Can the cure kill the patient? Corporate credit interventions and debt overhang*

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June 2020

Abstract

The corporate credit support programs recently announced by the US Treasury and the Federal Reserve involve a trade-off: while they may help firms avoid liquidation during the crisis, they could also lead to higher leverage among survivors, slowing down investment and growth in the recovery. We study this trade-off in a structural model of investment, financing and default. We highlight two main findings. First, without disruptions to financial markets during the crisis, credit support programs have either no effects, or, if support is offered at below-market rates, a detrimental effect. Second, if there are disruptions to financial markets, credit support programs help avoid a large wave of liquidations, and have relatively little downside: most firms in the model take on too little incremental leverage through the program to generate a large amount of debt overhang in the recovery.

Keywords: Investment, Leverage, Debt Overhang, Credit Programs.

JEL codes: G32, G33, H32, E58.

*First draft: June 2020. We thank Gadi Barlevy, François Gourio, Arvind Krishnamurthy and Jason Donaldson for very helpful discussions, and seminar participants at the Central Bank of Denmark and the Junior Financial Intermediation Working Group. The views expressed here are the authors' own and do not represent those of the Federal Reserve Bank of Chicago or the Federal Reserve system. Fabrice Tourre is affiliated with the Danish Finance Institute and kindly acknowledges its financial support.

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1 Introduction

The ongoing pandemic, and the social distancing measures implemented to address it, have led to severe disruptions to the normal course of operations of many US businesses. These disruptions have reduced many firms' current revenues and free cash flows, threatening their ability to cover operating costs, to honor debt payments, and to continue investing.

In order to avoid severe reductions in capacity, or even bankruptcy, many firms need to raise substantial new funding. The availability of this funding in financial markets may however be limited — precisely at the moment it is needed the most. Indeed, early on in the crisis, corporate bond credit spreads widened dramatically, while at the same time equity markets were selling off abruptly, curtailing firms' ability to these markets to raise funding.[1]

Against this backdrop of cash flow shock and disruptions to financial markets, Congress, Treasury, and the Federal Reserve have acted in concert in order to provide support to corporate credit markets. Three main programs have been launched: the Paycheck Protection Program (PPP), the Main Street Lending Program (MSLP), and the Corporate Credit Facilities (CCF). While these programs differ in a number of dimensions, they have three common features. First, their goal is to make new, government-provided funding available to firms. Second, this funding takes the form of debt. Third, with the exception of the CCF, loan terms (in particular, interest rates) are pre-specified by the programs, as opposed to being set through market mechanisms.[2]

These policy interventions have led to a public debate about the trade-offs involved in providing government-backed funding to corporations. On the positive side, these interventions are likely to help a number of firms avoid situations of financial distress and, at the extreme, default.[3] On the negative side, these interventions will leave firms with a higher burden of debt, as the crisis subsides and the recovery starts. In turn, this additional leverage may magnify the agency costs of debt, and in particular distort incentives to invest. Indeed, with higher leverage, the benefits of new investment do not accrue to shareholders in full (Myers, 1977). The resulting "debt overhang" may slow down the recovery, by making firms more guarded in their decisions to hire or invest.

There are two specific reasons to worry about the debt overhang effects associated with the programs. First, in general, debt overhang has been estimated to have potentially large effects on corporate valuations and corporate investment.[4] Second, US firms entered the crisis with relatively high leverage, compared to historical norms. Figure 1 shows the distribution of a common proxy for leverage, the ratio of debt to EBITDA, in the cross-section of US non-financial publicly traded firms. The share of aggregate sales of firms with a debt-to-EBITDA ratio above 2 was at at 15-year high in late 2019, potentially worsening the exposure of the typical US firm to debt overhang effects.

[1] See Haddad, Moreira and Muir (2020).

[2] For instance, PPP loans have a maturity of 2 years and an interest rate of 1%. MSLP loans have a maturity of up to four years, and an interest rate of LIBOR + 3%. Section 2 summarizes the key features of these programs in more detail.

[3] Default and the bankruptcy process may involve deadweight losses, while new private creditors may fail to internalize the gains from avoiding default, because these gains may accrue to other stakeholders (employees, customers, suppliers or existing lenders). Additionally, there may be other externalities (such as aggregate demand externalities) that may generate further benefits from these interventions. Our paper focuses on the former — deadweight losses — and not the latter — externalities associated with defaults.

[4] See, among others, Mello and Parsons (1992), Lang, Ofek and Stulz (1996), Hennessy (2004), Aivazian, Ge and Qiu (2005), Ahn, Denis and Denis (2006), Moyen (2007), and Kalemli-Ozcan, Laeven and Moreno (2018). We discuss the relationship between our paper and these earlier estiamtes of the importance of debt overhang for valuations and investment in Section 3.3.

In this paper, we study these programs through the lens of a structural model of corporate investment, financing and default. Our goal is to answer two questions in the context of our model. First, what is the net economic impact of the programs — in their current form — likely to be for participating firms? Second, would there be large benefits from designing the programs differently — for instance, by providing equity injections rather than loans, or by allowing for forbearance on existing obligations rather than offering new funding?

The model we construct deliberately stresses the interplay between debt and investment. It describes a partial equilibrium in an industry populated by firms who invest, borrow, issue equity, pay dividends to shareholders and make default decisions. An individual firm's problem has two main components. First, on the real side, the investment decisions of a firm follow a standard Q-theory model analogous to Hayashi (1982), where production has constant returns to scale with respect to a unique capital input. The marginal product of capital is assumed to be exogenous and constant, but firms are subject to temporary "capital quality shocks", which are i.i.d. across firms and over time. Second, on the financing side, firms have access to both debt and equity markets. The supply of debt and equity is infinitely elastic, and financial markets are frictionless. Debt is long-term, as in Leland et al. (1994), but it can be readjusted continuously and at no cost, as in DeMarzo and He (2016).[5] Firms choose to issue debt because it is tax-advantaged. However, debt is also defaultable, and default entails dead-weight losses.[6] The optimal debt issuance policy trades-off these two forces.

Firms in the model exhibit debt overhang in the sense that the optimal investment policy function has a negative slope with respect to leverage. As firms approach the default boundary (the highest leverage ratio for which equity values are positive), marginal q — the value of one incremental unit of capital, from shareholders' standpoint — declines, as default becomes more likely; as a result, investment falls.

Our assumptions conveniently lead to leverage being a sufficient statistic summarizing a firm's state. Aggregate moments of the economy then depend on the cross-sectional distribution of leverage and its dynamic evolution. In the absence of aggregate shocks, the economy is on a balanced-growth path, in which capital, investment and output all grow at the same endogenously-determined rate. Though the model is deliberately stylized, we show that it replicates key investment and financing moments of US public firms relatively well, matching, in particular, their overall level of indebtedness. The model-implied cross-sectional leverage distribution is also reasonably close to the data, though it features somewhat less dispersion.

We then use this model to study the effects of credit interventions following a crisis. We start by deriving the implications of the model in a laissez-faire equilibrium with no intervention. Specifically, we study the perfect foresight response of the economy to a temporary (six-month) decline in the average product of capital by 25%. We consider two scenarios: one in which financial (debt and equity) markets function normally during the crisis, and one where they shut down for the duration of the crisis.

We first study the case where the crisis does not involve disruptions to financial markets. Though the shock is large, it has relatively mild effects on investment: firms respond by issuing equity in order to smooth the temporary decline in cash flows and avoid having to cut back investment. Our main finding, in this case, is that funding programs will either leave aggregate investment unchanged relative to a

[5] The resulting model is analogous to Hennessy and Whited (2007), except that (a) it does not feature decreasing returns to scale, (b) we assume frictionless equity markets and a simpler corporate tax structure, and (c) we allow for long-term debt.

[6] For simplicity, we consider the case in which all firm value is destroyed in liquidation.

laissez-faire equilibrium, or, potentially, reduce it. Government-provided debt funding at market prices is "undone" by firms who have complete freedom to adjust their debt issuance policy; the resulting intervention leads to firms issuing less debt to private markets, while their investment and default choices are unchanged. When the government injects equity capital into firms in exchange for funds, ownership of firms by private investors declines, but debt financing, investment and default policies are once again unchanged. In both these cases, so long as the funding is provided at market levels, aggregate investment and economic growth remain identical to those in the laissez-faire environment. On the other hand, a debt funding program at below-market prices *reduces* aggregate investment. This is because the marginal benefit of an extra unit of debt, for firms, now includes an extra term: the wedge between shareholders' discount rate and the discount rate used by the government to price the program loans. When this wedge is positive, firms borrow more, have more leverage, and invest less.

We then turn to the case where the crisis involves both an exogenous decline in the average product of capital *and* a shut-down of external financing markets, both debt and equity. In the laissez-faire equilibrium, the combined effects of financial market disruptions and cash flow shocks now has very large real effects. On impact, the shock leads to a wave of liquidations, driven by firms that would normally rely on financial markets to roll over debt or raise equity capital. Additionally, during the crisis period, investment remains depressed, because firms cannot finance investment through equity issuance in the face of temporarily low cash flows. Upon the end of the crisis, investment resumes at a slightly higher pace than in steady-state, because surviving firms generally have lower leverage.

In this context, corporate credit interventions can potentially improve on the laissez-faire equilibrium. We start by considering an intervention where the government provides debt funding that exactly makes up for the decline in cash flows due to the drop in the average product of capital.

This program has two effects. First, it helps firms avoid liquidation during the crisis, particularly in its early stages. This is similar to the way in which equity issuance helps firms smooth out the shock in the case where private credit markets operate normally. Second, once the crisis is over, it leaves firms with higher leverage. This is different from the case where financial markets operate normally. In that case, funding would have been obtained predominantly via equity rather than debt issuances, leading to only small changes in leverage and normal investment rates (relative to the steady-state) during the recovery. The government program we consider thus involves a trade-off between reducing liquidations (during the crisis) and reducing investment (during the recovery).

However, we find that the effects of reduced liquidations on overall investment, capital, and output, easily overwhelm the debt overhang effects. The intervention reduces by approximately half the destruction of capital due to short-run defaults. On the other hand, while the additional leverage leads to investment rates that are indeed lower during the recovery, the difference relative to the laissez-faire equilibrium is minimal. The combination of large positive effect on the level of capital, and a limited negative effect on its growth rate implies than, 5 years after the shock, output and the capital stock is 10% lower with the intervention (compared to a no-shock scenario), compared to approximately 30% lower in the absence of intervention.

We then look at other designs for the policy intervention. Loan forbearance — allowing firms to delay and capitalize interest payments on debt for the duration of the crisis — has qualitative effects that are similar to the debt-funded earnings replacement policy. In particular, growth effects during the recovery are small. Loan forbearance leads to more liquidations on impact than the earnings replacement policy, though the difference is not substantial. We also consider government funding in exchange for an equity stake in the firm. We size the government equity stakes such that the fiscal cost of the intervention is identical to the fiscal cost of the intervention designed with debt funding. Our conclusions remain qualitatively and quantitatively the same: equity injections reduce the incremental debt overhang induced by government-provided debt funding, but those effects are very small.

Thus, our main result in this case is that while, qualitatively, the interventions do involve a trade-off between avoiding liquidations and reducing subsequent growth, quantitatively, this trade-off overwhelmingly seems to favor the intervention, relative to a laissez-faire equilibrium. Negative debt overhang effects are an order of magnitude smaller than positive effects on liquidation.

This naturally leads to the question of why the debt overhang channel *after the interventions* is small in our model, despite the fact that debt overhang has substantial effects in steady-state.[7] The main reason is that the bulk of firms in the model operate in a region where the *slope* of the investment policy function with respect to leverage is small. Moreover, the loan-funded government program has limited effects on overall leverage. A back of the envelope calculation is that it increases a firm's debt-to-EBITDA by $(1/2)$ (the duration of the shock, six months) multiplied by 25% (the size of the decline in productivity, and hence of the earnings replacement provided by the program), or approximately 0.1 (relative to a mean of approximately 2.7). Given the small slope of investment with respect to leverage in the region where most firms operate, this increase does not have large effects. The finding of a small debt overhang effect also explains why alternative funding programs, such as equity injections, would not lead to dramatically different investment behavior during the recovery.

There are two substantial reasons why the finding of a weak debt overhang channel might be incorrect. The first one, already hinted at above, is that the strength of the channel crucially depends on the elasticity of investment to leverage for the "modal" firm. We do not calibrate it directly in our model, because we are not aware of robust estimates of this elasticity. However, if it were the case that this elasticity is, in reality, substantially larger than implied by our model calibration, our conclusions might change. Second, our finding likely depends on the extreme form of financial disruption which we consider: a complete shut-down of equity and debt markets. Alternative approaches, in which, for instance, the cost of equity issuance increases (but does not become infinity), might lead to fewer liquidations in the short-run and a distribution of surviving firms that are more highly levered overall, potentially magnifying the debt overhang channel.

Aside from debt overhang — our focus in this paper —, the credit interventions have a number of other costs.[8] An important one is the phenomenon of "zombie firms" — firms that are kept alive by their lenders because these lenders have limited incentives to recognize loan losses.[9] While we do not consider distorted lender incentives explicitly, it is worth noting that firms in our model exhibit a behavior similar to those of "zombie" firms, in that they actively delay default when they are highly levered, in particular by issuing equity at an accelerated pace — a behavior that is made worse by the government interventions.

[7]For instance, the net growth rate of aggregate capital in the no-debt equilibrium is approximately 2.8%, while it is 0.9% in the steady-state of the model with debt.

[8]In particular, moral hazard or adverse selection on the part of the borrowers may also lead to distorted firm investment incentives and magnify the fiscal costs of the interventions. We do not consider these additional mechanisms in our framework, but we recognize their potential importance in assessing the costs and benefits of these programs.

[9]See Hoshi, Kashyap and Scharfstein (1990) and Caballero, Hoshi and Kashyap (2008) for a discussion of the phenomenon in the Japanese context.

Related Literature Our paper relates to four broad strands of literature.

First, it builds on a theoretical literature in corporate finance that studies how debt overhang affects investment. Building on the seminal insight of Myers (1977), this literature has developed dynamic models in which debt in place can affect firms' decisions to undertake new investment.[10] Our model more specifically builds on the continuous-time framework of DeMarzo and He (2016) and Admati et al. (2018). In that model, the tax deductibility of debt interest expense incentivizes firms to take on leverage in the first place.[11] Due to a lack of commitment, firms' managers make strategic default decisions, as in Leland et al. (1994) and Leland and Toft (1996). We combine this framework with a standard investment problem, analogous to Hayashi (1982) and Abel and Eberly (1994). The resulting investment decisions are distorted downward by the presence of debt, leading to a debt overhang problem. Our framework allows for a continuous adjustment of both (long-term) leverage and investment; by contrast, much of the existing work focuses on models in which long-term debt is fixed (e.g. Moyen 2007), or the investment decision focuses on whether to exercise a growth option (e.g. Childs, Mauer and Ott 2005).[12]

Second, our work speaks to an empirical literature that studies the effects of leverage on investment decisions.[13] In Section 3.3, we compare the steady-sate implications of our model to the findings in this literature. However, our focus on this paper is not on steady-state costs of debt overhang, but on its dynamic effects on investment following government interventions. One of our key findings is that while steady-state agency costs can be large, they do not substantially amplify the response of firms to these interventions.

Third, our work relates to a theoretical literature that studies the effects of corporate or sovereign debt overhang on macroeconomic activity (Krugman et al., 1988; Lamont, 1995; Philippon, 2010), and how policy can best address this overhang (Philippon and Schnabl, 2013). Relative to that literature, our goal in this paper is to provide a framework that can be used to quantify more precisely the effects of corporate debt overhang and the various policies to address them. Importantly, we allow for cross-sectional heterogeneity in leverage, which is generated by idiosyncratic capital quality shocks, as in Khorrami and Tourre (2020). This heterogeneity is crucial to understanding why the interventions we consider have limited negative aggregate effects: these effects are concentrated among small, relatively leveraged firms in our model.

Finally, our paper adds to work on the recent corporate credit market interventions by the Federal Reserve and the US Treasury. Brunnermeier and Krishnamurthy (2020) emphasize qualitatively the trade-off between dead-weight losses of bankruptcy and debt overhang in a one-period model of the firm, and distinguish policy interventions aimed at large firms that maintain access to financial markets vs. interventions aimed at small, manager-operated businesses. Instead, we take a more quantitative

[10] See, among others, Mello and Parsons (1992); Mauer and Ott (2000); Moyen (2007); Titman and Tsyplakov (2007); Manso (2008), and Diamond and He (2014).

[11] As in that environment, firms end up taking so much leverage that the tax benefits of debt end up fully dissipated by bankruptcy costs, so that enterprise value becomes identical to that of a firm in financial autarky.

[12] Though these papers are not primarily focused on the question of debt overhang, the model we study has many common features with Hennessy and Whited (2005), and Hennessy and Whited (2007). The two main differences with the latter paper, in particular, are (a) our model has constant returns to scale in production, allowing for easier aggregation; and (b) we assume frictionless equity markets and a simpler corporate tax structure; (c) we allow for long-term debt, potentially magnifying the effects of debt overhang; and (d) we study a continuous-time framework, which makes the computation of equilibrium price functions for debt considerably simpler.

[13] See, among many others, Whited (1992), Opler and Titman (1994), Lang, Ofek and Stulz (1996), Aivazian, Ge and Qiu (2005), Matsa (2011), or Wittry (2020).

approach, study firms in an infinite horizon environment, and focus on aggregate economic outcomes in different capital market scenarios. Some of our conclusions also differ: in particular, we highlight the potential for lending programs to distort downward investment if the cost of credit is subsidized. Our work is also related to English and Liang (2020), who study the structure of the Main Street Lending Program and who argue for a better targeting of such program, longer loan terms, smaller minimum loan sizes, and stronger incentives for banks to participate in it. Our model speaks directly to the argument that the government credit programs should be aimed at high-leverage firms who are in need of funds in order to avoid liquidation during the crisis.

The rest of the paper is organized as follows. Section 2 provides a brief overview of the recent corporate credit interventions in the US. Section 3 presents our model, and compares its steady-state implications to data on publicly traded US firms. Section 4 uses the model to analyze the potential effects of the crisis and the subsequent credit policy interventions. Section 5 concludes.

2 Corporate credit policy intervention: a brief overview

In response to the onset of the pandemic, Treasury and the Federal Reserve have acted in close coordination in order to provide support to corporate credit markets. This support has been organized around three main programs: the Corporate Credit Facilities (CCF); the Main Street Lending Program (MSLP); and the Paycheck Protection Program (PPP).[14]

The Corporate Credit Facilities (CCF) were initially launched on March 23rd, 2020, under section 13(3) of the Federal Reserve Act, which allows the Federal Reserve to lend to corporations in "unusual exigent circumstances".[15] The goal of the two facilities — the Primary Credit Market Facility (PMCCF) and the Secondary Credit Market Facility (SMCCF) — is to support the functioning of corporate bond markets. The Primary facility's mandate is to purchase new issuances of bonds by firms which held investment grade ratings up to and including March 22nd, 2020, while the Secondary facility's mandate is to purchase existing investment-grade bonds in the secondary market, as well as investment-grade and high-yield corporate bond exchange-traded funds (ETFs). The total size of the two facilities has been capped at \$750bn, with a \$75bn equity contribution from Treasury.[16]

The Main Street Lending Program (MSLP) was announced by the Federal Reserve on April 9th, 2020. Like the CCF, the program derives its funding and its authority from both the CARES act and section 13(3) of the Federal Reserve act. The goal of the program is to offer loans to small and medium-sized firms. The program consists of different facilities aimed at either buying new loans or refinancing existing ones. This lending is to be underwritten by private banks, who must also retain a participation in the loans issued through the facilities. Participation is subject to certain limits: for instance, borrowers may not have more than 15,000 employees or \$5bn in 2019 revenue; their debt-to-EBITDA ratio cannot exceed

[14] Additionally, a number of other direct or indirect actions have been taken, in particular by the Federal Reserve, in order to support the flow of credit to firms. Among others, these include setting up the Commercial Paper Funding Facility (CPFF), setting up the Term Asset-Backed Securities Loan Facility (TALF), and bank supervisory actions in order to facilitate lending. The model we study in this paper is however not suited to studying these other interventions.

[15] They were subsequently expanded through additional Treasury funding authorized by the Coronavirus Aids, Relief and Economic Stimulus (CARES) act.

[16] Official information on the Corporate Credit Facilities is available here and here; see Boyarchenko et al. (2020) for an overview of the key features of the program.

certain thresholds; and the loans themselves must also conform to certain notional limits. Altogether, the facilities may buy up to \$600bn of loans, backed by a \$75bn equity contribution from Treasury.[17]

The MSLP and the CCF share some features. Both programs aim to provide new funding to firms, to be financed through specific Special Purpose Vehicles (SPVs) backed by contributions from the Treasury and the Federal Reserve. In both programs, the funding offered to firms takes the form of debt. Finally, both programs have limitations in place that imply that safer or less leveraged firms are more likely to be able to participate (directly or indirectly). There are also important differences between the two programs. The CCF is primarily targeted at firms with access to bond markets, a very small fraction of the population of firms, but a large portion of overall corporate assets and corporate investment.[18] The MSLP can potentially reach a larger population of small and medium-sized firms. While CCF purchases will be done at prevailing market prices, MSLP loans are to be offered to borrowers at a fixed spread (300bps) over LIBOR. Finally, one of the two facilities of the CCF is explicitly aimed at promoting secondary market liquidity, while the MSLP facilities are only concerned with primary market purchases.

The Paycheck Protection Program (PPP) is a Small Business Administration program funded by Treasury, with liquidity support from the Fed, through its Paychek Protection Program Liquidity Facility (PPPLF). The CARES act initially endowed the SBA with \$349bn to fund loans to businesses, eventually expanding the sum to \$669bn. The program is meant to provide small businesses loans which can be used by firms to cover interest, payroll, rent, and utilities. The loans have a maturity of up to five years and an interest rate of 1%; their amount is calculated on the basis of the payroll costs of the borrower. Importantly — and different from the CCF and the MSLP — these loans can be partially or totally forgiven, provided the borrower meets certain criteria, particularly regarding employee retention. The PPP thus has some features of a (conditional) transfer program, whereas the CCF and MSLP are explicitly structured as credit policy programs.[19]

It should be noted that these corporate credit policy interventions have recent precedents in other countries. Some examples include the Financing Facilitation Act of 2009 in Japan (Yamori, 2019), the ECB's Coporate Sector Purchase Programme (CSPP), launched in 2016 (Ertan, Kleymenova and Tuijn, 2019), or the UK's Corporate Bond Purchase Scheme (Belsham, Rattan and Maher, 2017; D'Amico and Kaminska, 2019). In the US, the Federal Reserve in fact used its 13(3) powers to lend to some non-financial firms in the 1930s (Rosa, 1947; Sastry, 2018). While our aim in this paper is to provide a framework to quantify the potential impacts of the recent US programs, insights from our analysis may be relevant to other contexts in which this economic policy tool may be used.

3 Model

In this section, we develop a partial equilibrium model of investment, financing and default. We first describe the model, then discuss its key economic mechanisms, and finally, we calibrate it to data on US public non-financial corporations.

[17] Official information on the MSLP is available here and here; see Crouzet and Gourio (2020) for a discussion of the program.

[18] See, e.g., Faulkender and Petersen (2006) or Crouzet and Mehrotra (2020).

[19] Official information on the PPP is available here.

3.1 Model description

We first describe an individual firm's problem, and then turn to aggregation of firm decisions.

3.1.1 Firm problem

The problem of an individual firm extends the framework studied in DeMarzo and He (2016) by allowing for a continuous choice of investment rates subject to convex capital adjustment costs. Shareholders of the firm take the real interest rate as given. Their bond issuance decisions are motivated by the tax deductibility of the debt interest expense. Their investment decisions are distorted downwards due to the presence of long term debt. Creditors providing financing to the firm are competitive, and price the debt issued by such firm risk-neutrally.

Technology The production technology of firm j yields revenue $y_t^{(j)} = a_t k_t^{(j)}$ per unit of time. $k_t^{(j)}$ represents efficiency units of capital of firm j, while a_t is a measure of productivity. While $k_t^{(j)}$ is specific to a firm, a_t is identical across firms. A firm has at its disposal an investment technology with adjustment costs, such that $\Phi(g_t^{(j)})k_t^{(j)}dt$ spent allows the firm to grow its capital stock by $g_t^{(j)}k_t^{(j)}dt$, where Φ is increasing and convex. $g_t^{(j)}$ represents the rate of growth of firm j's capital stock (or equivalently, the net investment rate), while $\Phi(g_t^{(j)})$ represents the investment-to-capital ratio (per unit of time). The efficiency units of capital then satisfy

$$dk_t^{(j)} = k_t^{(j)} \left(g_t^{(j)} dt + \sigma dZ_t^{(j)} \right)$$

$Z_t^{(j)}$ is a Brownian motion, representing idiosyncratic shocks hitting the production technology of the firm; the shocks are identically and independently distributed across firms. In our numerical calculations, we will assume that

$$\Phi(g) := \delta + g + \frac{\gamma}{2} g^2.$$

The parameter δ will govern capital depreciations, while γ will be a measure of capital adjustment costs.

Capital Structure The firm has access to debt and equity markets. Both markets are frictionless. We note $b_t^{(j)}$ the principal amount of the firm j's debt. Its tax liability between t and $t + dt$ is equal to

$$\Theta \left(a_t k_t^{(j)} - \kappa b_t^{(j)} \right) dt$$

In the above, κ is the coupon rate (assumed to be constant) on the bonds issued by the firm, while Θ is the corporate tax rate. The motive for the firm to take on debt stems from the tax deductibility of the debt interest expense. Shareholders cannot commit to always repaying the debt issued by the firm, which is thus credit-risky. Upon default, the entire capital stock of the firm is destroyed. When firm j issues \$1 face value of bonds, it raises proceeds equal to $d_t^{(j)}$, which represents the (endogenous) debt price of the firm (per unit of face value). The dividends paid to shareholders of firm j at any time are equal to:

$$\pi_t^{(j)}k_t^{(j)}dt := \left[a_tk_t^{(j)} - \Phi\left(g_t^{(j)}\right)k_t^{(j)} - (\kappa+m)\,b_t^{(j)} + \iota_t^{(j)}k_t^{(j)}d_t^{(j)} - \Theta\left(a_tk_t^{(j)} - \kappa b_t^{(j)}\right)\right]dt$$

In the above, $\iota_t^{(j)}k_t^{(j)}dt$ is the notional amount of bonds issued between t and $t+dt$ by firm j (and sold at a price $d_t^{(j)}$ per unit of face value). m is the speed of debt amortization. Negative dividends represent share issuances executed by the firm. In this setting, the firm cannot commit to a particular debt issuance policy. This means that the debt balance $b_t^{(j)}$ satisfies

$$db_t^{(j)} = \left(\iota_t^{(j)}k_t^{(j)} - mb_t^{(j)}\right)dt$$

Levered Firm Problem From now on, we omit the firm's superscript j for notational simplicity. Shareholder value E is defined via:

$$E_t(k,b) := \sup_{g,\iota,\tau_d}\mathbb{E}^{k,b,t}\left[\int_t^{\tau_d} e^{-r(s-t)}\pi_s k_s ds\right] \tag{1}$$

The price of one unit of debt is defined via:

$$D_t(k,b) := \mathbb{E}^{k,b,t}\left[\int_t^{\tau_d} e^{-(r+m)(s-t)}(\kappa+m)ds\right] \tag{2}$$

We focus on equilibrium outcomes in which the equity value E_t is homogeneous of degree one in (k,b), and the debt price function D_t is homogeneous of degree zero in (k,b). For $x := b/k$, we can thus write $D_t(k,b) = d_t(x) := D_t(1,x)$ and $E_t(k,b) = ke_t(x) := kE_t(1,x)$. To write the recursive system for e_t and d_t, we need to scale by k_t. We show in appendix (A.1.1) that the firm problem can be re-written

$$e_t(x) = \sup_{\tau_d,g,\iota}\tilde{\mathbb{E}}^{x,t}\left[\int_t^{\tau_d} e^{-\int_t^s (r-g_u)du}\pi_s ds\right] \tag{3}$$

$$dx_t = \left[\iota_t - (g_t+m)\,x_t\right]dt - \sigma x_t d\tilde{Z}_t \tag{4}$$

In the above, $\tilde{Z}_t$ is a Brownian motion that is related to Z_t via $\tilde{Z}_t = Z_t - \sigma t$. Similarly, creditors price the debt rationally, anticipating the financing, investment and default strategy of shareholders, discounting cash-flows at the constant real interest rate r. This means that the debt price satisfies

$$d_t(x) = \mathbb{E}^{x,t}\left[\int_t^{\tau_d} e^{-(r+m)(s-t)}(\kappa+m)ds\right] \tag{5}$$

$$dx_t = \left[\iota_t - \left(g_t+m-\sigma^2\right)x_t\right]dt - \sigma x_t dZ_t$$

Optimal investment, issuance and default policies will only depend on (t,x) in the Markov equilibrium we are interested in. Leverage x_t is then the unique firm-level state variable. Shareholders will find it optimal to default according to a cutoff policy in x_t:

$$\tau_d = \inf\{t \geq 0 : \bar{x}_t \leq x_t\}$$

When the firm is highly levered, it chooses to raise equity capital from shareholders. However, at the leverage boundary $\bar{x}_t$, shareholders will find it too costly to continue injecting equity capital into the firm and will instead let it default.

Equity and Debt Valuation Equations Equity and debt satisfy the following pair of Hamilton-Jacobi-Bellman (HJB) equations:

$$0 = \max_{\iota, g} \Big[-(r-g)e_t(x) + a_t - \Phi(g) - (\kappa + m)\, x + \iota d_t(x) - \Theta\,(a - \kappa x) \\ + \partial_t e_t(x) + [\iota - (g+m)\, x]\, \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \Big] \quad (6)$$

and

$$(r+m)d_t(x) = \kappa + m + \partial_t d_t(x) + \left[\iota_t(x) - \left(g_t(x) + m - \sigma^2\right) x\right] \partial_x d_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} d_t(x). \quad (7)$$

These equations are coupled differential equations for e_t and d_t, valid on $(0, \bar{x}_t)$. The Feynman-Kac differential equation for d_t uses the Markov growth policy $g_t(x)$ and the Markov issuance policy $\iota_t(x)$ that result from the shareholder optimization problem. The time-consistency problem faced by shareholders (unable to tie their hands and commit to a particular capital structure policy) will however simplify our analysis, and will eventually lead to a decoupling of this system of partial differential equations. The boundary conditions are the following value-matching conditions:

$$e_t(\bar{x}_t) = 0 \quad (8)$$

$$d_t(\bar{x}_t) = 0 \quad (9)$$

Equations (8)-(9) describe what happens at bankruptcy: both equity holders and debt holders receive nothing, since all the capital of the firm is destroyed at such time.[20]

Optimality conditions In the class of equilibria we focus on, the first order condition of the shareholders' problem with respect to debt issuances leads to a relationship between debt and equity prices:

$$d_t(x) + \partial_x e_t(x) = 0 \quad (10)$$

As discussed in DeMarzo and He (2016), and as shown in appendix (A.1.2), condition (10) can be used to show that, for any leverage ratio x, the value of equity is the same as that of a firm whose shareholders can commit never to issue bonds again. We also show that the resulting firm debt issuance policy takes the following form:

$$\iota_t(x) = \frac{\Theta \kappa}{-\partial_x d_t(x)} \quad (11)$$

[20] In our setup, we are assuming that all the firm's capital is lost in bankruptcy. We could have instead assumed that only a fraction of the capital stock is destroyed during the bankruptcy process, while the remaining capital stock is split between creditors and shareholders through renegotiations. This alternative setup would qualitatively not change the nature of the equilibrium we focus on.

The debt issuance intensity is increasing in the tax shield, and decreasing in the slope of the bond price function. The optimal capital growth rate $g_t(x)$ follows a standard q-theory optimal rule:

$$\Phi'\left(g_t(x)\right) = e_t(x) - x\partial_x e_t(x)$$

Finally, in our setup, the equity value of a firm with leverage $x = 0$ is exactly equal to the unlevered firm value per unit of capital e_t^*, defined via

$$e_t^* = \sup_g \mathbb{E}^t \left[\int_t^\infty e^{-r(s-t)du} \left((1-\theta)\, a_s - \Phi\left(g_s\right) \right) \frac{k_s}{k_0} ds \right] \tag{12}$$

If we note g_t^* the optimal no-debt growth policy, then our model is a model in which the firm's growth rate is always strictly less than the no-debt firm growth rate, in other words $g_t(x) \leq g_t^*$ at all time t and for all leverage x. In addition, since the equity value is convex (see appendix (A.1.3)), investment and growth are dampened by a debt overhang channel, since

$$\partial_x g_t(x) := \frac{-\partial_{xx} e_t(x)}{\Phi''\left(g_t(x)\right)} < 0$$

Another way to illustrate the magnitude of the debt overhang channel in our model is to compare the optimal capital growth rate $g_t(x)$ to the growth rate $\tilde{g}_t(x)$ that would be chosen by a managers maximizing enterprise value, rather than equity value. If we define enterprise value (per unit of capital) as $v_t(x) := \left(E_t + b_t D_t\right)/k_t$, then $\tilde{g}_t(x)$ satisfies

$$\Phi'\left(\tilde{g}_t(x)\right) = v_t(x) - x\partial_x v_t(x) = e_t(x) - x\partial_x e_t(x) + x^2\partial_{xx} e_t(x)$$

Once again, the convexity of e_t shows that the investment rate for an enterprise-value maximizing firm is greater than the investment rate chosen by an equity-optimizing firm. In our calibration, we will use both the no-leverage capital growth rate g_t^*, as well as the enterprise-value maximizing growth rate $\tilde{g}_t(x)$, as compared to the actual growth rate $g_t(x)$, in order to quantify the degree of debt overhang in our model. Finally, using equation (10), we can show that in our model, marginal and average q are the same. Indeed,

$$\partial_k E_t = e_t(x) - x\partial_x e_t(x) = e_t(x) + x d_t(x) = v_t(x)$$

Default optimality takes the form of a smooth-pasting condition:

$$\partial_x e_t(\bar{x}_t) = 0 \tag{13}$$

Using our functional form specification for the adjustment cost technology, this condition implies that at the default boundary, the firm's capital growth rate is the minimal feasible investment rate: $g_t(\bar{x}_t) = -1/\gamma$. Finally, in appendix (A.1.4), we highlight a general result of this class of problems: the default boundary $\bar{x}_t$ for our firm is always higher than the default boundary of a firm that always uses the no-leverage optimal investment rule g_t^*. This means that a highly levered firm will sacrifice investments and instead pay dividends (or reduce the intensity of its share issuances) and postpone default. This decision, while privately optimal, is obviously socially inefficient.

3.1.2 Aggregation and balanced growth

Let $K_t := \int_0^{J_t} k_t^{(j)} dj$ be the aggregate capital stock in our economy, with J_t the measure of firms at time t. Note $\omega_t^{(j)} := k_t^{(j)}/K_t$ the share of aggregate capital owned by a particular firm, and note that $\int_0^{J_t} \omega_t^{(j)} dj = 1$. Note $f_t(x,\omega)$ the time-t joint density over leverage and capital shares, and note that $J_t = \int f_t(x,\omega) dx d\omega$. We assume that new firms, with capital stock equal to the average capital stock in the economy, enter at rate $\hat{\lambda}_t^n$, which we will specify exogenously. The dynamics of the aggregate capital stock are as follows:

$$dK_t = K_t \left[\underbrace{\int_0^{J_t} \omega_t^{(j)} g_t\left(x_t^{(j)}\right) dj dt}_{=\hat{g}_t dt} + \underbrace{\int_0^{J_t} \sigma \omega_t^{(j)} dZ_t^{(j)} dj}_{=0} - \underbrace{\int_0^{J_t} \omega_t^{(j)} dN_t^{d,(j)} dj}_{=\hat{\lambda}_t^d dt} + \hat{\lambda}_t^n dt \right] \tag{14}$$

In equation (14), the law of large numbers allows us to simplify the capital growth equation since (a) the aggregation of idiosyncratic shocks does not contribute to aggregate growth, while (b) capital destructions through default contribute a locally deterministic term $-\hat{\lambda}_t^d dt$, representing the capital-share weighted default rate of the firms in our economy. The aggregate capital stock thus grows at a deterministic rate $\mu_{K,t} = \hat{g}_t - \hat{\lambda}_t^d + \hat{\lambda}_t^n$. In appendix (A.1.5), we show that the capital-share-weighted default rate $\hat{\lambda}_t^d$ and the capital-share-weighted growth rate $\hat{g}_t$ can be computed using moments of the density $\hat{f}_t(x) := \int_\omega \omega f_t(x,\omega) d\omega$, which represents the percentage of the total capital stock at firms with leverage x. This density satisfies a modified Kolmogorov forward equation — an integro-differential equation that describes the dynamic properties of such density as a function of policy decisions made by firms. The capital-share-weighted default rate $\hat{\lambda}_t^d$ and the capital-share-weighted growth rate $\hat{g}_t$ can then be computed via:

$$\hat{\lambda}_t^d = -\frac{1}{2}\sigma^2 \bar{x}_t^2 \partial_x \hat{f}_t(\bar{x}_t), \qquad \hat{g}_t = \int g_t(x) \hat{f}_t(x) dx.$$

In a balanced-growth path, the capital stock K_t, the number of firms J_t, the aggregate outstanding debt $B_t := \int_0^{J_t} b_t^{(j)} dj$, all grow at constant rates, whereas the capital growth rate $\mu_{K,t}$, the capital-weighted default rate $\hat{\lambda}_t^d$, the growth rate $\hat{g}_t$, the investment rate $\hat{\Phi}_t := \int \Phi(g_t(x)) \hat{f}_t(x) dx$, and the injection rate $\hat{\lambda}_t^n$ are all constant.

3.2 Discussion

We next briefly discuss some of the salient features of the model.

First, firms in the model take on leverage aggressively in order to monetize the deductibility of debt interest expenses. Their inability to commit to a future financing strategy is detrimental for the total enterprise value. In fact, firms leverage up to the point where future default costs exactly wipe out the tax benefits of debt, so that a firm that has no debt outstanding has an equity value exactly equal to that of a firm that can never take on any leverage. This result is the focus of DeMarzo and He (2016). It is specific to the continuous-time framework studied here, and does not hold exactly in its discrete-time analog. However, it can be thought of as a limiting case of the more general insight that shareholders' inability to commit not to issue more debt in the future is "self-defeating", in that it undermines their ability to benefit from the tax shield. This limiting case is useful for our purposes, because it ensures that firms in the model actively take on debt, thus giving debt overhang the best possible chance to matter for investment decisions. By contrast, a model with commitment, such as Leland et al. (1994), tends to under-predict leverage relative to the data.

Second, the debt taken by the firm depresses investment: shareholders are less inclined to invest when the firm is highly levered, since this reduces their current cash-flows and since some of the value stemming from the related decrease in leverage is captured by creditors via higher debt prices. This leads to a debt overhang channel: the investment rate of a given firm is always lower than the investment rate of an unlevered firm, and the investment rate is a decreasing function of leverage. This debt overhang channel will potentially be amplified by government interventions involving credit extension to businesses.

Third, our model also ignores certain margins of adjustment that might play a role in the analysis we provide in the following section:

(a) The model does not feature cash holdings: cash inflows received by a firm (and related to sales and proceeds from debt issuances) are distributed to creditors, used for investment, with the balance distributed as dividends (rather than potentially stored as cash reserves). This means that a sudden decline in aggregate productivity a_t, accompanied by a sudden stop in capital markets, cannot be mitigated by cash reserves held by a firm, potentially exacerbating the effects of the financial market shock. Our decision to abstract from liquidity reserves stems from our desire to keep the model tractable, with a unique state variable (leverage) driving the ex-post firm-level heterogeneity.[21]

(b) The model also assumes that creditors lose their entire investment once a firm files for bankruptcy. This assumption is justified by both (a) a technical consideration[22], and (b) an empirical consideration[23]. Our zero recovery rate assumption impacts negatively the price of debt, thus exacerbating debt overhang near the default boundary. While this assumption is important for firm-level decisions, it does not necessarily change aggregate outcomes to the same extent. Indeed, if needed, one could separate the private losses suffered by creditors from the social loss, by assuming for example that the bankruptcy costs pushing creditors' recovery rate to zero are not purely deadweight losses, but instead represent transfers to other stakeholders.

(c) Third, our production technology omits labor and instead focuses on capital. A straightforward extension, as discussed in Khorrami and Tourre (2020), can be made in order to include such

[21] See Anderson and Carverhill (2012) for a model with liquidity in addition to debt as a firm state variable.

[22] If creditors' recovery in bankruptcy was strictly positive, the price of the firm's debt near the default boundary would be strictly positive, incentivizing shareholders of the firm to use an arbitrarily large intensity of debt issuance to pay an arbitrarily large dividend rate just before default. This would result in a dilution of creditors' debt claim, pushing the price of the firm's debt to zero, thus contradicting the possibility of a strictly positive debt price near the default boundary. This means that a "smooth" equilibrium – in which the firm's face value process is absolutely continuous – does not exist in such case.

[23] As mentioned previously, one could model the debt recovery rate to be strictly positive, at the condition that the firm's liquidation value is split between creditors and shareholders, as in proposition VIII in DeMarzo and He (2016). It is however empirically counterfactual to observe shareholders receiving a strictly positive payout in bankruptcy when creditors suffer a loss, given the absolute priority rule of Chapter 11 of the bankruptcy code.

factor of production in a Cobb-Douglas specification; such extension would not change firms' investment, financing and default policies, and is thus omitted from our analysis. Relatedly, our model features idiosyncratic shocks only, and abstracts from aggregate shocks and risk-premia. As section (3.1.2) shows, this assumption yields a well-defined balanced growth path, from which we start our experiments when analyzing the impact of the pandemic shock on aggregate outcomes. We however discuss potential extensions with risk premia in Section 4.1.3.

3.3 Steady-state properties

Calibration Table 1 reports our baseline calibration of the model. Our model has eight parameters; we calibrate six of them, and target moments for the remaining two. Sources and values for the six calibrated parameters (δ, γ, Θ, κ, m, and r) are reported in Table 1. The remaining two parameters are a, the marginal product of capital, and σ, the volatility of the capital quality shock. We choose the former so that the gross investment rate for an equity-only firm is $\Phi(g^*) = 13\%$, which correspond to the top quartile of the distribution of gross investment rates in the sample of non-financial firms we study below. Finally, given all other parameters, we choose σ so that along the balanced growth path, the average debt-to-EBITDA ratio, weighted by EBITDA, is 2.70, in line with its value in the sample of non-financial firms we study below.[24]

Policy functions Figure (2) reports equity values and the dividend issuance policy as a function of the ratio of debt-to-EBITDA x/a, where recall that $x = b/k$ denotes leverage.[25] Dividends are a decreasing function of leverage: with low levels of debt, the firm pays large dividends, mostly financed by the proceeds from debt issuances. Instead, at higher leverage levels, firm cash-flows are depressed by (a) high debt servicing costs, (b) lower levels of debt issuances and (c) lower prices obtained for each dollar face amount of debt issued. Even if shareholders cut investments, this latter force is not sufficient to offset the former effects, leading to dividends being a downward sloping function of leverage. Eventually, with a sufficiently high leverage, dividends become negative — in other words, the firm issues new shares. Our assumption that equity markets are open in the balanced-growth path is crucial at that point — absent open capital markets, a firm would have to default at lower levels of leverage, as will be discussed in section (4).

Figure (3) shows debt prices (declining) and credit spreads (increasing) as a function of the ratio of debt-to-EBITDA. Credit spreads when $x/a = 0$ are strictly positive, since the bond holders of a firm that is barely indebted take into account the fact that the firm will be issuing large amounts of debt, thus increasing future leverage and future default risk.

Figure (4) shows the bond issuance and investment policies. With low leverage, the slope $\partial_x d_t(x)$ of the debt price function is close to zero, leading to very high intensities of debt issuances. Finally, the optimal growth policy is strictly decreasing; it is equal to $g^* = 2.8\%$ when $x = 0$, whereas it is equal to $-1/\gamma = -17\%$ at the default boundary $\bar{x} = 1.94$ (corresponding to a maximum debt-to-EBITDA ratio of $\bar{x}/a = 8.1$).

[24] We choose to target the debt-to-EBITDA ratio, rather than book leverage, because k, in our model, represents efficiency units of capital, which may not be properly captured by measures of the book value of firms' productive assets.

[25] In all the figures, the dotted black line is the ergodic share-weighted density $\hat{f}(x)$.

Model fit Table 2 summarizes the model-implied first moments in our model, against their empirical counterpart computed using US non-financial firms from Compustat. Sample selection and variable construction are described in Appendix A.2.1. Because some of the programs we consider are explicitly restricted to firms with an investment grade (IG) rating, we compare the model's implications to moments in both the overall Compustat sample, and the sample restricted to IG firms. These firms are generally less leveraged, have higher interest coverage ratios, invest less, and pay out fewer dividends than the rest of Compustat firms.

In Table 2, only the first line is targeted by our calibration. The other moments we report are however relatively well matched by the model. (Because of our focus on aggregate outcomes in the next section, we compute first moments by weighting them by firm-level EBITDA, though the model fit is similar if observations are weighted by total or by fixed book assets.) In particular, the model predicts a somewhat higher debt issuance rate, and a somewhat lower interest coverage ratio than and dividend issuance rate than in the data. Interestingly, the model also predicts a higher aggregate investment rate than in the data, suggesting that among the model's high-leverage firms, investment rates are *higher* than among their data counterparts.

Aside from these moments, our calibration leads to an ergodic average economy's growth rate of 0.9%, which can be broken down between (a) an average capital growth rate due to investments equal to 1.9% annum, and (b) a capital destruction rate due to defaults of 1% per annum. Firms' average credit spreads are very high, equal to 4.50%; this is due to the commitment problem faced by shareholders, which leads them to issue bonds at a very high intensity (per annum average gross debt issuance rate is approximately 9.0% of a firm's capital stock; if one takes firm's debt amortization into account, per annum average net debt issuance rate is approximately 2.6% of a firm's capital stock). Finally, the ergodic weighted average default rate in our model is equal to 1.1% per annum.

Figure 5 compares the model's higher moments with their data counterparts. In order to do this, for each value x/a of debt-to-EBTIDA, we compute the cumulative share of different variables of interest (assets, k_t; EBITDA, ak_t; gross investment, $\Phi(g_t)$; and dividends, $\pi_t k_t$) for firms with a level of EBITDA that is lower than or equal to x/a. We do this in both the model and the data (where we only use data from fiscal year 2018). The top left panel reports this empirical CDF for total book assets.[26] These empirical CDFs are not targeted in our calibration; generally, they tend to suggest that the model underpredicts cross-sectional dispersion in assets, earnings, and investment relative to the data. The bottom right panel reports the empirical CDF for dividend issuance. The model-implied CDF rises above 100% because some firms in the model issue negative dividends. The empirical CDF also rises above 100%, for the same reason, but this is not clearly visible in the data. The model thus tends to overpredict the frequency with which firms (particularly those with high leverage) use equity issuances as a way to smooth revenue and continue debt payments. As discussed in Section 3.2, the lack of equity issuance costs, as well as the lack of ability for firms to hoard liquidity, contributes to this implication of the model. In our view, while counterfactual, this implication of the model is useful, because it magnifies the real effects of credit market shutdowns, and thus provides a form of upper bound on what the effects of these shutdowns (and the benefits of credit interventions) might be.

[26] Interpreting total book assets, in the data, as the empirical counterpart to k_t, the CDF reported for the model is then exactly the ergodic share-weighted density $\hat{f}(x)$, expressed over levels of debt-to-EBITDA rather than levels of leverage x.

Comparison with existing literature We conclude by briefly discussing the strength of debt overhang effects in our baseline calibration of the model, and comparing it with the data.

The ergodic pseudo-elasticity of the gross investment rate to leverage (that is, the average slope of gross investment rates to leverage) is -0.029; the number is -0.031 for net investment. In other words, net investment, for instance, falls by 0.031 percentage point for each percent point increase in leverage. This slope captures the strength of debt overhang in our baseline calibration of the model. Analogs to this number have been computed by a number of empirical papers estimating the strength of debt overhang. For instance, Lang, Ofek and Stulz (1996) find that net investment falls by 0.105 p.p. for each p.p. increase in leverage (see their Table 3, column 1); Ahn, Denis and Denis (2006) finds estimates of this number ranging from 0.038 to 0.135; Cai and Zhang (2011) estimates this number to be 0.0375 (see their Table 5, column 2); and Wittry (2020) estimates this number to be 0.038 (see his Table 10, Column (2)). Our model thus produces estimates of these pseudo-elasticities that are close (if slightly lower than) those estimates by the literature on this topic.

Though these numbers may appear small, they have a large impact on the overall investment rate along the balanced growth path of our model. For instance, the all-equity firm growth rate, in this model, is 2.8%. Comparing this to the aggregate growth rate of the capital stock of 0.9%, mentioned above, this indicates that leverage and defaults in this environment depress growth rate by 1.9% per annum.

Another way of quantifying the effects of debt overhang in our model is to look at the slope of equity valuations with respect to leverage. In a sample of Canadian firms, Wittry (2020) finds that a one-standard deviation increase in (non-collateralizable) debt is associated with firms either foregoing or delaying projects. He estimates that the total value of these delayed or foregone investment opportunities to be 6.34% of total equity value. By contrast, in our model, equity value is 30.5% lower, whereas enterprise value is 6.4% lower, when evaluated at a leverage that is one standard deviation above the ergodic mean. Thus, this measure of debt overhang strength suggests that our model "overshoots" its empirical counterpart.

Finally, one can compare the ergodic enterprise value per unit of capital (Tobin's average Q), $\bar{v} := \int v(x)\hat{f}(x)dx$, to the un-levered enterprise value e^*. The former is 1.111, while the latter is 1.166, suggesting that steady-state costs of debt overhang represent approximately 4.7% of total firm value. This measure of debt overhang cost is, in magnitude, higher than those estimated by Moyen (2007), who finds costs of 0.5% (when the benchmark is the un-levered enterprise value) or 4.7%-5.1% (when the benchmark is the firm value under the assumption that managers make investment decisions maximizing enterprise value). It is however worthwhile pointing out that those latter estimates use a model that features decreasing returns to scale, persistent productivity shocks, and some degree of commitment over financing decisions.

4 Crisis and Government Interventions

We model the crisis as a sudden shock to the productivity parameter a_t, which falls by 25% on impact. Productivity stays at such low level for 6 months, and then recovers linearly over the next 6 months. We compute different aggregate equilibrium objects along this perfect foresight path, and focus on different government policy responses. We analyze two very distinct environments. In the first part of our analysis, we assume that financial markets are operating normally during the crisis period. In the next part of our analysis, we instead make the stark assumption that both credit and equity markets are closed, due to some un-modeled friction to capital markets. Our analysis of aggregate outcomes when focusing on these two extreme capital markets' environments can be thought of as bounds on the actual outcome, in a financial market environment where debt and equity markets have been stressed without being completely shut.

4.1 Well-Functioning Financial Markets

In this section, our assumption is that debt and equity markets continue to be frictionless and open.

4.1.1 Laissez-Faire Economy

In the laissez-faire economy, the government does not intervene. With a surprise downward shock to productivity, the default rate increases dramatically, and reaches 7% per annum (note however that this is a flow rate, and the stock of firms defaulting during the crisis is a lot smaller than 7%). Firms deleverage slightly during the crisis, and the economy's pre-default growth rate is about 0.50% per annum lower than it would be without such shock. Firms end up financing their growth via share issuances, as illustrated in Figure (6b). 3 years after the shock, the capital stock ends up only 0.3% below its level without the shock.

4.1.2 Government Interventions: Some Irrelevance Results

We now study an environment where the government provides emergency funding to businesses during the crisis, and obtains either (a) loans or (b) equity stakes from companies receiving such funding. Crucially, we assume that the funding provided by the government is "at market rates", meaning that the market value of securities received by the government from a given firm is equal to the amount of funding provided by the government to such firm. In such circumstances, when debt and equity markets are operating without friction during the crisis, we show that such policies have no effects, as firms adjust their capital structure policy so as to "undo" the government intervention.

We first study a government that decides to provide emergency funding to businesses, in exchange for bonds that the firm would be issuing to the government. During the crisis, between t and $t + dt$, we assume that the government advances $s_t(x_t)k_t dt$ to a firm that has capital k_t. In exchange, the government obtains bonds issued by such firm, with balance $\iota_t^g(x_t)k_t dt = s_t(x)k_t dt/d_t(x_t)$. This formulation thus assumes that the intervention of the government can be conditioned on the leverage of the firm. The equity value for a given firm then satisfies the following Hamilton-Jacobi-Bellman equation:

$$0 = \max_{\iota,g} \Big[-(r-g)e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m)x + \iota d_t(x) - \Theta(a_t - \kappa x) \\ + \partial_t e_t(x) + \left[\iota_t^g(x) + \iota - (g+m)x\right]\partial_x e_t(x) + \frac{\sigma^2}{2}x^2 \partial_{xx} e_t(x)\Big] \quad (15)$$

Flow profits (per unit of capital) are thus increased by $s_t(x)$, while the drift rate of leverage is increased by the debt (per unit of capital) ι_t^g issued to the government. One can use the identity $\iota_t^g(x) = s_t(x)/d_t(x)$

to obtain

$$0 = \max_{\iota, g} \Big[-(r-g)e_t(x) + a_t - \Phi(g) - (\kappa + m)\,x + \left(\iota_t^g(x) + \iota\right) d_t(x) - \Theta\left(a_t - \kappa x\right) + \partial_t e_t(x) + \left[\iota_t^g(x) + \iota - (g+m)\,x\right] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \Big] \quad (16)$$

This formulation of equity holder's problem makes it clear that equation (16) is identical to equation (6). Thus, with this policy intervention financed by debt priced at market levels, the outcome is identical to the outcome in a laissez-faire environment. The debt issuance policy ι used by the firm to finance itself in private markets is tilted downwards, but the total debt issuance (public and private) is identical to the issuance in the laissez-faire environment. This result crucially relies on the assumption that the government funding comes at a price that is identical to what private markets would charge. It also relies on the assumption that the firm cannot commit to a particular debt capital structure policy and can adjust its issuances instantaneously and at no cost.

Imagine finally that the government decides to undertake a policy intervention under which (a) the government advances money to each firm, in exchange for (b) equity that the firm would be issuing to the government. Between t and $t + dt$, the government advances $s_t(x_t)k_t dt$ to a firm that has capital k_t and leverage x_t. In exchange, the government obtains equity issued by such firm; the number of shares the government needs to receive is equal to $\lambda_t(x_t)dt := s_t(x_t)dt/e_t(x_t)$, so that the government is "fairly" compensated for this cash advance. The equity value for a given firm then satisfies the following Hamilton-Jacobi-Bellman equation:

$$0 = \max_{\iota, g} \Big[-\left(r + \lambda_t(x) - g\right) e_t(x) + a_t + s_t(x) - \Phi(g) - (\kappa + m)\,x + \iota d_t(x) - \Theta\left(a_t - \kappa x\right) + \partial_t e_t(x) + \left[\iota - (g+m)\,x\right] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \Big] \quad (17)$$

We can then replace $\lambda_t(x)$ by its value to obtain exactly equation (6). In other words, when the government obtains an equity stake upon providing funding, it does not alter the equilibrium outcome through the crisis. Once again, providing such funding at market terms has no effect on outcomes. These results are summarized in the lemma below.

Lemma 1 *Suppose that financial markets continue to operate without any friction during the crisis period. Any government intervention financed (whether conditional on firm's leverage, or conditional on time, or unconditional) at market interest rate (whether financed by debt, equity claims, or even hybrid instruments) has no impact on aggregate investment and growth rates.*

4.1.3 Government Interventions: Debt Funding at Subsidized Rates

Imagine now that the government provides debt funding at an interest rate that is below the market rate. Let $\alpha_t > 0$ be the wedge between the interest rate at which private markets provide debt funding and where the government is accepting to provide such funding. For our numerical calculations, we will assume that at the start of the crisis, the government provides the funding at 1% below market rates, and that such subsidy then declines over time, as the crisis subsides. In this alternative setup, one can show that shareholders solve a problem identical to HJB equation (6), but in which the debt price obtained when issuing debt is calculated using the lower discount rate $r - \alpha_t$:

$$d_t(x) = \mathbb{E}^{t,x}\left[\int_t^{\tau_d} e^{-\int_t^s (r-\alpha_u+m)du}\,(\kappa+m)\,ds\right]$$

Thus, in this environment, the firm's investment and default policies are identical to those in the laissez-faire economy in which the government does not intervene. The improved debt pricing incentivizes firms to issue more debt, increasing leverage, and thus affecting aggregate investment rates and aggregate growth rates. The firm's optimal debt issuance policy is equal to

$$\iota_t(x) = \frac{\Theta\kappa + \alpha_t d_t(x)}{-\partial_x d_t(x)} \tag{18}$$

In other words, the subsidy α_t provides an incentive for firms to take on more debt, to increase leverage, which will exacerbate debt overhang. Note that this is not merely a numerical result, but a theoretical one: indeed, with a firm-level investment and default policy that is identical to the case where no subsidized funding is provided, the only impact of the funding subsidy is to increase the drift rate of leverage, pushing the cross-sectional leverage distribution towards higher leverage levels, thus decreasing aggregate investment.

Lemma 2 *Suppose that financial markets continue to operate without any friction during the crisis period. Any government intervention financed (whether conditional on firm's leverage, or conditional on time, or unconditional) by debt contracts priced at below market interest rates will lead to lower aggregate investment and growth rates at all time $t \geq 0$.*

Figure (7a) shows the time-path of the funding subsidy. Figure (7d) illustrates the fact that firms, confronted with debt funding at subsidized rates, increase debt issuance, resulting in higher aggregate leverage, as indicated in Figure (7c).

The analysis above assumes that the government intervention takes the form of providing financing at an interest rate $r - \alpha_t$ that is below the market interest rate r. One might argue that this modelling assumption is not entirely consistent with policy interventions such as the Primary or Secondary Corporate Credit Facilities — interventions through which the Federal Reserve purchases either primary or secondary bond offerings at market prices. Instead, one could have taken a modelling approach similar to that in DeMarzo, He and Tourre (2019), who specify exogenously the stochastic discount factor pricing the debt claims issued by firms. In that framework, a firm's capital stock is not only exposed to idiosyncratic shocks, but also to an aggregate shock that is priced – in other words, investors need to get paid an expected excess return to be exposed to price fluctuations of stocks and bonds. While this approach is not consistent with our aggregation analysis (which assumes that firms are only exposed to idiosyncratic shocks), it is nonetheless informative to think about firms' reactions to a sudden decrease in the price of risk — the most natural reduced form strategy to model the announcement by the Federal Reserve of the Corporate Credit Facilities.

In such environment, two cases need to be considered. First, if credit and equity markets are integrated, let π_t be the price of risk associated with the aggregate shock, and let ρ be the local correlation between the aggregate shock and a firm's capital quality shock. Shareholders are pricing their residual cash-flows under the risk-neutral measure, under which the dynamic evolution of a firm's leverage is corrected upwards by a drift adjustment $+\rho\sigma\pi_t x$. Thus, a reduction in risk prices triggered by a government policy announcement lowers the drift rate of leverage under the risk-neutral measure, leading to an increase in equity values e_t and the marginal value of capital $\partial_k E_t$, and thus an upward jump in investment rates. The financing policy of the firm is still driven by equation (11), and is only impacted by such change in risk price via the dependence of the issuance rate on the term $-\partial_x d_t(x) = \partial_{xx} e_t(x)$. Consider instead a situation where credit and equity markets are segmented, and note $\pi_{e,t}$ (resp. $\pi_{d,t}$) the Sharpe ratio of the aggregate shock in equity markets (resp. in credit markets). Once again, a decrease in equity markets' risk price $\pi_{e,t}$ does lead to an increase the marginal value of capital $\partial_k E_t$, and thus to an increase in investment on impact. However, the financing policy of the borrower becomes

$$\iota_t(x) = \frac{\Theta\kappa}{-\partial_x d_t(x)} - \rho\left(\pi_{d,t} - \pi_{e,t}\right)\sigma x \tag{19}$$

Thus, if the reduction in debt markets' risk price induced by the government intervention is not fully accompanied by a similar reduction in the equity markets' risk price, the firm increases its bond issuances, leading to a future deterioration of corporate leverage and thus a future reduction in investment and the economy's growth rate. Note also that the firms that are mostly levered (with high x) are those whose bond issuance rate increases the most — which are those exact firms suffering the most from debt overhang. Those results are summarized in the lemma below.

Lemma 3 *Suppose that financial markets continue to operate without any friction during the crisis period. When credit and equity markets are integrated, a decrease in the (common) risk price induced by a government announcement will, on impact, increase the marginal value of capital and investment. When credit and equity markets are segmented, if the decrease in credit market risk price induced by a policy announcement is larger in magnitude than the related decrease in equity market risk price, investment rates jump up on impact, but firms take on more leverage in the future, pushing down future investment via debt overhang effects.*

4.2 Financial Markets Shut-down

Government interventions either have neutral or negative consequences on aggregate investment and growth when financial markets are operating smoothly. However, as soon as frictions to credit and equity markets arise, an economic shock consistent with what we are studying can lead to a wave of bankruptcies and prolonged declines in investment and the level of the capital stock. In order to illustrate this, we now study an environment in which both credit and equity markets are completely shut down during the crisis period. Concretely, it means that firm's investment policies are constrained, as dividends must remain weakly positive. We will first study the environment where the government does not intervene in financial markets, and will then analyze what happens when the government provides debt or equity funding at such time.

4.2.1 No Government Intervention

When the government does not intervene, firm's investment policy is constrained as follows:

$$g_t \leq \bar{g}_t(x)$$

$$\bar{g}_t(x) := \Phi^{-1}\left(a_t - (\kappa + m)x - \Theta\left(a_t - \kappa x\right)\right)$$

Since Φ is strictly increasing on $[-1/\gamma, +\infty]$, the greater the leverage of the firm, the lower the bound $\bar{g}_t(x)$ on the capital growth rate. As $\bar{g}_t(x)$ reaches the capital growth lower bound $-1/\gamma$, the firm can no longer desinvest at a rate that is sufficiently high to avoid raising capital in financial markets, and since markets are shut during the crisis, firms at such level of indebtedness have to default. The HJB equation satisfied by the equity value $e_t(x)$ is then

$$0 = \max_{g \leq \bar{g}_t(x)} \Big[-(r-g)\, e_t(x) + a_t - \Phi\left(g\right) - (\kappa + m)\, x - \Theta\left(a_t - \kappa x\right) + \partial_t e_t(x) - (g+m)\, x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \Big] \quad (20)$$

This equation holds for $x \leq \bar{x}_t$, for which an expression is available in such case:

$$\bar{x}_t := \frac{(1-\Theta)a_t - \Phi\left(-1/\gamma\right)}{(1-\Theta)\kappa + m}$$

Finally, at the time the shock happens and financial markets shut down, a positive measure of firms defaults, and a fraction $1 - \hat{F}_{0-}(\bar{x}_0)$ of the capital stock is destroyed. This corresponds to the fraction of capital held by firms who are issuing shares on our balanced-growth path.

Figure (8) shows the time path of the capital stock and of the aggregate investment rate either (a) in the balanced-growth path, (b) through the crisis with financial markets functioning, and (c) through the crisis when financial markets are closed. The closure of financial markets has a dramatic impact on macroeconomic outcomes. As the firm default boundary suddenly shifts to a lower debt-to-EBIDTA level, a non-zero measure of firms (approximately 22% in our numerical calculations) defaults, and the aggregate investment rate during the sudden stop is severely reduced, as firms can no longer finance their investment with bond or equity issuances. Investment recovers dramatically as markets re-open, and the investment rate ends up above its steady state value, as the surviving firms, at that point, are less levered than in the balanced-growth path.

4.2.2 Emergency Debt Funding

We next turn to an environment that closely mimics the various credit market interventions implemented by the Federal Reserve and the US Treasury in response to the crisis. Specifically, we analyze a policy intervention according to which the government advances cashflows $s_t(x_t)k_t dt$ to any firm with capital k_t and leverage x_t during the crisis period. In return for such cash advance, the government receives $\nu_d(x_t)s_t(x)k_t dt$ principal amount of debt issued by such firm. Consider for example the case $\nu_d(x) = 1$ for all x. In such example, the government funding is subsidized given that the firm's debt always trades at a discount to par; since the debt price is decreasing in leverage, the more levered the firm, the greater the subsidy provided by the government. In the general case, the value of the subsidy is equal to

$$s_t(x_t)k_t\left(1 - \nu_d(x_t)d_t(x_t)\right)dt$$

In this environment, investment is still constrained, but such constraint is relaxed when compared to the case where the government does not intervene. The investment rate must satisfy

$$
\begin{aligned}
g_t &\leq \tilde{g}_t(x) \\
\tilde{g}_t(x) &:= \Phi^{-1}\left(a_t + s_t(x) - (\kappa + m)x - \Theta\left(a_t - \kappa x\right)\right)
\end{aligned}
$$

This means that the firm's problem becomes

$$
\begin{aligned}
0 = \max_{g \leq \tilde{g}_t(x)} \Big[&- (r-g)\, e_t(x) + a_t + s_t(x) - \Phi\left(g\right) - (\kappa + m)\, x - \Theta\left(a_t - \kappa x\right) \\
&+ \partial_t e_t(x) + \left[s_t(x)\nu_d(x) - (g+m)\, x\right] \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \Big] \quad (21)
\end{aligned}
$$

The value of the subsidy provided by the government to a firm with leverage x and capital k_0 at time zero is then

$$
S_0^d(x)k_0 := \mathbb{E}^{x,0}\left[\int_0^{\tau_d} e^{-\int_0^t r_s ds} s_t(x) k_t \left(1 - \nu_d(x_t) d_t(x_t)\right) dt\right]
$$

Appendix (A.1.6) discusses how to compute the value of such subsidy. The policy intervention represents a transfer to firms equal to $K_0 \int S_0^d(x)\hat{f}_0(x)dx$ in market value terms. In our numerical calculations, we assume that $s_t(x) = a_\infty - a_t$, so that the government is advancing money to firms in proportion to their revenue decline. We assume $\nu_d(x) = 1$, meaning that the government is effectively funding at par loans that are in fact worth less than par.

As figure (9) indicates, the initial capital destruction that follows the shock is less severe than in the laissez-faire environment, as the government funding allows highly-levered firms to continue operating without filing for bankruptcy. Investment is depressed (given that growth can no longer be financed via debt or equity issuances) but not as severely as in the laissez-faire environment, since the firm's current cashflows, boosted by the government funding, allow it to finance corporate investments at a higher rate. Debt overhang is present during the recovery – after 6 months, as financial markets reopen, firm's investment rate is lower than in the laissez-faire economy, as surviving firms have taken more debt with the government intervention. The debt overhang effect is however limited: the investment rate with government-extended debt funding is less than 0.50% lower than in the laissez-faire environment, and even if the debt-overhang is persistent, such persistence is not sufficient to overcome the level effect arising from the initial bankruptcies and related capital losses.

In our modelled policy intervention, the government extends $(a_\infty - a_t)k_t dt$ amount of funding in exchange for the same principal balance of a firm's debt. While such policy intervention scales with firm's size, it is independent of firm's leverage. It is thus not exactly consistent with the programs implemented by the Federal Reserve and the US Treasury. For one, the Corporate Credit Facilities are focused on investment grade corporate bonds[27], thus excluding high-yield bond issuers. Second, under the Main Street Lending Program, firms can receive funding priced at Libor + 300bps, so long as they have a pre-crisis debt-to-EBITDA ratio of 4x or below[28], and so long as a bank is willing to

[27] Both the Primary Market and the Secondary Market Corporate Credit Facilties' eligibility criteria specify that issuers must be rated at least BBB-/Baa3 as of the beginning of the crisis.

[28] Note that the debt-to-EBITDA limit of 4x is relevant for both the New Loan Facility and the Expanded Loan Facility of

extend funding at such price[29]. However, empirically as well as in our modelled environment, firms that have modest leverage when entering the crisis do not risk immediate default, and are less in need of emergency funding than firms entering the crisis with high levels of debt. Thus, both the CCF and MSLP target exactly those firms that need emergency funding the least. This suggests an alternative design for such credit market intervention, with identical (or lower) fiscal costs, but with the benefit of preventing the failure of a substantial fraction of highly leveraged firms that end up in need but not able to access liquidity during the crisis. Such alternative intervention would target highly leveraged firms, by setting $s_t(x) = \mathbb{I}_{\{x_t \geq \tilde{x}\}} \bar{s}_t$, for some carefully chosen leverage hurdle $\tilde{x}$ and some carefully chosen cash advance (per unit of capital) $\bar{s}_t$. Firms with leverage ratios lower than $\tilde{x}$ would not receive any government-provided funding, reducing the overall fiscal cost of the intervention for the government, or allowing the government to redeploy such savings towards the highly levered firms that need it the most. Such a targeted policy would be relatively straightforward to implement, as it relies on measures of firm leverage that are readily available in accounting data.

4.2.3 Other Government Interventions

In the previous section, the government injects funds into firms in exchange for debt claims, priced above market. While such intervention reduces the magnitude of the initial wave of defaults and its related dead-weight losses, surviving firms end up taking on more debt, which depresses investment during the market shut-down and even after markets reopen. To mitigate the resulting debt overhang effect, one could instead design a policy with identical fiscal costs, but according to which the government obtains shares of firms receiving such emergency funding.

Concretely, imagine that the government advances cashflows $s_t(x_t)k_t dt$ to any firm with capital k_t and leverage x_t during the crisis period, in return for receiving $\nu_e(x_t)s_t(x)k_t dt$ shares issued by such firm. The subsidy implicit in that scheme is equal to

$$s_t(x_t)k_t \left(1 - \nu_e(x_t)e_t(x_t)\right) dt$$

In this environment, investment is still constrained, but such constraint is relaxed in a way similar to what we described in section (4.2.2). One can compute the optimal default, investment and leverage policy of an individual firm, aggregate those decisions, and look at the resulting equilibrium outcomes, as suggested in Appendix (A.1.7). We continue to assume that the government advances $s_t(x) = a_\infty - a_t$, but in exchange for ν_e shares, where ν_e is sized so that the fiscal cost of the government intervention is identical to the fiscal cost computed for the debt intervention described in section (4.2.2).

The initial wave of bankruptcies is identical to the scenario where the government injects debt (instead of equity) funding, as illustrated in Figure (10). Firms however are less saddled by debt, and thus have higher investment rates than in connection with government-provided debt funding. However, the impact of such equity capital injection into the economy seems to provide limited additional benefits, in

the Main Street Lending Program. Instead, the Priority Loan Facility targets more risky borrowers and features a maximum debt-to-EBITDA ratio of 6x.

[29] Indeed, while the Main Street Lending Program is administered by the Federal Reserve Bank of Boston, US depository institutions are responsible for underwriting and originating the loans under such program, in exchange for origination fees, servicing fees, but also conditional on keeping 5% of the loan originated. This means that banks will only originate loans for which the market spread of borrowers is below L+300bps.

comparison to the credit market intervention: capital growth rates during the market freeze and after markets reopen differ by less than 25bps. This benefit has to be weighed against the un-modelled costs of such intervention: imposing that businesses "give up" equity stakes in exchange for funds can make firms' managers reluctant to accept such funds (since they would have to cease some control of the firm to a potential activist government), and renders its practical implementation more complex and potentially slower to execute, delaying the intervention at a critical time when firms need funds the most to avoid bankruptcy.

Instead of equity injections, the government could also consider broad-based debt forbearance policies, under which firms are allowed to delay their debt interest payments. Such debt interest payments are then capitalized, and remain payable as markets reopen when the crisis subsides. In other words, while markets are shut down, the debt balance of a given firm j evolves as follows

$$db_t^{(j)} = \left(\iota_t^{(j)} k_t^{(j)} - m b_t^{(j)} + \kappa b_t^{(j)}\right) dt$$

Instead, the firm's dividends (per unit of capital and per unit of time) are equal to

$$\pi_t^{(j)} = (1-\Theta) a_t - \Phi\left(g_t^{(j)}\right) - m x_t^{(j)}$$

This intervention has been used in the US mortgage market as a tool to mitigate the impact of the crisis on households. Under the CARES act, home mortgages that were purchased by one of the mortgage agencies after origination (i.e. close to 2/3 of outstanding residential mortgages in the US) can benefit from a temporary suspension of their required monthly payments. Note however that those payments are not forgiven; instead, they are owed later on, either as a lump-sum payment or smoothed over a certain time period. Such policy imposes potential costs onto creditors: since the unpaid interest gets capitalized, (a) creditors do not receive current interest on their debt, and (b) firm's leverage ends up being pushed higher, increasing the firm's default probability – *keeping constant the firm's default barrier*. This intervention however has the benefit of pushing the firms' default barrier to higher debt-to-EBITDA levels $\hat{x}_t$, since

$$\hat{x}_t = \frac{(1-\Theta)a_t - \Phi\left(-1/\gamma\right)}{m} > \frac{(1-\Theta)a_t - \Phi\left(-1/\gamma\right)}{(1-\Theta)\kappa + m} = \bar{x}_t$$

This means that some of the firms that would otherwise have defaulted at time zero – i.e. firms with $x_0 \in (\bar{x}_0, \hat{x}_0)$ – are able to survive the initial shock, and bond holders for those firms will benefit from the forbearance policy. This is not necessarily the case for less levered firms, for which a forbearance policy might reduce the price of a firm's legacy debt, compared to the counter-factual scenario where debt interest payments are made on a timely basis. Thus, the government on its own cannot mandate negatively impacted creditors to accept such debt forbearance policy without compensating them. In Appendix (A.1.7), we illustrate how to compute the fiscal cost of such policy intervention. Figure (10) illustrates the effect of such debt forbearance policy on aggregate outcomes. The impact of the pandemic shock, combined with the sudden stop in financial markets, ends up being dampened by the debt forbearance policy, since firms can save on current cashflows to stave off bankruptcy. The benefits of this intervention are however not as significant as those of the credit or equity market interventions, since the fiscal cost of such intervention are much more modest.

4.2.4 Welfare Comparisons

In order to compare the effect of the different policy interventions on aggregate outcomes, we use a simple welfare criterion, based on the net present value of aggregate consumption in our economy. Under the assumption that all debt and equity claims issued by firms belong to a representative household, and based on the assumption that tax payments made by firms are rebated to households, such household ends up consuming the aggregate output of the firms, minus the aggregate investments made by such firms:

$$C_t := \int_0^{J_t} a_t k_t^{(j)} dj - \int_0^{J_t} \Phi\left(x_t^{(j)}\right) k_t^{(j)} dj = \left(a_t - \hat{\Phi}_t\right) K_t,$$

where we have used $\hat{\Phi}_t := \int \Phi(x) \hat{f}_t(x) dx$ as the capital-share weighted investment-to-capital ratio. We compare our policy interventions using the statistic

$$W_0 := \int_0^{+\infty} e^{-rt} \left(a_t - \hat{\Phi}_t\right) K_t dt$$

Table (3) summarizes our results. As discussed previously, a policy focused on equity injections, fixing the fiscal cost of such intervention, dominates a policy focused on debt funding, as the former policy minimizes the investment distortions stemming from the debt overhang channel. Our welfare calculations however do not take into account the complexity and potential delays that would be almost unavoidable when implementing broad equity injections into US businesses. Similarly, loan forbearance, while improving welfare compared to the "laissez-faire" environment, is a less potent policy, compared to a credit market intervention. Moreover, such intervention involves subsidizing certain debt holders, while penalizing others – hence such policy is only implementable to the extent the government can compensate those creditors being hurt by the intervention. This latter consideration makes such policy less attractive when thinking about the US corporate sector than when thinking about the household sector, where a majority of US mortgage debt is guaranteed by the mortgage agencies – and thus, indirectly, by the federal government.

5 Conclusion

In response to the ongoing pandemic, the US Treasury and the Federal Reserve launched a series of interventions in corporate credit markets, with the goal of making funding widely available to firms for the duration of the crisis. These interventions involve lending to firms, at rates that are either market-determined or below market rates. These interventions raise two questions. First, what is their net economic impact likely to be, for participating firms? Second, would there be gains from structuring these interventions differently?

In this paper, we studied this question through the lens of a structural model that emphasizes one main potential cost of corporate credit interventions: they might depress investment during the recovery, because of increased debt overhang. Our two main findings are as follows. First, if financial (debt and equity) markets continue to function normally during the crisis, the net effect of these interventions is either zero (if debt is offered at market rates), or negative (if debt is offered at below-market rates), though the negative effect in the latter case is small. Second, if debt and equity markets are closed during the crisis, the interventions have large and positive effects. Though they do depress aggregate investment during the recovery, quantitatively, the effect is an order of magnitude smaller than their benefit, which is to avoid a wave of bankruptcies during the crisis. Because debt overhang effects during the recovery are small, the benefits from alternative policy designs, such as equity injections, are also small. While these conclusions apply to the policy interventions we consider, they speak more generally to the potential debt overhang costs of credit support programs during recessions — a policy tool that is becoming increasingly popular.

There are two main reasons why our conclusions regarding the relatively small effects of the debt overhang channel could be challenged. First, they hinge on a particular value of the elasticity of investment rates to leverage. Though our model implies elasiticities that are in line with those documented in the literature, the effects of the channel we consider would be amplified if this elasticity were in fact larger either on average or during downturns. Second, our conclusions also hinge on the fact that the crisis, in the model, triggers a large wave of exit, because of the lack of external funding (absent government interventions). Alternative versions of the model, with more moderate disruptions to financial markets during the crisis, would lead to a higher relative importance of the debt overhang channel. We leave these two questions for future research.

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Parameter	Value	Description	Target/source
δ	0.10	Depreciation rate	Average economic depreciation rate (BEA, 2020)
γ	6	Adjustment cost	Belo et al. (2019)
Θ	0.35	Corporate tax rate	Statutory rate pre-2017 (OECD, 2020)
κ	0.05	Debt coupon rate	Maximum bond price = par
$1/m$	10	Average debt maturity	Saretto and Tookes (2013)
r	0.05	Real rate	Leland et al. (1994)
a	0.24	Productivity	Optimal unconstrained gross investment rate = 13%
σ	0.30	TFP volatility	Weighted average Debt-to-EBITDA ratio = 2.70

Table 1: Calibration of the model of Section 3. Where applicable, the parameter values reported are annual. The rate of depreciation from the BEA fixed asset tables (BEA, 2020) is an average between the depreciation rate for physical assets and intellectual property products, weighted by the BEA's estimates of replacement costs. Additionally, we use the pre-2017 statutory corporate income tax rate, and we choose a coupon rate such that the maximum possible price of debt is par. The weighted average debt-to-ebitda ratio os obtained from the sample of Compustat firms used to compute the moments reported in Table 2 and described in Appendix A.2.1. The target for the optimal unconstrained gross investment rate is the top quartile of investment rates of firms in that sample.

		Model	Compustat (all firms)	Compustat (IG firms)
		Average, weighted by EBITDA		
Debt-to-EBITDA	$b_t/(ak_t)$	2.69	2.71	2.29
1/(Interest coverage ratio)	$\kappa b_t/(ak_t)$	0.135	0.116	0.084
Gross investment rate	$\Phi(g_t)$	0.121	0.095	0.089
Dividend rate	π_t	0.026	0.033	0.046
Gross debt issuance rate	ι_t	0.097	0.078	0.060
Net debt issuance rate	$\iota_t - mx_t$	0.026	0.010	0.008

Table 2: Model-implied vs. empirical first moments. The calibration used to construct these moments is described in Table 1. Appendix A.2.1 describes the sample selection and variable construction of the different moments reported in this table. The data is for Compustat firms incorporated in the US, excluding financials, utilies, and multinationals; all moments are weighted by firm EBITDA.

	$\frac{W_0}{W_0^{\textbf{laissez-faire}}}$	$\frac{W_0}{W_0^{\textbf{no-shock}}}$
laissez-faire	1	0.59
new loans	1.43	0.85
loan forbearance	1.36	0.81
equity injections	1.46	0.86

Table 3: Welfare calculations for different policy interventions. The first column compares the welfare of the particular policy intervention to the welfare in the "laissez-faire" environment (which assumes that financial markets are closed). The second column compares the welfare of those same policies to the welfare in the balance-growth path.

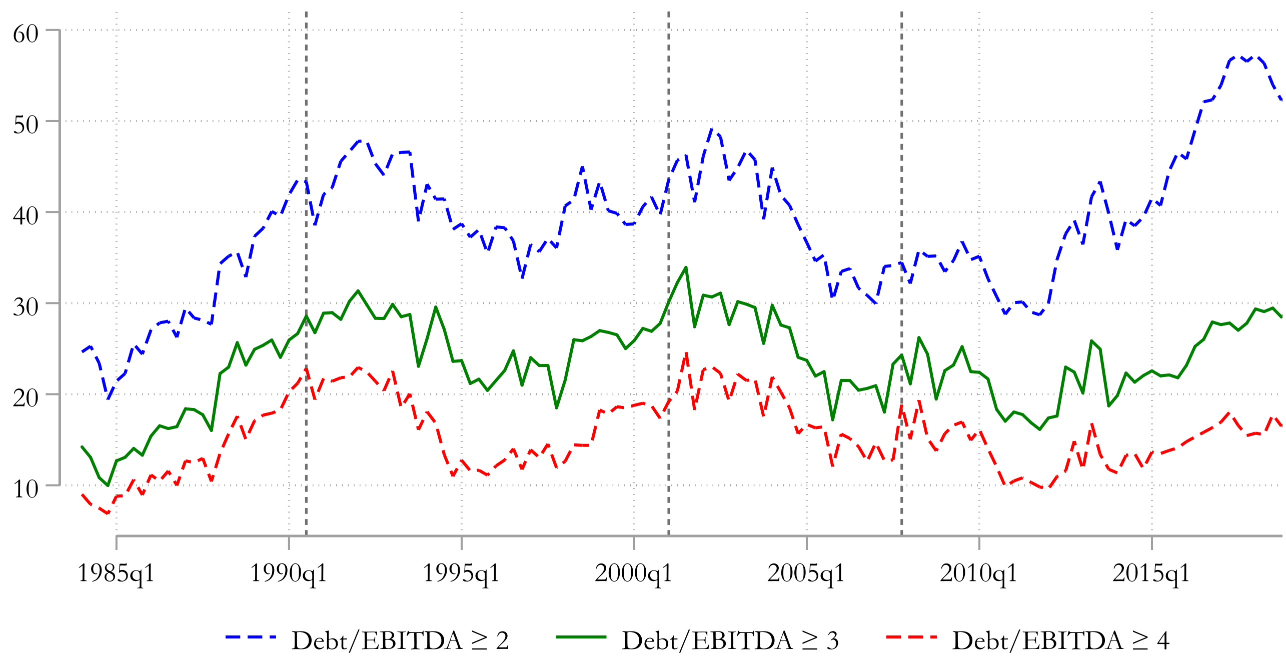
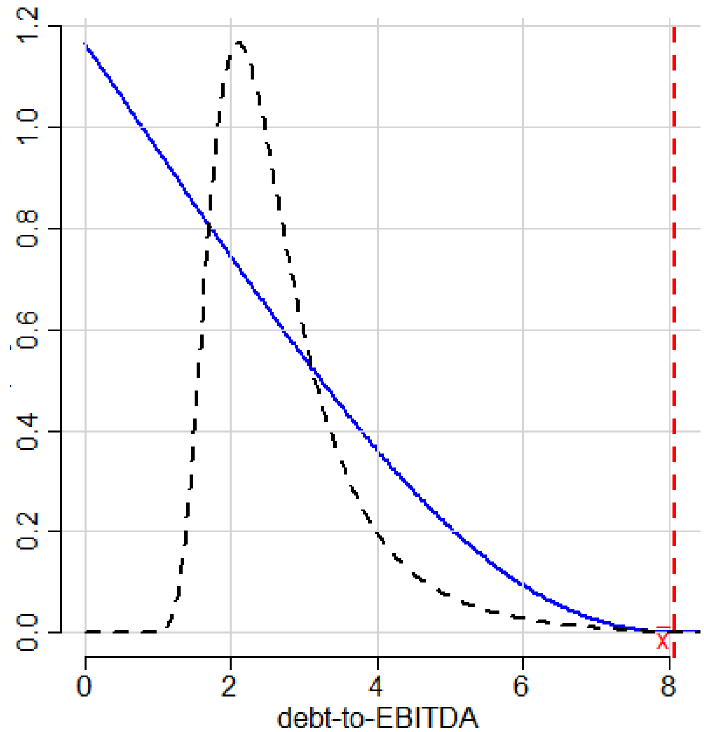


Figure 1: The distribution of gross leverage in the sample of US non-financial firms in Compustat. Gross leverage is measured as the ratio of gross debt to EBITDA. The figure reports the fraction of aggregate sales accounted for by firms with debt-to-EBITDA ratios above certain thresholds. The sample construction and variable definitions are reported in Appendix A.2.1.

(a): Equity Value $e(x)$



(b): Dividends $\pi(x)$

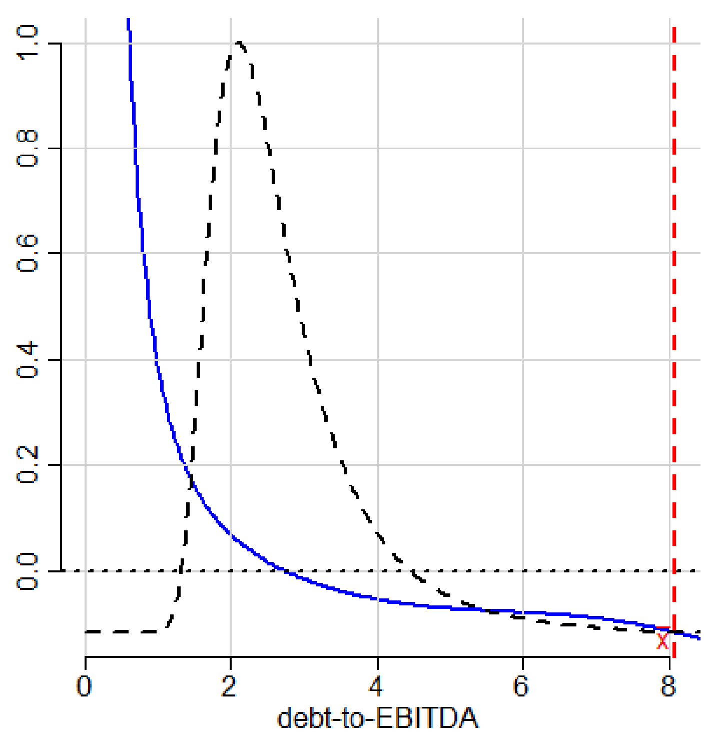
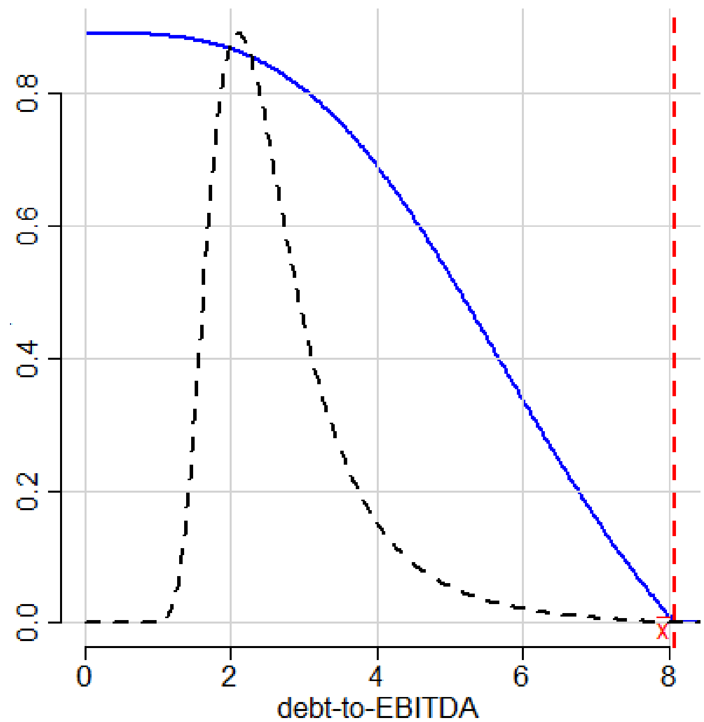


Figure 2: Equity value (top panel) and dividend policy (bottom panel) in the model of Section 3. The calibration used to produce these graphs is reported in Table 1.

(a): Debt price $d(x)$



(b): Credit Spreads $\varsigma(x)$

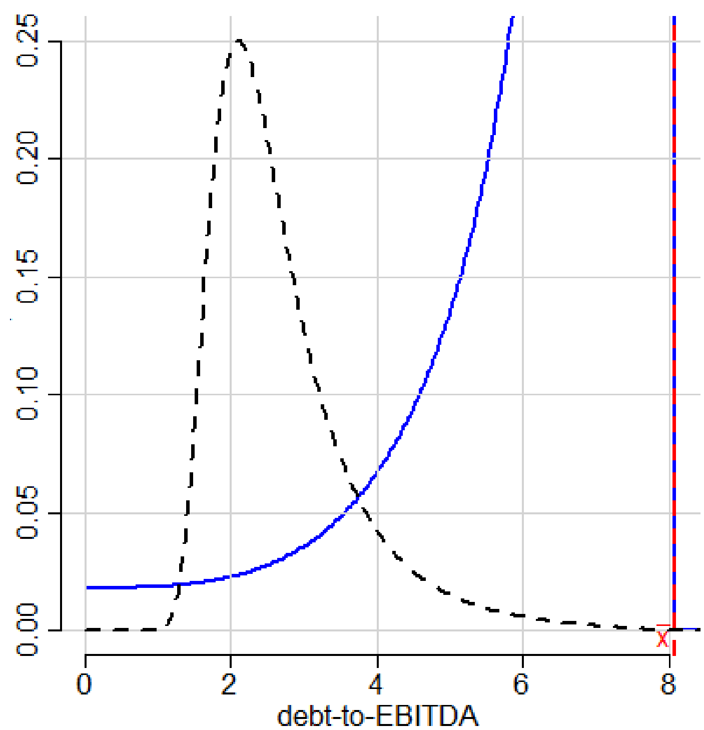
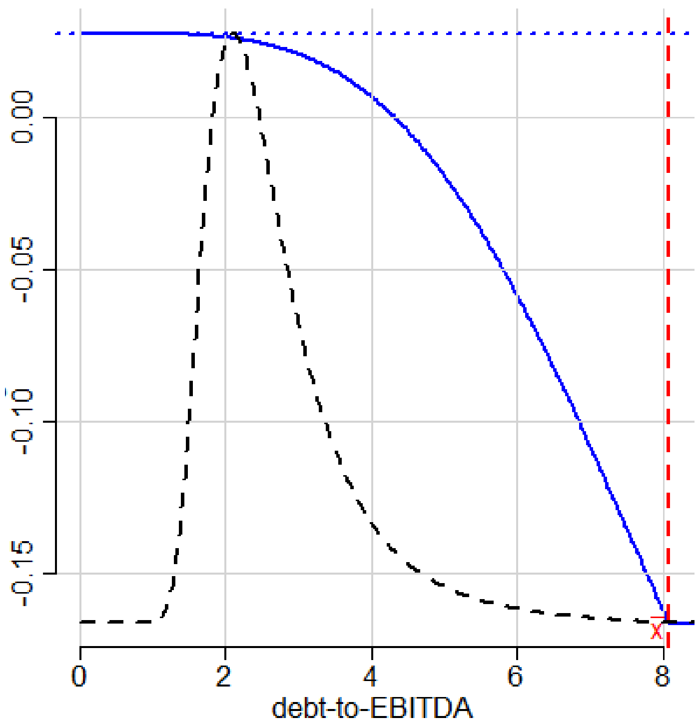


Figure 3: Debt prices (top panel) and credit spreads (bottom panel) in the model of Section 3. The calibration used to produce these graphs is reported in Table 1.

(a): Net investment rate $g(x)$



(b): Debt issuance rate $\varsigma(x)$

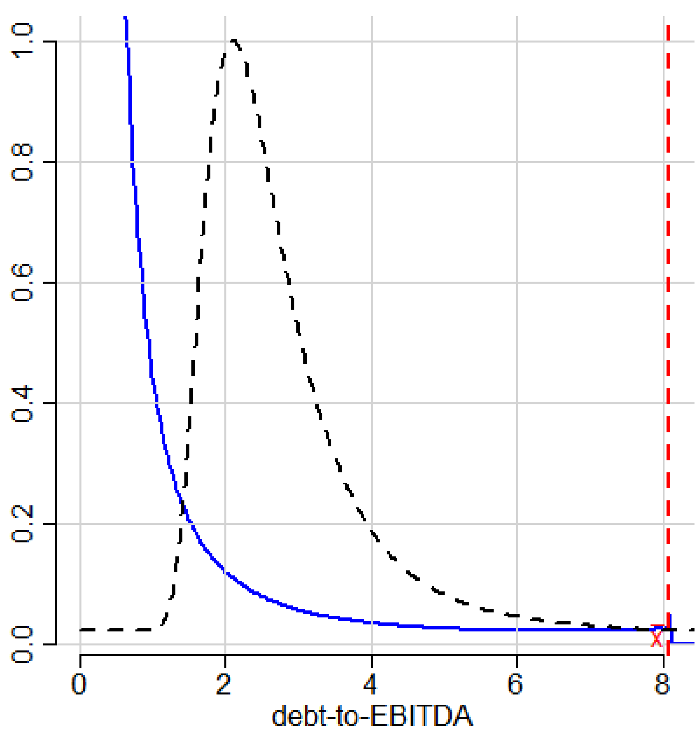


Figure 4: Net investment rate (top panel) and debt issuance rate (bottom panel) in the model of Section 3. The calibration used to produce these graphs is reported in Table 1.

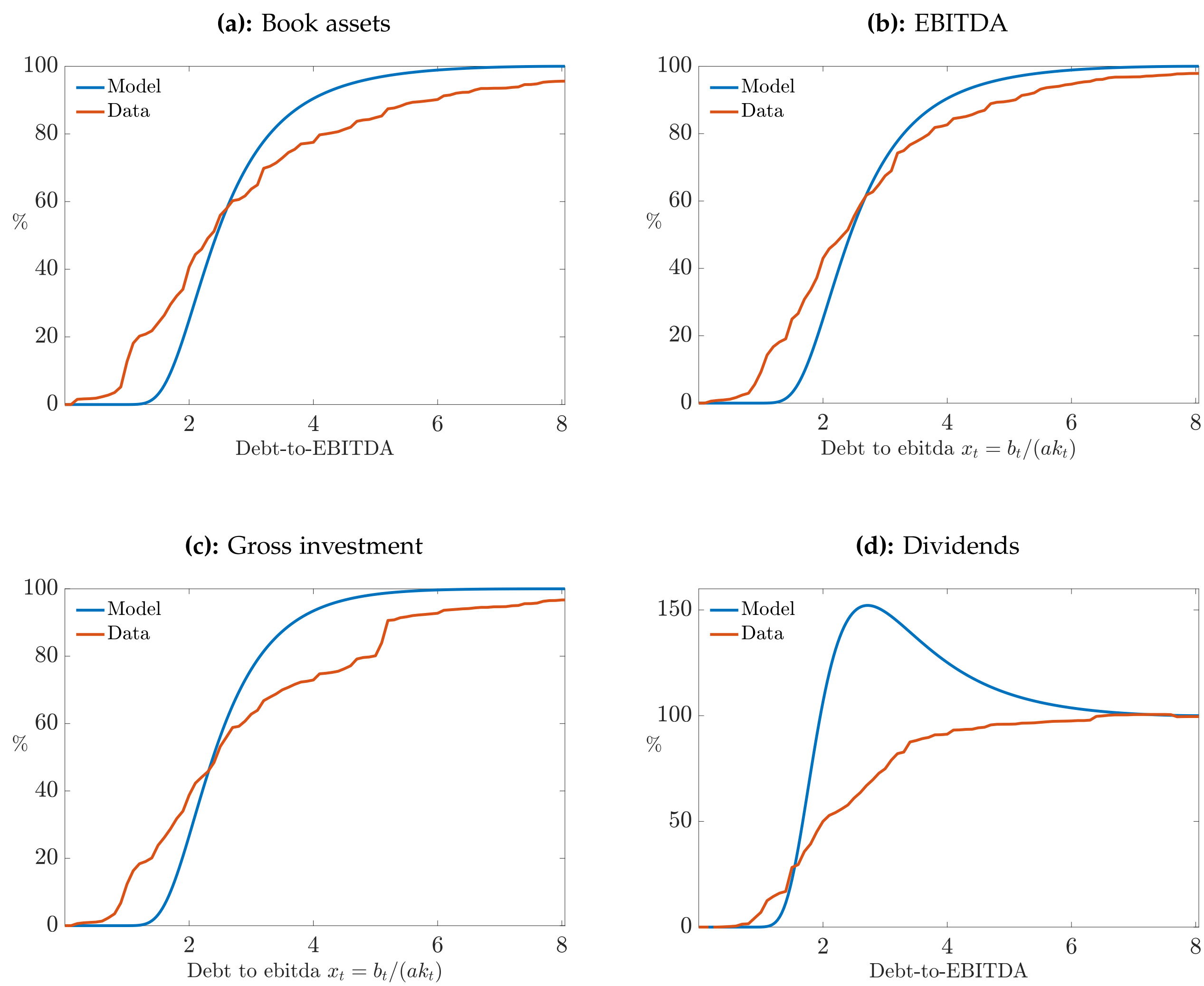


Figure 5: Comparison of empirical and model-implied cumulative distribution functions (CDF) for different variables. Each panel plots the cumulative share of a variable of interest (as a fraction of the aggregate value of that variable), as a function of the debt-to-EBITDA ratio. The plots are constructed using annual data for 2018 for the sample of Compustat US non-financial firms. Sample selection and variable construction are described in more detail in Appendix A.2.1.

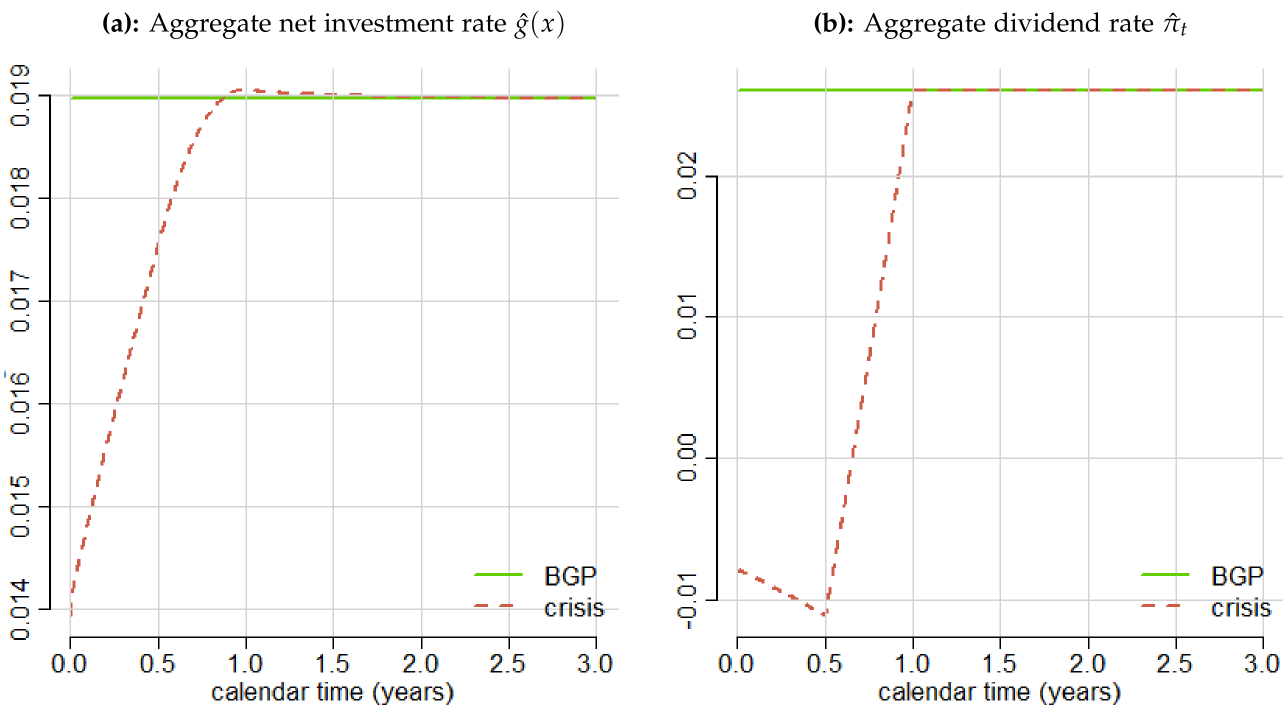


Figure 6: Crisis without financial market shutdown: the case of no intervention. Aggregate net investment rate (left panel) and dividend rate (right panel), following a temporary decline in productivity, in the case of well-functioning financial markets. The underlying shock is a 25% decline in the marginal product of capital a_t for a period of six months, followed by a linear recovery to its long-run steady-state level. See Section 4.1.1 for a description of this case.

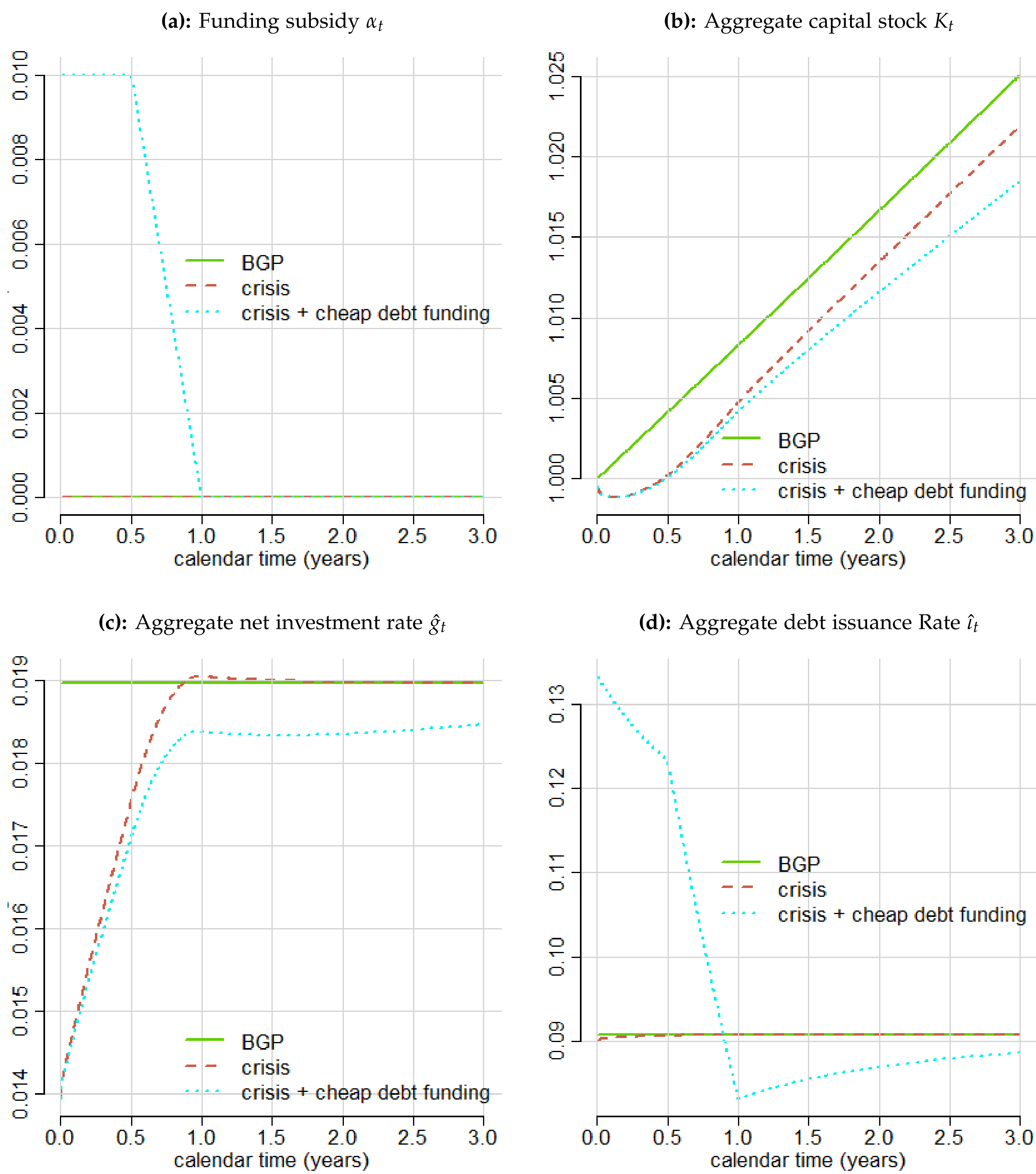


Figure 7: Crisis without financial market shutdown: the effect of debt funding at subsidized rates. Total funding subsidy (top left panel), aggregate capital stock (top right panel), aggregate net investment rate (bottom left panel) and aggregate debt issuance (bottom right panel) when the government provides funding at a subsidized interest rate during the crisis. These graphs are for the policy experiment of Section 4.1.3 in which financial markets function normally. The underlying shock is a 25% decline in the marginal product of capital a for a period of six months, followed by a linear recovery to its long-run steady-state level.

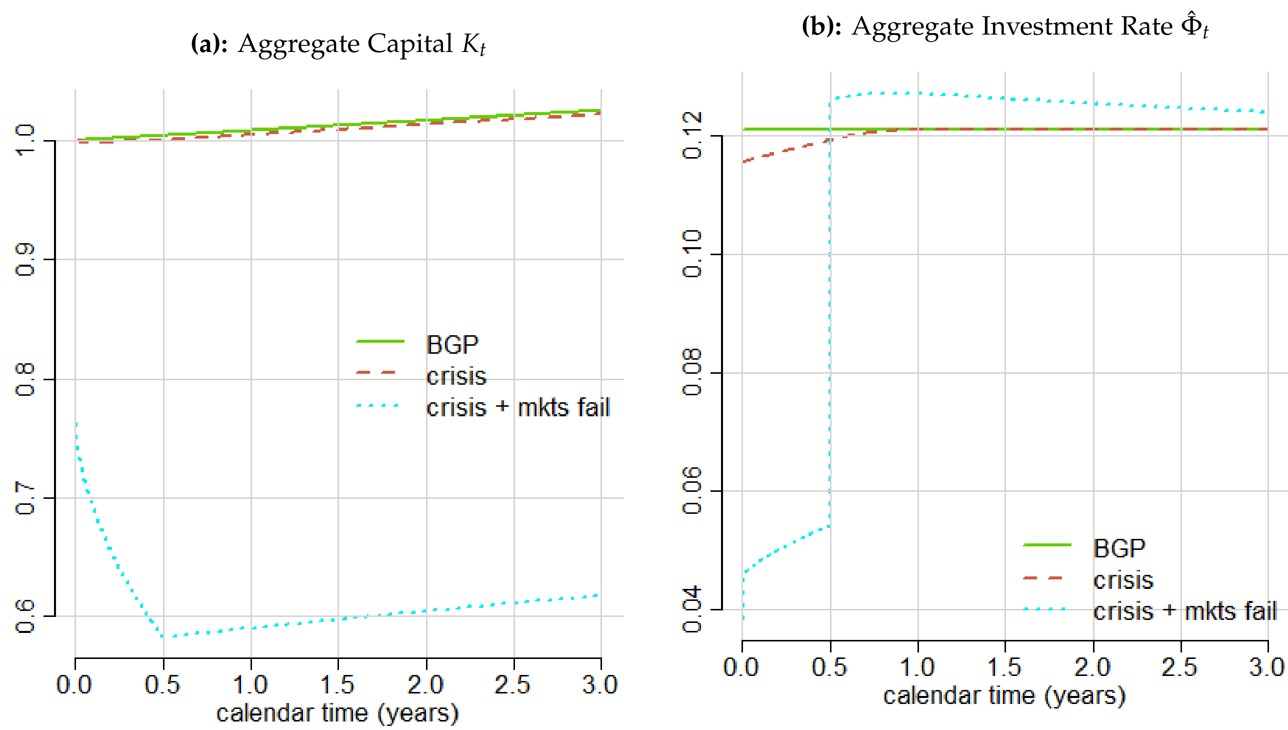


Figure 8: Crisis with financial market shutdown: the case of no intervention. Aggregate net investment rate (left panel) and dividend rate (right panel), following a temporary decline in productivity, with a shutdown in financial markets during the crisis. The underlying shock is a 25% decline in the marginal product of capital a_t for a period of six months, followed by a linear recovery to its long-run steady-state level. Additionally, equity and debt markets remain closed for the first six months of the crisis. See Section 4.2.1 for a description of this case.

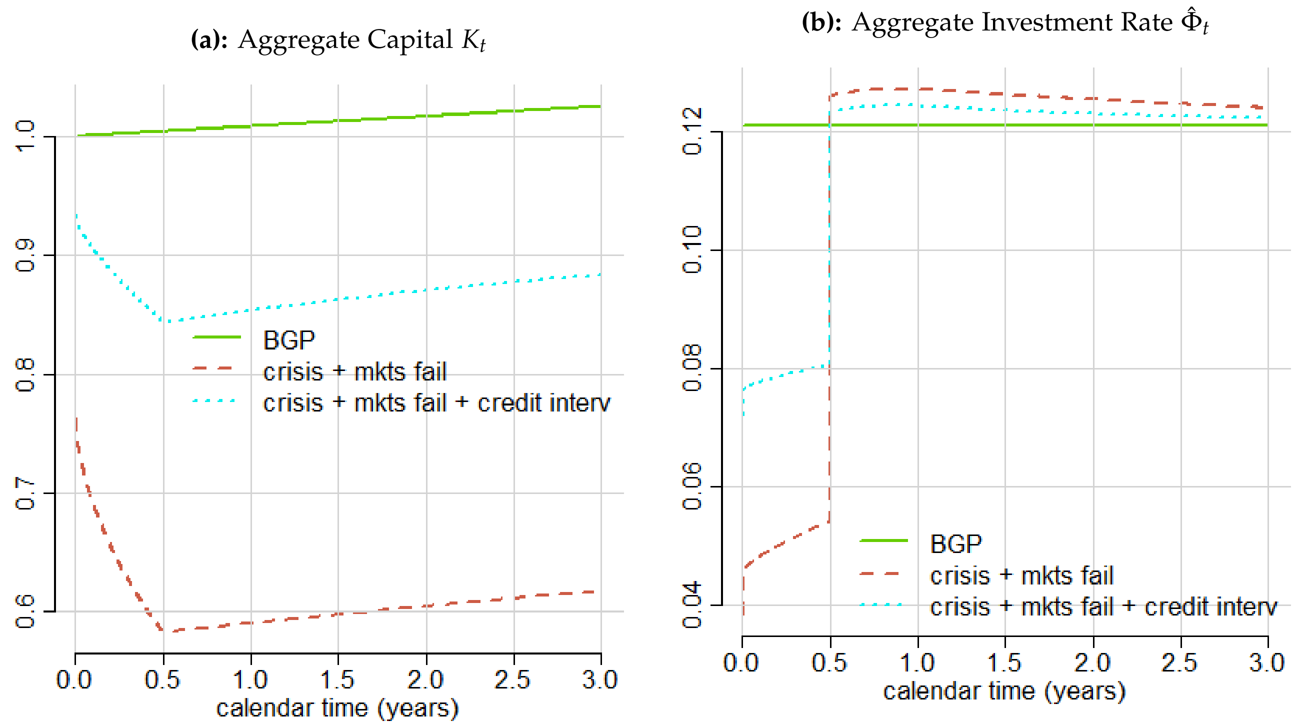


Figure 9: Crisis with financial market shutdown: the effect of emergency debt funding. Aggregate net investment rate (left panel) and dividend rate (right panel), following a temporary decline in productivity, with a shutdown in financial markets during the crisis. The underlying shock is a 25% decline in the marginal product of capital a_t for a period of six months, followed by a linear recovery to its long-run steady-state level. Additionally, equity and debt markets remain closed for the first six months of the crisis, but the government provides emergency debt funding to firms. See Section 4.2.2 for a description of this case.

(a): Aggregate Capital K_t **(b):** Aggregate Investment Rate $\hat{\Phi}_t$

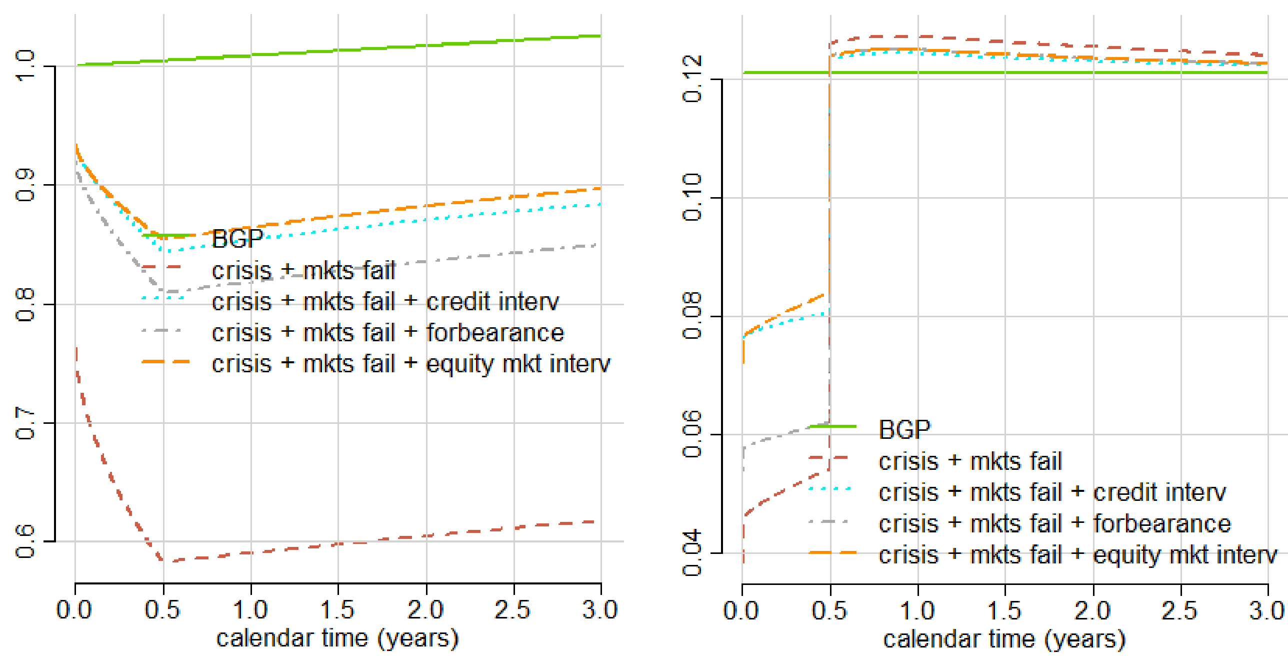


Figure 10: Crisis with financial market shutdown: the effect of alternative policy interventions. Aggregate net investment rate (left panel) and dividend rate (right panel), following a temporary decline in productivity, with a shutdown in financial markets during the crisis. The underlying shock is a 25% decline in the marginal product of capital a_t for a period of six months, followed by a linear recovery to its long-run steady-state level. Additionally, equity and debt markets remain closed for the first six months of the crisis. We consider two alternative policy interventions: equity injections, and loan forbearance. See Section 4.2.3 for a description of this case.

Appendix

Nicolas Crouzet and Fabrice Tourre

A.1 Theoretical Appendix

A.1.1 Rescaling

Let N_t^d be the counting process for default events. Notice that the effective capital k_t of a given firm can be written:

$$k_t = k_0 \exp\left(\int_0^t \left(g_s - \frac{\sigma^2}{2}\right) ds + \sigma Z_t\right) = k_0 \tilde{M}_t \exp\left(\int_0^t g_s ds\right).$$

In the above, we have introduced the martingale $\tilde{M}_t := e^{\sigma Z_t - \frac{1}{2}\sigma^2 t}$. This defines the change-of-measure $\tilde{\mathbb{P}}(A) = \mathbb{E}[\tilde{M}_t 1_A]$. We then have

$$E_t(k,b) = \sup_{g,\iota,\tau_d} \mathbb{E}^{k,b,t}\left[\int_t^{\tau_d} e^{-r(s-t)} \pi_s k_s ds\right] = k_t \sup_{g,\iota,\tau_d} \tilde{\mathbb{E}}^{x,t}\left[\int_t^{\tau_d} e^{-\int_t^s (r-g_u)du} \pi_s ds\right] = k_t e_t(x_t)$$

Under $\tilde{\mathbb{P}}$, $\tilde{Z}_t := Z_t - \sigma t$ is a standard Brownian motion, and x_t evolves according to

$$dx_t = \left[\iota_t - (g_t + m)\, x_t\right] dt - \sigma x_t d\tilde{Z}_t$$

A.1.2 Optimal Financing

When we use equation (10) and the firm's optimal growth policy $g_t(x)$ in the HJB equation (6) satisfied by the equity value, it becomes:

$$\begin{aligned}(r - g_t(x))\, e_t(x) = a_t - \Phi\left(g_t(x)\right) - (\kappa + m)\, x - \Theta\left(a - \kappa x\right) \\ + \partial_t e_t(x) - \left(g_t(x) + m\right) x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \quad \text{(A1)}\end{aligned}$$

Equation (A1) is the HJB equation for shareholders of a firm that can commit never to issue bonds ever again. If one were to differentiate equation (A1) w.r.t. x, substract equation (7), and use the first order condition (10), one can back out the issuance policy

$$\iota_t(x) = \frac{\Theta\kappa}{-\partial_x d_t(x)}$$

Equation (A1) makes it clear that in our setup, the equity value of a firm with leverage x can be computed as if such firm was allowing its bonds to amortize, and as if it could commit to never issuing bonds in the future.

A.1.3 Convexity

The convexity of e can be seen from the fact that shareholders always have the option to issue a non-zero measure of bonds. Indeed, take two arbitrary leverage ratios x_1, x_2, and $\lambda \in [0,1]$, with $x_\lambda = \lambda x_1 + (1-\lambda)x_2$. Consider feasible debt policies that make the firm's leverage jump from x_1 to x_λ, or from x_2 to x_λ. Then we have

$$\begin{aligned}e_t(x_1) &\geq e_t(x_\lambda) + (x_\lambda - x_1) d_t(x_\lambda) \\ e_t(x_2) &\geq e_t(x_\lambda) + (x_\lambda - x_2) d_t(x_\lambda)\end{aligned}$$

We can then take a weighted average of these inequalities to obtain $\lambda e_t(x_1) + (1-\lambda) e_t(x_2) \geq e_t(x_\lambda)$.

A.1.4 Default Boundary $\bar{x}_t$

Call $\hat{e}_t(x)$ the equity value for a firm that uses the no-leverage optimal investment rule g_t^*:

$$\hat{e}_t(x) := \sup_\tau \mathbb{E}^{t,x}\left[\int_t^\tau e^{-\int_t^s (r-g_u^*)du}\left(a_t - \Phi\left(g_s^*\right) - (\kappa+m)x_s - \Theta\left(a_s - \kappa x_s\right)\right)ds\right]$$

Note $\hat{x}_t$ the optimal default boundary associated with $\hat{e}_t(x)$, and $\Delta_t(x) := e_t(x) - \hat{e}_t(x)$. Then for $x < \min(\bar{x}_t, \hat{x}_t)$,

$$\begin{aligned}(r-g_t^*)\Delta_t(x) &= \Phi\left(g_t^*\right) - \Phi\left(g_t(x)\right) + \left(g_t(x) - g_t^*\right)\left(e_t(x) - x\partial_x e_t(x)\right) + \partial_t\Delta_t - (g_t^*+m)x\partial_x\Delta_t(x) + \frac{\sigma^2x^2}{2}\partial_{xx}\Delta_t(x) \\ &= \Phi\left(g_t^*\right) - \Phi\left(g_t(x)\right) + \left(g_t(x) - g_t^*\right)\Phi'\left(g_t(x)\right) + \partial_t\Delta_t - (g_t^*+m)x\partial_x\Delta_t(x) + \frac{\sigma^2x^2}{2}\partial_{xx}\Delta_t(x)\end{aligned}$$

Notice then that $\Phi\left(g_t^*\right) - \Phi\left(g_t(x)\right) + \left(g_t(x) - g_t^*\right)\Phi'\left(g_t(x)\right) \geq 0$ given that Φ is convex. Thus, noting $\tau := \inf\{t \geq 0 : x_t \geq \min(\hat{x}_t, \bar{x}_t)\}$, we have

$$\Delta_t(x) = \mathbb{E}^{t,x}\left[\int_t^\tau e^{-\int_t^s(r-g_u^*)du}\left(\Phi\left(g_s^*\right) - \Phi\left(g_s(x)\right) + \left(g_s(x) - g_s^*\right)\Phi'\left(g_s(x)\right)\right) + e^{-\int_t^\tau(r-g_u^*)du}\Delta_\tau(x_\tau)\right] \geq 0$$

This means that we must have $\bar{x}_t \geq \hat{x}_t$ – in other words, the firm delays its default decision, as it can reduce its investment rate and increase current cashflows when leverage is high.

A.1.5 Aggregation

Remember that the dynamics of the aggregate capital stock are as follows:

$$\begin{aligned}dK_t &= \int_0^{J_t} k_t^{(j)} g_t\left(x_t^{(j)}\right) dj dt + \int_0^{J_t} \sigma k_t^{(j)} dZ_t^{(j)} dj - \int_0^{J_t} k_t^{(j)} dN_t^{d,(j)} dj + \hat{\lambda}_t^n K_t dt \\ &= \left(\int_x\int_\omega g_t\left(x\right)\omega f_t(x,\omega) d\omega dx\right) K_t dt - \hat{\lambda}_t^d K_t dt + \hat{\lambda}_t^n K_t dt,\end{aligned}$$

where $\hat{\lambda}_t^d$ is the capital-share-weighted default rate, computed as follows:

$$\hat{\lambda}_t^d := \frac{1}{dt}\int_0^{J_t} \omega_t^{(j)} dN_t^{d,(j)} dj$$

We introduce $\hat{f}_t(x) := \int_\omega \omega f_t(x,\omega) d\omega$, which represents the percentage of the total capital stock at firms with leverage x. Then,

$$dK_t = \left(\hat{g}_t + \hat{\lambda}_t^n - \hat{\lambda}_t^d\right) K_t dt := \mu_{K,t} K_t dt,$$

where $\hat{g}_t := \int_x g_t\left(x\right)\hat{f}_t(x) dx$ is the average, pre-default and pre-injection aggregate capital growth rate. The law of motion for an individual firm's capital share is:

$$d\omega_t^{(j)} = \left(g_t\left(x_{t-}^{(j)}\right) - \hat{g}_t\right)\omega_{t-}^{(j)} dt + \sigma\omega_{t-}^{(j)} dZ_t^{(j)} - \omega_{t-}^{(j)}\left(dN_t^{d,(j)} - \hat{\lambda}_t^d dt\right) - \omega_{t-}^{(j)}\hat{\lambda}_t^n dt$$

The firm's capital share increases or decreases depending on whether its capital growth rate is greater or less than the weighted-average growth rate in the economy (the first term in the stochastic differential equation above). The firm's capital share also jumps down with default, due to bankruptcy costs. Finally, the firm's capital share decreases as new firms are injected into our economy. Introduce the coefficients

$\mu_{\omega,t}, \sigma_\omega$, such that:

$$d\omega_t^{(j)} := \mu_{\omega,t}\left(x_{t-}^{(j)}, \omega_{t-}^{(j)}\right) dt + \sigma_\omega\left(\omega_{t-}^{(j)}\right) dZ_t^{(j)} - \omega_{t-}^{(j)} dN_t^{d,(j)}$$

Remember that a firm's leverage follows:

$$\begin{aligned} dx_t^{(j)} &= \left[\iota_t\left(x_t^{(j)}\right) - \left(g_t\left(x_{t-}^{(j)}\right) + m - \sigma^2\right) x_{t-}^{(j)}\right] dt - \sigma x_{t-}^{(j)} dZ_t^{(j)} \\ &:= \mu_{x,t}\left(x_{t-}^{(j)}\right) dt + \sigma_x\left(x_{t-}^{(j)}\right) dZ_t^{(j)} \end{aligned} \tag{A2}$$

The Kolmogorov forward equation for the density $f_t(x,\omega)$ can be written, for $x \in (0, \bar{x}_t)$, $\omega \neq 1$:

$$\begin{aligned} \partial_t f_t(x,\omega) = &-\partial_x\left[\mu_{x,t}(x) f_t(x,\omega)\right] - \partial_\omega\left[\mu_{\omega,t}(x,\omega) f_t(x,\omega)\right] \\ &+ \frac{1}{2}\partial_{xx}\left[\sigma_x(x)^2 f_t(x,\omega)\right] + \frac{1}{2}\partial_{\omega\omega}\left[\sigma_\omega(\omega)^2 f_t(x,\omega)\right] + \partial_{x\omega}\left[\sigma_x(x)\sigma_\omega(\omega) f_t(x,\omega)\right] \end{aligned} \tag{A3}$$

Multiplying equation (A3) by ω, integrating over $\omega \in [0, +\infty)$ and using integration by parts, one can prove that the capital-share-weighted ergodic leverage density $\hat{f}_t(x)$ is solution to the following differential equation, for $x \in (0, \bar{x}_t)$:

$$\partial_t \hat{f}_t(x) = \left(g_t(x) - \hat{g}_t + \hat{\lambda}_t^d - \hat{\lambda}_t^n\right)\hat{f}_t(x) - \partial_x\left[\left(\iota_t(x) - \left(g_t(x) + m\right)x\right)\hat{f}_t(x)\right] + \frac{\sigma^2}{2}\partial_{xx}\left[x^2\hat{f}_t(x)\right], \tag{A4}$$

and

$$\hat{\lambda}_t^d = -\frac{1}{2}\sigma^2\bar{x}_t^2\partial_x\hat{f}_t(\bar{x}_t), \qquad \hat{g}_t = \int g_t(x)\hat{f}_t(x)dx, \qquad \hat{f}_t(\bar{x}_t) = 0$$

Since the endogenous default and growth rates $\hat{\lambda}_t^d$ and $\hat{g}_t$ depend on moments of the density $\hat{f}_t(x)$, equation (A4) is a nonlinear integro-differential equation. Finally, if we note $\lambda_t^d = -\frac{1}{2}\sigma^2\bar{x}_t^2\partial_x\int f_t(\bar{x}_t,\omega)d\omega$ the un-weighted firm default rate, the measure of firms J_t evolves as follows

$$\partial_t J_t = \hat{\lambda}_t^n - \lambda_t^d$$

A.1.6 Aggregate Subsidy Value

As a reminder, the subsidy received by a firm with leverage x and capital k_0 is equal to

$$S_0^d(x)k_0 := \mathbb{E}^{x,0}\left[\int_0^{\tau_d} e^{-\int_0^t r_s ds} s_t(x) k_t\left(1 - \nu_d(x_t)d_t(x_t)\right) dt\right]$$

It is then straightforward to show that the subsidy per unit of capital $S_t^d(x)$ satisfies the following Feynman-Kac PDE:

$$(r - g_t(x))\, S_t^d(x) = s_t(x)\left(1 - \nu_d(x)d_t(x)\right) + \partial_t S_t^d(x) + \left[s_t(x)\nu_d(x) - \left(g_t(x) + m\right)x\right]\partial_x S_t^d(x) + \frac{\sigma^2}{2}x^2\partial_{xx}S_t^d(x)\Big]$$

A.1.7 Alternative Interventions

A.1.7.1 Equity Injections

In that section, we describe the problem faced by managers of a firm, at a time when financial markets are completely closed. The government is injecting funds worth $s_t(x)$ per unit of time and per unit of capital into each firm, in exchange for $\nu_e(x)$ shares shares of the firm per unit of cash injected. The firm's problem becomes

$$0 = \max_{g \le \tilde{g}_t(x)} \Big[-\left(r + s_t(x)\nu_e(x) - g\right) e_t(x) + a_t + s_t(x) - \Phi\left(g\right) - \left(\kappa + m\right) x - \Theta\left(a_t - \kappa x\right) + \partial_t e_t(x) - \left(g + m\right) x \partial_x e_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} e_t(x) \Big] \tag{A5}$$

The value of the subsidy provided by the government to a firm with capital k_0 at time zero is then

$$S_0^e(x)k_0 := \mathbb{E}^{x,0}\left[\int_0^{\tau_d} e^{-\int_0^t r_s ds} s_t(x) k_t \left(1 - \nu_e(x_t) e_t(x_t)\right) dt\right]$$

A.1.7.2 Debt Forbearance

Let T be the length of time during which capital markets are closed. For $t \in [0, T]$, when a firm is allowed to defer its debt interest payments (which ends up being capitalized), the debt price $\hat{d}_t(x)$ satisfies

$$(r+m)\hat{d}_t(x) = m + \partial_t \hat{d}_t(x) - \left(g_t(x) + m - \kappa - \sigma^2\right) x \partial_x \hat{d}_t(x) + \frac{\sigma^2}{2} x^2 \partial_{xx} \hat{d}_t(x)$$

Instead, as soon as capital markets re-open, we assume that the forbearance period ends, such that debt prices satisfy equation (7). Thus, the initial cost of such intervention onto creditors is equal to

$$\int b_0^{(j)} \left(\hat{d}_0(x_0^{(j)}) - d_0(x_0^{(j)})\right) dj = K_0 \int x \left(\hat{d}_0(x) - d_0(x)\right) \hat{f}_0(x) dx$$

A.2 Empirics

A.2.1 Sample and variable construction

In order to construct the moments reported in Tables 1 and 2 and Figure 5, we use the Compustat Annual Fundamentals file. We focus on US-incorporated firms, with SIC codes not equal to 49 (utilities), 60 to 69 (financials), and above 91 (multinationals). We add screens to drop any remaining ADR, or any observation whose company name ends with the suffix "-REDH", "PRE FASB", "PRO FORMA", or "INDEX". Finally, we screen any firm-year observation with negative values for employment, sales, total assets current assets, current liabilities, cash and cash equivalents, and net property, plant and equipment.

In order to obtain ratings data, we merge the sample to the Mergent-FISD database.[30] We match each issue to a GVKEY in Compustat as follows: first, using the Mergent-FISD issuer CUSIP of the issue; if that fails, using the CUSIP associated with the parent id of the issue; and if that fails, using the ticker of the issuer. The resulting match covers approximately 92% of the nominal value of all issuances by US firms recorded in Mergent in 2017. Finally, we use only S&P ratings of issuances of senior unsecured debt, and classify a firm-year observation as investment-grade if it has at least one outstanding security rated BBB or higher.

Variable definitions for the moments reported in Tables 1 and 2 and Figure 5 are standard. The gross investment rate is defined as capital expenditures (`capx`) minus sales of property, plant equipment (`sppe`) when the latter is not missing, divided by property, plant and equipment (`ppent`). Total debt is defined as the sum of variables `dlc` and `dltt`; EBITDA is measured using the variable `oibdp`; interest expenses are measured using the variable `xint`; net payouts to shareholders (dividends, in the model) are defined as `dv+prstkc-sstk`; and net debt issuances are defined as `dltis+dltcc−dltr`. We winsorize the debt-to-EBITDA ratio, the interest coverage ratio, the gross investment rate, the dividend rate, and

[30] The reason for using Mergent-FISD is that records of bond ratings in the Compsutat files stop in 2017.

the gross and net debt issuance rates at the top 1% in each year. Finally, we focus on the 2000-2018 sample, because of the 2017 tax reform as well as the FASB reform of 2019 affecting the reporting of leases. All moments are computed year by year, weighted by EBITDA, and average across years, with the exception of the empirical CDFs reported in Figure 5, which are constructed using only 2018 data.

Figure 1 uses the quarterly Compustat files, instead of the annual files. We apply the same filters in order to select our sample. The measures of debt to EBITDA which we report in that figure use moving averages of the debt stock, and rolling sums of EBITDA, over the most recent four quarters over which we observe a firm.