The Economics of Intangible Capital*

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Abstract

Intangible assets are a large and growing part of firms' capital stocks. Intangibles are accumulated via investment – foregoing consumption today for output in the future – though they lack a physical presence. But rather than stopping with this "lack", we instead focus on the actual properties of intangibles that follow – in particular, non-rivalry and the need for storage. We model these properties in a simple way to demonstrate the economic implications, such as scalability and appropriability, that are often associated with intangibles. These implications coincide with a number of important issues and trends in macroeconomics and finance, including measurement of productivity, inequality, investment and valuation, rents and market power, and financing.

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1 Introduction

Intangible capital is generally defined as productive capital that lacks a physical presence. Like tangible capital, it is created through investment — that is, by foregoing current consumption in order to increase output in the future. However, unlike tangible capital, the investment produces no particular equipment or structure, but rather a new piece of knowledge that can be used to increase production. Familiar and important examples include patents, software and databases, trademarks, customer lists, franchise agreements, and human capital.

Our goal is to analyze the properties that affirmatively characterize intangibles—instead of defining them (by what they lack) in contrast with tangible capital. We focus on two main properties. First, since intangibles lack a physical presence they require a *storage medium*. The medium can be a piece of physical capital, like a computer (for software), or a document (for a patent or a design), or a person (for a method or an innovation). The form of the storage medium an intangible can be stored in determines who can appropriate its marginal product. Second, intangibles can be simultaneously stored (copied) in multiple places, which leads to *non-rivalry in use*. Since the same intangible can be used simultaneously in many instances in production, intangibles allow for economies of scale.

These two properties—the need for storage and non-rivalry—are at the core of our analysis. Importantly, whether intangibles can be stored in a physical medium (that is, outside of their inventor), and the degree of non-rivalry, is determined by the interaction of the storage technology and the institutions that exist to create exclusivity. For instance, an algorithm can be stored in a book which can be copied multiple times; whether its creator chooses to do so largely depends on whether she can appropriate a significant share of its benefits. As a result of this interaction, different types of intangibles are differentiated by their effective degree of non-rivalry while the need for storage can take various forms and be subject to different types of frictions. We discuss how these two properties relate to familiar types of intangibles. We then propose a simple model to capture the economic implications of these properties. Finally, using the model, we show how these concepts can shed light on some important recent macroeconomic and financial trends, including declining measured productivity growth, rising market power, rising valuations, and declining tangible investment rates.

Existing work has largely defined intangibles as a variant within the traditional capital framework: the investment process is more uncertain; they depreciate slower (or faster, or more randomly) than physical capital; they have a different relative price; and so on. Intangibles then often amount to 'missing' or 'unmeasured' capital. The scope for measuring them is therefore inherently narrower and tends to exclude significant components (for instance, intangibles stored in employees). A further consequence of modeling them as (unmeasured) physical capital is that their implications for key macroeconomic and financial quantities is not particularly distinct. By starting from a more affirmative description of intangibles bypasses these boundaries and leads to novel implications for the theory and measurement of intangibles.

Section 2 defines and discusses the two key properties of intangibles. Section 3 provides a simple model capturing these two properties and relating to notions of scalability and appropriability often discussed in the literature on intangibles. Section 4 draws their implications for productivity growth measurement, income distribution, the relationship between valuations and investment, market power, and financing. Section 5 concludes by recommending areas for future research.

2 Intangibles as assets

2.1 Fundamental properties of intangibles

Like machinery or structures, intangibles are capital, in the sense that creating them requires foregoing output today (investment) in order to achieve more output in the future. Unlike machinery or structures, however, intangibles lack a physical presence. This endows them with two fundamental properties.

Property 1: Intangibles must be stored (P1)

By definition, intangibles lack a physical presence. As a result, the information they contain must be stored or codified. This may be done using different media: writing, drawing, software, recordings, or other technologies, as well as simply human memory.

For instance, an algorithm is stored using code or software; a logo is stored using drawings; a managerial process is stored in key talent or in written instructions specifying the operational rules of a firm.

Intangibles are sometimes conflated with their storage medium. For example, the value of a book is not the paper and cover, but rather the information it holds. Similarly, algorithms and data can be written down or coded, but the value derives from the information, not the software per se.

Property 2: Intangibles are non-rival in use (P2)

Intangibles can be stored simultaneously in multiple places. As a result, intangibles can be used as an input in the production of *multiple* instances of a particular good or service at the *same* time. We refer to this first fundamental property as “non-rivalry in use”, as opposed to simply “non-rivalry”, in order to stress that intangibles are production inputs.[1]

For instance, the same algorithm can be copied (stored in multiple places) and used in multiple simultaneous instances to produce, say, search results. The same design for a logo can be drawn and then copied and used in multiple simultaneous instances to brand clothing products. Managerial processes can be used in multiple simultaneous instances in different parts of the same organization, or across firms around the world.

2.2 Turning intangibles into assets

Properties **P1** and **P2** are broad: they describe, for instance, public goods that can be used as capital inputs, such as an open-source operating system, or a recipe for ketchup. However, we will use ”intangible assets” to describe intangible capital inputs whose value can be captured and privately appropriated. This depends on technology, and institutions.

Technology determines how intangibles are stored. Prior to the development of writing (including images), knowledge was passed down from one person to another. Writing allowed for intangibles to be stored independently of individuals, thereby becoming *alienable*. As technology has progressed, the scope of which intangibles can be stored externally has expanded. For instance, digital media can store larger amounts of information than writing, allowing for complex intangibles such as genomic sequences or consumer databases to be stored effectively. Recording a lecture is more effective than writing notes. These advances may explain the rapid rise in measured intangibles since the 1990s.

Institutions create extrinsic value from stored intangible assets by enforcing excludability. Since intangibles are non-rival, excludability limits the set of agents with the right to use the asset and capture its value.

The institutions that give rise to excludability and property rights assignment may be informal or formal. For instance, voluntary limits to the disclosure of ideas, such as trade secrets, create excludability, and therefore value. Trade organizations agree to product standards or they may arise in a market equilibrium. However, excludability is often formal and relies on enforcement through the legal system. Legal systems have had to evolve as new forms of storage for intangibles have emerged, including the patent system, copyright enforcement, and non-compete clauses. Note that enforcement institutions may be centralized (eg. the patent system), or bilateral (eg. through non-compete contracts).

Technology and institutions interact to determine the equilibrium value of intangible assets. For example, the fact that the same piece of software can be copied and used simultaneously undermines

[1] By contrast, the public economics literature uses the expression “non-rival” to refer to consumption goods.

its value unless IP rights are enforced by institutions. Further, as the technology to store intangibles evolves, it can displace the value of intangibles stored in now obsolete technologies—for example, lossless streaming has largely displaced compact discs.

Importantly, both technology and institutions are endogenous, so there is feedback between them. Institutions affect the incentive to store intangibles. For example, if others cannot be excluded from using the innovation for free, inventors have less incentive to innovate and share their ideas. Conversely, the degree of legal protection depends on the storage technology. If firms cannot codify management practices, it can be harder to exclude others from using them, say by copying its practices or hiring away key talent.

2.3 Parallels with physical capital

While properties **P1** and **P2** are specific to intangible capital, intangibles also share properties in common with physical capital. Fundamentally, it is an accumulated factor that requires foregoing output today for output in the future. Hence it is an accumulated factor; it depreciates; it can be firm-specific to varying degrees. We next briefly discuss these parallels.

(i) **Accumulation** Like physical capital, intangibles typically require investment, say in R&D, marketing, or skill accumulation. The mapping from investment to accumulated capital may be less certain than for physical capital—the mapping between effort and idea production may not be simple—which is historically why accounting has not capitalized R&D in the same way as capital expenditures. Nevertheless, investing in intangibles typically implies a resource cost and therefore foregone output today in exchange for capital accumulation and output in the future. Of course, the mapping for physical capital can also be uncertain. Expenditure on fiber optic cables, for example, does not accumulate in a simple way to the current value nor productive use of the current stock of fiber optic cable. In general, investment which reverses depreciation from wear and tear has a more certain outcome than investment aimed at overcoming obsolescence, and intangibles may be subject to more dramatic obsolescence shocks.

(ii) **Depreciation** Just like physical capital, intangibles depreciate over time. For physical capital, depreciation can be caused by (1) wear and tear and (2) obsolescence. However, for intangibles, non-rivalry implies that there cannot be wear and tear — only obsolescence. Obsolensence, like wear and tear, may be partial, however reversing depreciation due to obsolescence requires investment that involves innovation rather than straightforward replacement.

Obsolescence can take many forms and can result either from lack of continued investment (maintenance) in intangibles; the arrival of new/or better vintages of intangibles; or other reasons exogenous to the firm. For instance, brand value may be forgotten if marketing expenses are not kept up; management and production processes may become obsolete as new methods appear; data that is not up-to-date becomes less relevant.

In addition, just like physical capital may be destroyed as a result of a natural disaster, intangibles can be destroyed by other forms of disasters: corporate scandals; violation of intellectual property laws (expropriation) by governments; key employees leaving the organization; changes in laws; or shifts in consumer tastes (sports teams change their names in response to shifting cultural norms).

(iii) **Specificity** Like physical capital, intangibles may be more or less specific to a particular combination of labor and capital (a production stream, firm or industry). Examples from the literature include aerospace plants, which are specific to the firm that first builds it (Ramey and Shapiro, 2001) or a railroad track that embeds particular standards (Benmelech, 2009).

For intangibles, specificity can modulate the impact of non-rivalry. A scalable and non-specific input can be scaled in multiple dimensions. For example, a brand may apply to all the goods produced by a firm, not just particular products. Firms may invest to broaden the span of an intangible such as brands or management and production processes. A retail platform may house many types of goods, not just books or clothing. When an intangible production input is not narrowly specific, non-rivalry has greater potential to affect the entire span of the firm.

2.4 Examples

	Storage	**Property-Rights Institution**
Patents	Patent application	Patent system
Software	Computers	Copyright system
Video and audio material	Audiovisual media	Copyright system
Franchise agreements	Contract	Contract enforcement
Consumer lists	Digital media, contracts, or in Labor	Contract enforcement
Organization capital	Key talent, manuals	Non-compete clauses, trade secrets
Brands	Consumers, Trademark media	Trademark system

Table 1 Some familiar intangible assets and how they fit in the framework. A subset of these are drawn from IAS 38, available at `https://www.iasplus.com/en/standards/ias/ias38`.

Properties **P1** and **P2** map to commonly recognized types of intangible capital. We use our approach to explain why these intangibles are understood to have value: how they are stored and how property rights are generated. We start with a set of examples in which excludability is generated by laws and regulations, and then provide additional cases in which private contracts play a key role. Table 1 summarizes this discussion.

(i) **Patents and Blueprints:** Patents are inventions stored in writing and diagrams. Patents can represent a new production method or a new final or intermediate good. The non-rivalry of the patent arises from the fact that it can be used repeatedly as a blueprint for production. The patent is made excludable by a set of laws and regulations composing the patent system.

(ii) **Software and databases:** Software is a set of computer instructions, or a set of data, stored in a digital medium. Non-rivalry follows from the fact that the instructions or data can be copied and re-used at zero marginal cost. Copyright laws provide excludability in some cases, while private contracts protect others.

(iii) **Video and Audio material:** a set of cultural content (words, sounds, images) that is stored on various types of audiovisual media (computers, film reel, recordings). Non-rivalry arises because the content can be reproduced on new media. Excludability results from copyright laws and protection, but can become part of the public domain (in which case it remains an intangible, but ceases to have private value).

(iv) **Brand value:** Brands refer to the reputation that a particular groups of products or services have among customers, traditionally identified to a particular name or symbol identifying the group of products or services. This reputation is non-rival in the sense that many customers can simultaneously share the same assessment of the products or services. The trademark and copyright system protect excludability. Brand values are typically traded together with the product line it represents, suggesting that the storability of brands is only partial.[2]

(v) **Franchise agreements:** Franchise agreements provide a set of instructions to franchisees, along with the right to use a brand name and potentially a supply chain. It is stored in codified rules (e.g. instruction manuals for McDonald's owners), as well as in workers. Non-rivalry is driven by the fact that the same set of instructions and brand can be sold simultaneously to many franchisees. Excludability is made possible by the enforceability of contracts (supply agreements, agreements granting the right to use logos, and so on) signed with the franchisees.

[2] Firms do spin off product lines; Seiko recently spun off their Grand Seiko line of watches with no loss to brand value. Occasionally, brand value is sold separately from products, particularly in firm liquidations.

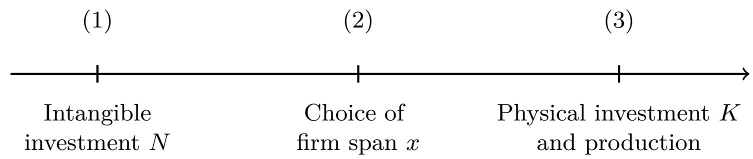

Figure 1: Timeline of the model.

(vi) **Consumer lists and purchase agreements:** a list of consumers that have purchased a product previously, or have agreed to purchase a particular product in the future. Existing consumer lists may be stored in a database (eg. dental or hair stylist clients). For these intangibles, non-rivalry might be more limited, though the same customer list can be used to generate repeated transactions. As with franchise agreements, excludability is dependent upon the enforceability of contracts (such as purchase agreements).

(vii) **Management/organization capital:** a set of processes or instructions stored in formal (written) practices or in the human capital of managers and key talent. Non-rivalry follows from the fact that the same processes or instructions can be used repeatedly. Key talent generally cannot be excluded from using the processes or instructions outside the original firm, so organization capital is transferred by buying a firm or hiring key talent.

3 Production with intangible capital

In this section, we present a simple model to exposit the ideas discussed in the previous section – non-rivalry, storability, and limits to excludability – and derive their implications for production and investment.

3.1 A simple model of production with intangibles

We focus on a single firm which operates for a single period. The firm is managed by an entrepreneur, who makes operating and investment choices in order to maximize the terminal value of the profits she receives. Though when describing the model we use the word 'entrepreneur' to describe the owner of the intangible asset, broader interpretations are possible. The term encapsulates all parties that participate in the creation, dissemination, and use of intangible assets. Examples include highly skilled personnel, who are responsible for the creation of new inventions or business ideas; entrepreneurs, managers, and startup employees.

The model includes two types of capital, tangible and intangible. Both types of capital can be deployed across multiple production streams, which could be different product lines, physical locations, or market segments, for example. Together, these streams determine the span, x, of the firm. To highlight the role of intangibles, we abstract from all frictions affecting physical investment. Tangible capital can be rented at some constant user cost. By contrast, intangibles in the model exhibit the defining characteristics emphasized above. In particular, the model captures the implications of non-rivalry in use **P2**, and limits to excludability. We discuss how limits to excludability can arise from the storage requirement for intangibles **P1** and from related property-rights enforcement issues.

For ease of exposition, we split the model into three stages. Figure 1 sketches the timeline of decisions in the model. We next discuss the choice of physical capital, firm span, and intangibles in the reverse order in which they occur, starting with the production stage.

Stage 3: Production

In the production stage, the firm's input N of intangible capital and its span of production x are taken as given. Profit maximization is described by:

$$V(N,x) = \max_{\{N(s),K(s)\}_{s\in[0,x]},K} \int_0^x N(s)^{1-\zeta}K(s)^{\zeta}ds - RK$$

$$\text{s.t.} \quad \int_0^x K(s)\,ds \leq K \tag{1}$$

$$\left(\int_0^x N(s)^{\frac{1}{1-\rho}}\,ds\right)^{1-\rho} \leq N \tag{2}$$

The firm thus chooses the amount of tangible capital and the allocation of tangible and intangible capital to each stream of production. Each $s \in [0,x]$ indexes a different production stream. Within each stream, production uses two inputs, intangible capital, $N(s)$ and physical capital $K(s)$, and has constant returns to scale:

$$\forall s \in [0,x], \quad Y(s) = N(s)^{1-\zeta}K(s)^{\zeta}, \tag{3}$$

where $\zeta \in]0,1[$ is the elasticity of output with respect to physical capital. A production stream could represent an establishment, a product, a market segment, a geography, so long as production satisfies constant returns within that stream.

The difference between the two constraints (1) and (2) illustrates the first fundamental property of intangible capital: because intangibles are non-rival in use, they are scalable in production.

In particular, the constraint limiting the deployment of physical capital K takes a familiar form, where the same unit of physical capital (say a machine) cannot be simultaneously used in multiple streams. In particular, the firm first decides on the total stock of physical capital K it wants to employ, and then rents it at the constant and exogenous price R on input markets. It then allocates that amount K across different production streams indexed by s, subject to the constraint (1).

Similarly, the firm's input of intangible capital N is allocated across streams subject to the constraint (2). Unlike the standard adding up constraint (1), however, the resource constraint for intangibles (2) is non-linear, capturing the idea that the same intangible can be used simultaneously in different streams.[3] The parameter ρ captures the degree of non-rivarly of the intangible input in production within the firm and across production streams.

Assumption 1 (non-rivalry within the firm)**.** $0 < \rho \leq 1$.

To see why ρ captures non-rivalry within the firm, consider two extreme cases. When $\rho = 0$, then there is no difference between tangible and intangible capital: the two constraints (2) and (1) are identical and both types of capital are rival within the firm. By contrast, when ρ approaches 1, constraint (2) now becomes:

$$\lim_{\rho \to 1} \left(\int_0^x N(s)^{\frac{1}{1-\rho}} ds \right)^{1-\rho} = \max_{s \in [0,x]} N(s) \leq N. \tag{4}$$

In this case, the same intangible can be used in every production stream—that is, N becomes completely scalable, since the firm can now set $N(s) = N$ for all streams.

More generally, the marginal rate of technical substitution of intangibles across any two production streams $N(s)$ and $N(s')$ is:

$$\nu(N(s), N(s'); \rho) = \left(\frac{N(s')}{N(s)} \right)^{\frac{\rho}{1-\rho}}.$$

This marginal rate of substitution is equal to 1 when $\rho = 0$, so that increasing $N(s')$ by a marginal unit requires reducing $N(s)$ by a marginal unit. When $\rho \to 1$, on the other hand, the marginal rate of substitution converges to:

$$\lim_{\rho \to 1} \nu(N(s), N(s'); \rho) = \begin{cases} +\infty & \text{if} \quad N(s') > N(s) \\ 0 & \text{if} \quad N(s') < N(s) \end{cases}$$

In this case, so long as $N(s') < N(s)$, increasing $N(s')$ by a marginal unit is costless: it does not require reducing $N(s)$ at all.

Our model allows for partial non-rivalry: $0 < \rho < 1$. In this case, increasing $N(s')$ by a small amount does require reducing $N(s)$, but less than one-for-one. In other words, increasing the intangible input in one production stream is not entirely costless, but it does not necessarily reduce

[3] Non-rivalry in use goes beyond than assuming that the intangible input is not specific to a particular production stream. Rather, it implies that using the input across different streams does not incur an additional cost.

its availability for other production streams, either. Partial non-rivalry will arise either as a result of imperfections in the storage technology (say, difficult to communicate knowledge effectively) or as a result of institutions enforcing excludability (say, the firm needs to pay a licensing cost to use the same intangibles across different production streams).

Given our assumption that the marginal revenue product of all streams s is the same, the optimal allocation of capital across streams is symmetric:

$$\forall s \in [0,x], \quad N(s) = x^{-(1-\rho)} N,$$

$$K(s) = \left(\frac{A}{1-\zeta}\right)^{\frac{1}{\zeta}} N(s), \qquad \text{where} \quad A \equiv (1-\zeta)\left(\frac{\zeta}{R}\right)^{\frac{\zeta}{1-\zeta}}$$

As a result, the total demand for physical capital K and the value of the firm are equal to:

$$K(N,x) = \left(\frac{A}{1-\zeta}\right)^{\frac{1}{\zeta}} N x^{\rho}, \tag{5}$$

and

$$V(N,x) = A N x^{\rho}. \tag{6}$$

Examining (6), we can immediately see that the quantity of the intangible N and the scope of implementation x are complements, which can allow for increasing returns to scale if the firm can increase both N and x. We next introduce a tradeoff between these two choices.

Stage 2: Storage choice and firm span

We next discuss the choice of scale, or span, x. For now, let us take as given the initial endowment of intangible capital N. Given that initial endowment, the initial creator of the intangible asset (the entrepreneur) needs to make a choice regarding the optimal scale/span of production indexed by x.

Our key assumption is that in order to increase the span of the firm, the entrepreneur may need to give up some of the firm's endowment in intangibles. We model this choice as follows. Let $0 \leq N_e \leq N$ denote the portion of intangible capital that the entrepreneur retains, and let:

$$\theta \equiv \frac{N_e}{N}.$$

Since V is linear in N, total value accruing to the entrepreneur is:

$$V(N_e, x; \theta) = \theta V(N,x).$$

We make the following assumption about the relationship between firm span x and ownership θ.

Assumption 2 (Limited appropriability)**.** *The span of the firm is related to the share of intangibles retained by the entrepreneur through:*

$$x(\theta) = -\frac{1}{\delta}\log\left(\frac{\theta}{\delta}\right). \tag{7}$$

where $\delta \in (0,1]$ *is a fixed parameter.*

Assumption 2 states that retaining a higher ownership share of the intangible, θ, requires the entrepreneur to choose a smaller span, since $x'(\theta) = -\frac{1}{\delta\theta} < 0$. The strength of this effect is governed by the parameter δ. When δ is close to zero, the entrepreneur can increase span without giving up a large portion of her endowment of intangibles. When δ is large, on the other hand, increasing higher span requires forfeiting more intangibles. Thus, δ captures limits to the appropriability of the returns generated by the entrepreneur's intangibles.[4]

Under this assumption, the entrepreneur jointly chooses span and the share of the intangible endowment to retain, θ, as follows:

$$\hat{V}_e(N) \;=\; \max_{\theta\in[0,1],\; x\geq 0} \quad \theta V(N,x) \quad \text{s.t.} \quad x(\theta) = -\frac{1}{\delta}\log\left(\frac{\theta}{\delta}\right). \tag{8}$$

The solution is:

$$\hat{x} \;=\; \frac{\rho}{\delta}, \quad \hat{\theta} \;=\; \delta e^{-\rho},$$

and

$$\hat{V}_e(N) \;=\; A\,N\,\delta e^{-\rho}\left(\frac{\rho}{\delta}\right)^{\rho}.$$

A high degree of non-rivalry (ρ close to 1) is associated with high firm span (high $\hat{x}$) but low retention of intangibles by the entrepreneur (low $\hat{\theta}$). Strong limits to appropriability, or high costs of storing the intangible externally (δ close to 1) is associated with low firm span (low $\hat{\theta}$) but high retention of intangibles by the entrepreneur (high $\hat{\theta}$).

Consistent with the discussion of Section 2, Assumption 2 captures the idea that the non-rival nature of intangibles, along with frictions in their storage, imply that fully appropriating their returns may be difficult. But Assumption 2 goes beyond this quality by specifying that limited appropriability is exacerbated by x, the span of the firm. There are (at least) two interpretations of this assumption.

1. **Imitation:** As the firm adds new production streams, it uses its intangibles across more production facilities, markets, or geographies. More workers and more consumers are exposed

[4] Note that, with this particular functional for $x(\theta)$, complete retention by the entrepreneur would imply that span is $x(1) = \log(\delta)/\delta < 0$. Thus the constraint that $x \geq 0$, in the entrepreneur's problem, requires that the entrepreneur retain no more than a fraction $\theta \leq \delta$ of the intangible capital stock.

to the firm's intangibles. As a result, competitors may find it easier to replicate the intangible and appropriate some of the returns it generates, as described above. A higher value of δ would imply that imitation and replication process erodes firm rents faster as scale expands.

Under this interpretation, $\theta(x) = \delta e^{-\delta x}$ denotes the total amount of the intangible asset the firm can effectively appropriate given its choice of scale x. As such, the parameter δ is a function not only of the institutional environment (for example, the degree of intellectual property rights protection) but also could vary with the type of intangible. For instance, δ would be higher for intangibles that are easy to store (copy) and not well protected by specific legal institutions, like software.

This interpretation does not emphasize a distinction between an entrepreneur and outsiders that potentially own a part of the firm, but rather, between private and social returns to the intangible. Interpreting $\hat{V}_e(N)$ as the value of the firm to its owners, $V(N, \hat{x}) - \hat{V}_e(N)$ would then denote the social or public value of the intangible which can accrue to agents outside the firm (i.e. the value of positive spillovers).

2. **Incomplete contracts:** Increasing the span of production entails upfront costs. Markets are imperfect so outside investors require collateral. Under this interpretation, we can think of the intangible asset N as initially stored only within the inventor or entrepreneur herself — it is, say, an idea for new drug, a TV show, or an improved production process. In order for the intangible to serve as collateral, it must first be stored and codified outside of the entrepreneur. Once stored, returns to the intangible can be partly appropriated by the investors. The net result is that the entrepreneur only retains rights to cashflows generated by an effective amount $N_e < N$ of the original quantity of intangibles. She is willing to accept 'storing' an amount $N - N_e$ externally (in pledgable, tangible capital, for example) in order to secure outside financing. Incomplete contracts imply that when the entrepreneur makes the scale decisions x she only maximizes her own payoff, which depends on N_e as that determines the amount she can appropriate.

   Under this interpretation, a portion $\hat{\theta} V(N, \hat{x})$ of the value of the firm accrues to the entrepreneur, and the remainder, $(1 - \hat{\theta}) V(N, \hat{x})$, accrues to outside investors. The parameter δ would be higher when the financing needs of the entrepreneur are greater, or the pledgeability of their intangible capital is low. This interpretation fits intangibles whose storage technology makes appropriation by outside parties (like financiers) difficult, for instance intangibles stored in key talent, like organization capital or technical advances that are difficult for outside parties to evaluate.

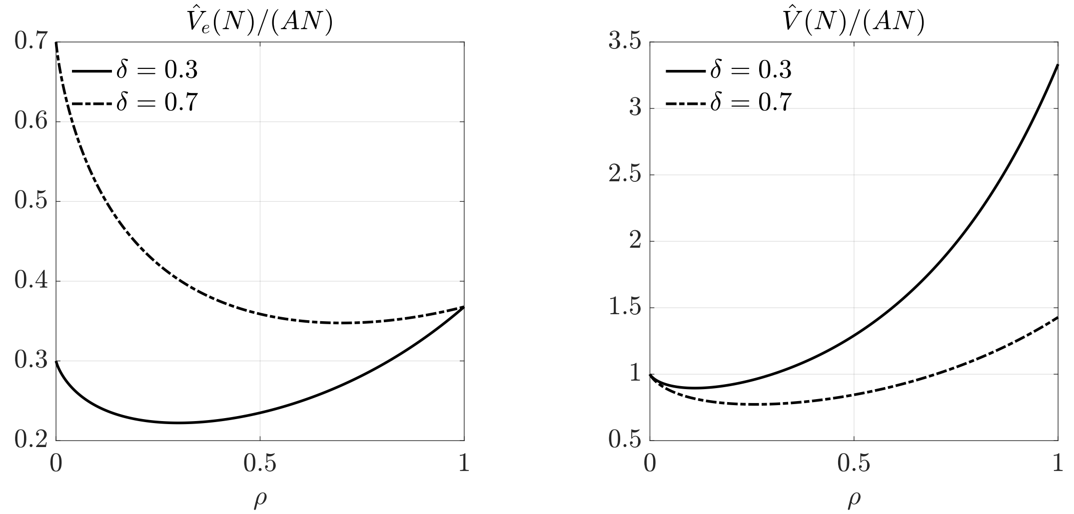

Figure 2: Comparative statics with respect to ρ and δ. The left panel reports the entrepreneur's value function, $\hat{V}_e(N)$, normalized by AB, where $A = (1-\zeta)(\zeta/R)^{\frac{\zeta}{1-\zeta}}$ and N is the intangible stock. The bottom right panel report total enterprise value $\hat{V}(N)$, normalized by AN. In the case $\delta = 0.3$, the entrepreneur chooses $\hat{\theta} = 1$ for all values of x. By contrast, when $\delta = 0.7$, the entrepreneur chooses to give up controlf over some of the intangibles, $\hat{\theta} < 1$, when scalability is sufficiently high (ρ sufficiently close to 1).

Under either interpretation, notice that by our assumption of limited appropriability of the full amount of returns associated with N, the choice of scale by the entrepreneur is inefficient. This follows from the fact that the entrepreneur chooses x to maximize (8), rather than $V(N, x)$. The latter value function, which corresponds to either the social value of the intangible (interpretation 1) or the enterprise value of the firm (interpretation 2), can be written (given the optimal choice of the entrepreneur $\hat{x}$) as

$$\hat{V}(N) = A\,N\,\hat{x}^{\rho} = A\,N\,\left(\frac{\rho}{\delta}\right)^{\rho}.$$

Because of limited appropriability of the surplus associated with N, the entrepreneur always chooses a smaller span than the span that maximizes enterprise value ($x = +\infty$).

To obtain some further intuition about the interaction of non-rivalry with limited appropriability, Figure 2 illustrates the comparative statics of the model with respect to ρ, non-rivalry, and δ, the limits to appropriablity.

The optimal span $\hat{x} = \rho/\delta$ increases with the degree of non-rivalry. However, for given limits to appropriability, δ, a higher degree of non-rivalry does not necessarily make the entrepreneur better off. The left panel of Figure 2 illustrates this. When appropriability is high (δ is low), given the option to adopt an intangible-intensive technology with a high non-rivalry (ρ to 1), versus using only rival capital inputs ($\rho = 0$), the entrepreneur would generally pick the former, and operate at high scale. However, when appropriability is low, that is not the case: the entrepreneur might instead pick a technology with rival capital inputs, $\rho = 0$, and focus on a single production stream ($\hat{x} = 0$).

This property demonstrates the complementarity we emphasized in Section 2.2 between non-rivalry and limits to appropriability: non-rivalry may generate value, but the entrepreneur will only value the associated intangible asset to the extent that the benefits are appropriable. To see this, note that when $\delta > e^{-\rho}$, the cross-partial derivative of the entrepreneur's value function is equal to:

$$\frac{\partial^2 \log(\hat{V}_e(N))}{\partial \rho \, \partial \delta} = -1/\delta < 0,$$

so that non-rivalry (high ρ) and appropriability (low δ) are complements.

Finally, the comparison of the left and right panels of Figure 2 further illustrate the fact that the entrepreneur's scale choices may be inefficient. In the right panel, for all values of δ, a non-rival technology (ρ close to 1) yields higher social value (under interpretation 1) or enterprise value (under interpretation 2). This conflicts with the preferences of the entrepreneur, who would rather choose a technology with non-scalable inputs when δ is sufficiently high.

Stage 1: Intangible investment/creation

The last step is to determine the initial amount of the intangible asset N. To do so, we need to take the perspective of the entrepreneur who invests in producing new intangibles. More specifically, the entrepreneur exerts effort ι subject to a convex cost $c(\iota)$ to generate a new intangible asset. The process of generating new intangibles can be risky: exerting effort ι yields intangible capital $N \sim f(N; \iota)$. Exerting higher effort yields ex-ante better outcomes: we assume that if $\iota' > \iota$ then $f(N; \iota') \overset{\text{fosd}}{>} f(N; \iota)$. Given these assumptions, the entrepreneur solves

$$\max_{\iota} \int \hat{V}_e(N) f(N; \iota) dN - c(\iota) \tag{9}$$

which after substituting for (9) yields the optimality condition

$$A \left[\delta e^{-\rho} \left(\frac{\rho}{\delta} \right)^{\rho} \right] \frac{\partial}{\partial e} E[N; \hat{\iota}] = \frac{\partial}{\partial \iota} c(\hat{\iota}). \tag{10}$$

Examining (10), we see that the dependence of the entrepreneur's optimal effort choice on ρ and δ is determined by how the term in brackets depends on ρ and δ.

We can immediately see that $\partial \hat{\iota} / \partial \delta < 0$; if the entrepreneur needs to give up more value in order to scale her firm (δ is higher) then her marginal valuation of the intangible N is lower, which leads to lower ex-ante investment in generating intangibles. By contrast, the comparative statics with respect to the non-rivalry parameter ρ are more subtle. It turns out that the term in brackets is increasing in ρ if $\rho > \delta$, and decreasing otherwise. This is again related to the complementarity we emphasized in Section 2.2 between non-rivalry and appropriability. Scalability may generate value, but the entrepreneur will only value the associated intangible asset to the extent that the benefits are sufficiently appropriable. Otherwise, if the entrepreneur cannot appropriate a significant share of the rents (δ is high enough) she will exert less effort in generating new intangibles for local increases in non-rivalry ρ.

Additionally, the model features underinvestment in innovation since the entrepreneur's effort choice depends on her private value of the intangible, which in general is lower than the social value. This is similar to models of endogenous growth with spillovers. Perhaps surprisingly, however, we see that the degree of under-investment can be greater for intangibles that are highly scalable (higher ρ) if appropriability is low enough (δ is high).

Last, it is important to emphasize the distinction between ex-post rents to the entrepreneur

$$A\left[\delta e^{-\rho}\left(\frac{\rho}{\delta}\right)^{\rho}\right]N - c(\hat{\iota}), \tag{11}$$

with ex-ante rents

$$A\left[\delta e^{-\rho}\left(\frac{\rho}{\delta}\right)^{\rho}\right]E[N;\hat{\iota}] - c(\hat{\iota}). \tag{12}$$

If there is selection on which entrepreneurs enter the market (or equivalently if failure $N = 0$ is a feasible outcome despite the amount of effort involved) then focusing on ex-post compensation to entrepreneurs (11) will overstate their payoff. Put simply, ex-post rents (11) can be positive even if rents are zero ex-ante (12) due to free entry of entrepreneurs.

3.2 Implications and relation to other approaches

We conclude by discussing the relationship between our model, and the way in which the key properties of intangibles have been discussed in existing work.

Scalability

A common assumption in the literature is that intangible capital can contribute to ”higher returns to scale”, or more generally, that intangibles are ”scalable”. This property is often thought to be connected to the non-rival nature of intangible capital as a production input. To explain this, a distinction is usually made between the cost of *creating* new intangible assets, and the cost of *using* existing intangible assets.

Like tangible assets, creating new intangible assets is costly, in the sense that it requires investment on the part of the firm together with inputs that may be in scarce supply. However, once a new intangible asset is created, the cost of *using* it repeatedly is quite low or even zero. The low usage cost within the firm is analogous to non-rivalry (**P2**), and Assumption 1 in the model. The fact an intangible asset is being used to produce one unit of a good or service for a customer does not fully preclude it from being used to produce the same good or service for another customer.

In existing work, this is often captured by assuming that intangibles involves high fixed costs, but leads to lower marginal cost of production for the firm. As a result, production at intangible-intensive firms may be characterized by increasing returns to scale, at least locally.[5]

Instead of assuming a particular cost structure with increasing returns, our model starts from a formal definition of non-rivarly in use, Assumption 1. Interestingly, the notion of ”scalability” that results from this approach is somewhat different from the existing literature. Namely, ”scalability” in our model is best understood as the fact intangible capital and firm span are *complements*. To see why, recall that firm value at Stage 3 is given by $V(N, x) = A\, N\, x^{\rho}$. When $\rho = 0$, we have:

$$\frac{\partial V(N,x)}{\partial x} = 0, \qquad \frac{\partial V(N,x)}{\partial N} = A.$$

In this case, the marginal value of intangible capital is independent of the span of the firm x, and thus intangibles do not benefit from greater scale. Similarly, increasing the amount of intangibles deployed in the firm does not affect the benefit of increasing scale x. By contrast, when $\rho \to 1$,

$$\frac{\partial V(N,x)}{\partial x} = A\, N, \qquad \frac{\partial V(N,x)}{\partial N} = A\, x.$$

In this case, firm scale and the stock of intangibles become complements: the value of the intangible asset increases when it can be employed in multiple segments. Similarly, the higher the value of the firm's intangible asset, the greater is the benefit of expanding the span x of operations.

Thus, from the assumption of non-rivalry in use, our model derives a novel characterization of ”scalability” or ”returns to scale” for intangibles, namely that they act as complements with firm span – and more so, the stronger is non-rivalry (i.e., the higher is ρ).

Limits to appropriability

The fact that intangibles can be stored externally and copied (the fundamental properties **P1** and **P2**), combined with imperfections in the enforcement of exclusion, imply that the returns generated by a particular intangible asset may be difficult to fully appropriate by the firm or agent that created it. Non-rivalry means that an existing intangible can be copied and used at relatively low cost, while

[5] Naturally, locally increasing returns are not entirely specific to intangibles. For instance, the production of power from nuclear plants primarily uses tangible inputs, but has the same feature of high fixed, low marginal cost.

storability makes it possible for another party to partly appropriate the returns of the intangible.

For instance, competitors can imitate a firm's process by copying their key patent, algorithm, or software. Because of the non-rivalry of intangibles, this process is substantially simpler than for replicating, say, the competitor's production facilities. Additionally, this imitation process is facilitated by environments with weak intellectual property rights protection. Eventually, this replication can erode the returns generated by the intangible for its original creator. In our model, this process is captured by the assumption that $\delta > 0$, so that the entrepreneur can only appropriate the returns generated by a portion of the intangibles she initially owns.

We note that the flipside of limits to appropriability is that intangibles generate spillovers outside of the firm. Ideas that are stored and widely disseminated can be used effectively in production—or even spur the development of better ideas. As a result, the use of a specific intangible asset by one firm may indirectly increase productivity in other firms who can potentially adopt the same intangible. This notion of positive spillovers is present in many models of endogenous growth. Though it is not fully fleshed out in our model, we note that it is somewhat captured by the fact that the overall social or welfare value of the project, $\hat{V}(N)$, can exceed the value that is privately appropriable by the entrepreneur, $\hat{V}_e(N)$.

Here, we should emphasize that negative spillovers are also possible. The same forces that lead to wide dissemination and adoption of new ideas imply that older ideas become more easily obsolete. A new and more efficient method of production can be licensed to many firms, leading to a drop in the value of the intangible asset representing the old production method (e.g. the patent). The assumption that $\delta > 0$ could also capture this process of "external" depreciation of the firm's assets.

4 Economic implications of intangible capital

In this section we discuss how accounting for intangibles and understanding their unique characteristics can help shed light on key trends of the data. In particular, the decades since the 1990's have been characterized by relatively low growth in total factor productivity, a decline in the labor share, weak tangible investment and rising valuations, an increase in economic rents, and rising inequality.

4.1 Aggregate Productivity and Factor Shares

We begin with a discussion of how challenges in measuring intangibles can lead to skewed inference regarding aggregate productivity and factor shares.

Consider the model in section 3.1. If we assume that there is no heterogeneity in ρ and δ and then clear the market for physical capital to determine the equilibrium interest rate R, we can write aggregate output Y with some abuse of notation as

$$Y = \left(\frac{\rho}{\delta}\right)^{\rho\,(1-\zeta)} \, N^{1-\zeta}\, K^{\zeta}. \tag{13}$$

As we examine (13), it is useful to keep in mind that the simple model in Section 3.1 has constant total factor productivity. As such, aggregate output is a function of the economies' stock of physical capital K, the quantity of intangibles N, with an adjustment for the fact that intangibles are non-rival (the first term). Here, for simplicity, we ignore the effort cost e to generate new intangibles when constructing output (consistent with the data).

In the data, economists typically measure output and the stock of physical capital relatively well, or at least better than the stock of intangible assets. Indeed, recall from property **P1** that measuring the stock of intangibles as an input to production is quite challenging, since they can be embodied not only in physical assets but also in labor.[6] To fix ideas, consider the extreme case where the only measurable input to production is physical capital, that is, aggregate statistics can only measure Y_t and K_t. Taking logs of (13), we can define measured TFP (in logs) as

$$tfp \equiv \log Y - \zeta \log K = \rho\,(1-\zeta)\;(\log\rho - \log\delta) + (1-\zeta)\log N. \tag{14}$$

Examining equation (14), we see that measured productivity depends not only on the 'stock' of intangible capital N but also on their degree of non-rivalry and appropriability (i.e. ρ and δ). Specifically, the non-rivalrous nature of intangibles can imply that once an intangible asset is developed, output can increase rapidly as the intangible capital is applied in many locations or applications simultaneously (which depends on ρ). Similarly, the optimal scale of deployment is a function of appropriability (determined by δ).

Understanding the Productivity Slowdown

One can take two approaches in interpreting the productivity slowdown through the lens of equation (14). One approach would be to take the stance that aggregate productivity only changes as a result of intangibles. In this case, the *entirety* of the Solow residual—the change in (14)—is

[6] There are several practical reasons why intangibles are difficult to measure well. Indeed, even though there is considerable effort devoted to measuring intangible assets by the Bureau of Economic Analysis (BEA), many economists agree that intangibles are underestimated in both national accounts and corporate balance sheets. For example, McGrattan and Prescott (2010b) point out that the 1990s in particular show a puzzling increase in hours and corporate capital gains at the same time that measured total factor productivity (TFP) and wage income was low. They interpret these facts by noting that intangible capital is typically expensed (i.e. treated as a production cost), or financed by employees' sweat equity. They show how the resulting underestimation of output and income mechanically leads to lower measured labor productivity growth. The effect on TFP growth is less clear due to the fact that the underestimated intangible capital is both an output (raising actual TFP) and an input (lowering actual TFP).

driven by incomplete capital measurement. In this view, intangibles and their properties are the 'dark matter' that explain the wedge tfp_t between measured output and inputs. this is essentially the view adopted by models of endogenous growth (see Jones, 2021, for a recent survey). A positive Solow residual then reflects faster net growth in the unmeasured portion of capital inputs (intangibles). Under that view, slow productivity growth could result from lower investment in intangibles N or changes in the degree of non-rivalry ρ or limited appropriability δ. Put differently, if a firm invests in a process that increases its productivity, economic modelers must decide whether to model the increase in output as coming from an increase in inputs or an increase in the productivity of existing inputs.

Under the first view, the properties of the Solow residual are closely tied to the dynamics of N_t and shifts in the parameters ρ and δ. Under that view, the slowdown in the growth rate of measured productivity can be rationalized by

- *Time-to-build.* Intangible capital may involve substantial time-to-build, and its effects may resemble types of tangible capital with high fixed costs of installation, but with negligible variable costs. An example of this is logistics-optimization software. Such software may require substantial development time, but once it is successfully operational it can improve delivery times for all of a firm's production units. New production methods may take time to be adopted; further may effectiveness may be low at first, but subsequently increase due to learning by doing. To the extent that these up-front costs are not recognized as investment, they can generate a slowdown in productivity over the short run even as long-run productivity is higher (Brynjolfsson, Rock, and Syverson, 2021).

- *Depreciation.* The depreciation and obsolescence process for intangible capital can also impact total factor productivity growth. Because best-ideas and processes can be superceded by innovation, installed intangible capital can become quickly and potentially fully obsolete. Combined with long development lags or learning by doing (time-to-build), displacement of existing capital during periods of rapid innovation can exacerbate the drop in measured output in the short run (Greenwood and Jovanovic, 1999).

- *Investment.* The fact that the investing firm or entrepreneur cannot capture all of the value of intangible investment may also lead to a lower incentive to create intangible capital internally. As we discuss in the last part of Section 3.1, shifts in the degree of non-rivalry ρ or degree of appropriability δ not only affect output directly as in equation (13), but they also indirectly affect the incentive to exert effort to create new intangibles. Variations in the effective rate of appropriability can also lead to fluctuations in measured TFP, as is the case for instance in the model of Kondo, Li, and Papanikolaou (2021).

Under the second view, there is a conceptual difference between total factor productivity and the stock of intangible capital—Covid19 is an example of a productivity shock that is conceptually distinct from intangibles. Under that view, researchers aim to construct more "complete" measures of capital—here, estimate N_t directly—and adjust the Solow residual correspondingly. This is the approach followed in the seminal contributions of Basu, Fernald, Oulton, and Srinivasan (2003) and Corrado, Hulten, and Sichel (2005), and also explored in McGrattan and Prescott (2010a), McGrattan and Prescott (2010b), and Crouzet and Eberly (2021a). This approach proceeds by identifying expenditures and prices of excluded intangible investment, and then using them to construct estimates of N_t, sometimes with the help of an equilibrium model.

One important lesson from this literature is that unmeasured intangibles can increase, or decrease, the fraction of the Solow residual attributable to aggregate productivity growth (Basu et al., 2003; Corrado, Hulten, and Sichel, 2009; McGrattan and Prescott, 2010b; Crouzet and Eberly, 2021a). A key open issue is how to appropriately construct price indices to deflate investment expenditures in intangibles. A further complication is that, output, intangibles, and measured productivity in the U.S. may be further mis-measured due to the fact that U.S. corporations have a tax incentive to book income from intangible assets abroad (Guvenen, Mataloni Jr, Rassier, and Ruhl (2021)). The income that is booked offshore lowers gross operating surplus in the U.S., thereby reducing measured value added. The impact on measured total factor productivity depends on the countervailing effects of missing intangible inputs and missing income.

An important limitation of this literature is that it has so far treated intangible investment as an input that has similar economic characteristics to physical capital; more specifically, it effectively assumes that it is a Cobb-Douglas substitute for physical capital. However, a fundamental property of intangibles is non-rivalry, which together with limited appropriability, can lead to positive spillovers. Spillovers imply that measuring the 'stock' of intangibles N_t is insufficient: as we can see in equation (14), the spillover parameter ρ together with the parameter governing limited appropriability δ also affect measured productivity. This implies that in order to measure the full contribution of intangibles to economic output, researchers need to not only obtain measures of the intangible stock N_t but also accounts for the value of spillovers.[7]

[7]To the extent that aggregate market values are used to measure the price of intangibles included in national accounts, some of these spillovers may be included in existing estimates of the intangible capital stock. But aggregate market values for intangibles are scarce, and even when these measures exist, it is not always clear how to allocate spillover returns across sectors (Moylan and Okubo, 2020). Further, market values may underestimate the contribution of intangibles if these are partly stored in key labor inputs (Eisfeldt and Papanikolaou, 2014).

Factor Income Shares

Recent work has documented a secular decline in the labor share in national accounting data, both in the United States and globally (Elsby, Hobijn, and Şahin, 2013; Karabarbounis and Neiman, 2014). When interpreting these trends, however, it is useful to keep in mind that current national accounts either omit certain types of intangibles (organization capital, for example) or allocate intangibles as part of physical capital. The fact that intangibles are typically hard to measure implies that factor shares are also mismeasured. Depending on the implicit assumptions researchers make, the share of output that would accrue to intangibles is can be allocated to either physical capital, labor, or 'rents', where the latter is defined as monopoly profits.

As an illustration, let us now re-interpret the fixed factor K in the model as a composite input good consisting of physical capital M and labor L, both in fixed supply,

$$K \equiv M^{\alpha} L^{1-\alpha}. \tag{15}$$

In this case, the share of output that accrues to 'intangibles' (i.e. not to K) is equal to

$$\frac{Y - R\,K}{Y} = 1 - \zeta, \tag{16}$$

while the factor share of physical capital and labor is $\alpha\,\zeta$ and $(1-\alpha)\,\zeta$, respectively.

Now, the question is whether the factor share of intangibles (16) should be allocated to labor or capital. The model in Section 3.1 helps shed some light—under the second interpretation of Assumption 2. Recall that, under that interpretation, the limits to appropriability δ are motivated by incomplete markets: the entrepreneur internalizes that if she expands the scale of the intangible she needs to 'store' some of the intangible externally, which implies giving up some rents. In that case, we can interpret as

$$\frac{N_e}{N} = \delta e^{-\rho} \qquad \text{and} \qquad \frac{N - N_e}{N} = 1 - \delta e^{-\rho} \tag{17}$$

as the share of intangible share that accrues to the entrepreneur and outside investors, respectively. Given our model setup, (17) only depends on the degree of non-rivalry: the greater the degree of non-rivalry (higher ρ) the smaller is the share that accrues to the entrepreneur, as she chooses to give up a larger fraction of her rents to achieve a higher scale.

Equation (17) gives some guidance on how the factor shares of intangibles should be allocated between capital and labor. The entrepreneur's share N_e/N should likely be considered labor income if it is the case that human capital is the key input in the production of new intangibles. The residual part $1 - N_e/N$, however, is the part that outside investors have a claim to, which could (though it need not be) be part of capital income.

Any decomposition of Y into factor shares that ignores this distinction will misallocate (16) to either physical capital, labor, or monopoly rents. For instance, Barkai (2020) measures monopoly rents as output minus labor expenses minus the stock of physical capital times its user cost. As a result, this measure of monopoly rents will include the share of output due to intangibles.[8] Alternatively, if one estimates the capital share as one minus the wage payments to labor (Elsby et al., 2013; Karabarbounis and Neiman, 2014), this factor share will include income from intangibles. If labor is a key input in the production of intangibles, part of that income should in fact be classified as labor income. To the extent that intangibles are 'stored' in key employees (**P1**) this choice can understate the labor share. In particular, some capital income and profits are actually equity compensation for high-skilled labor inputs and are thus partially misclassified. Smith, Yagan, Zidar, and Zwick (2019), Bhandari and McGrattan (2021) show how private firm ownership leads to compensation of labor inputs with capital income. Eisfeldt, Falato, and Xiaolan (2021) document the large fraction of labor compensation in the form of equity-based pay in recent decades.

A further complication in interpreting the trends in the labor share is that total output is also mismeasured if expenditures on intangibles are recorded as a cost of production rather than investment. Indeed, expenditures on intangibles tend to be recorded either as payments to labor or purchases of intermediate inputs, such as consulting services or lab equipment.[9] Recent revisions to the U.S. National Income and Product Accounts capitalized certain intangible assets (especially intellectual property products (IPP)) and added these "produced" assets to gross operating surplus and as income to capital. In particular, Koh, Santaeulàlia-Llopis, and Zheng (2020) show how this allocation of intellectual property products (IPP) to the capital account in the U.S. National Income and Product Accounts generated most of the measured decline in the labor share.

This raises the question of whether the payments to intangibles should be allocated to labor or capital. The allocation likely varies across different types of intangibles. Indeed, the distribution of income from intangible capital depends on how it is stored, and then on the property rights to the storage type. For example, managers may accrue income from intangible assets such as organization processes or corporate culture. Similarly, capital owners (shareholders) may accrue income from software, patents, or brands.

Further, it is highly likely that, because of the ambiguity in ownership of intangibles, cashflows

[8]Given that many intangible assets (for example, patents, copyrights, and trademarks) are associated with exclusivity and hence monopoly power, the conceptual distinction between monopoly rents and the factor share of intangibles is not obvious.

[9]A substantial fraction of R&D investment is labor compensation. For example, Adobe's 2019 R&D expenditures, and Sanderson Farm's 2019 breakdown of Selling and General Administrative (SG&A) Expenses (see the firms' respective 10-Ks) show that the majority of their research and SG&A expenditures are in fact labor compensation.

from intangibles flow to both labor (key talent) as well as owners of tangible capital (shareholders). This has implications for the measurement of intangibles that relies on market valuations of firms. In general, the full value of intangible capital will be partly on the market value balance sheet of firms, and partly on the market value balance sheet (wealth) of key talent. Variation in bargaining power between the two parties can lead to insiders appropriating a larger or smaller share. As a result, imputing the stock of intangibles based on market valuation ratios will likely under-estimate the value of intangibles (Eisfeldt and Papanikolaou, 2014).

In summary, because current practice tends to either omit intangibles from national accounts or allocate their payments primarily to capital, the actual labor share is likely higher than the share computed using NIPA data, in particular after the revisions in 1999 and 2013. Depending on the severity of this mis-classification of payments to intangibles, it can generate mismeasurement and trends in the observed labor share.

4.2 Inequality

Under certain conditions, the existence of intangible assets can lead to inequality in income and wealth. The conditions govern the allocation of the returns and potential rents from intangible capital to different agents in the economy. Specifically, we discuss below the conditions where (i) rents that may accrue to inventors and entrepreneurs, (ii) rents may accrue to key employees, and (iii) intangibles may exacerbate capital-skill complementarity.

Rents accrue to inventors and entrepreneurs

When inventors and entrepreneurs conceive and develop a new intangible, as described in our earlier model, for example, they can appropriate a fraction of the value generated by intangible capital and innovation. As such, equation (16) can be used to shed light on inequality in income and wealth. Recall that N_e/N is the share of intangibles that accrue to the entrepreneur whereas $1 - N_e/N$ is the share that accrues to outsiders (for example, outside investors, or the economy more broadly). The key difference between these two parts is that the entrepreneur's share is concentrated and not easily tradeable ex-ante (the entrepreneur cannot pre-sell claims to future intangibles she has yet to produce, otherwise her incentive to exert effort would be low). By contrast, by codifying (i.e. storing) the intangible, the entrepreneur can convert an amount $N - N_e$ to an asset, to which outsiders can lay a claim. Most importantly, these claims on $N - N_e$ can be diffusely held as investors build diversified portfolios.

This key distinction between the two shares (what is stored with the entrepreneur and hence owned by her versus what is owned by outside stakeholders) can help interpret patterns in inequality.

Kogan, Papanikolaou, and Stoffman (2020) explore these ideas in a general equilibrium model. The key feature of their model is incomplete markets: each period, a small measure of agents—the 'inventors'—are randomly endowed with a blueprint for a new project. Kogan et al. (2020) interpret these inventors broadly as encapsulating all parties that share the rents from new investment opportunities besides the owners of the firm's publicly-traded securities. Since these blueprints are valuable, yet inventors cannot contract ex-ante to share these rents with the rest of the economy, their model implies that firm owners reap only part of the benefits, yet all of the costs, of creative destruction. As a result, the arrival of new technologies is associated with greater income and wealth inequality, together with a motive to insure against states of the world with rapid technological innovation.

Rents accrue to key employees

The discussion above implicitly assumed that the share $1 - N_e/N$ was owned by shareholders. This need not necessarily be the case. It is true that part of it is likely codified in media that can be intellectually protected, but another part could be stored with key employees, as with N_e. An example would be management practices: a textbook or a business school can prescribe 'best practices' but it does not immediately follow that everyone who takes the class or reads the book becomes an effective manager. Further, since the value of these intangibles are likely to be partly specific to the firm, the cash flows generated by intangible capital are often shared between shareholders and key talent.

As intangibles grow in importance in terms of firms' capital stocks and value, the importance of key labor inputs in contributing to and operating these assets may lead to inequality between key labor and other workers (McGrattan and Prescott, 2010b; Smith et al., 2019; Eisfeldt et al., 2021; Bhandari and McGrattan, 2021). Further, the share of rents that accrues to key talent need not be constant, as it will depend on their outside option, which introduces a further source of inequality (Eisfeldt and Papanikolaou, 2013).

Intangibles exacerbate capital-skill complementarity

Intangibles can have an impact on income inequality even when intangibles are all stored externally, as long as their output is complementary with certain worker skills. Thus, even though the model we developed in Section 3.1 has no explicit labor inputs, it is relatively straightforward to include different types of labor into the model, possibly using a CES framework similar to Krusell, Ohanian, Ríos-Rull, and Violante (2000) and Eisfeldt et al. (2021).

A common view of technological progress in the 20-th century is that it is primarily 'skill biased', that is, it is more complementary to high-skill workers but was a substitute for low-skill workers (Goldin and Katz, 2008). As a result, if intangibles increase the marginal productivity of high-skilled labor inputs, the income of this segment of the labor force benefits. Thus, a rise in the importance of intangible over the last few decades may have contributed to rising inequality. In support of this view, (Eisfeldt et al., 2021) find that pay to high-skilled labor, and in particular equity pay, grew fastest in recent decades within industries which were most exposed to declining investment goods prices, as a proxy for the growth in intangibles stored in capital goods.

More generally, however, the relation between intangibles and the level of income inequality can be ambiguous if new intangibles are also associated with skill displacement. Put differently, an increase in between-group inequality need not translate into an increase in between-worker inequality if workers transition across groups. Using detailed data on patent inventions and occupation task descriptions, Kogan, Papanikolaou, Schmidt, and Seegmiller (2021) document workers in occupations most exposed to technology improvements experience declines in wage earnings. Importantly, they find that the workers most adversely affected are the highest-paid workers, which can be consistent with technology-skill complementarity only as long as workers can also lose part of their human capital when technology improves.

4.3 Tangible Investment and Tobin's Q

In recent decades, investment in physical capital has been declining among US firms. At the same time, Tobin's Q for physical capital — the ratio of enterprise value W_t to the replacement cost of physical capital, $Q_{K,t} = W_t/K_t$ — has been rising. Figure 3 illustrates this phenomenon, drawing on accounting and valuation data for non-financial, publicly traded US firms in Compustat.[10] Gutiérrez and Philippon (2016) note that this pattern is a puzzle for standard investment models in which average Q coincides with the marginal return to investment, or maginal q.

One interpretation is that average $Q_{K,t}$ and marginal $q_{K,t}$ are distinct, and more importantly, the difference has increased over time. The model of Section 3.1 can shed light on the role that intangibles may have played in that process. In order to make the connection to Q-theory clearer, we assume that the total stock of physical capital is fixed, instead of rented on markets at a fixed marginal cost R. A firm with installed physical capital K, intangibles N, and span x has value:

$$W(K,N,x) \quad = \quad \max_{\{N(s),K(s)\}_{s\in[0,x]}} \quad \int_0^x N(s)^{1-\zeta} K(s)^{\zeta} ds \tag{18}$$

[10]The figure estimates physical investment $I_{K,t}$ as the aggregated capital expenditures of firms in the sample; the stock of physical capital K_t as the aggregated value of their gross property, plant and equipment; and enterprise value W_t as the aggregated market value of their equity, plus the aggregated book value of their debt, minus the aggregated book value of their cash holdings. Data sources and sample construction the same as in Crouzet and Eberly (2021b).

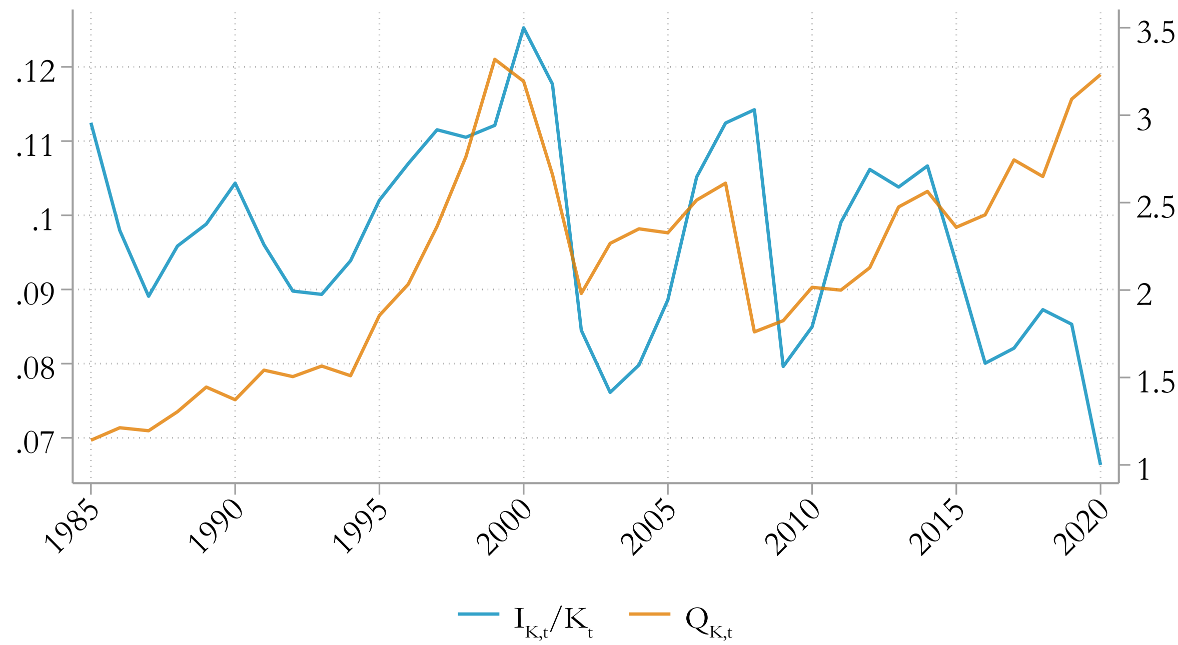

Figure 3: Time series for the gross physical investment rate, $\frac{I_{K,t}}{K_t}$, and Tobin's Q for physical capital, $Q_{K,t} = \frac{W_t}{K_t}$. The data source is the sample of US non-financial corporations in Compustat. $I_{K,t}$ is measured as the sum of all capital expenditures (Compustat item `capx`); K_t is measured as the gross value of the stock of property, plant and equipment (Compustat item `ppegt`); and enterprise value, W_t is computed as the market value of equity (the product of Compustat items `prc` and `shrout`), plus the book value of debt (the sum of Compustat items `dlc` and `dltt`), minus cash and cash equivalent (Compustat item `che`). Details on sample selection are reported in the Online Appendix of Crouzet and Eberly (2021b).

$$\text{s.t.} \quad [q_K] \quad \int_0^x K(s)\,ds \leq K \tag{19}$$

$$[q_N] \quad \left(\int_0^x N(s)^{\frac{1}{1-\rho}}\,ds\right)^{1-\rho} \leq N, \tag{20}$$

where q_K and q_N are the Lagrange multipliers on the physical and intangible capital allocation constraints, that is, the marginal increase in firm value associated with a marginal increase in K or in N. Following steps similar to Section 3.1, the solution for enterprise value and these shadow values is:

$$W(K,N,x) = N^{1-\zeta}K^{\zeta}x^{\rho(1-\zeta)}, \tag{21}$$

$$q_K = \zeta\left(\frac{N}{K}\right)^{1-\zeta}x^{\rho(1-\zeta)}, \tag{22}$$

$$q_N = (1-\zeta)\left(\frac{K}{N}\right)^{\zeta}x^{\rho(1-\zeta)}. \tag{23}$$

Note that, if we impose $q_K = R$, we obtain:

$$K = \left(\frac{\zeta}{R}\right)^{\frac{1}{1-\zeta}} N x^{\rho} \quad \text{and} \quad V(N,x) = W(K,N,x) - RK = (1-\zeta)\left(\frac{\zeta}{R}\right)^{\frac{\zeta}{1-\zeta}} N x^{\rho},$$

that is, the same solution as in Section 3. Thus the model of Section 3 can be thought of as a particular case of the fixed-capital model, if q_K is set fixed to $q_K = R$.

The solution in Equations (21)-(23) implies the following firm value decomposition:

$$W(K,N,x) = q_K K + q_N N. \tag{24}$$

This decomposition offers some insight into the growing disconnect between average Q_K and marginal q_K for physical capital. Rewriting Equation (24) as:

$$\frac{W}{K} \equiv Q_K = q_K + q_N \frac{N}{K}, \tag{25}$$

we see that intangibles introduce a wedge between average Q_K and marginal q_K. The wedge, $Q_K - q_K = \frac{N}{K} q_N$, depends positively on the ratio of intangible to physical capital, N/K, since:

$$Q_K - q_K = q_N \frac{N}{K} = (1-\zeta)\left(\frac{N}{K}\right)^{1-\zeta}\left(\frac{\rho}{\delta}\right)^{\rho(1-\zeta)}. \tag{26}$$

In a dynamic extension of this model, physical investment rates would be an increasing function of the marginal value of physical capital q_K. Thus, the growing disconnect between average Q_K and physical investment rates (a function of marginal q_k could be explained by an rising ratio of intangible to physical capital N/K.

This explanation is explored in more detail in Crouzet and Eberly (2019), who show that a relationship similar (26) holds in a more general, dynamic investment model where firms accumulate both intangibles and physical capital. Crouzet and Eberly (2019) apply this idea to cross-sectional data, and find that augmenting standard investment-Q regressions with estimates of N_t/K_t helps reduces the firm-level gap between investment and $Q_{K,t}$ by about 30%.[11]

The analysis in Crouzet and Eberly (2019) assumes that intangibles are similar to physical capital in that they are not scalable within the firm, and their value is fully appropriable by the entrepreneur. By allowing for these properties, equation (26) highlights how non-rivalry and limits

[11] An alternative approach is to use Equation (26) as an indirect way to measure N_t/K_t. Measures of W_t and K_t are can be obtained from market valuations and balance sheet information, while $q_{K,t}$ and $q_{N,t}$ can be estimated from measures of investment rates in physical and intangible capital. The stock of intangibles to the physical capital stock is then the residual $(W_t - q_{K,t}K_t)/q_{N,t}$. This "residual" approach is followed, for instance, by Hall (2001), who calls it the "quantity revelation principle". Eisfeldt and Papanikolaou (2014) point to the shortcomings of this approach if N is not fully reflected in stock market valuations, consistent with the discussion in Sections 4.1 and 4.2.

to appropriability could affect the wedge $Q_K - q_K$. For instance, for sufficiently low δ (that is, when appropriability is sufficiently strong), technological advances that increase the non-rivalry of intangibles within the firm, ρ, will lead to a growing wedge $Q_K - q_K$. An example could be the growing availability (and declining price of) of digital media, which facilitate the replication and scaling of intangibles stored in software. Analogously, changes in the institutional environment that improve the appropriation of intangible returns — say, improved patent protection — will generally magnify the disconnect between Q_K and investment. Note that both mechanisms are distinct from a decline in ζ. Such a decline would also lead to an increase in the wedge $Q_K - q_K$, but not because of scale or appropriation effects; instead, it would capture better substitutability between intangible and physical capital in production.

More generally, this approach allows other interpretations of the wedge. For example, if the arrival of new intangible assets (say new blueprints for capital goods) displaces the value of existing capital, then average Q can fall even as marginal Q rises. The evidence in Greenwood and Jovanovic (1999), namely that the arrival of information technology led to a fall in the value of existing firms is consistent with that view. In a model with technology shocks embodied in new capital goods (as in Papanikolaou, 2011, for example), improvements in new capital goods can lead to a decline in average Q if the replacement value of the installed capital stock (the denominator in Q) does not adjust fully to offset the decline in the market value of incumbent firms.

4.4 Rents and Market structure

We define rents as returns generated by an asset in excess of its marginal (user) cost. Like physical capital, intangible capital can generate rents for its owners. Rents relate to market structure because they often arise in situations where the capital (whether physical or intangible) is used to produce a good for which there are poor substitutes. For instance, a drug formula produces rents to the extent that it is difficult to produce a generic substitute. These conditions can lead to imperfect competition and in some cases, to greater market concentration.

In situations where intangible capital generates rents, it is important to note that the intangible asset itself is not the rent. The rent is the stream of appropriable returns generated by the intangible in excess of its user cost. In the example of franchises, in Table 1, the intangible asset is the combination of the brand with the logistical and organizational instructions provided to the franchisee. In order to generate rents, the asset must produce a marginal return above its user cost. It may be possible for the owner to appropriate this return, for instance through enforceable franchise agreements. The fact that rents result from intangibles is important for measuring rents, as firms can invest in generating rents, but these investments and the intangibles that result, are distinct from rents.

Conceptually, the rents generated by the intangible asset should be measured separately from the intangible asset itself. To illustrate the distinction, consider again the fixed-capital model described in Equations (18)-(20), but assume that total sales are given by:

$$\left(\int_0^x N(s)^{1-\zeta} K(s)^{\zeta}\right)^{\frac{1}{\mu}},$$

where $\mu > 1$ is a fixed parameter that creates a wedge between the average and marginal revenue product of both K and N. The wedge is a simple way to capture the rents associated with production, and could be microfounded, for instance, as a markup of output prices over marginal cost in a monopolistic setting.[12] In this case, firm value W, and marginal q_K and q_N are given by:

$$W(K,N,x) = x^{\frac{(1-\zeta)\rho}{\mu}} N^{\frac{1-\zeta}{\mu}} K^{\frac{\zeta}{\mu}} \tag{27}$$

$$q_K = \zeta \left(\frac{N}{K}\right)^{1-\zeta} \left(\frac{K^{\zeta}}{N^{1-\zeta}}\right)^{1-\frac{1}{\mu}} x^{\rho(1-\zeta)} \tag{28}$$

$$q_N = (1-\zeta) \left(\frac{K}{N}\right)^{\zeta} \left(\frac{K^{\zeta}}{N^{1-\zeta}}\right)^{1-\frac{1}{\mu}} x^{\rho(1-\zeta)} \tag{29}$$

This solution implies a more general version of the firm value decomposition in Equation (24):

$$W(K,N,x) = \overbrace{q_K K + q_N N}^{(a)} + \underbrace{(\mu-1) q_K K + (\mu-1) q_N N}_{(b)}. \tag{30}$$

This decomposition highlights two broad sources of firm value. Term (a) is the value of installed capital, which makes up all of firm value in the absence of rents ($\mu = 1$), as was the case in Equation (24). Term (b) is the net value of rents, which is positive only when $\mu > 1$. Additionally, each of these two terms (value of installed assets, and rents) can be decomposed between a contribution of physical and a contribution of intangible capital. Consistent with the idea that the intangible asset itself is not the rent, this decomposition assumes no specific relationship between μ (a fixed parameter) and the stock of intangibles (N, which we take as given in this application of the model).

Crouzet and Eberly (2021b) show that a decomposition similar to Equation (30) holds in a much broader class of dynamic investment models in which capital inputs, both physical and intangible,

[12] We choose to introduce this wedge to the sum of all production across streams. If, instead, it applied individually to each stream, i.e. $Y(s) = \left(N(s)^{1-\zeta} K(s)^{\zeta}\right)^{1/\mu}$, then the firm would have a motive to increase its span even when $\rho = 0$, because a higher span would counterbalance decreasing returns at the stream level. We leave this mechanism out of the model, since it is independent of the non-rivalry of intangibles, but it is possible to include it in the model. It generally leads to a higher choice of span, all other things equal.

can generate rents.[13] Moreover, they describe how to estimate the components of the decomposition using a simple set of moments, including Tobin's Q for physical assets, flow returns to physical capital, and estimate of the ratio of intangible to physical capital, N/K. The key finding is that rents associated with intangible assets have contributed a sharply rising share in the growth total enterprise value of US businesses since the early 1990s, from approximately 15% in the mid-1980s, to up to 40% in 2015, depending on how broadly intangibles are measured.

However, Crouzet and Eberly (2021b) do not allow for non-rivalry of intangibles, $\rho > 0$, or for limits to their appropriability, $\delta > 0$. To see the effect of these properties on rents, define total rents per unit of physical capital:

$$\Gamma(K, N, x) \quad \equiv \quad (\mu - 1)\,(q_K + q_N N)\,, \tag{31}$$

and it is straightforward to see that:

$$\Gamma(K, N, x) = \frac{\mu - 1}{\mu}\left(\frac{N}{K}\right)^{1-\zeta}\left(\frac{K^{\zeta}}{N^{1-\zeta}}\right)^{1-\frac{1}{\mu}} x^{\rho(1-\zeta)}. \tag{32}$$

Of these rents, only $\Gamma^e(K, N, x)$ are appropriable by the entrepreneur, where:

$$\Gamma^e(K, N, x) \quad \equiv \quad \delta e^{-\frac{\delta(1-\zeta)x}{\mu}}\Gamma(K, N, x), \tag{33}$$

As in Section 3.1, we assume that the entrepreneur chooses the span x to maximize her claim, which, in this model, is proportional to the value of rents she can appropriate. The optimal span is again the same as in Section 3.1, namely $\hat{x} = \frac{\rho}{\delta}$, implying:

$$\hat{\Gamma}(K, N) \quad = \quad \frac{\mu - 1}{\mu}\left(\frac{N}{K}\right)^{1-\zeta}\left(\frac{K^{\zeta}}{N^{1-\zeta}}\right)^{1-\frac{1}{\mu}}\left(\frac{\rho}{\delta}\right)^{\frac{(1-\zeta)\rho}{\mu}}, \tag{34}$$

$$\hat{\Gamma}^e(K, N) \quad = \quad \delta e^{-\frac{\delta(1-\zeta)x}{\mu}}\hat{\Gamma}(K, N). \tag{35}$$

Relative to a model with $\rho = 0$ and $\delta = 0$, this expression shows that both limits to appropriability and non-rivalry affect both the total size of rents, and the fraction retained by the entrepreneur. As in Section 3.1, in cases where $0 < \rho < \delta$, limits to appropriability dominate, and rents are *smaller* than if intangibles had the same economic properties as physical capital, i.e. full appropriability and rival within the fFirm. On the other hand, when $0 < \delta < \rho$, non-rivalry dominates, and rents are higher than they would be if intangibles had the same economic properties as intangibles. Additionally,

[13] The model of Crouzet and Eberly (2021b) is dynamic, so the rent term (b) in Equation 30 is a present value, instead of a one-period flow, as in the simple model of this paper.

higher non-rivalry ρ generally leads to a lower share of rents accruing to the entrepreneur.

While the modeling above has focused on market power and rents, market concentration is a potential outcome of intangibles because of their non-rivalry within the firm. As discussed in Section 2, scale economies within the firm are implied by property **P2**, non-rivalry in use. Scale economies will manifest themselves as higher fixed but lower marginal costs for more intangible-intensive firms. As a result, all other things equal, these firms will take up a larger share of their markets. This is closely related to the theory of natural monopolies, which also emphasizes that markets featuring some firms with high-fixed, low-marginal cost structures will have high equilibrium concentration (Baumol, 1977).[14]

Thus, more intangible-intensive industries may naturally be more concentrated; more intangible-intensive firms should command higher market shares; and within firm, intangible intensity should be correlated with market share. Indeed, Crouzet and Eberly (2019) provide reduced-form evidence consistent with these predictions, using data on publicly traded US firms.[15]

4.5 Implications for Corporate Finance and the Cost of Capital

Firms' capital structure, or how firms are financed, also depends on the fraction of the firm's capital stock that is intangible and where those intangibles are stored. The first fundamental property of intangible assets (**P1**) implies that they can be stored in capital, labor, or even in customers. As a result, intangibles are less likely to be pledgable to outside investors, suggesting that debt is unlikely to be the preferred form of financing (Falato, Kadyrzhanova, Sim, and Steri, 2020). Sun and Xiaolan (2019) argues instead that intangibles are primarily financed by employees, who then have an implicit claim on them. The simple model we developed in Section 3.1 has a simplified version of this tradeoff: in order for the entrepreneur to be able to expand the scale of operations (increase x) she needs to give up property rights on a share of her initial stock of intangibles N, likely by codifying (i.e. storing) them externally, which enables them to be pledged to outside investors and thus provide financing. Related to this point, the capital structure of new firms with substantial intangibles appears to include more concentrated control in the form of dual-class shares whereby initial owners, founders, and employees retain more voting rights than outside equity holders (Ritter, 2020).

[14] We consider an approach to closing the model that allows for entry and hence endogenizes industry size and composition. This approach is more explicit in developing this intuition on market concentration.

[15] Separate from economies of scale, intangibles might be conducive to market concentration if they are associated economies of scope. By "economies of scope", we refer to the idea that rather than having two firms separately create and use the same intangible asset, it may be more efficient for the two firms to merge and share the fixed cost of creating the intangible asset. For instance, it may be more efficient for two retailers to merge and operate under the same brand, than to create and use their own brands separately. More broadly, intangibles may matter for the determination of firm boundaries.

The presence of intangibles can also affect the cost of capital. Given that intangibles may be stored in non-capital inputs, the sharing rule for cash flows generated by intangible capital may also expose investors in firms relying on intangible assets to unique risk factors (Eisfeldt and Papanikolaou, 2013; Eisfeldt, Kim, and Papanikolaou, 2020). Further, if the economic value that is generated by new ideas cannot be fully pledged and hence diversified to outside investors, then states of the world with rapid technological innovation will be associated with higher inequality, creating a demand for insurance against these states and leading to a lower cost of capital for fast-growing firms (Kogan et al., 2020).

5 Implications for Future Research

Our intent in describing intangibles more affirmatively – rather than simply a lack of physical form – is to understand their role in the production of goods, services, and value more clearly. In particular, intangibles have more economic meaning for what they are, than for what they are not. A lack of physical form means that intangibles can be used without rivalry, and hence create the basis for economies of scale and of scope. There are limits to this property however, for as intangibles are used more broadly or intensively, their use may be compromised, creating trade-offs that bound the firm.

These properties have broad implications, which have only begun to foster research. We conclude by emphasizing a few areas, among many, for future exploration.

We discussed the role of intangibles in measured productivity, which can be positive or negative, through a variety of channels, include direct measurement of intangibles inputs as well as biases in the factor income shares. Understanding how measured productivity can remain so weak in the face of seemingly continuous innovation remains puzzling. Further research on measurement and the interaction between intangibles and other factors, especially through non-rivalry, may shed light on this apparent contradiction.

In addition to productivity measurement, factor shares are important for understanding the distribution of income. Intangibles are particularly difficult to allocate to capital or labor, or within types of labor. Further refinement of the allocation of rents may illuminate the sources of inequality, as well as the productivity dynamics that drive overall growth.

One explanation for weak productivity and investment is the rise of market power, and accumulation of rents within the largest firms - driving concentration. We described the properties of intangibles and preliminary evidence consistent with scale economies and rents derived from intangible capital. Further exploration of this connection between intangibles, market power, and industry concentration is a rich area for future research.

Finally, intangibles create interesting challenges for finance. Some intangibles have lively secondary markets, for example in licensing of patents and copyrights. This suggests the possibility of collateralizing intangibles and their use as financial assets separate from the rest of firm. Other intangibles are less separable from the rest of firm, creating challenges for financing. Similarly, the risks associated with intangibles have historically excluded them from standard capital accounting, which is an institutional challenge in many markets (in addition to the risk itself). Strengthening institutions is an important area of research and policy development, as firms rely more on intangibles for production, yet may not be able to rely on it as collateral for financing.

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