

Internet Appendix for
“Can the cure kill the patient?
Corporate credit interventions and debt
overhang”

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A.1 Corporate Credit Interventions

This Appendix provides examples of corporate credit interventions that have been implemented in advanced economies. We define corporate credit interventions as fiscal or monetary programs consisting of outright purchases of, or guarantees on, securities or loans issued by non-financial corporations. We start with the US interventions of 2020, and then discuss interventions implemented in several other advanced economies, both before and during 2020.

A.1.1 Corporate credit interventions in the US

The key features of the 2020 US corporate credit interventions are summarized in Appendix [Table A-1](#). In what follows, we give more detail on each program, summarize their common design features, and discuss the distinction between the effects of the announcement of these programs, and their actual implementation.

A.1.1.1 Main programs

On March 23rd, 2020, the Fed announced its intention to directly intervene in corporate credit markets. The announcement marked an unprecedented step in its commitment to support the flow of credit to firms.⁴⁷ Two Fed credit programs were created, the Corporate Credit Facilities (CCF), and the Main Street Lending Program (MSLP). Both programs stopped new purchases on December 31st, 2020.

The Corporate Credit Facilities (CCF) Two Corporate Credit Facilities (CCF) were initially announced on March 23rd, 2020.⁴⁸ The Primary Market Corporate Credit Facility

⁴⁷To our knowledge, the Federal Reserve had only used its 13(3) powers to directly lend to non-financial firms once before, in a small-scale program implemented by the New York Fed during the Great Depression ([Rosa, 1947](#); [Sastry, 2018](#)). Some of the Great Recession programs that the Fed resumed in February and March of 2020 were meant to support indirectly credit supply, including the Commercial Paper Funding Facility, the Term Asset-Backed Securities Loan Facility, as well as bank supervisory actions meant to facilitate lending. In this paper, we focus on programs involving direct interventions in corporate credit markets.

⁴⁸They were subsequently expanded through additional Treasury funding authorized by the Coronavirus Aids, Relief and Economic Stimulus (CARES) act.

(PMCCF) was created to allow the Fed to purchase corporate bonds and syndications on the primary market, while the Secondary Market Corporate Credit Facility (SMCCF) was meant to allow the Fed to participate in the secondary markets for single-name corporate bond and bond ETFs. Both were created under section 13(3) of the Federal Reserve Act.

The PMCCF and the SMCCF were each initially funded with a \$10bn equity investment from the Treasury's Exchange Stabilization Fund (ESF), with the Fed providing an additional \$90bn, allowing for total purchases of up to \$200bn. After the passage of the Coronavirus Aid, Relief and Economic Security (CARES) act, on March 27th, the ESF was expanded to \$454bn. On April 9th, some of these additional funds were earmarked to allow the total scale of the facilities to reach up to \$750bn. The announcement is available [here](#).

The CCF targeted firms with access to bond markets.⁴⁹ Initially, the program was restricted to investment-grade rated firms, but the April 9th announcement expanded eligibility to firms that had been downgraded since March 22nd, and also allowed purchases of ETFs with exposure to high-yield bonds. Other than credit ratings, neither the PMCCF nor the SMCCF had other eligibility criteria. All Fed purchases were done at market prices, without caps on the size of individual purchases, and participation was not subject to restrictions on the use of funds. The CCF facilities could not hold more than 25% of an individual issuance (for the PMCCF) or 10% of total issuances (for the SMCCF) of a particular firm. Note that the PMCCF allowed new issuances to be used to refinance existing debt. Official term sheets for the Corporate Credit Facilities are available [here](#) (for the PMCCF) and [here](#) (for the SMCCF). [Boyarchenko et al. \(2020\)](#) provides a more detailed discussion of the program.

The Main Street Lending Program (MSLP) The Main Street Lending Program (MSLP) was first announced on April 9th, at the same time as the expansion of the CCF. The announcement is available [here](#). The aim of this program was to provide loans to small and medium-size firms. Participating borrowers had to meet certain size criteria (fewer

⁴⁹While rated firms make up a small fraction of the firm population, the account for a large fraction of corporate assets and investment; see, e.g., [Faulkender and Petersen \(2006\)](#) or [Crouzet and Mehrotra \(2020\)](#).

than 15,000 employees, or less than \$5bn revenue in 2019).

Three facilities were created: two of them allowed the Fed to buy newly issued loans, while the third allowed it to fund the upsizing of existing loans. Two additional facilities, for non-profit entities, were added on June 15th, 2020; the announcement is available [here](#). The facilities were funded with a \$75bn equity contribution from the Treasury's ESF, allowing, in principle, total purchases to reach \$600bn. As of January 31st, 2021, the average size of loans outstanding was \$9.54mm, with a minimum of \$0.10mm and a maximum of \$300mm. Loan-specific information on MSLP purchases is available [here](#).

The low take-up relative to the potential scale of the program may partly reflect the requirements imposed on both lenders and borrowers. Under the terms of the program, all MSLP loans had to be originated by banks, in exchange for a fee. Originating lenders had to retain a 5% participation in the loan. Loan size and leverage were subject to caps. The minimum loan size, initially set at \$0.5mm, was eventually lowered to \$0.1mm; the maximum loan size varied from \$30mm to \$300mm across facilities. Borrower eligibility was also subject to a leverage constraint: firms could borrow up to no more than four times 2019 EBITDA (or 6, in the case of the Priority facility), effectively excluding firms that had high debt to EBITDA ratios in 2019 from participating. Interest rates on MSLP loans are set at LIBOR plus 3%, regardless of the underlying financial conditions of the borrower. All loans made were five-year maturity, with a two-year grace period for principal payment. MSLP loans also had to be contractually senior to other forms of debt, though security was not explicitly required. Finally, borrowers' use of funds was tightly restricted. Borrowers were prevented from using proceeds from MSLP loans to refinance existing debt, and they must also follow the limits on distributions to shareholders outlined in the CARES act. Borrowers had to make "commercially reasonable efforts" to maintain employment for the time during which the MSLP loan is outstanding. Official information on the MSLP is available [here](#); see also [Crouzet and Gourio \(2020\)](#) for a discussion of the program.

A.1.1.2 Program design

As highlighted in Appendix Table in [Table A-1](#), the US 2020 corporate credit interventions differ along four main dimensions: the form of government participation; the eligibility conditions; the pricing; and the constraints on participating firms.

1. **Form of government participation** Both programs involved the central bank buying and holding *debt* securities of non-financial firms, either directly, or indirectly, through ETFs. The Primary Market Corporate Credit Facility (PMCCF) focused on purchases of corporate bonds and syndications on the primary market, while the Secondary Market Corporate Credit Facility (SMCCF) focused on buying single-name corporate bonds or bond ETFs in secondary markets. The MSLP involved the Fed buying and holding loans originated by participating private banks.
2. **Eligibility conditions** Only investment-grade firms, or firms downgraded after March 22nd, 2020 were eligible for direct purchases under the CCF.⁵⁰ Besides this rating criterion, the CCF did not feature any other eligibility restrictions. The MSLP was subject to tighter restrictions, most importantly a size limit (participating firms were to have fewer than 15,000 employees, or less than \$5bn revenue, in 2019), and a leverage constraint (firms could borrow up to no more than 4 to 6 times 2019 EBITDA, depending on the MSLP facility, effectively excluding firms that were highly levered coming into the crisis). Loans purchased by the MSLP also had to be contractually senior to other forms of debt.
3. **Pricing** The two programs also differ in their pricing of the government participations. All CCF purchases were to be done at market prices. By contrast, loans purchased under the MSLP were effectively subsidized, with interest rates set at LIBOR plus 3%.
4. **Constraints on participating firms** Inclusion in CCF purchases did not require the issuer of the security to meet particular constraints. There was also no cap on CCF purchases from individual issuers.⁵¹ By contrast, the MSLP put much tighter restrictions on participating borrowers. Borrowers were prevented from using proceeds from MSLP loans to refinance existing debt, and also had to follow limits on distributions to shareholders outlined in the CARES act. On top of the

⁵⁰High-yield firms were indirectly eligible, to the extent that bond ETFs purchased under the SMCCF had exposure to high-yield firms.

⁵¹The CCF facilities could not hold more than 25% of an individual issuance (for the PMCCF) or 10% of total issuances (for the SMCCF) of a particular firm. Additionally, the PMCCF allowed new issuances to be used to refinance existing debt.

overall leverage constraint, loans were capped at \$30m to \$300m, depending on the facility.

Thus, the main similarities between the CCF and MSLP is that they both involved purchases of debt securities subject to eligibility conditions (though the extent of these conditions differed across programs). On the other hand, (a) MSLP pricing was explicitly capped, whereas CCF rates were not; (b) MSLP participation put constraints on borrowers, whereas inclusion in CCF purchases did not. Our model helps shed light on whether these design variations matter for the effects and effectiveness of these programs, and why.

A.1.1.3 Announcement vs. implementation effects

While the CCF and the MSLP were set up to large scales (the CCF could purchase up to \$750bn of securities, and the MSLP up to \$600bn of loans), effective purchases were much smaller. The bottom panel of [Figure A-1](#) reports cumulative purchases under the CCF (only the SMCCF was effectively used by the Fed). The SMCCF purchases through December 31st, 2020 reached approximately \$14bn, two-thirds of it in the form of ETFs.⁵² Likewise, total purchases under the MSLP fell far short of the program's capacity. As of January 31st, 2021, the program had bought participations in 1830 loans, for a total principal amount of \$17.4bn.

The fact that take-up rates in the Fed programs were small does not necessarily indicate that they had a limited impact. The March 23rd *announcement* of the Fed's decision to intervene in corporate credit markets — as opposed to the actual purchases — had an appreciable effect on the cost of debt of corporations. The bottom panel of [Figure A-1](#) shows that on the two announcement dates of March 23rd (the initial announcement of the facilities) and April 9th (their expansion), corporate credit spreads for BBB rated firms fell substantially, despite the fact that purchases under the CCF did not begin until May 12th (for ETFs) and June 16th (for single-name bonds).⁵³ [Gilchrist et al. \(2024\)](#) analyze in detail the effect of the Fed's announcements on bond markets.

⁵²The bottom panel of [Figure A-1](#) reports cumulative gross purchases. As of March 11th, 2020, the SMCCF portfolio consisted of ETFs shares with a total market value of at \$8.59bn, and single-name bonds with \$5.13bn total par value.

⁵³The March 23rd announcement is available [here](#), and the April 9th announcement is available [here](#).

Comparing changes in spreads across eligible and ineligible bond issues, they conclude that the announcements reduced corporate bond spreads by 70bps on average.⁵⁴ By contrast, they estimate the effect of actual purchases on credit spreads to have been approximately 5bps. Thus, the CCF may have had an impact on credit conditions even without a large take-up rate.⁵⁵

In the main text, we propose an analysis of these "announcement" effects (as opposed to "implementation" or take-up effects) through the lens of our model, by analyzing the effects of a change in risk prices in credit markets on leverage and investment rates.⁵⁶

A.1.1.4 Corporate credit interventions in other advanced economies

In many other advanced economies, corporate credit interventions were an important part of the policy response to the 2020 events. In some countries, they had already been in use prior to those events.

Programs implemented following the 2020 crisis In response to the events of 2020, a number of European economies developed and implemented ad-hoc programs in order to provide credit support to firms. In the five largest economies European economies (Germany, the United Kingdom, France, Italy, and Spain), *government participations* under these programs generally involved loan guarantees, as opposed to direct lending.⁵⁷ The guarantees were generally underwritten by public financial institutions (such as Germany's KfW or France's BPI) with implicit backing from their respective fiscal authorities. *Eligibility* to specific programs was determined on the basis of firm size, with countries offering programs designed for SMEs (such as the Bounce Back Loan Scheme in the UK) and for large firms (such as the Coronavirus Large Business Interruption Loan Scheme, or CLBILS, in the UK). Access to the guarantee schemes were also restricted by a European Union directive, adopted in March 2020, which barred firms that already

⁵⁴In a sample of eligible and ineligible bonds from the same issuers, they find smaller effects (20bps).

⁵⁵Similar evidence on this point is provided in Haddad, Moreira and Muir (2021).

⁵⁶Because data on the cost of credit for smaller, private firms is generally unavailable in the US, it is unclear whether the MSLP had announcement effects separate from its implementation.

⁵⁷Anderson, Papadia and Véron (2021) estimate that 92% of the funds initially committed by these five countries to business credit support programs were tied to loan guarantee schemes, though some grant programs were also implemented. For instance, Germany provided €50bn in loan guarantees and €50 in direct grants.

were "in difficulty" as of 12/31/2019 from participating, with the notion of "difficulty" defined in, particular, in terms of existing leverage. Both *pricing* and *constraints on participating firms* varied across programs and countries. For instance, in France, the loan guarantee program constrained interest rates to be very low (specifically, tied to banks' liquidity costs, which was close to zero during the period), and it also restricted participating firms from engaging in share buybacks or dividend payments. By contrast, the UK's CLBILS allowed for market pricing of loans and put no restrictions on participating borrowers.⁵⁸

Overall, the design of these interventions was similar to the Fed's CCF and MSLP. The public guarantee advantages were passed through to banks and borrowers, either in terms of higher volumes, riskier portfolios, or lower interest rates, resulting in what amounted to subsidized lending programs subject to eligibility and participation restrictions, like the MSLP and the CCF.

Take-up in these programs was generally much larger than in the MSLP or CCF. Anderson, Papadia and Véron (2021) estimate a total take-up €503bn (or about 4.2% of 2019 GDP) for the five economies mentioned above as of the end of 2020.

Programs pre-dating the 2020 crisis In the Eurozone and in the UK, programs similar to the CCF had been implemented by monetary authorities prior to the 2020 crisis. The ECB's Corporate Sector Purchase Program (CSPP) was launched in 2016; it involved secondary market purchases of corporate bonds issued by investment-grade firms and meeting certain eligibility criteria. One of the explicit goals of the program was to reduce the cost of capital for issuers of eligible bonds (De Santis et al., 2018). Consistent with the large announcement effects of the CCF, the announcement of the CSPP led to an increase in bond prices for both ineligible and eligible issuers and bonds, at least in part through a reduction in risk premia in corporate bond markets, as documented by Pegoraro and Montagna (2020). In the UK, the Bank of England launched the Corporate Bond Purchase Scheme (CBPS) in August of 2016; purchases of eligible bonds ran from September 2016 to April 2017. D'Amico and Kaminska (2019) discuss the effects of the announcement on bond prices, and find that it led to a reduction in credit spreads.

⁵⁸Funding caps also varied across countries and programs; for instance, the French program was capped on the basis of past annual turnover, while there were no explicit caps for the CLBILS.

They also document an increase of corporate bond issuances in the wake of the program's announcement. Thus, similar to the CCF, the EU and the UK bond purchase programs had substantial announcement effects on yields and issuance, separate from implementation effects.⁵⁹

⁵⁹Additionally, the Bank of Japan (BoJ) conducted annual purchases of corporate bonds ranging from 0.5 to 3.2 trillion yen from February of 2009 to October of 2012 (see [Suganuma, Ueno et al. 2018](#)); over this period of time, [Arai \(2017\)](#) shows that corporate bond yields typically declined following BoJ announcements, though the announcements were not necessarily specific to the corporate bond-buying scheme.

A.2 Theory

A.2.1 No leverage firm

The average (and marginal) value of capital e^* of a firm that cannot take on debt is

$$ke^* := \sup_g \mathbb{E}^k \left[\int_0^{+\infty} \frac{\tilde{\xi}_{e,t}}{\tilde{\xi}_{e,0}} [a - \theta(a - \delta) - \Phi(g_t)] k_t dt \right]$$

With the usual change in measure, e^* must satisfy the HJB equation

$$0 = \max_g -(r + \rho\sigma v_e - g)e^* + a - \theta(a - \delta) - \Phi(g),$$

where $r + \rho\sigma v_e$ is the risk-adjusted expected return, which takes into account capital's exposure to priced aggregate shocks. The optimal investment rule can thus be written

$$g^* = (\Phi')^{-1}(e^*) = \frac{e^* - 1}{\gamma},$$

where the last equality follows from the assumption we made for the adjustment cost function Φ . Reinjecting into the HJB satisfied by e^* yields a system of 2 equations in 2 unknown

$$e^* = \frac{a - \theta(a - \delta) - \Phi(g^*)}{r + \rho\sigma v_e - g^*} \quad e^* = \Phi'(g^*) \quad (\text{A1})$$

This system can be re-written

$$r + \rho\sigma v_e = g^* + \frac{1}{\Phi'(g^*)} [a - \theta(a - \delta) - \Phi(g^*)]$$

Since the function Φ is increasing and convex (for $g > -1/\gamma$, where $-1/\gamma$ is the minimum achievable capital growth rate), the right hand side of this equation is a decreasing function of g^* (so long as the steady state cash-flow rate $a - \theta(a - \delta) - \Phi(g^*)$ is positive). In order for a solution $g^* < r + \rho\sigma v_e$ to this non-linear equation to exist, we must impose $\Phi(r + \rho\sigma v_e) > a - \theta(a - \delta)$. In such case, the right hand side of the equation above, evaluated at $g^* = r + \rho\sigma v_e$, is strictly less than $r + \rho\sigma v_e$, whereas the right hand side of

the equation above, evaluated at $g^* \rightarrow -1/\gamma$, diverges to $+\infty$ under the assumption that $\Phi(-1/\gamma) < a - \theta(a - \delta)$. Thus, under the parameter condition

$$\Phi(r + \rho\sigma v_e) > a - \theta(a - \delta) > \Phi(-1/\gamma), \quad (\text{A2})$$

there is a unique stationary investment rule g^* and a unique equity value (per unit of capital) e^* satisfying the system of equations (A1). Our functional form for Φ allows us to determine g^* analytically, as the smallest solution to the quadratic equation

$$\frac{\gamma}{2} (g - (r + \rho\sigma v_e))^2 + a - \theta(a - \delta) - \Phi(r + \rho\sigma v_e) = 0$$

This yields

$$g^* = r + \rho\sigma v_e - \left[\frac{2}{\gamma} (\Phi(r + \rho\sigma v_e) - (a - \theta(a - \delta))) \right]^{1/2}$$

A.2.2 Firms with leverage and constant investment rate

Imagine that the firm can commit to a specific investment policy g , but cannot commit to a debt issuance policy. This is a version of the model studied by DeMarzo and He (2021), with a slightly different default settlement mechanism. We solve this version of the model as it will be used as one of our benchmark models to quantify the strength of the debt overhang channel (see Table 3). Let e_g be the equity value (per unit of capital); by definition,

$$e_g(x) := \sup_{\tau, \iota} \mathbb{E}^x \left[\int_0^{+\infty} e^{-(r + \rho\sigma v_e - g)t + n_t \ln \alpha_k} (a - \Phi(g) + \iota_t d(x_t) - (\kappa + m)x_t - \theta(a - \delta - \kappa x_t)) dt \right]$$

$$dx_t = \iota_t - (g + m - \rho\sigma v_e) x_t dt - \sigma x_t d\tilde{Z}_t + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) x_{t-} dn_t,$$

with n_t the default counting process corresponding to the default policy τ . Since financial markets are frictionless, the insight of DeMarzo and He (2021) holds, i.e. the equity

value e_g is equal to the no-trade value:

$$e_g(x) = \sup_{\tau} \mathbb{E}^x \left[\int_0^{+\infty} e^{-(r+\rho\sigma\nu_e-g)t+n_t \ln \alpha_k} (a - \Phi(g) - (\kappa + m)x_t - \theta(a - \delta - \kappa x_t)) dt \right] \quad (\text{A3})$$

$$dx_t = -(g + m - \rho\sigma\nu_e) x_t dt - \sigma x_t d\tilde{Z}_t + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) x_t dn_t.$$

Thus, e_g satisfies the ODE, for $x \leq \bar{x}_g$ (the default boundary):

$$(r + \rho\sigma\nu_e - g) e_g(x) = a - \Phi(g) - (\kappa + m)x - \theta(a - \delta - \kappa x) - (g + m - \rho\sigma\nu_e) x e'_g(x) + \frac{(\sigma x)^2}{2} e''_g(x)$$

Let η_g be the positive root of the characteristic polynomial

$$\frac{\sigma^2}{2} \eta(\eta - 1) - (g + m - \rho\sigma\nu_e) \eta - (r + \rho\sigma\nu_e - g) = 0$$

Then the equity value (per unit of capital) satisfies

$$e_g(x) = e_g^* \left[1 - \left(\frac{1 - \alpha_k}{1 - \alpha_k \left(\frac{\alpha_b}{\alpha_k} \right)^{\eta_g}} \right) \left(\frac{x}{\bar{x}_g} \right)^{\eta_g} \right] - x d_g^* \left[1 - \left(\frac{1 - \alpha_b}{1 - \alpha_k \left(\frac{\alpha_b}{\alpha_k} \right)^{\eta_g}} \right) \left(\frac{x}{\bar{x}_g} \right)^{\eta_g - 1} \right],$$

where the zero-leverage equity value e_g^* and the zero-leverage debt price d_g^* are equal to

$$e_g^* := \frac{(1 - \theta)a + \theta\delta - \Phi(g)}{r - g + \rho\sigma\nu_e} \quad d_g^* := \frac{\kappa(1 - \theta) + m}{r + m}.$$

One can verify that our analytic expression for e_g satisfies the boundary condition at default:

$$e_g(\bar{x}_g) = \alpha_k e_g \left(\frac{\alpha_b}{\alpha_k} \bar{x}_g \right)$$

The smooth pasting condition delivers $\bar{x}_g$:

$$\bar{x}_g = \frac{\eta_g}{\eta_g - 1} \left(\frac{1 - \alpha_k}{1 - \alpha_b} \right) \frac{e_g^*}{d_g^*} \quad (\text{A4})$$

We can back out the debt price and issuance policies via:

$$d_g(x) = d_g^* \left[1 - \left(\frac{1 - \alpha_b}{1 - \alpha_b \left(\frac{\alpha_b}{\alpha_k} \right)^{\eta_g - 1}} \right) \left(\frac{x}{\bar{x}_g} \right)^{\eta_g - 1} \right]$$

$$\iota_g(x) = \frac{\theta \kappa \bar{x}_g}{(\eta_g - 1) d_g^*} \left(\frac{1 - \alpha_b \left(\frac{\alpha_b}{\alpha_k} \right)^{\eta_g - 1}}{1 - \alpha_b} \right) \left(\frac{x}{\bar{x}_g} \right)^{2 - \eta_g}$$

A.2.3 Rescaling

Let $n_t^{(j)}$ be the counting process for firm j 's defaults. The product $\tilde{\zeta}_{e,t} k_t^{(j)}$ satisfies

$$\tilde{\zeta}_{e,t} k_t^{(j)} = \tilde{\zeta}_{e,0} k_0^{(j)} \tilde{M}_t^{(j)} \exp \left(\int_0^t \left(g_s^{(j)} - (r + \rho \sigma v_e) \right) ds + \ln \alpha_k \int_0^t dn_s^{(j)} \right).$$

In the above, we have introduced the martingale $\tilde{M}_t^{(j)} := e^{(\rho \sigma - v_e) Z_t + \sqrt{1 - \rho^2} \sigma Z_t^{(j)} - \frac{1}{2} (v_e^2 + \sigma^2 - 2 \rho \sigma v_e) t}$.

This defines the change-of-measure $\tilde{\mathbb{P}}(A) = \mathbb{E}[\tilde{M}_t^{(j)} 1_A]$. We then have

$$E(k, b) = \sup_{g, \iota, \tau} \mathbb{E}^{k, b} \left[\int_0^{+\infty} \frac{\tilde{\zeta}_{e,t}}{\tilde{\zeta}_{e,0}} \pi_t k_t dt \right] = k \sup_{g, \iota, \tau} \tilde{\mathbb{E}}^x \left[\int_0^{+\infty} e^{-\int_0^t (r + \rho \sigma v_e - g_u) du + \int_0^t \ln \alpha_k dn_u} \pi_t dt \right] = ke(x)$$

Under $\tilde{\mathbb{P}}$, x_t evolves according to

$$dx_t = [\iota_t - (g_t + m - \rho \sigma v_e) x_t] dt - \sigma x_t d\tilde{Z}_t + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) x_t - dn_t,$$

with $\tilde{Z}_t$ a Brownian motion under $\tilde{\mathbb{P}}$.

A.2.4 Optimal financing in frictionless benchmark

When we use equation (18) and the firm's optimal growth policy $g(x)$ in the variational inequality (13) satisfied by the equity value, we then have, for $x \in [0, \bar{x}]$:

$$(r + \rho\sigma v_e - g(x)) e(x) = a - \Phi(g(x)) - (\kappa + m)x - \theta(a - \delta - \kappa x) - (g(x) + m - \rho\sigma v_e) x e'(x) + \frac{\sigma^2 x^2}{2} e''(x) \quad (\text{A5})$$

Equation (A5) is the HJB equation for shareholders of a firm that can commit never to issue bonds ever again. If one were to differentiate equation (A5) w.r.t. x , subtract equation (15), and use the first order condition (18), one can back out the issuance policy

$$\iota(x) = \frac{\theta\kappa}{-d'(x)} + \rho\sigma x(v_e - v_d).$$

Without equity issuance frictions, equation (A5) shows that the equity value of a firm with leverage x can be computed as if such firm was allowing its bonds to amortize, and as if it could commit to never issuing bonds in the future. With equity issuance frictions, the simplifications of this section are no longer relevant.

A.2.5 Debt prices and credit spreads

As discussed in the main text, the debt price satisfies

$$D_t = \mathbb{E}_t \left[\int_t^{+\infty} \frac{\xi_{d,s}}{\xi_{d,t}} \alpha_b^{n_s - n_t} e^{-m(s-t)} (\kappa + m) ds \right] = \hat{\mathbb{E}}_t \left[\int_t^{+\infty} \alpha_b^{n_s - n_t} e^{-(r+m)(s-t)} (\kappa + m) ds \right],$$

where we have used the change in measure $\hat{\mathbb{P}}(A) = \mathbb{E}[\hat{M}_t 1_A]$, with $\hat{M}_t := e^{-v_d Z_t - \frac{1}{2} v_d^2 t}$ a martingale. Under $\hat{\mathbb{P}}$, x_t evolves according to

$$dx_t = \left[\iota(x_t) - (g(x_t) + m - \sigma^2 - \rho\sigma v_d) x_t \right] dt - \sigma x_t d\hat{Z}_t + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) \bar{x} dn_t,$$

with $\hat{Z}_t$ a Brownian motion under $\hat{\mathbb{P}}$.

We then define the credit spread $cs(x)$ of a firm with leverage x as the spread over the

risk-free benchmark at which risk-free bond cashflows need to be discounted in order to recover the bond price $d(x)$:

$$d(x) := \int_0^\infty e^{-(r+m+cs(x))t} (\kappa + m) dt = \frac{\kappa + m}{r + m + cs(x)} \quad \Rightarrow \quad cs(x) = \frac{\kappa + m}{d(x)} - (r + m)$$

Since the debt price is decreasing in x , the credit spread is an increasing function of x . Lastly, we note that the bond price and credit spreads, when the firm has no leverage, satisfy

$$d(0) = \frac{\kappa + m}{r + m} - \frac{\theta\kappa}{r + m} \quad \Rightarrow \quad cs(0) = \frac{(r + m)\theta\kappa}{\kappa + m - \theta\kappa} > 0$$

Credit spreads are thus bounded from below by $cs(0) > 0$ since creditors anticipate the fact that the firm will be issuing bonds in the future, therefore increasing corporate default risk.

A.2.6 Convexity

The convexity of e can be seen from the fact that shareholders always have the option to issue a non-zero measure of bonds. Indeed, take two arbitrary leverage ratios x_1, x_2 , and $\lambda \in [0, 1]$, with $x_\lambda = \lambda x_1 + (1 - \lambda)x_2$. Consider feasible debt policies that make the firm's leverage jump from x_1 to x_λ , or from x_2 to x_λ . Then we have

$$e(x_1) \geq e(x_\lambda) + (x_\lambda - x_1)d(x_\lambda)$$

$$e(x_2) \geq e(x_\lambda) + (x_\lambda - x_2)d(x_\lambda)$$

Take a weighted average of these inequalities to obtain $\lambda e(x_1) + (1 - \lambda)e(x_2) \geq e(x_\lambda)$. Thus e is convex in x . If we take equation (17) and differentiate w.r.t. x , we obtain

$$g'(x) = \frac{-e''(x)}{\Phi''(g(x))} < 0.$$

A.2.7 Default boundary $\bar{x}$

Since the case with “reinjections” is more subtle to treat, in this section we analyze the simpler environment in which, at the time of default, shareholders are totally wiped out – i.e. $e(\bar{x}) = 0$ at the default boundary. This can be thought of as the special case where $\alpha_k = 0$ or $\alpha_b = \alpha_k < 1$. We use the notation of [Section A.2.2](#): e_g^* is the equity value (per unit of capital) for a firm that cannot commit to a particular capital structure policy, but that can commit to always using the no-leverage optimal investment rule $g = g^*$. Let $\bar{x}_g^*$ be the default boundary for the above problem; it satisfies equation (A4). Since the investment policy g^* and the default policy that solved (A3) are feasible for managers of a firm that can adjust its investment policy freely, we must have $e(x) \geq e_g^*(x)$. In particular, at the boundary $\bar{x}_g^*$, we must have $e(\bar{x}_g^*) \geq e_g^*(\bar{x}_g^*) = 0$. Thus, it must be the case that $\bar{x} \geq \bar{x}_g^*$.

A.2.8 Risk premia

It is worthwhile noting that in the presence of aggregate shocks, equity risk premia are non-zero. In fact, the expected equity return is equal to

$$\mathbb{E} \left[\frac{\pi_t k_t dt + dE(k_t, b_t)}{E(k_t, b_t)} \right] = \left[r + \rho v_e \sigma \left(1 - \frac{x_t e'(x_t)}{e(x_t)} \right) \right] dt$$

A.2.9 Aggregation

The dynamics of the aggregate capital stock are as follows:

$$\frac{dK_t}{K_t} = \underbrace{\int_0^1 \omega_t^{(j)} g(x_t^{(j)}) dj dt}_{=\hat{g}_t dt} + \underbrace{\sigma \sqrt{1 - \rho^2} \int_0^1 \omega_t^{(j)} dz_t^{(j)} dj}_{=0} + \underbrace{\sigma \rho \int_0^1 \omega_t^{(j)} dZ_t dj}_{=\sigma \rho dZ_t} - (1 - \alpha_k) \underbrace{\int_0^1 \omega_t^{(j)} dn_t^{(j)} dj}_{=\hat{\lambda}_t^d dt}$$

In particular, the aggregate capital stock is exposed to aggregate shocks, so that

$$\frac{dK_t}{K_t} = \left(\hat{g}_t - (1 - \alpha_k) \hat{\lambda}_t^d \right) dt + \rho \sigma dZ_t,$$

with $\hat{g}_t$ (resp. $\hat{\lambda}_t^d$) the capital-share-weighted investment rate (resp. default rate):

$$\hat{g}_t := \int_0^1 \omega_t^{(j)} g(x_t^{(j)}) dj \quad \hat{\lambda}_t^d := \frac{1}{dt} \int_0^1 \omega_t^{(j)} dn_t^{(j)} dj$$

Consider $\hat{f}_t(x) := \int_{\omega} \omega f_t(x, \omega) d\omega$, which represents the percentage of the total capital stock at firms with leverage x . The law of motion for an individual firm's capital share is:

$$d\omega_t^{(j)} = \left(g(x_{t-}^{(j)}) - \hat{g}_t \right) \omega_{t-}^{(j)} dt + \sigma \sqrt{1 - \rho^2} \omega_{t-}^{(j)} dz_t^{(j)} - (1 - \alpha_k) \omega_{t-}^{(j)} \left(dn_t^{(j)} - \hat{\lambda}_t^d dt \right)$$

The firm's capital share (i) increases or decreases depending on whether its capital growth rate is greater or less than the weighted-average growth rate in the economy, (ii) increases or decreases with idiosyncratic capital quality shocks, and (iii) jumps down with default, due to bankruptcy costs. Introduce the coefficients $\mu_{\omega,t}, \sigma_{\omega}$, such that:

$$d\omega_t^{(j)} := \mu_{\omega,t} \left(x_{t-}^{(j)}, \omega_{t-}^{(j)} \right) dt + \sigma_{\omega} \left(\omega_{t-}^{(j)} \right) dz_t^{(j)} - (1 - \alpha_k) \omega_{t-}^{(j)} dn_t^{(j)}$$

Remember that a firm's leverage follows:

$$\begin{aligned} dx_t^{(j)} &= \left[\iota \left(x_t^{(j)} \right) - \left(g \left(x_{t-}^{(j)} \right) + m - \sigma^2 \right) x_{t-}^{(j)} \right] dt - \sigma x_{t-}^{(j)} \left(\rho dZ_t + \sqrt{1 - \rho^2} dz_t^{(j)} \right) + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) \bar{x} dn_t^{(j)} \\ &:= \mu_{x,t} \left(x_{t-}^{(j)} \right) dt + \sigma_x^{idio} \left(x_{t-}^{(j)} \right) dz_t^{(j)} + \sigma_x^{agg} \left(x_{t-}^{(j)} \right) dZ_t + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) \bar{x} dn_t^{(j)} \end{aligned}$$

The Kolmogorov forward equation for $f_t(x, \omega)$ is written, for $x \in (0, \bar{x})$, $x \neq \frac{\alpha_b}{\alpha_k} \bar{x}$:

$$\begin{aligned} df_t(x, \omega) &= -\partial_x [\mu_{x,t}(x) f_t(x, \omega)] dt - \partial_{\omega} [\mu_{\omega,t}(x, \omega) f_t(x, \omega)] dt \\ &+ \frac{1}{2} \partial_{xx} [\sigma_x^{idio}(x)^2 f_t(x, \omega)] dt + \frac{1}{2} \partial_{\omega\omega} [\sigma_{\omega}(\omega)^2 f_t(x, \omega)] dt + \partial_{x\omega} [\sigma_x^{idio}(x) \sigma_{\omega}(\omega) f_t(x, \omega)] dt \\ &+ \partial_x [\sigma_{x,t}^{agg}(x) f_t(x, \omega)] dZ_t \quad (\text{A6}) \end{aligned}$$

Multiplying equation (A6) by ω , integrating over $\omega \in [0, +\infty)$ and using integration by parts, one can prove that the capital-share-weighted leverage density $\hat{f}_t(x)$ is solution to

the following stochastic differential equation, for $x \in (0, \bar{x})$, $x \neq \frac{\alpha_b}{\alpha_k} \bar{x}$:

$$d\hat{f}_t(x) = (g(x) - \mu_{K,t}) \hat{f}_t(x) dt - \partial_x \left[(\iota(x) - (g(x) + m)x) \hat{f}_t(x) \right] dt \\ + \frac{(1 - \rho^2)\sigma^2}{2} \partial_{xx} \left[x^2 \hat{f}_t(x) \right] dt + \rho\sigma \partial_x \left[x \hat{f}_t(x) \right] dZ_t \quad (\text{A7})$$

In addition,

$$\hat{\lambda}_t^d = -\frac{1}{2}\sigma^2 \bar{x}^2 \partial_x \hat{f}_t(\bar{x}), \quad \hat{g}_t = \int g(x) \hat{f}_t(x) dx, \quad \hat{f}_t(\bar{x}) = 0, \quad \mu_{K,t} = \hat{g}_t - (1 - \alpha_k) \hat{\lambda}_t^d$$

Define the long run distribution of capital-weighted leverage via

$$\hat{f}_\infty(x) dx := \lim_{T \rightarrow +\infty} \frac{1}{T} \int_0^T \omega_s 1_{\{x_s \in [x, x+dx]\}} ds, \quad (\text{A8})$$

then a version of equation (A7) exists for the density $\hat{f}_\infty(x)$:

$$\mu_K \hat{f}_\infty(x) = g(x) \hat{f}_\infty(x) - \partial_x \left[(\iota(x) - (g(x) + m)x) \hat{f}_\infty(x) \right] + \frac{\sigma^2}{2} \partial_{xx} \left[x^2 \hat{f}_\infty(x) \right] \quad (\text{A9}) \\ := \mathcal{L}^* \hat{f}_\infty(x),$$

where $\mathcal{L}$, the adjoint of the operator $\mathcal{L}^*$, is defined in (24); $\mathcal{L}$ is the capital growth operator, which will be useful for the computation of expected future macroeconomic aggregates.

A.2.10 A crisis state

A.2.10.1 Firm problem

We model the crisis as an unanticipated change, for all firms, in (a) the productivity parameter, which decreases from a to $a_c < a$, and (b) the price of risk, which increases from ν to $\nu_c > \nu$. The crisis lasts an exponentially distributed time, with parameter χ . Equity (per unit of capital) e_c satisfies, during such crisis time:

$$0 = \max \left[\alpha_k e_c \left(\frac{\alpha_b}{\alpha_k} x \right) - e_c(x), \max_{\iota, g} \left[- (r + \rho \sigma v_c + \chi - g) e_c(x) + a_c - \Phi(g) - (\kappa + m) x \right. \right. \\ \left. \left. - \theta (a - \delta - \kappa x) + \iota d_c(x) + \chi e(x) + [\iota - (g + m - \rho \sigma v_c) x] e'_c(x) + \frac{\sigma^2}{2} x^2 e''_c(x) \right] \right]$$

This equation is the counterpart to equation (13), adjusted to reflect Poisson transitions (at rate χ) back to the "normal" state of the economy. Instead, debt prices d_c satisfy, during such crisis time:

$$(r + m + \chi) d_c(x) = \kappa + m + \chi d_c(x) + \left[\iota_c(x) - (g_c(x) + m - \sigma^2 - \rho \sigma v_c) x \right] d'_c(x) + \frac{(\sigma x)^2}{2} d''_c(x) \\ d_c(\bar{x}_c) = \alpha_b d_c \left(\frac{\alpha_b}{\alpha_k} \bar{x}_c \right).$$

In the above, we have used notation $\iota_c(x), g_c(x)$ for the optimal issuance and growth policies followed by the firm during the crisis. Similarly, $\bar{x}_c$ is the optimal default barrier of the firm during such crisis. A reasoning identical to what was previously discussed allows us to derive the optimal investment and issuance policies during such crisis time:

$$\Phi'(g_c(x)) = e_c(x) - x e'_c(x) := q_c(x) \\ \iota_c(x) = \frac{\theta \kappa}{-d'_c(x)}$$

A.2.10.2 Expected future aggregate variables

In this section, we discuss our computation of expected future macroeconomic aggregates upon the occurrence of a transient aggregate shock. If T is the exponentially distributed crisis length, the expected aggregate capital stock then satisfies

$$\mathbb{E}[K_t] = \int_j \mathbb{E}[k_t^{(j)}] dj \\ = \int_j k_0^{(j)} \tilde{\mathbb{E}} \left[\alpha_k^{n_t^{(j)}} \exp \left(\int_0^{t \wedge T} g_c(x_u^{(j)}) du + \int_{t \wedge T}^t g(x_u^{(j)}) du \right) \right] dj \\ = K_0 \iint \omega h_c^{(k)}(x, t) f_0(x, \omega) d\omega dx \\ = K_0 \int h_c^{(k)}(x, t) \hat{f}_0(x) dx$$

In the above, we have implicitly defined the function $h_c^{(k)}(\cdot, \cdot)$ as follows

$$h_c^{(k)}(x, t) := \tilde{\mathbb{E}} \left[\alpha_k^{n_t} \exp \left(\int_0^{t \wedge T} g_c(x_u) du + \int_{t \wedge T}^t g(x_u) du \right) | x_0 = x \right]$$

$$dx_u = [\iota_c(x_u) - (g_c(x_u) + m) x_u] du - \sigma x_u d\tilde{Z}_u + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) x_u - dn_u, \quad u \leq T$$

$$dx_u = [\iota(x_u) - (g(x_u) + m) x_u] du - \sigma x_u d\tilde{Z}_u + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) x_u - dn_u, \quad u \geq T$$

The function $h_c^{(k)}$ satisfies the PDE

$$\partial_t h_c^{(k)}(x, t) = \mathcal{L}_c h_c^{(k)}(x, t) + \chi \left(h^{(k)}(x, t) - h_c^{(k)}(x, t) \right) \quad (\text{A10})$$

$$h_c^{(k)}(x, 0) = 1 \quad \forall x \leq \bar{x}_c$$

$$h_c^{(k)}(x, t) = \alpha_k^{n_c(x)} h_c^{(k)} \left(\left(\frac{\alpha_b}{\alpha_k} \right)^{n_c(x)} x, t \right) \quad \forall t, x > \bar{x}_c$$

$n_c(x)$, defined for $x > \bar{x}_c$, represents the number of consecutive debt restructuring during crisis times, i.e.:

$$n_c(x) := 1 + \left\lfloor \frac{\ln(x/\bar{x}_c)}{\ln(\alpha_k/\alpha_b)} \right\rfloor$$

In the above, the operator $\mathcal{L}_c$ is the crisis-time counterpart of the capital growth operator $\mathcal{L}$, defined in (24). In other words, $\mathcal{L}_c$ is defined, for $x \leq \bar{x}_c, t \geq 0$, via:

$$\mathcal{L}_c \varphi(x, t) := g_c(x) \varphi(x, t) + [\iota_c(x) - (g_c(x) + m) x] \partial_x \varphi(x, t) + \frac{\sigma^2 x^2}{2} \partial_{xx} \varphi(x, t),$$

where φ is an arbitrary smooth function. Similarly, the function $h^{(k)}(\cdot, \cdot)$ is defined as follows

$$h^{(k)}(x, t) := \tilde{\mathbb{E}} \left[\alpha_k^{n_t} \exp \left(\int_0^t g(x_u) du \right) | x_0 = x \right]$$

$$dx_u = [\iota(x_u) - (g(x_u) + m) x_u] dt - \sigma x_u d\tilde{Z}_u + \left(\frac{\alpha_b}{\alpha_k} - 1 \right) \bar{x} dn_t, \quad x_0 = x$$

The function $h^{(k)}$ satisfies the PDE

$$\begin{aligned} \partial_t h^{(k)}(x, t) &= \mathcal{L} h^{(k)}(x, t) \\ h^{(k)}(x, 0) &= 1 & \forall x \leq \bar{x} \\ h^{(k)}(x, t) &= \alpha_k^{n(x)} h^{(k)}\left(\left(\frac{\alpha_b}{\alpha_k}\right)^{n(x)} x, t\right) & \forall t, x > \bar{x}, \end{aligned}$$

where the operator $\mathcal{L}$ is defined in equation (24), and where $n(x)$, defined for $x > \bar{x}$, represents the number of consecutive debt restructuring, i.e.:

$$n(x) := 1 + \lfloor \frac{\ln(x/\bar{x})}{\ln(\alpha_k/\alpha_b)} \rfloor$$

Using a similar reasoning, we can compute the aggregate future output, investment, growth, debt issuance, profits, and bonds outstanding, via:

$$\begin{aligned} \mathbb{E}[Y_t] &= \int_j \mathbb{E}\left[a_t k_t^{(j)}\right] dj = K_0 \int \underline{h}_y(x, t) \hat{f}_0(x) dx \\ \mathbb{E}[G_t] &= \int_j \mathbb{E}\left[g\left(x_t^{(j)}\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_g(x, t) \hat{f}_0(x) dx \\ \mathbb{E}[I_t] &= \int_j \mathbb{E}\left[\Phi\left(g\left(x_t^{(j)}\right)\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_\Phi(x, t) \hat{f}_0(x) dx \\ \mathbb{E}[\mathcal{I}_t] &= \int_j \mathbb{E}\left[\iota\left(x_t^{(j)}\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_\iota(x, t) \hat{f}_0(x) dx \\ \mathbb{E}[DIV_t] &= \int_j \mathbb{E}\left[\pi\left(x_t^{(j)}\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_\pi(x, t) \hat{f}_0(x) dx \\ \mathbb{E}[B_t] &= \int_j \mathbb{E}\left[b_t^{(j)}\right] dj = K_0 \int \underline{h}_b(x, t) \hat{f}_0(x) dx \\ \mathbb{E}[E_t] &= \int_j \mathbb{E}\left[e\left(x_t^{(j)}\right) k_t^{(j)}\right] dj = K_0 \int \underline{h}_e(x, t) \hat{f}_0(x) dx \end{aligned}$$

The functions $\underline{h}_y, \underline{h}_g, \underline{h}_\Phi, \underline{h}_\iota, \underline{h}_\pi, \underline{h}_b, \underline{h}_e$ can be computed in a similar way to $\underline{h}_k$: they all satisfy the same equation (A10), and only differ in their initial condition at $t = 0$:

$$\begin{aligned} \underline{h}_y(x, 0) &= a \\ \underline{h}_g(x, 0) &= g(x) \end{aligned}$$

$$\underline{h}_\Phi(x, 0) = \Phi(g(x))$$

$$\underline{h}_\iota(x, 0) = \iota(x)$$

$$\underline{h}_\pi(x, 0) = \pi(x)$$

$$\underline{h}_b(x, 0) = x$$

$$\underline{h}_e(x, 0) = e(x)$$

A.2.11 Market frictions

In this section, we discuss two market environments that feature market frictions: (i) when all financial markets suffer a sudden-stop and (ii) when firms incur equity issuance costs.

A.2.11.1 Sudden stop

Imagine that during the crisis, all financial markets are shut down. For simplicity, we assume that firms cannot buy back their own debt, i.e. we set $\iota = 0$. We assume that at intensity χ , the crisis ends and we revert back to the steady state. The equity value e_c in the crisis state satisfies

$$\begin{aligned} \max_g & - (r + \rho\sigma\nu_c - g) e_c(x) + a_c - \Phi(g) - (\kappa + m)x - \theta(a - \delta - \kappa x) \\ & + \chi(e(x) - e_c(x)) - (g + m + \rho\sigma\nu_c)xe'_c(x) + \frac{(\sigma x)^2}{2}e''_c(x) \\ \text{s.t. } & a_c - \Phi(g) - (\kappa + m)x - \theta(a - \delta - \kappa x) \geq 0 \end{aligned}$$

The cashflow constraint yields a maximum possible investment rate, equal to

$$\bar{g}(x) = \Phi^{-1}(a_c - (\kappa + m)x - \theta(a - \delta - \kappa x))$$

The optimal investment policy is thus

$$g(x) = \min \left[\bar{g}(x), (\Phi')^{-1}(q(x)) \right]$$

When the government offers up to ι_g notional amount of loans at a price of d_g , the decision for a firm to accept such loans is based on:

$$\max_{\iota \in [0, \iota_g]} \iota (d_g + e'_c(x))$$

Thus, the firm's optimal public-sector borrowing decision is $\iota^* = \iota_g 1_{\{d_g > -e'_c(x)\}}$.

A.2.11.2 Equity issuance costs

In this section, we extend the model to incorporate equity issuance costs, building on the framework introduced in [Section 5.3](#). We consider a more general setting than that presented in [Section 5.3](#), which facilitates numerical solution of the model. Specifically, we assume that firms face costs of the form $c(\pi_t) k_t$, where the cost function $c(\cdot)$ is twice continuously differentiable, decreasing, and convex, with $c'(\pi) \in (-1, 0)$. Net payouts to shareholders, expressed per unit of capital k_t , are given by $\pi(x; \iota, g) - c(\pi(x; \iota, g))$. For a given debt price $d(\cdot)$, the equity value (per unit of capital) solves the variational inequality

$$0 = \max \left[\alpha_k e \left(\frac{\alpha_b}{\alpha_k} x \right) - e(x); \max_{g, \iota} \left[-(r + \rho \sigma v_e - g) e(x) + \pi(x; \iota, g) - c(\pi(x; \iota, g)) + \tilde{\mathcal{L}}_{g, \iota} e(x) \right] \right].$$

Given Markov controls $g(\cdot), \iota(\cdot)$ and a default policy $\bar{x}$, the debt price must satisfy [\(15\)](#)-[\(16\)](#). An MPE is simply defined as a fixed point of this system of functional equations — one HJB for e , and one martingale condition — effectively an ODE — for d , with appropriate boundary conditions. Numerically, we will be iterating on these two equations to find a fixed point and thus an MPE of this model extension.

The optimality condition for debt issuances ι is:

$$(1 - c'(\pi(x))) d(x) + e'(x) = 0 \tag{A11}$$

Similarly, the optimality condition for investment g is:

$$q(x) - (1 - c'(\pi(x))) \Phi'(g(x)) = 0 \tag{A12}$$

It is straightforward to verify that the second order conditions for (A11)-(A12) are automatically satisfied since Φ and c are convex functions, and c is decreasing. Equations (A11) and (A12) can be combined to obtain

$$\Phi'(g(x)) = \left(\frac{d(x)}{-e'(x)} \right) q(x)$$

In other words, equity issuance costs add a distortion to both the investment margin and the debt issuance margins. In summary, the optimal policies can be written as a function of $e(x), e'(x), d(x)$:

$$g(x) = (\Phi')^{-1} \left(\left(\frac{d(x)}{-e'(x)} \right) q(x) \right) \quad (\text{A13})$$

$$\pi(x) = (c')^{-1} \left(1 + \frac{e'(x)}{d(x)} \right) \quad (\text{A14})$$

$$\iota(x) = \frac{1}{d(x)} [\pi(x) - (a - (\kappa + m)x - \Phi(g(x)) - \theta(a - \delta - \kappa x))] \quad (\text{A15})$$

Finally, we discuss our choice of the cost function c . We will use

$$c(\pi) = \frac{\psi}{\eta} \log(1 + e^{-\eta\pi})$$

This function is smooth, strictly decreasing, strictly convex, with slope $c'(\pi) \in (-\psi, 0)$. We built this function so that for $\eta \rightarrow +\infty$, we have

$$\begin{aligned} c(\pi) &\approx -\psi\pi, & \pi < 0 \\ c(\pi) &\approx 0, & \pi > 0 \end{aligned}$$

$\eta > 0$ calibrates the degree of “kinkiness” of the function $c(\cdot)$ at $\pi = 0$. The parameter ψ pins down the proportional equity issuance costs when $\pi < 0$.

A.2.12 Fiscal cost calculations

The subsidy received by a firm with initial leverage x_0 and capital k_0 is equal to

$$S_d(x_0)k_0 := \mathbb{E}^x \left[\int_0^T e^{-rt} k_t \iota^*(x_t) (d_g(x_t) - d(x_t)) dt \right],$$

where $\iota^*(x) := \iota_g \mathbb{1}(d_g > -e'_c(x))$ is the firm's public sector borrowing decision. It is straightforward to show that the subsidy per unit of capital $S_d(x)$ satisfies the following Feynman-Kac equation:

$$\begin{aligned} (r + \rho\sigma v_e - g(x) + \chi) S_d(x) &= \iota^*(x) (d_g(x) - d(x)) \\ &+ [\iota^*(x) + \iota(x) - (g(x) + m)x] S'_d(x) + \frac{\sigma^2}{2} x^2 S_d(x) \end{aligned}$$

The total tax subsidy is then equal to $K_0 \int S_d(x) \hat{f}_0(x) dx$.

A.3 Calibration and estimation

In this section, we discuss how we calibrate and estimate the model in our baseline exercise of [Section 3](#). We also discuss how our results change when we calibrate certain parameters differently, or target different moments, as outlined in [Section 5](#).

A.3.1 Calibration of asset and debt recovery rates

In the model, when a firm defaults on its debt payments, it is restructured. The restructuring involves a reduction in the firm’s productive capital stock, k_t , by a factor of $1 - \alpha_k$, as well as a reduction in the par value of the firm’s outstanding stock of debt, b_t , by a factor of $1 - \alpha_b$. These parameters must satisfy $0 < \alpha_b < \alpha_k < 1$. These assumptions are meant to capture, in a simple manner, reorganizations in which the firm survives as a going concern, but may nevertheless incur direct or indirect costs of financial distress (as captured by the assumption that $1 > \alpha_k$), and change capital structure (as captured by the assumption that $\alpha_b < \alpha_k < 1$).

In practice, such reorganizations can occur through private workouts, or under court supervision, in the context of a Chapter 11 bankruptcy. In order to calibrate the parameters α_k and α_b , we take a more specific stance. We interpret the restructuring process in the model as the outcome of a Chapter 11 bankruptcy in which the firm was successfully reorganized and continued operating as an independent entity after the reorganization. Moreover, consistent with the rest of our estimation, we interpret the model as representing publicly traded firms, and assume that book assets accurately measure k_t .

With these assumptions in mind, we use data from the Bankruptcy Research Database ([LoPucki and Doherty 2007](#); BRD, in what follows). The BRD contains information on all bankruptcies filed by public companies between 1979 and 2022, provided the company had at least \$100m in book assets in constant 1980 dollars at the time of the filing.⁶⁰ In what follows, we first describe these data, and then discuss how we map them to our model.

Before proceeding, we acknowledge that this calibration relies on a particular interpretation of the restructuring process in our model — successful Chapter 11 reorgani-

⁶⁰This threshold corresponds to approximately \$310m in 2019 dollars, and is calculated by the BRD using book assets reported in Schedule A of the filing petition.

zations of public companies —, which may understate the direct and indirect costs of financial distress and restructuring. As a robustness check, in [Section 5.1.3](#) and [Appendix A.3.6.3](#), we estimate a version of the model where deadweight losses in default are higher, drawing on other evidence from the literature on the costs of financial distress.

A.3.1.1 The Bankruptcy Research Database (BRD)

The BRD contains information on the universe of all bankruptcy filings since October 1st, 1979, made by corporations that were publicly traded and had book assets greater than \$100m, in 1980 dollars, at the time of filing.⁶¹ The database is publicly available.⁶²

We first apply the formatting code provided by the BRD website (`BRD import.do`) to process the raw data. Out of 1218 filings in the raw data, 25 were Chapter 7 filings; these are dropped, and there remain 1193 Chapter 11 filings. Out of these, we select those identified by the BRD as reorganizations, defined as a situation in which at least one company emerged from Chapter 11 and continued operations as a stand-alone business. This corresponds to 817 cases, or 68.5% of total Chapter 11 filings recorded in the database.⁶³

We then further restrict the sample to those cases in which the following variables are non-missing: book assets in the last 10K filing before filing (variable `AssetsBefore`); book assets in the first 10K filing after emergence (variable `AssetsEmerging`); book liabilities (book assets minus book equity) before filing (variable `LiabBefore`); book liabilities after emergence (variable `LiabEmerging`). This leaves us with 415 bankruptcy cases. The reduction in sample size primarily appears to be driven by firms that emerge as private corporations, and are therefore not required to file 10Ks after emergence.

⁶¹After adjusting for inflation using the CPI-U index, this corresponds to approximately \$265m in 2010 dollars, and \$310m in 2019 dollars.

⁶²https://lopucki.law.ufl.edu/download_cases_table_terms.php. We used the January 2023 version of the database, containing all bankruptcy filings of publicly traded firms with assets above \$100m in 1980 dollars from October 1st, 1979, to December 31st, 2022.

⁶³Specifically, we focus on the cases where the BRD variable `Emerge` is coded as "yes". The file `Protocols.pdf`, provided by the BRD, specifies that "This field is "yes" if at least one operating company emerged from Field is complete bankruptcy." In particular, emergence can correspond either to a reorganization according to a confirmed plan, or to a 363 asset sale of the business as a going concern.

For each case we then define:

$$\begin{aligned}\widehat{\alpha}_k &\equiv \frac{\text{Book assets from 10K after emergence}}{\text{Book assets from 10K before filing}} \\ \frac{\widehat{\alpha}_b}{\alpha_k} &\equiv \frac{\text{Book leverage after emergence}}{\text{Book leverage before filing}}\end{aligned}\tag{A16}$$

where:

$$\text{Book leverage after emergence} = \frac{\text{Book liabilities from 10K after emergence}}{\text{Book assets from 10K after emergence}}, \tag{A17}$$

and book leverage before filing is defined in a similar manner.

Table A-2 reports the results, along with additional information extracted from the BRD data. We also report: book leverage at filing in the same sample ($N = 415$); book leverage at filing in the broader sample of Chapter 11 bankruptcies that complete a reorganization, but for which data on capital structure at emergence is not necessarily available; book leverage at emergence; and an estimate of the recovery rate for unsecured claims, for cases where the information is available in the BRD database. The recovery rate estimate is defined as the ratio of distributions to unsecured claimants, `DistribUnsec`, to the face value of these claims at filing, `ClaimsUnsec`. The information is only available for 53 cases. However, other research using Moody’s Universal Recovery Database or similar sources for security-level recovery rates finds family recovery rates for debt claims that are of the same magnitude. For instance, [Dou et al. \(2021, Table 1, Panel B\)](#) report a weighted average debt recovery rate of 46% across all classes of claims (senior and junior), while [Antill \(2022, Table 1\)](#) reports a debt recovery rate of 49% for the cases in his sample. Note that, for both papers, these cases include bankruptcies eventually leading to liquidations, opposed to successful reorganizations, to which our sample is restricted.

A.3.1.2 Mapping the data to asset and debt recovery rates

In the model, α_k is equal to the ratio of book assets immediately after restructuring, to the ratio of book assets before bankruptcy. In the BRD sample described above, the

average reduction in book assets from filing to emergence was:

$$\widehat{\alpha}_k = 0.720. \quad (\text{A18})$$

Additionally, the average ratio of book leverage (book debt to book assets) after emergence, relative to filing, was:

$$\frac{\widehat{\alpha}_b}{\alpha_k} = 0.805. \quad (\text{A19})$$

Table A-2 reports these values along with the distribution of these ratios across Chapter 11 cases. We use these averages to calibrate the values of α_b and α_k to:

$$\alpha_k = \widehat{\alpha}_k = 0.720, \quad \alpha_b = \frac{\widehat{\alpha}_b}{\alpha_k} \times \widehat{\alpha}_k = 0.580. \quad (\text{A20})$$

A.3.2 Estimation methodology

We next describe the method we use to estimate the remaining model parameters.

A.3.2.1 Estimating equations

In our estimation, we use the following reduced-form panel regressions:

$$\Phi_{i,t} = \widehat{\Phi} + \varepsilon_{i,t}^{(1)} \quad (\text{A21})$$

$$dep_{i,t} = \widehat{dep} + \varepsilon_{i,t}^{(2)} \quad (\text{A22})$$

$$tax_{i,t} = \widehat{tax} + \varepsilon_{i,t}^{(3)} \quad (\text{A23})$$

$$int_{i,t} = \widehat{int} + \varepsilon_{i,t}^{(4)} \quad (\text{A24})$$

$$x_{i,t} = \widehat{x} + \varepsilon_{i,t}^{(5)} \quad (\text{A25})$$

$$x_{i,t} = \widehat{c}^{(x)} + \widehat{\rho}x_{i,t-1} + \varepsilon_{i,t}^{(6)} \quad (\text{A26})$$

$$\Phi_{i,t} = \widehat{c}^{(\Phi)} + \widehat{\Gamma}x_{i,t} + \varepsilon_{i,t}^{(7)} \quad (\text{A27})$$

to target the seven following moments:

$$\hat{\beta} = \left\{ \widehat{\Phi}, \widehat{dep}, \widehat{tax}, \widehat{int}, \widehat{x}, \widehat{\rho}_x, \widehat{\Gamma} \right\}, \quad (\text{A28})$$

We also estimate the five additional reduced-form regressions:

$$roa_{i,t} = \widehat{roa} + \varepsilon_{i,t}^{(8)} \quad (\text{A29})$$

$$iss_{i,t} = \widehat{iss} + \varepsilon_{i,t}^{(9)} \quad (\text{A30})$$

$$pay_{i,t} = \widehat{pay} + \varepsilon_{i,t}^{(10)} \quad (\text{A31})$$

$$sq_{i,t}^{(\Phi)} = \widehat{var}^{(\Phi)} + \varepsilon_{i,t}^{(11)} \quad (\text{A32})$$

$$sq_{i,t}^{(x)} = \widehat{var}^{(x)} + \varepsilon_{i,t}^{(12)} \quad (\text{A33})$$

where:

$$sq_{i,t}^{(\Phi)} \equiv \Phi_{i,t}^2 - \mathbb{E}(\Phi_{i,t})^2 \quad (\text{A34})$$

$$sq_{i,t}^{(x)} \equiv x_{i,t}^2 - \mathbb{E}(x_{i,t})^2. \quad (\text{A35})$$

We then use the seven following additional non-targeted moments to assess model fit:

$$\left\{ \widehat{roa}, \widehat{iss}, \widehat{pay}, \widehat{var}^{(\Phi)}, \widehat{var}^{(x)}, \widehat{c}^{(x)}, \widehat{c}^{(\Phi)} \right\}. \quad (\text{A36})$$

These represent, respectively, the average ratio of EBITDA to book assets, the average gross rate of debt issuance, the average payout rate, and the average squared within-firm deviation of gross investment and leverage, and the intercepts from Equations (A26) and (A27), respectively.

In our baseline exercise, prior to estimating the panel regression models, we apply the within-firm transformation to the data. For any raw variable $\tilde{z}$, we construct:

$$z_{i,t} \equiv \tilde{z}_{i,t} - \hat{\mathbb{E}}_i [\tilde{z}_{i,t}] + \hat{\mathbb{E}} [\tilde{z}_{i,t}], \quad (\text{A37})$$

where $\hat{\mathbb{E}}_i$ denotes the within-firm mean and $\hat{\mathbb{E}}$ denotes the unconditional mean. Thus the empirical moments $\{\hat{\Phi}, \widehat{dep}, \widehat{tax}, \widehat{int}, \widehat{x}, \widehat{roa}, \widehat{iss}, \widehat{pay}\}$ are unconditional (or sample) averages, while the empirical moments $\{\hat{\rho}, \hat{\Gamma}, \widehat{var}^{(\Phi)}, \widehat{var}^{(x)}\}$ are within-firm averages or coefficients. We treat the data in this way in our baseline because the model features no firm fixed effects, and is best interpreted as describing the within-firm evolution of leverage and investment. However, we discuss the impact of this data transformation in our estimates below.

Finally, because we are interested in the implications of policy interventions on the aggregate dynamics of investment, we weight all of our observations in estimating the panel data model described above, using the firm-year observation's share of total book assets in that year's sample as the weight. Likewise, below, when estimating the corresponding regressions in the model, we also weight observations by their relative assets share. Thus all the moments we report and use should be interpreted as asset-weighted moments.

A.3.2.2 Estimator

We estimate $\Theta = (a, \delta, \theta, \kappa, \sigma, m, \gamma)$ by solving:

$$\hat{\Theta} \in \arg \min_{\Theta \in \Omega} Q(\Theta), \quad Q(\Theta) \equiv \left(\hat{\beta} - \beta(\Theta) \right)' \widehat{\mathbf{W}} \left(\hat{\beta} - \beta(\Theta) \right), \quad (\text{A38})$$

where $\widehat{\mathbf{W}}$ is a weighting matrix, $\beta(\Theta)$ are the model-implied counterparts to the data moments, and Ω is the compact subset of $\mathbb{R}^{N_\Theta}$ given by:

$$\Omega = \left\{ \Theta \in \mathbb{R}^{N_\Theta} \text{ s.t. } \Phi(r + \rho\sigma v_e) > a - \theta(a - \delta) > \Phi(-1/\gamma) \text{ and } \forall k, \Theta_k \in [\underline{\Theta}_k, \overline{\Theta}_k] \right\},$$

and where $\underline{\Theta}_k < \overline{\Theta}_k$ are finite upper and lower bounds on the k^{th} structural parameter, which we impose throughout the estimation. The parameter restriction is consistent with (A2) and it ensures that the model with no debt issuance always has a solution for the set of parameters that we search over.

The model-implied values $\beta(\Theta)$ of the reduced-form data moments we target, $\hat{\beta}$, are not immediately available from the model in closed form. A natural approach would be to use simulation, yielding a Simulated Method of Moments estimator (Pakes and

Pollard, 1989). Indeed, SMM estimators are the most commonly used approach for estimating dynamic capital structure and investment models.⁶⁴ However, these methods require simulating the model, which can be computationally intensive and which adds simulation noise to the computed values of $\hat{\beta}$.

In [Appendix A.3.7](#) below, we show how to compute the model-implied moments without simulating the model. In this respect, our approach is not new: it builds on the non-stochastic, distribution-based methods introduced by [Young \(2010\)](#) and now widely used in quantitative models of firm dynamics (e.g., [Terry, 2017](#); [Terry, Whited and Zakolyukina, 2023](#)). Rather than simulating a panel of firms, these methods represent the cross-sectional distribution on the model’s grid and propagate it using the transition implied by the optimal policies, which avoids cross-sectional sampling noise.⁶⁵ Our specific contribution is to exploit the fact that the moments we target are coefficients of dynamic panel regressions evaluated under the model’s ergodic distribution. We show that these coefficients can be computed in closed form by combining the ergodic distribution with the transition kernel $\mathbf{H}(\tau) = \exp(\tau\mathbf{L})$ evaluated at the relevant horizons τ , without iterating the distribution forward in time. Heuristically, these are the moments that would be obtained by simulating data from the model for an infinite number of periods, and estimating the panel data regressions in Equations (A21)-(A33) on the resulting dataset. Relative to non-stochastic simulation, which marches the distribution forward period by period (as is necessary, for instance, when the cross-section is non-stationary because of aggregate shocks, as in [Terry, Whited and Zakolyukina 2023](#)), this delivers the targeted regression coefficients directly, increasing the speed of estimation and eliminating the impact of simulation noise on $\beta(\Theta)$, thus making the variation of model-implied moments with respect to Θ smoother.

[Appendix A.3.7](#) discusses the details of this method, and shows that it can be applied generally. Additionally, that section provides results on consistency and asymptotic normality for the resulting estimator, and it discusses the choice of $\hat{\mathbf{W}}$. In our estimation, we use $\hat{\mathbf{W}} = \hat{\mathbf{S}}^{-1}$, where $\hat{\mathbf{S}}$ is a consistent estimator of the variance-covariance matrix of data moments. For $\hat{\mathbf{S}}$ we use the [Arellano et al. \(1987\)](#) estimator of the asymptotic

⁶⁴See [Hennessy and Whited \(2005\)](#), [Hennessy and Whited \(2007\)](#), or [Ottonello and Winberry \(2020\)](#).

⁶⁵In a continuous-time setting, the relevant object is the intensity matrix (or infinitesimal generator) implied by the optimal policies; see [Achdou et al. \(2014\)](#).

variance for each equation in (A21)-(A33), ensuring that our standard errors are robust to heteroskedasticity (over time and across firms) and arbitrary correlation structures of error terms within firms.

A.3.3 Data for baseline estimation

Our data source in our baseline estimate is the sample of Compustat non-financial firms for the period 2010-2019. In what follows we describe: variable definitions and mapping to model variables; sample selection criteria; summary statistics and comparison to existing estimates of similar moments in the literature.

A.3.3.1 Variable construction

Table A-3 lists the variables we use in the computation of both our targeted and non-targeted moments, their definition in the model, and their definition in the data. All data items are drawn from the Compustat fundamental annual files. Our treatment of the Compustat data is standard and follows other work (Whited, 1992; Frank and Goyal, 2003; Hennessy, 2004; Hennessy and Whited, 2007), with some exceptions, which we now discuss.

First, our measure of leverage $x_{i,t}$ is the ratio of book debt to book assets. In particular, we do not subtract cash from debt or assets, so that our baseline measure represents gross leverage. In the model firms do not hold cash, while in reality they do, so that gross and net leverage ratios can differ. We use gross leverage because it facilitates comparison with existing studies; because it is always positive, as in the model; and because, in the BRD, which we use to calibrate the parameters (α_k, α_b) , only gross leverage ratios are available. In Appendix A.3.6.4 we re-estimate the model using net leverage measures instead.

Second, because we use book leverage to measure $x_{i,t}$, we are implicitly taking the stance that book assets are the empirical counterpart to $k_{i,t}$ in our model. The rest of our measurement approach, in particular the way in which we weight observations, is consistent with this. The exception is our counterpart to gross investment in the model, $\Phi(g(x_{i,t}))$. For this, we use the ratio of capital expenditures to gross property, plant and equipment (PP&E). We tolerate this tension between the measurement of leverage

and investment because capital expenditures from cash flow statements are generally thought to be accurate measures of investment in physical assets, even if physical assets do not account for all of the firm's productive capital. Our measure would be correct if the gross investment rate in PP&E reported by firms coincided with their actual overall investment rate in productive capital. In [Appendix A.3.6.4](#), we discuss an investment measure (net investments, from the cash flow statement) that is consistent with book assets being a proxy for $k_{i,t}$, but which potential includes investments in financial assets.

A.3.3.2 Sample selection

Having constructed the variables described in [Table A-3](#), we then apply sequentially the following sample selection criteria:

1. drop firm-year observations not incorporated in the USA (`fic = "USA"`)
2. drop firm-year observations whose two-digit SIC code (`sic`) is between 60 and 69 (financials), between 91 and 99 (multinationals), or equal to 49 (utilities);
3. drop observations whose name contains the strings "-REDH", "PRE FASB", "PRO FORMA", "INDEX", "-ADR", "-ADS", or has a non-missing and strictly positive value for `adrr`⁶⁶;
4. drop firm-year observations with negative entries for one of the following variables: `sale`, `at`, `oibdp`, `dlc`, `dltt`, `ppeg`;
5. drop firm-year observations with missing values for any of the ratios reported in [Table A-3](#), as well as missing values for the lag of book leverage, $x_{i,t-1}$;
6. keep firm-year observations between 2010 and 2019.

Note that we restrict the sample to firms with strictly positive EBITDA (measured using the Compustat item `oibdp`), consistent with our model. In 2019, negative EBITDA firms account for 2.1% of total assets and 2.0% of total revenue of non-financial Compustat firms, and their EBITDA was 2.4% of aggregate EBITDA (in absolute value). Given

⁶⁶A positive value for this variable indicates that the observation corresponds to an American Depositary Receipt. The other filters used similarly indicate either foreign entities, or stale or redundant observations.

that our moments are weighted by total assets, this screen has little quantitative impact. Finally, in our baseline exercise, we also require firms to be in sample for at least three consecutive years, in order to be able to estimate firm fixed effects. Our final selected sample has 22405 firm-year observations for 3319 different firms.

A.3.3.3 Summary statistics, targeted moments

We winsorize all the variables described in [Table A-3](#) at the 1% and 99% within the selected sample. Resulting summary statistics are reported in [Table A-4](#).

As mentioned above, in the estimation we weight all observations by their share of book assets in a given year. That is, for each firm-year observation, we define:

$$w_{i,t} = \frac{N_t \times k_{i,t}}{\sum_j k_{j,t}}, \quad (\text{A39})$$

where N_t is the number of observations in sample for that year. The first column of [Table A-4](#) reports the weighted average of each variable, that is:

$$\hat{z}_t = \frac{1}{N_t} \sum_i w_{i,t} z_{i,t}.$$

The other moments reported in [Table A-4](#) refer to the unweighted distribution.

In [Table 1](#), we report the targeted and non-targeted moments which we use in the estimation of the model, as described in [Section 3](#) and [Appendix A.3.2.1](#) above. These moments are obtained by estimating the reduced form model described in [A.3.2.1](#), where we weight all regressions using the share of book assets. Specifically, we multiply each data observation in the system of panel regressions by $\sqrt{w_{i,t}}$, where $w_{i,t}$ is the weight described in Equation (A39). Note that in computing the corresponding model-implied moments, we also weight all observations by their share of book assets, as described in [Appendix A.3.7.2](#).

A.3.3.4 Comparison to existing evidence

The targeted and non-targeted moments reported in [Table 1](#) may differ from existing estimates from the literature for two main reasons: first, we only keep firms with positive

EBITDA in sample; second, moments are weighted by assets. Both of these choices imply that our moments will tend to reflect the characteristics of the larger firms in our sample.

Targeted moments The average physical investment rate in our sample is 11.6%, close to estimates from prior work using similar data sources. In his sample of non-financial public firms, [Hennessy \(2004\)](#) documents an average gross investment rate of 12% per year, as a fraction of capital at replacement cost (Table I), while [Hennessy and Whited \(2005\)](#) report a gross investment rate of 7.9% per year, as a fraction of book assets, in their baseline sample.

Average book leverage in our sample is 30%. This number is consistent with estimates of gross book leverage for non-financial firms reported in the literature. For instance, [Lemmon, Roberts and Zender \(2008\)](#) document that gross leverage is 27% in their sample of nonfinancial firms between 1965 and 2003 (see their Table I). Two other measures of leverage are commonly used in the literature: net leverage, which subtracts cash from debt; and the ratio of debt to EBITDA. In our sample, the ratio of net debt to EBITDA is 19% on average. By comparison, [Hennessy and Whited \(2007\)](#) target a net debt to assets ratio of 12.04% (Table I) for their baseline estimation, and 14.52% (Table III) in their restricted large firm sample. Finally, in our sample, the ratio of gross debt to EBITDA is 3.12, on average; by contrast, in a sample of rated firms, [Baghai, Servaes and Tamayo \(2014\)](#) report an average gross debt to EBITDA ratio of 3.7 for 2009.⁶⁷

An important moment in our estimation is the covariance between investment and leverage, $\hat{\Gamma} = -11.1\%$. Recall that, in our baseline estimation, this number is estimated using firm fixed effects, so that it represents a within-firm covariance. The economic interpretation is that investment declines by $10p.p. \times -11.1\% = -1.11p.p.$ for every $10p.p.$ increase in leverage.⁶⁸ We discuss existing estimates of this elasticity in more detail in [Section 3.3](#) and [Appendix A.3.5.2](#) below.

Another important moment is the autocorrelation of book leverage, which we use to discipline leverage persistence. Our baseline estimate of $\hat{\rho}_x = 0.67$ represents the within-firm autocorrelation of leverage. It can be compared to the evidence from [Lemmon,](#)

⁶⁷Our sample weighted average ratio of net debt to EBITDA is 1.93.

⁶⁸Note that in our sample, the average, within-firm standard deviation of leverage, defined as the squared root of the residual from Equation (A35) is approximately 18p.p.

Roberts and Zender (2008), Section IV. The authors first residualize book leverage on firm fixed effects, along with time effects and firm controls, and then estimate the same AR(1) process for the residuals as we do in Section 3, Equation (25). This effectively represents a within-firm autocorrelation of leverage, consistent with our baseline; their point estimate is 0.66, very close to ours. In Section 3.3 and Appendix A.3.5.2, we compare the evidence of Lemmon, Roberts and Zender (2008) on leverage persistence in the cross-section of firms with the implications of our model in more detail.

Another recent study of the persistence of leverage is DeAngelo, Gonçalves and Stulz (2018). In their Table 1, the authors report that the median annual change in leverage for firms with book leverage between 0.30 and 0.40 is -0.010 . These estimates do not include controls for firm-level fixed effects. Using Equation (25), we have:

$$\mathbb{E}_{t-1} [\Delta x_{i,t}] = (1 - \rho_x) (\hat{x} - x_{i,t-1}),$$

where $\hat{x}$ is the unconditional average book leverage. Consider a firm with $x_{i,t-1} = 0.40$ (or 10p.p. higher leverage than the mean). Then, using our point estimate of $\rho_x = 0.94$ obtained without firm fixed effects, the decline in leverage predicted by the reduced-form model of Equation (25) is $(1 - 0.94) \times (-0.10) = -0.006$, somewhat smaller than their estimates. By contrast, if we use our baseline autocorrelation — estimated within firm — we obtain a predicted leverage decline of -0.033 . Thus our two estimation procedures (with and without firm fixed effects) bracket the speed of mean-reversion implicit in their data.

Finally, our estimate of the ratio of depreciation expenses to EBITDA is 37.15%, equivalent to approximately $15\% \times 37.15\% = 5.5\%$ of book assets. Given that the average ratio of physical capital (measured by ppeg) to book assets is approximately 56% in our sample, this is also equivalent to approximately $5.5\%/56\% = 9.8\%$ of productive capital. Hennessy and Whited (2007), page 1722, report a ratio of depreciation expenses to capital of 15%, somewhat higher than this number. By contrast, Cooper and Haltiwanger (2006), p627, report estimates of the rate of economic depreciation for plants in the manufacturing sector ranging from 6.3% to 7.6%. Our estimates are closer to the latter two values, though they remain higher, presumably because, in many instance, accounting rules allow firms to accelerate depreciation relative to the economic life of the asset. We

estimate the average ratio of taxes paid to EBITDA to be 12.06%, reflecting the interest and depreciation expenses tax shields (as discussed below, our point estimate for the average marginal tax rate is $\theta = 25.53\%$, closer to average headline rates for the sample we consider.) Finally, the average ratio of interest expenses to EBITDA we measure, 13.72%, implies an interest coverage ratio (defined as the ratio of EBITDA to interest expense) of approximately 7. By contrast, the numbers cited in [Frank and Goyal \(2009\)](#) (Table AII) imply an interest coverage ratio of approximately 5; this difference is qualitatively consistent with declining interest costs for the period 2010-2019 (our sample) relative to their period (2000-2003).

Non-targeted moments Among our non-targeted moments, the weighted average the return on assets, $\widehat{roa}$, is 15.1%. As a point of comparison, [Hennessy and Whited \(2005\)](#) report a value of 14.6% for this ratio in their sample (see their Table II). Using aggregate data from the Flow of Funds, [Crouzet and Eberly \(2023\)](#) find that the aggregate ratio of EBITDA to physical assets for non-financial firms is 22.1% for the 2001-2017 period. Given that the average ratio of physical capital to book assets is approximately 56% in our sample, the resulting aggregate ratio of EBITDA to physical capital is 26.7%, somewhat higher than the Flow of Funds estimate. The average equity payout rate is approximately 2.7% per year. By comparison, [Hennessy and Whited \(2005\)](#) and [Hennessy and Whited \(2007\)](#) respectively report payout rates of 4.2% (Table II) and 8.9% (Table I), and [Frank and Goyal \(2003\)](#) (Table 2) report payout rates of approximately 5% per year in the 1975-1990 period, and 10% per year in the 1990-2000 period. These rates are expressed as a fraction of book assets, whereas we express ours as a fraction of the lagged market value of equity, so as to be able to interpret them as a rate of return. Relative to book assets, the average payout rate in our sample is 4.8%, consistent with the findings of other work described above. For the average debt issuance rate, [Frank and Goyal \(2003\)](#) define debt issuance rates as the ratio of net issuance of long-term debt relative to book assets $(dltis - dltr)/at$, finding an average debt issuance rate of 3.4% for 1998 (Table 2). Defined in the same way as [Frank and Goyal \(2003\)](#), the weighted average debt issuance rate in our sample is 4.5%, in line with their estimates.

A.3.4 Baseline results

Here, we provide a more detailed discussion of our baseline estimation results, focusing in particular on identification, as well as on how our estimates compare to existing results in the literature.

A.3.4.1 Intuition for identification

Our baseline approach estimates seven parameters using seven moments, and is therefore exactly identified. Four moments are key to our estimation procedure: the average investment rate, $\hat{\Phi}$; average leverage, $\hat{x}$; the within-firm autocorrelation of leverage, $\hat{\rho}_x$; and the within-firm sensitivity of investment to leverage, $\hat{\Gamma}$. **Figure A-2** shows how these four moments vary with the four main parameters they help identify, which are, respectively, the return to capital, a , the volatility of capital quality shocks, σ , the convexity of capital adjustment costs, γ , and the maturity of debt at issuance, m . The main intuition for this graph is that identification in this model is “almost recursive”. Put differently, **Figure A-2** is “almost” lower-diagonal, in the sense that the sensitivity of moments to parameters in the upper triangular portion of **Figure A-2** is relatively small.

The top row of **Figure A-2** shows that variation in the model’s average gross investment rate primarily identifies the average product of capital a . The reason is that, for a given discount rate and a given tax rate, a is the key determinant of Tobin’s q in this model, which in turn determines the steady-state gross investment rate. The slope of the average investment rate with respect to γ is negative but relatively small, while the average investment rate is almost insensitive to the volatility of capital quality shocks σ or to the maturity of debt, m .⁶⁹ Investment adjustment costs γ also only have small effects on the level of investment (consistent with the assumption that they depend on the square of the expected net growth rate of the capital stock).

Given a value for a , the second row of **Figure A-2** shows that the average leverage falls with the volatility of capital quality shocks, σ , while the sensitivity of investment

⁶⁹An increase in the volatility of capital quality shocks shifts the default boundary to the right, because the option value of continuing for equity holders increases. This tends to increase average leverage and lower investment rates. At the same time, a higher volatility increases (in absolute value) the slope of the debt price function with respect to leverage, so that debt issuance is lower with higher volatility. This tends to lower average leverage and increase investment. Quantitatively, these effects approximately offset each other.

with respect to leverage varies very little with σ . The average leverage therefore helps identify the parameter σ . A higher volatility leads to lower average leverage because the debt price function becomes steeper as volatility increases. On the other hand, the debt price function is relatively insensitive to capital adjustment costs γ because these have only a small (local) impact on the average net growth rate of capital. As a result, variation in γ has a limited impact on overall leverage. Likewise, the variation in m (the inverse of debt maturity), has relatively little impact on leverage.

Given values for a and σ , the third and fourth rows of [Figure A-2](#) show that the autocorrelation of leverage and the sensitivity of investment to leverage help identify m and γ . The autocorrelation of leverage decreases with m , as longer term debt makes leverage more persistent. On the other hand, the autocorrelation of leverage is relatively insensitive to γ , the convexity of investment adjustment costs. However, the sensitivity of investment to leverage decreases sharply (in absolute value) with the magnitude of adjustment costs. All else equal, in response to a capital quality shock that increases their leverage, firms cut back investment less when investment adjustment costs are higher.

Finally, note that for a given value of a , identification for the other three parameters of the model (the tax rate θ , the depreciation rate δ , and the coupon rate κ) comes more directly from the three remaining moments we choose to target (the ratio of taxes to EBITDA, the ratio of depreciation expenses to EBITDA, and the ratio of interest payments to EBITDA).

A.3.4.2 Point estimates and comparison to existing literature

Estimation results are reported in [Table 1](#). Three parameters are particularly important to determine the quantitative effects of aggregate shocks: the volatility of capital quality shocks, σ ; the curvature of capital adjustment costs, γ ; and the debt amortization rate, m . Our estimates of these parameters relate to the existing literature as follows.

Because we assume that the average return to capital, a , is constant, the volatility of capital quality shocks, σ , can be compared to estimates of the volatility of shocks to revenue or operating profits in related models of investment with financing or real frictions. Moreover, recall that σ is the total annualized volatility of these shocks (both idiosyncratic and aggregate). Using a similar (but older) panel of firms as ours, the estimates from [Hennessy and Whited \(2005\)](#) and [Hennessy and Whited \(2007\)](#) imply annualized

standard deviations of approximately 0.12; however, these are net of time effects, so they only represent the idiosyncratic component of profitability shocks. Likewise, the point estimates for the capital quality shocks in [Ottonello and Winberry \(2020\)](#) implied annualized standard deviations of approximately 0.12, but they are also net of time effects. [Chen \(2010\)](#) calibrates his model to match both the aggregate volatility of corporate profits in NIPA data and historical estimates of default rates and finds an overall annualized volatility of log operating profits of $\sqrt{0.14^2 + 0.24^2} = 0.28$. Finally, in their analysis of manufacturing plants in the LBD, [Cooper and Haltiwanger \(2006\)](#) document a total annual standard deviation of log innovations to operating profits of 0.35, somewhat higher than our point estimate of 0.50 (see their p.623). Thus our estimate is generally in line with the literature.

Our point estimate for the adjustment cost parameter γ is 4.4. There is a fair amount of variation in the values of this parameter in the existing literature; our point estimate is within the range of estimates. Recently, [Falato et al. \(2022\)](#) report estimates of γ ranging from approximately 2 to 20, with higher values obtained in versions of their model that do not allow for intangible capital, as is the case in our baseline model. [Belo et al. \(2022\)](#) also report a similar range of estimates. Finally, [Ottonello and Winberry \(2020\)](#) use an (aggregate) adjustment cost corresponding to a value of γ of approximately 4.⁷⁰ However, note that these articles use the slope of investment with respect to q in order to estimate γ , whereas we use the slope of investment with respect to leverage (the underlying state variable, in the model).

Finally, our point estimate of the debt amortization rate is $m = 0.18$, imply an average debt maturity of $1/0.18 = 5.56$ years at issuance. [Gomes, Jermann and Schmid \(2016\)](#) use a similar model of geometrically maturing long-term debt; their calibration implies an average debt maturity of $1/(4 \times 0.05) = 5.00$ years at issuance, close to ours. [Dangl and Zechner \(2021\)](#), in their theoretical analysis of the effect of debt maturity on leverage dynamics, focus on maturities ranging from 10 to 30 years, closer to the typical maturities

⁷⁰Other estimates include: [Hayashi \(1982\)](#), who finds a value of approximately $\gamma = 20$ using aggregate data on the US corporate sector; [Gilchrist and Himmelberg \(1995\)](#), who estimate a value of approximately $\gamma = 3$ in their sample of rated firms; and [Hall \(2001\)](#), who considers values ranging from $\gamma = 2$ to $\gamma = 8$. [Cooper and Haltiwanger \(2006\)](#) estimates a much lower value of γ , but in a model with concavity in the revenue function, and without using estimates of the investment-Q relationship. Their estimated model implies an investment-Q slope of approximately 0.2, consistent with $\gamma = 5$.

of corporate bonds for US large non-financial firms. By contrast, our five-year point estimate is closer to the maturity of syndicated loans at issuance (see, for instance, [Mian and Santos 2018](#), Figure 3).

The remaining point estimates are also within the range of typical estimates in the literature. In particular, the corporate income tax rate of $\theta = 0.26$, for instance, is close to, though somewhat lower than, the mean (29%) and media (33%) values obtained using simulation methods for the Compustat panel from 1980-2007 in [Blouin, Core and Guay \(2010\)](#); the discrepancy may be accounted for by the fact that our moment is asset-weighted.

A.3.4.3 Model fit to non-targeted moments

[Table 1](#) also reports the model-implied values of seven other moments at our baseline point estimate, and compares them with their data counterparts. These seven non-targeted moments are described in Equations (A29)-(A35). The model fits reasonably well the gross debt issuance rate, while it somewhat over-estimates both the equity payout rate and the rate of return on assets, relative to the data. In terms of second moments, the model tends to underestimate the within-firm dispersion of investment rate, while it tends to somewhat over-estimate the within-firm dispersion of leverage. The higher empirical dispersion of leverage is likely due to the fact that we abstract from fixed investment costs, which can amplify the dispersion of firm-level investment rates (as in, e.g., [Caballero and Engel 1999](#)). The model-implied ergodic standard deviation of leverage is approximately 0.109, while the data-implied counterpart is 0.077. We note that because these are within-firm estimates, short panel dimensions for some firms may lead to a downward bias in the estimated value of this moment. In our model, an important source of leverage dispersion stems from our default settlement mechanism. In details, given our calibration and our point estimate, firms are re-injected relatively far away from the model-implied average leverage. In our baseline model, at reinjection, firms have book leverage of approximately 0.70 (consistent with exit occurring at $\bar{x} = 0.88$, and a calibrated ratio $\alpha_b/\alpha_k = 0.80$), while the modal level of leverage in the (weighted) stationary distribution is approximately 0.24. This combination of parameters implies that default will be followed by periods of high leverage, even after restructuring, increasing the overall level of leverage dispersion in the stationary distribution.

A.3.5 External validation

Here, we discuss our external validation exercises.

A.3.5.1 Leverage persistence

Data moments For this exercise, we extend the sample of firms to include all firm-year observations of Compustat non-financial firms from 1979 to 2019. We apply the same sample selection criteria as those described in [Appendix A.3.3.2](#).

We construct average leverage for firms of different quartiles of initial leverage by following the methodology described in [Lemmon, Roberts and Zender \(2008\)](#). Specifically, in each year t , we form four portfolios of firms, corresponding to the four quartiles of the empirical distribution of book leverage. We call year t the “initial year”, and the corresponding portfolios the “initial leverage” portfolios. We then compute the average leverage in each initial leverage portfolio in each year $t + k$, $k \geq 0$. This gives us four time series for each year t , one for each initial leverage portfolio. We then compute the equal-weighted average of these time series across all the years in our sample, to obtain the plots reported in [Figure A-3](#), where these data moments are reported in black. In each graph, the horizontal axis is the time since portfolio formation (i.e. year t), which we refer to as “event time”.

There are four different panels in [Figure A-3](#). The bottom left panel reports the moments obtained without applying the fixed effects transformation to firm-level leverage, and when we do not weight observations according to their relative size. The bottom right panel reports the same moments, when we first remove a firm fixed effect from each firm, and then add back the unconditional (sample) mean of leverage, as in [Section 3](#).

Both top panels of [Figure A-3](#) repeat this exercise, but weight each observation by their relative size, consistent with our baseline estimation exercise described in [Section 3](#). In order to do this, we first bin observations in 101 equally spaced bins of book leverage, from 0 to 1, where observations with leverage larger than 1 are allocated to the highest bin. Fix an initial year t . In each subsequent year $t + k$, $k \geq 0$, we compute the share of assets of firms in each leverage bin, as a fraction of total assets of firm-year observations across all four portfolios. This gives us a weight $w_{b,t,t+k}$ for each bin b for initial year t and event-time $t + k$. For initial year t and event-time $t + k$, we then compute average

leverage in each portfolio by weighting each observation according to its corresponding weight $w_{b,t,t+k}$.

Model moments In order to compute the model counterpart to the data moments reported in [Figure A-3](#), we use two different estimates of the model: our baseline estimate, which uses weighted moments and is net of firm fixed effects (bottom right panel); and an alternative estimate, which uses unweighted moments but is also net of firm fixed effects (top right panel). The point estimates and model fit are reported in [Table A-7](#).

In each case, we use the results of [Appendix A.3.7](#), described below, to compute average expected leverage by initial leverage quantile. Specifically, these results indicate that for any initial value of leverage x_0 , we can compute the expected leverage at horizon τ by:

$$\mathbb{E}_{\Theta}(x_{\tau}|x_0 = x_i) = (\mathbf{H}(\tau)\mathbf{X})_i, \quad i = 1, \dots, N_x.$$

Here, N_x is the number of points on the grid for leverage used to solve the model, $\mathbf{X}$ is an $N_x \times 1$ vector with entries equal to the grid points $(x_i)_{i=1}^{N_x}$, Θ is a particular set of structural parameters for which we have solved the model, and $\mathbf{H}(\tau)$ is a matrix whose expression is reported in [Appendix A.3.7](#). We then compute the expected leverage in a particular quantile of the distribution of leverage, defined by two boundaries $\underline{x} < \bar{x}$, as:

$$\mathbb{E}_{\Theta}(x_{\tau}|x_0 \in [\underline{x}, \bar{x}]) = \frac{\sum_{x_i \in [\underline{x}, \bar{x}]} \mathbb{E}_{\Theta}(x_{\tau}|x_0 = x_i) f_i}{\sum_{x_i \in [\underline{x}, \bar{x}]} f_i},$$

where $(f_i)_{i=1}^{N_x}$ denotes the stationary distribution of the model. As explained in [Appendix A.3.7](#), this can be done either for the unweighted or for the asset-weighted versions of the stationary distribution f and the Markov intensity matrix $\mathbf{H}$.

A.3.5.2 The magnitude of debt overhang costs in steady-state

Elasticity of investment with respect to leverage [Table 2](#) compares the model-implied elasticity of investment to leverage to existing reduced-form evidence characterizing this elasticity. We report three different measures of the model-implied elasticity: the baseline case; a case with adjustment costs fixed to $\gamma = 2.5$; and a case with adjustment costs fixed to $\gamma = 7.5$. [Table A-5](#), [Table A-6](#) and [Table A-7](#) report the corresponding model estimates.

Note that our baseline estimate is such that the model-implied elasticity is lower than the data counterpart that we target. This is for two reasons. First, while this elasticity is related to the adjustment cost parameter γ , the value of this parameter also influences other moments targeted in the estimation. Second, among the moments we target, this elasticity is the most imprecisely estimated, so that it is the moment the estimator puts the least weight on.

In reporting the coefficients from prior work in [Table 2](#), where appropriate, we scale them so that they are comparable to the model-implied moment that we are interested in assessing. A simple adjustment is that, because we measure investment rates in percentage points (e.g. 12p.p., instead of 0.12) but leverage ratios in absolute units (e.g. 0.3 instead of 30%), we multiply the elasticities by 100 when the corresponding paper uses the same units for both investment rates and leverage. Additionally, [Ahn, Denis and Denis \(2006\)](#) scale capital expenditures by sales, not assets. They do not report the ratio of sales to assets in their sample. In our sample, that equal-weighted average of that ratio is 0.92, so we scale the coefficient by that ratio. [Opler and Titman \(1994\)](#) do not directly estimate an elasticity with respect to leverage, but rather the differential investment rate between the bottom 8 deciles and the top 2 deciles of their sample. They report a 13.2% difference in leverage between the top 2 deciles and the median leverage ratio, so we scale the coefficient by that number. [Wittry \(2021\)](#) measures leverage relative to the market value of assets, so we scale the coefficient by the inverse of Tobin's Q , which is equal to 1.1 in his sample.

Effects on firm value In [Table 2](#), we also compare the magnitude of debt overhang costs on enterprise value in our model with other estimates in the literature. We again do this for three values of adjustment costs, the baseline estimate, and calibrated low and high values. Our benchmark for computing the relative loss in enterprise value in [Table 2](#) is the no-debt model. Specifically, the moments we report are:

$$\Delta V := \frac{\int_0^{\bar{x}} (e(x) + xd(x)) \hat{f}_\infty(x) dx - e^*}{e^*}, \quad (\text{A40})$$

where e^* denotes enterprise value per unit of capital in the no-debt case. Because we integrate enterprise value per unit of capital under the capital-weighted distribution $\hat{f}(x)$,

ΔV should be interpreted as the ratio of (ergodic average) aggregate enterprise value to aggregate capital in the baseline estimate, divided by the ratio of (ergodic average) aggregate enterprise value to aggregate capital in the no-debt benchmark.⁷¹

Note that the benchmark used to compute agency costs in [Mello and Parsons \(1992\)](#), [Mauer and Ott \(2000\)](#), [Childs, Mauer and Ott \(2005\)](#), [Titman and Tsyplakov \(2007\)](#), and [Moyen \(2007\)](#) is the value of a levered firm that maximizes the total value of equity plus debt. Making this direct comparison in our model is difficult for at least two reasons. First, because our firms do not have operating leverage, an enterprise-value maximizing firm would never choose to default. Second, since our model features continuous debt issuance, if shareholders in our environment were maximizing enterprise value — rather than equity value — the debt issuance policy would be indeterminate, as the cash payments from creditors to shareholders (and vice-versa) wash out for the purpose of enterprise value. For these reasons, we choose to use as a benchmark the value of an unlevered firm, which is an upper bound on firm value in the enterprise-value maximizing case. Thus our reported costs of debt overhang should be thought of as an upper bound.⁷²

Effects on aggregate growth Next, we discuss our quantitative assessment of the steady-state debt overhang costs in the estimated model of [Section 3](#). [Table 3](#) contains the main quantitative results.

Focusing first on Panel A, the column marked "(1)" shows the aggregate growth rate of the capital stock (or equivalently, of aggregate EBITDA) in our baseline estimate. We report two different measures: the first line shows the growth rate including deadweight losses due to exit and restructuring ($\hat{g} - (1 - \alpha_k)\hat{\lambda}_d$, in the notation of [Section 2](#)), while the bottom line focuses on the growth rate of continuing firms only ($\hat{g}$). The column marked "(2)" shows the growth rate in a model where firms are constrained to issue no debt. The column marked "(2)-(1)" shows the difference between this first-best model

⁷¹Recall that the model has balanced growth, so enterprise values must be expressed in intensive units in order to make the comparison.

⁷²Another potential benchmark would be a firm that endogenously chooses debt, but keeps investment at its first-best level; again, the value of an unlevered firm is an upper bound to equity values in this case. This benchmark isolates the effect of debt overhang costs coming from downward distortions in investment. We consider this alternative benchmark in the discussion of aggregate growth effects below.

and the baseline estimate.

Considering the overall growth rate first, the table indicates that growth is 1.69p.p. higher in the first-best model with no debt than in the baseline model. Of this difference, 1.13p.p., or about two-thirds, comes from firms that are away from the default boundary investing at a lower rate (the debt overhang effect), and 0.56p.p., or about one-third comes from deadweight losses associated with default.

The columns marked "(3)" and "(3)-(1)" in [Table 3](#) report a comparison between our baseline model and a model in which, rather than eliminating leverage altogether, we allow firms to borrow, but fix their investment rate at its first-best level. By construction, the growth rate differential for continuing firms with respect to the baseline model is the same (1.13p.p.). The overall growth difference (including the effects of restructuring) is however notably smaller (1.05p.p., versus 1.69p.p.). The reason for this difference is that, in the model with fixed investment, firms systematically default earlier (i.e. at a lower default threshold) than in the model with endogenous investment. Thus deadweight losses of default have a more negative impact on growth than in our baseline model; in other words, direct costs of leverage associated with restructuring are *lower* in our baseline model than in the model with fixed investment rates.

Panel B repeats this exercise, but in a model where we calibrate the convexity of investment adjustment costs to be $\gamma = 7.5$. In order to allow for a fair comparison between the baseline model and the two alternative version of the model (no debt and debt but fixed investment), we fully re-estimate the model, but do not target the elasticity of investment with respect to leverage. Results from this estimation are reported in [Table A-7](#). The implied equilibrium aggregate growth rate of the model is 2.73p.p., and 3.37p.p. when focusing only on the weighted growth rate of continuing firms. With high adjustment costs of capital, Panel B. shows that debt overhang effects are markedly more muted. In fact, focusing only on continuing firms, the growth rate is numerically very close between the baseline model and the two alternative models (the difference is less than one basis point per annum). All of the real effects of leverage on growth are driven by default and restructuring costs, as opposed to investment distortions away from the default boundary.

Overall, the message from this analysis is that the debt overhang costs we obtain are quantitatively large in our baseline estimation, in the order of 1.00% per annum when

focusing strictly on effects net of any direct deadweight losses associated with default and restructuring. These costs are much smaller, or virtually eliminated, for higher value of adjustment costs.

A.3.6 Alternative estimates of the baseline model

We now turn to a discussion of how our baseline estimates change when targeting different data moments, or when changing calibrated parameter values. [Table A-5](#), [Table A-6](#) and [Table A-7](#) contain detailed results.

A.3.6.1 Small firms

We first explore whether in the sample of small firms, the key moments that we target in our estimation differ substantially, and if so, whether these differences have a large quantitative impact on our key counterfactuals.

To do so, we re-estimate our model on the sub-sample of our panel made up of the firm-quarter observations with book assets below the median. This leaves us with a sample of 11094 observations for 1984 firms.⁷³ To get a sense of the size differences between these two groups of firms, the average undeflated value of book assets in this subsample is \$348m, while in the baseline sample including both small and large firms, it is \$6092bn.

Results of the estimation on the small firm subsample are reported in [Table A-5](#), in the column marked "(A1): small firms". Two main differences with the baseline estimation results stand out. First, in the data, the autocorrelation of leverage is lower among small firms (the difference with the same moment in the overall sample is statistically significant) — most likely due to shorter term loan debt used by these smaller firms, relative to the typical long-term bond debt used by large firms. As a result, the point estimate for m , the rate at which debt matures, is higher, and statistically different from

⁷³Note that, in principle, a firm could be above the median in some years, and below the median above the others, creating gaps in the sequence of observations for that particular firm. We tolerate this issue in order to make our definition of small firms as simple as possible. However, we note that this issue is not pervasive; in our small firm subsample, 248 firms have at least one observation that also belongs to the large firm subsample — observations above the median in particular quarter —, and 270 observations are immediately preceded by an observation belonging to the large firm subsample

our baseline estimate. According to this estimate, shocks that affect leverage in the short-run are thus less likely to persist. Second, in the data, the within-firm sensitivity of investment to leverage is smaller (more negative) in the small firm sample. This leads to a lower point estimate for the adjustment cost γ , indicating that, in this version of the model, investment is more likely to respond to short-run changes in leverage. A caveat is that the point estimates for γ in the two models are economically similar, and their difference is not statistically significant, so the differences in this respect may be muted.

Other differences between the two estimates are notable. First, small firms have substantially higher estimated values for a , consistent with higher measured investment rates (and with higher measured values of return on assets, though we do not target this moment). Second, they face capital quality shocks that have a higher volatility σ , leading them to take on less leverage in steady-state. Third, in the data their interest payments per dollar of EBITDA are substantially larger than large firms, implying, through the lens of the model, coupon rates that are substantially above the discount rate r . Finally, in the data, these firms pay fewer dollar of taxes per dollar of EBITDA than in our baseline sample. But their interest and depreciation expenses (relative to EBITDA) are both substantially higher. Given the deductibility of these expenses, our estimation implies an effective corporate income tax rate that is higher in this sample than in our baseline sample, and at the upper end of the range of estimates of marginal corporate income tax rates reported in the literature (see, e.g., [Blouin, Core and Guay 2010](#)).

A.3.6.2 Residualized investment rates

A generic concern with our approach is model misspecification. More specific to the question we are interested in, a potential concern is that there are determinants of investment which (a) are ignored in our model, and (b) are correlated with leverage, leading to potential biases in the point estimate of the sensitivity of leverage to investment, one of the key moments which we target in our estimation. If our estimate is biased upward (in absolute value), we may be over-stating the elasticity of investment to leverage in our model, leading to exaggerated debt overhang effects.

In what follows, we make progress in addressing this issue by re-estimating our model after residualizing investment rates on two potential additional determinants of investment that our model does not capture. First, firms in our model do not hold

cash, whereas a large literature has shown that investment may respond positively to unexpected movements in firms' cash buffers.⁷⁴ Second, firms in our model do not face time-varying investment opportunities (in fact, investment opportunities are identical across firms, though our baseline estimation method addresses fixed heterogeneity in investment rates across firms by introducing firm fixed effects.)

In a first stage, we therefore estimate the following regression on our panel:

$$\Phi_{i,t} = \alpha_i + \beta_c c_{i,t-1} + \beta_Q Q_{i,t-1} + \eta_{i,t}, \quad (\text{A41})$$

where $\Phi_{i,t}$ is the gross investment rate, and $c_{i,t-1}$ and $Q_{i,t-1}$ are measures of the cash-to-asset ratio and of Tobin's Q for firm i (lagged one year). For $c_{i,t-1}$, we use the measured ratio of EBITDA to book assets (denoted $roa_{i,t-1}$ in Table A-4). For Tobin's Q , we use the ratio of the book value of debt plus the market value of equity to the book value of assets, consistent with the rest of our measurement strategy.⁷⁵

After estimating Equation (A41), we recover the residuals $\eta_{i,t}$. We then apply to them the same within-firm transformation as in our baseline, i.e. we demean them within firm and add the unconditional mean investment rate, so that, by construction, the sample and within-firm means of $\eta_{i,t}$ is the sample average of gross investment rates, $\Phi_{i,t}$. Finally note that throughout we weight observations by assets, again consistent with our baseline approach.

We then repeat our baseline estimation exercise, replacing $\tilde{\Phi}_{i,t}$ in Equations (25)-(31) with the (re-centered) residuals $\eta_{i,t}$ from Equation (A41). The resulting moments and parameter estimates are reported in Table A-5, columns marked "(A3)". All the targeted moments are the same as in our baseline model, with the exception of the estimated slope of investment with respect to leverage is -10.39 , instead of -11.10 in our baseline. As a result, the point estimates from our model are largely unchanged, as shown in Table (A-5).

To explain these results, in Table A-8, we report results from the following panel regressions:

$$\Phi_{i,t} = \alpha_i + \beta_x x_{i,t-1} + \beta_c c_{i,t-1} + \beta_Q Q_{i,t-1} + \eta_{i,t}, \quad (\text{A42})$$

⁷⁴For a critical review of the evidence on investment cash-flow sensitivities, see Chen and Chen (2012).

⁷⁵See Table A-4 for more details.

where $x_{i,t-1}$ is the lagged book leverage. Column (1) corresponds to our baseline specification; the coefficient is the same as the sensitivity of investment to leverage that we target in that specification, with a one percentage point change in leverage (measured by the ratio of book debt to book assets) leading to a $0.01 \times (-11.10) = 0.11\%$ change in gross investment rate. Adding the control for Cash (column 2), the sensitivity falls to -9.03 . Finally, adding both controls for cash and for Tobin's Q , the sensitivity is -10.99 . None of the two point estimates are statistically different from our baseline. In other words, from the standpoint of the regression we focus on in our estimation, the covariance structure between leverage, cash-to-assets, and Tobin's Q does not induce a significant bias in the estimated investment-leverage sensitivity.⁷⁶ Note that, consistent with the existing literature, the sensitivities of investment to cash and Tobin's Q are positive and statistically different from zero, indicating that the cash-flow sensitivity and investment- Q findings are present in these data even after controlling for leverage.⁷⁷ Thus, residualizing investment essentially does not affect our key moment of interest.

As a robustness check, Table A-8 also reports the variation in the investment-leverage sensitivity when we do not include firm fixed effects in the estimating regressions, in columns (4)-(6). Consistent with columns (4)-(6), there is no statistically significant change in the magnitude of the investment-leverage sensitivity in this case either.

A.3.6.3 Higher deadweight losses from restructuring

A concern with our baseline calibration is that, for some firms, default may lead to liquidation rather than restructuring. Chapter 7 liquidation generally involves higher deadweight losses than Chapter 11 reorganization.⁷⁸ Thus our baseline calibration may understate the deadweight losses associated with default.

⁷⁶The point estimate in column (3) of Table A-8 is different from the targeted investment-leverage sensitivity with residualized investment in Table A-5 because there is some correlation between $x_{i,t-1}$, $roa_{i,t-1}$, and $Q_{i,t-1}$, though as mentioned above it is insufficient to substantially bias our point estimates.

⁷⁷Note that the model does generate within-firm variation in Tobin's average Q and marginal q , even though investment opportunities are fixed, because of debt overhang. This is however distinct from misspecification concerns related to time-varying investment opportunities; moreover, because the sufficient state for the firm's problem is leverage, our key targeted moment, the slope of investment with respect to leverage, should absorb this effect.

⁷⁸Among others, Anttil (2022) uses an instrument based on judge assignment to show that liquidations generate 50% lower family debt recovery rates than reorganizations.

To address this, we re-estimate the model using a higher value for α_k , the parameter characterizing the size of deadweight losses in bankruptcy in our model. We use evidence from [Kermani and Ma \(2023\)](#), which studies a sample of Chapter 11 bankruptcy filings. During Chapter 11 reorganizations, the bankruptcy estate is required to get outside appraisals of the liquidation value of the firm’s existing assets, should they be sold piecemeal. These liquidation values help assess potential asset recovery rates should the firm enter Chapter 7, and are then used to inform the court-ordered restructuring. We view this as an upper bound on the size of deadweight losses that a firm in distress might face.

We use a value of $\alpha_k = 0.45$. This is the weighted average across all asset types for firms in the Chapter 11 sample reported by [Kermani and Ma \(2023, p.207\)](#).⁷⁹ Additionally, we calibrate α_b such that the reduction in leverage post- to pre-bankruptcy is 0.805, consistent with our previous exercise. This leads to a value of $\alpha_b = 0.362$. [Table A-5](#) shows that the resulting estimates of the model are largely unchanged, except for the volatility of capital quality shocks, which is somewhat higher, and the rate at which debt mature, which is also somewhat higher.⁸⁰

A.3.6.4 More alternative estimates

[Table A-6](#) and [Table A-7](#) contain results from more alternative estimation procedures.

The columns marked (A4) in [Table A-6](#) report results when we do not include firm fixed effects in the computation of the panel data moments. In our baseline, we applied the within-firm transformation in such a way that sample averages would be preserved, so panel data moments corresponding to sample averages (i.e. intercepts in reduced-form regressions) are unchanged by construction. The two moments that change are the autocorrelation of leverage, and the sensitivity of investment to leverage. The autocorrelation of leverage increases substantially; the coefficient becomes $\hat{\rho}_x = 0.93$ (we can reject

⁷⁹This is consistent with our treatment of book assets as our measure of productive capital. However it includes the recovery values of items like inventory, cash, receivables, and identifiable intangibles. The recovery rate for PPE reported by [Kermani and Ma \(2023\)](#) is 35%.

⁸⁰This result highlights why the size of deadweight losses in liquidation are difficult to identify from moments derived from the panel alone. The panel does not allow us to observe the frequency and effects of restructurings consistently, and the moments we can measure outside of restructuring episodes are uninformative about the value of α_k .

the null that $\hat{\rho}_x = 1$ at the 5 and 1% levels). This higher leverage persistence, relative to the within-firm specification would be consistent with the presence of firm-specific fixed effects in leverage in the data (which our model does not contain). Additionally, the sensitivity of investment to leverage is smaller in the specification without fixed effects. This would also be indicative of permanent differences in investment rates across firms, say because of differences in a , the return on assets (which would create permanent differences in Tobin's q).⁸¹ Given these two differences, and following the intuition described in Section 3.2, the resulting model has a higher maturity of debt at issuance, m , and a (somewhat) higher investment adjustment cost γ . Thus, in this alternative estimate of the model, movements in leverage are more persistent, but they translate to smaller movements in investment.

The columns marked (A5) in Table A-6 report results when we measure leverage using net debt, instead of gross debt. More specifically, in this exercise we redefine book leverage as the ratio of debt net of cash (Compustat item che), to the ratio of book assets net of cash, so as to obtain a balance sheet statement net of liquid financial assets. This leads to a decrease in our measured average leverage, from 0.30 in the baseline to 0.21 in this exercise. The redefinition of leverage also affects estimates of the sensitivity of leverage to investment and of the autocorrelation of leverage, but the effects are quantitatively more muted.⁸² As a result, in this alternative approach, we estimate a higher volatility of capital quality shocks than we do in the baseline (0.54, compared to 0.35 in our baseline exercise). The point estimate for m is also somewhat higher, imply an initial maturity at issuance of 3.02 years instead of 5.53 years in our baseline.

The columns marked (A6) in Table A-6 report results when we measure gross investment flows using the negative of investing cash flows (compustat item ivncf), instead of capital expenditures only. In this case we then measure the gross investment rate as the ratio of the negative cash flows, to book assets. This measure of investment is con-

⁸¹Heuristically, if the true data generating process had fixed effects in investment, that is, $\Phi_{i,t} = \Phi_i + \Gamma x_{i,t} + \varepsilon_{i,t}$, then without fixed effects the estimated coefficient would be $\hat{\Gamma} = \Gamma + \frac{\widehat{cov}(\Phi_i, x_{i,t})}{\widehat{var}(x_{i,t})}$. Thus the fixed effects would have to be such that firms with high investment relative to average, that is, high Φ_i , also typically have high leverage, so that $\widehat{cov}(\Phi_i, x_{i,t}) > 0$.

⁸²Note that this re-definition also changes the definition of total assets we use (they are now net of cash). Because all of our moments are weighted by assets, this implies that some of the first moments that are measured independently from leverage, like investment, slightly change as well; but these changes are very small economically and statistically indistinguishable from the baseline exercise.

sistent with the rest of our measurement approach, which uses book assets as the proxy for k in the model. This is because investment cash flows (an item of the cash flows) contain not only capital expenditures on property, plant and equipment, but also any net investments in financial instruments (including cash), and any asset acquisitions or divestment, which may impact goodwill. Thus changes in overall book assets (including cash, goodwill, and other long-term assets) are more likely to be correlated with this measure of investment. Table A-6 shows that the resulting estimates of gross investment rates are lower, and that the sensitivity of investment to leverage is somewhat higher with this measure. Both effects imply that the estimated adjustment cost parameter, γ , is lower than in our baseline.⁸³

The columns marked (A7) in Table A-7 report results obtained when constraining the convex component of investment adjustment costs, γ , to be high. As noted in Section 3.2, our point estimate of investment adjustment costs is at the lower end of existing estimates. In Table A-7, we constrain the adjustment cost parameter to be equal to $\gamma = 7.5$, at the upper end of the range of existing estimates. Additionally, we do not target the sensitivity of investment with respect to leverage (the main moment which identifies γ in our baseline estimation). The implied sensitivity of investment with respect to leverage is -3.28 in the estimated model, versus -5.63 in the baseline estimate. The main difference with the baseline estimate of the model is that the estimated average return to capital is even larger than in our baseline estimation; intuitively, this is needed to match the high gross investment rates in the model. In this version of the model, because investment adjustment costs are large, the steady-state magnitude of debt overhang costs is lower than in our baseline estimate. Aggregate growth is 1.30% in steady-state, and 2.08% in a steady-state with no debt, a 0.78% difference, which is smaller than the 0.99% difference implied by our baseline calibration.⁸⁴

The columns marked (A8) in Table A-7 report results obtained when estimating a version of the model in which firms can endogenously choose leverage, but where in-

⁸³Recall that in the model, gross investment is $\Phi_{i,t} = g_{i,t} + \delta + \frac{1}{2}\gamma g_{i,t}^2$, so that gross investment includes adjustment costs to the capital stock. Thus generally, a lower value of γ will reduce gross investment rates, all else equal.

⁸⁴Among continuing firms, in this version of the model, there is almost no difference in growth rates in the no-debt and with-debt version; quantitatively, almost all of the negative effect of debt overhang comes from the deadweight losses associated with occasional default.

vestment is fixed at its first-best (frictionless value). We re-estimate the model without targeting the sensitivity of investment to leverage given that investment is fixed withing firm. Note that the level of the (fixed) investment rate is estimated (and matches the aggregate investment rate), as opposed to being calibrated to match the values in the baseline version of the model. Point estimates of the parameters in the model are close to our baseline version. By construction, however, this version of the model does not feature any endogenous debt overhang effects for firms away from the default boundary.

A.3.7 Details of the estimation methodology

Here we give the details on the estimator we use in Equation (A38). Our method of moments estimator leverages the solution to the model, and in particular, the Markov intensity matrix, to compute directly the model-implied values of the regression coefficients we target. We note that, although we use it in the context of a continuous-time model, it is straightforward to adapt this approach to discrete-time models in which the Markov transition matrix associated with the optimal policies is easy to compute.

We start by describing the reduced-form data moments, then the computation of their model counterparts, and finally the extremum estimator and its properties. The part of the procedure that departs from standard practice is the computation of the model counterparts to the reduced-form data moments: building on the non-stochastic, distribution-based approach of Young (2010), we obtain them in closed form from the model's ergodic distribution and transition kernel rather than by simulation. The rest is standard. We describe the method in general terms, as it can be applied to any situation where the targeted moments can be expressed as coefficients in a set of panel regressions, including lags of arbitrary length.

A.3.7.1 Data moments

We assume that the moments targeted in the estimation can be represented as a system of panel data regressions of the form:

$$\tilde{Y}_{i,t} = \tilde{\mathbf{X}}_{i,t} \hat{\beta}_N + \varepsilon_{i,t}. \quad (\text{A43})$$

Here, $\tilde{Y}_{i,t}$ is a column vector of size $N_Y \times 1$, where N_Y is the number of equations to be estimated; $\tilde{X}_{i,t}$ is a matrix of size $N_Y \times K$, where K is the number of reduced-form coefficients to be estimated; and $\hat{\beta}_N$ is the vector of reduced-form coefficients, of size $K \times 1$. Note that $\tilde{X}$ could contain both current and lagged variables.

We form the OLS estimate of these data moments as:

$$\hat{\beta}_N = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{Y}. \quad (\text{A44})$$

Here, the matrix $\tilde{X}$ is of size $(N_Y N_f N_t) \times K$, where N_f is the number of firms and N_t the number of periods in the data, and likewise, the matrix $\tilde{Y}$ is of size $(N_Y N_f N_t) \times 1$. We form each matrix as follows: for each equation in the panel data system (A43), we stack vertically the blocks of observations corresponding to each firm, ordering the time dimension first; and we then stack the resulting observations vectors across equations. Finally, $N = N_f N_t$ denotes the total number of firm by time observations in the panel data.

To clarify this notation, consider for instance the first equation in the panel data system (A43), and let $Y_{i,t}^{(1)}$ denote the independent variable in this equation and $X_{i,t}^{(1)}$ (a vector of size $1 \times K$) the corresponding observations of the independent variables. The first N_t elements of $\tilde{Y}$ correspond to observations 1 through N_t of variable $Y_{i,t}^{(1)}$ for firm $i = 1$; the second N_t elements of $\tilde{Y}$ correspond to observations 1 through N_t of variable $Y_{i,t}^{(1)}$ for firm $i = 2$; and so on. The matrix $\tilde{X}$ is constructed in the same way.

Note that once the matrices $\tilde{X}$ and $\tilde{Y}$ have been formed, we eliminate any entries with missing data in either $\tilde{X}$ or $\tilde{Y}$, which may occur if the underlying panel data is unbalanced. In this case, the number of observations, N , will be strictly less than $N_f N_t$.

A.3.7.2 Model-implied moments

Next, we discuss how to construct model counterparts to the data moments described in Equation (A44). We first rewrite the panel data model in Equation (A43) more explicitly as:

$$Y(x_{i,t}) = \sum_{l=1}^L X_l(x_{i,t-\tau_l})\beta_l + \varepsilon_{i,t} \quad (\text{A45})$$

where:

$$Y : \mathbb{R} \rightarrow \mathbb{R}^{N_Y \times 1}, \quad x \mapsto Y(x),$$

and where, if $L \geq 2$,

$$0 \leq \tau_1 < \tau_2 \leq \tau_3 \leq \dots \leq \tau_L,$$

and for any $1 \leq l \leq L$,

$$X_l : \mathbb{R} \rightarrow \mathbb{R}^{N_Y \times K_l}, \quad x \mapsto X_l(x).$$

Note that this is the same as Equation (A43), except that we have more explicitly separated the independent variables according to their lag order, so that:

$$\sum_{l=1}^L K_l = K. \quad (\text{A46})$$

We also made explicit the dependence of firm-specific variables on the firm's state, $x_{i,t}$.

The panel data model (A43) could in principle be estimated on data simulated from the model. Consider the population value of the associated OLS estimator for the reduced-form moments $\{\beta_l\}_{l=1}^L$. It is given by:

$$\beta(\Theta) \equiv \left(\mathbb{E}_\Theta \left[\begin{pmatrix} X_1'(x_{i,t-\tau_1})X_1(x_{i,t-\tau_1}) & \cdots & X_1'(x_{i,t-\tau_1})X_L(x_{i,t-\tau_L}) \\ \vdots & & \vdots \\ X_L'(x_{i,t-\tau_L})X_1(x_{i,t-\tau_1}) & \cdots & X_L'(x_{i,t-\tau_L})X_L(x_{i,t-\tau_L}) \end{pmatrix} \right] \right)^{-1} \mathbb{E}_\Theta \left[\begin{pmatrix} X_1'(x_{i,t})Y(x_{i,t}) \\ \vdots \\ X_L'(x_{i,t-\tau_L})Y(x_{i,t}) \end{pmatrix} \right]. \quad (\text{A47})$$

Here, we used the notation $\beta(\Theta) = \{\beta_l(\Theta)\}_{l=1}^L$ for the population values, where Θ , a vector Θ of size $p \times 1$, refers to the structural parameters of the model that are being estimated. Additionally, we used the notation $\mathbb{E}_\Theta$ to refer to the ergodic distribution of the state variable $\{x_{i,t}\}$ in the stationary steady-state of the model. Heuristically, $\beta(\Theta)$ corresponds to the value of the point estimates one would obtain by estimating the panel data model in Equation (A45) on simulated data from the model, if the simulation size were arbitrarily large.

In what follows, we explain how to compute $\beta(\Theta)$ for an arbitrary set of structural parameters. This computation leverages the numerical solution of the model described above, allowing us to avoid simulation altogether. Note that the numerical objects obtained from the discretized solution to the model all implicitly depend on Θ , the vector of structural parameters. However, in the results that follow, we omit this dependence to alleviate notation.

Preliminaries Let N_x be the size of the grid for x and denote by $\{x_n\}_{n=1}^{N_x}$ its elements. Let $\tau \geq 0$. Let $\mathbf{L}$ be the intensity matrix from the approximate solution to the model, and define the matrix $\mathbf{H}(\tau)$ as:

$$\mathbf{H}(\tau) = \exp(\tau \mathbf{L}) \quad : \quad N_x \times N_x \quad (\text{A48})$$

to which we also make appropriate adjustments to account for re-injections after exit (that is, move the mass of firms that exit within the period τ to the reinjection point). Also define:

$$\mathbf{F} = \text{diag}(f) \quad : \quad N_x \times N_x \quad (\text{A49})$$

where $\text{diag}(f)$ is the matrix with the diagonal equal to the vector f and zeros elsewhere, and where f a vector representing the stationary distribution of the model in the approximate solution:

$$\forall 1 \leq n \leq N_x, \quad f_n \geq 0, \quad \text{and} \quad \sum_{n=1}^{N_x} f_n = 1, \quad (\text{A50})$$

with f_n denoting the n -th element of f . Let $\mathbf{1}_{N_x}$ be the $N_x \times 1$ vector with entries all equal to 1. Then we have:

$$\mathbf{1}_{N_x}' \mathbf{H}(\tau)' \mathbf{F} = f'. \quad (\text{A51})$$

To see this, note that the matrix $\mathbf{H}(\tau)' \mathbf{F}$ has entries:

$$\mathbf{H}(\tau)' \mathbf{F} = \begin{pmatrix} f_1 \mathbf{H}_{1,1}(\tau) & \cdots & f_{N_x} \mathbf{H}_{N_x,1}(\tau) \\ \vdots & & \vdots \\ f_1 \mathbf{H}_{1,N_x}(\tau) & \cdots & f_{N_x} \mathbf{H}_{N_x,N_x}(\tau) \end{pmatrix}$$

where $\mathbf{H}_{n,m}(\tau)$ is the (n,m) -th element of $\mathbf{H}(\tau)$. Using the fact that because $\mathbf{H}(\tau)$ is a Markov transition matrix, $\sum_m \mathbf{H}_{n,m}(\tau) = 1$ for any $n = 1, \dots, N_x$ gives the result.⁸⁵

The first lemma describes how we approximate the conditional expectation of an arbitrary real-valued function of the state of the model, x_t .

Lemma 1 *Let:*

$$X : \mathbb{R} \rightarrow \mathbb{R}, \quad x \mapsto X(x), \quad (\text{A52})$$

and define the $N_x \times 1$ vector:

$$\mathbf{X} = (X(x_n))_{n=1}^{N_x} \quad : \quad N_x \times 1. \quad (\text{A53})$$

Then we can approximate the conditional expectations

$$\forall n = 1, \dots, N_x, \quad \mathbb{E}_\Theta [X(x_\tau) | x_0 = x_n] \quad : \quad 1 \times 1 \quad (\text{A54})$$

by the matrix product:

$$\begin{pmatrix} \mathbb{E}_\Theta [X(x_\tau) | x_0 = x_1] \\ \vdots \\ \mathbb{E}_\Theta [X(x_\tau) | x_0 = x_{N_x}] \end{pmatrix} = \mathbf{H}(\tau) \mathbf{X} \quad : \quad N_x \times 1. \quad (\text{A55})$$

The second lemma describes how to generalize this to the conditional expectation of an arbitrary matrix-valued function of the state of the model, x_t .

⁸⁵Note that Equation (A51) for any arbitrary vector f , not necessarily one representing a distribution.

Lemma 2 *Let:*

$$X : \mathbb{R} \rightarrow \mathbb{R}^{N_Y \times K}, \quad x \mapsto X(x), \quad (\text{A56})$$

and define the matrix:

$$\mathbf{X} = (X(x_n))_{n=1}^{N_x} \quad : \quad (N_x N_Y) \times K, \quad (\text{A57})$$

that is, the matrix obtained by vertically stacking the (matrix) values of X at each point on the grid for x . Additionally, define the matrix:

$$\hat{\mathbf{H}}(\tau) = \mathbf{H}(\tau) \otimes I_{N_Y \times N_Y}, \quad (\text{A58})$$

where $\otimes$ is the Kronecker product and $I_{N_Y \times N_Y}$ is the identity matrix of size N_Y . Then we can approximate the conditional expectations

$$\forall n = 1, \dots, N_x, \quad \mathbb{E}_{\Theta} [X(x_\tau) | x_0 = x_n] \quad : \quad N_Y \times K \quad (\text{A59})$$

by the matrix product:

$$\begin{pmatrix} \mathbb{E}_{\Theta} [X(x_\tau) | x_0 = x_1] \\ \vdots \\ \mathbb{E}_{\Theta} [X(x_\tau) | x_0 = x_{N_x}] \end{pmatrix} = \hat{\mathbf{H}}(\tau) \mathbf{X} \quad : \quad (N_x N_Y) \times K. \quad (\text{A60})$$

Next, we describe how to use these results to compute the expectation of the inner product of two arbitrary conforming matrix-valued functions of the state variable x_t .

Lemma 3 *Let X_1 and X_2 be two arbitrary, matrix-valued functions, such that:*

$$X_1 : \mathbb{R} \rightarrow \mathbb{R}^{N_Y \times K_1}, \quad x \mapsto X_1(x), \quad (\text{A61})$$

$$X_2 : \mathbb{R} \rightarrow \mathbb{R}^{N_Y \times K_2}, \quad x \mapsto X_2(x). \quad (\text{A62})$$

Define the matrices:

$$\mathbf{X}_1 = (X_1(x_n))_{n=1}^{N_x} \quad : \quad (N_x N_Y) \times K_1 \quad (\text{A63})$$

$$\mathbf{X}_2 = (X_2(x_n))_{n=1}^{N_x} \quad : \quad (N_x N_Y) \times K_2 \quad (\text{A64})$$

that is, the matrices obtained by vertically stacking the (matrix) values of X_1 and X_2 at each point on the grid for x . Let $\tau_1, \tau_2 \geq 0$ and without loss of generality, $\tau_1 \leq \tau_2$, and define:

$$\tau = \tau_2 - \tau_1. \quad (\text{A65})$$

Finally, define the matrix $\hat{\mathbf{H}}(\tau)$ as in Equation (A58), and the matrix $\hat{\mathbf{F}}$ as:

$$\hat{\mathbf{F}} = \mathbf{F} \otimes I_{N_Y \times N_Y}. \quad (\text{A66})$$

Then we can approximate the unconditional expectation of the inner product:

$$\mathbb{E}_\Theta [X_1(x_{t-\tau_1})' X_2(x_{t-\tau_2})] \quad : \quad K_1 \times K_2 \quad (\text{A67})$$

by the matrix product:

$$\mathbb{E}_\Theta [X_1(x_{t-\tau_1})' X_2(x_{t-\tau_2})] = \mathbf{X}_1' (\hat{\mathbf{H}}(\tau)' \hat{\mathbf{F}}) \mathbf{X}_2 \quad : \quad K_1 \times K_2. \quad (\text{A68})$$

To prove this lemma, we use the fact that

$$\begin{aligned} \mathbb{E}_\Theta [X_1(x_{t-\tau_1})' X_2(x_{t-\tau_2})] &= \mathbb{E}_\Theta [X_1(x_{t-\tau_1})' X_2(x_{t-\tau_2})] \\ &= \mathbb{E}_\Theta [X_1(x_{s+\tau})' X_2(x_s)] \\ &= \mathbb{E}_\Theta [\mathbb{E} [X_1(x_{s+\tau}) | x_s]' X_2(x_s)] \\ &= \int_x \mathbb{E}_\Theta [X_1(x_\tau) | x_0 = x]' X_2(x) f(x) dx, \end{aligned}$$

where f is the ergodic distribution of the state variable, x .⁸⁶ We then apply Lemma 2.

As a sanity check, consider a case where $N_Y = 1$ and $K_1 = 1$, and:

$$\tau_1 = 0, \quad \tau_2 = \tau > 0, \quad \text{and} \quad \forall x, \quad X_1(x) = 1 \quad : \quad 1 \times 1. \quad (\text{A69})$$

⁸⁶This distribution depends on Θ , but to alleviate notation, we omit this dependence.

Then, $\mathbf{X}_1 = \mathbf{1}_{N_Y \times 1}$. So the result of Lemma 3 says that:

$$\mathbb{E}_\Theta [X_2(x_{t-\tau_2})] = \mathbb{E} [X_1(x_{t-\tau_1})' X_2(x_{t-\tau_2})] = \mathbf{X}_1' (\mathbf{H}(\tau)' \mathbf{F}) \mathbf{X}_2 \quad : \quad 1 \times K_2, \quad (\text{A70})$$

where $\mathbf{X}_1$, in this particular case, is a vertical matrix of size $N_x \times 1$ containing only ones. Therefore, using Equation (A51), our approximation implies that:

$$\mathbb{E}_\Theta [X_2(x_{t-\tau_2})] = f' \mathbf{X}_2 \quad : \quad 1 \times K_2, \quad (\text{A71})$$

which is the natural approximation of the simple expectation in our setup.

Finally, note that using the properties of the Kronecker product, we can rewrite the inner matrix product in Equation (A68) as:

$$\begin{aligned} \widehat{\mathbf{H}}(\tau)' \widehat{\mathbf{F}} &= (\mathbf{H}(\tau) \otimes I_{N_Y \times N_Y})' (\mathbf{F} \otimes I_{N_Y \times N_Y}) \\ &= (\mathbf{H}(\tau)' \otimes I_{N_Y \times N_Y}) (\mathbf{F} \otimes I_{N_Y \times N_Y}) \\ &= (\mathbf{H}(\tau)' \mathbf{F}) \otimes I_{N_Y \times N_Y}. \end{aligned}$$

Computing $\beta(\Theta)$ The main result is described in the following Lemma, which follows immediately from Lemmas 1-3.

Lemma 4 *The population values of the OLS estimator associated with the panel data model described in Equation (A47) can be numerically approximated by:*

$$\beta(\Theta) = \mathbf{A}^{-1} \mathbf{B} \quad (\text{A72})$$

where, using the same notation as in Lemma 3,

$$\mathbf{A} = \begin{pmatrix} \mathbf{X}_1' \widehat{\mathbf{F}} \mathbf{X}_1 & \cdots & \mathbf{X}_1' \widehat{\mathbf{H}}(\tau_L - \tau_1)' \widehat{\mathbf{F}} \mathbf{X}_L \\ \vdots & & \vdots \\ \mathbf{X}_L' \widehat{\mathbf{F}} \widehat{\mathbf{H}}(\tau_L - \tau_1) \mathbf{X}_1 & \cdots & \mathbf{X}_L' \widehat{\mathbf{F}} \mathbf{X}_L \end{pmatrix} \quad : \quad K \times K, \quad (\text{A73})$$

$$\mathbf{B} = \begin{pmatrix} \mathbf{X}'_1 \widehat{\mathbf{F}} \widehat{\mathbf{H}}(\tau_1) \mathbf{Y} \\ \vdots \\ \mathbf{X}'_L \widehat{\mathbf{F}} \widehat{\mathbf{H}}(\tau_L) \mathbf{Y} \end{pmatrix} : K \times 1. \quad (\text{A74})$$

Here, we used the fact that $\widehat{\mathbf{H}}(0) = I_{N_x N_Y \times N_x N_Y}$. Note that the block of the matrix $\mathbf{A}$ corresponding to the inner product $X_n(x_{i,t-\tau_n})' X_m(x_{i,t-\tau_m})$, where $n > m$, is given by:

$$\mathbf{X}'_n \widehat{\mathbf{F}} \widehat{\mathbf{H}}(\tau_n - \tau_m) \mathbf{X}_m. \quad (\text{A75})$$

Finally, we consider both unweighted and weighted versions of the estimator. In the weighted version of the estimator, the intensity matrix $\mathbf{H}$ and the stationary density vector f used in the computation of $\beta(\Theta)$ are the weighted versions of these objects obtained in the discretized solution to the model. For reduced-form data moments, in order to obtain weighted versions, we replace $\tilde{Y}_{i,t}$ and $\tilde{\mathbf{X}}_{i,t}$ by:

$$\tilde{Y}_{i,t}^{(w)} = \sqrt{w_{i,t}} \tilde{Y}_{i,t}, \quad \tilde{\mathbf{X}}_{i,t}^{(w)} = \sqrt{w_{i,t}} \tilde{\mathbf{X}}_{i,t} \quad (\text{A76})$$

where $w_{i,t}$ is a positive scalar that satisfies $\sum_i w_{i,t} = 1$ for all t .

A.3.7.3 Estimator

We now describe how we construct point estimates, confidence intervals, and test statistics, using an extremum estimator that minimizes the distance between $\hat{\beta}_N$, the reduced-form data moments, and $\beta(\Theta)$, their population values under the ergodic distribution of the model.

Estimator We estimate Θ using the following method of moments estimator:

$$\hat{\Theta} \in \arg \min_{\Theta \in \Omega} Q_N(\Theta), \quad Q_N(\Theta) \equiv (\hat{\beta}_N - \beta(\Theta))' \hat{\mathbf{W}} (\hat{\beta}_N - \beta(\Theta)). \quad (\text{A77})$$

Here, $\hat{\mathbf{W}}$ is a symmetric positive-definite matrix, which may depend on the data; we come back to the choice of $\hat{\mathbf{W}}$ below, and Ω is a compact set containing possible param-

eter values, as described above.

For this estimator, we now provide sufficient conditions for consistency and asymptotic normality. For consistency we have the following result.

Lemma 5 (Consistency) *Assume that the following conditions hold:*

A1.1. *The function $\beta(\cdot) : \mathbb{R}^p \rightarrow \mathbb{R}^K$ defined in Lemma (4) is continuous on Ω ;*

A1.2. *The data $\{Y_{i,t}, X_{i,t}\}$ are ergodic stationary;*

A1.3. *The matrix $\hat{\mathbf{W}}$ converges in probability to a symmetric positive definite matrix $\mathbf{W}$;*

A1.4. *There exists $\Theta_0 \in \mathring{\Omega}$ such that:*

$$\beta(\Theta_0) = \mathbb{E} [\tilde{\mathbf{X}}'_{i,t} \tilde{\mathbf{X}}_{i,t}]^{-1} \mathbb{E} [\tilde{\mathbf{X}}'_{i,t} \tilde{Y}_{i,t}], \quad (\text{A78})$$

where $\mathbb{E}[\cdot]$ denotes the expectations operator under the true data-generating process; and moreover, for all $\Theta \in \Omega$, $\Theta \neq \Theta_0$ implies that $\beta(\Theta) \neq \beta(\Theta_0)$.

Then the estimator in Equation (A44) is well-defined, and moreover, it is consistent:

$$\hat{\Theta} \xrightarrow[N \rightarrow +\infty]{p} \Theta_0. \quad (\text{A79})$$

This Lemma is a direct application of standard results on extremum estimators (Hayashi, 2011, Chapter 7). No explicit conditions on uniform convergence are needed. Indeed, let:

$$\beta_0 \equiv \mathbb{E} [X'_{i,t} X_{i,t}]^{-1} \mathbb{E} [X'_{i,t} Y_{i,t}]. \quad (\text{A80})$$

Moreover, let $Q_0(\Theta) \equiv (\beta_0 - \beta(\Theta))' \mathbf{W} (\beta_0 - \beta(\Theta))$. Clearly, uniform convergence in Θ of $Q_N(\Theta)$ to $Q_0(\Theta)$ holds if:

$$\sup_{\Theta \in \Omega} \|\{\hat{\beta}_N - \beta(\Theta)\} - \{\beta_0 - \beta(\Theta)\}\| \xrightarrow[N \rightarrow +\infty]{p} 0, \quad (\text{A81})$$

where $\|\cdot\|$ is the Euclidean norm. But this is the same as:

$$\sup_{\Theta \in \Theta} \|\hat{\beta}_N - \beta_0\| \xrightarrow[N \rightarrow +\infty]{p} 0, \quad (\text{A82})$$

which, using Assumption A1.2., follows from the ergodic theorem.

Next, for asymptotic normality, we have the following result.

Lemma 6 (Asymptotic normality) *Assume that, besides the conditions in Lemma (5), the following conditions hold:*

A2.1. *The function $\beta(\cdot) : \mathbb{R}^p \rightarrow \mathbb{R}^K$ defined in Lemma (4) is continuously differentiable on $\mathring{\Omega}$, and the Jacobian matrix of size $K \times p$:*

$$\mathbf{G} = \frac{\partial \beta}{\partial \Theta}(\Theta_0) \quad (\text{A83})$$

is full column rank;

A2.2. *The data-generating process is such that:*

$$\sqrt{N}(\hat{\beta}_N - \beta(\Theta_0)) \xrightarrow[N \rightarrow +\infty]{d} \mathbf{N}(0, \mathbf{S}), \quad (\text{A84})$$

where $\xrightarrow{d}$ denotes convergence in distribution, and $\mathbf{S}$ is a symmetric positive definite matrix.

Then the estimator in Equation (A44) is asymptotically normal:

$$\begin{aligned} \sqrt{N}(\hat{\Theta} - \Theta_0) &\xrightarrow[N \rightarrow +\infty]{d} \mathbf{N}(0, \text{Avar}(\hat{\Theta})), \\ \text{Avar}(\hat{\Theta}) &= \mathbf{D}_1^{-1} \mathbf{D}_2 \mathbf{D}_1^{-1}, \end{aligned} \quad (\text{A85})$$

$$\mathbf{D}_1 = \mathbf{G}' \mathbf{W} \mathbf{G},$$

$$\mathbf{D}_2 = \mathbf{G}' \mathbf{W} \mathbf{S} \mathbf{W} \mathbf{G}.$$

The proof of this lemma follows from applying the mean value theorem to the necessary first-order condition for an optimum in Equation (A44); see for instance Hayashi (2011, Proposition 7.10). A sufficient condition for Assumption A2.2. to hold is that under the data-generating process, the moments:

$$\mathbb{E} \left[\left(\tilde{\mathbf{X}}_{i,t}^{(y_1, k)} \tilde{Y}_{i,t}^{(y_2)} \right)^2 \right] \quad (\text{A86})$$

exist and are finite for any $y_1, y_2 = 1, \dots, Y$ and $k = 1, \dots, K$, in which case the Lindberg-Lévy central limit theorem holds.

Additionally, if $\hat{\mathbf{S}}$ is a consistent estimator of $\mathbf{S}$, then $\text{Avar}(\hat{\Theta})$ can be consistently estimated by:

$$\begin{aligned}\widehat{\text{Avar}}(\hat{\Theta}) &= \hat{\mathbf{D}}_1^{-1} \hat{\mathbf{D}}_2 \hat{\mathbf{D}}_1^{-1}, \\ \hat{\mathbf{D}}_1 &= \hat{\mathbf{G}}' \hat{\mathbf{W}} \hat{\mathbf{G}}, \\ \hat{\mathbf{D}}_2 &= \hat{\mathbf{G}}' \hat{\mathbf{W}} \hat{\mathbf{S}} \hat{\mathbf{W}} \hat{\mathbf{G}}, \\ \hat{\mathbf{G}} &= \frac{\partial \beta}{\partial \Theta}(\hat{\Theta})\end{aligned}\tag{A87}$$

In this case, a consistent estimate of the asymptotic standard deviation of $\hat{\Theta}_k$ is:

$$\hat{\sigma}_k = \sqrt{N^{-1} \widehat{\text{Avar}}(\hat{\Theta})_{k,k}}, \quad k = 1, \dots, K.\tag{A88}$$

Additionally, if the choice of the weighting matrix $\hat{\mathbf{W}}$ is efficient, that is, $\hat{\mathbf{W}} = \hat{\mathbf{S}}^{-1}$, then:

$$J_N \equiv N Q_N(\hat{\Theta}) \xrightarrow[N \rightarrow +\infty]{d} \chi^2(K - p),\tag{A89}$$

which can be used to test for over-identifying restrictions when $K > p$.

In order to construct a consistent estimate of $\mathbf{S}$, we proceed as follows. For each $m = 1, \dots, N_Y$, where N_Y is the total number of equations in the system, let:

$$\hat{\varepsilon}^{(m)} = \tilde{Y}^{(m)} - \tilde{\mathbf{X}}^{(m)} \hat{\beta}_N^{(m)}.\tag{A90}$$

Here $\hat{\varepsilon}^{(m)}$ is an $N \times 1$ vector of residuals, $\tilde{Y}^{(m)}$ is an $N \times 1$ vector containing the observations of the dependent variable in Equation m ; $\tilde{\mathbf{X}}^{(m)}$ is an $N \times K_m$ matrix containing the observations of the independent variables in Equation m , and K_m is the number of such variables; and $\beta_N^{(m)}$ is the subset of reduced-form coefficients in $\hat{\beta}_N$ that enter equation

m . Furthermore, define:

$$\begin{aligned}
\hat{\mathbf{B}}^{(m)} &= \text{diag} \left(\hat{\varepsilon}^{(m)} \right) & : & \quad N \times N \\
\hat{\mathbf{V}}^{(m)} &= N^{-1} (\tilde{\mathbf{X}}^{(m)})' \hat{\mathbf{B}}^{(m)} \tilde{\mathbf{X}}^{(m)} & : & \quad K_m \times K_m \\
\mathbf{S}_{XX}^{(m)} &= N^{-1} (\tilde{\mathbf{X}}^{(m)})' \tilde{\mathbf{X}}^{(m)} & : & \quad K_m \times K_m \\
\widehat{\text{Avar}}(\hat{\beta}_N^{(m)}) &= \left(\mathbf{S}_{XX}^{(m)} \right)^{-1} \hat{\mathbf{V}}^{(m)} \left(\mathbf{S}_{XX}^{(m)} \right)^{-1} & : & \quad K_m \times K_m
\end{aligned} \tag{A91}$$

Following [Arellano et al. \(1987\)](#), for each $m = 1, \dots, N_Y$, this provides an estimator of the asymptotic variance of $\hat{\beta}_N^{(m)}$ that is robust to the presence of heteroskedasticity (over time and across firms), and arbitrary correlation in error terms over time within firm. We then stack the resulting variance-covariance matrices $\widehat{\text{Avar}}(\hat{\beta}_N^{(m)})$ into a single, block-diagonal matrix, of size $K \times K$, to obtain the estimator $\hat{\mathbf{S}}$. Note that the resulting estimator implicitly assumes that the error terms are uncorrelated across equations. Additionally, it assumes that error terms are not correlated within time across firms.

A.3.7.4 Numerical implementation

We use Matlab's `patternsearch` routine to minimize numerical objectives, with 50 randomly drawn starting points for each objective function. For each randomly drawn starting point, we first find the solution to Equation (A44). We then select the overall point estimate to be the one for which the terminal value of the objective function is minimized across all ten random starting points.

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	Government participation	Borrower eligibility	Interest rate	Restrictions on borrowers	Funding cap
CCF	Debt (any seniority)	Based on credit rating	Prevailing market rates	None	None
MSLP	Debt (senior)	Size + Debt/EBITDA ≤ 4 or 6	LIBOR + 3%	Dividends + debt repurchases	Up to debt/EBITDA ≤ 4 or 6 + max loan size

Table A-1: Summary of key features of the 2020 corporate credit interventions in the United States. For the CCF, only investment-grade or firms downgraded after March 22nd, 2020 were eligible for direct purchases; ETFs with high-yield exposure were also eligible. The CCF could only purchase up to 25% of individual issuances or 10% of the total debt outstanding of particular firms, but there were no caps on total issuance by the borrower. Size restrictions for the MSLP were based on 2019 revenue or employment. Funding caps depended on the facility used (new loan, priority loan, or expanded loan). For more details, see the main text; see also, on the CCF, [Boyarchenko et al. \(2020\)](#); and on the MSLP, [Crouzet and Gourio \(2020\)](#).

Moment	Description	Mean	Median	p_{25}	p_{75}	N
$\widehat{\alpha}_k$	$\frac{\text{Book assets at emergence}}{\text{Book assets at filing}}$	0.720	0.689	0.475	0.939	415
$\frac{\widehat{\alpha}_b}{\alpha_k}$	$\frac{\text{Book leverage at emergence}}{\text{Book leverage at filing}}$	0.805	0.742	0.538	0.961	415
$\widehat{\bar{x}}$	Book leverage at filing	1.072	0.977	0.817	1.187	415
$\widehat{\bar{x}}$	Book leverage at filing (all cases)	1.090	0.965	0.814	1.201	803
$\frac{\widehat{\alpha}_b}{\alpha_k} \widehat{\bar{x}}$	Book leverage at emergence	0.760	0.728	0.593	0.872	415
$\widehat{d(\bar{x})}$	Recovery rate on unsecured claims	0.483	0.474	0.154	0.799	53

Table A-2: Data moments from the sample of reorganizations. The data are from the Bankruptcy Research Database (LoPucki and Doherty 2007) and cover the universe of Chapter 11 filings of publicly traded firms with at least \$100m in book assets in 1980 dollars from 1979 to 2022. We restrict the sample to firms that are successfully reorganized during Chapter 11, either through a confirmed plan or through a 363 sale of the business as a going concern. Furthermore, we restrict attention to filings for which information on book assets and book liabilities after emergence is publicly available, except for line marked "Book leverage at filing (all cases)", where we include all successful reorganizations with information on pre-filing capital structure. Variable definitions are reported in Section A.3.1.1.

Ratio	Description	Model	Data
$\Phi_{i,t}$	Gross investment rate	$100 \times \Phi(g(x_{i,t}))$	$100 \times \frac{\text{capx}}{11.\text{ppegt}}$
$dep_{i,t}$	Ratio of depreciation to EBITDA	$100 \times \frac{\delta}{a}$	$100 \times \frac{\text{dp}}{\text{oibdp}}$
$tax_{i,t}$	Ratio of taxes paid to EBITDA	$100 \times \theta \left(1 - \frac{\kappa x_{i,t} + \delta}{a} \right)$	$100 \times \frac{\text{txt}}{\text{oibdp}}$
$int_{i,t}$	Ratio of interest paid to EBITDA	$100 \times \frac{\kappa x_{i,t}}{a}$	$100 \times \frac{\text{xint}}{\text{oibdp}}$
$x_{i,t}$	Book leverage	$x_{i,t}$	$\frac{11.\text{dlc} + 11.\text{dltt}}{11.\text{at}}$
$roa_{i,t}$	Return on assets	$100 \times a$	$100 \times \frac{\text{oibdp}}{11.\text{at}}$
$iss_{i,t}$	Gross debt issuance rate	$100 \times \frac{\iota(x_{i,t})k_{i,t}}{b_{i,t}}$	$100 \times \frac{\text{dltis} + \text{dlcch}}{11.\text{dlc} + 11.\text{dltt}}$
$pay_{i,t}$	Equity payout rate	$100 \times \frac{\text{div}(x_{i,t})}{e(x_{i,t})}$	$100 \times \frac{\text{dv} + \text{prstk} - \text{sstk}}{11.\text{mkvalt}}$
$Q_{i,t}$	Tobin's Q	$e(x_{i,t}) + x_{i,t}$	$\frac{11.\text{dlc} + 11.\text{dltt} + 11.\text{mkvalt}}{11.\text{at}}$

Table A-3: Definition of financial and investment ratios in terms of model variables and data items. All data variables are drawn from the Compustat Fundamentals Annual file, and referred to by their Compustat mnemonics. The prefix 11. denotes the annual lag of a variable; we use beginning-of-period (end-of-preceding-year) when normalizing by book assets or debt.

Ratio	Description	Mean (weighted)	Mean (unweighted)	Median	p_{25}	p_{75}	N	# firms
$\Phi_{i,t}$	Gross investment rate	11.59	12.53	8.73	5.29	14.87	22405	3319
$dep_{i,t}$	Ratio of depreciation to EBITDA	37.15	53.81	31.77	18.98	52.94	22405	3319
$tax_{i,t}$	Ratio of taxes paid to EBITDA	12.06	9.94	13.11	1.35	23.10	22405	3319
$int_{i,t}$	Ratio of interest paid to EBITDA	13.72	19.74	8.61	1.97	21.31	22405	3319
$x_{i,t}$	Book leverage	0.30	0.27	0.23	0.06	0.40	22405	3319
$roa_{i,t}$	Return on assets	15.08	14.76	13.00	8.58	18.58	22405	3319
$iss_{i,t}$	Gross debt issuance rate	40.55	58.59	21.08	-2.77	81.49	22405	3319
$pay_{i,t}$	Equity payout rate	2.71	-9.03	0.88	0.00	3.89	22405	3319
$Q_{i,t}$	Tobin's Q	1.39	1.49	1.13	0.76	1.75	22405	3319

Table A-4: Summary statistics. The sample is the panel of Compustat non-financial US firms from 2010-2019. Sample selection is described in [Appendix A.3.3.2](#). Variable definitions and construction are described in [Appendix A.3.3.1](#) and [Table A-3](#). In the column marked "Mean (weighted)", observations are weighted by their share of book assets in the same year.

A. Parameters	Baseline results				Alternative results											
					(A1): Small firms				(A2): High deadweight losses				(A3): Residualized investment			
	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.
a	✓	0.25	0.014	[0.23, 0.28]	✓	0.37	0.029	[0.33, 0.43]	✓	0.29	0.028	[0.25, 0.34]	✓	0.25	0.012	[0.24, 0.28]
δ	✓	0.09	0.006	[0.09, 0.11]	✓	0.22	0.018	[0.19, 0.25]	✓	0.11	0.011	[0.09, 0.13]	✓	0.09	0.005	[0.09, 0.11]
τ_π	✓	0.26	0.011	[0.24, 0.28]	✓	0.39	0.027	[0.35, 0.44]	✓	0.25	0.011	[0.24, 0.28]	✓	0.26	0.012	[0.24, 0.28]
κ	✓	0.12	0.008	[0.11, 0.13]	✓	0.30	0.026	[0.26, 0.35]	✓	0.13	0.014	[0.12, 0.16]	✓	0.12	0.007	[0.11, 0.13]
σ	✓	0.35	0.017	[0.33, 0.39]	✓	0.51	0.045	[0.44, 0.59]	✓	0.46	0.065	[0.36, 0.57]	✓	0.35	0.015	[0.33, 0.38]
m	✓	0.18	0.015	[0.16, 0.21]	✓	0.37	0.043	[0.30, 0.44]	✓	0.32	0.094	[0.17, 0.47]	✓	0.18	0.016	[0.16, 0.22]
γ	✓	4.39	1.96	[1.17, 7.63]	✓	3.24	0.772	[1.98, 4.52]	✓	1.84	0.056	[1.75, 1.94]	✓	3.89	1.450	[1.51, 6.28]
r	✗		0.03		✗		0.03		✗		0.03		✗		0.03	
ρ	✗		0.50		✗		0.50		✗		0.50		✗		0.50	
ν	✗		0.42		✗		0.42		✗		0.42		✗		0.42	
α_k	✗		0.72		✗		0.72		✗		0.45		✗		0.72	
α_b	✗		0.58		✗		0.58		✗		0.36		✗		0.58	
B. Model fit	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.
$\hat{\Phi}$	✓	11.69	11.59	[11.22, 11.97]	✓	13.99	14.04	[13.76, 14.33]	✓	11.61	11.59	[11.22, 11.97]	✓	11.68	11.59	[11.22, 11.97]
$\widehat{dep}$	✓	37.29	37.15	[36.09, 38.21]	✓	57.46	58.16	[56.38, 59.94]	✓	37.39	37.15	[36.09, 38.21]	✓	37.41	37.15	[36.09, 38.21]
$\widehat{tax}$	✓	12.49	12.06	[11.24, 12.90]	✓	9.21	9.78	[9.13, 10.45]	✓	12.45	12.06	[11.24, 12.90]	✓	12.55	12.06	[11.24, 12.90]
$\widehat{int}$	✓	13.77	13.72	[13.25, 14.20]	✓	19.07	19.73	[18.90, 20.58]	✓	13.77	13.72	[13.25, 14.20]	✓	13.79	13.72	[13.25, 14.20]
$\hat{x}$	✓	0.30	0.30	[0.30, 0.31]	✓	0.24	0.23	[0.23, 0.25]	✓	0.30	0.30	[0.30, 0.31]	✓	0.30	0.30	[0.30, 0.31]
$\hat{\rho}_x$	✓	0.67	0.67	[0.63, 0.71]	✓	0.56	0.53	[0.50, 0.57]	✓	0.64	0.67	[0.63, 0.71]	✓	0.67	0.67	[0.63, 0.71]
$\hat{\Gamma}$	✓	-5.63	-11.10	[-14.99, -7.21]	✓	-6.11	-11.89	[-14.17, -9.60]	✓	-12.29	-11.10	[-14.99, -7.21]	✓	-6.34	-10.39	[-14.04, -6.72]
$\widehat{roa}$	✗	25.27	15.08	[14.82, 15.36]	✗	37.45	15.00	[14.81, 15.20]	✗	28.70	15.08	[14.82, 15.36]	✗	25.17	15.08	[14.82, 15.36]
$\widehat{iss}$	✗	27.63	40.55	[34.65, 46.47]	✗	46.73	60.03	[53.68, 66.39]	✗	44.88	40.55	[34.65, 46.67]	✗	27.98	40.55	[34.65, 46.67]
$\widehat{pay}$	✗	11.28	2.71	[0.80, 4.63]	✗	29.51	-39.96	[-107.58, 27.67]	✗	14.88	2.71	[0.80, 4.63]	✗	11.31	2.71	[0.80, 4.63]
$\widehat{var}^{(\Phi)}$	✗	0.93	27.69	[13.69, 41.70]	✗	1.00	79.09	[65.49, 92.70]	✗	5.24	27.69	[13.69, 41.70]	✗	1.16	160.14	[153.13, 167.17]
$\widehat{var}^{(x)}$	✗	0.01	0.01	[0.01, 0.01]	✗	0.01	0.01	[0.01, 0.02]	✗	0.02	0.01	[0.01, 0.01]	✗	0.01	0.01	[0.01, 0.01]
$\widehat{c}^{(x)}$	✗	0.10	0.11	[0.10, 0.12]	✗	0.11	0.12	[0.11, 0.13]	✗	0.11	0.11	[0.10, 0.12]	✗	0.10	0.11	[0.10, 0.12]
$\widehat{c}^{(\Phi)}$	✗	13.37	14.76	[13.62, 15.91]	✗	15.45	16.70	[16.15, 17.27]	✗	15.25	14.76	[13.62, 15.91]	✗	13.57	14.56	[13.48, 15.64]
Nr. obs		22405				11094				22405				22405		
Nr. firms		3319				1984				3319				3319		
Firm f.e.		yes				yes				yes				yes		
Subsample		all firms				small firms				all firms				all firms		
Leverage measure		debt/assets				debt/assets				debt/assets				debt/assets		
Investment measure		100 × capex/PPE				100 × capex/PPE				100 × capex/PPE				100 × capex/PPE, residualized		

Table A-5: Alternative estimation results. The columns labeled "Baseline results" report results from our baseline estimation exercise. The columns marked "(A1): Small firms" report results when our targeted panel data moments are computed in the sample restricted to firm-year observations with assets below the median. The columns marked "(A2): High deadweight losses" report results when calibrate deadweight losses of restructuring to a higher value (or equivalently, recovery rates α_k to a lower value) than in our baseline. The columns marked "(A3): Residualized investment rates" report results when residualize investment rates on cash balances and Tobin's Q before estimating the sensitivity of investment to leverage. See [Section 5.1](#) and [Appendix A.3.6.4](#) for more details.

					Alternative results											
Baseline results					(A4): No firm fixed effects				(A5): Net leverage				(A6): Investing cash flows			
A. Parameters	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.
a	✓	0.25	0.014	[0.24, 0.28]	✓	0.20	0.059	[0.11, 0.31]	✓	0.31	0.012	[0.30, 0.33]	✓	0.22	0.017	[0.20, 0.26]
δ	✓	0.09	0.006	[0.09, 0.11]	✓	0.08	0.023	[0.04, 0.12]	✓	0.12	0.006	[0.11, 0.13]	✓	0.08	0.007	[0.08, 0.10]
τ_π	✓	0.26	0.011	[0.24, 0.28]	✓	0.25	0.011	[0.24, 0.27]	✓	0.24	0.012	[0.23, 0.26]	✓	0.26	0.012	[0.24, 0.28]
κ	✓	0.12	0.008	[0.11, 0.13]	✓	0.09	0.027	[0.05, 0.14]	✓	0.21	0.013	[0.19, 0.24]	✓	0.10	0.009	[0.09, 0.12]
σ	✓	0.35	0.017	[0.33, 0.39]	✓	0.24	0.12	[0.05, 0.44]	✓	0.53	0.025	[0.50, 0.58]	✓	0.33	0.031	[0.28, 0.38]
m	✓	0.18	0.015	[0.16, 0.21]	✓	0.06	0.075	[-0.06, 0.19]	✓	0.33	0.026	[0.29, 0.38]	✓	0.19	0.028	[0.15, 0.25]
γ	✓	4.39	1.962	[1.17, 7.63]	✓	4.76	0.765	[3.50, 6.02]	✓	2.34	0.430	[1.64, 3.06]	✓	2.09	0.724	[0.91, 3.29]
r	✗		0.03		✗		0.03		✗		0.03		✗		0.03	
ρ	✗		0.50		✗		0.50		✗		0.50		✗		0.50	
ν	✗		0.42		✗		0.42		✗		0.42		✗		0.42	
α_k	✗		0.72		✗		0.72		✗		0.72		✗		0.72	
α_b	✗		0.58		✗		0.58		✗		0.58		✗		0.58	
B. Model fit	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.
$\hat{\Phi}$	✓	11.69	11.59	[11.22, 11.97]	✓	11.40	11.59	[11.22, 11.97]	✓	11.20	11.22	[10.88, 11.57]	✓	8.87	8.79	[8.45, 9.13]
$\widehat{dep}$	✓	37.29	37.15	[36.09, 38.21]	✓	37.72	37.15	[36.09, 38.21]	✓	37.90	37.86	[36.77, 38.95]	✓	37.45	37.15	[36.09, 38.21]
$\widehat{tax}$	✓	12.49	12.06	[11.24, 12.90]	✓	12.21	12.06	[11.24, 12.90]	✓	11.51	11.85	[11.01, 12.69]	✓	12.48	12.06	[11.24, 12.90]
$\widehat{int}$	✓	13.77	13.72	[13.25, 14.20]	✓	13.49	13.72	[13.25, 14.20]	✓	14.13	14.15	[13.66, 14.64]	✓	13.79	13.72	[13.25, 14.20]
$\hat{x}$	✓	0.30	0.30	[0.30, 0.31]	✓	0.30	0.30	[0.30, 0.31]	✓	0.21	0.21	[0.20, 0.22]	✓	0.30	0.30	[0.30, 0.31]
$\hat{\rho}_x$	✓	0.67	0.67	[0.63, 0.71]	✓	0.91	0.93	[0.92, 0.95]	✓	0.59	0.58	[0.55, 0.62]	✓	0.70	0.67	[0.63, 0.71]
$\hat{\Gamma}$	✓	-5.63	-11.10	[-14.99, -7.21]	✓	-10.64	-5.99	[-8.34, -3.62]	✓	-8.42	-8.57	[-10.98, -6.16]	✓	-12.22	-17.68	[-22.88, -12.48]
$\widehat{roa}$	✗	25.27	15.08	[14.82, 15.36]	✗	20.45	15.08	[14.82, 15.36]	✗	31.01	17.26	[16.91, 17.62]	✗	22.41	15.08	[14.82, 15.36]
$\widehat{iss}$	✗	27.63	40.55	[34.65, 46.47]	✗	12.89	40.55	[34.65, 46.47]	✗	49.39	42.95	[37.15, 48.77]	✗	26.49	40.55	[34.65, 46.47]
$\widehat{pay}$	✗	11.28	2.71	[0.80, 4.63]	✗	7.26	2.71	[0.80, 4.63]	✗	18.23	2.60	[0.54, 4.67]	✗	12.61	2.71	[0.80, 4.63]
$\widehat{var}^{(\Phi)}$	✗	0.93	27.69	[13.69, 41.70]	✗	2.30	91.19	[70.79, 111.69]	✗	2.32	26.12	[13.25, 39.01]	✗	3.20	102.22	[87.05, 117.40]
$\widehat{var}^{(x)}$	✗	0.01	0.01	[0.01, 0.01]	✗	0.02	0.03	[0.03, 0.04]	✗	0.09	0.10	[0.09, 0.11]	✗	0.01	0.01	[0.01, 0.01]
$\widehat{c}^{(x)}$	✗	0.10	0.11	[0.10, 0.12]	✗	0.03	0.03	[0.03, 0.04]	✗	0.09	0.10	[0.09, 0.11]	✗	0.09	0.11	[0.10, 0.12]
$\widehat{c}^{(\Phi)}$	✗	13.37	14.76	[13.62, 15.91]	✗	14.57	13.30	[12.37, 14.24]	✗	12.95	12.83	[12.33, 13.34]	✗	12.56	13.84	[12.39, 15.30]
Nr. obs		22405				22405				22405				22405		
Nr. firms		3319				3319				3319				3319		
Firm f.e.		yes				no				yes				yes		
Subsample		all firms				all firms				all firms				all firms		
Leverage measure		debt/assets				debt/assets				(debt-cash)/(assets-cash)				(debt-cash)/(assets-cash)		
Investment measure		100 × capex/PPE				100 × capex/PPE				100 × capex/PPE				-100 × (inv. cash flow)/assets		

Table A-6: Alternative estimation results. The columns labeled "Baseline results" report results from our baseline estimation exercise. The columns marked "(A4): No firm fixed effects" report results when our targeted panel data moments are estimated without including firm fixed effects. The columns marked "(A5): Net leverage" report results when we use net leverage, defined as the ratio of book debt minus cash to book assets minus cash, as our measure of leverage. The columns marked "(A6): Investing cash flows" report results when we use the ratio of the negative of investing cash flows (from the cash flow statement) to book assets as our measure of investment. See [Section 5.1](#) and [Appendix A.3.6.4](#) for more details.

A. Parameters	Baseline results				Alternative results											
					(A7): High adjustment costs				(A8): Fixed investment				(A9): Low adjustment costs			
	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.	Estimated?	Est.	Std.	90% c.i.
a	✓	0.25	0.014	[0.23, 0.28]	✓	0.26	0.007	[0.26, 0.28]	✓	0.25	0.008	[0.24, 0.27]	✓	0.25	0.012	[0.24, 0.28]
δ	✓	0.09	0.006	[0.09, 0.11]	✓	0.10	0.04	[0.10, 0.11]	✓	0.09	0.004	[0.09, 0.10]	✓	0.09	0.005	[0.09, 0.11]
τ_π	✓	0.25	0.011	[0.24, 0.28]	✓	0.25	0.011	[0.24, 0.27]	✓	0.26	0.011	[0.24, 0.28]	✓	0.25	0.011	[0.24, 0.27]
κ	✓	0.12	0.008	[0.11, 0.13]	✓	0.12	0.005	[0.12, 0.13]	✓	0.12	0.005	[0.11, 0.13]	✓	0.12	0.007	[0.11, 0.13]
σ	✓	0.35	0.017	[0.33, 0.39]	✓	0.36	0.013	[0.34, 0.39]	✓	0.35	0.013	[0.33, 0.38]	✓	0.35	0.017	[0.33, 0.39]
m	✓	0.18	0.015	[0.16, 0.21]	✓	0.18	0.015	[0.16, 0.21]	✓	0.17	0.015	[0.15, 0.20]	✓	0.17	0.018	[0.15, 0.21]
γ	✓	4.39	1.962	[1.17, 7.63]	✗		7.50		✗		4.39		✗		2.50	
r	✗		0.03		✗		0.03		✗		0.03		✗		0.03	
ρ	✗		0.50		✗		0.50		✗		0.50		✗		0.50	
ν	✗		0.42		✗		0.42		✗		0.42		✗		0.42	
α_k	✗		0.72		✗		0.72		✗		0.72		✗		0.72	
α_b	✗		0.58		✗		0.58		✗		0.58		✗		0.58	
B. Model fit	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.	Targeted?	Model	Data	90% c.i.
$\hat{\Phi}$	✓	11.69	11.59	[11.22, 11.97]	✓	11.63	11.59	[11.22, 11.97]	✓	11.66	11.59	[11.22, 11.97]	✓	15.86	11.59	[11.22, 11.97]
$\widehat{dep}$	✓	37.29	37.15	[36.09, 38.21]	✓	37.22	37.15	[36.09, 38.21]	✓	37.33	37.15	[36.09, 38.21]	✓	36.82	37.15	[36.09, 38.21]
$\widehat{tax}$	✓	12.49	12.06	[11.24, 12.90]	✓	12.21	12.06	[11.24, 12.90]	✓	12.58	12.06	[11.24, 12.90]	✓	12.32	12.06	[11.24, 12.90]
$\widehat{int}$	✓	13.77	13.72	[13.25, 14.20]	✓	13.73	13.72	[13.25, 14.20]	✓	13.74	13.72	[13.25, 14.20]	✓	14.25	13.72	[13.25, 14.20]
$\hat{x}$	✓	0.30	0.30	[0.30, 0.31]	✓	0.30	0.30	[0.30, 0.31]	✓	0.30	0.30	[0.30, 0.31]	✓	0.31	0.30	[0.30, 0.31]
$\hat{\rho}_x$	✓	0.67	0.67	[0.63, 0.71]	✓	0.66	0.67	[0.63, 0.71]	✓	0.64	0.67	[0.63, 0.71]	✓	0.64	0.67	[0.63, 0.71]
$\hat{\Gamma}$	✓	-5.63	-11.10	[-14.99, -7.21]	✗	-3.28	-11.10	[-14.99, -7.21]	✗	0.00	-11.10	[-14.99, -7.21]	✗	-10.32	-11.10	[-14.99, -7.21]
$\widehat{roa}$	✗	25.27	15.08	[14.82, 15.36]	✗	26.25	15.08	[14.82, 15.36]	✗	24.88	15.08	[14.82, 15.36]	✗	25.28	15.08	[14.82, 15.36]
$\widehat{iss}$	✗	27.63	40.55	[34.65, 46.47]	✗	27.39	40.55	[34.65, 46.47]	✗	26.30	40.55	[34.65, 46.47]	✗	31.16	40.55	[34.65, 46.47]
$\widehat{pay}$	✗	11.28	2.71	[0.80, 4.63]	✗	11.77	2.71	[0.80, 4.63]	✗	10.98	2.71	[0.80, 4.63]	✗	7.34	2.71	[0.80, 4.63]
$\widehat{var}^{(\Phi)}$	✗	0.93	27.69	[13.69, 41.70]	✗	0.36	27.69	[13.69, 41.70]	✗	0.00	27.69	[13.69, 41.70]	✗	2.98	27.69	[13.69, 41.70]
$\widehat{var}^{(x)}$	✗	0.01	0.01	[0.01, 0.01]	✗	0.01	0.01	[0.01, 0.01]	✗	0.01	0.01	[0.01, 0.01]	✗	0.01	0.01	[0.01, 0.02]
$\widehat{c}^{(x)}$	✗	0.10	0.11	[0.10, 0.12]	✗	0.10	0.11	[0.10, 0.12]	✗	0.11	0.11	[0.10, 0.12]	✗	0.11	0.11	[0.10, 0.12]
$\widehat{c}^{(\Phi)}$	✗	13.37	14.76	[13.62, 15.91]	✗	12.61	14.76	[13.62, 15.91]	✗	11.66	14.76	[13.62, 15.91]	✗	19.04	14.76	[13.62, 15.91]
Nr. obs		22405				22405				22405				22405		
Nr. firms		3319				3319				3319				3319		
Firm f.e.		yes				yes				yes				yes		
Subsample		all firms				all firms				all firms				all firms		
Leverage measure		debt/assets				debt/assets				debt/assets				(debt-cash)/(assets-cash)		
Investment measure		100 × capex/PPE				100 × capex/PPE				100 × capex/PPE				-100 × (inv. cash flow)/assets		

Table A-7: Alternative estimation results. The columns labeled "Baseline results" report results from our baseline estimation exercise. The columns marked "(A7): High adjustment costs" report results when we fix the size of investment adjustment costs to $\gamma = 10$, and re-estimate the model without targeting the empirical OLS slope of investment with respect to leverage. The columns marked "(A8): Fixed investment" report results when investment is fixed to its first-best level, leverage evolves endogenously, and the model is re-estimated without targeting the empirical OLS slope of investment with respect to leverage. See [Section 5.1](#) and [Appendix A.3.6.4](#) for more details.

	Dependent variable: investment rate ($\Phi_{i,t}$)											
	(1)		(2)		(3)		(4)		(5)		(6)	
Leverage ($x_{i,t-1}$)	-11.10	(2.365)	-9.03	(2.109)	-10.39	(2.125)	-5.99	(1.433)	-5.02	(1.243)	-5.23	(1.206)
Cash flow ($c_{i,t-1}$)			0.18	(0.034)	0.15	(0.032)			0.23	(0.044)	0.10	(0.041)
Tobin's Q ($Q_{i,t-1}$)					2.20	(0.280)					2.67	(0.314)
Firm fixed effects	yes		yes		yes		no		no		no	
Nr. obs	22405		22405		22405		22405		22405		22405	
Nr. firms	3319		3319		3319		3319		3319		3319	

Table A-8: Residualizing investment on measures of cash flow and Tobin's Q. The table reports results from estimating variations of the following reduced-form model: $\Phi_{i,t} = \alpha_i + \beta_x x_{i,t-1} + \beta_c c_{i,t-1} + \beta_Q Q_{i,t-1} + \varepsilon_{i,t}$, where $\Phi_{i,t}$ is the investment rate, $x_{i,t}$ is book leverage, $c_{i,t}$ is return on assets (the ratio of EBITDA to book assets, and our proxy for cash-flows), and $Q_{i,t}$ is Tobin's Q, or the ratio of enterprise value, to book assets. Units are as follows: column (1) indicates that a 0.01 point increase in book leverage (e.g. from 0.30 to 0.31) is associated with a 0.11p.p. decline in gross investment rates (e.g. from 12.00% to 11.89%); column (2) indicates that a 1 p.p. increase in our cash flow measure (e.g. from 15% to 16%) is associated with a 0.18p.p. increase in gross investment rates (e.g. from 12.00% to 12.18%); ; column (3) indicates that a 1 unit increase increase Tobin's Q (e.g. from 1 to 2) is associated with a 2.2p.p. increase gross investment rates (e.g. from 12.00% to 14.00%). See [Table A-3](#) for a definition of each variable in terms of Compustat data items. Columns (1) is the same regression as in our baseline estimation approach, that is, Equation (26) in the main text. The specifications in columns (2) and (3) contain firm fixed effects. The specifications in columns (4) to (6) are the same as in (1) to (3), but without fixed effects. Standard errors clustered by firm are reported in parentheses.

Description	Baseline	Small firms	Low α_k	Resid. invest.	No firm FE	Net leverage	Invest. cashflows	High γ	Fixed invest.	Low γ
μ_K	2.12%	-12.31%	0.27%	1.71%	3.15%	-1.29%	0.02%	1.29%	1.23%	5.71%
μ_K first best	3.17%	-7.53%	1.95%	2.77%	4.26%	0.31%	1.48%	2.08%	2.26%	6.92%
avg. enterprise value $\mathbb{E}[e(x) + xd(x)]$	1.10	0.71	1.02	1.08	1.16	0.99	1.01	1.13	1.07	1.15
avg. enterprise value first best e^*	1.13	0.76	1.04	1.11	1.20	1.01	1.03	1.16	1.10	1.17
$\frac{\Delta K}{K}$ credit access restoration	-1.76%	-8.38%	-2.53%	-1.79%	-1.67%	-2.62%	-2.27%	-1.42%	-1.75%	-1.89%
$\frac{\Delta K}{K}$ credit mkt. price-of-risk stabilization	-1.87%	-4.79%	-2.55%	-1.88%	-1.85%	-2.28%	-2.41%	-1.58%	-1.86%	-1.99%
$\frac{\Delta K}{K}$ mkt. price-of-risk stabilization	4.34%	5.57%	9.47%	4.77%	3.56%	7.91%	7.38%	2.82%	4.28%	7.60%
Δx moral hazard, credit access rest.	0.93%	0.60%	0.71%	0.86%	1.50%	0.63%	0.70%	0.96%	0.90%	0.80%
$\Delta \mu_K$ moral hazard, credit access rest.	-0.06%	-0.19%	-0.06%	-0.05%	-0.14%	-0.06%	-0.08%	-0.04%	-0.05%	-0.07%
Δx moral hazard, credit risk px. stab.	0.37%	0.11%	0.25%	0.31%	0.79%	0.20%	0.30%	0.32%	0.34%	0.33%
$\Delta \mu_K$ moral hazard, credit risk px. stab.	-0.04%	-0.09%	-0.04%	-0.03%	-0.10%	-0.04%	-0.06%	-0.02%	-0.04%	-0.05%
Δx moral hazard, mkt. risk px. stab.	1.53%	0.49%	1.49%	1.58%	0.21%	0.89%	1.48%	1.77%	1.49%	1.63%
$\Delta \mu_K$ moral hazard, mkt. risk px. stab.	0.77%	0.38%	1.26%	0.77%	0.91%	0.96%	1.12%	0.44%	0.68%	1.36%

Table A-9: Debt overhang effects of credit market interventions across model estimations. For each estimation strategy, the table reports: (a) the balanced-growth-path growth rate μ_K and its first-best counterpart; (b) the ergodic average enterprise value $\mathbb{E}[e(x) + xd(x)]$ and its first-best counterpart; (c) the long-run debt-overhang distortions generated by government interventions, measured as the percentage deviation of the long-run capital stock relative to the laissez-faire benchmark under the “credit access restoration,” “credit market price-of-risk stabilization,” and “market price-of-risk stabilization” scenarios; and (d) the effects of moral hazard on average leverage and the long-run growth rate under the “credit access restoration,” “credit market price-of-risk stabilization,” and “market price-of-risk stabilization” scenarios.

	Baseline	Equity issuance frictions
Ergodic values		
avg. leverage $\mathbb{E}[x]$	29.77%	28.88%
avg. investment rate $\mathbb{E}[\Phi(g(x))]$	11.66%	11.30%
avg. Tobin's q $\mathbb{E}[q(x)]$	110.52%	110.19%
avg. credit spreads $\mathbb{E}[cs(x)]$	4.11%	4.13%
default rate $\hat{\lambda}_d$	1.40%	1.38%
growth rate μ_K	2.12%	1.79%
Distortions from credit market interventions		
$\frac{\Delta K}{K}$ credit price stabilization	-2.04%	-1.96%
$\frac{\Delta K}{K}$ credit access restoration	-1.95%	-3.90%
$\frac{\Delta K}{K}$ equity market spill over	4.74%	6.46%

Table A-10: Equity issuance frictions. First part of the table shows some of the ergodic statistics of (a) the baseline model and (b) the model extension that features equity issuance frictions, under the same model parameters. Second part of the table shows the percentage deviation of the long-run capital stock relative to the laissez-faire benchmark under the “credit price stabilization,” “credit access restoration,” and “equity market spillover” scenarios. “Baseline” refers to our baseline frictionless model, whereas “Equity issuance frictions” refers to our model with equity issuance costs.

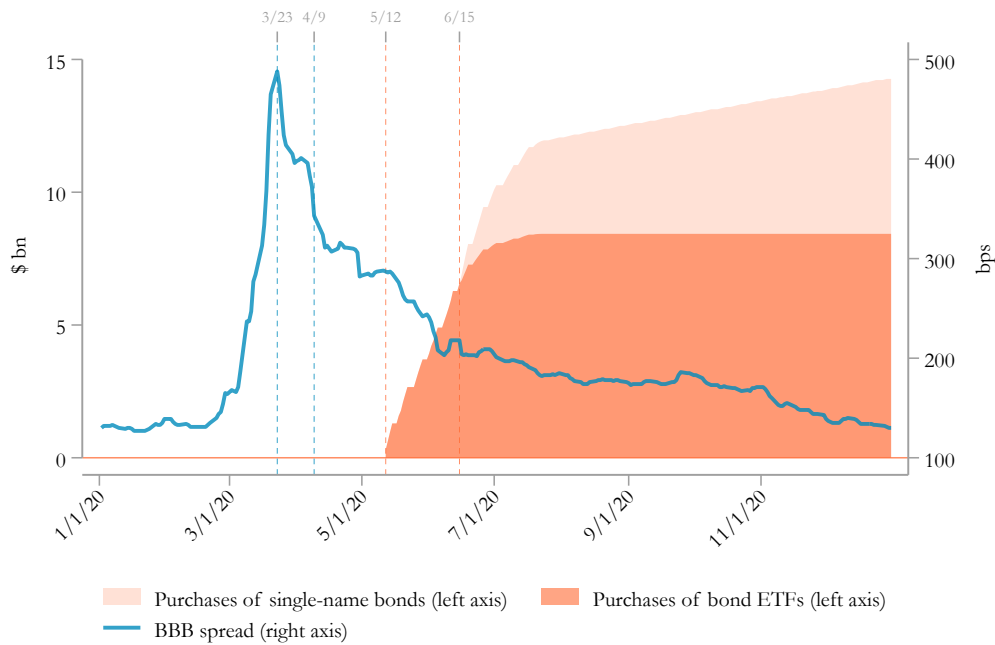


Figure A-1: Effects of the Secondary Market Corporate Credit Facility. The shaded areas report the cumulative gross purchases of bond ETFs (dark orange) and single-name bonds (light orange) realized under the SMCCF until its closure, on 12/30/2020. The blue line reports the ICE/BofA BBB corporate option-adjusted spread, which measures the option-adjusted spread of an index of corporate securities rates BBB over the corresponding spot Treasury rates (FRED series BAMLCOA4CBBB). The two dates highlighted with blue lines correspond to the Fed's announcement of the Corporate Credit Facilities, on March 23rd, 2020, and of their extension, on April 9th, 2020. The two dates highlighted in orange correspond to the start of the purchases of ETFs (May 12th, 2020) and single-name bonds (June 16th, 2020).

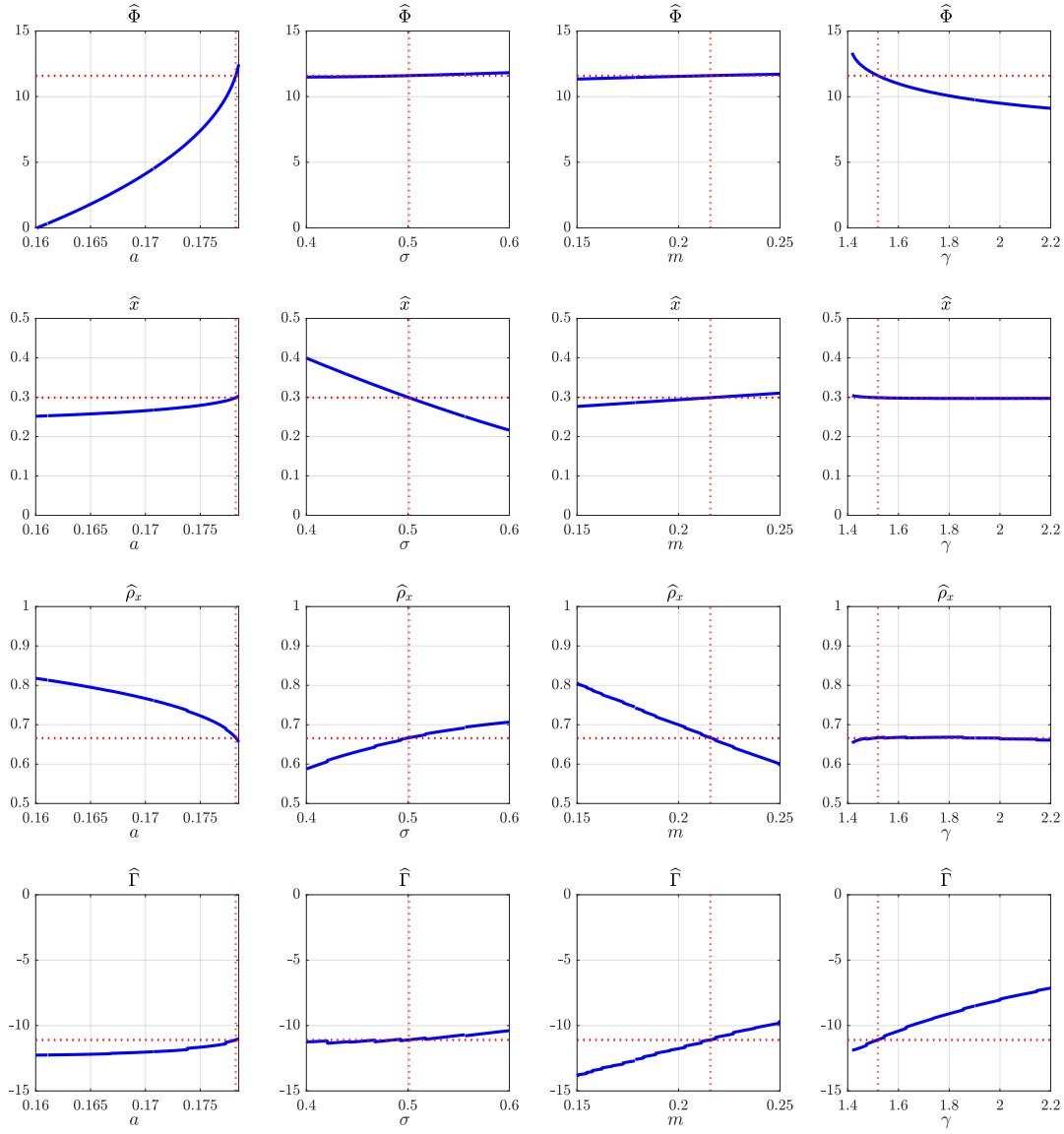


Figure A-2: Intuition for identification. Each row of panels corresponds to a different moment: the top row is $\hat{\Phi}$, the average gross investment rate; the second row from the top is $\hat{x}$, average book leverage; the third row from the top is $\hat{\rho}_x$, the autocorrelation of leverage; and the bottom row is the sensitivity of gross investment with respect to leverage, $\hat{\Gamma}$. The columns correspond to different parameters: from left to right, a , σ , m and γ , respectively. In each graph, the solid blue line is the value of the moment as a function of the corresponding parameter. The dashed red lines represent the targeted moment and the estimated parameter in our baseline exercise, the results of which are reported in Table 1.

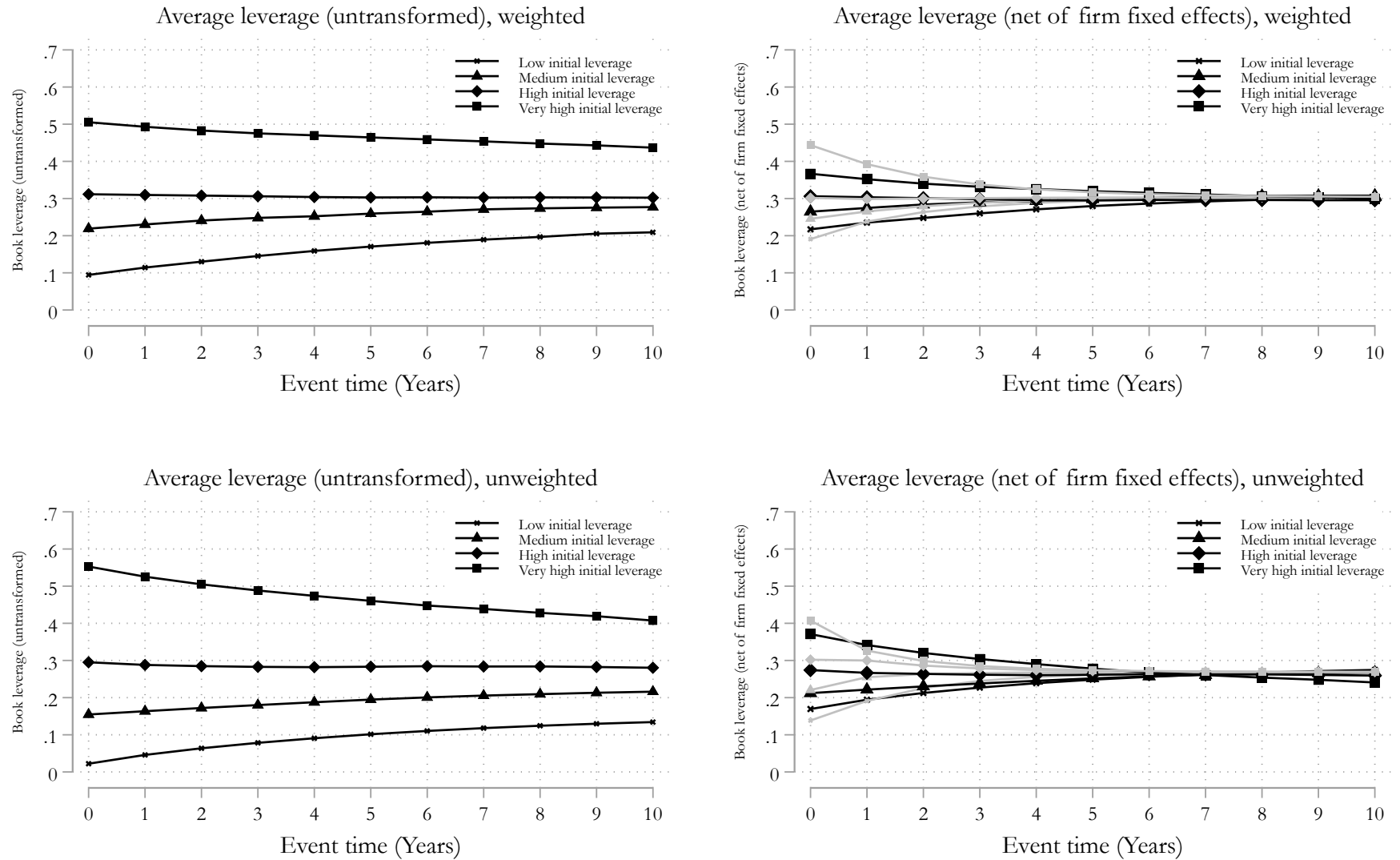


Figure A-3: External validation: leverage persistence. In each panel, black lines refer to data moments, and grey lines refer to their model-implied counterparts. The vertical axis refers to book leverage. In each panel, firms are sorted by initial leverage quartile are formed. In the data, initial leverage is computed year by initial year, for 1979 to 2019. In each subsequent year, average leverage conditional on the initial leverage quartile is computed. The lines reported are simple averages of these moments across initial years. The horizontal axis reports time since the portfolio is formed. On the right column, the moments reported are the raw data. On the left column, leverage is first de-meant within firm. On the top line, moments are weighted by book assets. On the bottom line, moments are unweighted. Appendix A.3.5.1 provides more detail on the construction of these moments and on their model counterparts.