Can the cure kill the patient?
Corporate credit interventions and debt overhang*

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Abstract

Business credit programs (BCPs) are an important component of the public policy response to the recent recession. We develop and estimate a model to quantify the impact of BCPs on corporate leverage and investment. Our framework highlights the potential debt overhang effects of BCPs. Following a real shock, if private capital markets continue functioning normally, BCPs incentivize firms to take on too much leverage, leading to underinvestment in the recovery. However, if the shock is accompanied by a sudden stop in private capital markets, BCPs help limit the output losses caused by forced liquidations. Quantitatively, we show that this positive "level" effect outweighs the milder negative "growth" effects of debt overhang among surviving firms. Alternative designs (like equity injections) create only small gains over a simple loan program.

Keywords: Investment, Leverage, Debt Overhang, Credit Programs.

JEL codes: G32, G33, H32, E58.

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1 Introduction

Starting in March 2020, as the potential economic impact of the pandemic came into focus, the Federal Reserve moved to provide aggressive support to the US economy. After lowering the upper range of the Fed Funds rate to 0.25%, the Fed redeployed some of the unconventional asset purchase programs it had used during the Great Recession. But it didn’t stop there. Starting in the second half of March 2020, as the US corporate debt and equity markets showed continued signs of strain, the Fed announced a new kind of program: direct purchases of bonds of non-financial corporations, and direct lending to non-financial firms. By late March, after the passage of the CARES act, Treasury followed suit and announced its own program to provide direct funding to non-financial firms.

The goal of this paper is to provide a framework to quantify the impact of credit support policies such as these, which we refer to as "Business Credit Programs", or BCPs, for short. We develop a dynamic model of investment and financing with heterogeneous firms, and estimate the model using data on publicly traded US non-financial firms in late 2019. We then use the model for two purposes: to quantify the potential effect on corporate investment and leverage of BCPs designed similarly to the Fed’s programs; and to estimate potential gains from designing BCPs differently.

Our framework deliberately focuses on one potential downside of BCPs: by creating debt overhang, they could eventually depress investment, particularly during the recovery, as corporate borrowers will have to work through large amounts of legacy debt. We use our model to weigh this potential downside effect of BCPs on growth, against their positive “level” effect: by providing emergency funding to firms, they may help prevent unnecessary liquidations, and avoid the deadweight losses associated with these liquidations.

The main insights from our analysis are the following. We first consider a scenario where the economy is hit by a real aggregate shock (such as, say, an exogenous decline in productivity), but private capital markets continue to function normally. In this case, we show that BCPs have at best no effect, and are, in general, distortionary. BCPs are distortionary whenever they provide funding that is subsidized, relative to the cost of capital prevailing in private markets. The subsidy encourages firms to over-leverage, eventually leading to underinvestment. As a result, in this scenario, BCPs are best avoided. The result is intuitive; our model has the added value of showing that the investment distortion is quantitatively small. A loan program priced 100bps below market rates only depresses investment during the recovery by about 0.2 p.p. per year (on 12 p.p. basis), relative to laissez-faire.

Second, we consider a scenario where, on top of the real aggregate shock, debt and equity markets undergo a "sudden stop" in the early stages of the crisis. BCPs then provide a substitute source of funding for firms, and help avoid some (though not all) of the liquidations that the sudden stop would otherwise cause. Our main quantitative insight is that this positive "level" effect dominates the negative "growth" effect of higher leverage among surviving firms. With a loan-based BCP, 3 years after the crisis, aggregate capital is 5% below the no-crisis counterfactual; without BCP, the gap is 15%. Thus, long-run debt overhang effects are quantitatively modest. Finally, an equity injection program is only marginally better than the simple loan program, precisely because marginal debt overhang effects in the simple loan program are small. (A loan forbearance program is worse, as it involves a smaller implicit subsidy that the loan program.)
BCPs could create other distortions than those related to debt overhang. However, we think that isolating and quantifying those related to debt overhang is useful for at least two reasons. First, a large literature in corporate finance and macroeconomics has emphasized that debt overhang can distort investment downward and, in the corporate context, destroy firm value. Second, leverage in the corporate sector was unusually elevated coming into the crisis. Figure 1 shows the distribution of total revenue among US non-financial public firms up to 2019; it indicates that the aggregate importance of relatively high leverage firms (those debt-to-ebitda ratios above 3) was at a 15-year high when the recession started. This high initial leverage makes the incremental BCP debt all the more likely to worsen investment distortions. It also echoes some concerns, voiced in the popular press and in policy circles, that BCPs may give birth to a cohort of “zombie firms”, characterized by high debt burdens and low investment.

The model we construct deliberately stresses the interplay between debt and investment. It describes a partial equilibrium in an industry populated by firms who invest, borrow, issue equity, pay dividends to shareholders and make default decisions.

An individual firm’s problem has two main components. First, on the real side, the investment decisions of a firm follow a standard Q-theory model analogous to Hayashi (1982), where production has constant returns to scale with respect to a unique capital input. The marginal product of capital is assumed to be exogenous and constant, but firms are subject to temporary “capital quality shocks”, which are i.i.d. across firms and over time. Second, on the financing side, firms have access to both debt and equity markets. The supply of debt and equity is infinitely elastic, and financial markets are frictionless. Debt is long-term, as in Leland et al. (1994), but it can be readjusted continuously and at no cost, as in DeMarzo and He (2016). Firms choose to issue debt because it is tax-advantaged. However, debt is also defaultable, and default entails dead-weight losses. The optimal debt issuance policy trades-off these two forces.

Firms in the model exhibit debt overhang in the sense that the optimal investment policy function has a negative slope with respect to leverage. As firms approach the default boundary, marginal $q$ — the value of one incremental unit of capital, from shareholders’ standpoint — declines, as default becomes more likely; as a result, investment falls.

Our assumptions conveniently lead to leverage being a sufficient statistic summarizing a firm’s state. Aggregate moments of the economy then depend on the cross-sectional distribution of leverage and its dynamic evolution. In the absence of aggregate shocks, the economy is on a balanced-growth path, in which capital, investment and output all grow at the same endogenously-determined rate.

We then use data on US public firm’s investment and capital structure in 2019 to estimate the model. We fit three particular moments that are important to quantify the effects of BCPs: the aggregate investment rate; leverage; and the sensitivity of investment to leverage, that is, the marginal effect of an increase in leverage on investment rates. In the data, this sensitivity is approximately $-1$, meaning that a one-unit increase in debt-to-ebitda is, on average, associated with a decline in...

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1In particular, moral hazard or adverse selection on the part of the borrowers may also lead to distorted firm investment incentives, and also magnify the fiscal costs of the interventions. We do not consider these additional mechanisms in our framework, though we recognize their potential importance in assessing the costs and benefits of these programs.

2We discuss these papers, and how our work relates to them, in the literature review section below.

3See, among many others, Financial Times (2020) and Group of 30 (2020).
investment of 1 p.p.. Our model is able to replicate his moment well (for otherwise standard values of structural parameters), and moreover, it matches closely a number of non-targeted financial moments. In our estimated model, though the marginal effect of debt overhang might appear small, its average effect is substantial: aggregate (real) growth is approximately 0.5% per year, compared to 2.3% in a no-debt economy. The main empirical shortfall of the model is that it underestimates cross-sectional dispersion in investment and leverage rates; but it does so both in the left (low-leverage) and right (high-leverage) tails of the distribution, which does not bias the magnitude of debt overhang in an obvious way. Additionally, we show that the main features of our estimated model are robust to a number of variations in the measurement of leverage, investment rates, or the leverage-investment sensitivity, as well as to the sample of firms (all public firms, or rated firms only) that we consider.

We then use this model to study the effects of credit interventions following a crisis. Specifically, we study the perfect foresight response of the economy to a temporary (six-month) decline in the average product of capital by 25%. We consider two scenarios: one in which private capital (debt and equity) markets function normally during the crisis, and a “sudden stop” scenario, where private equity and debt markets shut down in the early stages of the crisis.

We first study the case where the crisis does not involve disruptions to private capital markets. Though the shock is large, it has relatively mild effects on investment: firms respond by issuing equity in order to smooth the temporary decline in cash flows and avoid having to cut back investment. Our main finding, in this case, is that funding programs will either leave aggregate investment unchanged relative to a laissez-faire equilibrium, or, potentially, reduce it. Government-provided debt funding at market prices is “undone” by firms who have complete freedom to adjust their debt issuance policy; the resulting intervention leads to firms issuing less debt to private markets, while their investment and default choices are unchanged. When the government injects equity capital into firms in exchange for funds, ownership of firms by private investors declines, but debt financing, investment and default policies are once again unchanged. In both these cases, so long as the funding is provided at market levels, aggregate investment and economic growth remain identical to those in the laissez-faire environment. On the other hand, a debt funding program at below-market prices reduces aggregate investment. This is because the marginal benefit of an extra unit of debt, for firms, now includes an extra term: the wedge between shareholders’ discount rate and the discount rate used by the government to price the program loans. When this wedge is positive, firms borrow more and end up with more leverage. The higher leverage takes time to disappear, and leads to a prolonged period of depressed investment in the recovery. Quantitatively, however, the effect is small. For a 100 bps subsidy to loans, investment rates are only 0.2 p.p. lower than in the laissez-faire equilibrium, in years 1 through 3 after the crisis.

We then turn to the case where the crisis involves both an exogenous decline in the average product of capital and a shut-down of private capital markets, both debt and equity. In the laissez-faire equilibrium, the combined effects of financial market disruptions and cash flow shocks now has large real effects. On impact, the shock leads to a wave of liquidations, driven by firms that would normally rely on financial markets to roll over debt or raise equity capital. Additionally, during the crisis period, investment remains depressed, because firms cannot finance investment through equity issuance in the face of temporarily low cash flows. Upon the end of the crisis, investment resumes at
a slightly higher pace than in steady-state, because surviving firms generally have lower leverage.

In this context, corporate credit interventions can potentially improve on the laissez-faire equilibrium. We start by considering an intervention where the government provides debt funding that exactly makes up for the decline in cash flows due to the drop in the average product of capital.

This program has two effects. First, it helps firms avoid liquidation during the crisis, particularly in its early stages. This is similar to the case where private capital markets continue operating normally, and equity issuance helps firms smooth out the shock in the case where private credit markets operate normally. Second, once the crisis is over, it leaves firms with higher leverage. This is different from the case where private capital markets continue operating normally. In that case, funding would have been obtained predominantly via equity rather than debt issuances, leading to only small changes in leverage and normal investment rates (relative to the steady-state) during the recovery. The government program we consider thus involves a trade-off between reducing liquidations (during the crisis) and reducing investment (during the recovery).

However, we find that the effects of reduced liquidations on overall investment, capital, and output, is larger than the debt overhang effects. The intervention reduces by more than two-thirds the destruction of capital due to short-run defaults. On the other hand, while the additional leverage leads to investment rates that are indeed lower during the recovery, the difference relative to the laissez-faire equilibrium is small. The combination of large positive effect on the level of capital, and a limited negative effect on its growth rate implies that, 5 years after the shock, output and the capital stock is 5% lower with the intervention (compared to a no-shock scenario), compared to approximately 15% lower in the absence of intervention.

We then look at other designs for the policy intervention. Loan forbearance — allowing firms to delay and capitalize interest payments on debt for the duration of the crisis — generally has smaller effects than the simple loan program. Forbearance reduces liquidations by a magnitude comparable to the loan program; but it leaves firms with less cash on hand than the loan program, so that investment is more persistently depressed during the early stages of the crisis. The flipside of the fact that the intervention has more limited real effects is its that it also has a much smaller fiscal cost; we show that the implicit fiscal transfer in this program is substantially smaller than in the loan program.

We also consider government funding in exchange for an equity stake in the firm. We size the government equity stakes such that the fiscal cost of the intervention is identical to the fiscal cost of the intervention designed with debt funding. Our conclusions remain qualitatively and quantitatively the same: equity injections reduce the incremental debt overhang induced by government-provided debt funding, but those effects are very small.

Thus, our main result in the sudden stop case is that the trade-off between reducing liquidations and creating debt overhang generally seems to favor a simple loan program, relative to a laissez-faire equilibrium. This naturally leads to the question of why the debt overhang channel is small in our model, despite the fact that debt overhang has substantial effects in steady-state. The main reason is that the bulk of firms in the model operate in a region where the slope of the investment policy function with respect to leverage is small. Moreover, the loan-funded government program has limited effects on overall leverage. A back of the envelope calculation is that it increases a firm’s debt-to-EBITDA by \((1/2)\) (the duration of the shock, six months) multiplied by 25% (the size of
the decline in productivity, and hence of the earnings replacement provided by the program), or approximately 0.1 (relative to a mean of approximately 2.1). Given the small slope of investment with respect to leverage in the region where most firms operate, this increase does not have large effects. The finding of a small debt overhang effect also explains why alternative funding programs, such as equity injections, would not lead to dramatically different investment behavior during the recovery.

There are two substantial reasons why the finding of a weak debt overhang channel might be incorrect. The first one, already hinted at above, is that the strength of the channel crucially depends on the elasticity of investment to leverage for the “modal” firm. Our estimation ensures that the model generates an empirically correct average elasticity of investment to leverage. However, if it were the case that this elasticity is, for certain groups of firms, or in certain periods of time (such as recessions), substantially larger, our quantitative conclusions might change. Second, our finding depends on the extreme form of financial disruption which we consider: a complete shut-down of equity and debt markets. Alternative approaches, in which, for instance, the cost of equity issuance increases (but does not become infinity), might lead to fewer liquidations in the short-run and a distribution of surviving firms that are more highly levered overall, potentially magnifying the debt overhang channel.

Finally, our results also speak to the phenomenon of “zombie firms” — firms that are kept alive by their lenders because these lenders have limited incentives to recognize loan losses. While we do not consider distorted lender incentives explicitly, it is worth noting that firms in our model exhibit a behavior similar to those of “zombie” firms, in that they actively delay default when they are highly levered, in particular by issuing equity at an accelerated pace — a behavior that is made worse by the government interventions, though our quantitative results suggest this is insufficient to substantially slow down investment in the recovery.

**Related Literature** Our paper relates to four strands of literature.

First, it builds on a theoretical literature in corporate finance that studies how debt overhang affects investment. Building on the seminal insight of Myers (1977), this literature has developed dynamic models in which debt in place can affect firms’ decisions to undertake new investment. Our model more specifically builds on the continuous-time framework of DeMarzo and He (2016) and Admati et al. (2018). In that model, the tax deductibility of debt interest expense incentivizes firms to take on leverage in the first place. Due to a lack of commitment, firms’ managers make strategic default decisions, as in Leland et al. (1994) and Leland and Toft (1996). We combine this framework with a standard investment problem, analogous to Hayashi (1982) and Abel and Eberly (1994). The resulting investment decisions are distorted downward by the presence of debt, leading to a debt overhang problem. Our framework allows for a continuous adjustment of both (long-term) leverage and investment; by contrast, much of the existing work focuses on models in which long-

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4See Hoshi, Kashyap and Scharfstein (1990) and Caballero, Hoshi and Kashyap (2008) for a discussion of the phenomenon in the Japanese context.
5See, among others, Mello and Parsons (1992); Mauer and Ott (2000); Moyen (2007); Titman and Tsypaklov (2007); Manso (2008), and Diamond and He (2014).
6As in that environment, firms end up taking so much leverage that the tax benefits of debt end up fully dissipated by bankruptcy costs, so that enterprise value becomes identical to that of a firm in financial autarky.
term debt is fixed (e.g. Moyen 2007), or the investment decision focuses on whether to exercise a growth option (e.g. Childs, Mauer and Ott 2005). Two papers adopting closely related models for debt overhang are Hennessy (2004) and Perla, Pflueger and SzkuP (2020), though neither studies aggregation and the transmission of shocks that affect the distribution of corporate leverage.\(^7\)

Second, our work speaks to an empirical literature that studies the effects of leverage on investment decisions.\(^8\) In Section 4, we compare the steady-sate implications of our model to the findings in this literature. However, our focus on this paper is not on steady-state costs of debt overhang, but on its dynamic effects on investment following government interventions. One of our key findings is that while steady-state agency costs can be large, they do not substantially amplify the response of firms to these interventions.

Third, our work relates to a theoretical literature that studies the effects of corporate or sovereign debt overhang on macroeconomic activity (Krugman et al., 1988; Lamont, 1995; Philippon, 2010), and how policy can best address this overhang (Philippon and Schnabl, 2013). Relative to that literature, our goal in this paper is to provide a framework that can be used to quantify more precisely the effects of corporate debt overhang and the various policies to address them. Importantly, we allow for cross-sectional heterogeneity in leverage, which is generated by idiosyncratic capital quality shocks, as in Khorrami and Tourre (2020). This heterogeneity is crucial to understanding why the interventions we consider have limited negative aggregate effects: these effects are concentrated among small, relatively leveraged firms in our model. Relatedly, recent empirical work (Jordà et al., 2020) also questions whether corporate debt overhang substantially lowers investment during recoveries, using long-run aggregate data on a panel of countries.

Finally, our paper adds to work on the recent corporate credit market interventions by the Federal Reserve and the US Treasury. Brunnermeier and Krishnamurthy (2020) emphasize qualitatively the trade-off between dead-weight losses of bankruptcy and debt overhang in a one-period model of the firm, and distinguish policy interventions aimed at large firms that maintain access to financial markets vs. interventions aimed at small, manager-operated businesses. Hanson et al. (2020) also discuss the potential effects of the Fed’s program. They do not focus on debt overhang, but rather highlight the benefits of BCPs when there are externalities (aggregate demand or otherwise) associated with firm default. Our approach in this paper is both more quantitative, and more specifically focused on debt overhang, which we think is an important dimension of BCPs, particularly for larger, publicly-traded firms. Some of our conclusions also differ: in particular, we highlight the potential for lending programs to distort investment downward if the cost of credit is subsidized.\(^9\)

\(^7\)Additionally, though these papers are not primarily focused on the question of debt overhang, the model we study also has many common features with Hennessy and Whited (2005) and Hennessy and Whited (2007). The main differences with the latter paper, in particular, are (a) our model has constant returns to scale in production, allowing for easier aggregation; and (b) we assume frictionless equity markets and a simpler corporate tax structure; (c) we allow for long-term debt, potentially magnifying the effects of debt overhang; and (d) we study a continuous-time framework in which the computation of equilibrium price functions for debt is considerably simpler.


\(^9\)Our work is also related to English and Liang (2020), who study the structure of the Main Street Lending Program and who argue for a better targeting of such program, longer loan terms, smaller minimum loan sizes, and stronger incentives for banks to participate in it. Our model speaks directly to the argument that the government credit programs should be aimed at high-leverage firms who are in need of funds in order to avoid liquidation during the crisis. Greenwood, Iverson
2 A brief overview of Business Credit Programs (BCPs) in the US

US equity and credit markets started showing signs of stress in late February 2020, as the potential economic consequences of the pandemic came into focus. From its peak on February 19th, to March 23rd, the S&P 500 lost approximately one-third of its value, while corporate credit spreads, even among investment-grade firms, rose substantially (see Figure 2). In response, from March 3rd to March 23rd, the Federal Reserve rapidly deployed both conventional and unconventional policy responses. The upper range of the target for the Fed Funds rate was cut to 0.25% by March 15th, and by March 23rd, the Fed had resumed most of the key asset purchase programs that were part of the unconventional response to the 2007-2009 crisis, including large-scale purchases in Treasury and MBS markets, which had shown signs of stress in early March.

The March 23rd announcement, however, went beyond the unconventional tools used in 2007-2009, as it signaled the Federal Reserve’s intention to provide direct support to corporate credit markets. Treasury soon followed with similar plans. We next briefly summarize the key features of the three main business credit support programs (BCPs) that were implemented as a result: the Corporate Credit Facilities (CCF), the Main Street Lending Program (MSLP), and the Paycheck Protection Program (PPP).

The Corporate Credit Facilities (CCF) Two Corporate Credit Facilities (CCF) were initially announced on March 23rd, 2020. The Primary Market Corporate Credit Facility (PMCCF) was created to allow the Fed to purchase corporate bonds and syndications on the primary market, while the Secondary Market Corporate Credit Facility (SMCCF) was meant to allow the Fed to participate in the secondary markets for single-name corporate bond and bond ETFs. Both were created under section 13(3) of the Federal Reserve Act.

The PMCCF and the SMCCF were each initially funded with a $10bn equity investment from the Treasury’s Exchange Stabilization Fund (ESF), with the Fed providing an additional $90bn, allowing for total purchases of up to $200bn. After the passage of the Coronavirus Aid, Relief and Economic Security (CARES) act, on March 27th, the ESF was expanded to $454bn. On April 9th, some of these additional funds were earmarked to allow the total scale of the facilities to reach up to $750bn.

Figure 2 shows that on the two announcement dates of March 23rd (the initial announcement of...
the facilities) and April 9th (their expansion), corporate credit spreads fell substantially. Purchases under the CCFs did not begin until May 12th (for ETFs) and June 16th (for single-name bonds), and only the SMCCF was actively used. As indicated by Figure 2, the SMCCF purchases through November 30th, 2020 reached approximately $14bn, two-thirds of it in the form of ETFs.

The CCF targets firms with access to bond markets. Initially, the program was restricted to investment-grade rated firms, but the April 9th announcement expanded eligibility to firms that had been downgraded since March 22nd, and also allowed purchases of ETFs with exposure to high-yield bonds. Other than credit ratings, neither the PMCCF nor the SMCCF had other eligibility criteria. All Fed purchases were done at prevailing market prices, and there were no caps on the size of individual purchases. Finally, participation was not subject to restrictions on the use of funds by participating firms.

**The Main Street Lending Program (MSLP)** The Main Street Lending Program (MSLP) was first announced on April 9th, at the same time as the expansion of the CCF. The aim of this program is to provide loans to small and medium-size firms, and participating borrowers must meet certain size criteria (fewer than 15000 employees, or less than $5bn revenue in 2019).

Three main facilities were created: the first two would allow the Fed to buy newly issued loans, while the third would allow it to fund the upsizing of existing loans. The facilities were funded with a $75bn equity contribution from the Treasury’s ESF, allowing, in principle, total purchases across all facilities to reach $600bn. As of November 30th, 2020, however, the program had only bought 646 loans, totaling $6.32bn; average loan size was $9.79mn, with a minimum of $0.23mn and a maximum of $300mn.

The low take-up relative to the potential scale of the program may partly reflect the requirements imposed on both lenders and borrowers. Under the terms of the program, all MSLP loans must be originated by private banks, in exchange for a fee. Originating lenders must retain a 5% participation in the loan. Loan size is subject to caps. The minimum loan size, initially set at $0.5mn, was eventually lowered to $0.1mn. The maximum loan size varies from $30 mn to $300mn across facilities, but is also subject to a leverage constraint: firms cannot borrow up to more than four times 2019 EBTIDA (or 6, in the case of the Priority facility), effectively excluding firms that had high debt to EBITDA ratios in 2019 from participating. Interest rates on MSLP loans are set at LIBOR plus 3%, regardless of the underlying financial conditions of the borrower. All loans made are five-year maturity, with a

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16 Gilchrist et al. (2020) analyze in detail the effect of the Fed’s announcements on bond markets.
17 Figure 2 reports cumulative gross purchases. As of November 30th, 2020, the SMCCF portfolio consisted of ETFs valued at $8.77bn, and single-name bonds of $4.86bn total par value.
18 As is well known, rated firms make up a small fraction of the firm population, but a large fraction of corporate assets and corporate investment; see, e.g., Faulkender and Petersen (2006) or Crouzet and Mehrotra (2020).
19 The only limit is that the facilities may not hold more than 25% of an individual issuance (for the PMCCF) or 10% of total issuances (for the SMCCF) of a particular firm.
20 The PMCCF allowed, in particular, new issuances to be used to refinanced existing debt. Official term sheets for the Corporate Credit Facilities is available here (for the PMCCF) and here (for the SMCCF). See Boyarchenko et al. (2020) for a more detailed discussion of the program.
21 The announcement is available here.
22 Two additional facilities were added on June 15th, 2020; the announcement is available here.
23 Loan-specific information on MLSP purchases is available here.
two-year grace period for principal payment. MSLP loans must also be contractually senior to other forms of debt, though security is not explicitly required. Finally, borrowers are restricted from using loans to refinance existing debt, and they must also follow the limits on distributions to shareholders outlined in the CARES act.\textsuperscript{24}

**The Paycheck Protection Program (PPP)** The CARES act, adopted on March 27th, included the creation of the Paycheck Protection Program (PPP). This program is meant to allow Treasury to lend to small firms (those with less than 500 employees) via the Small Business Administration. PPP loans were made through private banks, but were fully guaranteed by the SBA. The Fed’s involvement in the program was minimal: it only provided funding to banks originating the loans, through its Paychek Protection Program Liquidity Facility (PPPLF). The CARES act initially endowed the SBA with $349bn to fund loans to businesses, eventually expanding the sum to $669bn on April 24th, given the high demand by small businesses for funds under that program. A total of approximately $525bn of loans across approximately 5mn lenders were made through the program.\textsuperscript{25}

The main goal of the program was to make loans to small businesses, with loan proceeds that could be used by firms to cover interest, payroll, rent, and utilities. Requirement on participating borrowers and lenders for the PPP were much lighter than for MSLP loans. Loans had a maturity of up to five years and an interest rate of 1%. Loan caps were calculated on the basis of past payroll costs of the borrower. Most importantly, loans could be partially or totally forgiven, provided the borrower met certain criteria, particularly regarding employee retention.\textsuperscript{26} Granja et al. (2020), Hubbard, Strain et al. (2020), and Lutz et al. (2020) discuss the take-up and the effects of the PPP in detail.

**Summary** Table 1 provides a summary of the main features of each of the three programs. Because PPP loans are forgivable, the PPP resembles a (conditional) transfer program. By contrast, the CCF and MSLP are explicitly structured as loan programs. While the CCF and MSLP share some common features, they target different groups of firms. Moreover, the eligibility criteria and borrower requirements of the CCF are looser than those of the CCF. On the other hand, the CCF is implemented at prevailing market rates, while MSLP loans carry a fixed spread over LIBOR. We will return to these differences in design as we discuss the potential effects of direct lending programs in Section 5.

### 3 Model

In this section, we develop a partial equilibrium model of investment, financing and default. We first describe the model, and then discuss the key mechanisms it helps highlight, as well as its most important limitations.

\textsuperscript{24}Borrowers must also make “commercially reasonable efforts” to maintain employment for the time during which the MSLP loan is outstanding. Official information on the MSLP is available [here](#); see Crouzet and Gourio (2020) for a discussion of the program.

\textsuperscript{25}Information on the program is available [here](#).

\textsuperscript{26}Official information on the PPP is available [here](#).
3.1 Model description

We first describe an individual firm’s problem, and then turn to aggregation of firm decisions.

3.1.1 Firm problem

The problem of an individual firm extends the framework studied in DeMarzo and He (2016) by allowing for a continuous choice of investment rates subject to convex capital adjustment costs and by modifying the consequences of a firm default. Shareholders of the firm take the real interest rate $r$ as given. Their bond issuance decisions are motivated by the tax deductibility of the debt interest expense. Their investment decisions are distorted downwards due to the presence of long term debt. Creditors providing financing to the firm are competitive, and price the debt issued by such firm risk-neutrally.

**Technology**  The production technology of firm $j$ yields revenue $y_t^{(j)} = a_t k_t^{(j)}$ per unit of time. $k_t^{(j)}$ represents efficiency units of capital of firm $j$, while $a_t$ is a measure of productivity. While $k_t^{(j)}$ is specific to a firm, $a_t$ is identical across firms. A firm has at its disposal an investment technology with adjustment costs, such that $\Phi(g_t^{(j)}) k_t^{(j)} dt$ spent allows the firm to grow its capital stock by $g_t^{(j)} k_t^{(j)} dt$, where $\Phi$ is increasing and convex. $g_t^{(j)}$ represents the rate of growth of firm $j$’s capital stock (or equivalently, the net investment rate), while $\Phi(g_t^{(j)})$ represents the investment-to-capital ratio (per unit of time). The efficiency units of capital then satisfy

$$dk_t^{(j)} = k_t^{(j)} \left( g_t^{(j)} dt + \sigma dZ_t^{(j)} \right).$$

$Z_t^{(j)}$ is a Brownian motion, representing idiosyncratic shocks hitting the production technology of the firm; the shocks are identically and independently distributed across firms. In our numerical calculations, we will assume that

$$\Phi(g) := \delta + g + \frac{\gamma}{2} g^2.$$

The parameter $\delta$ governs the depreciation of capital, while $\gamma$ governs the magnitude of capital adjustment costs.

**Capital Structure**  The firm has access to debt and equity markets. Both markets are frictionless. We note $b_t^{(j)}$ the principal amount of the firm $j$’s debt. Its tax liability between $t$ and $t + dt$ is equal to

$$\Theta \left( a_t k_t^{(j)} - \kappa b_t^{(j)} \right) dt$$

In the above, $\kappa$ is the coupon rate (assumed to be constant) on the bonds issued by the firm, while $\Theta$ is the corporate tax rate. The motive for the firm to take on debt stems from the tax deductibility of the debt interest expense.

Shareholders cannot commit to always repaying the debt issued by the firm, which is thus credit-risky. Upon default of firm $j$ at time $\tau$, bankruptcy costs cause the efficient units of capital to jump
downwards by a factor \( \alpha_k \), so that \( k^{(j)}_t = \alpha_k k^{(j)}_{t-} \). At the same time, shareholders and creditors renegotiate the firm’s debt, resulting in the firm emerging from bankruptcy with a lower debt burden \( b^{(j)}_t = \alpha_b b^{(j)}_{t-} \). In what follows, we impose the parameter condition \( 0 < \alpha_b < \alpha_k < 1 \), so that bankruptcy costs are strictly positive and so that the restructured firm’s debt-to-ebitda ratio is strictly lower than its pre-bankruptcy value. We will also denote \( N^d_t \) the related default counting process.

When firm \( j \) issues $1 face value of bonds, it raises proceeds equal to \( d_t^{(j)} \), which represents the (endogenous) debt price of the firm (per unit of face value). The dividends paid to shareholders of firm \( j \) at any time are equal to:

\[
\pi_t^{(j)} k_t^{(j)} dt := \left[ a_t k_t^{(j)} - \Phi \left( g_t^{(j)} \right) k_t^{(j)} - (\kappa + m) b_t^{(j)} + t_t^{(j)} k_t^{(j)} d_t^{(j)} - \Theta \left( a_t k_t^{(j)} - \kappa b_t^{(j)} \right) \right] dt.
\]

In the above, \( t_t^{(j)} k_t^{(j)} dt \) is the notional amount of bonds issued between \( t \) and \( t + dt \) by firm \( j \) (and sold at a price \( d_t^{(j)} \) per unit of face value). \( m \) is the speed of debt amortization. Negative dividends represent share issuances executed by the firm.

In this setting, the firm cannot commit to a particular debt issuance policy, so that the evolution of the debt balance \( b_t^{(j)} \) is given by:

\[
db_t^{(j)} = \left( t_t^{(j)} k_t^{(j)} - mb_t^{(j)} \right) dt
\]

**Levered Firm Problem** From now on, we omit the firm’s superscript \( j \) for notational simplicity. Shareholder value \( E \) is defined via:

\[
E_t(k, b) := \sup_{g_t, \tau_d} \mathbb{E}^{k, b, t} \left[ \int_t^{+\infty} e^{-r(s-t)} \pi_s k_s ds \right]
\]

(1)

The price of one unit of debt is defined via:

\[
D_t(k, b) := \mathbb{E}^{k, b, t} \left[ \int_t^{\tau_d} e^{-(r+m)(s-t)} (\kappa + m) ds \right]
\]

(2)

We focus on equilibrium outcomes in which the equity value \( E_t \) is homogeneous of degree one in \((k, b)\), and the debt price function \( D_t \) is homogeneous of degree zero in \((k, b)\). For \( x := b/k \), we can thus write \( D_t(k, b) = d_t(x) := D_t(1, x) \) and \( E_t(k, b) = k e_t(x) := k E_t(1, x) \). To write the recursive system for \( e_t \) and \( d_t \), we need to scale by \( k_t \). We show in appendix (A.1.2) that the shareholders’ problem can be re-written as:

\[
e_t(x) = \sup_{g_t, \delta_t} \mathbb{E}^{x, t} \left[ \int_t^{+\infty} e^{-\int_t^{+\infty} (r-g_s) ds + \int_t^{+\infty} \ln a_t dN_t^d} \pi_s k_s ds \right],
\]

(3)

\[
dx_t = \left[ t_t - (g_t + m) x_t \right] dt - \sigma x_t d\tilde{Z}_t + \left( \frac{\alpha_b}{\alpha_k} - 1 \right) dN_t^d.
\]

(4)

In the above, \( \tilde{Z}_t \) is a Brownian motion that is related to \( Z_t \) via \( \tilde{Z}_t = Z_t - \sigma t \). Similarly, creditors price the debt rationally, anticipating the financing, investment and default strategy of shareholders,
discounting cash-flows at the constant real interest rate $r$. This means that the debt price satisfies

$$
d_t(x) = \mathbb{E}^x \left[ \int_t^{\infty} \alpha_b \mathbb{N}^d_{\mathbb{N}_t} \mathbb{N}^d e^{-(r + m)(s-t)}(\kappa + m)ds \right]
$$

(5)

$$
dx_t = \left[t_t - (g_t + m - \sigma^2) x_t \right] dt - \sigma x_t dZ_t + \left(\frac{\alpha_b}{\alpha_k} - 1\right) dN^d_t
$$

Optimal investment, issuance and default policies will only depend on $(t, x)$ in the Markov equilibrium we are interested in. Leverage $x_t$ is then the unique firm-level state variable. Shareholders will find it optimal to default according to a cutoff policy in $x_t$:

$$
\tau_d = \inf\{t \geq 0 : \bar{x}_t \leq x_t\}
$$

When the firm is highly levered, it chooses to raise equity capital from shareholders. However, at the leverage boundary $\bar{x}_t$, shareholders will find it too costly to continue injecting equity capital into the firm and will instead let it default.

**Equity and Debt Valuation Equations** Equity and debt satisfy the following pair of Hamilton-Jacobi-Bellman (HJB) equations:

$$
0 = \max_{\iota, g} \left[ -(r - g)e_t(x) + a_t - \Phi(g) - (\kappa + m)x + ud_t(x) - \Theta(a_t - \kappa x) + \partial_t e_t(x) + \left[ t_t - (g_t + m) x \right] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]
$$

(6)

and

$$
(r + m)d_t(x) = \kappa + m + \partial_t d_t(x) + \left[ t_t(x) - (g_t(x) + m - \sigma^2)x \right] \partial_x d_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} d_t(x).
$$

(7)

These equations are coupled differential equations for $e_t$ and $d_t$, valid on $(0, \bar{x}_t)$. The Feynman-Kac differential equation for $d_t$ uses the Markov growth policy $g_t(x)$ and the Markov issuance policy $\iota_t(x)$ that result from the shareholder optimization problem. The time-consistency problem faced by shareholders (unable to tie their hands and commit to a particular capital structure policy) will however simplify our analysis, and will eventually lead to a decoupling of this system of partial differential equations. The boundary conditions are the following value-matching conditions:

$$
e_t(\bar{x}_t) = \alpha_k e_t \left(\frac{\alpha_b}{\alpha_k} \bar{x}_t\right)
$$

(8)

$$
d_t(\bar{x}_t) = \alpha_b d_t \left(\frac{\alpha_b}{\alpha_k} \bar{x}_t\right)
$$

(9)

Equations (8)-(9) describe what happens at bankruptcy: the efficiency units of capital suffer a dead-weight loss equal to a fraction $1 - \alpha_k$ of the capital stock, while creditors suffer a haircut equal to $1 - \alpha_b$ of their notional balance.
Optimality conditions In the class of equilibria we focus on, the first order condition of the shareholders’ problem with respect to debt issuances leads to a relationship between debt and equity prices:

\[ d_t(x) + \partial_x e_t(x) = 0. \] (10)

As shown in appendix (A.1.3), condition (10) implies that the debt issuance policy is:

\[ \iota_t(x) = \frac{\Theta \kappa}{-\partial_x d_t(x)}. \] (11)

The debt issuance intensity is increasing in the tax shield, and decreasing in the slope of the bond price function. The optimal capital growth rate \( g_t(x) \) follows a standard q-theory optimal rule:

\[ \Phi' (g_t(x)) = e_t(x) - x \partial_x e_t(x). \]

Finally, default optimality takes the form of a smooth-pasting condition:

\[ \partial_x e_t(\bar{x}_t) = \alpha_b \partial_x e_t \left( \frac{a_b}{a_k} \bar{x}_t \right). \] (12)

3.1.2 Aggregation and balanced growth

Let \( K_t := \int_0^h k_t^{(i)}dj \) be the aggregate capital stock in our economy, with \( J_t \) the measure of firms at time \( t \). Note \( \omega_t^{(i)} := k_t^{(i)}/K_t \) the share of aggregate capital owned by a particular firm, and note that \( \int_0^h \omega_t^{(i)}dj = 1 \). Note \( f_t(x, \omega) \) the time-\( t \) joint density over leverage and capital shares, and note that \( J_t = \int f_t(x, \omega)dxd\omega \). The dynamics of the aggregate capital stock are as follows:

\[
\begin{align*}
d K_t &= K_t \left[ \int_0^h \omega_t^{(i)} g_t \left( x_t^{(i)} \right) dj dt + \int_0^h \sigma \omega_t^{(i)} dZ_t^{(i)} dj - (1 - \alpha_k) \int_0^h \omega_t^{(i)} dN_t^{d,(i)}dj \right] \\
&= \hat{g}_t dt - (1 - \alpha_k) \hat{\lambda}_t^d dt.
\end{align*}
\] (13)

In equation (13), the law of large numbers allows us to simplify the capital growth equation since (a) the aggregation of idiosyncratic shocks does not contribute to aggregate growth, while (b) capital destructions through default contribute a locally deterministic term \(-(1 - \alpha_k) \hat{\lambda}_t^d dt\), representing the capital-share weighted credit loss rate in our economy. The aggregate capital stock thus grows at a deterministic rate \( \mu_{K,t} = \hat{g}_t - (1 - \alpha_k) \hat{\lambda}_t^d \).

In appendix (A.1.6), we show that the capital-share-weighted default rate \( \hat{\lambda}_t^d \) and the capital-share-weighted growth rate \( \hat{g}_t \) can be computed using moments of the density \( f_t(x) := \int_\omega \omega f_t(x, \omega)d\omega \), which represents the percentage of the total capital stock at firms with leverage \( x \). This density satisfies a modified Kolmogorov forward equation — an integro-differential equation that describes the dynamic properties of such density as a function of policy decisions made by firms. The capital-share-weighted default rate \( \hat{\lambda}_t^d \) and the capital-share-weighted growth rate \( \hat{g}_t \) can then be computed.
via:

$$\lambda_t^d = -\frac{1}{2} \sigma^2 \hat{X}_t^2 \partial_x \hat{f}_t(\hat{x}_t), \quad \hat{g}_t = \int g_t(x) \hat{f}_t(x) dx.$$  

In a balanced-growth path, the capital stock $K_t$ and the aggregate outstanding debt $B_t := \int^h_0 b^{(i)}_t dj$ both grow at constant rates, whereas the capital growth rate $\mu_{K,t}$, the capital-weighted default rate $\hat{\lambda}_t^d$ and the growth rate $\hat{g}_t$ are constant.

### 3.2 Discussion

**Investment and debt overhang** In this model, the debt taken on by firms depresses investment. Shareholders are less inclined to invest when the firm is highly levered, since this reduces their current cash-flows and since some of the value stemming from the related decrease in leverage is captured by creditors via higher debt prices. This leads to a debt overhang channel: the investment rate of a given firm is always lower than the investment rate of an unlevered firm, and the investment rate is a decreasing function of leverage.

Formally, in Appendix (A.1.4), we show that the equity value is convex. Using the first-order condition for investment, we then obtain:

$$\partial_x g_t(x) := -\frac{\partial_x e_t(x)}{\Phi''(g_t(x))} < 0$$

Investment is therefore always decreasing with leverage. Appendix (A.1.4) establishes a second, related result. Let $g^*_t$ be the optimal growth policy of a firm that is born with no debt and subsequently never borrows. Then,

$$g_t(x) \leq g^*_t \quad \forall x \geq 0,$$

that is, levered firms always invest less than the unlevered firm. Thus, debt overhang in this model reduces investment at the margin (as leverage increases), and overall (relative to a no-debt firm).\(^{27}\)

Figure 3 illustrates these two properties graphically. It reports the optimal gross investment rate, $\Phi(x)$, as a function of the ratio of debt to ebitda (which is proportional to book leverage $b/k$ in our model), as well as the stationary distribution of firms across levels of debt to ebitda, in the calibration that will be used for analysis in section 5.\(^{28}\) Investment rapidly declines with leverage, and is lowest at the default boundary.\(^{29}\) This debt overhang channel will potentially be amplified by government interventions involving loans to businesses.

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\(^{27}\) Another way to illustrate the magnitude of the debt overhang channel in our model is to compare the optimal capital growth rate $\bar{g}_t(x)$ to the growth rate $\hat{g}_t(x)$ that would be chosen by a manager maximizing enterprise value, rather than equity value. If we define enterprise value (per unit of capital) as $v_t(x) := (E_t + b_tD_t)/k_t$, then $\bar{g}_t(x)$ satisfies

$$\Phi'(\bar{g}_t(x)) = v_t(x) - x\partial_x v_t(x) - e_t(x) - x\partial_x e_t(x) + x^2 \partial_{xx} e_t(x)$$

Once again, the convexity of $e_t$ shows that the investment rate for an enterprise-value maximizing firm is greater than the investment rate chosen by an equity-optimizing firm.

\(^{28}\) We report policies as a function of the debt to ebitda ratio $z = b/(ka)$ instead of leverage $x = b/k$ because debt to ebitda is the leverage metric we use in the estimation of the model, as it allows us to sidestep the question of how to measure productive capital $k$ in the data.

\(^{29}\) Using our functional form specification for the adjustment cost technology, this condition implies that at the default
"Zombie" firms  The investment behavior of firms in this model can be used to think about the concept of "zombie" firms. In appendix (A.1.5), we highlight a general result of this class of problems: the default boundary $\bar{x}_t$ for our firm is always higher than the default boundary of a firm that commits to using the no-leverage optimal investment rule $g_t^*$ (but is otherwise free to choose its capital structure). This means that a highly levered firm will sacrifice investments and instead pay dividends (or reduce the intensity of its share issuances) and postpone default. This decision is privately optimal, but causes investment to be depressed, and some firms to remain active at higher leverage rates than they otherwise would, had they followed the no-leverage investment rule $g_t^*$.

Financial policies  Figure (4) reports equity values and the dividend issuance policy as a function of the ratio of debt-to-ebitda. Dividends are a decreasing function of leverage: with low levels of debt, the firm pays large dividends, mostly financed by the proceeds from debt issuances. Instead, at higher leverage levels, firm cash-flows are depressed by (a) high debt servicing costs, (b) lower levels of debt issuances and (c) lower prices obtained for each dollar face amount of debt issued. Even if shareholders cut investments, this latter force is not sufficient to offset the former effects, leading to dividends being a downward sloping function of leverage. Eventually, with a sufficiently high leverage, dividends become negative — in other words, the firm issues new shares. Our assumption that equity markets are open in the balanced-growth path is crucial at that point — absent open capital markets, a firm would have to default at lower levels of leverage. This will provide a transmission channel for “sudden stops” in capital markets to investment decisions, as discussed in section (5).

Figure (5) shows debt prices $d_t$ and debt issuance rates as a function of the ratio of debt-to-ebitda. Credit spreads are strictly positive (i.e., $d_t < 1$) even for the no-leverage firm ($z = 0$), since the bondholders of a firm that is barely indebted take into account the fact that the firm will be issuing large amounts of debt, thus increasing future leverage and future default risk. With low leverage, the slope $\partial_x d_t(x)$ of the debt price function is close to zero, leading to very high intensities of debt issuances for low-debt firms. These firms primarily use the proceeds from these issuances to pay out dividends, as illustrated in Figure 4.

The role of limited commitment  Firms in the model take on leverage aggressively in order to monetize the deductibility of debt interest expenses. Their inability to commit to a future financing strategy is detrimental for the total enterprise value. In fact, firms leverage up to the point where future default costs exactly wipe out the tax benefits of debt, so that a firm that has no debt outstanding has an equity value exactly equal to that of a firm that can never take on any leverage. This result is the focus of DeMarzo and He (2016). As shown in appendix (A.1.3), condition (10) can be used to show that, for any leverage ratio $x$, the value of equity is the same as that of a firm whose shareholders can commit to follow a policy where they issue bonds again, and only pay down existing debt. As a boundary, the firm’s capital growth rate satisfies

\[ g_t(\bar{x}_t) = a_k \bar{x}_t \left( \frac{a_k}{a_t} \right) + (1 - a_k) (-1/\gamma) \]

In other words, the gross investment rate at default is a weighted average of (i) the gross investment rate at exit from bankruptcy, and (ii) the minimum feasible gross investment rate $-1/\gamma$. 

result, the equity value of a firm with leverage \( x = 0 \) is exactly equal to the unlevered firm value per unit of capital \( e^*_t \), defined via:

\[
e^*_t = \sup_g \mathbb{E}^t \left[ \int_t^\infty e^{-r(s-t)} du \left( (1 - \theta) a_s - \Phi(g_s) \frac{k_s}{k_0} ds \right) \right]
\]  

(14)

This result illustrates the more general insight that shareholders’ inability to commit not to issue more debt in the future is “self-defeating”, in that it undermines their ability to benefit from the tax shield.\(^{30}\) The result is particularly useful for our purposes, because it ensures that firms in the model actively take on debt, thus giving debt overhang the best possible chance to matter for investment decisions. (By contrast, a model with commitment, such as Leland et al. (1994), tends to under-predict leverage relative to the data.)

**Default resolution** Our model assumes that upon default, the firm does not exit, but that it is instead restructured. The restructuring involves deadweight losses in productive capital, i.e. the restructured firm has less productive capital than the defaulting firm (\( \alpha_k < 1 \)). A large literature documents and measures deadweight losses associated with defaults and liquidations; in section 4, we use this literature to calibrate the value of \( \alpha_k \). Avoiding these deadweight losses is also the main reason why credit interventions may be beneficial following an aggregate shock. Additionally, the restructuring of the firm involves creditors accepting a haircut (\( \alpha_b < 1 \)), though equityholders are not completely wiped out from the restructured firm. Strictly speaking, this violates the absolute priority rule (APR). This assumption is motivated by a theoretical consideration: zero recovery rates for equityholders, but positive recovery rates for bondholders would not be consistent with a smooth debt issuance equilibrium.\(^{31}\) However, Bris, Welch and Zhu (2006) show that 12.2% of the Chapter 11 reorganizations they consider, APR is violated. Weiss (1990) also documents violations of APR in the majority (29 of 37) of the bankruptcy cases in his sample. Our bankruptcy resolution protocol is consistent with these APR violations.

**Cash holdings** The model does not feature cash holdings: cash inflows received by a firm (and related to sales and proceeds from debt issuances) are either distributed to creditors or used for investment, and any balance is distributed as dividends (rather than potentially stored as cash reserves). This means that a sudden decline in aggregate productivity \( \alpha_t \), accompanied by a sudden stop in capital markets, cannot be mitigated by cash reserves held by a firm, potentially exacerbating the effects of the financial market shock. Our decision to abstract from liquidity reserves stems from our desire to keep the model tractable, with a unique state variable (leverage) driving the ex-post

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\(^{30}\) The exact result only obtains in continuous-time, and is only approximate in the discrete-time analog model.

\(^{31}\) If creditors’ recovery in bankruptcy was strictly positive, the price of the firm’s debt near the default boundary would be strictly positive, incentivizing shareholders of the firm to use an arbitrarily large intensity of debt issuance to pay an arbitrarily large dividend rate just before default. This would result in a dilution of creditors’ debt claim, pushing the price of the firm’s debt to zero, thus contradicting the possibility of a strictly positive debt price near the default boundary. This means that a “smooth” equilibrium – in which the firm’s face value process is absolutely continuous – does not exist in such case. An alternative is to assume that both creditors lose their entire investment in bankruptcy. This would be inconsistent with evidence on recovery rates in bankruptcy, and also magnify the extent of deadweight losses, potentially exaggerating the benefits of credit interventions following a shock that leads to inefficient liquidations.
Potential extensions  Our production technology omits labor and instead focuses on capital. A straightforward extension, as discussed in Khorrami and Tourre (2020), can be made in order to include such factor of production in a Cobb-Douglas specification; such extension would not change firms’ investment, financing and default policies, and is thus omitted from our analysis. Relatedly, our model features idiosyncratic shocks only, and abstracts from aggregate shocks and risk-premia. As section (3.1.2) shows, this assumption yields a well-defined balanced growth path, from which we start our experiments when analyzing the impact of the pandemic shock on aggregate outcomes. We however discuss potential extensions with risk premia in Section 5.1.3.

4 Estimation

In this section, we describe how we fit the model of section 3 to data on public non-financial corporations. Since our goal is to use the model to describe the potential effects of shocks and government interventions that started in late 2020Q1, we fit the model to 2019 data.

4.1 Data

Sources  Our main data source is the Compustat fundamentals annual file. We focus on firm-year observations from fiscal year 2019, as our goal is to fit the ergodic distribution of the model to that year. Aside from Compustat, we also use the BEA’s Fixed Assets Tables in order to construct measures of the growth rate in prices of capital goods, \( \Pi_K \), and the aggregate, real growth rate of the capital stock of non-financial corporations, as well NIPA data to construct a value added deflator \( \Pi \) for the output of non-financial firms. Appendix A.2 reports the details. Finally, we report default rates for rated firms from S&P (2019), and estimates of debt recovery rates from Ou, Chiu and Metz (2011), though we do not target either of these moments in our estimation.

Compustat variable definitions  With some exceptions, which we discuss below, our treatment of the Compustat data is standard and follows other work (Whited, 1992; Frank and Goyal, 2003; Hennessy, 2004; Hennessy and Whited, 2005). We define the following variables from Compustat items (denoted by their mnemonic, in bold font):

\[
\text{ebitda} = \text{oibdp} \quad \text{(operating income before depreciation)}
\]

\[
\text{gross debt} = \text{dlc} + \text{dltt} \quad \text{(short- plus long-term debt)}
\]

\[
\text{net debt} = \text{gross debt} - \text{che} \quad \text{(gross debt minus cash)}
\]

\[
\text{market value of equity} = \text{mkvalt} \quad \text{(market value of common shares)}
\]

\[32\text{See Anderson and Carverhill (2012) for a model with liquidity in addition to debt as a firm state variable.}\]
equity payouts = dv + prstkc - sstk  
(cash dividends plus net stock purchases)

gross issuance of debt = dltis + dlcc  
(LT debt issuance plus change in ST debt)

In the definitions above, oibdp is operating income before depreciation (from income statements), dlc and dltt are short and long-term debt outstanding (from balance sheets), che are cash and cash equivalents, dv are cash dividend payments (from cash flow statements), prstkc are repurchases of stock (from cash flow statements), sstk are sales of stock (from cash flow statements), mkvalt is the market value of common shares (replaced by prcc_f × csho if missing), dltis is the gross issuance of long-term debt (from cash flow statements), and dlcc is the net change in short-term debt (from cash flow statements).33

Our baseline empirical mesure for the investment rate in the model, \( \Phi(x) \), is:

\[
\Phi(x) \equiv \frac{\text{capx}}{\Pi_K \times (l . ppegt)} \quad \text{(gross investment rate)}
\]

Here, \( l \) indicates that we use beginning-of-period values (that is, values from the close of the preceding fiscal year). Additionally, capx is capex (from cash flow statements), ppegt is gross property, plant and equipment at book value (from balance sheets), and \( \Pi_K \) is the gross change in a price index for property, plant and equipment, defined below. We map key financial ratios from the model to the data as follows:

\[
\begin{align*}
z(x) & \equiv \frac{\text{net debt}}{\text{ebitda}} \quad \text{(debt/ebitda ratio)} \\
\kappa z(x) & \equiv \frac{\text{xint}}{\text{ebitda}} \quad \text{(inverse interest coverage ratio)} \\
\pi(x)/e(x) & \equiv \frac{\text{equity payouts}}{l^{(1/2)} . (\text{market value of equity})} \quad \text{(payout rate on equity)} \\
l(x)/x & \equiv \frac{\text{gross issuance of debt} - \text{chech}}{l^{(1/2)} . (\text{net debt})} \quad \text{(gross issuance rate of debt)} \\
x & \equiv \frac{\text{net debt}}{\text{at}} \quad \text{(book leverage)}
\end{align*}
\]

Here, xint are interest and other related expenses (from income statements), at are total book assets (from balance sheets), and chech is change in cash and cash equivalents (from cash flow statements). Additionally, we define \( l^{(1/2)} . x \equiv (1/2)(\Pi \times l . x + x) \), the average of prior-period (appropriately reflated by the inflation rate \( \Pi \)) and current period values for variable \( x \). (We use this average in order to reduce the influence of observations with extremely large gross debt issuance rates due to very low 2018 debt stocks.)

In mapping data and model variables, we made certain choices that impact the values of the moments used in the estimation. We briefly highlight these choices here, and discuss their impact on key moments in a separate robustness section below.

---

33We treat missing variable fields as follows: if one of dv, prstkc or sstk is missing but at least one of the two others is not, we replace the missing value by 0. We do the same for dltis and dlcc, and for dlc and dltt.
First, we use net, not gross debt measures, because, as mentioned above, firms in our model do not hold cash. Below, we discuss results obtained using gross debt measures.

Second, we do not subtract \( \text{dltr} \) (reduction in LT debt, from cash flow statements) from the computation of the gross issuance of debt in levels. The Compustat manual defines this cash flow item as “a reduction in long-term debt caused by its maturation, payments of long-term debt, and the conversion of debt to stock”. Thus, this item contains principal payments on maturing debt, \( mbdt \) in our model. Our goal is to construct a measure of gross issuance, \( n_t dt = \iota k_t dt \), so we omit this term from our empirical measure.

Third, we use gross, not net stocks of PP&E, because the latter produces investment rates that are both very high and very volatile, likely driven by the difference between accounting and economic measures of depreciation. We consider a perpetual inventory estimate of the PP&E stock in our robustness checks.

Fourth, we define the equity payout rate as a fraction of the market value of equity, \( \pi(x)/e(x) = \pi(b/k)k/E(k,b) \), instead of the book value of productive capital, \( \pi(x) = \pi(b/k)k/k \). Likewise, we measure the rate of gross issuance of debt as a fraction of existing debt, \( \iota(x)/x = \iota(b/k)k/b \), instead of the book value of productive capital, \( \iota(x) = \iota(b/k)k/k \). This is because, to the extent possible, we prefer not to rely too much on a particular measure of the book value of productive capital \( k \), since it is not obvious how to measure productive capital \( k \) from accounting data. We however acknowledge that our baseline measure of investment rates does take a stance on what productive capital is (PP&E), and so we also provide a robustness check that defines investment including intangible capital.

Finally, we also report measures of book leverage \( x \) defined as the ratio of book debt to book assets. Conceptually, the analog in our model is \( x = b/k \). Our empirical measure uses book assets in the denominator, thus implicitly assuming that book assets is a measure of productive capital, at odds with the definition of \( k \) used in investment rates. We report this moment, despite this tension with our other measures, in order to allow comparisons with other papers using similar data. We do not target this moment in our estimation.

**Sample selection and summary statistics**  Sample selection criteria are described in detail in Appendix A.2. They are standard, except for two. First, either short-term (\( d1c \)) or long-term (\( d1t \)) debt must be strictly positive in both 2018 and 2019 for an observation to be included in the sample, since firms in our model all have strictly positive debt. Second, we restrict the sample to firms with strictly positive ebitda, since, as we explain below, the moments we target are weighted by ebitda. In 2019, firms with negative ebitda account for 2.1% of total assets and 2.0% of total revenue of non-financial Compustat firms, and their ebitda was 2.4% of aggregate ebitda (in absolute value).

Our selected sample has 1589 observations. We winsorize the variables \( \Phi(x), z(x), \kappa z(x), \pi(x)/e(x), \iota(x)/x \), and \( x \) at their 1st and 99th percentile within the selected sample. Finally, by analogy with the model, we define the weight associated with an observation \( i \) as:

\[
    w_i = \frac{N \times \text{ebitda}_i}{\sum_j \text{ebitda}_j},
\]

where the sum is over the selected sample, and \( N \) is the size of that sample. Unweighted summary
statistics for the resulting sample are reported in Appendix Table A-1.

**Moments used in the estimation**  The first column of Table 2 reports the data moments that we target in our baseline estimation. With one exception, these moments are weighted averages of the variables defined above, computed as:

\[
\hat{y} \equiv \frac{1}{N} \sum_i w_i y_i,
\]

where \(y_i\) is observation of variable \(y\) for firm \(i\), with:

\[
y \in \{ \Phi(x), z(x), \kappa z(x), \pi(x)/e(x), \iota(x)/x, x \}.
\]

The exception is the third line, which reports the slope of investment with respect to debt/ebitda. This slope is computed using the observation weights \(w_i\):

\[
\hat{\Gamma} \equiv \frac{\text{cov}(100 \cdot \Phi(x), z(x))}{\text{var}(z(x))},
\]

\[
\text{cov}(100 \cdot \Phi(x), z(x)) \equiv \frac{1}{N} \sum_i w_i (\Phi z_i - \hat{\Phi} \hat{z}),
\]

\[
\text{var}(z(x)) \equiv \frac{1}{N} \sum_i w_i (z_i - \hat{z})^2.
\]

The first three moments in the first column of Table 2, \(\hat{\Phi}, \hat{z}\) and \(\hat{\Gamma}\), are those targeted in the estimation. The rest of the moments are untargeted but will serve to evaluate model fit. The other columns of the table report alternative values of the moments we consider when different variable definitions or sample selection criteria are used. We come back to this in our robustness section below.

**Comparison to other evidence**  We next briefly compare the moments reported in Table 2 to existing estimates in the literature. Our estimates may differ from the literature for at least two reasons: first, we select a sample of firms with positive ebitda; second, moments are weighted by ebitda. We make both of these data choices in order to remain consistent with the model we use. Both of them imply that our moments will tend to reflect the characteristics of the larger firms in our sample. We focus our discussion on two moments that are central to our model: average leverage, and the sensitivity of investment to leverage.\(^{34}\)

Average book leverage in our sample (\(\hat{x}\)) is somewhat more elevated than existing estimates. For instance, in our sample, average gross book leverage (gross debt divided by gross book assets; Column (3) in Table 2) is approximately 38%, whereas Lemmon, Roberts and Zender (2008) document that gross leverage is 27% in their sample of nonfinancial firms between 1965 and 2003 (see their Table I). Aside from weighting and sample selection, mentioned above, two other reasons explain why our measures of leverage are somewhat more elevated than existing estimates.\(^{35}\) First, book leverage

\(^{34}\)Appendix A.2 further compares investment rates, equity payout rates, and debt issuance rates in our sample to existing evidence in the literature.

\(^{35}\)In our sample, value-weighting pushes up average leverage: for instance, gross leverage is 33% on average in our
has trended up over the past two decades, and particularly so after 2010. Second, FASB Accounting Standards Update 2017-02 mandated the inclusion of operating leases in measures of book debt, creating an upward jump in book values of debt in Compustat starting in the first quarter of 2019. The same factors likely explain why our measures of net book leverage (gross debt minus cash, divided by book assets) are more elevated than existing estimates. For instance, Hennessy and Whited (2007) target a net debt to assets ratio of 12.04% (Table I) for their baseline estimation, and 14.52% (Table III) in their restricted large firm sample, whereas our baseline estimate in Table 2 is 25.58%. Our estimates of debt/ebitda are likewise elevated, though other evidence report similar magnitudes: in a sample of rated firms, Baghai, Servaes and Tamayo (2014) report an average gross debt to ebitda ratio of 3.7 for 2009 (our sample average is 2.13).

The sensitivity of the gross investment rate to the ratio of debt to ebitda in our sample is $-1.04$. This moment is important because, as explained below: it captures the marginal effect of debt overhang in our model. This number should be interpreted as follows: gross investment falls by 1.04 percentage point (p.p.) for each unit increase in the ratio of debt to ebitda (that is, for instance, for an increase from 2.13 to 3.13). Analogs to this number have been estimated in empirical work on debt overhang, though the pseudo-elasticity measured is generally defined differently. For instance, Lang, Ofek and Stulz (1996) find that investment falls by 0.105 p.p. for each p.p. increase in book leverage (as opposed to the ratio of debt to ebitda) (see their Table 3, column 1). Using the ratio of ebitda to book assets in our data, this number can be translated into an increase in net investment of 2.62 p.p. for each unit increase in debt-to-ebita. Similarly, the estimates of Ahn, Denis and Denis (2006) imply a sensitivity of net investment ranging from 0.95 p.p to 3.35 p.p. per unit increase in ebitda; the estimates of Cai and Zhang (2011) imply a sensitivity of 0.93; and the estimates of Wittry (2020) imply a sensitivity of 0.95. Thus, the sensitivity of investment to leverage in our sample is consistent with existing work, though it is at the lower range of existing estimates. We compare other estimates of the strength of debt overhang from this literature to our the implications of our estimated model below.

4.2 Estimation

Methodology The construction of the equilibrium of the model, and the solution method, both rely on the computation of the stationary distorted distribution $\hat{f}$. For any set of structural parameters, this distorted distribution is fast and simple to compute. We therefore estimate the model using cross-sectional moments under this distorted distribution, which are themselves fast and simple to compute. This allows us to avoid having to resort to model simulation, which introduces noise and poses problems of convergence to steady-state.

We use a cross-sectional two-step feasible Generalized Method of Moments estimator (see, e.g., unweighted sample. Measures of aggregate leverage also tend to be higher: for instance, using aggregate data from the Flow of Funds, Gomes, Jermann and Schmid (2016) document an aggregate book leverage of 42%.

36In our model, a 1 p.p. change in $x = b/k$ (say, from 0.50 to 0.51) translates into a $1/(100 \times a)$ unit change in debt to ebitda $b/(ak)$, where $a$ is the ratio of ebitda to book assets. In our model, this ratio is approximately $a = 0.25$. Thus, a one unit change in debt to ebitda is associated with a $100 \times a = 25$ p.p. change in $x = b/k$.

37Ahn, Denis and Denis (2006) find estimates of the pseudo-elasticity of investment to book leverage ranging from 0.038 to 0.135 (see their Table 4). Cai and Zhang (2011) find an estimate of 0.0375 (see their Table 5). Wittry (2020) finds an estimate of 0.038 (his Table 10).
Erickson and Whited 2000), with appropriately weighted observations, to reflect the fact that we target moments under the distorted distribution. The method is standard and is described in Appendix A.2.2.

The numerical solution method for the model is described in Appendix [TBA]. We use 101 gridpoints in $x$ in our numerical solution routine, with maximum leverage set to $x^{max} = 3.0$; we check that this maximum is never binding when solving the model. In order to estimate the model, we use 10 random starting points, and report estimation results from the starting point achieving the lowest value for the GMM objective. We use Matlab’s patternsearch algorithm to find minima in the two-step GMM procedure, and we constraint the search so that the unconstrained optimal investment rate is well-defined, that is, $(a, \gamma)$ such that:

$$a \leq (1 - \Theta)^{-1} \left( r + \delta + \frac{1}{2} \gamma r^2 \right).$$

**Intuition for identification** The model has ten parameters, $\{r, \kappa, m, \delta, \Theta, a_k, a_b, a, \sigma, \gamma\}$. We normalize $\kappa = r$, so that the price of risk-free debt is equal to 1. We then calibrate six parameters, $\{r, m, \delta, \Theta, a_k, a_b\}$, to values drawn from prior evidence. (We describe below the sources for these parameters’ values).

We use an exactly identified approach to estimate the remaining three parameters, $a$ (the average product of capital), $\sigma$ (the volatility of capital quality shocks), and $\gamma$ (the convexity of capital adjustment costs). We match three data moments: the average gross investment rate $100 \cdot \hat{\Phi}$; the average ratio of debt to ebitda $\hat{z}$; and the cross-sectional sensitivity of investment to the ratio of debt to ebitda, $\hat{\Gamma}$.

Figure 6 reports how each of these moments vary with each of the three parameters. The main intuition for this graph is that identification in this model is “almost recursive”. Put differently, Figure 6 is “almost” lower-diagonal, in the sense that the sensitivity of moments to parameters in the upper triangular portion of Figure 6 is relatively small.

The top row of Figure 6 shows that variation in the model’s average gross investment rate primarily identifies the average product of capital $a$. The slope of the average investment rate with respect to $\gamma$ is negative but relatively small, while the average investment rate is almost insensitive to the volatility of capital quality shocks. An increase in the volatility of capital quality shocks shifts the default boundary to the right, because the option value of continuing for equityholders increases. This tends to increase average leverage and lower investment rates. At the same time, a higher volatility increases (in absolute value) the slope of the debt price function with respect to leverage, so that debt issuance is lower with higher volatility. This tends to lower average leverage and increase investment. Quantitatively, these effects approximately offset each other. Investment adjustment costs $\gamma$ also only have small effects on the level of investment (consistent with the assumption that they depend on the square of the expected net growth rate of the capital stock).

Given a value for $a$, the second row of Figure 6 shows that the average debt to ebitda ratio falls with the volatility of capital quality shocks, $\sigma$, while the sensitivity of investment with respect to leverage varies very little with $\sigma$. The average debt to ebitda ratio therefore helps identify the parameter $\sigma$. A
higher volatility leads to lower average leverage because the debt price function becomes steeper as volatility rises. The debt price function is relatively insensitive to capital adjustment costs $\gamma$ because those have only a small (local) impact on the average net growth rate of capital.

Finally, given values for $a$ and $\sigma$, the third row of Figure 6 shows that the slope of investment with respect to leverage helps identify the adjustment cost parameter $\gamma$. This slope is negative everywhere, as debt overhang causes investment to decline with leverage. Moreover, the magnitude of the slope decreases (in absolute value) with the magnitude of adjustment costs. All else equal, in response to a capital quality shock that increases their leverage, firms cut back investment less when investment adjustment costs are higher. Thus, the marginal effect of leverage on investment helps identify adjustment costs $\gamma$.

Values for calibrated parameters  Values for the six calibrated parameters, along with the sources for these values, are reported in the top panel of Table 3. We note two brief comments about their values.

First, we use a value of $r = 0.05$, drawn from Crouzet and Eberly (2020), who estimate $r$ as the discount rate required to account for observed values of Tobin’s $Q$ in the non-financial corporate sector (as opposed to computing it from measures of the risk-free rate). Their approach is consistent with the face that $r$, in the model, represents the discount rate equity- and debtholders, and not necessarily the return on riskless (government) securities.

Second, we use a recent estimate of deadweight losses on capital from Kermani and Ma (2020). This estimate is close to the median post- to pre- liquidation value of book assets of 38% reported by Bris, Welch and Zhu (2006) in a sample of 61 chapter 7 liquidations. These authors find creditor recovery rates in liquidation (defined as the ratio of amount recovered to par value of debt owed) ranging from 5.4% to 27.4%, depending on assumptions about collateral recovery by secured creditors. We use the intermediate value of $a_b = 0.15$ in our calibration.

4.3 Results

Baseline point estimates  Estimation results are reported in Table 3, Panel B. The three parameters $(a, \sigma, \gamma)$ are precisely estimated, and the targeted moments are well matched.

The point estimate for capital adjustment costs is $\gamma = 7.16$. There is a fair amount of variation in values of this parameter in the existing literature, even within those that use a $Q$-theoretic approach to estimating $\gamma$. Hayashi (1982) estimates a value of approximately $\gamma = 20$ using aggregate data on the US corporate sector, while Gilchrist and Himmelberg (1995) estimate a value of approximately $\gamma = 3$ in their sample of rated firms. Hall (2001) considers values ranging from $\gamma = 2$ to $\gamma = 8$.\footnote{Cooper and Haltiwanger (2006) estimates a much lower value of $\gamma$, but in a model with concavity in the revenue function, and without using estimates of the investment-$Q$ relationship. As they point out, their estimated model implies an investment-$Q$ slope of approximately 0.2, consistent with $\gamma = 5$.}

Recently, Falato et al. (2020) report estimates of $\gamma$ ranging from approximately 2 to 20, with higher values obtained in versions of their model that do not allow for intangible capital, as is the case in our baseline model. Our point estimate lies in toward the middle of this range. However, we note that different from these papers, which use the slope of investment with respect to $q$ in order to estimate
\( \gamma \), our paper uses the slope of investment with respect to leverage (the underlying state variable, in the model) instead.

The point estimate for \( a \) implies that the average before-tax returns to (physical) capital in the model are 24% per year. Using data from the Flow of Funds, Crouzet and Eberly (2020) find an average return to capital (defined in the same way as this paper) of 22.1% on average for the 2001-2017 period (Table 1). Other recent work on the rise in corporate profits have also finds returns to capital of the same magnitude (see, e.g. Barkai 2020).

Existing estimates of the capital quality shocks \( \sigma \) are more difficult to find. In the context of a real business cycle model with disaster risk, Gourio (2012) estimates a volatility for capital quality shocks of \( \sigma = 0.092 \), though this estimate is not comparable because he considers a process that has a persistent component.

**Non-targeted moments** Table 2, Panel C compares model and data values for non-targeted moments. Interest coverage ratios in the model are close to their data counterparts. The average equity payout rate is somewhat lower than in the data, though it is closer if distributions are measured only as cash dividends (see Table 2, Column 1). The average debt issuance rate is also somewhat smaller in the model than in the data. The empirical dispersion of the debt issuance rates is very large (see Appendix Table A-1), making the empirical average potentially imprecise.

The ratio of debt to capital, \( x \), is approximately 50% on average in the model, whereas in our sample, the ratio of book debt to book assets is only half that. As noted above, given the definition of capital in our baseline approach (PP&E), the correct empirical counterpart for \( x \) is the ratio of debt to PP&E, which is 64%, on average, in our sample. Thus, whether the model underestimates or overstate book leverage depends on one’s choice of empirical proxy for productive assets. This is why we do not target this ratio in our baseline approach.

Table 2, Panel C also compares the model’s implications for default rates, recovery rates, and aggregate growth, to the data. The model somewhat overstates default rates, and understates aggregate growth, relative to the data. Default rates are somewhat higher than empirical default rates for 2018 among rated firms (1.5% vs. 1.0%), though we also note that they are lower than default rates among non-investment grade rates firms (which are 2% in the data).\(^{39}\) We define debt recovery rates, in the model, as the value of debt claims at the default boundary, \( d(\bar{x}) \). The data provided in Table 2 computes recovery rates in a similar fashion, as enterprise value divided by total debt, or \( e(\bar{x})/\bar{x} + d(\bar{x}) \) in our model.\(^{40}\) The main difference is that, as discussed above, the resolution of default in our model assumes APR is violations, so that \( e(\bar{x}) > 0 \). This may partly explain the low recovery rates in our model are low relative to those in the data, though this is also likely due to the low value of \( \alpha_b \) used in our calibration. Finally, we note that the model some under-estimates the aggregate growth rate of the total capital stock, where the empirical counterpart is computed using the Fixed Assets tables. This reflects, in part, the relatively high default rate implied by the calibration compared to the data.

\(^{39}\)Default rates are from S&P (2019) Table 1. These only include rated firms, which are presumably less likely to default than non-rated firms in our sample.

\(^{40}\)Recovery rates are from Ou, Chiu and Metz (2011), Exhibit 9, for subordinated debt. Moody’s recovery rates are measured as enterprise value divided by total debt owed at time of resolution.
Finally, Figure 7 reports information on the higher moments of the data and the model. Specifically, the figure reports the cumulative share of different variables of interest (assets, \( k_t \); EBITDA, \( a_k t \); gross investment, \( \Phi(g_t) \); and dividends, \( \pi_t k_t \)) by level of debt-to-EBITDA, both in the model and the sample used for model estimation. These empirical CDFs are not targeted in our calibration. They suggest that the model underpredicts cross-sectional dispersion in EBITDA and investment or assets, relative to the data. Note that the model underpredicts the importance of both very high- and very low-leverage firms, so that this does not bias the effects of debt overhang on investment in one particular direction.

The bottom right panel reports the empirical CDF for dividend issuance. The model-implied CDF rises above 100% because some firms in the model issue negative dividends.\(^{41}\) The model thus tends to overpredict the frequency with which firms (particularly those with high leverage) use equity issuances as a way to smooth revenue and continue debt payments. The lack of equity issuance costs, as well as the lack of ability for firms to hoard liquidity (discussed in Section 3.2) contributes to this implication of the model. In our view, while counterfactual, this implication of the model is useful, because it magnifies the real effects of credit market shutdowns, and thus provides a form of upper bound on what the effects of these shutdowns (and the benefits of credit interventions) might be.

**Implications for the strength of debt overhang**  Figure 3, along with the investment policy function, reports the steady-state distribution of debt-to-EBITDA in the model. By construction, the policy function and the distribution are such that the model matches the average marginal effect of leverage on investment — the moment \( \hat{\Gamma} \) in our notation above. The value of this moment indicates that a one unit increase in the ratio of debt-to-EBITDA lowers investment, on average, by 1.04 percentage point; alternatively, a one standard-deviation (approximately 0.64 units of debt-to-EBITDA) increase in debt-to-EBITDA lowers the investment rate by 0.67%. Thus, the marginal effect of leverage in the estimated model are relatively small. Of course, these effects are an average: the local slope of investment with respect to leverage for high-debt firms is much steeper, as indicated by Figure 3; and, whereas gross investment rates for firms with leverage below two is close to 12%, it rapidly declines for firms with debt to EBITDA ratios above 3, and is equal to 4% for firms at the default boundary.

Despite the apparently small “marginal effects” of leverage and debt overhang, the presence of debt on firms’ balance sheet substantially depresses investment. At the estimated parameters, the growth rate of the economy would be 2.1% per annum in a version of the model in which firms never borrow. By contrast, the equilibriumFor instance, the all-equity firm growth rate, in this model, is 2.8%. Comparing this to the aggregate growth rate of the capital stock of 0.5%, reported in Table 3, this indicates that debt overhang depresses growth rate by 2.3% per annum in the model.

Another way of quantifying the effects of debt overhang in our model is to look at the slope of equity valuations with respect to leverage. In a sample of Canadian firms, Wittry (2020) finds that a one-standard deviation increase in (non-collateralizable) debt is associated with firms either foregoing or delaying projects. He estimates that the total value of these delayed or foregone investment opportunities to be 6.34% of total equity value. By contrast, in our model, equity value is 21.93%.

\(^{41}\)The empirical CDF also rises above 100%, for the same reason, though this is not clearly visible in the graph.
lower, whereas enterprise value is 4.37% lower, when evaluated at a leverage that is one standard deviation above the ergodic mean.

Finally, one can compare the ergodic enterprise value per unit of capital (Tobin’s average Q), $\bar{v} := \int v(x) \hat{f}(x) dx$, to the un-levered enterprise value $e^*$. The former is 1.111, while the latter is 1.166, suggesting that steady-state costs of debt overhang represent approximately 4.7% of total firm value. This measure of debt overhang cost is, in magnitude, higher than those estimated by Moyen (2007), who finds costs of 0.5% (when the benchmark is the un-levered enterprise value) or 4.7%-5.1% (when the benchmark is the firm value under the assumption that managers make investment decisions maximizing enterprise value).\footnote{It is however worthwhile pointing out that those latter estimates use a model that some degree of commitment over financing decisions.}

**Robustness** The columns marked (1) to (9) in Table 2 report key data moments under nine alternative variable definitions and sample selection criteria, and Table 3 reports estimation results obtained using these alternative sets of moments. We briefly highlight some results here, leaving the rest of the discussion to Appendix A.2.3.

Column (2) reports moments when adjusting for changes in the treatment of operating leases in 2019. As mentioned above, FASB Accounting Standards Update 2017-02 mandated the inclusion of operating leases in measures of book debt, creating an upward jump in book values of debt in Compustat starting in 2019.\footnote{For details on the effect of the change on book leverage in Compustat, see Palazzo and Yang (2019).} We correct for this change by using reported past rental commitments (details are in Appendix A.2.3). This lowers the debt-to-ebitda ratio by about 10%, and book leverage by about 3 percentage points. The sensitivity of investment to leverage is almost unchanged, and parameter estimates also change very little, except for the slightly higher dispersion of idiosyncratic shocks.

Column (3) reports moments when gross debt is used to define leverage ratios and issuance rates. By construction, leverage ratios are higher. The sensitivity of investment to leverage remains negative, but its magnitude falls by about half. As a result, the point estimate of $\sigma$ is lower, and that of $\gamma$ is larger. This latter would mitigate potential debt overhang effects of an increase in leverage.

Column (6) reports moments when we replace $\text{ppeg}$ by a perpetual inventory estimate of the current cost stock of property, plant and equipment for each firm in our sample. This perpetual inventory method is described in detail in Appendix A.2.3. This approach results in an average investment rate that is higher by about 3 percentage points (14.86% instead of 11.28%), and a somewhat larger slope of investment with respect to leverage, in absolute value (1.19 instead of 1.09). The resulting estimate of the marginal product of capital, $a$, is higher, but the overall calibration remains close to the baseline.

Columns (7) to (9) in Table 2 deal with adjustments related to intangible capital and intangible investment, which are described in detail in Appendix A.2.3. Column (8) in Table 3 reports results when a narrow definition of intangible capital — capitalized past R&D expenditures — is used in the adjustment. The main difference with the baseline is that investment in this case appears to be much more sensitive to leverage; as a result, the adjustment cost parameter is lower. This would potentially
magnify the effects of debt overhang following a shock.

Finally, Column (10) uses the same variable definitions and sample selection criteria as Column (1), but restricts the sample to firms with an S&P credit rating (short or long-term). This is relevant to the extent that certain credit support programs were restricted to rated firms. With the exception of leverage (which is somewhat higher), moments in the rated sample are very close to the baseline, with only higher investment and debt issuance rates. This is intuitive, given the fact that rated firms tend to be larger, and that moments are weighted by ebitda.

5 The impact of BCPs

We model the real effects of the crisis as a sudden shock to the productivity parameter $a_t$, which falls by 25% on impact. Productivity stays at such low level for 6 months, and then recovers linearly over the next 6 months. We compute different aggregate equilibrium objects along this perfect foresight path, and focus on different government policy responses.

We analyze the effect of this shock, and of potential government interventions, in two very distinct environments. In the first part of our analysis, we assume that financial markets are operating normally during the crisis period. In the next part of our analysis, we instead make the stark assumption that both credit and equity markets are closed, due to some un-modeled friction to capital markets. Our analysis of aggregate outcomes when focusing on these two extreme capital markets’ environments can be thought of as bounds on the actual outcome, in a financial market environment where debt and equity markets have been stressed without being completely shut.

5.1 Well-Functioning Financial Markets

In this section, our assumption is that debt and equity markets continue to be frictionless and open.

5.1.1 Laissez-Faire Economy

In the laissez-faire economy, the government does not intervene. With a surprise downward shock to productivity, the default rate increases dramatically, and reaches 7% per annum (note however that this is a flow rate, and the stock of firms defaulting during the crisis is a lot smaller than 7%). Firms deleverage slightly during the crisis, and the economy’s pre-default growth rate is about 0.50% per annum lower than it would be without such shock. Firms end up financing their growth via share issuances, as illustrated in Figure (8b). 3 years after the shock, the capital stock ends up only 0.2% below its level without the shock, as illustrated in Figure (8a). Thus, despite the relatively large shock, firms’ to access capital markets allows them to smooth the shock and limit its impact on capital growth.

5.1.2 Government Interventions: Some Irrelevance Results

We next assume that, in this environment where the government provides emergency funding to businesses during the crisis, and obtains either (a) loans or (b) equity stakes from companies receiving
such funding. Crucially, we assume that the funding provided by the government is "at market rates", meaning that the market value of securities received by the government from a given firm is equal to the amount of funding provided by the government to such firm. In such circumstances, when debt and equity markets are operating without friction during the crisis, we show that such policies have no effects, as firms adjust their capital structure policy so as to "undo" the government intervention.

We first study a government that decides to provide emergency funding to businesses, in exchange for bonds that the firm would be issuing to the government. During the crisis, between $t$ and $t + dt$, we assume that the government advances $s_t(x_t)k_t dt$ to a firm that has capital $k_t$. In exchange, the government obtains bonds issued by such firm, with balance $\lambda_t(x_t)k_t dt = s_t(x)k_t dt / d_t(x_t)$. This formulation thus assumes that the intervention of the government can be conditioned on the leverage $ι_t$.

Imagine finally that the government decides to undertake a policy intervention under which (a) the government advances money to each firm, in exchange for (b) equity that the firm would be issuing to the government. One can use the identity $\lambda_t(x_t) = s_t(x_t) / d_t(x_t)$ to obtain

$$0 = \max_{ι, g} \left[ - (r - g) e_t(x) + a_t + s_t(x) - Φ(g) - (κ + m) x + i d_t(x) - Θ(a_t - κx) 
+ \partial_t e_t(x) + [ι_t^g(x) + i - (g + m) x] \partial_x e_t(x) + \frac{σ^2}{2} x^2 \partial_{xx} e_t(x) \right]$$

Flow profits (per unit of capital) are thus increased by $s_t(x)$, while the drift rate of leverage is increased by the debt (per unit of capital) $ι_t^g$ issued to the government. One can use the identity $ι_t^g(x) = s_t(x) / d_t(x)$ to obtain

$$0 = \max_{ι, g} \left[ - (r - g) e_t(x) + a_t - Φ(g) - (κ + m) x + (ι_t^g(x) + i) d_t(x) - Θ(a_t - κx) 
+ \partial_t e_t(x) + [ι_t^g(x) + i - (g + m) x] \partial_x e_t(x) + \frac{σ^2}{2} x^2 \partial_{xx} e_t(x) \right]$$

This formulation of equity holder’s problem makes it clear that equation (16) is identical to equation (6). Thus, with this policy intervention financed by debt priced at market levels, the outcome is identical to the outcome in a laissez-faire environment. The debt issuance policy $ι$ used by the firm to finance itself in private markets is tilted downwards, but the total debt issuance (public and private) is identical to the issuance in the laissez-faire environment. This result crucially relies on the assumption that the government funding comes at a price that is identical to what private markets would charge. It also relies on the assumption that the firm cannot commit to a particular debt capital structure policy and can adjust its issuances instantaneously and at no cost.

Imagine finally that the government decides to undertake a policy intervention under which (a) the government advances money to each firm, in exchange for (b) equity that the firm would be issuing to the government. Between $t$ and $t + dt$, the government advances $s_t(x_t)k_t dt$ to a firm that has capital $k_t$ and leverage $x_t$. In exchange, the government obtains equity issued by such firm; the number of shares the government needs to receive is equal to $λ_t(x_t)dt := s_t(x_t)dt / e_t(x_t)$, so that the government is “fairly” compensated for this cash advance. The equity value for a given firm then satisfies the following Hamilton-Jacobi-Bellman equation:
0 = \max_{t,g} \left[ - (r + \lambda_t(x) - g) e_t(x) + a_t + s_t(x) - \Phi (g) - (\kappa + m) x + ud_t(x) - \Theta (a_t - \kappa x) + \partial_t e_t(x) + \left[ t - (g + m) x \right] \partial x e_t(x) + \frac{\sigma^2}{2} x^2 \partial x e_t(x) \right] \tag{17}

We can then replace \( \lambda_t(x) \) by its value to obtain exactly equation (6). In other words, when the government obtains an equity stake upon providing funding, it does not alter the equilibrium outcome through the crisis. Once again, providing such funding at market terms has no effect on outcomes. These results are summarized in the lemma below.

**Lemma 1** Suppose that financial markets continue to operate without any friction during the crisis period. Any government intervention financed (whether conditional on firm’s leverage, or conditional on time, or unconditional) at market interest rate (whether financed by debt, equity claims, or even hybrid instruments) has no impact on aggregate investment and growth rates.

5.1.3 Government Interventions: Debt Funding at Subsidized Rates

Imagine now that the government provides debt funding at an interest rate that is below the market rate. Let \( \alpha_t > 0 \) be the wedge between the interest rate at which private markets provide debt funding and where the government is accepting to provide such funding. For our numerical calculations, we will assume that at the start of the crisis, the government provides the funding at 1% below market rates, and that such subsidy then declines over time, as the crisis subsides. In this alternative setup, one can show that shareholders solve a problem identical to HJB equation (6), but in which the debt price obtained when issuing debt is calculated using the lower discount rate \( r - \alpha_t \):

\[
d_t(x) = \mathbb{E}^{t,x} \left[ \int_t^{\infty} e^{-\int_t^s (r - \alpha_u + m) du + \int_t^u \alpha_u dN^d_s} (\kappa + m) ds \right]
\]

Thus, in this environment, the firm’s investment and default policies are identical to those in the laissez-faire economy in which the government does not intervene. The improved debt pricing incentivizes firms to issue more debt, increasing leverage, and thus affecting aggregate investment rates and aggregate growth rates. The firm’s optimal debt issuance policy is equal to

\[
\iota_t(x) = \frac{\Theta \kappa + a_t d_t(x)}{-\partial x d_t(x)} \tag{18}
\]

In other words, the subsidy \( \alpha_t \) provides an incentive for firms to take on more debt, to increase leverage, which will exacerbate debt overhang. Note that this is not merely a numerical result, but a theoretical one: indeed, with a firm-level investment and default policy that is identical to the case where no subsidized funding is provided, the only impact of the funding subsidy is to increase the drift rate of leverage, pushing the cross-sectional leverage distribution towards higher leverage levels, thus decreasing aggregate investment.

**Lemma 2** Suppose that financial markets continue to operate without any friction during the crisis period. Any government intervention financed (whether conditional on firm’s leverage, or conditional on time, or...
unconditional) by debt contracts priced at below market interest rates will lead to lower aggregate investment and growth rates at all time $t \geq 0$.

Figure (9a) shows the time-path of the funding subsidy. Figure (9b) illustrates the fact that firms, confronted with debt funding at subsidized rates, increase debt issuance, resulting in higher aggregate leverage, as indicated in Figure (9c).

The analysis above assumes that the government intervention takes the form of providing financing at an interest rate $r - \alpha_t$ that is below the market interest rate $r$. One might argue that this modelling assumption is not entirely consistent with policy interventions such as the Primary or Secondary Corporate Credit Facilities — interventions through which the Federal Reserve purchases either primary or secondary bond offerings at market prices. Instead, one could have taken a modelling approach similar to that in DeMarzo, He and Tourre (2019), who specify exogenously the stochastic discount factor pricing the debt claims issued by firms. In that framework, a firm’s capital stock is not only exposed to idiosyncratic shocks, but also to an aggregate shock that is priced — in other words, investors need to get paid an expected excess return to be exposed to price fluctuations of stocks and bonds. While this approach is not consistent with our aggregation analysis (which assumes that firms are only exposed to idiosyncratic shocks), it is nonetheless informative to think about firms’ reactions to a sudden decrease in the price of risk — the most natural reduced form strategy to model the announcement by the Federal Reserve of the Corporate Credit Facilities.

In such environment, two cases need to be considered. First, if credit and equity markets are integrated, let $\pi_t$ be the price of risk associated with the aggregate shock, and let $\rho$ be the local correlation between the aggregate shock and a firm’s capital quality shock. Shareholders are pricing their residual cash-flows under the risk-neutral measure, under which the dynamic evolution of a firm’s leverage is corrected upwards by a drift adjustment $+\rho \sigma x_t$. Thus, a reduction in risk prices triggered by a government policy announcement lowers the drift rate of leverage under the risk-neutral measure, leading to an increase in equity values $e_t$ and the marginal value of capital $\partial_k E_t$, and thus an upward jump in investment rates. The financing policy of the firm is still driven by equation (11), and is only impacted by such change in risk price via the dependence of the issuance rate on the term $-\partial_x d_t(x) = \partial_{xx} e_t(x)$. Consider instead a situation where credit and equity markets are segmented, and note $\pi_{e,t}$ (resp. $\pi_{d,t}$) the Sharpe ratio of the aggregate shock in equity markets (resp. in credit markets). Once again, a decrease in equity markets’ risk price $\pi_{e,t}$ does lead to an increase the marginal value of capital $\partial_k E_t$, and thus to an increase in investment on impact. However, the financing policy of the borrower becomes

$$t_t(x) = \frac{\Theta_k}{-\partial_x d_t(x)} - \rho (\pi_{d,t} - \pi_{e,t}) \sigma x$$

Thus, if the reduction in debt markets’ risk price induced by the government intervention is not fully accompanied by a similar reduction in the equity markets’ risk price, the firm increases its bond issuances, leading to a future deterioration of corporate leverage and thus a future reduction in investment and the economy’s growth rate. Note also that the firms that are mostly levered (with high $x$) are those whose bond issuance rate increases the most — which are those exact firms suffering
the most from debt overhang. Those results are summarized in the lemma below.

**Lemma 3** Suppose that financial markets continue to operate without any friction during the crisis period. When credit and equity markets are integrated, a decrease in the (common) risk price induced by a government announcement will, on impact, increase the marginal value of capital and investment. When credit and equity markets are segmented, if the decrease in credit market risk price induced by a policy announcement is larger in magnitude than the related decrease in equity market risk price, investment rates jump up on impact, but firms take on more leverage in the future, pushing down future investment via debt overhang effects.

5.2 Financial Markets Shut-down

Government interventions either have neutral or negative consequences on aggregate investment and growth when financial markets are operating smoothly. However, as soon as frictions to credit and equity markets arise, an economic shock consistent with what we are studying can lead to a wave of bankruptcies and prolonged declines in investment and the level of the capital stock. In order to illustrate this, we now study an environment in which both credit and equity markets are completely shut down during the crisis period. Concretely, it means that firm’s investment policies are constrained, as dividends must remain weakly positive. We will first study the environment where the government does not intervene in financial markets, and will then analyze what happens when the government provides debt or equity funding at such time.

5.2.1 No Government Intervention

When the government does not intervene, firm’s investment policy is constrained as follows:

\[ g_t \leq \bar{g}_t(x) \]

\[ \bar{g}_t(x) := \Phi^{-1} \left( a_t - (\kappa + m)x - \Theta (a_t - \kappa x) \right) \]

Since \( \Phi \) is strictly increasing on \([-1/\gamma, +\infty]\), the greater the leverage of the firm, the lower the bound \( \bar{g}_t(x) \) on the capital growth rate. As \( \bar{g}_t(x) \) reaches the capital growth lower bound \(-1/\gamma\), the firm can no longer desinvest at a rate that is sufficiently high to avoid raising capital in financial markets, and since markets are shut during the crisis, firms at such level of indebtedness have to default. The HJB equation satisfied by the equity value \( e_t(x) \) is then

\[
0 = \max_{g \leq \bar{g}_t(x)} \left[ - (r - g) e_t(x) + a_t - \Phi (g) - (\kappa + m) x - \Theta (a_t - \kappa x) \right. \\
\left. + \partial_t e_t(x) - (g + m) x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]
\]

This equation holds for \( x \leq \bar{x}_t \), for which an expression is available in such case:

\[
\bar{x}_t := \frac{(1 - \Theta) a_t - \Phi(-1/\gamma)}{(1 - \Theta) \kappa + m}
\]
Finally, at the time the shock happens and financial markets shut down, a positive measure of firms defaults, and a fraction $1 - \hat{F}_0 - (\tilde{x}_0)$ of the capital stock is destroyed. This corresponds to the fraction of capital held by firms who are issuing shares on our balanced-growth path.

Figure (10) shows the time path of the capital stock and of the aggregate investment rate either (a) in the balanced-growth path, (b) through the crisis with financial markets functioning, and (c) through the crisis when financial markets are closed. The closure of financial markets has a dramatic impact on macroeconomic outcomes. As the firm default boundary suddenly shifts to a lower debt-to-EBIDTA level, a non-trivial measure of firms defaults, and the aggregate investment rate during the sudden stop is severely reduced, as firms can no longer finance their investment with bond or equity issuances. Investment recovers dramatically as markets re-open, and the investment rate ends up above its steady state value, as the surviving firms, at that point, are less levered than in the balanced-growth path. Nevertheless, the large liquidation wave, combined with depressed investment during the crisis period, means that shock has a long-run level effect of about 15% on the capital stock.

5.2.2 Emergency Debt Funding

We next turn to an environment that closely mimics the various credit market interventions implemented by the Federal Reserve and the US Treasury in response to the crisis. Specifically, we analyze a policy intervention according to which the government advances cashflows $s_t(x_t)k_t dt$ to any firm with capital $k_t$ and leverage $x_t$ during the crisis period. In return for such cash advance, the government receives $v_d(x_t)s_t(x_t)k_t dt$ principal amount of debt issued by such firm. Consider for example the case $v_d(x) = 1$ for all $x$. In such example, the government funding is subsidized given that the firm’s debt always trades at a discount to par; since the debt price is decreasing in leverage, the more levered the firm, the greater the subsidy provided by the government. In the general case, the value of the subsidy is equal to

$$s_t(x_t)k_t (1 - v_d(x_t)\nu_t(x_t)) dt$$

In this environment, investment is still constrained, but such constraint is relaxed when compared to the case where the government does not intervene. The investment rate must satisfy

$$g_t \leq \bar{g}_t(x)$$

$$\bar{g}_t(x) : = \Phi^{-1} (a_t + s_t(x) - (\kappa + m)x - \Theta (a_t - \kappa x))$$

This means that the firm’s problem becomes

$$0 = \max_{0 \leq g \leq \bar{g}_t(x)} \left[ -(r - g) e_t(x) + a_t + s_t(x) - \Phi (g) - (\kappa + m)x - \Theta (a_t - \kappa x) \right.$$  
$$+ \partial_t e_t(x) + [s_t(x)v_d(x) - (g + m)x] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]$$  

$$0 = \max_{g \leq \bar{g}_t(x)} \left[ -(r - g) e_t(x) + a_t + s_t(x) - \Phi (g) - (\kappa + m)x - \Theta (a_t - \kappa x) \right.$$  
$$+ \partial_t e_t(x) + [s_t(x)v_d(x) - (g + m)x] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right]$$  

(21)
The value of the subsidy provided by the government to a firm with leverage $x$ and capital $k_0$ at time zero is then

$$S'_d(x)k_0 := E^{x,0} \left[ \int_0^{+\infty} e^{-\int_0^t r_s ds} s_t(x)k_t \left(1 - \nu_d(x_t)d_t(x_t)\right) dt \right]$$

Appendix (A.1.7) discusses how to compute the value of such subsidy. The policy intervention represents a transfer to firms equal to $K_0 \int S'_d(x) f_0(x) dx$ in market value terms. In our numerical calculations, we assume that $s_t(x) = a_\infty - a_t$, so that the government is advancing money to firms in proportion to their revenue decline. We assume $\nu_d(x) = 1$, meaning that the government is effectively funding at par loans that are in fact worth less than par.

As figure (11) indicates, the initial capital destruction that follows the shock is less severe than in the laissez-faire environment, as the government funding allows highly-levered firms to continue operating without filing for bankruptcy. Investment is depressed (given that growth can no longer be financed via debt or equity issuances) but not as severely as in the laissez-faire environment, since the firm’s current cashflows, boosted by the government funding, allow it to finance corporate investments at a higher rate. Debt overhang is present during the recovery – after 6 months, as financial markets reopen, firm’s investment rate is lower than in the laissez-faire economy, as surviving firms have taken more debt with the government intervention. The debt overhang effect is however limited: the investment rate with government-extended debt funding is less than 0.50% lower than in the laissez-faire environment, and even if the debt-overhang is persistent, such persistence is not sufficient to overcome the level effect arising from the initial bankruptcies and related capital losses.

In our modelled policy intervention, the government extends $(a_\infty - a_t)k_t dt$ amount of funding in exchange for the same principal balance of a firm’s debt. While such policy intervention scales with firm’s size, it is independent of firm’s leverage. It is thus not exactly consistent with the programs implemented by the Federal Reserve and the US Treasury. For one, the Corporate Credit Facilities are focused on investment grade corporate bonds, thus excluding high-yield bond issuers. Second, under the Main Street Lending Program, firms could receive funding priced at Libor + 300bps, so long as they had a pre-crisis debt-to-EBITDA ratio of 4x or below, and so long as a bank is willing to extend funding at such price. However, empirically as well as in our modelled environment, firms that have modest leverage when entering the crisis do not risk immediate default, and are less in need of emergency funding than firms entering the crisis with high levels of debt. Thus, both the CCF and MSLP target exactly those firms that need emergency funding the least. This suggests an alternative design for such credit market intervention, with identical (or lower) fiscal costs, but with the benefit of preventing the failure of a substantial fraction of highly leveraged firms that end

\[44 \text{Both the Primary Market and the Secondary Market Corporate Credit Facilities’ eligibility criteria specify that issuers must be rated at least BBB-/Baa3 as of the beginning of the crisis.}\]

\[45 \text{Note that the debt-to-EBITDA limit of 4x is relevant for both the New Loan Facility and the Expanded Loan Facility of the Main Street Lending Program. Instead, the Priority Loan Facility targets more risky borrowers and features a maximum debt-to-EBITDA ratio of 6x.}\]

\[46 \text{Indeed, while the Main Street Lending Program is administered by the Federal Reserve Bank of Boston, US depository institutions are responsible for underwriting and originating the loans under such program, in exchange for origination fees, servicing fees, but also conditional on keeping 5% of the loan originated. This means that banks will only originate loans for which the market spread of borrowers is below L+300bps.}\]
up in need but not able to access liquidity during the crisis. Such alternative intervention would target highly leveraged firms, by setting \( s_t(x) = \mathbb{I}_{\{x_t \geq \bar{x}_t\}} \tilde{s}_t \), for some carefully chosen leverage hurdle \( \bar{x} \) and some carefully chosen cash advance (per unit of capital) \( \tilde{s}_t \). Firms with leverage ratios lower than \( \bar{x} \) would not receive any government-provided funding, reducing the overall fiscal cost of the intervention for the government, or allowing the government to redeploy such savings towards the highly levered firms that need it the most. Such a targeted policy would be relatively straightforward to implement, as it relies on measures of firm leverage that are readily available in accounting data.

5.2.3 Other Government Interventions

In the previous section, the government injects funds into firms in exchange for debt claims, priced above market. While such intervention reduces the magnitude of the initial wave of defaults and its related dead-weight losses, surviving firms end up taking on more debt, which depresses investment during the market shut-down and even after markets reopen. To mitigate the resulting debt overhang effect, one could instead design a policy with identical fiscal costs, but according to which the government obtains shares of firms receiving such emergency funding.

Concretely, imagine that the government advances cashflows \( s_t(x_t)k_t dt \) to any firm with capital \( k_t \) and leverage \( x_t \) during the crisis period, in return for receiving \( v_e(x_t)s_t(x)k_t dt \) shares issued by such firm. The subsidy implicit in that scheme is equal to

\[
 s_t(x_t)k_t (1 - v_e(x_t)e_t(x_t)) dt
\]

In this environment, investment is still constrained, but such constraint is relaxed in a way similar to what we described in section (5.2.2). One can compute the optimal default, investment and leverage policy of an individual firm, aggregate those decisions, and look at the resulting equilibrium outcomes, as suggested in Appendix (A.1.8). We continue to assume that the government advances \( s_t(x) = a_{\infty} - a_t \), but in exchange for \( v_e \) shares, where \( v_e \) is sized so that the fiscal cost of the government intervention is identical to the fiscal cost computed for the debt intervention described in section (5.2.2).

The initial wave of bankruptcies is very similar to the scenario where the government injects debt (instead of equity) funding, as illustrated in Figure (12). Firms however are less saddled by debt, and thus have higher investment rates than in connection with government-provided debt funding. The benefits of this intervention are visible particularly during the period during which financial markets are closed; investment rates are similar in the period during when markets re-open.

These benefits has to be weighed against the un-modelled costs of such intervention: imposing that businesses "give up" equity stakes in exchange for funds can make firms’ managers reluctant to accept such funds (since they would have to cease some control of the firm to a potential activist government), and renders its practical implementation more complex and potentially slower to execute, delaying the intervention at a critical time when firms need funds the most to avoid bankruptcy.

Instead of equity injections, the government could also consider broad-based debt forbearance policies, under which firms are allowed to delay their debt interest payments. Such debt interest payments are then capitalized, and remain payable as markets reopen when the crisis subsides. In
other words, while markets are shut down, the debt balance of a given firm $j$ evolves as follows

$$db_t^{(j)} = \left( i_t^{(j)}k_t^{(j)} - mb_t^{(j)} + \kappa b_t^{(j)} \right) dt$$

Instead, the firm’s dividends (per unit of capital and per unit of time) are equal to

$$\pi_t^{(j)} = (1 - \Theta)a_t - \Phi \left( s_t^{(j)} \right) - mx_t^{(j)}$$

This intervention has been used in the US mortgage market as a tool to mitigate the impact of the crisis on households. Under the CARES act, home mortgages that were purchased by one of the mortgage agencies after origination (i.e. close to 2/3 of outstanding residential mortgages in the US) can benefit from a temporary suspension of their required monthly payments. Note however that those payments are not forgiven; instead, they are owed later on, either as a lump-sum payment or smoothed over a certain time period. Such policy imposes potential costs onto creditors: since the unpaid interest gets capitalized, (a) creditors do not receive current interest on their debt, and (b) firm’s leverage ends up being pushed higher, increasing the firm’s default probability – keeping constant the firm’s default barrier. This intervention however has the benefit of pushing the firms’ default barrier to higher debt-to-EBITDA levels $\hat{x}_t$, since

$$\hat{x}_t = \frac{(1 - \Theta)a_t - \Phi (-1/\gamma) m}{(1 - \Theta)\kappa + m} = \hat{x}_t$$

This means that some of the firms that would otherwise have defaulted at time zero – i.e. firms with $x_0 \in (\hat{x}_0, \hat{x}_0)$ – are able to survive the initial shock, and bond holders for those firms will benefit from the forbearance policy. This is not necessarily the case for less levered firms, for which a forbearance policy might reduce the price of a firm’s legacy debt, compared to the counter-factual scenario where debt interest payments are made on a timely basis. Thus, the government on its own cannot mandate negatively impacted creditors to accept such debt forbearance policy without compensating them. In Appendix (A.1.8), we illustrate how to compute the fiscal cost of such policy intervention. Figure (12) illustrates the effect of such debt forbearance policy on aggregate outcomes. The impact of the pandemic shock, combined with the sudden stop in financial markets, ends up being somewhat dampened by the debt forbearance policy, since firms can save on current cashflows to stave off bankruptcy. However, the benefits of this intervention are however not as significant as those of the credit or equity market interventions, in particular for firms’ ability to maintain investment during the crisis period. The smaller effects of the intervention are, in part, due to the fact that its fiscal costs are much more modest than that of equity injections or direct loans.

5.2.4 Welfare Comparisons

In order to compare the effect of the different policy interventions on aggregate outcomes, we use a simple welfare criterion, based on the net present value of aggregate consumption in our economy. Under the assumption that all debt and equity claims issued by firms belong to a representative household, and based on the assumption that tax payments made by firms are rebated to house-
holds, such household ends up consuming the aggregate output of the firms, minus the aggregate investments made by such firms:

\[ C_t := \int_0^t a_t k_t^{(j)} dj - \int_0^t \Phi \left( x_t^{(j)} \right) k_t^{(j)} dj = (a_t - \Phi_t) K_t, \]

where we have used \( \Phi_t := \int \Phi (x) \hat{f}_t (x) dx \) as the capital-share weighted investment-to-capital ratio. We compare our policy interventions using the statistic

\[ W_0 := \int_0^{+\infty} e^{-rt} (a_t - \Phi_t) K_t dt \]

Table (5) summarizes our results. As discussed previously, a policy focused on equity injections, fixing the fiscal cost of such intervention, dominates a policy focused on debt funding, as the former policy minimizes the investment distortions stemming from the debt overhang channel. Our welfare calculations however do not take into account the complexity and potential delays that would be almost unavoidable when implementing broad equity injections into US businesses. Similarly, loan forbearance, while improving welfare compared to the “laissez-faire” environment, is a less potent policy, compared to a credit market intervention. Moreover, such intervention involves subsidizing certain debt holders, while penalizing others – hence such policy is only implementable to the extent the government can compensate those creditors being hurt by the intervention. This latter consideration makes such policy less attractive when thinking about the US corporate sector than when thinking about the household sector, where a majority of US mortgage debt is guaranteed by the mortgage agencies – and thus, indirectly, by the federal government.

6 Conclusion

Business Credit Programs (BCPs) are a novel feature of the public policy response to the crisis. In this paper, we developed a model to quantify the potential impact of this type of program on corporate leverage and investment. The model stresses debt overhang as a potential long-run distortion created by BCPs. Our two main findings are as follows. First, if capital markets continue to function normally during the crisis, BCPs have either zero effect on investment (if debt is offered at market rates), or negative effects (if debt is offered at below-market rates), though the negative effect in the latter case is small. Second, if the crisis is accompanied by a sudden stop in financial markets, BCPs offer a substitute source of funding for firms and thereby help avoid large output losses driven by forced liquidations. This “level” effect is substantially larger than the negative “growth” effect of higher debt levels among surviving firms as a result of the BCP. Because debt overhang effects during the recovery are small, the benefits from alternative policy designs, such as equity injections, are also small.

While these conclusions apply to the particular policy interventions we consider (motivated by the Fed’s two main corporate credit support programs), they are also useful to think through the size of the potential effect of debt overhang created by corporate lending programs in other countries. More generally, our results also speak to the view that the high amount of debt issuance of firms since the beginning of the crisis (particularly among rated, publicly traded firms) portends a slow recovery and
lackluster investment. The quantitative results from our model suggest that the increase in leverage necessary for this to occur would need to be very large, because most firms in the model operate in a part of the state-space where investment is not very sensitive to leverage.

There are two main reasons why our quantitative conclusion that the debt overhang channel is unlikely to have large effects could be challenged. First, our conclusion hinges on a particular value of the elasticity of investment rates to leverage. Though our model implies elasticities that match those we estimate in the data, and are in line with the existing literature, the effects of the channel we consider would be amplified if this elasticity were in fact larger, particularly during downturns. Second, our conclusion also hinges on the fact that the crisis, in the model, triggers a large wave of exit, because of the lack of external funding (absent government interventions). Alternative versions of the model, with more moderate disruptions to financial markets during the crisis, causing less exit, would lead to higher estimate of the force of the debt overhang channel. We leave these two questions for future research.
References


Table 1: Summary of key features of the 2020 Business Credit Programs (BCPs) in the United States. For the CCF, only investment-grade or firms downgraded after March 22nd, 2020 were eligible for direct purchases; ETFs with high-yield exposure were eligible. The CCF may only purchase up to 25% of individual issuances or 10% of the total debt outstanding of particular firms, but there are no caps on total issuance by the borrower. Size restrictions for the MSLP are based on 2019 revenue or employment. Funding caps depend on the facility used (new loan, priority loan, or expanded loan). For the PPP, grant convertibility is based on the firm using loan proceeds to cover payroll costs, rent, and payments on utilities. For more details, see the main text; see also, on the CCF, Boyarchenko et al. (2020); on the MSLP, Crouzet and Gourio (2020); and on the PPP, Granja et al. (2020).
<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>Baseline</th>
<th>(1) Only cash dividends</th>
<th>(2) Leasing adjustment</th>
<th>(3) Gross debt</th>
<th>(4) Asset-weighting</th>
<th>(5) Only LT debt</th>
<th>(6) ( k ) from PIM</th>
<th>(7) 2017 sample</th>
<th>(8) 2017 intan 1</th>
<th>(9) 2017 intan 2</th>
<th>(10) Rated firms</th>
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<tr>
<td>( 100 \cdot \Phi )</td>
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<td>11.28</td>
<td>11.28</td>
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<td>( \hat{z} )</td>
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<td>2.13</td>
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<td>1.79</td>
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<td>( \hat{\gamma} )</td>
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<td>−1.04</td>
<td>−1.09</td>
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<td>( 100 \cdot \kappa \hat{z} )</td>
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<td>11.26</td>
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<td>25.74</td>
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Table 2: Moments used in the estimation. The sample used is all Compustat non-financial firms (both rated and unrated). The moments reported are defined in the text. The third column, marked "Baseline", reports the moments used in the estimation of the model in our baseline sample from 2019; the baseline ebitda weights are used to compute them. Columns marked (1)–(10) report the values of these moments using alternative sample selection criteria and variable definitions. Column (1) defines equity payouts as only cash dividends. Column (2) adjusts for the changes in the accounting treatment of leases that occurred in 2019Q1. Column (3) uses gross debt, instead of net debt, in the definition of leverage ratios and debt issuance rates. Column (4) weighs observations by book assets instead of ebitda. Column (5) excludes short-term debt from the definition of leverage ratios and debt issuance rates. Column (6) uses a measure of the physical capital stock obtained using the perpetual inventory method. Columns (7) to (9) deal with adjustments for intangibles. Data for adjustments related to intangible is only available up to 2017, so Column (7) first reports the moments constructed in the same way as the baseline (with no intangible adjustment), but for the 2017 sample. Column (8) then adjusts for intangible investment in R&D, while Column (9) adjusts for intangible investment in both R&D and organization capital. Column (10) reports moments obtained when restricting the sample to firms with a credit rating.
Panel A. Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>$r$</td>
<td>risk-free rate</td>
<td>0.05</td>
<td>Crouzet and Eberly (2020) (Figure A3)</td>
</tr>
<tr>
<td>$m$</td>
<td>debt amortization rate</td>
<td>0.10</td>
<td>Saretto and Tookes (2013) (Table I, Panel B)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.10</td>
<td>Hennessy and Whited (2005) (Table III)</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>corporate income tax rate</td>
<td>0.35</td>
<td>OECD (2020) (Table II.1)</td>
</tr>
<tr>
<td>$\alpha_k$</td>
<td>1−deadweight losses</td>
<td>0.33</td>
<td>Kermani and Ma (2020) (Table I, Panel A)</td>
</tr>
<tr>
<td>$\alpha_b$</td>
<td>1−debt haircut</td>
<td>0.15</td>
<td>Bris, Welch and Zhu (2006) (Table XIII)</td>
</tr>
</tbody>
</table>

Panel B. Estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Point estimate</th>
<th>Standard error</th>
<th>[5, 95] normal confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>average product of capital</td>
<td>0.24</td>
<td>0.01</td>
<td>[0.22, 0.25]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>volatility of capital quality shock</td>
<td>0.31</td>
<td>0.03</td>
<td>[0.27, 0.35]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>curvature of capital adjustment cost</td>
<td>7.16</td>
<td>0.35</td>
<td>[6.59, 7.74]</td>
</tr>
</tbody>
</table>

Panel C. Model fit

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>Source</th>
<th>Targeted?</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100 \cdot \hat{\Phi}$</td>
<td>gross investment rate</td>
<td>Compustat sample</td>
<td>✓</td>
<td>11.28</td>
<td>11.28</td>
</tr>
<tr>
<td>$\hat{z}$</td>
<td>debt/ebitda</td>
<td>Compustat sample</td>
<td>✓</td>
<td>2.13</td>
<td>2.14</td>
</tr>
<tr>
<td>$\hat{\Gamma}$</td>
<td>slope of inv. wrt debt/ebitda</td>
<td>✓</td>
<td>-1.04</td>
<td>-1.04</td>
<td></td>
</tr>
<tr>
<td>$100 \cdot \hat{\kappa \hat{z}}$</td>
<td>inverse interest cov. ratio</td>
<td>x</td>
<td>11.3</td>
<td>10.7</td>
<td></td>
</tr>
<tr>
<td>$100 \cdot \hat{i}/\hat{e}$</td>
<td>equity payout rate</td>
<td>Compustat sample</td>
<td>x</td>
<td>4.6</td>
<td>3.0</td>
</tr>
<tr>
<td>$100 \cdot \hat{r}/\hat{x}$</td>
<td>debt issuance rate</td>
<td>x</td>
<td>25.7</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>$100 \cdot \hat{x}$</td>
<td>book leverage</td>
<td>x</td>
<td>25.6</td>
<td>51.0</td>
<td></td>
</tr>
<tr>
<td>$100 \cdot d(\hat{x})$</td>
<td>debt recovery rate</td>
<td>Ou, Chiu and Metz (2011)</td>
<td>x</td>
<td>29.3</td>
<td>12.1</td>
</tr>
<tr>
<td>$100 \cdot \lambda_d$</td>
<td>default rate</td>
<td>S&amp;P (2019)</td>
<td>x</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$100 \cdot G$</td>
<td>aggregate growth rate</td>
<td>BEA Fixed Assets</td>
<td>x</td>
<td>1.9</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3: Baseline estimation. Panel A reports the sources for the model parameters which we calibrate. The tax rate is the US pre-2017 statutory corporate income tax rate, drawn from the OECD’s Tax Database. Additionally, we set $\kappa = r$, so that the price of risk-free debt is normalized to 1. Panel B reports values for the three estimated parameters. We use a two-step feasible GMM approach, and data on non-financial public firms in 2019, as described in Section 4. Panel C reports model and data moments: targeted moments (first three lines); non-targeted moments drawn from the same sample used for the estimation (lines 4 through 7); and non-targeted moments drawn from other sources (lines 8 through 10).
### Panel A. Estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(8)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.25</td>
<td>0.31</td>
</tr>
<tr>
<td>([0.22, 0.25])</td>
<td>[0.22, 0.25]</td>
<td>[0.22, 0.27]</td>
<td>[0.20, 0.29]</td>
<td>[0.22, 0.25]</td>
<td>[0.23, 0.26]</td>
<td>[0.29, 0.32]</td>
<td>[0.21, 0.26]</td>
<td></td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.31</td>
<td>0.35</td>
<td>0.21</td>
<td>0.24</td>
<td>0.37</td>
<td>0.39</td>
<td>0.36</td>
<td>0.26</td>
</tr>
<tr>
<td>([0.27, 0.35])</td>
<td>[0.31, 0.40]</td>
<td>[0.12, 0.29]</td>
<td>[0.09, 0.38]</td>
<td>[0.34, 0.41]</td>
<td>[0.34, 0.44]</td>
<td>[0.30, 0.42]</td>
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<tr>
<td>(\gamma)</td>
<td>7.16</td>
<td>6.24</td>
<td>15.00</td>
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<td>10.35</td>
<td>3.33</td>
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<tr>
<td>([6.59, 7.74])</td>
<td>[5.70, 6.79]</td>
<td>[13.31, 16.69]</td>
<td>[13.87, 16.12]</td>
<td>[5.28, 6.32]</td>
<td>[10.25, 10.47]</td>
<td>[3.32, 3.36]</td>
<td>[6.52, 8.52]</td>
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### Panel B. Model fit

<table>
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<th>Moment</th>
<th>Baseline</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(8)</th>
<th>(10)</th>
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</thead>
<tbody>
<tr>
<td>(100 \cdot \Phi)</td>
<td>11.28</td>
<td>11.28</td>
<td>11.28</td>
<td>11.28</td>
<td>10.89</td>
<td>14.86</td>
<td>17.95</td>
<td>16.98</td>
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<tr>
<td>(\hat{\xi})</td>
<td>2.13</td>
<td>2.14</td>
<td>1.90</td>
<td>2.93</td>
<td>2.70</td>
<td>2.63</td>
<td>2.03</td>
<td>2.06</td>
</tr>
<tr>
<td>(\hat{\Gamma})</td>
<td>−1.04</td>
<td>−1.04</td>
<td>−1.09</td>
<td>−1.10</td>
<td>−0.62</td>
<td>−0.70</td>
<td>−0.61</td>
<td>−1.13</td>
</tr>
<tr>
<td>(100 \cdot \kappa\hat{\xi})</td>
<td>11.3</td>
<td>10.7</td>
<td>11.7</td>
<td>14.6</td>
<td>14.0</td>
<td>13.2</td>
<td>11.3</td>
<td>9.0</td>
</tr>
<tr>
<td>(100 \cdot \pi/e)</td>
<td>4.6</td>
<td>3.0</td>
<td>4.6</td>
<td>3.1</td>
<td>4.6</td>
<td>3.1</td>
<td>4.6</td>
<td>3.1</td>
</tr>
<tr>
<td>(100 \cdot \iota/x)</td>
<td>25.7</td>
<td>17.9</td>
<td>33.2</td>
<td>23.2</td>
<td>28.4</td>
<td>14.9</td>
<td>31.8</td>
<td>15.4</td>
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<tr>
<td>(100 \cdot \delta)</td>
<td>25.6</td>
<td>51.0</td>
<td>22.5</td>
<td>45.2</td>
<td>37.5</td>
<td>71.5</td>
<td>27.0</td>
<td>63.8</td>
</tr>
<tr>
<td>(100 \cdot d(x))</td>
<td>29.3</td>
<td>12.1</td>
<td>29.3</td>
<td>10.5</td>
<td>29.3</td>
<td>16.8</td>
<td>29.3</td>
<td>15.4</td>
</tr>
<tr>
<td>(100 \cdot \lambda_d)</td>
<td>1.0</td>
<td>1.5</td>
<td>1.0</td>
<td>1.4</td>
<td>1.0</td>
<td>1.9</td>
<td>1.0</td>
<td>1.8</td>
</tr>
<tr>
<td>(100 \cdot G)</td>
<td>1.9</td>
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<td>1.9</td>
<td>0.3</td>
<td>1.9</td>
<td>−0.1</td>
<td>1.9</td>
<td>−0.3</td>
</tr>
</tbody>
</table>

Table 4: Robustness checks on baseline estimate. The top panel reports point estimates, and the bottom panel reports measures of model fit. In each panel, columns report estimates of the model when different moments are targeted than in the baseline. The targeted moments are the first three reported in Panel B (\(100 \cdot \Phi, \hat{\xi}, \hat{\Gamma}\)). Columns are indexed with the same numbers as in Table 2. Compustat sample moments reported in the columns marked "Data" are the same as in that table. Non-Compustat moments are the same as in Table 3, except for Column (8), where the growth rate of the quantity index for the net stock of capital \(G_{tot,t}\) is used, and in Column (10), where the default rate for investment-grade firms from S&P (2019) is used. We omit the case of cash dividends because targeted moments are the same, and we only include one of the two robustness checks on intangibles to save space. Calibrated parameters are identical across all estimations, and equal to those reported in Table 3, Panel A, with the exception of column (8), where we set \(\delta = 15\%\) in order to account for the higher depreciation rates of intangible capital (Li and Hall, 2020).
Table 5: Welfare calculations for different policy interventions. The first column compares the welfare of the particular policy intervention to the welfare in the "laissez-faire" environment (which assumes that financial markets are closed). The second column compares the welfare of those same policies to the welfare in the balance-growth path.

<table>
<thead>
<tr>
<th>Policy Intervention</th>
<th>$W_0/W_0$ (laissez-faire)</th>
<th>$W_0/W_0$ (no-shock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>laissez-faire</td>
<td>1</td>
<td>0.78</td>
</tr>
<tr>
<td>new loans</td>
<td>1.11</td>
<td>0.89</td>
</tr>
<tr>
<td>loan forbearance</td>
<td>1.10</td>
<td>0.85</td>
</tr>
<tr>
<td>equity injections</td>
<td>1.12</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Figure 1: The distribution of gross leverage in the sample of US non-financial firms in Compustat. Gross leverage is measured as the ratio of gross debt to EBITDA. The figure reports the fraction of aggregate sales accounted for by firms with debt-to-EBITDA ratios above certain thresholds. The sample is US publicly traded non-financial corporations. Variable definitions are reported in Appendix A.2.
Figure 2: The size and effect of the Secondary Market Corporate Credit Facility (SMCCF). The shaded areas report the cumulative gross purchases of bond ETFs (dark orange) and single-name bonds (light orange) realized under the SMCCF. As of 11/30/2020, the par value of single-name bonds held was $4.86bn, and the market value of ETF shares held was $8.77bn. The blue line reports the ICE/BofA BBB corporate option-adjusted spread, which measures the option-adjusted spread of an index of corporate securities rates BBB over the corresponding spot Treasury rates (FRED series BAMLC0A4CBBB). The two dates highlighted with blue lines correspond to the Fed’s announcement of the Corporate Credit Facilities, on March 23rd, 2020, and of their extension, on April 9th, 2020. The two dates highlighted in orange correspond to the start of the purchases of ETFs (May 12th, 2020) and single-name bonds (June 16th, 2020).
Figure 3: The gross investment rate in the model of Section 3. The shaded blue area represents the steady-state asset-weighted (or distorted) distribution $\tilde{f}(x)$, and the red line to the right of the graph represents the default threshold. The dashed green line indicates the no-debt investment rate. The calibration used is reported in Table 3.
(a): Equity value per unit of capital, $e(x)$

(b): Dividend rate, $\pi(x)$

Figure 4: Equity value per unit of capital (top panel) and dividend rate (bottom panel) in the steady-state of the model of Section 3. In both graphs, the shaded blue area represents the steady-state asset-weighted (or distorted) distribution $\tilde{f}(x)$, and the red line to the right of the graph represents the default threshold. The calibration used is reported in Table 3.
Figure 5: Debt price function (top panel) and credit spreads (bottom panel) in the steady-state of the model of Section 3. In both graphs, the shaded blue area represents the steady-state asset-weighted (or distorted) distribution $\tilde{f}(x)$, and the red line to the right of the graph represents the default threshold. The calibration used is reported in Table 3.
Figure 6: Comparative statics of the model of Section 3, for the three moments targeted in the baseline estimate. Each row of panels corresponds to a different moment: the top row corresponds to $100 \cdot \hat{\Phi}$, the average gross investment rate; the middle row corresponds to $\hat{z}$, the average debt/ebitda ratio; the bottom row corresponds to $\hat{\Gamma}$, the sensitivity of gross investment with respect to leverage. Each column corresponds to a different parameter: the first column corresponds to $a$, the average return to productive capital; the second column correspond to $\sigma$, the volatility of capital quality shocks; and the third column corresponds to $\gamma$, the parameter governing the convexity of investment adjustment costs. In each graph, the solid blue line is the value of the moment reported in the graph, and the two red lines highlight the value of the parameter and the moment at the point estimate in our baseline estimation, which corresponds to $a = 0.24$, $\sigma = 0.31$, and $\gamma = 7.16$ for the parameters, and $100 \cdot \hat{\Phi} = 11.28$, $\hat{z} = 2.13$, and $\hat{\Gamma} = -1.04$ for the moments.
Figure 7: Comparison of empirical and model-implied cumulative distribution functions (CDF) for different variables. Each panel plots the cumulative share of a variable of interest (as a fraction of the aggregate value of that variable), as a function of the debt-to-ebitda ratio. The plots are constructed using the sample used in the estimation of the model.
Figure 8: Crisis without financial market shutdown: the case of no intervention. Path of aggregate capital (top panel) and dividend rate (bottom panel), following a temporary decline in productivity, in the case of well-functioning financial markets. The underlying shock is a 25% decline in the marginal product of capital \(a_t\) for a period of six months, followed by a linear recovery to its long-run steady-state level. The vertical dashed red line indicated the date at which productivity begins to recover, and the vertical dashed green line indicates the date at which productivity has recovered to its long-run steady-state value. See Section 5.1.1 for a description of this case.
Figure 9: Crisis without financial market shutdown: the effect of debt funding at subsidized rates. Total funding subsidy (top left panel), aggregate debt issuance (top right panel), aggregate gross investment rate (bottom left panel), and aggregate capital stock (bottom right panel), when the government provides funding at a subsidized interest rate during the crisis. The underlying shock is a 25% decline in the marginal product of capital $a$ for a period of six months, followed by a linear recovery to its long-run steady-state level. The vertical dashed red line indicated the date at which productivity begins to recover, and the vertical dashed green line indicates the date at which productivity has recovered to its long-run steady-state value. See Section 5.1.3 for a description of this case.
Figure 10: Crisis with financial market shutdown: the case of no intervention. Aggregate capital stock (top panel) and aggregate investment rate (bottom panel), following a temporary decline in productivity, with a shutdown in financial markets during the crisis. The underlying shock is a 25\% decline in the marginal product of capital $a_t$ for a period of six months, followed by a linear recovery to its long-run steady-state level. Additionally, funding (equity and debt) markets remain closed for the first six months of the crisis. The vertical dashed red line indicates the date at which productivity begins to recover and funding markets reopen, and the vertical dashed green line indicates the date at which productivity has recovered to its long-run steady-state value. See Section 5.2.1 for a description of this case.
Figure 11: Crisis with financial market shutdown: the effect of emergency debt funding. Aggregate capital stock (top panel) and aggregate investment rate (bottom panel), following a temporary decline in productivity, with a shutdown in financial markets during the crisis. The underlying shock is a 25% decline in the marginal product of capital $a_t$ for a period of six months, followed by a linear recovery to its long-run steady-state level. Additionally, equity and debt markets remain closed for the first six months of the crisis, but the government provides emergency debt funding to firms. The vertical dashed red line indicates the date at which productivity begins to recover, the government subsidy stops, and funding markets reopen; and the vertical dashed green line indicates the date at which productivity has recovered to its long-run steady-state value.
Figure 12: Crisis with financial market shutdown: the effect of alternative policy interventions. Aggregate capital stock (top panel) and aggregate investment rate (bottom panel), following a temporary decline in productivity, with a shutdown in financial markets during the crisis. The underlying shock is a 25% decline in the marginal product of capital $a_t$ for a period of six months, followed by a linear recovery to its long-run steady-state level. Additionally, equity and debt markets remain closed for the first six months of the crisis. We consider two alternative policy interventions: equity injections, and loan forbearance. The vertical dashed red line indicates the date at which productivity begins to recover, the government programs stop, and funding markets reopen; and the vertical dashed green line indicates the date at which productivity has recovered to its long-run steady-state value.
Appendix

Nicolas Crouzet and Fabrice Tourre
A.1 Model

A.1.1 No Leverage Firm

The average and marginal value \( e_t^* \) of a zero-leverage firm satisfies the equation

\[
0 = \max_g - (r - g) e_t^* + (1 - \theta) a_t - \Phi(g) + \partial_t e_t^*
\]

The optimal investment rule can thus be written

\[
g_t^* = (\Phi')^{-1}(e_t^*) = \frac{e_t^* - 1}{\gamma},
\]

where the last equality follows from the functional form assumption we made for the adjustment cost function \( \Phi \). The value (per unit of capital) \( e_t^* \) can then be computed via

\[
e_t^* = E_t \left[ \int_t^{+\infty} e^{-\int_t^s (r - g_u) du} \left[ (1 - \theta) a_s - \Phi(g_s^*) \right] ds \right]
\]

In a stationary environment, \( g^* \) and \( e^* \) solve a system of 2 equations in 2 unknown

\[
e^* = \frac{(1 - \theta)a - \Phi(g^*)}{r - g^*} \quad e^* = \Phi'(g^*) \quad (A1)
\]

This system can be re-written

\[
r = g + \frac{1}{\Phi'(g^*)} [(1 - \theta)a - \Phi(g^*)]
\]

Since the function \( \Phi \) is increasing and convex (for \( g > -1/\gamma \), where \( -1/\gamma \) is the minimum achievable capital growth rate), the right hand side of this equation is a decreasing function of \( g^* \) (so long as the steady state cash-flow rate \( (1 - \theta)a - \Phi(g^*) > 0 \)). In order for a solution \( g^* < r \) to this non-linear equation to exist, we must impose \( \Phi(r) > (1 - \theta)a \). In such case, the right hand side of the equation above, evaluated at \( g^* = r \), is strictly less than \( r \), whereas the right hand side of the equation above, evaluated at \( g^* \to -1/\gamma \), diverges to +\( \infty \) under the assumption that \( \Phi(-1/\gamma) < (1 - \theta)a \). Thus, under the parameter condition

\[
\Phi(r) > (1 - \theta)a > \Phi(-1/\gamma),
\]

There is a unique stationary investment rule \( g^* \) and a unique equity value (per unit of capital) \( e^* \) satisfying the system of equations (A1). Our function form for \( \Phi \) allows us to determine \( g^* \) analytically, as the smallest solution to the quadratic equation

\[
\frac{\gamma}{2} (g - r)^2 + (1 - \theta)a - \Phi(r) = 0
\]

This yields

\[
g^* = r - \left[ \frac{2}{\gamma} (\Phi(r) - (1 - \theta)a) \right]^{1/2}
\]
A.1.2 Rescaling

Let \( N_t^d \) be the counting process for default events. Notice that the effective capital \( k_t \) of a given firm can be written:

\[
k_t = k_0 \exp \left( \int_0^t \left( g_s - \frac{\sigma^2}{2} \right) ds + \sigma Z_t \right) = k_0 \tilde{M}_t \exp \left( \int_0^t g_s ds \right).
\]

In the above, we have introduced the martingale \( \tilde{M}_t := e^{\sigma Z_t - \frac{1}{2} \sigma^2 t} \). This defines the change-of-measure \( \tilde{P}(A) = \mathbb{E}[\tilde{M}_1 1_A] \). We then have

\[
E_t(k, b) = \sup_{g, \lambda, \tau_d} \mathbb{E}^{k, b, t} \left[ \int_t^{\infty} e^{-r(s-t)} \pi_s k_s ds \right] = k_t \sup_{g, \lambda, \tau_d} \mathbb{E}^{x, t} \left[ \int_t^{\infty} e^{-\int_t^s (r-g_u) du + \int_t^s \ln a_u dN_u^d} \pi_s ds \right] = k_t e_t(x_t)
\]

Under \( \tilde{P} \), \( \tilde{Z}_t := Z_t - \sigma t \) is a standard Brownian motion, and \( x_t \) evolves according to

\[
dx_t = [\mu_t - (g_t + m) x_t] dt - \sigma x_t d\tilde{Z}_t + \left( \frac{\lambda_b}{\lambda_k} - 1 \right) dN_t^d
\]

A.1.3 Optimal Financing

When we use equation (10) and the firm’s optimal growth policy \( g_t(x) \) in the HJB equation (6) satisfied by the equity value, it becomes:

\[
(r - g_t(x)) e_t(x) = a_t - \Phi(g_t(x)) - (\kappa + m) x - \Theta(a - \kappa x)
\]

\[
+ \partial_x e_t(x) - (g_t(x) + m) x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \tag{A2}
\]

Equation (A2) is the HJB equation for shareholders of a firm that can commit never to issue bonds ever again. If one were to differentiate equation (A2) w.r.t. \( x \), substract equation (7), and use the first order condition (10), one can back out the issuance policy

\[
i_t(x) = \frac{\Theta \kappa}{-\partial_x e_t(x)}
\]

Equation (A2) makes it clear that in our setup, the equity value of a firm with leverage \( x \) can be computed as if such firm was allowing its bonds to amortize, and as if it could commit to never issuing bonds in the future.

A.1.4 Convexity

The convexity of \( e \) can be seen from the fact that shareholders always have the option to issue a non-zero measure of bonds. Indeed, take two arbitrary leverage ratios \( x_1, x_2, \) and \( \lambda \in [0, 1] \), with \( x_\lambda = \lambda x_1 + (1 - \lambda) x_2 \). Consider feasible debt policies that make the firm’s leverage jump from \( x_1 \) to \( x_\lambda \), or from \( x_2 \) to \( x_\lambda \). Then we have

\[
e_t(x_1) \geq e_t(x_\lambda) + (x_\lambda - x_1) d_t(x_\lambda)
\]

\[
e_t(x_2) \geq e_t(x_\lambda) + (x_\lambda - x_2) d_t(x_\lambda)
\]

We can then take a weighted average of these inequalities to obtain \( \lambda e_t(x_1) + (1 - \lambda) e_t(x_2) \geq e_t(x_\lambda) \).
A.1.5 Default Boundary \( \hat{x}_t \)

Call \( \hat{e}_t(x) \) the equity value for a firm that uses the no-leverage optimal investment rule \( g^*_t \):

\[
\hat{e}_t(x) := \sup_\tau E^{t, x} \left[ \int_t^T e^{-\int_s^t (r-g^*_s) du} (a_t - \Phi(g^*_s) - (\kappa + m)x_s - \Theta(a_s - \kappa x_s)) ds \right]
\]

Note \( \hat{x}_t \) the optimal default boundary associated with \( \hat{e}_t(x) \), and \( \Delta_t(x) := e_t(x) - \hat{e}_t(x) \). Then for \( x < \min(\hat{x}_t, \hat{x}) \),

\[
(r - g^*_t)\Delta_t(x) = \Phi(g^*_t) - \Phi(g_t(x)) + (g_t(x) - g^*_t)(e_t(x) - x\partial_x e_t(x)) + \partial_t \Delta_t - (g^*_t + m)x\partial_x \Delta_t(x) + \frac{\sigma^2 x^2}{2} \partial_{xx} \Delta_t(x)
\]

\[
= \Phi(g^*_t) - \Phi(g_t(x)) + (g_t(x) - g^*_t)\Phi'(g_t(x)) + \partial_t \Delta_t - (g^*_t + m)x\partial_x \Delta_t(x) + \frac{\sigma^2 x^2}{2} \partial_{xx} \Delta_t(x)
\]

Notice then that \( \Phi(g^*_t) - \Phi(g_t(x)) + (g_t(x) - g^*_t)\Phi'(g_t(x)) \geq 0 \) given that \( \Phi \) is convex. Thus, noting \( \tau := \inf\{t \geq 0 : x_t \geq \min(\hat{x}_t, \hat{x})\} \), we have

\[
\Delta_t(x) = E^{t, x} \left[ \int_t^\tau e^{-\int_s^t (r-g^*_s) du} \left( \Phi(g^*_s) - \Phi(g_s(x)) + (g_s(x) - g^*_s)\Phi'(g_s(x)) \right) + e^{-\int_s^\tau (r-g^*_s) du} \Delta_t(x) \right] \geq 0
\]

This means that we must have \( \hat{x}_t \geq \hat{x}_t \) in other words, the firm delays its default decision, as it can reduce its investment rate and increase current cashflows when leverage is high.

A.1.6 Aggregation

Remember that the dynamics of the aggregate capital stock are as follows:

\[
dK_t = \int_0^t k_t^{(j)} g_t \left( x_t^{(j)} \right) d\omega_t + \int_0^t \sigma k_t^{(j)} dZ_t^{(j)} dj - (1 - \alpha_k) \int_0^t k_t^{(j)} dN_t^{d(i)}dj + \hat{\lambda}_t^n K_t dt
\]

\[
= \left( \int_x \int_\omega g_t(x) \omega f_t(x, \omega) d\omega dx \right) K_t dt - (1 - \alpha_k) \hat{\lambda}_t^n K_t dt + \hat{\lambda}_t^n K_t dt,
\]

where \( \hat{\lambda}_t^n \) is the capital-share-weighted default rate, computed as follows:

\[
\hat{\lambda}_t^n := \frac{1}{dt} \int_0^t \omega_t^{(j)} dN_t^{d(i)} dj
\]

We introduce \( \hat{f}_t(x) := \int_\omega \omega f_t(x, \omega) d\omega \), which represents the percentage of the total capital stock at firms with leverage \( x \). Then,

\[
dK_t = \left( \hat{g}_t + \hat{\lambda}_t^n - (1 - \alpha_k) \hat{\lambda}_t^n \right) K_t dt := \mu_{K,t} K_t dt,
\]

where \( \hat{g}_t := \int_x g_t(x) \hat{f}_t(x) dx \) is the average, pre-default and pre-injection aggregate capital growth rate. The law of motion for an individual firm’s capital share is:

\[
d\omega_t^{(j)} = \left( g_t \left( x_t^{(j)} \right) - \hat{g}_t \right) \omega_t^{(j)} dt + \omega_t^{(j)} dZ_t^{(j)} - (1 - \alpha_k) \omega_t^{(j)} dN_t^{d(i)} - \hat{\lambda}_t^n \omega_t^{(j)} dt
\]

The firm’s capital share increases or decreases depending on whether its capital growth rate is greater or less than the weighted-average growth rate in the economy (the first term in the stochastic differential equation above). The firm’s capital share also jumps down with default, due to bankruptcy costs. Finally, the firm’s capital share decreases as new firms are injected into our economy. Introduce the
In that section, we describe the problem faced by managers of a firm, at a time when financial markets are completely closed. The government is injecting funds worth $s_t(x)$ per unit of time and per unit
of capital into each firm, in exchange for $\nu_e(x)$ shares shares of the firm per unit of cash injected. The firm’s problem becomes

$$0 = \max_{g \leq g_t(x)} \left[ -(r + s_t(x)\nu_e(x) - g) e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m) x - \Theta(a_t - \kappa x) + \partial_t e_t(x) - (g + m) x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \right] \quad (A6)$$

The value of the subsidy provided by the government to a firm with capital $k_0$ at time zero is then

$$S_0^0(k_0) := \mathbb{E}^x_0 \left[ \int_0^{+\infty} e^{-r_t s_t(x)} \left( 1 - \nu_e(x_t) e_t(x_t) \right) dt \right]$$

A.1.8.2 Debt Forbearance

Let $T$ be the length of time during which capital markets are closed. For $t \in [0,T]$, when a firm is allowed to defer its debt interest payments (which ends up being capitalized), the debt price $\hat{d}_t(x)$ satisfies

$$(r + m)\hat{d}_t(x) = m + \partial_t \hat{d}_t(x) - (g_t(x) + m - \kappa - \sigma^2) x \partial_x \hat{d}_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} \hat{d}_t(x)$$

Instead, as soon as capital markets re-open, we assume that the forbearance period ends, such that debt prices satisfy equation (7). Thus, the initial cost of such intervention onto creditors is equal to

$$\int b_0^{(i)} \left( \hat{d}_0(x_0^{(i)}) - d_0(x_0^{(i)}) \right) d\gamma = K_0 \int x \left( \hat{d}_0(x) - d_0(x) \right) f_0(x) dx$$

A.2 Estimation

A.2.1 Data

Growth rates of prices of capital goods We construct a rate of inflation in capital goods prices, $\Pi_{K,t}$, using data from the BEA’s Fixed Assets tables, as follows. First, we define the gross rate of change in the quantity of equipment and structures for the non-financial corporate sector as:

$$G_t = \left( \frac{K_{\text{struct},t}}{K_t} C_{\text{struct},t}^{-1} + \frac{K_{\text{equip},t}}{K_t} G_{\text{equip},t}^{-1} \right)^{-1}.$$  

Here, $K_{\text{struct},t}$ is the current-cost net stock of non-residential structures in the non-financial corporate sector (Fixed Assets Table 4.1, line 39), $G_{\text{struct},t}$ is the growth rate in the chain-type quantity index for the net stock of non-residential structures in the non-financial corporate sector (Fixed Assets Table 4.2, line 39), $K_{\text{equip},t}$ and $G_{\text{equip},t}$ are similarly defined, but for equipment, and $K_t = K_{\text{struct},t} + K_{\text{equip},t}$ is the current-cost total stock of equipment and structures. The reason why we need to construct this index from underlying Fixed Assets data is that the gross rate of change in the quantity of capital for the non-financial corporate sector (Fixed Assets Table 4.2, line 37) includes intellectual property products, which our baseline measures of investment rate, which is limited to PP&E, does not capture. When including intangibles in our investment measure, we use the deflator $\Pi_{\text{tot},t} = K_{\text{tot},t} / K_{\text{tot},t-1} G_{\text{tot},t}^{-1}$, where $K_{\text{tot},t}$ is the current-cost net stock of capital in the non-financial corporate sector (Fixed Assets Table 4.1, line 37), and $G_{\text{tot},t}$ is the growth rate in the chain-type quantity index for the net stock of capital in the non-financial corporate sector (Fixed Assets Table 4.2, line 37).
We then define the rate of inflation in capital prices $\Pi_K$ as:

$$\Pi_{K,t} = \frac{K_t}{K_{t-1}} G_t^{-1}. \quad (1)$$

Finally, we define the inflation rate $\Pi$ as:

$$\Pi_t = \frac{Y_{\text{nom},t}/Y_{\text{real},t}}{Y_{\text{nom},t-1}/Y_{\text{real},t-1}}, \quad (2)$$

where $Y_{\text{nom},t}$ is gross value added of of the non-financial corporate business sector in current dollars (NIPA table 1.14, line 17), and $Y_{\text{real},t}$ is gross value added of of the non-financial corporate business sector in chained 2012 dollars (NIPA table 1.14, line 41).

**Compustat sample selection criteria** We apply sequentially the following sample selection criteria:

1. drop firm-year observations not incorporated in the USA ($\text{fic} = \text{"USA"}$)
2. drop firm-year observations whose two-digit SIC code ($\text{sic}$) is between 60 and 69 (financials), between 91 and 99 (multinationals), or equal to 49 (utilities);
3. drop observations whose name contains the strings "-REDH", "PRE FASB", "PRO FORMA", "INDEX", "-ADR", "-ADS", or has a non-missing and strictly positive value for $\text{adrr}$\footnote{A positive value for this variable indicates that the observation corresponds to an American Depositary Receipt. The other filters used similarly indicate either foreign entities, or stale or redundant observations.};
4. drop firm-year observations with negative entries for one of the following variables: $\text{sale}$ (total revenue, from income statements) and $\text{che}$;
5. drop firm-year observations with strictly negative ebitda in 2019;
6. drop firm-year observations with zero short and long-term debt $\text{dlc}$ and $\text{dltt}$ in 2018 or 2019;
7. keep observations for fiscal year ($\text{fyear}$) 2019;
8. after computing the key ratios of interest, keep observations such that $\Phi(x), z(x), \kappa z(x), \pi(x)/e(x), i(x)/x$, and $x$ are non-missing.

Unweighted summary statistics after winsorizing all moments at the top 99% and bottom 1% are reported in Appendix Table A-1.

**Comparison to existing evidence** Our baseline estimates of physical investment rate are consistent with previous research using similar data sources. In particular, in his sample of non-financial public firms, Hennessy (2004) documents an average gross investment rate of 12% per year, as a fraction of capital at replacement cost (Table I), while Hennessy and Whited (2005) report a gross investment rate of 7.9% per year, as a fraction of book assets, in their baseline sample. The total (physical plus intangible) investment rates we consider in our robustness section are also similar to (though slightly smaller than) those reported by Peters and Taylor (2017). Compared to aggregate investment rates, we find somewhat smaller numbers than estimates of gross investment rates that would be obtained using the Fixed Assets Table. For instance, Crouzet and Eberly (2020), Figure A8, report an aggregate gross physical investment rate of approximately 9% for 2017. The difference between the two may be driven by sample selection and by weighting, as Crouzet and Eberly (2020) show that aggregate
gross investment rates in the Fixed Assets Table match those of Compustat data, in levels, when the latter are computed using gross property, plant and equipment in the denominator (see their Internet Appendix IA.2).

The average equity payout rate, in our baseline sample, is approximately 4.6% per year. This is in the range of the moments used by Hennessy and Whited (2005) and Hennessy and Whited (2007), who, respectively, report an equity issuance rate of 4.2% (Table II) and 8.9% (Table I), and Frank and Goyal (2003) (Table 2) report equity issuance rates in the order of 5% per year in the 1975-1990 period, and 10% per year in the 1990-2000 period. These equity issuance rates are expressed as a fraction of book assets; equity issuance rates defined as a fraction of book assets are somewhat lower in our data, approximately 4% per year. We choose to express them as a fraction of the market value of equity for the reasons discussed above.

Our estimate of the average debt issuance rate in our baseline sample is also relatively high. As a point of comparison, Frank and Goyal (2003) define debt issuance rates as the ratio of net issuance of long-term debt to book assets (dltris/atl), an average debt issuance rate of 3.4% for 1998 (Table 2). Defined in the same way as Frank and Goyal (2003), the weighted average debt issuance rate in our sample is 3.7%, in line with their estimates.

A.2.2 Estimation method

Let $Y$ be the vector of estimated structural parameters, of size $N_p \times 1$. Let $\{X_i\}_{i=1}^{N}$ be a set of data vectors, each of size $N \times 1$. Additionally, each observation is attached a particular empirical weight $w_i$, a scalar, which satisfies: $\sum_{i=1}^{N} w_i = N$, where $N$ is the total number of observations in the cross-section. The mapping $\Xi(.) : \mathbb{R}^{N} \times \mathbb{R} \rightarrow \mathbb{R}^{N_m}$. Here, $N_m$ is the number of moments to be matched from the data, and the function $\Xi$ describes how these moments are computed from data observations. Finally, define the mapping $\Xi_m : \mathbb{R}^{N_p} \rightarrow \mathbb{R}^{N_m}$. Here, $N_p$ is the number of structural parameters in the model, and $\Xi_m$ describes how the theoretical moments are computed from these structural parameters.

For instance, $X_i = \{x_i\}_{i=1}^{N}$ could be a vector of observed leverage ratios $x_i$ in the cross-section, weighted by their relative share of total ebitda: $w_i = \frac{\text{ebitda}_i}{\sum_{j=1}^{N} \text{ebitda}_j}$. To match weighted average leverage, on would use $\Xi(X_i, w_i) = w_i x_i$ and $\Xi_m(Y) = \int_{x \leq \tilde{x}(Y)} f(x; Y) dx$, where $f(., Y)$ is the stationary distorted density and $\tilde{x}(.,)$ is the default threshold.

Define the functions:

$$g(Y, X, w) = \Xi(X, w) - \Xi_m(Y), \quad G(Y, \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N}) = \frac{1}{N} \sum_{i=1}^{N} g(Y, X_i, w_i).$$

We obtain an initial point estimate for $Y$, $\tilde{Y}$, as:

$$\tilde{Y} = \arg \min_Y G(Y; \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N})' G(Y; \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N}).$$

We then compute an estimate of the optimal weighting matrix, $\hat{W}$, as:

$$\hat{W} = \left[ \frac{1}{N} \sum_{n=1}^{N} g(\tilde{Y}, X_i, w_i) g(\tilde{Y}, X_i, w_i)' \right]^{-1}.$$

Finally, we compute the point estimate of $Y$, $\hat{Y}$, as:

$$\hat{Y} = \arg \min_Y G(Y; \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N})' \hat{W} G(Y; \{X_i\}_{i=1}^{N}, \{w_i\}_{i=1}^{N}).$$
The asymptotic distribution of \( \hat{\Upsilon} \) is given by:
\[
\sqrt{N} (\hat{\Upsilon} - \Upsilon_0) \sim N(0, \Omega),
\]
where an estimate of the variance-covariance matrix \( \Omega \) is given by:
\[
\hat{\Omega} = \left( \left( \frac{\partial G}{\partial \Upsilon} (Y, \{X_i\}_{i=1}^N, \{w_i\}_{i=1}^N) \right) \prime \hat{W} \left( \frac{\partial G}{\partial \Upsilon} (Y, \{X_i\}_{i=1}^N, \{w_i\}_{i=1}^N) \right) \right)^{-1}.
\]

The Jacobian of \( G(\cdot) \) must be approximated using numerical differentiation. To assess model fit, a test statistic for over-identifying restrictions in the case \( N_m > N_p \) is:
\[
J = G(Y, \{X_i\}_{i=1}^N, \{w_i\}_{i=1}^N) \prime \hat{W} G(Y, \{X_i\}_{i=1}^N, \{w_i\}_{i=1}^N),
\]
which is distributed as a \( \chi^2 \)-squared with \( N_m - N_p \) degrees of freedom under the null that the over-identifying restrictions hold.

### A.2.3 Robustness

Here, we discuss key moments obtained under alternative variable definitions or sample selection criteria, and reported in Table 2, and the implied estimates for parameter values. We focus on the robustness exercises not discussed in the main text.

Column (1) reports moments when equity payouts are measured using only dividends (as opposed to dividends plus net stock repurchases). This only affects the equity payout rate, which falls to 2.33%, closer to historical estimates of the dividend-price ratio for the S&P500 of around 2% (Shiller, 2015).

Column (2) reports moments when adjusting for changes in the treatment of operating leases in 2019. In order to correct for the change in accounting rules, we subtract rental commitments (variable \( \text{mrct} \) in Compusatat), when they are reported in 2019, from our definition of total debt. When they are not reported, but are reported in either 2018 or 2017, we subtract the lagged values, applying a 10% annual growth rate. (The value of 10% is somewhat arbitrary; we choose it to minimize the discontinuity in median book leverage over the 2010-2019 sample.)

Column (4) reports moments when the weight of an observation is defined as its total book assets relative to average book assets in the sample. This definition is only consistent with the model to the extent that book assets, \( \at \), are a good proxy for a firm’s productive assets. With this weighting, leverage ratios are higher, investment rates are somewhat lower, and the sensitivity of investment to leverage is less than half of our baseline estimate, though it remains negative.

Column (5) reports moments after excluding short-term debt and the change in short-term debt from all the definitions of debt-related variables, since the model only allows for long-term debt of fixed maturity. This lowers leverage ratios, but leaves the sensitivity of investment rates to leverage approximately unchanged.

Column (6) reports moments when we replace \( \text{ppegt} \) by a perpetual inventory estimate of the current cost stock of property, plant and equipment for each firm in our sample. We compute this estimate using the following recursion:
\[
\tilde{k}_{j,t+1} = \Pi_{K,t}(1 - \delta_{j,t})\tilde{k}_t + \text{capx}_{j,t},
\]
where \( j \) indexes a firm, \( \tilde{k}_{j,t+1} \) is the end of year \( t \), current cost estimate of the stock of property, plant and equipment, \( \Pi_{K,t} \) is the gross rate of change in capital prices derived above, \( \text{capx}_{j,t} \) are capital expenditures reported by firm \( j \) in year \( t \), and \( \delta_{j,t} \) is the rate of economic depreciation of the stock of
property, plant and equipment.

Given an initial estimate of a firm’s capital stock $\tilde{k}_{j,t_0(j)}$, we iterate on the relationship above. For the depreciation component, we set: $\delta_{j,t} = \delta = 10\%$, consistent with our calibration of the model, described below.\footnote{Unlike Hennessy and Whited (2007), Riddick and Whited (2009), Belo et al. (2020) and Falato et al. (2020), among others, we do not use reported accounting depreciation in Compustat, dp to measure depreciation expenses, i.e. $dp_{j,t} = \delta_j \Pi_{K_j} \tilde{k}_t$. First, because firms may use accelerated accounting depreciation, estimates of $\delta_j$ based on accounting depreciation may overstate the true rate of economic depreciation. Second, in our sample, there are frequent occurrences of firms reporting depreciation in excess capital expenditures despite a growing net PP&E stock. The difference is due to acquisitions, but it implies that perpetual inventory method estimates of $\tilde{k}_{j,t}$ obtained this way can be negative. This is the case for instance with Amazon, which reports depreciation in excess of capex for 6 of the 25 years that it is present in Compustat.}

We compute an initial value for the capital stock $\tilde{k}_{j,t_0(j)}$ as follows. Assuming that the rate of inflation in capital prices and the growth rate of investment are constant for $t \leq t_0(j)$, we obtain:

$$\tilde{k}_{j,t_0(j)} = \frac{1 + g_{j,t,t_0(j)}}{g_{1,t(t_0(j) + \delta - (\Pi_{K_0(j)} - 1) (1 - \delta)}^\text{capx}_{j,t_0(j)}} - 1. \quad (A7)$$

We then set $g_{j,t,t_0(j)}$ equal to the 10-year backward-looking moving average of $g_{1,t}$, where:

$$g_{1,t} \equiv \frac{I_{\text{struct},t} + I_{\text{equip},t}}{I_{\text{struct},t-1} + I_{\text{equip},t-1}} - 1. \quad (A8)$$

Here, $I_{\text{struct},t}$ is investment in non-residential structures in the non-financial corporate sector (Fixed Assets Table 4.7, line 39), and $I_{\text{equip},t}$ is similarly defined, but for equipment.\footnote{An alternative approach for initializing the perpetual inventory method consists of using, for $g_{j,t,t_0(j)}$, the sample average growth rate of capx$_{j,t}$ for $t \leq t_0(j)$ for firm $j$. Empirically, the relative small sample and the highly variable investment rates imply that the measure of $g_{j,t_0(j)}$, obtained this way will be noisy, and can lead to values that violate the transversality condition needed for condition (A7) to hold.} Finally, for each firm, we define $t_0(j)$ as the earliest year, before 2019, such that capx$_{j,t}$ is observed continuously from year $t_0(j) - 1$ to 2019, and we only keep firms for which $t_0(j)$ is lower than or equal to 2010, so that at least 10 years of data is used to compute the estimate of the capital stock. The sample is therefore smaller than in our baseline (1241 observations, instead of 1589).

This somewhat more complicated approach results in an average investment rate that is higher by about 3 percentage points (14.86\% instead of 11.28\%), and a somewhat larger slope of investment with respect to leverage, in absolute value (1.19 instead of 1.09).

Columns (7) to (9) deal with adjustments related to intangible capital and intangible investment. In order to adjust for intangibles, we follow the approach developed by Peters and Taylor (2017). The data for these adjustments is only available up to 2017. In Column (7), we therefore start by computing the moments of interest in the 2017 sample, with no adjustments for intangibles. Additionally, we require that firms in the sample have all the data required to make the adjustments for intangibles in Columns (8) and (9), so that the sample used in Columns (7)-(9) can be kept constant.

Column (7) indicates that even without intangible adjustments, leverage is lower, and debt issuance rates higher, in the 2017 sample.\footnote{The difference between 2017 and 2019 leverage is partly due to the upward trend in leverage, and partly to the changes implemented in 2019Q1 of the FASB rules for capitalizing leases mentioned above.} Column (8) then adjusts for intangible investment in R&D. Specifically, we adjust our ebitda measure as:

$$\text{ebitda} = \text{ebitda} + x_{\text{rd}},$$

where $x_{\text{rd}}$ are R&D expenditures (from income statements), which are treated as operating expenditures for accounting purposes. (We additionally impose that $x_{\text{rd}}$ be weakly smaller than $x_{xsga}$, sales, general and administrative expenses (from income statements), of which $x_{\text{rd}}$ is a subcomponent, and...
we replace missing values of xrd by zero.) Additionally, we make the following adjustments before computing investment rates:

\[
\begin{align*}
\text{ppegt} &= \text{ppegt} + k_{\text{int\_know}} \\
\text{capx} &= \text{capx} + xrd
\end{align*}
\]

Here, \(k_{\text{int\_know}}\) is the capitalized value of past R&D expenditures, as computed by Peters and Taylor (2017).\(^{51}\) Note that, with this adjustment, the proxy for productive capital \(k\) used in the definition of investment rates and in the computation of dividend and debt issuance ratios is the same; productive capital is defined as \(\text{ppegt} + k_{\text{int\_know}}\).\(^{52}\) Finally, in this case, we use the deflator \(\Pi_{\text{tot},t}\) defined above in the computation of investment rates. This rate of price change is appropriate because the BEA Fixed Assets tables define private nonresidential fixed assets inclusive of intellectual property products which, in the Fixed Assets tables, primarily consist of R&D capital.

In Column (8), note first that debt to ebitda ratios are lower than in Column (7). This is because adjusting ebitda for R&D expenditures increases ebitda. Additionally, note that the investment rate is higher than in the baseline case. Finally, the resulting sensitivity of investment to leverage is substantially higher than in the baseline.\(^{53}\)

Column (9) repeats the same exercise, but using an adjustment for both R&D capital and organization capital. In this adjustment, we define total productive capital as \(\text{ppegt} + k_{\text{int\_know}} + k_{\text{int\_org}}\), where the estimate of the value of organization capital, \(k_{\text{int\_org}}\), is again obtained from Peters and Taylor (2017). We use the same series \(\Pi_{\text{tot},t}\) defined above in order to deflate the lagged capital stock. This does not allow for different inflation rates of organization vs. R&D capital, but we are not aware of good estimates for the rate of change in the price of organization capital. Following the definition of that paper, we measure total investment as \(\text{capx} + xrd + 0.3 \times (xsga - xrd)\). Relative to the case with only R&D, leverage ratios are even lower, investment rates even higher, and the sensitivity of leverage to investment rates even higher (in absolute value).

Finally, Column (10) restricts the sample to firms with a credit rating. To retrieve the credit rating, we use Capital IQ’s \text{wrds\_erating} file on WRDS. After applying the different sample selection criteria, we link the remaining 2019 Compustat observations to the history of their ratings using the \text{wrds\_gvkey} file on WRDS, which maps Capital IQ/S&P ratings to Compustat gvkey. We designate a firm in our sample as rated when (a) the \text{gvkey} received at least one rating (variable \text{ratingsymbol}) between 2017 and 2019, and (b) the rating was different from "NR" (the value which corresponds to unrated securities in the \text{wrds\_erating} file). This merge leaves 443 firms in sample, compared to 1589 in our baseline sample.\(^{54}\)

---

\(^{51}\)We obtain its value from the latest download of the Total Q file on WRDS.

\(^{52}\)This adjustment is correct if intangibles and physical capital are perfect substitutes, but it may not be correct if they are not; in that case, not straightforward notion of total capital or total investment rate exists, as highlighted by Crouzet and Eberly (2020).

\(^{53}\)Some of the moments not directly affected by the modification of ebitda also change relative to Column (7), but this is because adjustments to ebitda also affect each observation’s weight.

\(^{54}\)This number somewhat higher than other estimates of the relative importance of rated firms, who make up approximately 20% of the population of public firms (see, e.g. Faulkender and Petersen 2005). This is due to our sample selection criteria, which tend to eliminate smaller firms. Additionally, firms in the rated sample account for approximately 50% of aggregate book assets, relative to the baseline sample.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>mean</th>
<th>s.d.</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>N</th>
</tr>
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<tr>
<td>$100 \cdot \Phi(x)$</td>
<td>gross investment rate</td>
<td>11.07</td>
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<td>5.42</td>
<td>8.36</td>
<td>13.34</td>
<td>1589</td>
</tr>
<tr>
<td>$z(x)$</td>
<td>debt/ebitda</td>
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<td>5.34</td>
<td>0.86</td>
<td>2.44</td>
<td>4.44</td>
<td>1589</td>
</tr>
<tr>
<td>$100 \cdot \kappa z(x)$</td>
<td>inverse interest coverage ratio</td>
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<td>42.24</td>
<td>7.02</td>
<td>14.00</td>
<td>26.65</td>
<td>1589</td>
</tr>
<tr>
<td>$100 \cdot \pi(x)/e(x)$</td>
<td>payout rate on equity</td>
<td>2.43</td>
<td>6.23</td>
<td>0.07</td>
<td>1.61</td>
<td>4.53</td>
<td>1589</td>
</tr>
<tr>
<td>$100 \cdot i(x)/x$</td>
<td>gross issuance rate of debt</td>
<td>45.75</td>
<td>230.45</td>
<td>2.82</td>
<td>26.41</td>
<td>87.43</td>
<td>1589</td>
</tr>
<tr>
<td>$100 \cdot x$</td>
<td>book leverage</td>
<td>27.83</td>
<td>28.44</td>
<td>9.98</td>
<td>27.52</td>
<td>44.03</td>
<td>1589</td>
</tr>
<tr>
<td>$w_i$</td>
<td>ebitda (rel. to average)</td>
<td>1.00</td>
<td>3.50</td>
<td>0.04</td>
<td>0.17</td>
<td>0.60</td>
<td>1589</td>
</tr>
</tbody>
</table>

**Table A-1:** Summary statistics in the baseline sample. Variable definitions, sample selection, and other adjustments are described in Section 4 and Appendix A.2. All moments are unweighted. The data are for 2019. The sample is restricted to include only firms with strictly positive ebitda in 2019 and strictly positive total gross debt in 2018 and 2019. Some variables are scaled by a factor of 100 in order to facilitate interpretation.