Can the cure kill the patient?
Corporate credit interventions and debt overhang

Nicolas Crouzet and Fabrice Tourre

Northwestern University and Copenhagen Business School
Introduction

Motivation: Business credit support programs (BCSPs)
Introduction

Motivation: Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]
Introduction

Motivation: Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?

BCSPs as "supplements" to markets:
- Depress $i/k$

BCSPs as "substitutes" for markets:
- Less excess exit, $i/k >>$ depressed

Small gains from alternative designs
Introduction

Motivation: Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?

short run (exit)
Introduction

Motivation: Business credit support programs (BCSPs)  
[Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?
short run (exit) vs. long run (debt overhang, low $i/k$)
Introduction

Motivation: Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]

Q1  Effect on firm decisions?
    short run (exit) vs. long run (debt overhang, low $i/k$)

Q2  Gains from alternative designs?
Introduction

Motivation: Business credit support programs (BCSPs)  
[Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?  
short run (exit) vs. long run (debt overhang, low $i/k$)

Q2 Gains from alternative designs?  
equity? forbearance on existing debt?
Introduction

Motivation: Business credit support programs (BCSPs)

[Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?
   short run (exit) vs. long run (debt overhang, low $i/k$)

Q2 Gains from alternative designs?
   equity? forbearance on existing debt?

What we do: study BCSPs in a dynamic model with heterogeneous firms
Introduction

Motivation: Business credit support programs (BCSPs)  
[Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?  
short run (exit) vs. long run (debt overhang, low $i/k$)

Q2 Gains from alternative designs?  
equity? forbearance on existing debt?

What we do: study BCSPs in a dynamic model with heterogeneous firms

BCSPs as "supplements" to markets:
Introduction

Motivation: Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?
short run (exit) vs. long run (debt overhang, low $i/k$)

Q2 Gains from alternative designs?
equity? forbearance on existing debt?

What we do: study BCSPs in a dynamic model with heterogeneous firms

BCSPs as “supplements” to markets: depress $i/k$
Introduction

Motivation: Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?
   short run (exit) vs. long run (debt overhang, low $i/k$)

Q2 Gains from alternative designs?
   equity? forbearance on existing debt?

What we do: study BCSPs in a dynamic model with heterogeneous firms

BCSPs as “supplements” to markets: depress $i/k$

BCSPs as “substitutes” for markets:
Introduction

Motivation: Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?
   short run (exit) vs. long run (debt overhang, low $i/k$)

Q2 Gains from alternative designs?
   equity? forbearance on existing debt?

What we do: study BCSPs in a dynamic model with heterogeneous firms

   BCSPs as “supplements” to markets: depress $i/k$

   BCSPs as “substitutes” for markets: less excess exit
Introduction

**Motivation:** Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]

**Q1** Effect on firm decisions?
- short run (exit) vs. long run (debt overhang, low $i/k$)

**Q2** Gains from alternative designs?
- equity? forbearance on existing debt?

**What we do:** study BCSPs in a dynamic model with heterogeneous firms

- BCSPs as “supplements” to markets: depress $i/k$
- BCSPs as “substitutes” for markets: less excess exit $\gg$ depressed $i/k$
Introduction

Motivation: Business credit support programs (BCSPs) [Fed: CCF, MSLP; Treasury: PPP]

Q1 Effect on firm decisions?
short run (exit) vs. long run (debt overhang, low \( i/k \))

Q2 Gains from alternative designs?
equity? forbearance on existing debt?

What we do: study BCSPs in a dynamic model with heterogeneous firms

BCSPs as “supplements” to markets: depress \( i/k \)

BCSPs as “substitutes” for markets: less excess exit \( \Rightarrow \) depressed \( i/k \)
small gains from alternative designs
1. Model
Key elements

- Investment:
  - AK model with convex adjustment costs (Hayashi, 1982)

- External financing:
  - equity or tax-advantaged debt (Leland, 1994; DeMarzo and He, 2016)
  - deadweight losses in default

- Industry equilibrium:
  - $\infty$ elastic supply of capital at rate $r$
  - exit following default; entry at exogenous rate
  - balanced growth path with heterogeneous firms
Individual firm problem

- Law of motion for capital

\[ dk_t = k_t [g_t \, dt + \sigma \, dZ_t], \quad \text{capex} = \Phi (g_t) k_t \, dt \]
Individual firm problem

- Law of motion for capital

\[ dk_t = k_t [g_t dt + \sigma dZ_t], \quad \text{capex} = \Phi(g_t)k_t dt \]
Individual firm problem

- Law of motion for capital

\[ dk_t = k_t [g_t dt + \sigma dZ_t], \quad \text{capex} = \Phi(g_t)k_t dt \]

- Law of motion for debt

\[ db_t = (\iota_t k_t - mb_t) dt \]
Individual firm problem

- Law of motion for capital
  \[ dk_t = k_t [g_t dt + \sigma dZ_t] , \quad \text{capex} = \Phi(g_t) k_t dt \]

- Law of motion for debt
  \[ db_t = (\nu_t k_t - mb_t) dt \]

- Dividends
  \[ \pi_t k_t := a_t k_t - \Phi(g_t) k_t + \nu_t k_t dt - (\kappa + m) b_t - \Theta (a_t k_t - \kappa b_t) \]
Individual firm problem

- Law of motion for capital
  \[ dk_t = k_t \left[ g_t dt + \sigma dZ_t \right], \quad \text{capex} = \Phi(g_t)k_t dt \]

- Law of motion for debt
  \[ db_t = (\nu_t k_t - mb_t) dt \]

- Dividends
  \[ \pi_t k_t := a_t k_t - \Phi(g_t)k_t + \nu_t k_t dt - (\kappa + m) b_t - \Theta(a_t k_t - \kappa b_t) \]
Individual firm problem

- Law of motion for capital
  \[ dk_t = k_t [g_t dt + \sigma dZ_t], \quad \text{capex} = \Phi(g_t)k_t dt \]

- Law of motion for debt
  \[ db_t = (\iota_t k_t - mb_t) dt \]

- Dividends
  \[ \pi_t k_t := a_t k_t - \Phi(g_t)k_t + \iota_t k_t d_t - (\kappa + m) b_t - \Theta(a_t k_t - \kappa b_t) \]

- Value of equity and debt
  \[ E_t(k_t, b_t) = \sup_{g, \iota, \tau_d} \mathbb{E} \left[ \int_t^{\tau_d} e^{-r(s-t)} \pi_s k_s ds \right], \quad \tilde{d}_t(b_t, k_t) = \mathbb{E} \left[ \int_t^{\tau_d} e^{-(r+m)(s-t)} (\kappa + m) ds \right] \]
Default, investment, and financing policies

- Leverage is a sufficient state

\[ x_t := b_t/k_t, \quad E_t(k_t, b_t) = k_t e_t(x_t) \]
Default, investment, and financing policies

- Leverage is a sufficient state

\[ x_t := \frac{b_t}{k_t}, \quad E_t(k_t, b_t) = k_t e_t(x_t) \]

- Default threshold

\[ \partial x e_t(\bar{x}_t) = 0 \]
Default, investment, and financing policies

- Leverage is a sufficient state

\[ x_t := \frac{b_t}{k_t}, \quad E_t(k_t, b_t) = k_t e_t(x_t) \]

- Default threshold

\[ \partial_x e_t(\bar{x}_t) = 0 \]

- Q-theory

\[ \Phi'(g_t) = \partial_k E_t(x_t) \]
Default, investment, and financing policies

- Leverage is a sufficient state

\[ x_t := \frac{b_t}{k_t}, \quad E_t(k_t, b_t) = k_t e_t(x_t) \]

- Default threshold

\[ \partial_x e_t(\bar{x}_t) = 0 \]

- Q-theory

\[ \Phi'(g_t) = \partial_k E_t(x_t) \]
Debt overhang

\[
\text{Investment rate } \Phi(g(x))
\]

\[
\Phi(g^*)
\]

\[
\tilde{z} = \frac{\bar{x}}{a}
\]

Debt to ebitda \( z = \frac{x}{a} \)

0 2 4 6 8

0 2 4 6 8 10 12 14

%
Default, investment, and financing policies

- Leverage is a sufficient state
  \[ x_t := \frac{b_t}{k_t}, \quad E_t(k_t, b_t) = k_t e_t(x_t) \]

- Default threshold
  \[ \partial x e_t(\bar{x}_t) = 0 \]

- Q-theory
  \[ \Phi'(g_t) = \partial_k E_t(x_t) \]

- Trade-off theory
  \[ \iota_t = \frac{\Theta K}{-\partial_x d_t(x_t)} \]
Financing policies

Debt issuance rate $\iota(x)$

\[ \tilde{z} = \frac{x}{a} \]

Dividend issuance rate $\pi(x)$

\[ \tilde{z} = \frac{x}{a} \]
Aggregation

- Solve jointly for:

  capital-weighted density  \( \hat{f}_t(x) := \int \omega f_t(x, \omega) d\omega, \quad \omega_t^{(j)} \equiv \frac{k_t^{(j)}}{K_t} \)  

  aggregate growth rate  \( \hat{g}_t = \int g_t(x) \hat{f}_t(x) dx \)

  default rate  \( \hat{\lambda}_t^d = -\frac{1}{2} \sigma^2 \bar{x}_t^2 \partial_x \hat{f}_t(\bar{x}_t) \)
Aggregation

- Solve jointly for:

  - capital-weighted density
    \[ \hat{f}_t(x) := \int \omega f_t(x, \omega) d\omega, \quad \omega_t^{(j)} \equiv \frac{k_t^{(j)}}{K_t} \]  
  
  [Kolmogorov forward equation]

  - aggregate growth rate
    \[ \hat{g}_t = \int g_t(x) \hat{f}_t(x) dx \]

  - default rate
    \[ \hat{\lambda}_t^d = -\frac{1}{2} \sigma^2 \bar{x}_t^2 \partial_x \hat{f}_t(\bar{x}_t) \]
Stationary distribution

\[ \Phi(g^*) \]

\[ \bar{z} = \frac{\bar{x}}{a} \]

Capital-weighted distribution \( \hat{f}(x) \) (rhs, %)

Investment rate \( \Phi(g(x)) \) (lhs, %)

Debt to ebitda \( z = \frac{x}{a} \)
Estimation

· Calibrate 4 parameters: $\kappa = 0.04$, $\delta = 0.10$, $\Theta = 0.35$, $m = 0.10$.

· Estimate 3 parameters: $[\text{GMM details}]$
  - $a$ (average product of capital)
  - $\sigma$ (vol. of idiosyncratic shocks)
  - $\gamma$ (curv. of investment adjustment costs)

<table>
<thead>
<tr>
<th>Moment Description</th>
<th>Targeted?</th>
<th>Data Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\Phi}$ average investment rate</td>
<td>9.48</td>
<td>9.47</td>
</tr>
<tr>
<td>$\hat{z}$ average debt/ebitda</td>
<td>2.71</td>
<td>2.71</td>
</tr>
<tr>
<td>$\text{cov}(\Phi(x), z(x))$</td>
<td>-3.66</td>
<td>-3.66</td>
</tr>
<tr>
<td>$\text{var}(z(x))$</td>
<td>slope of inv. w.r.t debt/ebitda</td>
<td>-3.66</td>
</tr>
</tbody>
</table>
Estimation

- Calibrate 4 parameters:

\[
 r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10.
\]
Estimation

- Calibrate 4 parameters:
  \[ r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10. \]

- Estimate 3 parameters:
  \[ a \text{ (average product of capital)} \]
  \[ \sigma \text{ (vol. of idiosyncratic shocks)} \]
  \[ \gamma \text{ (curv. of investment adjustment costs)} \]

[GM details]
Estimation

- Calibrate 4 parameters:

$$r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10.$$  

- Estimate 3 parameters:

  \(a\) \hspace{1cm} \text{(average product of capital)}

  \(\sigma\) \hspace{1cm} \text{(vol. of idiosyncratic shocks)}

  \(\gamma\) \hspace{1cm} \text{(curv. of investment adjustment costs)}

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>Targeted?</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{\Phi})</td>
<td>average investment rate</td>
<td>✓</td>
<td>9.48</td>
<td>9.47</td>
</tr>
<tr>
<td>(\hat{z})</td>
<td>average debt/ebitda</td>
<td>✓</td>
<td>2.71</td>
<td>2.71</td>
</tr>
<tr>
<td>(\frac{\text{cov}(\Phi(x),z(x))}{\text{var}(z(x))})</td>
<td>slope of inv. w.r.t debt/ebitda</td>
<td>✓</td>
<td>−3.66</td>
<td>−3.66</td>
</tr>
</tbody>
</table>
Estimation

- Calibrate 4 parameters:

\[ r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10. \]

- Estimate 3 parameters:

\[ a \quad \text{(average product of capital)} \]
\[ \sigma \quad \text{(vol. of idiosyncratic shocks)} \]
\[ \gamma \quad \text{(curv. of investment adjustment costs)} \]

Effect of debt overhang on investment:
Estimation

- Calibrate 4 parameters:
  \[ r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10. \]

- Estimate 3 parameters:
  \[ a \quad \text{(average product of capital)} \]
  \[ \sigma \quad \text{(vol. of idiosyncratic shocks)} \]
  \[ \gamma \quad \text{(curv. of investment adjustment costs)} \]

Effect of debt overhang on investment:

At the margin: debt/ebitda \( 3 \rightarrow 4 \quad \implies \quad i/k \quad 12\% \rightarrow 8.34\% \)

[Evidence from other papers]
Estimation

- Calibrate 4 parameters:

\[ r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10. \]

- Estimate 3 parameters:

\( a \) \hspace{1em} (average product of capital)
\( \sigma \) \hspace{1em} (vol. of idiosyncratic shocks)
\( \gamma \) \hspace{1em} (curv. of investment adjustment costs)

Effect of debt overhang on investment:

At the margin: \( \text{debt/ebitda} \ 3 \to 4 \implies \frac{i}{k} \ 12\% \to 8.34\% \)

On average: \( \text{growth rate of all-equity firm} = 2.8\% \)

\( \text{aggregate growth rate of } K_t = 0.9\% \)
Estimation

· Calibrate 4 parameters:

\[ r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10. \]

· Estimate 3 parameters:

\[ a \quad \text{(average product of capital)} \]
\[ \sigma \quad \text{(vol. of idiosyncratic shocks)} \]
\[ \gamma \quad \text{(curv. of investment adjustment costs)} \]

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>Targeted?</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\Phi} )</td>
<td>average investment rate</td>
<td>✔️</td>
<td>9.48</td>
<td>9.47</td>
</tr>
<tr>
<td>( \hat{z} )</td>
<td>average debt/ebitda</td>
<td>✔️</td>
<td>2.71</td>
<td>2.71</td>
</tr>
<tr>
<td>( \frac{\text{cov}(\Phi(x),z(x))}{\text{var}(z(x))} )</td>
<td>slope of inv. w.r.t. debt/ebitda</td>
<td>✔️</td>
<td>-3.66</td>
<td>-3.66</td>
</tr>
</tbody>
</table>
Estimation

- Calibrate 4 parameters:

\[ r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10. \]

- Estimate 3 parameters:

\( a \) (average product of capital)
\( \sigma \) (vol. of idiosyncratic shocks)
\( \gamma \) (curv. of investment adjustment costs)

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>Targeted?</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa \hat{\nu} )</td>
<td>average (inverse) interest coverage ratio</td>
<td>( \times )</td>
<td>11.61</td>
<td>13.53</td>
</tr>
<tr>
<td>( \hat{\pi} )</td>
<td>average dividend issuance rate</td>
<td>( \times )</td>
<td>3.32</td>
<td>3.49</td>
</tr>
<tr>
<td>( \hat{i} - m\hat{x} )</td>
<td>average net debt issuance rate</td>
<td>( \times )</td>
<td>0.96</td>
<td>1.06</td>
</tr>
</tbody>
</table>
Estimation

- Calibrate 4 parameters:

  \[ r = \kappa = 0.04, \quad \delta = 0.10, \quad \Theta = 0.35, \quad m = 0.10. \]

- Estimate 3 parameters:

  \( a \) (average product of capital)
  \( \sigma \) (vol. of idiosyncratic shocks)
  \( \gamma \) (curv. of investment adjustment costs)

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>Targeted?</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{var}(z(x)) )</td>
<td>variance of debt/ebitda</td>
<td>( \times )</td>
<td>3.08</td>
<td>0.90</td>
</tr>
<tr>
<td>( \hat{F}(z(x) \leq 1) )</td>
<td>total asset share, debt/ebitda ( \leq 1 )</td>
<td>( \times )</td>
<td>9.21</td>
<td>0.00</td>
</tr>
<tr>
<td>( \hat{F}(z(x) &gt; 3) )</td>
<td>total asset share, debt/ebitda &gt; 3</td>
<td>( \times )</td>
<td>32.53</td>
<td>22.06</td>
</tr>
</tbody>
</table>
2. BCSPs as a supplements to private capital markets
The shock

\( \frac{(a_t - a_0)}{a_0} \)
Aggregate response without credit support programs

\[
\frac{(K_t - K_0)}{K_0}
\]
Dividend payments during the crisis

- \( \hat{\pi}_t \)

- No shock
- Shock + no intervention

years (\( t \))
BCSPs at market prices: an irrelevance result

Suppose that the government offers funding priced using the market discount rate \( r \). Then, relative to the no-intervention equilibrium, all outcomes are unchanged. 

[Proof]

Funding program can consist of debt, equity, any hybrid instrument be unconditional or conditional on leverage.
BCSPs at market prices: an irrelevance result

Result

*Suppose that the government offers funding priced using the market discount rate *r*. Then, relative to the no-intervention equilibrium, all outcomes are unchanged.*
Result

Suppose that the government offers funding priced using the market discount rate $r$.

Then, relative to the no-intervention equilibrium, all outcomes are unchanged.

Funding program can

consist of debt, equity, any hybrid instrument
be unconditional or conditional on leverage
BCSPs below market rates are distortionary
BCSPs below market rates are distortionary

**Result**

Suppose the government offers loans priced using the discount rate:

\[ \tilde{r}_t = r - \alpha_t, \quad \alpha_t > 0. \]

Then, relative to the no-intervention equilibrium, debt issuance is strictly higher, and aggregate growth is strictly lower.

[Proof]
Suppose the government offers loans priced using the discount rate:

\[ \tilde{r}_t = r - \alpha_t, \quad \alpha_t > 0. \]

Then, relative to the no-intervention equilibrium, debt issuance is strictly higher, and aggregate growth is strictly lower.
BCSPs below market rates are distortionary

Result

Suppose the government offers loans priced using the discount rate:

$$\tilde{r}_t = r - \alpha_t, \quad \alpha_t > 0.$$  

Then, relative to the no-intervention equilibrium, debt issuance is strictly higher, and aggregate growth is strictly lower.

More issuance $\implies$ distribution $\hat{f}_t(x)$ shifts right
Aggregate investment

\[ \hat{\Phi}_t \]
3. BCSPs as substitutes for private capital markets
Financial market disruptions

· Assume that during the first 6 months, debt and equity markets are shut down
Financial market disruptions

- Assume that during the first 6 months, debt and equity markets are shut down
- No equity issuance:

\[ \pi_t k_t = [a_t - \Phi(g_t) - (\kappa + m) x_t - \Theta (a_t - \kappa x_t)] k_t \geq 0 \]
Financial market disruptions

- Assume that during the first 6 months, debt and equity markets are shut down
- No equity issuance:
  \[ \pi_t k_t = [a_t - \Phi(g_t) - (\kappa + m)x_t - \Theta(a_t - \kappa x_t)] k_t \geq 0 \]
- Investment is constrained by net income:
  \[ g_t \leq \bar{g}_t(x) := \Phi^{-1}(a_t - (\kappa + m)x_t - \Theta(a_t - \kappa x_t)) \]
Financial market disruptions

- Assume that during the first 6 months, debt and equity markets are shut down

- No equity issuance:
  \[
  \pi_t k_t = [a_t - \Phi(g_t) - (\kappa + m) x_t - \Theta (a_t - \kappa x_t)] k_t \geq 0
  \]

- Investment is constrained by net income:
  \[
  g_t \leq \bar{g}_t(x) := \Phi^{-1} (a_t - (\kappa + m) x_t - \Theta (a_t - \kappa x_t))
  \]

- Low net income can trigger default:
  \[
  \bar{x}_t = \frac{(1 - \Theta)a_t - \Phi_{\text{min}}}{(1 - \theta)\kappa + m}
  \]
Aggregate capital

\[
\frac{(K_t - K_0)}{K_0}
\]

- **No shock**
- **Shock + normal funding markets**
- **Shock + disrupted funding markets**

Years (t): 0 to 3
Aggregate investment
Intervention 1: earnings replacement, financed by debt

- Funding extended in exchange for loans priced at par
Intervention 1: earnings replacement, financed by debt

- Funding extended in exchange for loans priced *at par*

- Firms can use the funding up to:

\[
\text{ebitda} + \text{proceeds from government funding} = \text{pre-crisis ebitda}
\]
Intervention 1: earnings replacement, financed by debt

- Funding extended in exchange for loans priced at par

- Firms can use the funding up to:

  \[ \text{ebitda} + \text{proceeds from government funding} = \text{pre-crisis ebitda} \]

- Implicit flow fiscal cost of government funding (per unit of capital):

  \[ s_t(x)(1 - d_t(x)), \quad s_t(x) \equiv \text{rate of borrowing from gov't program} \]
Intervention 1: earnings replacement, financed by debt

• Funding extended in exchange for loans priced \textit{at par}

• Firms can use the funding up to:

\[
\text{ebitda} + \text{proceeds from government funding} = \text{pre-crisis ebitda}
\]

• Implicit flow fiscal cost of government funding (per unit of capital): 

\[
s_t(x)(1 - d_t(x)), \quad s_t(x) \equiv \text{rate of borrowing from gov't program}
\]

[Derivations]
Aggregate capital

\( \frac{(K_t - K_0)}{K_0} \)

- **No shock**
- **Shock + disrupted funding markets**
- **Shock + disrupted funding markets + subsidized loans**

years (t)
Aggregate investment rate

![Graph showing the aggregate investment rate over time with different scenarios: No shock, Shock + disrupted funding markets, and Shock + disrupted funding markets + subsidized loans. The x-axis represents years (t) from 0 to 3, and the y-axis represents the investment rate from 4% to 14%. The graph includes a vertical dashed line at t = 1, indicating a point of interest or change.]
Why is debt overhang so small?

BCPS loans moves the debt/ebitda ratio:

\[ z_t = \frac{b_t}{ak_t} \rightarrow z'_t = \frac{b_t + (1/2)a_t^Rk_t}{ak_t} \approx 2.70 = z_t + \frac{1}{2} \frac{a_t^R}{a} = z_t + 0.125 \approx 2.82 \]
Why is the effect of debt overhang after the crisis so small?

Gov loan moves the debt/ebitda ratio:

\[ z_t = \frac{b_t}{ak_t} \quad \rightarrow \quad z_t' = \frac{b_t + \left(\frac{1}{2}\right)a_t^R k_t}{ak_t} \]

\[ = \frac{2.70}{2} + \frac{1}{2} \left(\frac{1}{2}\right) = 2.82 \]

Small move, in a region where the slope of investment is not steep.
Other interventions

\[
\frac{(K_t - K_0)}{K_0}
\]

Graph showing the impact of different interventions on \( (K_t - K_0)/K_0 \) over time, with lines representing scenarios like 'Shock + disrupted funding markets + subsidized loans', 'Shock + disrupted funding markets', and 'No shock'. The x-axis represents time in years, and the y-axis represents the percentage change in \( (K_t - K_0)/K_0 \).
Other interventions

\[
\frac{(K_t - K_0)}{K_0}
\]

- Shock + disrupted funding markets + forbearance
- Shock + disrupted funding markets + subsidized loans
- Shock + disrupted funding markets
- No shock

years (t)
Other interventions

\( \frac{(K_t - K_0)}{K_0} \)

years (t)
Conclusion
Conclusion

Q1  Effect of BCSPs on firm decisions?
Conclusion

Q1 Effect of BCSPs on firm decisions?

as "supplements" to private markets: depress $i/k$
Q1 Effect of BCSPs on firm decisions?

as “supplements” to private markets: depress $i/k$

as “substitutes” for private markets: avoid excess exit $\Rightarrow$ depress $i/k$
Conclusion

Q1  Effect of BCSPs on firm decisions?

as "supplements" to private markets: depress $i/k$

as "substitutes" for private markets: avoid excess exit $\Rightarrow$ depress $i/k$

Q2  Gains from alternative designs?
Conclusion

**Q1** Effect of BCSPs on firm decisions?
- as "supplements" to private markets: depress $i/k$
- as "substitutes" for private markets: avoid excess exit $\gg$ depress $i/k$

**Q2** Gains from alternative designs?
- second order, unless $\partial(i/k)/\partial x$ is much higher ...
A (very) brief overview of the lending programs

1. Corporate Credit program — $750bn
   - (mostly) investment-grade firms
   - bonds and ETFs at market prices
   - as of August 31:
     - corporate bonds: $3.8bn (3yr avg maturity, yield spread ≈ 85bps p.a.)
     - ETFs: $8.7bn (87% IG, 13% HY)

2. Main Street Lending program — $600bn
   - all firms (less than 15,000 employees, or less than $5bn 2019 revenues)
   - 5-year term loans, with one participating bank, at LIBOR + 300bps
   - as of August 31: $1.07bn (118 borrowers, via 41 lenders)

3. Paycheck Protection program — $670bn
   - payroll loan/grants, through SBA, with liquidity backing from Fed
Secondary market corporate credit facilities’ ("SMCCF") purchases
Leverage in the run-up to the crisis

Percent of aggregate sales

1985q1 1990q1 1995q1 2000q1 2005q1 2010q1 2015q1 2020q1

Debt/EBITDA ≥ 2
Debt/EBITDA ≥ 3
Debt/EBITDA ≥ 4

[Net leverage] [Interest coverage ratios] [Days of cash on hand] [Projected firms with zero cash]
Leverage in the run-up to the crisis: net debt

![Graph showing leverage trends](image)
Interest coverage ratios in the run-up to the crisis

![Interest coverage ratios graph](graph.png)
Days of cash on hand in the run-up to the crisis

Percent of aggregate sales

1985q1 1990q1 1995q1 2000q1 2005q1 2010q1 2015q1 2020q1

Days of cash on hand ≤ 60

Days of cash on hand ≤ 90

Days of cash on hand ≤ 120

[Back]
Projected firms with zero cash
Aggregation

- Solve for capital-share weighted density:

\[
\hat{f}_t(x) := \int \omega f_t(x, \omega) d\omega, \quad \omega_i^{(j)} \equiv \frac{k_i^{(j)}}{K_t}.
\]

- Modified KF equation

\[
\partial_t \hat{f}_t(x) = \left(g_t(x) - \hat{g}_t + \hat{\lambda}_t^d - \hat{\lambda}_t^n\right) \hat{f}_t(x) - \partial_x \left[(\bar{u}_t(x) - (g_t(x) + m) x) \hat{f}_t(x)\right] + \frac{\sigma^2}{2} \partial_{xx} \left[x^2 \hat{f}_t(x)\right]
\]

- Aggregate capital-share-weighted moments

  Default rate \( \hat{\lambda}_t^d = \frac{1}{2} \sigma^2 \bar{x}_t^2 \partial_x \hat{f}_t(\bar{x}_t) \)

  Growth rate \( \hat{g}_t = \int g_t(x) \hat{f}_t(x) dx \)

- Aggregate capital stock dynamics

\[
\begin{align*}
dK_t &= \int_0^t k_t^{(j)} g_t (x_t^{(j)}) dj dt + \int_0^t \sigma k_t^{(j)} dZ_t^{(j)} dj - \int_0^t k_t^{(j)} dN_t^{d,(j)} dj + \hat{\lambda}_t^n K_t dt \\
&= (\hat{g}_t - \hat{\lambda}_t^d + \hat{\lambda}_t^n) K_t dt
\end{align*}
\]
Intervention 1: earnings replacement, financed by debt

- Funding extended in exchange for loans priced at par: \( s_t(x) = \nu_t^g(x) \)

\[
0 = \max_{\nu, g} \left[ - (r - g)e_t(x) + a_t - \Phi(g) - (\kappa + m)x + u d_t(x) - \Theta(a_t - \kappa x) + \partial_t e_t(x) + [u - (g + m)x] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]
\]
Intervention 1: earnings replacement, financed by debt

· Funding extended in exchange for loans priced at par: \( s_t(x) = \nu_t^g(x) \)

\[
0 = \max_{g \leq \bar{g}(x)} \left[ - (r - g)e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m) x - \Theta(a_t - \kappa x) \\
+ \partial_t e_t(x) + [\nu_t^g(x) - (g + m) x] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]
\]
Intervention 1: earnings replacement, financed by debt

- Funding extended in exchange for loans priced at par: \( s_t(x) = \nu^g_t(x) \)

\[
0 = \max_{g \leq \bar{g}_t(x)} \left[ -(r - g)e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m)x - \Theta(a_t - \kappa x) \right.
\]
\[+ \partial_t e_t(x) + \left[ \nu^g_t(x) - (g + m)x \right] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \]

- Assume for simplicity that funding extended is \( s_t = a_\infty - a_t \)
Intervention 1: earnings replacement, financed by debt

- Funding extended in exchange for loans priced at par: \( s_t(x) = \nu_t^g(x) \)

\[
0 = \max_{g \leq \bar{g}_t(x)} \left[ -(r - g)e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m)x - \Theta(a_t - \kappa x) \right. \\
+ \partial_t e_t(x) + \left[ \nu_t^g(x) - (g + m)x \right] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \]

- Assume for simplicity that funding extended is \( s_t = a_\infty - a_t \)

- Cost of fiscal package

\[
K_0 \int \left( \mathbb{E}^{x,0} \left[ \int_0^{\tau_d} e^{-\int_0^s (r - g_u(x))du} s_t(x_t) (1 - d_t(x_t)) \, dt \right] \right) \hat{f}_0(x) \, dx
\]
Intervention 2: loan forbearance

· Moratorium on interest payments on debt, which ends up being capitalized

\[
0 = \max_{t, g} \left[ - (r - g) e_t(x) + a_t - \Phi(g) - (\kappa + m) x + \lambda d_t(x) - \Theta(a_t - \kappa x) + \partial_t e_t(x) + [\nu - (g + m) x] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]
\]
Intervention 2: loan forbearance

· Moratorium on interest payments on debt, which ends up being capitalized

\[
0 = \max_{g \leq \bar{g}(x)} \left[ -(r - g) e_t(x) + a_t - \Phi(g) - mx - \Theta a_t \\
+ \partial_t e_t(x) + [\kappa - (g + m)] x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]
\]
Intervention 3: earnings replacement, financed by equity injections

- Each dollar of funding extended in exchange for $\nu_e$ shares

$$0 = \max_{\nu, g} \left[ - (r - g)e_t(x) + a_t - \Phi(g) - (\kappa + m)x + \nu d_t(x) - \Theta(a_t - \kappa x) \ight.$$  
$$+ \partial_t e_t(x) + [\nu - (g + m)x] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]$$
Intervention 3: earnings replacement, financed by equity injections

- Each dollar of funding extended in exchange for $\nu_e$ shares

$0 = \max_{g \leq \bar{g}(x)} \left[ -(r + \nu_es_t(x) - g)e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m)x - \Theta(a_t - \kappa x) \\
+ \partial_t e_t(x) - (g + m)x\partial_x e_t(x) + \frac{\sigma^2}{2}x^2\partial_{xx} e_t(x) \right]$
Intervention 3: earnings replacement, financed by equity injections

- Each dollar of funding extended in exchange for $\nu_e$ shares

\[
0 = \max_{g \leq \hat{g}(x)} \left[ -(r + \nu_e s_t(x) - g)e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m) x - \Theta(a_t - \kappa x) \right. \\
\left. + \partial_t e_t(x) - (g + m) x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]
\]

- Assume for simplicity that funding extended is $s_t = a_\infty - a_t$
Intervention 3: earnings replacement, financed by equity injections

- Each dollar of funding extended in exchange for $\nu_e$ shares

$$0 = \max_{g \leq \tilde{g}_t(x)} \left[ - (r + \nu_e s_t(x) - g)e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m) x - \Theta(a_t - \kappa x) 
+ \partial_t e_t(x) - (g + m) x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]$$

- Assume for simplicity that funding extended is $s_t = a_\infty - a_t$

- Cost of fiscal package

$$K_0 \int \left( \mathbb{E}^{x,0} \left[ \int_0^{\tau_d} e^{-\int_0^t (r - g_u(x))du} s_t(x_t) (1 - \nu_e e_t(x_t)) dt \right] \right) \hat{f}_0(x) dx$$
Intervention 3: earnings replacement, financed by equity injections

- Each dollar of funding extended in exchange for $\nu_e$ shares

\[
0 = \max_{g \leq \bar{g}(x)} \left[ - (r + \nu_e s_t(x) - g) e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m)x - \Theta(a_t - \kappa x) \\
+ \partial_t e_t(x) - (g + m)x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]
\]

- Assume for simplicity that funding extended is $s_t = a_\infty - a_t$

- Cost of fiscal package

\[
K_0 \int \left( \mathbb{E}^{x,0} \left[ \int_0^T e^{-\int_0^t (r - g_u(x)) du} s_t(x_t) (1 - \nu_e e_t(x_t)) dt \right] \right) \hat{f}_0(x) dx
\]

- in order to make programs comparable, $\nu_e$ chosen so that the total expected costs of the intervention equals the total expected cost of the earnings replacement program financed by debt at par.
A simple welfare comparison

\[ W_0 = \int_0^{+\infty} e^{-rt} \left( a_t - \hat{\Phi}_t \right) K_t dt \]

<table>
<thead>
<tr>
<th></th>
<th>( W_0 ) laissez-faire</th>
<th>( W_0 ) no shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>laissez-faire</td>
<td>1</td>
<td>0.59</td>
</tr>
<tr>
<td>new loans</td>
<td>1.43</td>
<td>0.85</td>
</tr>
<tr>
<td>loan forbearance</td>
<td>1.36</td>
<td>0.81</td>
</tr>
<tr>
<td>equity injections</td>
<td>1.46</td>
<td>0.86</td>
</tr>
</tbody>
</table>
Aggregation and balanced-growth path

- Solve for capital-share weighted density:

\[ \hat{f}_t(x) := \int \omega f_t(x, \omega) d\omega, \quad \omega_t^{(j)} \equiv \frac{k_t^{(j)}}{K_t}. \]

- Modified KF equation

\[
\partial_t \hat{f}_t(x) = \left( g_t(x) - \hat{g}_t + \lambda_t^d - \lambda_t^u \right) \hat{f}_t(x) - \partial_x \left[ (\tau_t(x) - (g_t(x) + m) x) \hat{f}_t(x) \right] + \frac{\sigma^2}{2} \partial_{xx} \left[ x^2 \hat{f}_t(x) \right]
\]

- Aggregate capital-share-weighted moments

 Default rate \[ \hat{\lambda}_t^d = -\frac{1}{2} \sigma^2 \bar{x}_t^2 \partial_x \hat{f}_t(\bar{x}_t) \]

 Growth rate \[ \hat{g}_t = \int g_t(x) \hat{f}_t(x) dx \]

- Aggregate capital stock dynamics

\[
dK_t = \int_0^{\hat{g}_t} k_t^{(j)} g_t \left( x_t^{(j)} \right) djdt + \int_0^{\hat{\lambda}_t^d} \sigma k_t^{(j)} dZ_t^{(j)} dj - \int_0^{\hat{\lambda}_t^u} k_t^{(j)} dN_t^{d,(j)} dj + \hat{\lambda}_t^u K_t dt = (\hat{g}_t - \hat{\lambda}_t^d + \hat{\lambda}_t^u) K_t dt
\]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Point estimate</th>
<th>Standard error</th>
<th>[5, 95] normal CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>average product of capital</td>
<td>0.223</td>
<td>0.001</td>
<td>[ 0.231, 0.235]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>volatility of idiosyncratic shock</td>
<td>0.236</td>
<td>0.010</td>
<td>[ 0.219, 0.253]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>curvature of capital adjustment cost</td>
<td>2.550</td>
<td>0.643</td>
<td>[ 1.493, 3.608]</td>
</tr>
</tbody>
</table>
### GMM (exactly identified case)

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>Targeted?</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100 \times \hat{\Phi}$</td>
<td>average investment rate</td>
<td>✓</td>
<td>9.48</td>
<td>9.47</td>
</tr>
<tr>
<td>$\hat{\nu}$</td>
<td>average debt-to-ebitda</td>
<td>✓</td>
<td>2.71</td>
<td>2.71</td>
</tr>
<tr>
<td>$100 \times \frac{\text{cov}(\Phi(x),\nu(x))}{\text{var}(\nu(x))}$</td>
<td>slope of inv. w.r.t debt-to-ebitda</td>
<td>✓</td>
<td>−3.66</td>
<td>−3.66</td>
</tr>
<tr>
<td>$100 \times \kappa\hat{\nu}$</td>
<td>average (inverse) interest coverage ratio</td>
<td>✗</td>
<td>11.61</td>
<td>13.53</td>
</tr>
<tr>
<td>$100 \times \hat{\pi}$</td>
<td>average dividend issuance rate</td>
<td>✗</td>
<td>3.32</td>
<td>3.49</td>
</tr>
<tr>
<td>$100 \times \hat{i}$</td>
<td>average gross debt issuance rate</td>
<td>✗</td>
<td>10.21</td>
<td>7.38</td>
</tr>
<tr>
<td>$100 \times (\hat{i} - m\hat{x})$</td>
<td>average net debt issuance rate</td>
<td>✗</td>
<td>0.96</td>
<td>1.06</td>
</tr>
<tr>
<td>$\text{var}(\nu(x))$</td>
<td>variance of debt-to-ebitda</td>
<td>✗</td>
<td>3.08</td>
<td>0.90</td>
</tr>
<tr>
<td>$\text{var}(100 \times \Phi(x))$</td>
<td>variance of investment rate</td>
<td>✗</td>
<td>23.36</td>
<td>13.32</td>
</tr>
<tr>
<td>$100 \times \hat{F}(\nu(x) \leq 1)$</td>
<td>total asset share, debt-to-ebitda $\leq 1$</td>
<td>✗</td>
<td>9.21</td>
<td>0.00</td>
</tr>
<tr>
<td>$100 \times \hat{F}(\nu(x) \leq 2)$</td>
<td>total asset share, debt-to-ebitda $\leq 2$</td>
<td>✗</td>
<td>43.00</td>
<td>19.89</td>
</tr>
<tr>
<td>$100 \times \hat{F}(\nu(x) \leq 3)$</td>
<td>total asset share, debt-to-ebitda $\leq 3$</td>
<td>✗</td>
<td>67.47</td>
<td>77.94</td>
</tr>
</tbody>
</table>
The strength of the debt overhang channel

Average growth:

Growth rate of all-equity firm = 2.8%
Aggregate growth rate of $K_t$ = 0.9%

Marginal effects:

$$\frac{\partial (i/k)_t}{\partial x_t}$$

<table>
<thead>
<tr>
<th></th>
<th>$(i/k)_t = \text{Gross investment}$</th>
<th>$(i/k)_t = \text{Net investment}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>-0.094</td>
<td>-0.106</td>
</tr>
<tr>
<td>Lang, Ofek, Stulz (1996)</td>
<td></td>
<td>-0.105</td>
</tr>
<tr>
<td>An, Denis, Denis (2006)</td>
<td></td>
<td>-0.086</td>
</tr>
<tr>
<td>Cai, Zhang (2011)</td>
<td>-0.038</td>
<td></td>
</tr>
<tr>
<td>Wittry (2020)</td>
<td>-0.038</td>
<td></td>
</tr>
</tbody>
</table>