligned}$$

where τ_H is the precision of the hedgers' beliefs,

$$\tau_H \equiv \operatorname{var}(d \mid p)^{-1} = \frac{a_1^2}{a_2^2} f_Z^{-1} + f_D^{-1} \tag{363}$$

L.3 Expected utility

From above, expected utility conditional on prices and z_h is

$$k_h z_h + k_h p + E[(k_h + q_h)(d - p) \mid p] - \frac{\rho_H^{-1}}{2} (k_h + q_h)^2 \operatorname{var}[(d - p) \mid p, z_h] - \frac{\rho_H^{-1}}{2} k_h^2 \sigma_x^2 \tag{364}$$

Multiplying the k_h and q_h FOCs by k_h and q_h then using them to substitute out the variances, then inserting the solutions for k_h and $k_h + q_h$, utility becomes

$$\begin{aligned} \frac{1}{2} (k_h z_h + E[k_h d + q_h (d - p) \mid p]) &= \frac{1}{2} (k_h (z_h + p) + (k_h + q_h) E[d - p \mid p]) && (365) \\ &= \frac{1}{2} \left(\frac{\rho_H E[d - p \mid p]^2}{\operatorname{var}(d - p \mid p)} + \frac{\rho_H (z_h + p)^2}{\sigma_x^2} \right) && (366) \end{aligned}$$

The second term represents the utility gained from exposure to x, which is obtained by going long the private investment and short an equal amount of the fundamental, leaving pure exposure to $\varepsilon_{h,x}$. The first term is the usual utility gained from investing in the fundamental. These two investments are completely independent of each other.

The law of total variance, as in the main results, gives us

$$\begin{aligned} \operatorname{var}[d-p] &= \operatorname{var}[E[d-p \mid p]] + E[\operatorname{var}[d-p \mid p]] && (367) \\ &= E\left[E[d-p \mid p]^2\right] + \operatorname{var}(d-p \mid p) && (368) \end{aligned}$$

which we can substitute in for the first term. For the second term, we have

$$\begin{aligned} E\left[(z_h+p)^2\right] &= \operatorname{var}(z_h+p) && (369) \\ &= \operatorname{var}((z_h-z)+z+a_1 d-a_2 z) && (370) \\ &= \sigma_z^2+(1-a_2)^2 f_Z+a_1^2 f_D && (371) \end{aligned}$$

Substituting back into the equation for expected utility,

$$\begin{aligned} E\left[\frac{1}{2}\left(\frac{\rho_H E[d-p \mid p]^2}{\operatorname{var}(d-p \mid p)}+\frac{\rho_H (z_h+p)^2}{\sigma_x^2}\right)\right] &= \frac{1}{2}\rho_H\left(\frac{\operatorname{var}[d-p]-\operatorname{var}(d-p \mid p)}{\operatorname{var}(d-p \mid p)}+\frac{\sigma_z^2+(1-a_2)^2 f_Z+a_1^2 f_D}{\sigma_x^2}\right) && (372) \\ &= \frac{1}{2}\rho_H \frac{(1-a_1)^2 f_D+a_2^2 f_Z-\tau_H^{-1}}{\tau_H^{-1}}+\frac{1}{2}\rho_H \frac{\sigma_z^2+(1-a_2)^2 f_Z+a_1^2 f_D}{\sigma_x^2} && (373) \\ &= \frac{1}{2}\left\{\begin{array}{c} \left(\tau_H(1-a_1)^2+\sigma_x^{-2}a_1^2\right) f_D+ \\ \left(\tau_H a_2^2+\sigma_x^{-2}(1-a_2)^2\right) f_Z-1+\rho_H\sigma_x^{-2}\sigma_z^2 \end{array}\right\} && (374) \end{aligned}$$

L.4 Equilibrium

Now suppose there are unit masses of both the informed investors and the hedgers. This is without loss of generality as their influence can be controlled by shifting ρ_H and ρ (where the latter remains the risk tolerance of the sophisticated investors from the main analysis). The equilibrium condition is

$$0=\rho_H\left(\frac{a_1}{a_2^2}f_Z^{-1}-\tau_H-\sigma_x^{-2}\right)(a_1d-a_2z)-\rho_H\int_h z_h\sigma_x^{-2}dh+\int_i\rho\left(f_i^{-1}d+\left(\frac{a_1}{a_2^2}f_Z^{-1}-\tau_{avg}\right)(a_1d-a_2z)\right)di \tag{375}$$

Matching coefficients on z and d and using the law of large numbers so that $\int_h z_h dh = z$,

$$0 = \rho_H\left(\frac{a_1}{a_2^2}f_Z^{-1}-\tau_H-\sigma_x^{-2}\right)+\rho_H\sigma_x^{-2}a_2^{-1}+\rho\left(\frac{a_1}{a_2^2}f_Z^{-1}-\tau_{avg}\right) \tag{376}$$

$$0 = \rho_H\left(\frac{a_1}{a_2^2}f_Z^{-1}-\tau_H-\sigma_x^{-2}\right)+\rho f_{avg}^{-1}a_1^{-1}+\rho\left(\frac{a_1}{a_2^2}f_Z^{-1}-\tau_{avg}\right) \tag{377}$$

Equating the right hand sides of those two equations yields

$$\frac{\rho f_{avg}^{-1}}{\rho_H\sigma_x^{-2}}=\frac{a_1}{a_2} \tag{378}$$

Inserting that formula into the second equation yields

$$0 = \rho_H\left(a_1^{-1}\frac{a_1^2}{a_2^2}f_Z^{-1}-\tau_H-\sigma_x^{-2}\right)+\rho f_{avg}^{-1}a_1^{-1}+\rho a_1^{-1}\frac{a_1^2}{a_2^2}f_Z^{-1}-\rho\tau_{avg} \tag{379}$$

$$a_1 = \frac{(\rho_H+\rho)\left(\frac{\rho f_{avg}^{-1}}{\rho_H\sigma_x^{-2}}\right)^2 f_Z^{-1}+\rho f_{avg}^{-1}}{\rho_H\left(\tau_H+\sigma_x^{-2}\right)+\rho\tau_{avg}} \tag{380}$$

L.5 Restricting speculators

The main text refers to the agents able to gather information as "sophisticates" as opposed to the unsophisticated noise traders. Here we describe them as speculators, who are making pure bets on the fundamental, as opposed to hedgers, who hold the fundamental (partly) to hedge their private investments.

The main text considers the experiment of restricting trading by the sophisticates. Here, if only the hedgers can trade, and not the speculators, the market clearing condition is

$$0=\rho_H\left(\frac{a_1}{a_2^2}f_Z^{-1}-\tau_H-\sigma_x^{-2}\right)(a_1d-a_2z)-\rho_H z_i\sigma_x^{-2} \tag{381}$$

Again matching coefficients,

$$0 = \rho_H \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_H - \sigma_x^{-2} \right) a_1 \tag{382}$$

$$0 = -\rho_H \left(\frac{a_1}{a_2^2} f_Z^{-1} - \tau_H - \sigma_x^{-2} \right) a_2 - \rho_H \sigma_x^{-2} \tag{383}$$

This immediately implies $a_1 = 0$, and hence

$$a_2 = \frac{\sigma_x^{-2}}{f_D^{-1} + \sigma_x^{-2}} \tag{384}$$

So we again get that result, not surprisingly, that prices are uninformative when the investors who have access to information about fundamentals are no longer allowed to invest.

That does not mean, though, that the welfare benefits in this case go to zero, since the hedgers can still trade with each other. In fact, in both cases, they can always perfectly hedge the idiosyncratic part of z_i, since prices depend only on aggregate z – each individual agent has no effect on prices.

Expected utility when speculators cannot trade is

$$\frac{1}{2} \left\{ \frac{f_D^{-1} \sigma_x^{-2}}{f_D^{-1} + \sigma_x^{-2}} f_Z + \rho_H \sigma_x^{-2} \sigma_z^2 \right\} \tag{385}$$

More generally, expected utility is

$$EU_H \equiv \frac{1}{2} \left\{ \begin{array}{c} \left(\tau_H (1 - a_1)^2 + \sigma_x^{-2} a_1^2 \right) f_D + \\ \left(\tau_H a_2^2 + \sigma_x^{-2} (1 - a_2)^2 \right) f_Z - 1 + \rho \sigma_x^{-2} \sigma_z^2 \end{array} \right\} \tag{386}$$

L.6 Speculator profits

The formulas for utility and expected profits go through in this case unchanged since they depend just on the optimization of the speculators, taking a_1 and a_2 as given.

L.7 Results

This section describes how the results from the main text are affected by the replacement of the noise traders with hedgers.

L.7.1 Solution 1

There continues to be a linear solution, but in this case the coefficients are

$$a_1 = \frac{(\rho_H + \rho)\left(\frac{\rho f_{avg}^{-1}}{\rho_H f_x^{-1}}\right)^2 f_Z^{-1} + \rho f_{avg}^{-1}}{\rho_H \left(\tau_O + \sigma_x^{-2}\right) + \rho \tau_{avg}} \tag{387}$$

$$a_2 = a_1 \frac{\rho_H \sigma_x^{-2}}{\rho f_{avg}^{-1}} \tag{388}$$

L.7.2 Lemma 2

The derivation of this result depends only on the information structure and the existence of a linear equilibrium, so the utility of the speculators is the same here as in the main text.

L.7.3 Solution 2

The solution follows directly from the linearity of utility. As before, tedious algebra confirms that $\lambda'(\cdot) < 0$.

L.7.4 Result 1

The fact that the trade restrictions affect only targeted frequencies follows directly from the separability of the model across frequencies, so is unchanged here. The formula for prices in the case where speculators cannot trade is

$$p_j = -\frac{\sigma_x^{-2}}{f_D^{-1} + \sigma_x^{-2}} z_j \tag{389}$$

L.7.5 Corollary 1.1

The lack of informativeness at restricted frequencies follows trivially from the pricing function at those frequencies. The lack of any change in informativeness at unrestricted frequencies follows from the fact that the pricing function at those frequencies is unaffected.

L.7.6 Corollaries 1.2–1.4

These results all are driven entirely by the properties of the frequency transformation and are therefore unaffected by the choice of noise traders versus hedgers.

L.7.7 Corollary 1.5

The volatility of returns in the absence of speculators is

$$f_D + \left(\frac{\sigma_x^{-2}}{f_D^{-1} + \sigma_x^{-2}}\right)^2 f_Z \tag{390}$$

When speculators are present but uninformed, the pricing function is

$$p = \frac{f_D \rho_H}{f_D \rho_H + (\rho + \rho_H)\,\sigma_x^2} z \tag{391}$$

$$\operatorname{var}(r) = f_D + \left(\frac{f_D \rho_H}{f_D \rho_H + (\rho + \rho_H)\,\sigma_x^2}\right)^2 f_Z \tag{392}$$

Straightforward algebra shows that $\frac{f_D \rho_H}{f_D \rho_H + (\rho+\rho_H)\sigma_x^2} < \frac{\sigma_x^{-2}}{f_D^{-1} + \sigma_x^{-2}}$, which implies that return volatility is lower with uninformed speculators than with a trading restriction.

L.7.8 Result 2

The exact form of the formula for profits of speculators no longer holds. However, the nonnegativity does hold. The specific corollaries are more important and are discussed further below.

L.7.9 Corollary 2.1

$$\left.\frac{d}{d\psi_j} E_{-1}\left[\tilde{q}_{LF,j} r_j\right]\right|_{\psi_j=\lambda_j(0)^-} > 0 \tag{393}$$

$$\left.\frac{d}{d\psi_j} E_{-1}\left[\sum_t \tilde{Q}_{LF,t}\left(D_t - P_t\right)\right]\right|_{\psi_j=\lambda_j(0)^-} > 0 \tag{394}$$

$$\left.\frac{d}{d\psi_j} E_{-1}\left[U_{LF,0}\right]\right|_{\psi_j=\lambda_j(0)^-} > 0 \tag{395}$$

The problem faced by the sophisticated investors is the same in the sense that they continue to acquire information to the point that $\lambda_j\left(f_{avg,j}^{-1}\right) = \psi_j$, unless $\lambda_j(0) \leq \psi_j$, in which case they acquire no information. A marginal decline in ψ_j then leads to a marginal increase in $f_{avg,j}^{-1}$.

To obtain the derivative of $E_{-1}\left[\tilde{q}_{LF,j} r_j\right]$, simply use the formulas for speculator profits from the main analysis. That result then immediately implies the derivative in the second line, due to the separability across frequencies. Similarly, it remains the case that speculator utility is equal to $\sum_j Var(r_j)\tau_{i,j}$, and differentiation of $Var(r_j)\tau_{i,j}$ with respect to $f_{avg,j}^{-1}$ yields the desired result.

L.7.10 Corollary 2.2

From above, we have

$$\operatorname{var}\left[d_j \mid p_j\right] = \left(\frac{\rho f_{avg,j}^{-1}}{\rho_H \sigma_x^{-2}} f_{Z,j}^{-1} + f_{D,j}^{-1}\right)^{-1}, \tag{396}$$

which is obviously decreasing in f_{avg}^{-1}. It is also possible to confirm that return volatility is decreasing in $f_{avg,j}^{-1}$.

L.7.11 Corollary 2.3

This result follows from the fact that each frequency independently contributes nonnegatively to the profits and utility of speculators, so it continues to hold here.

Similarly, the profits and utility of the hedgers must weakly fall under an investment restriction since they always have the option of not investing at any particular frequency.

L.7.12 Corollaries 2.4 and 2.5

The formula for the earnings of noise traders does not apply to the hedgers. Moreover, their earnings are not simply the negative of those of the speculators since they also have their private investment opportunity.

It remains the case that at any frequency where $\lambda_j(0) \leq \psi$, there is no information acquisition in equilibrium. That immediately implies that restricting speculators from trading still has no impact on price informativeness, since prices are uninformative in any case.

However, there is an important change in the result for utility. We now have, in the case where prices are already uninformative

$$\text{when } f_{avg}^{-1} = 0, \ \frac{d}{d\rho} EU_H > 0 \tag{397}$$

That is, when the speculators are acting purely to provide insurance to the hedgers, any increase in their risk-bearing capacity increases the expected utility of the hedgers. So whereas in the case of noise traders, restricting trade at a frequency where no information was being acquired was beneficial, with hedgers it actually is socially harmful.

L.7.13 Section 4.4.3

It remains the case that

$$\lim_{f_{avg}^{-1}} a_1 = 1 \tag{398}$$

$$\lim_{f_{avg}^{-1}} a_2 = 0 \tag{399}$$

Furthermore, straightforward algebra shows that, writing the expected utility of the hedgers as a function of f_{avg}^{-1}, we have

$$\lim_{f_{avg}^{-1} \to \infty} EU_H \left(f_{avg}^{-1} \right) > EU_H (0) \tag{400}$$

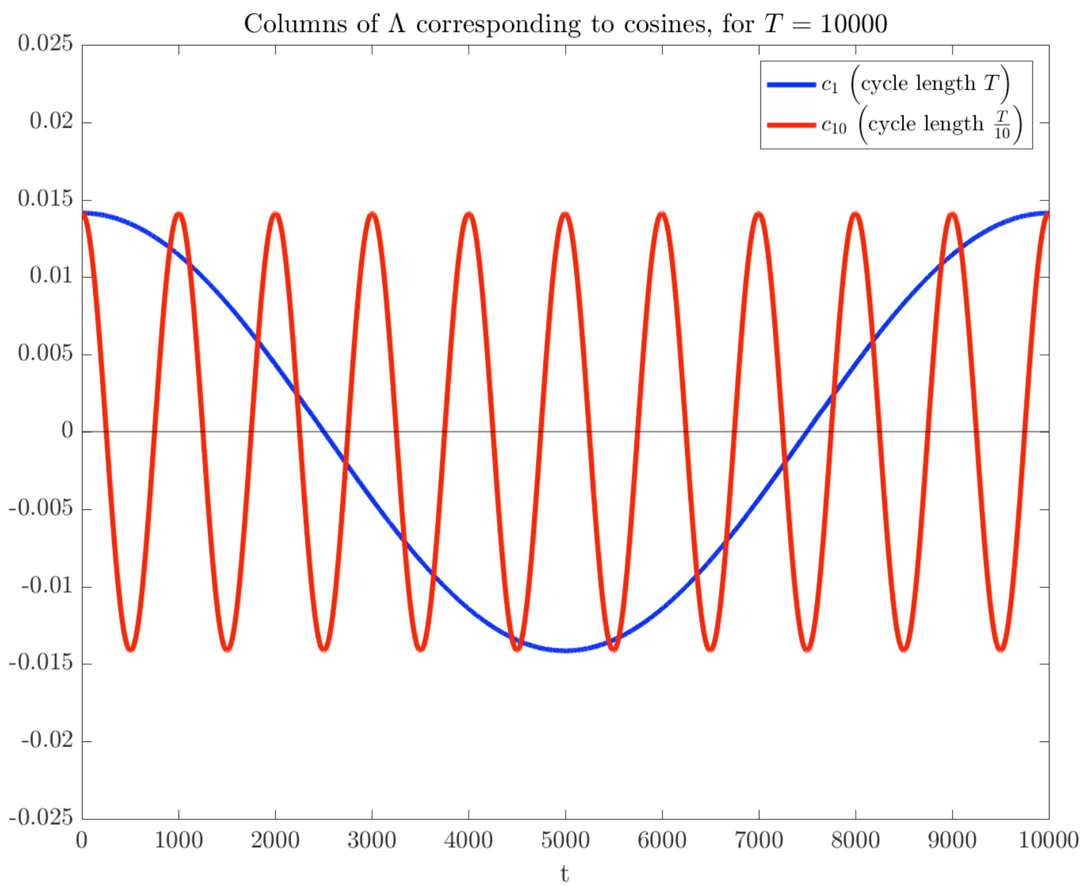
which shows that the hedgers are better off in a fully informative equilibrium than in the alternative uninformative case.

Table 1. Information losses relative to benchmark

	Short-term rest.	Low-freq. cost
Equity price	0.00	4.06
1-week difference	1.59	0.00
1-month difference	0.46	1.08
1-year difference	0.05	2.89
Monthly second difference	0.61	-0.11

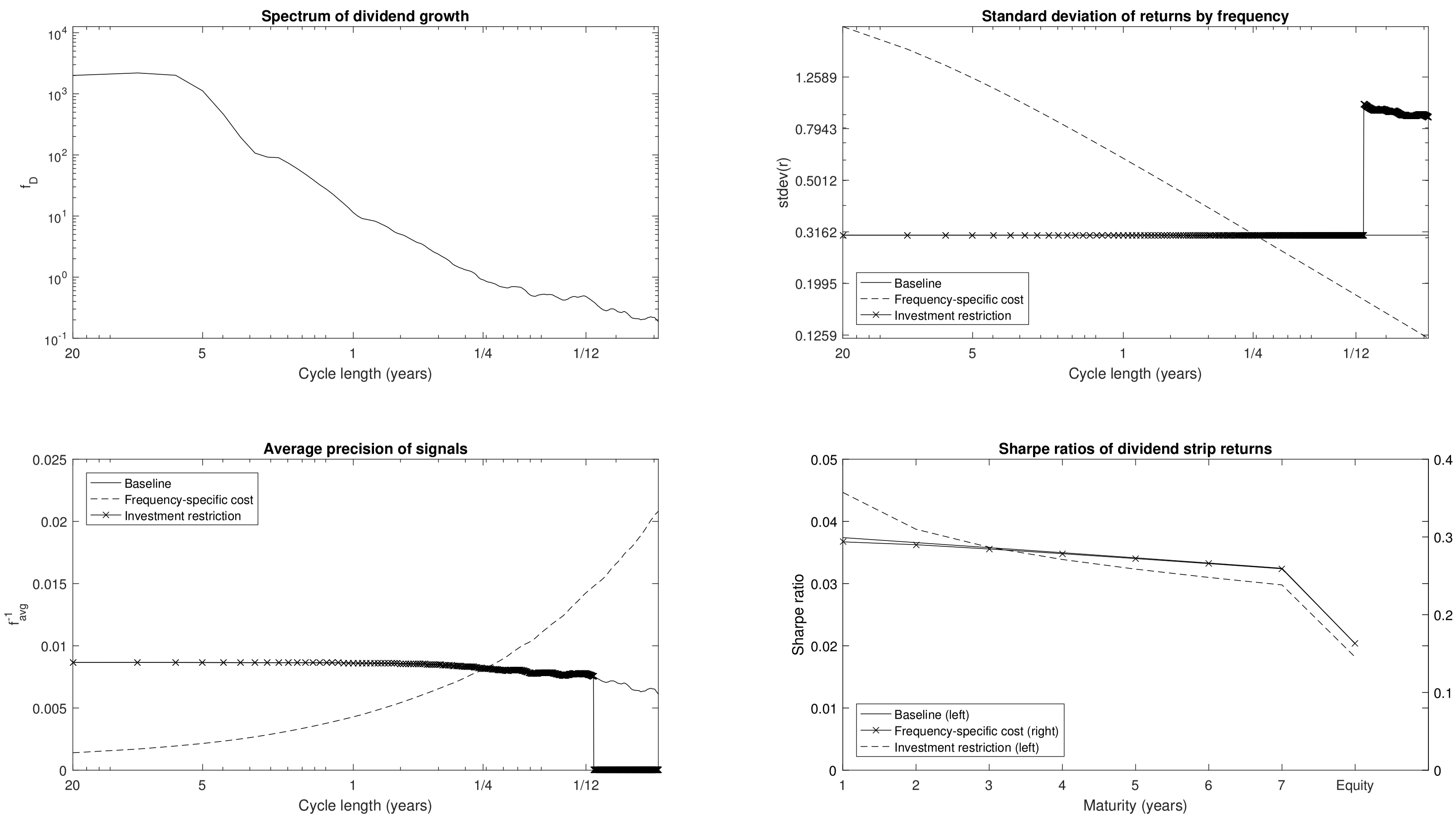
Note: log differences in precision of functions of equity prices for fundamentals between the baseline model and the short-term restriction or alternative cost function that is high at low frequencies. The first row is for the level of the equity price on date 1. The second is for the difference in the price of equity in week 1 and week 2 (which isolates just the dividend in the first week). The third and fourth rows are for the difference between the price of equity on the first date and one month and one year later, respectively. Finally, the bottom row is the efficiency of $P_9^{equity} - 2P_5^{equity} + P_1^{equity}$, which measures the change in price growth across the first and second months.

Figure 1: Portfolio weights



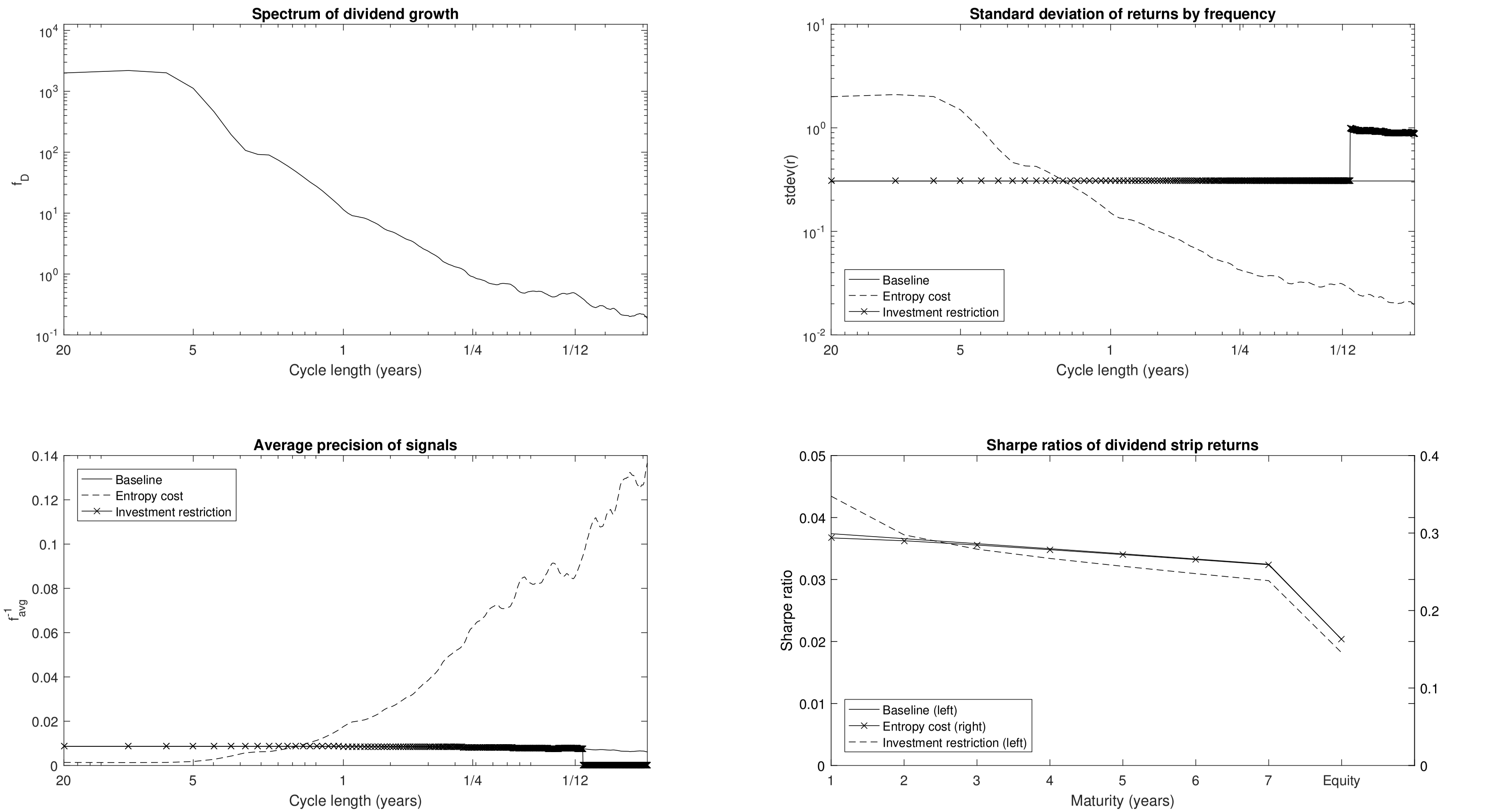
Notes: Portfolio weights for the cosing frequency portfolios c_1 and c_10, as defined in the main text. The horizontal axis is time, or the maturity of the corresponding futures contract. The vertical axis is the weight which each portfolio puts on that futures contract.

Figure 2: Numerical example



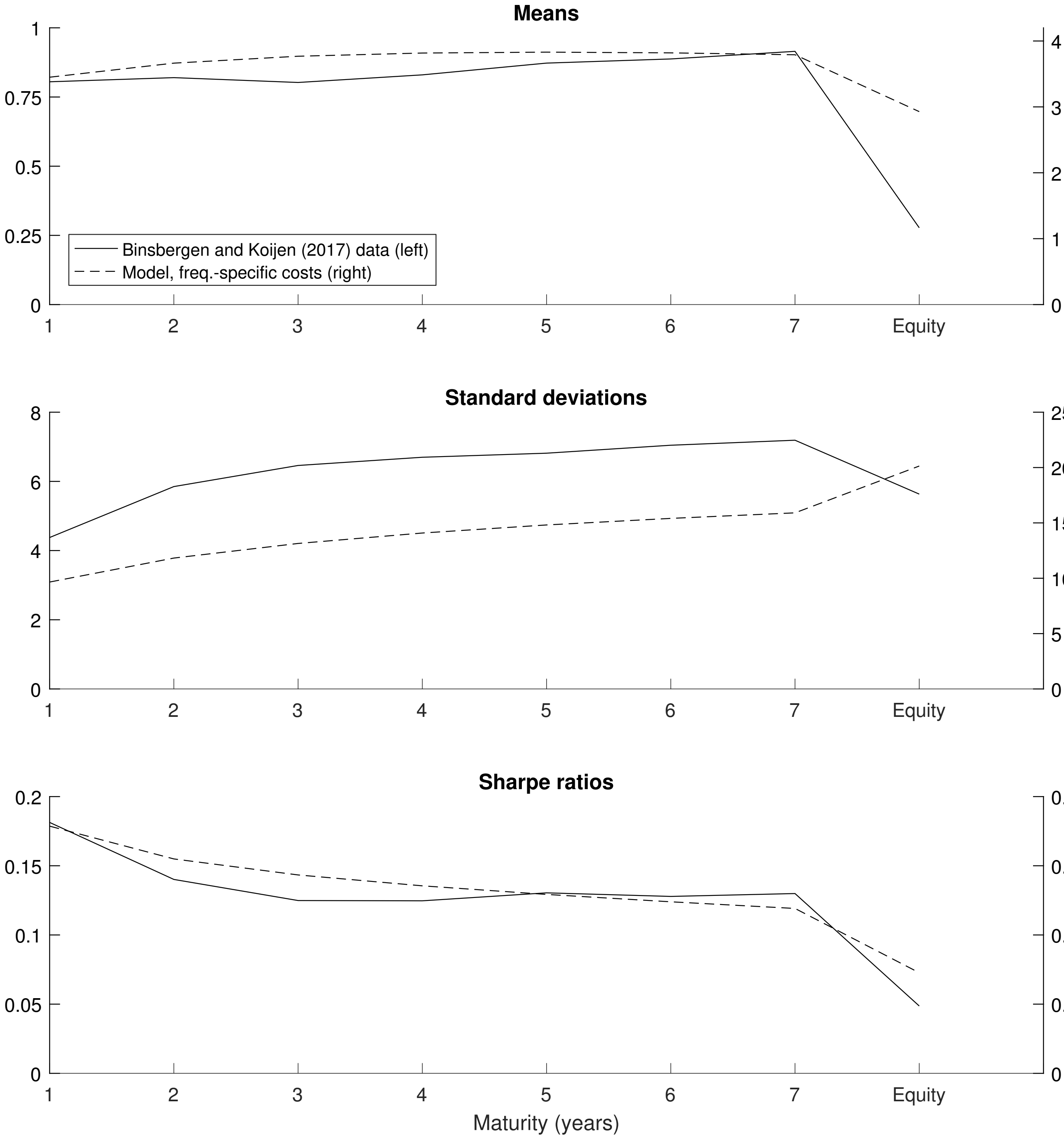
Notes: The four panels plot results for the numerical example. The frequency-specific cost case is where the cost of information at frequency ω_j is proportional to $(\omega_j + \omega_1)^{-1}$. The investment restriction says that sophisticated investors cannot hold portfolios that fluctuate at frequencies corresponding to cycles lasting one month or less. The Sharpe ratios for the dividend strip returns are calculated based on per-period returns as discussed in appendix F and then annualized.

Figure A.1: Numerical example with entropy cost



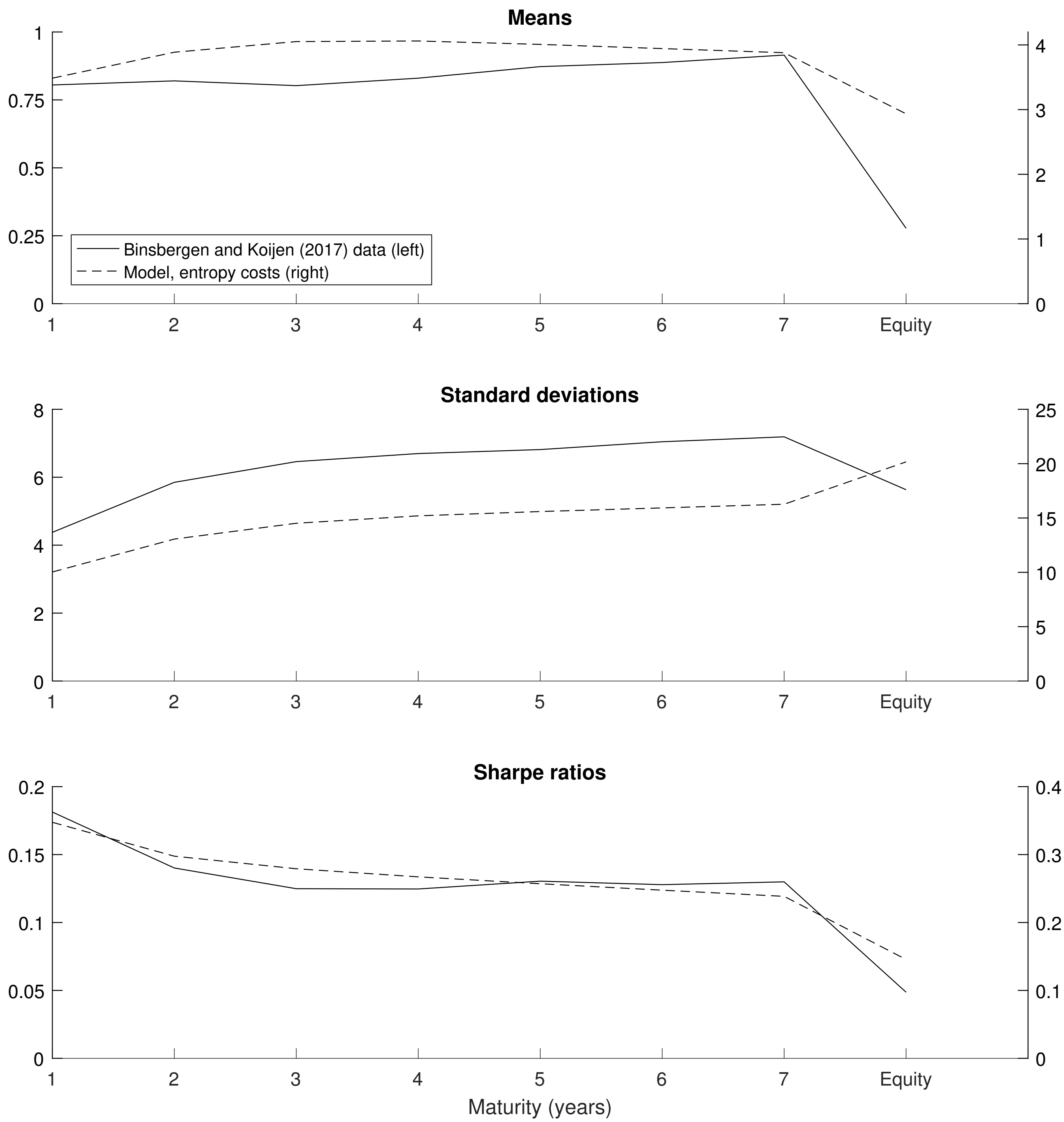
Notes: See figure 2. The only change is that the frequency-specific information cost case is replaced by the case where information flow is measured by entropy. That setup is analyzed in appendix K.3.

Figure A.2: Details of dividend strips returns, frequency-specific information cost case



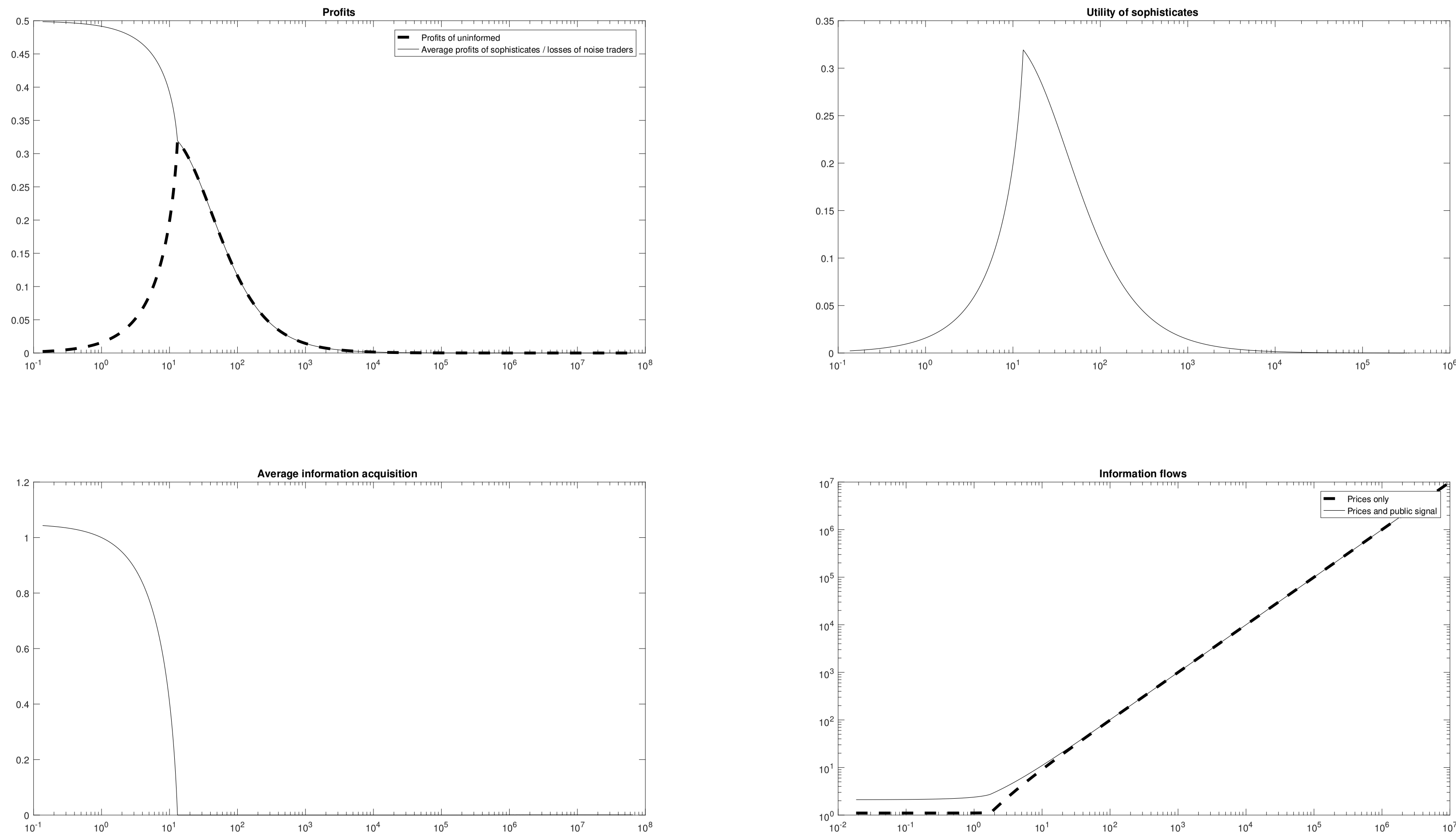
Notes: The three panels report per-period characteristics of dividend strip returns in the model with the frequency-specific cost specification along with empirical moments reported by Binsbergen and Koijen (2017) (averaged across the four markets they examine). All values are annualized.

Figure A.3: Details of dividend strips returns, entropy cost case



Notes: See figure A.2. This figure reports results for the case where information flow is measured by entropy instead of precision.

Figure A.4: Numerical example in public signal model



Notes: Numerical results for the model described in appendix G in which investors observe a public signal about fundamentals. The outputs are all for a single frequency, taking advantage of separability. The x-axis represents the precision of the public signal. Average information acquisition in the bottom-left panel is f_{avg}^{-1}. Utility in the top-right panel is net of information acquisition costs ($psi_j f_i^{-1}$). Information flows in the bottom-right panel are the precision obtained from conditioning either on prices alone or prices and the public signal.