Can the cure kill the patient?
Corporate credit interventions and debt overhang

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Business credit programs as countercyclical tools

Sudden, large contraction \Rightarrow increase in corporate default risk.

Novel policy response: business credit programs. $750bn Corporate Credit Facilities (“CCF”) $600bn Main Street Lending Program (“MSLP”)

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Q2 Benefits from alternative program designs? new loans vs. forebearance on existing debt vs. equity injections vs. ...
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Leverage in the run-up to the crisis

[Net leverage] [Interest coverage ratios] [Day of cash on hand] [Projected firms with zero cash]
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Overview

Structural model:

- Q-theory + trade-off theory

Crisis:

- Cash-flow shock + risk price shock + sudden stop

1. Perfect capital markets:

   - Lending programs have ambiguous effects on investment:
     - Any funding at market rates: neutral (irrelevance result)
     - Debt at subsidized prices: negative (↑ leverage, ↓ investment)
     - Intervention reducing cost of equity capital: positive (↑ Tobin's q, ↑ investment)

2. Sudden stop:

   - Short-run positive effects on investment dominate:
     - Weak debt overhang channel
     - Second-order gains from alternative designs
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1. Model
Model building blocks

- $ak$ production with convex adjustment cost function $\Phi$ (Hayashi, 1982)

- Permanent, Brownian shocks to efficiency units of capital $k_j(t)$ (Brunnermeier and Sannikov, 2014)

- Financing via either tax-advantaged exponentially amortizing debt $b_j(t)$ or equity

- No commitment over bond issuances or default policy (DeMarzo and He, 2020)

- At default, bankruptcy costs and firm restructuring with debt haircut (DeMarzo, He and Tourre, 2021)

- Exogenous SDF(s) $\rightarrow$ “industry” (partial) equilibrium

- Partially idiosyncratic, partially aggregate shock $\rightarrow$ cross-sectional distribution over $(b, k)$
Model building blocks

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Key model outcomes

- leverage $x := b/k$ sufficient statistic for a given firm

$$E(k, b) = ke(x) \quad D(k, b) = d(x) \quad G(k, b) = kg(x) \quad I(k, b) = k\nu(x)$$
Key model outcomes

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  \[ E(k, b) = ke(x) \quad D(k, b) = d(x) \quad G(k, b) = kg(x) \quad I(k, b) = k\iota(x) \]

- defaults when leverage reaches cutoff $\bar{x}$

\[
\tilde{R}_d(x) - R_d(x) < 0 \quad \text{debt issuance rate (per unit of capital): trade-off theory with a twist}
\]

\[
\Phi'(g(x)) = \partial_k E = q(x)
\]

\[
\rho^\bullet < 0 \quad \text{arbitrage motive}
\]

\[
\tilde{R}_d(x) - R_d(x)\quad \text{debt expected return wedge (between equity and credit market investors)}
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- firm-level growth rate $g(x)$ satisfies $q$-theory rule $\Phi'(g(x)) = \partial_k E := q(x)$
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- debt overhang: \( g'(x) < 0 \) and \( g(x) < g^* \)
Debt overhang

Investment rate $\Phi(g(x))$

Debt to ebitda $z = \frac{x}{a}$

$\bar{z} = \frac{\bar{x}}{\bar{a}}$
Key model outcomes

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- debt issuance rate (per unit of capital): trade-off theory with a twist

\[ \iota(x) = \frac{\Theta_K}{-d'(x)} + \frac{\left( \tilde{R}_d(x) - R_d(x) \right) d(x)}{-d'(x)} \]

\[ \text{tax motive, arbitrage motive} \]

- $\tilde{R}_d(x) - R_d(x)$: debt expected return wedge (between equity and credit market investors)
Financing policies

Debt issuance rate $\nu(x)$

\[ \bar{z} = \frac{x}{a} \]

Dividend issuance rate $\pi(x)$

\[ \bar{z} = \frac{x}{a} \]
Aggregation

- Aggregate capital stock $K_t := \int k_t^{(j)} dj$
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- Capital-share weighted distribution $\hat{F}_t(x) := \int \frac{k_t^{(j)}}{K_t} \mathbb{I}(x_t^{(j)} \leq x) dj$
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- Aggregate capital-share-weighted moments

  Default rate $\hat{\lambda}_t^d = -\frac{1}{2} \sigma^2 \bar{x}^2 \partial_x \hat{f}(\bar{x})$

  Average growth $\hat{g}_t = \int g(x) \hat{f}_t(x) dx$
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  Average growth \( \hat{g}_t = \int g(x) \hat{f}_t(x)dx \)

- Aggregate growth \( \mu_{K,t} := \hat{g}_t - (1 - \alpha_k)\hat{\lambda}_t \) and aggregate capital stock dynamics

  \[ dK_t = \mu_{K,t} K_t dt + \rho \sigma K_t dZ_t \]
Long run distribution

\[
\tilde{z} = \frac{x}{a}
\]

- Capital-weighted distribution \( \hat{f}(x) \) (rhs, \%)
- Investment rate \( \Phi(g(x)) \) (lhs, \%)

Debt to ebitda \( z = \frac{x}{a} \)
Estimation

Calibrate 4 parameters:
- $r = \kappa = 5\%$
- $\delta = 10\%$
- $\Theta = 35\%$
- $1/m = 10$ years

Estimate 3 parameters:
- $a$ (average product of capital)
- $\sigma$ (TFP shock vol.)
- $\gamma$ (curv. of investment adjustment costs)

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<thead>
<tr>
<th>Moment Description</th>
<th>Targeted?</th>
<th>Data Model</th>
<th>$\hat{\Phi}$ average investment rate</th>
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<th>$\text{cov}(\Phi(x), z(x))$</th>
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<td>Average investment rate</td>
<td>9.48%</td>
<td>9.47%</td>
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<tr>
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\[ \hat{z} \] average debt/ebitda
\[ \text{cov}(\hat{\Phi}(x), \hat{z}(x)) \] slope of inv. w.r.t debt/ebitda
\[ \hat{\kappa} \] average (inverse) interest coverage ratio
\[ \hat{\pi} \] average dividend issuance rate
\[ \hat{\iota} - \hat{\mu} \] average net debt issuance rate
\[ \text{var}(\hat{z}(x)) \] variance of debt/ebitda
\[ \hat{F}(\hat{z}(x) \leq 1) \] total asset share, debt/ebitda \leq 1
\[ \hat{F}(\hat{z}(x) > 3) \] total asset share, debt/ebitda > 3
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<td>$\hat{i} - \hat{m}$</td>
<td>average net debt issuance rate</td>
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<td>$\text{var}(z(x))$</td>
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2. The crisis as a temporary aggregate shock
Economic and financial shock

- Transient aggregate shock with exponentially distributed length (1 year expected length)
Economic and financial shock

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- Productivity drop during crisis, from $a$ to $a = 0.75a$
  - approx similar to ebitda drop in Compustat from Q4 2019 to Q2 2020
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- Increase in risk prices, from $\nu = 0$ to $\nu = 85\%$
  - S&P 500 dropped 34% between Feb 20, 2020 and March 23, 2020
  - IG credit spreads went from 133bps to 488bps between Feb 20 and March 23
  - HY credit spreads went from 362bps to 1,087bps between Feb 20 and March 23
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- Outcomes of focus: expected future macroeconomic aggregates

\[
E[K_t] = E \left[ \int_j k_t^{(j)} dj \right] \quad E[Y_t] = E \left[ \int_j a_t k_t^{(j)} dj \right] \quad E[C_t] = E \left[ \int_j (a_t - \Phi(g_t^{(j)})) k_t^{(j)} dj \right]
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Aggregate capital response during crisis
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Aggregate investment response during crisis
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3. Theoretical results with perfect financial markets
Result 1: irrelevance theorem

Suppose that (a) the government offers funding at (private) market prices and (b) the intervention does not alter any investors' SDF. Then, relative to the laissez-faire, all outcomes are unchanged. Funding program can consist of debt, equity, any hybrid instrument, be implemented via (fairly priced) government-backed credit guarantees, and be unconditional or conditional on leverage.
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- consist of debt, equity, any hybrid instrument
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- be unconditional or conditional on leverage
Result 2: subsidized loans are distortionary

Suppose that the government intervention decreases the required return on debt $R_d(x)$, without altering equity markets' SDF. Then, relative to the laissez-faire, future debt issuance is higher and future investment is lower.

$$\tilde{\iota}(x) = \Theta \kappa - d'(x)$$

$\text{tax motive}$

$$\tilde{R}_d(x) - R_d(x)$$

$\text{arbitrage motive}$

More issuance $\Rightarrow$ distribution $\hat{f}_t(x)$ shifts right $= \Rightarrow$ lower investment
Result 2: subsidized loans are distortionary

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Then, relative to the laissez-faire, future debt issuance is higher and future investment is lower.
Result 2: subsidized loans are distortionary

Result

Suppose that the government intervention decreases the required return on debt $R_d(x)$, without altering equity markets’ SDF.

Then, relative to the laissez-faire, future debt issuance is higher and future investment is lower.

\[ \tilde{\iota}(x) = \Theta \kappa \frac{\tilde{R}_d(x) - R_d(x)}{-d'(x)} d(x) > \iota(x) \]

- **Tax motive**: $\Theta \kappa \frac{\tilde{R}_d(x) - R_d(x)}{-d'(x)} d(x)$
- **Arbitrage motive**: $\frac{\tilde{R}_d(x) - R_d(x)}{-d'(x)} d(x)$
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$$
\tilde{\iota}(x) = \frac{\Theta \kappa}{-d'(x)} + \frac{\left(\tilde{R}_d(x) - R_d(x)\right) d(x)}{-d'(x)} > \iota(x)
$$

More issuance $\implies$ distribution $\hat{f}_t(x)$ shifts right $\implies$ lower investment
Aggregate leverage when $\tilde{R}_d - R_d = 2\%$
Aggregate leverage when $\tilde{R}_d - R_d = 2\%$
Result 3: expansionary announcement effects

Suppose that the government intervention decreases the effective cost of equity capital for firms. Then, relative to the laissez-faire, aggregate investment and growth is higher on impact.

Intervention: either conventional MP (↓\(r_e\)), or unconventional via announcement (↓\(\nu_e\)).

Caveat: with segmented markets, if ↓\(\in R_d(x)\) is larger than ↓\(\in  \tilde{R}_d(x)\)...

\[\iota(x) = \Theta \kappa - d'(x)\]

\[\Rightarrow\]

\[\text{tax motive} + (\tilde{R}_d(x) - R_d(x))d(x) - d'(x)\]

\[\Rightarrow\]

\[\text{arbitrage motive} \cdot\]

On impact, so long as \(R_e(x)\) falls, investment stimulated as Tobin's \(q\) jumps up; Over time, corporate leverage increases, pushing down investment.
Result 3: expansionary announcement effects

**Result**

Suppose that the government intervention decreases the effective cost of equity capital for firms. Then, relative to the laissez-faire, aggregate investment and growth is higher on impact.
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\[
\iota(x) = \frac{\Theta \kappa}{-d'(x)} + \frac{\left(\tilde{R}_d(x) - R_d(x)\right) d(x)}{-d'(x)}
\]

\text{tax motive} \quad \text{arbitrage motive}
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- tax motive
- arbitrage motive

· On impact, so long as $R_e(x)$ falls, investment stimulated as Tobin’s $q$ jumps up;
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Suppose that the government intervention decreases the effective cost of equity capital for firms. Then, relative to the laissez-faire, aggregate investment and growth is higher on impact.

Intervention: either conventional MP ($\downarrow r_e$), or unconventional via announcement ($\downarrow \nu_e$).

Caveat: with segmented markets, if $\downarrow$ in $R_d(x)$ is larger than $\downarrow$ in $\tilde{R}_d(x)$...

$$\tau(x) = \frac{\Theta \kappa}{-d'(x)} + \frac{\left(\tilde{R}_d(x) - R_d(x)\right)}{-d'(x)} d(x)$$

\begin{align*}
\text{tax motive} & \quad \text{arbitrage motive} \\
\frac{\Theta \kappa}{-d'(x)} & \quad \frac{\left(\tilde{R}_d(x) - R_d(x)\right)}{-d'(x)} d(x)
\end{align*}

- On impact, so long as $R_e(x)$ falls, investment stimulated as Tobin’s $q$ jumps up;
- Over time, corporate leverage increases, pushing down investment.
Aggregate investment with expansionary announcement effects
4. Disruption in financial markets
External financing during the crisis period

- Two possible sudden stop being examined

- Shut down in equity markets only ($\pi_t \geq 0$)
- Shut down in all financial markets ($\pi_t \geq 0, \iota_t \leq 0$)
- Large increase in corporate default rate (default boundary $\bar{x}$)
- Investment curtailed due to lack of external financing available
- Credit market intervention are uncontroversially beneficial for expected future aggregate capital and output, even if they increase corporate leverage (relative to laissez-faire)
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Why are debt overhang effects of credit market interventions so small?

Investment rate $\Phi(g(x))$

BCP loans move the debt/ebitda ratio:

$$z_t = \frac{b_t}{ak_t} \rightarrow z'_t = \frac{b_t + \left(\frac{1}{\chi} \right) (a - a) k_t}{ak_t}$$

$$\approx 2.20$$

$$= z_t + \frac{1}{\chi} \left( 1 - \frac{a}{a} \right)$$

$$= z_t + 0.25 \approx 2.45$$
Why are debt overhang effects of credit market interventions so small?

BCP loans move the debt/ebitda ratio:

\[
z_t = \frac{b_t}{ak_t} \quad \rightarrow \quad z'_t = b_t + \left(\frac{1}{\chi} \right) \left( \frac{a - a}{ak_t} \right) \approx 2.20
\]

Small move, in a region where the slope of investment is not steep.
Other interventions

- Debt funding extended at price $d_g < 1$
  - In that case, firms with leverage $x < x^*(d_g)$ do not take loan
  - Close proxy to Main Street Lending Program (loans @ $L + 300$)
Other interventions

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- Debt funding with dividends/share buy-back restrictions
  - Conditions required for participation in Main Street Lending Program
  - Constraint that addresses commitment problem at the same time

- Debt forbearance program
  - Similar to what US implemented in connection with agency mortgages
  - Difficult to implement in practice given required lender compensation
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Key take-aways
Conclusion

Fed + Treasury providing credit to firms during the crisis.

1. What will the \textbf{net} economic impact of these programs be?

2. Would there be large gains to doing things differently?
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   - Weak debt overhang effects during the recovery

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   - $> 0$ with perfect capital markets and decrease in risk prices
   - $\gg 0$ with capital markets’ disruptions
   - Weak debt overhang effects during the recovery

2. Would there be large gains to doing things differently?
   - Not really!  Unless $\partial g / \partial x$ is much larger ...
More
Secondary market corporate credit facilities’ ("SMCCF") purchases

- Purchases of single-name bonds (left axis)
- Purchases of bond ETFs (left axis)
- BBB spread (right axis)

Key dates:
- 3/23
- 4/9
- 5/12
- 6/15

Timeline:
- 1/1/20
- 3/1/20
- 5/1/20
- 7/1/20
- 9/1/20
- 11/1/20
Leverage in the run-up to the crisis: net debt

Percent of aggregate sales

1985q1 1990q1 1995q1 2000q1 2005q1 2010q1 2015q1 2020q1

Net debt/EBITDA ≥ 1
Net debt/EBITDA ≥ 2
Net debt/EBITDA ≥ 3
Interest coverage ratios in the run-up to the crisis

[Back]
Days of cash on hand in the run-up to the crisis

- Days of cash on hand ≤ 60
- Days of cash on hand ≤ 90
- Days of cash on hand ≤ 120

Percent of aggregate sales

1985q1 1990q1 1995q1 2000q1 2005q1 2010q1 2015q1 2020q1
Projected firms with zero cash

<table>
<thead>
<tr>
<th>Quarter</th>
<th>No shock</th>
<th>Shock + no payouts to equityholders</th>
<th>Shock + stop investing</th>
<th>Shock + all adjustments</th>
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<td>2022Q4</td>
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</tr>
</tbody>
</table>
Model of the firm

- Technology with adjustment costs: $\Phi (g_t) k_t dt$ spent allows capital to grow by $g_t k_t dt$

$$dk_t^{(j)} = k_t^{(j)} \left[ g_t^{(j)} dt + \sigma \left( \rho dZ_t + \sqrt{1 - \rho^2} dZ_t^{(j)} \right) + (\alpha_k - 1) dN_t^{(j)} \right]$$
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  \]

- Financing via long term debt with notional \( b_t^{(j)} \) that satisfies: \( db_t^{(j)} = \left( t_t^{(j)} k_t^{(j)} - m b_t^{(j)} \right) dt \)
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\]

· Financing via long term debt with notional \( b_t^{(j)} \) that satisfies: \( db_t^{(j)} = \left( \iota_t^{(j)} k_t^{(j)} - mb_t^{(j)} \right) dt \)

· Dividends to shareholders of firm \( j \)

\[
\pi_t^{(j)} k_t^{(j)} := ak_t^{(j)} - \Phi \left( g_t^{(j)} \right) k_t^{(j)} + \iota_t^{(j)} k_t^{(j)} D_t^{(j)} - (\kappa + m) b_t^{(j)} - \Theta \left( ak_t^{(j)} - \kappa b_t^{(j)} \right)
\]
Model of the firm

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\[
dk_t^{(j)} = k_t^{(j)} \left[ g_t^{(j)} \, dt + \sigma \left( \rho dZ_t + \sqrt{1 - \rho^2} dZ_t^{(j)} \right) + (\alpha_k - 1) \, dN_t^{(j)} \right]
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\pi_t^{(j)} = \underbrace{ak_t^{(j)} - \Phi \left( g_t^{(j)} \right) k_t^{(j)}}_{\text{ebitda - capex}} + \underbrace{\iota_t^{(j)} k_t^{(j)} D_t^{(j)}}_{\text{net debt issuance}} - \underbrace{(\kappa + m) b_t^{(j)}}_{\text{taxes}} - \underbrace{\Theta \left( ak_t^{(j)} - \kappa b_t^{(j)} \right)}_{\text{net debt issuance}}
\]

- Investor \( n \) (\( n \in \{e, d\} \)) with SDF \( \xi_{n,t} \) that satisfies \( \frac{d\xi_{n,t}}{\xi_{n,t}} = -r_n \, dt - \nu_n \, dZ_t \)
Model of the firm

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· Shareholder problem and debt valuation

\[
E(k_t, b_t) = \sup_{g, \iota, \tau} \mathbb{E}^{Q_e} \left[ \int_t^{+\infty} e^{-r_{(s-t)} \pi_s k_s} ds \right] \quad D(k_t, b_t) = \mathbb{E}^{Q_d} \left[ \int_t^{+\infty} e^{-(r_d + m)(s-t)} \alpha_b N_t (\kappa + m) ds \right]
\]
Mathematical derivations

· HJB equation for shareholders

\[ 0 = \max_{\nu, g} \left[ - (r - g)e(x) + a - \Phi(g) - (\kappa + m)x + \nu d(x) - \Theta(a - \kappa x) \right. \]

\[ + \left. \left[ \nu - (g + m)x \right] e'(x) + \frac{\sigma^2}{2} x^2 e''(x) \right] \]
Mathematical derivations

· HJB equation for shareholders

\[ 0 = \max_{\lambda, g} \left[ - (r - g) e(x) + \lambda - \Phi (g) - (\kappa + m) x + \nu d(x) - \Theta (a - \kappa x) \right. \]
\[ \left. + [\nu - (g + m) x] e'(x) + \frac{\sigma^2}{2} x^2 e''(x) \right] \]

· Feynman-Kac equation for debt price

\[(r + m) d(x) = \kappa + m + \left[ \nu(x) - \left( g(x) + m - \sigma^2 \right) x \right] d'(x) + \frac{\sigma^2}{2} x^2 d''(x). \]
Mathematical derivations

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· First order conditions for optimality

\[ d(x) + e'(x) = 0 \Rightarrow \iota(x) = \frac{\Theta \kappa}{-d'(x)} + \frac{R_d(x) - \tilde{R}_d(x)}{-d'(x)} d(x), \quad q(x) := e(x) - xe'(x) = \Phi'(g(x)) \]
Mathematical derivations

· HJB equation for shareholders

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· First order conditions for optimality

\[ d(x) + e'(x) = 0 \Rightarrow \nu(x) = \frac{\Theta \kappa}{-d'(x)} + \frac{\left( R_d(x) - \tilde{R}_d(x) \right)}{-d'(x)} d(x), \quad q(x) := e(x) - xe'(x) = \Phi'(g(x)) \]

· Expected debt returns \( R_d \) and \( \tilde{R}_d \) and equity returns \( R_e \)

\[ R_d(x) = r_d - \rho \nu_d \sigma \frac{xd'(x)}{d(x)}, \quad \tilde{R}_d(x) = r_e - \rho \nu_e \sigma \frac{xd'(x)}{d(x)}, \quad R_e(x) = r_e - \rho \nu_e \sigma \left[ 1 - \frac{xe'(x)}{e(x)} \right] \]
### GMM (exactly identified case)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Point estimate</th>
<th>Standard error</th>
<th>[5, 95] normal CI</th>
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</thead>
<tbody>
<tr>
<td>$a$</td>
<td>average product of capital</td>
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<td>0.001</td>
<td>[ 0.231, 0.235]</td>
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<td>$\sigma$</td>
<td>volatility of idiosyncratic shock</td>
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<tr>
<td>$\gamma$</td>
<td>curvature of capital adjustment cost</td>
<td>2.550</td>
<td>0.643</td>
<td>[ 1.493, 3.608]</td>
</tr>
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</table>
### GMM (exactly identified case)

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>Targeted?</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
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<td>$100 \times \hat{\Phi}$</td>
<td>average investment rate</td>
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<td>9.47</td>
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<tr>
<td>$\hat{z}$</td>
<td>average debt-to-ebitda</td>
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<td>2.71</td>
<td>2.71</td>
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<td>$100 \times \frac{\text{cov}(\hat{\Phi}(x), z(x))}{\text{var}(z(x))}$</td>
<td>slope of inv. w.r.t debt-to-ebitda</td>
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<td>-3.66</td>
<td>-3.66</td>
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<tr>
<td>$100 \times \kappa \hat{z}$</td>
<td>average (inverse) interest coverage ratio</td>
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<td>$100 \times \hat{\pi}$</td>
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<tr>
<td>$100 \times (\hat{i} - m\hat{x})$</td>
<td>average net debt issuance rate</td>
<td>x</td>
<td>0.96</td>
<td>1.06</td>
</tr>
<tr>
<td>$\text{var}(z(x))$</td>
<td>variance of debt-to-ebitda</td>
<td>x</td>
<td>3.08</td>
<td>0.90</td>
</tr>
<tr>
<td>$\text{var}(100 \times \Phi(x))$</td>
<td>variance of investment rate</td>
<td>x</td>
<td>23.36</td>
<td>13.32</td>
</tr>
<tr>
<td>$100 \times \hat{F}(z(x) \leq 1)$</td>
<td>total asset share, debt-to-ebitda $\leq 1$</td>
<td>x</td>
<td>9.21</td>
<td>0.00</td>
</tr>
<tr>
<td>$100 \times \hat{F}(z(x) \leq 2)$</td>
<td>total asset share, debt-to-ebitda $\leq 2$</td>
<td>x</td>
<td>43.00</td>
<td>19.89</td>
</tr>
<tr>
<td>$100 \times \hat{F}(z(x) \leq 3)$</td>
<td>total asset share, debt-to-ebitda $\leq 3$</td>
<td>x</td>
<td>67.47</td>
<td>77.94</td>
</tr>
</tbody>
</table>
The strength of the debt overhang channel

Average growth:

- Growth rate of all-equity firm = 2.8%
- Aggregate growth rate of $K_t$ = 0.9%

Marginal effects:

$$\frac{\partial (i/k)_t}{\partial x_t} \begin{array}{ll}
(i/k)_t = \text{Gross investment} & (i/k)_t = \text{Net investment} \\
\text{Model} & -0.094 & -0.106 \\
\text{Lang, Ofek, Stulz (1996)} & & -0.105 \\
\text{An, Denis, Denis (2006)} & & -0.086 \\
\text{Cai, Zhang (2011)} & -0.038 & \\
\text{Wittry (2020)} & -0.038 & \\
\end{array}$$