

Aggregate Implications of Corporate Debt Choices

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This article studies the transmission of financial shocks in a model where corporate credit is intermediated via both banks and bond markets. In choosing between bank and bond financing, firms trade-off the greater flexibility of banks in case of financial distress against the lower marginal costs of large bond issuances. I find that, in response to a contraction in bank credit supply, aggregate bond issuance in the corporate sector increases, but not enough to avoid a decline in aggregate borrowing and investment. Keeping leverage constant while retiring bank debt would expose firms to a higher risk of financial distress; they offset this by reducing total borrowing. A calibration of the model to the Great Recession indicates that this precautionary mechanism can account for one-third of the total decline in investment by firms with access to bond markets.

Key words: Banks, Bonds, Financial structure, Financial frictions, Firm dynamics, Output, Investment, Productivity risk.

JEL Codes: E22, E23, G21, G33

1. INTRODUCTION

How do financial frictions affect aggregate investment? In the aftermath of the 2007–9 recession, which was marked by a dramatic 24.2% collapse in fixed capital formation, the question has attracted renewed attention among policymakers and economists.¹ Because debt is the main source of external financing for investment projects, the literature on the topic has focused on the effects of frictions affecting the issuance of debt. Moreover, models capturing these frictions generally assume that all borrowing takes place via a single financial intermediary, typically thought of as a bank. In these models, firms effectively face a single borrowing constraint and use a single type of debt.

While this approach may be useful to capture the constraints faced by small firms, it ignores the fact that larger firms in fact have access to, and make use of, various types of debt instruments. Bank loans only accounted for 31.3% of total debt of the non-financial corporate sector in the U.S. on the eve of the Great Recession, with bonds making up the lion's share of the remainder (63.0%). Moreover, there is substantial evidence that the debt mix used by firms with access to public debt markets varies, both in the cross section and over time. In the cross section, [Rauh and Sufi \(2010\)](#) study a sample of rated U.S. firms, and show that the majority of them borrow simultaneously from banks and bond markets, with only the very largest specializing in bond borrowing. Over the

1. Appendix A contains a description of data sources.

business cycle, [Adrian *et al.* \(2012\)](#) and [Becker and Ivashina \(2014\)](#) find evidence of substitution between bank loans and corporate bonds among rated firms as credit conditions tighten. Yet, while debt composition is shaped by idiosyncratic and aggregate factors, its implications for aggregate investment elude standard models, where total borrowing is the only margin of choice. This is not an omission of minor importance: in 2007, firms with access to public debt markets—those for which debt composition was a relevant choice—accounted for 63.7% of the total investment of the non-financial corporate sector.²

Given this evidence, the goal of this article is to understand whether, and how, the conclusions of standard models of financial frictions change when firms have access to different debt instruments. I address three specific questions. First, what drives cross-sectional differences in debt choices, and how do they relate to firm-level investment? Second, how do aggregate shocks, and in particular shocks to bank credit supply, affect aggregate investment, when firms can endogenously adjust debt composition? Third, can policy affect the way firms choose their financing mix, say by encouraging bond issuance in lieu of bank borrowing, and if so, are such policies necessarily beneficial to aggregate investment?

To answer these questions, I develop a macroeconomic model with heterogeneous firms in which both the scale and composition of debt are endogenous. Section 2 describes this model. It is the first model of firm dynamics with financial frictions to endogenize jointly borrowing composition, investment choices, and firm growth. In this economy, firms can finance investment either internally (through the accumulation of retained earnings) or by issuing two types of debt: bank loans and market debt. Credit is constrained by the fact that firms have limited liability, and default entails deadweight losses of output. The central assumption of the model is that banks and market lenders differ in their ability to deal with financial distress. Specifically, I assume that bank loans can be restructured when firms' revenues are low, whereas market liabilities cannot. In this sense, banks offer more flexibility than market lenders when a firm is in financial distress. On the other hand, outside of financial distress, I assume that bank lending is more restrictive than market lending. In the model, this difference is captured through differences in intermediation costs between bank and market lenders. The higher intermediation costs of banks are reflected in the equilibrium terms of lending contracts, which firms must honour outside of financial distress. In choosing the scale and composition of borrowing, firms therefore trade-off the greater flexibility of bank debt in financial distress against the lower costs associated with market financing during normal times.³

Section 3 analyses firms' financial policies in steady state. The model endogenously generates a distribution of firms across levels of internal finance, and along this distribution, firms differ in their choices of debt composition and investment. The model has two key predictions. First, firms sort into two financial regimes: a "mixed finance" regime, in which they borrow simultaneously from banks and bond markets, and a "market finance" regime, in which they borrow only from public debt markets. Second, the transition between these two regimes occurs as a firm's internal finance increases and its credit risk falls. The joint determination of debt structure and investment is crucial to understanding these results. First, the endogeneity of investment creates a complementarity between bank and market finance: as market debt issuance increases, firms are able to invest more and operate at larger scales; this improves their liquidation

2. This ratio compares total investment of rated non-financial firms in Compustat with total investment of non-financial corporate businesses from the Flow of Funds. Appendix A contains more details on the construction of these ratios.

3. In Section 2, I discuss the empirical support for the view that banks are more flexible creditors than markets in financial distress. I also provide evidence in support of the assumption that during normal times, bank borrowing is more costly and places more constraints on firms than market borrowing.

value, which may in turn relax their bank borrowing constraint. This complementarity is what drives some firms to use a combination of bank and market debt. Second, because of decreasing returns, firms have an unconstrained optimal scale of operation, which is independent of internal resources. As firms grow and accumulate internal resources, they require less leverage to approach that scale. Lower leverage implies a smaller probability of financial distress, and therefore smaller gains associated with bank debt flexibility. When internal finance is large enough, relative to the optimal scale of investment, these gains become negligible and firms only use market debt.

These cross-sectional predictions line up very closely with recent evidence on the debt structure of rated firms, both qualitatively and quantitatively. [Rauh and Sufi \(2010\)](#) study the debt structure of rated U.S. firms, and emphasize two key findings: first, the majority of firms in their sample (68.3%) use a combination of bank loans and publicly traded debt, with the remainder using only publicly traded debt; second, firms move from the former to the latter group as their credit quality increases.⁴ In the model, the key driver of a firm's choice between mixed finance and market finance is its ex-ante probability of financial distress, which declines as it grows and accumulates internal finance. As a result, there is a tight connection between measures of risk, such as the ex-ante liquidation probability, or the volatility of profits, and debt composition, as is the case in the data. Moreover, the model provides a quantitatively accurate counterpart to the data. In the baseline calibration of the model, which matches the *aggregate* ratio of bank debt to total debt of medium and large U.S. firms, 57.1% of firms use a mixed debt structure. The steady state of the model thus captures both the intensive and extensive margin aspects of debt choices.

Given the model's ability to match key cross-sectional facts about debt structure, the remainder of the article uses it as a laboratory to study the aggregate implications of debt choices. Section 4 focuses on the transmission of financial shocks. I construct the perfect foresight response of the economy to a bank credit supply shock, which is captured as a permanent increase in the intermediation costs of banks. The magnitude of this shock is chosen so that the model matches the large and persistent drop in the aggregate bank share of total debt for medium and large U.S. firms during the Great Recession, which fell from 24.0% in 2007Q3 to 19.8% in 2011Q3. Matching this drop quantitatively requires an increase in banks' intermediation costs of roughly 12% of their initial difference with market intermediation costs in steady state. In addition to its effects on the bank share, the shock also generates a large recession, with investment contracting by 7.9% over the first three years. In contrast, in Compustat data, investment by rated firms dropped by 21.1% between 2007 and 2010. The shock is thus sufficient to account approximately 40% of the observed contraction in investment by firms with access to public debt markets observed in that dataset.

The shock affects investment in two ways, one already present in existing macroeconomic models of frictional investment, and one new to this article. On one hand, firms react to the permanently higher bank intermediation costs by reducing their borrowing from banks. This is the traditional "bank lending channel" of financial shocks. In this model, the effect accounts for about two-thirds of the response of investment and is limited to mixed finance firms, which relied on bank loans before the shock. The remaining third of the response of output is accounted for by a novel propagation mechanism. This mechanism is confined to firms that, prior to the shock, had accumulated sufficient internal finance to be close to the market finance regime, but were still relying on bank debt. The bank credit supply shock causes these firms to make the

4. In related work, [Colla et al. \(2013\)](#) document similar patterns of debt choices among rated firms. Section 3 discusses this evidence in more detail.

switch between financial regimes. In other words, these firms retire all outstanding bank debt and issue bonds instead. The key finding is that this substitution is less than one for one. As a result, the amounts borrowed and invested by switching firms drop relative to their pre-shock investment levels. The negative effect of this “debt substitution channel” can be interpreted as a precautionary response of firms. Moving from the mixed to the market finance regime strips firms of the flexibility that bank debt normally offers. As a result, for a comparable leverage, switching firms are more prone to liquidation risk. Switching between financial regimes increases the financial fragility of these firms. To offset this increase in financial fragility, they choose to deleverage relative pre-shock levels.⁵ The quantitative exercise of this article suggests that the “debt substitution channel” played an important role in the contraction of investment during the Great Recession.

Section 5 concludes by studying the real effects of policies aimed at incentivizing firms to rely more heavily on market credit. I analyse two specific examples: German efforts to develop a bond market specifically targeted at medium-sized firms, and an Italian fiscal reform extending tax deductibility of interest payments to bond issues by private firms. As for the propagation of financial shocks, the effects of these policies are best understood in terms of a “lending channel” and a “substitution channel” effect. On the one hand, they boost aggregate investment by lowering the cost of market debt issuance (the “lending channel” effect, applied to market lending rather than banks, in this case). On the other hand, they induce medium-sized firms that were previously partially bank financed to switch entirely to market finance. As a result of their increased fragility, these firms borrow less, and their output and investment falls (the “substitution channel” effect). The net effect of the policy on aggregate investment and output in steady state is in general positive, but this comes at the expense of a precautionary reduction in leverage and activity of medium-sized firms.

The results of this article speak to the idea that bond markets could act as a substitute to traditional, bank-based intermediation at times when the latter is impaired. Providing support to corporate bond markets has indeed been one of the explicitly stated goals of recent quantitative easing policies, for example, in the U.K. and the U.S.⁶ In the framework of this article, bank and bond debt are imperfect substitutes. While firms modify their debt structure in response to relative increase in the cost of bank credit, the substitution towards bond financing has adverse effects on the investment of firms of intermediate size and credit risk. This is because bond financing is inherently more fragile, so that a change in debt structure also requires lower leverage and lower investment. Absent other mechanisms to offset this precautionary deleveraging, the model thus suggests that these policies could inadvertently have had a contractionary effect on the investment decisions of firms of intermediate size.

1.1. *Related literature*

This article builds on the extensive literature on corporate debt structure, following the seminal contributions of [Diamond \(1991\)](#), [Rajan \(1992\)](#), [Besanko and Kanatas \(1993\)](#) and [Bolton and Scharfstein \(1996\)](#). The assumption that banks and bondholders differ in their degree

5. The term “precautionary” is used here somewhat loosely, as firms value the stream of future dividends in a risk-neutral manner. However, the combination of decreasing returns and borrowing constraints implies that firms’ valuation of internal finance flows is concave. This is a common feature of this class of models, and it is explored in depth by [Cooley and Quadriini \(2001\)](#).

6. For a description of the quantitative easing policies of the Bank of England and their effects on corporate bond issuance, see [Joyce et al. \(2011\)](#). For the effects of the Federal Reserve’s Large-Scale Asset Purchase programmes on corporate bond yields, see [Gilchrist and Zakrajšek \(2013\)](#).

of flexibility in times of financial distress builds on the insight of [Bolton and Scharfstein \(1996\)](#) that the dispersion of bondholders reduces individual incentives to renegotiate debt payments and may create holdout problems that impede efficient restructuring. The description of banks as flexible creditors adopted in this article is closest to the continuous-time framework of [Hackbarth *et al.* \(2007\)](#). This article, along with most of the theoretical literature on the topic, focuses on the structure of financing, given a fixed scale of investment; in contrast, in this article, the scale of investment is also determined endogenously, which allows for cross-sectional and aggregate implications to be drawn from the model.

This article is also related to the literature on firm growth and financial frictions. The key friction is limited liability, as in [Cooley and Quadrini \(2001\)](#), [Clementi and Hopenhayn \(2006\)](#), or [Hennessy and Whited \(2007\)](#). In particular, the connection between firms' optimal financial policies and their steady state growth dynamics follows closely [Cooley and Quadrini \(2001\)](#). This article contributes to this literature by introducing an endogenous choice of debt composition and illustrating its implications for firm growth and the distribution of firms across levels of internal finance in steady state.

The macroeconomic implications of debt heterogeneity have been addressed by relatively few papers. [Bolton and Freixas \(2006\)](#), in the context of a static model, show that, by affecting the spread of bank loans over corporate bonds, monetary policy can lower banks' equity-capital base, in turn leading to a contraction in corporate credit. This is in contrast to the traditional view that the "bank lending channel" operates through reductions in bank reserves. The model of this article does not distinguish between the causes of contractions in bank lending; its focus is purely and squarely on their consequences for firm-level and aggregate investment. Closest to this framework, [De Fiore and Uhlig \(2011, 2014\)](#) construct an asymmetric information model of bond and bank borrowing and show that it accounts well for long-run differences between the Euro Area and the U.S. to the extent that information availability on corporate risk is lower in Europe. They also provide a model-based assessment of the changes in corporate debt composition in the U.S. during the Great Recession, relying on a combination of different shocks, including an increase in firm-level uncertainty and in the intermediation costs of banks. Aside from the differences in the role that banks play in our respective setups, their work addresses different aspects of debt choices. In their model, firms choose to be financed either entirely by loans or entirely by bonds; thus, their results speak to the extensive margin of participation in public debt markets. Instead, this article focuses on the debt composition choices of firms with access to public debt markets.

2. A DYNAMIC MODEL OF INVESTMENT AND DEBT COMPOSITION

This section describes a model of firm investment with financial frictions. Heterogeneous firms with limited internal finance must raise outside funding from financial intermediaries.⁷ Time is discrete. A firm's stock of internal finance at the beginning of period t is denoted by e_t ; it represents total retained earnings accumulated by the firm up to time t . The novelty of the model is that outside funding can take two forms: bank debt (b_t) or market debt (m_t). Firms are infinitely lived, but all debt contracts mature within one period.

In this economy, investment is constrained for two reasons. First, firms have limited liability, and liquidation involves deadweight losses. Second, equity issuance is costly. In the baseline version of the model, firms only issue equity at birth; no issuance of new shares is allowed thereafter. (Section 3.5 extends the model to allow active firms to issue new shares). If issuance

7. Throughout, the term "firm" will, strictly speaking, refer to equityholders.

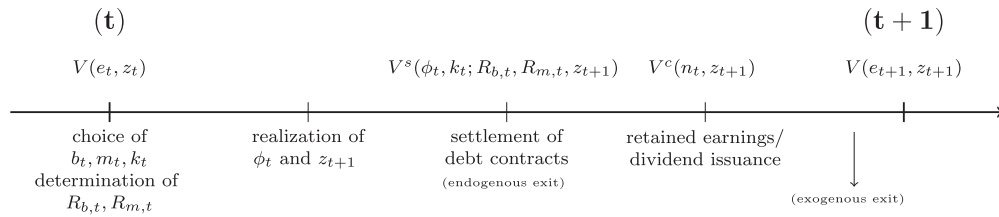


FIGURE 1
Timing of an individual firm's problem.

of new shares were costless, or if deadweight liquidation losses were zero, investment would be unconstrained. With frictions to both new equity and debt issuance, the firm will progressively accumulate internal finance to limit its reliance on debt.

For clarity, Sections 2.1–2.8 describe the details of the model, while Section 2.9 discusses the key model assumptions.

2.1. Overview of an individual firm's problem

Figure 1 summarizes the timing of an individual firm's problem. There are three stages within each period: first, the choice of investment and debt structure; second, the settlement of debt contracts; third, the dividend issuance/retained earnings decision.

At the beginning of the period, a firm is characterized by its stock of internal finance e_t and by its average (or expected) productivity level in the current period, z_t . The value of the firm at this stage is denoted by $V(e_t, z_t)$; its managers are risk-neutral and value future dividend payments using the discount factor β .

The firm operates a decreasing returns to scale technology that takes capital k_t as the sole input. The firm's total resources after production are given by

$$\pi_t = \phi_t k_t^\zeta + (1 - \delta)k_t,$$

where $0 < \zeta < 1$ denotes the degree of returns to scale, $0 < \delta < 1$ denotes the rate of depreciation of capital, and $\phi_t = z_t \epsilon_t$ denotes the realized productivity of the firm. The shock ϵ_t is drawn independently of z_t from a distribution $F_\epsilon(\cdot)$ with mean standardized to 1 and support on the positive real line. Simultaneously with ϵ_t , the firm's next-period productivity z_{t+1} is also drawn from a discrete-state Markov chain with transition probabilities $p(\cdot | z_t)$, defined over the state-space \mathcal{Z} .

For a firm borrowing $b_t \geq 0$ and $m_t \geq 0$ from bond markets, total investment is given by:

$$k_t = e_t + b_t + m_t.$$

The two debt contracts specify a gross promised repayment to banks, $R_{b,t}$, and to market-based intermediaries, $R_{m,t}$. The debt structure (b_t, m_t) is chosen by the firm before ϕ_t or z_{t+1} are known, so the contractual promises $R_{b,t}$ and $R_{m,t}$ cannot be indexed by either shock. Instead, they only depend on b_t, m_t (the amounts borrowed) and (e_t, z_t) (the firm's state variables).⁸

8. In the model described in this section, firms are not allowed to save within a period. Section 3.5 relaxes this assumption.

Debt contracts are settled in an interim stage. At this stage, the value of a firm capital in place k_t , current productivity ϕ_t , next-period productivity z_{t+1} and liabilities $(R_{b,t}, R_{m,t})$ is denoted by $V^s(\phi_t, k_t; R_{b,t}, R_{m,t}, z_{t+1})$. The firm can choose to either fulfill its promise to repay $(R_{b,t}, R_{m,t})$ to its creditors, or attempt to renegotiate down the payments. If the realization of ϕ_t is sufficiently bad, the firm may also be forced to liquidate. The details of the settlement process are spelled out below.

Firms that are not liquidated during debt settlement allocate remaining funds, n_t , between dividends and next-period internal finance, e_{t+1} . These funds represent the firm's net worth after debt settlement:

$$n_t = \pi_t - \tilde{R}_{b,t} - \tilde{R}_{m,t},$$

where $\tilde{R}_{b,t}$ and $\tilde{R}_{m,t}$ denote the (potentially renegotiated) payments to banks and market-based intermediaries, respectively. After this dividend issuance choice, only a fraction $(1 - \eta)$ of firms survive; the rest receive an exogenous exit shock. These firms consume a fraction $\xi \in [0, 1[$ of e_{t+1} before exiting.

A recursive formulation of the firm's problem can be obtained by solving the model backwards within each period, starting with the dividend-issuance stage.

2.2. Dividend issuance

Given the value function V , the dividend-issuance problem can be written as:

$$\begin{aligned} V^c(n_t, z_{t+1}) &= \max_{d_t, e_{t+1}} d_t + (1 - \eta)\beta V(e_{t+1}, z_{t+1}) + \eta\xi e_{t+1} \\ \text{s.t.} \quad &d_t + e_{t+1} \leq n_t, \quad d_t \geq 0. \end{aligned} \quad (1)$$

Here, d_t denotes dividends issued, and η denotes the probability of exogenous exit. Conditional on exogenous exit, the firm is allowed to consume a fraction ξ of its net worth. The dividend issuance problem is analogous to [Cooley and Quadrini \(2001\)](#). As in that model, the optimal dividend policy of the firm will be characterized by a unique threshold, denoted by $\bar{e}(z_{t+1})$, such that a firm with net worth n_t and next-period productivity z_{t+1} will issue dividends if $n_t > \bar{e}(z_{t+1})$, and will otherwise reinvest n_t and issue no dividends.⁹ The threshold $\bar{e}(z_{t+1})$ is such that the marginal value net worth n is strictly larger than 1 for any $n < \bar{e}(z_{t+1})$, and smaller than 1 for any $n \geq \bar{e}(z_{t+1})$. An important implication of this dividend issuance rule is that surviving firms with productivity z_t never start a period with more than $\bar{e}(z_t)$ in internal finance. This limits the relevant state-space of the firm's problem to $[0, \bar{e}(z_t)]$, for any productivity level z_t .

2.3. Debt settlement

At the debt-settlement stage, the firm has three options: liquidation, debt restructuring, or full payment of its liabilities. Let V_t^L , V_t^R , and V_t^P denote the respective values of the firm under each option. The firm faces the discrete choice problem:

$$V^s(\phi_t, k_t; R_{b,t}, R_{m,t}, z_{t+1}) = \max_{L,R,P} (V_t^L, V_t^R, V_t^P). \quad (2)$$

9. The optimality of this dividend policy is formally established as a part of the proof of Proposition 3.

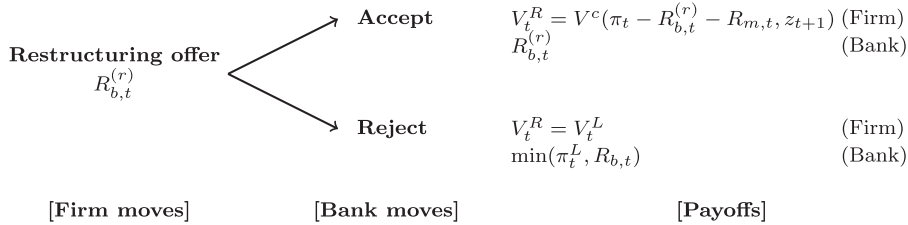


FIGURE 2

Two-stage game for debt restructuring.

Liquidation (V_t^L) In liquidation, the firm is shut down and its resources π_t are seized by creditors.

Assumption 1. *The liquidation value of the firm is $\pi_t^L = \chi \pi_t$, where $0 \leq \chi \leq 1$.*

When $\chi < 1$, the transfer of the firm's resources to creditors involves a deadweight loss. In that case, lenders charge the firm a liquidation premium. Absent this loss (when $\chi = 1$), investment would be unconstrained.

With multiple creditors, one must take a stance on how liquidation resources are allocated among stakeholders. I assume that a claim to π_t^L can only be honoured if all stakeholders placed higher in the priority structure have first been made whole, similar to the Absolute Priority Rule (APR) which governs chapter 7 corporate bankruptcies in the U.S. In this model, there are three stakeholders: bank lenders, market lenders, and the firm itself. The firm is the residual claimant. I furthermore assume that bank lenders are senior to market lenders in the priority structure. Payoffs to stakeholders are then given by:

$$\begin{aligned} \tilde{R}_{b,t} &= \min(R_{b,t}, \pi_t^L) && \text{(bank lenders)} \\ \tilde{R}_{m,t} &= \min(\max(0, \pi_t^L - R_{b,t}), R_{m,t}) && \text{(market lenders)} \\ V_t^L &= \max(0, \pi_t^L - R_{b,t} - R_{m,t}) && \text{(firm)} \end{aligned} \quad (3)$$

Restructuring (V_t^R) In restructuring, firms attempt to renegotiate their liabilities, conditional on the realization of their resources π_t . The crucial distinction between banks and market lenders lies with their ability to carry out this restructuring.

Assumption 2. (Debt flexibility). *Only bank debt can be restructured.*

Restructuring is modelled as a two-stage game between the firm and the bank. This game is summarized in Figure 2. The firm and its bank bargain over the potential surplus generated by avoiding liquidation. The firm moves first and offers to repay the bank the amount $R_{b,t}^{(r)}$, instead of the promised amount $R_{b,t}$. The bank can choose to accept or reject the offer. If the offer is rejected, liquidation ensues, and all parties receive the liquidation payoffs described by (3). In this process, market lenders only have an indirect role; their liabilities must remain untouched in any successful restructuring agreement.

The optimal action of the bank is to accept the firm's offer, if and only if it exceeds its reservation value, i.e., if and only if $R_{b,t}^{(r)} \geq \min(R_{b,t}, \pi_t^L)$. The value of an offer $R_{b,t}^{(r)}$ to the firm is

therefore

$$\tilde{V}^R(R_{b,t}^{(r)}) = \begin{cases} V^c(\pi_t - R_{b,t}^{(r)} - R_{m,t}, z_{t+1}) & \text{if } R_{b,t}^{(r)} \geq \min(R_{b,t}, \pi_t^L) \\ V_t^L & \text{if } R_{b,t}^{(r)} < \min(R_{b,t}, \pi_t^L) . \end{cases}$$

The firm chooses the restructuring offer, $R_{b,t}^{(r)}$, to maximize this value, subject to a feasibility constraint. Specifically, we make the following assumption.

Assumption 3. (Transfers to creditors). *The total transfers to creditors (bank and market) at the debt settlement stage cannot exceed:*

$$\pi_t^P = \phi_t k_t^\zeta + \chi_P (1 - \delta_k) k_t, \quad \chi_P \in [\chi, 1].$$

The parameter χ_P captures the firm's ability to sell capital to pay down debt. If $\chi_P = 1$, depreciated capital is effectively fungible with operating revenue; it can be liquidated at no loss to settle debt payments. If $\chi_P = \chi$, instead, the firm incurs the full extent of liquidation losses when it sells capital to pay down debt. I discuss Assumption 3 in more detail in Section 2.9.

With this assumption, the debt settlement problem can be written as:

$$\begin{aligned} V_t^R &= \max_{R_{b,t}^{(r)}} \tilde{V}_t^R(R_{b,t}^{(r)}) \\ \text{s.t. } &\pi_t^P - R_{b,t}^{(r)} - R_{m,t} \geq 0. \end{aligned} \quad (4)$$

Payment (V_t^P) A firm can only pay its creditors in full if resources available for repayment, π_t^P , exceed total liabilities; otherwise, it is liquidated. Therefore,

$$V_t^P = \begin{cases} V^c(\pi_t^P - R_{m,t} - R_{b,t}, z_{t+1}) & \text{if } \pi_t^P \geq R_{b,t} + R_{m,t} \\ V_t^L & \text{if } \pi_t^P < R_{b,t} + R_{m,t}. \end{cases} \quad (5)$$

Debt-settlement outcomes Given the values of ϕ_t , k_t , $R_{m,t}$, and $R_{b,t}$, a solution to problem (2), subject to (3), (4), and (5), is called a *debt settlement outcome*. The following proposition describes the equilibrium debt-settlement outcomes.

Proposition 1. (Debt-settlement outcomes). *Let $\gamma \equiv (1 - \chi_P)(1 - \delta)$. Then,*

- **when** $\frac{R_{b,t}}{\chi} \geq \frac{R_{m,t} + \gamma k_t}{1 - \chi}$, *the firm chooses to repay its creditors in full if and only if $\pi_t \geq \frac{R_{b,t}}{\chi}$. It successfully restructures its debt if and only if $\frac{R_{m,t} + \gamma k_t}{1 - \chi} \leq \pi_t < \frac{R_{b,t}}{\chi}$, and it is liquidated when $\pi_t < \frac{R_{m,t} + \gamma k_t}{1 - \chi}$;*
- **when** $\frac{R_{b,t}}{\chi} < \frac{R_{m,t} + \gamma k_t}{1 - \chi}$, *the firm repays its creditors in full if and only if $\pi_t \geq R_{b,t} + R_{m,t} + \gamma k_t$, and it is liquidated otherwise.*

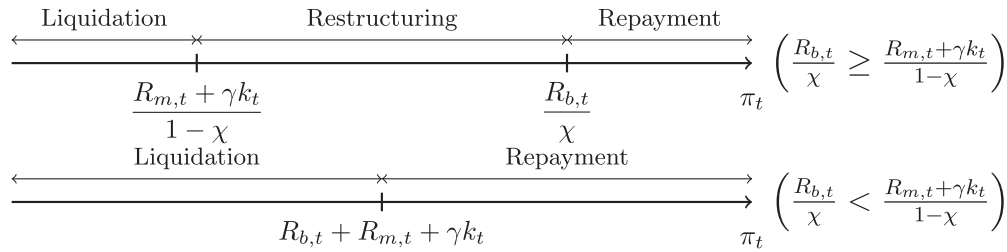


FIGURE 3

Debt settlement outcomes, as a function of the realization of π_t .

Moreover, in all debt-settlement outcomes resulting in restructuring, the bank obtains its reservation value $\chi\pi_t$, and in all debt-settlement outcomes resulting in liquidations, $V_t^L = 0$.

Figure 3 gives a graphical representation of the two possible situations described in Proposition 1. The logic underlying the conditions of this proposition is the following. In restructuring, the bank always obtains at most its reservation payoff, $\chi\pi_t$. So, restructuring is only advantageous for the firm if $n_t^R = \pi_t - \chi\pi_t - R_{m,t} \geq n_t^P = \pi_t - R_{b,t} - R_{m,t}$, or equivalently, if $\pi_t \leq \frac{R_{b,t}}{\chi}$. At the same time, restructuring bank debt payments to $\chi\pi_t$ is only *feasible* if it leaves the firm able to pay off bondholders: $\pi_t^P \geq \chi\pi_t + R_{m,t}$, or, since $\pi_t^P = \pi_t - \gamma k_t$, $\pi_t \geq \frac{R_{m,t} + \gamma k_t}{1 - \chi}$. Thus, for restructuring to occur, it must be the case that the liabilities of the firm are such that $\frac{R_{m,t} + \gamma k_t}{1 - \chi} \leq \frac{R_{b,t}}{\chi}$, and that the realization of income π_t falls between these boundaries.

Two points are worth noting about the results of Proposition 1. First, the firm does not necessarily prefer restructuring to repayment. As mentioned above, the firm will only find restructuring profitable if the surplus it can extract from it, $R_{b,t} - \chi\pi_t$, is positive. The bank's ability to force liquidation effectively limits the firm's incentives to use the bargaining process.

Second, restructuring will not always save the firm from liquidation. This is because of the indirect role of market lenders in the restructuring process. When market liabilities are relatively large ($\frac{R_{b,t}}{\chi} \leq \frac{R_{m,t} + \gamma k_t}{1 - \chi}$), even though the firm can extract a positive surplus from the bank in restructuring, that surplus is never sufficient to avoid liquidation. On the other hand, when market liabilities are relatively small ($\frac{R_{b,t}}{\chi} \geq \frac{R_{m,t} + \gamma k_t}{1 - \chi}$), restructuring can be the best option for the firm. In some cases ($\frac{R_{m,t} + \gamma k_t}{1 - \chi} \leq \pi_t \leq R_{b,t} + R_{m,t} + \gamma k_t$), it is because restructuring allows the firm to avoid liquidation. In others ($R_{b,t} + R_{m,t} + \gamma k_t \leq \pi_t \leq \frac{R_{b,t}}{\chi}$), the firm restructures opportunistically: it could pay in full both creditors, but instead decides to use its bargaining power to extract surplus from the bank.

2.4. Debt pricing and feasible debt structures

Banks and market lenders are perfectly competitive financial intermediaries. Unit lending costs are given by r_b (for banks) and r_m (for markets); here, I treat them as given, and come bank to their equilibrium determination in Section 2.7. Perfect competition implies that lenders make zero expected profits on each loan. Therefore, equilibrium promised repayments $R_{b,t}$ and $R_{m,t}$

must satisfy:

$$\sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t \geq 0} \tilde{R}_b(\phi_t, \overbrace{e_t + b_t + m_t}^{=k_t}, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t | z_t) \right) = (1 + r_b) b_t, \quad (6)$$

$$\sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t \geq 0} \tilde{R}_m(\phi_t, e_t + b_t + m_t, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t | z_t) \right) = (1 + r_m) m_t.$$

Here, $\tilde{R}_i(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1})$, $i = b, m$, is short-hand notation for the gross return on lending for each type of financial intermediary. This return depends on the realization of the shock $\phi_t = z_t \epsilon_t$, as well as on the firm's debt structure, since both determine the firms' resources π_t and the associated debt-settlement outcomes. Moreover, the debt contracts may in principle depend on the realization of next-period productivity z_{t+1} , since this realization influences the firm's continuation value, and hence its default decision.¹⁰

The *lending menu* $\mathcal{S}(e_t, z_t)$ is defined as the set of all debt structures $(b_t, m_t) \in \mathbb{R}_+^2$ for which there exists a unique solution to (6). It is the set of feasible debt structures for a firm with internal finance e_t and productivity z_t .

Proposition 2. *The lending menu $\mathcal{S}(e_t, z_t)$ is non-empty and compact for all $e_t \geq 0$. Moreover, $\mathcal{S}(e_t, z_t)$ can be partitioned into two non-empty and compact subsets $\mathcal{S}_K(e_t, z_t)$ and $\mathcal{S}_R(e_t, z_t)$, such that:*

- The lending terms $(R_{b,t}, R_{m,t})$ satisfy $\frac{R_{b,t}}{\chi} \geq \frac{R_{m,t} + \gamma k_t}{1 - \chi}$ if and only if $(b_t, m_t) \in \mathcal{S}_R(e_t, z_t)$;
- The lending terms $(R_{b,t}, R_{m,t})$ satisfy $\frac{R_{b,t}}{\chi} < \frac{R_{m,t} + \gamma k_t}{1 - \chi}$ if and only if $(b_t, m_t) \in \mathcal{S}_K(e_t, z_t)$.

Proposition 2 provides a partition of the feasible set of debt structures according to the nature of the debt settlement outcomes that they lead to. In the first subset, $\mathcal{S}_R(e_t, z_t)$, debt structures (b_t, m_t) imply liabilities $(R_{b,t}, R_{m,t})$ such that restructuring is sometimes successful during debt settlement. On the other hand, in the second subset, $\mathcal{S}_K(e_t, z_t)$, debt structures are such that restructuring never occurs during debt settlement. The subsets $\mathcal{S}_K(e_t, z_t)$ and $\mathcal{S}_R(e_t, z_t)$ are depicted in Figure 4. Their relative location conveys the same intuition that underpins Proposition 1: *ex-post*, debt structures tend to lead to successful restructuring when they are tilted towards bank loans (*i.e.* towards the lower-right part of the positive orthant in Figure 4). On the other hand, for debt structures tilted towards market debt (*i.e.* towards the upper-left part of the positive orthant in Figure 4), firms never successfully restructure once productivity is realized; instead, they either repay debt, or are liquidated.

10. For example, when $\frac{R_{m,t} + \gamma k_t}{1 - \chi} \leq \frac{R_{b,t}}{\chi}$, the gross lending return function for market lenders is given by

$$\tilde{R}_m(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) = \begin{cases} 0 & \text{if } \pi(\phi_t, k_t) \leq \frac{R_{m,t} + \gamma k_t}{1 - \chi} \\ R_{m,t} & \text{if } \pi(\phi_t, k_t) > \frac{R_{m,t} + \gamma k_t}{1 - \chi} \end{cases}.$$

The first case corresponds to a realization of productivity that is sufficient for the firm to be forced into liquidation. The second case corresponds to a realization of productivity such that the firm will either choose to pay its creditors in full or will be able to successfully restructure debt payments with the bank.

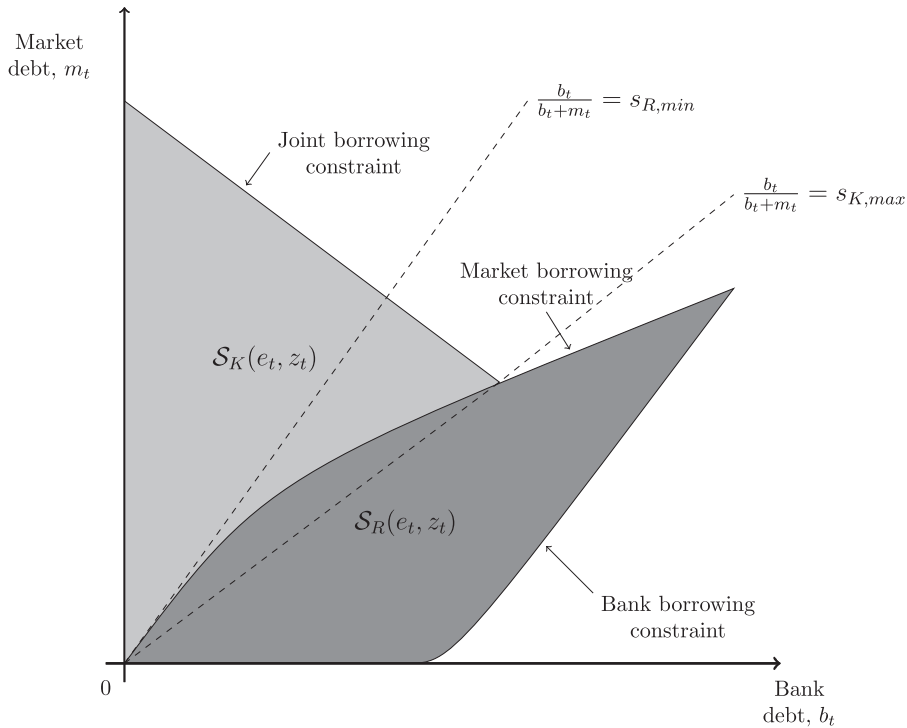


FIGURE 4
Generic shape of the lending menu.

2.5. The firm's dynamic debt structure problem

The firm's problem can now be written recursively as:

$$V(e_t, z_t) = \max_{(b_t, m_t) \in \mathcal{S}(e_t, z_t)} \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t \geq \phi^l(k_t, R_{b,t}, R_{m,t})} V^s(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t | z_t) \right) \quad (7)$$

$$\begin{aligned} \text{s.t.} \quad k_t &= e_t + b_t + m_t && \text{(Capital structure)} \\ \pi(\phi_t, k_t) &= \phi_t k_t^\chi + (1 - \delta)k_t && \text{(Production)} \\ (1 + r_b)b_t &= \sum_{z_t \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t} \tilde{R}_b(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t | z_t) \right) && \text{(ZPC bank)} \\ (1 + r_m)m_t &= \sum_{z_t \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t} \tilde{R}_m(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t | z_t) \right) && \text{(ZPC market)} \\ V^s(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) &= V^c(\max(n_t^P, n_t^R), z_{t+1}) && \text{(Debt settlement)} \\ n_t^P &= \pi(\phi_t, k_t) - R_{b,t} - R_{m,t} && \text{(Repayment)} \\ n_t^R &= \pi(\phi_t, k_t) - \chi \pi(\phi_t, k_t) - R_{m,t} && \text{(Restructuring)} \\ \pi(\phi^l(k_t, R_{b,t}, R_{m,t}), k_t) &= \min \left(\frac{R_{m,t} + \gamma k_t}{1 - \chi}, R_{b,t} + R_{m,t} + \gamma k_t \right) && \text{(Liquidation)} \\ V^c(n_t, z_{t+1}) &= \max_{0 \leq e_{t+1} \leq n_t} n_t - e_{t+1} + (1 - \eta)\beta V(e_{t+1}, z_{t+1}) + \eta \xi e_{t+1} && \text{(Dividend issuance)} \end{aligned}$$

This formulation incorporates results from the three stages of the firm's problem discussed in the previous paragraphs. First, the debt structure chosen by the firm at the beginning of the period must be feasible: $(b_t, m_t) \in \mathcal{S}(e_t, z_t)$. Second, the expression for the value of the firm at the debt-settlement stage, $V^s(\phi_t, k_t, R_{m,t}, R_{b,t}, z_{t+1})$, uses the results of Proposition 1. Finally,

the value of the firm at the dividend-issuance stage is the sum of the current value of dividends issued and the discounted value of future dividends.

Additionally, note that liquidation in the model can be triggered by negative realizations of net income n_t after debt settlement (whether or not a renegotiation occurred). In this sense, debt introduces a bias towards liquidation, in that indebted firms may exit even though they may have profitable growth options available (as embodied by next-period productivity z_{t+1}). Liquidation decisions are thus cash-flow driven and not strategic, in contrast to models that build upon the work of [Leland \(1994\)](#), or those that build upon [Hennessy and Whited \(2005, 2007\)](#). This helps to isolate the problem of the determination of the lending menu from the firm's dynamic investment problem, as described in more detail in Section 2.8.

A solution to this problem is characterized by thresholds for internal finance $\{\bar{e}(z_t)\}_{z_t \in \mathcal{Z}}$ as well as a value function V and policy functions \hat{b} and \hat{m} for borrowing levels and their promised repayments counterparts, \hat{R}_b , and \hat{R}_m . All these functions are defined on $B = [0, \bar{e}(\bar{z})] \times \mathcal{Z}$, where $\bar{z} = \max_{z \in \mathcal{Z}} z$.

2.6. Entry and exit

2.6.1. Exit. There are two sources of firm exit in this economy. First, some firms are endogenously liquidated at the debt-settlement stage. The fraction of existing firms with internal finance e_t and productivity z_t that are liquidated is given by $F\left(\phi^l\left(\hat{k}(e_t, z_t), \hat{R}_b(e_t, z_t), \hat{R}_m(e_t, z_t)\right) \middle| z_t\right)$. Here, $\phi^l(k, R_b, R_m)$ is the liquidation threshold defined in (7), and $\hat{k}(e_t, z_t) \equiv e_t + \hat{b}(e_t, z_t) + \hat{m}(e_t, z_t)$ is the optimal investment decision of the firm. Second, a fraction η of firms exogenously exit after the dividend-issuance stage.

Let μ_t denote the measure of firms over B at the beginning of period t .¹¹ The total mass of firms exiting during period t is given by

$$\delta^e(\mu_t) = \int_{(e_t, z_t) \in B} \left(\overbrace{F\left(\phi^l\left(\hat{k}(e_t, z_t), \hat{R}_b(e_t, z_t), \hat{R}_m(e_t, z_t)\right) \middle| z_t\right)}^{\text{liquidations}} + \underbrace{\left(1 - F\left(\phi^l\left(\hat{k}(e_t, z_t), \hat{R}_b(e_t, z_t), \hat{R}_m(e_t, z_t)\right) \middle| z_t\right)\right)\eta}_{\text{exogenous exits}} \right) d\mu_t(e_t, z_t).$$

2.6.2. Entry. Each period, there are a fixed number, N_e , of potential entering firms. Each potential entrant is endowed with a productivity level z_{t+1} . Productivities of new entrants are independently drawn from the stationary distribution $p(z_{t+1})$ of the Markov chain for productivity. Additionally, potential entrants differ by their fixed cost of entry κ_t . This cost is drawn from a distribution $G(\cdot)$, with support on a subset of \mathbb{R}_+ ; the draw is independent from the potential entrant's productivity, z_{t+1} .

Conditional on having incurred the fixed entry cost κ_t , a new firm chooses its initial net worth to maximize the present value of discounted dividends:

$$e^e(z_{t+1}) = \arg \max_e \beta V(e, z_{t+1}) - (1 + \gamma_e)e. \quad (8)$$

11. Formally, μ_t is a measure on the measurable space (B, \mathcal{B}) , where \mathcal{B} is the σ -algebra generated by subsets of B of the form $[0, e[\times z$, for $z \in \mathcal{Z}$ and $e \in [0, \bar{e}(\bar{z})]$; see Appendix B for more details.

Let $E(z_{t+1}) = \beta V(e^e(z_{t+1}), z_{t+1}) - (1 + \gamma_e)e^e(z_{t+1})$ denote the present value of operating for such a firm. A potential entrant will choose to incur the fixed entry cost, and start operating, if and only if,

$$E(z_{t+1}) \geq \kappa_t. \quad (9)$$

As a result, the mass of firms entering with productivity z_{t+1} is given by $Nep(z_{t+1})\mathbf{P}(\kappa_t \leq E(z_{t+1})) = Nep(z_{t+1})G(E(z_{t+1}))$. This entry process repeats every period. Finally, the entry decision occurs at the end of period t , after financial contracts between existing firms and intermediaries have been settled.¹²

2.6.3. Evolution of firm measure. Given the firm's optimal policy functions and the above assumptions about entry, the law of motion for the distribution of firms on B can be expressed as $\mu_{t+1} = M(\mu_t) + \nu_t$. $M(\mu_t)([0, e_{t+1}[, z_{t+1})$ represents the measure of firms that survive in a given period and transition to a subset $([0, e_{t+1}[, z_{t+1}) \subset B$ of the state-space in the following period, given the optimal policy functions of firms and the current firm measure μ_t .¹³ Additionally, $\nu_t \in \mathcal{M}(B)$ represent the mass of new firms that enter according to the process described above.

2.7. Financial intermediation

Intermediaries raise funds to extend credit to firms. I assume that they face an identical opportunity cost of funds, but different intermediation costs.

Assumption 4. (Financial intermediation costs). Banks and market lenders have a common opportunity cost of funds, given by $r = \frac{1}{\beta} - 1$. Their intermediation costs per unit of credit are given γ_b and γ_m . The wedge between bank and market-specific intermediation costs is strictly positive: $\gamma_b - \gamma_m > 0$.

Equilibrium financial intermediation costs for banks and markets are thus given by

$$r_m = r + \gamma_m, \quad r_b = r + \gamma_b, \quad r = \frac{1}{\beta} - 1. \quad (10)$$

The restriction that $r = \frac{1}{\beta} - 1$ can be thought of as a general equilibrium outcome. Indeed, it would hold in a model in which intermediaries raise deposits from a representative, risk-neutral household. In such a model, perfect competition in the market for deposits would impose that $\beta(1+r) = 1$. I discuss Assumption 4 in more detail in Section 2.9.

2.8. Equilibrium

Definition 1. (Recursive competitive equilibrium). A recursive competitive equilibrium of this economy is given by value functions V , V^s , and V^c , dividend issuance thresholds $\{\bar{e}(z_t)\}_{z_t \in \mathcal{Z}}$,

12. So long as the productivity distribution of new entrants is unconstrained, the assumption of stochastic entry cost is not necessary to obtain the results that follow. I restrict the entry distribution to the invariant distribution $p(z_t)$ for simplicity. Under this restriction, the assumption of a stochastic entry cost helps ensure that there is always firm entry at every productivity level. I thank an anonymous referee for pointing this out.

13. Formally, M is a self-map over $\mathcal{M}(B)$, the set of measures defined on (B, \mathcal{B}) . The detailed definition of M in terms of the firm's optimal policies is reported in Appendix B.

policy functions $\hat{d}(e_t, z_{t+1})$, $\hat{b}(e_t, z_t)$, $\hat{m}(e_t, z_t)$, equilibrium lending costs r_b and r_m , equilibrium lending functions $R_b(e_t, z_t, b_t, m_t)$ and $R_m(e_t, z_t, b_t, m_t)$, entry sizes $\{e^e(z_t)\}_{z_t \in \mathcal{Z}}$, a measure of entrants ν , a firm measure μ and a transition mapping M , such that:

- the value functions solve problem (7), and $\{\bar{e}(z_t)\}_{z_t \in \mathcal{Z}}$, \hat{d} , \hat{b} and \hat{m} , are the associated policies;
- equilibrium lending costs satisfy (10);
- equilibrium lending terms satisfy the zero-profit conditions of intermediaries (6);
- M is consistent with firms' optimal policies;
- ν and $\{e^e(z_t)\}_{z_t \in \mathcal{Z}}$ are consistent with optimal decisions of new entrants, (8)-(9);
- given ν and M , the firm measure μ is invariant.

Proposition 3. (Existence of a recursive competitive equilibrium). *There exists a recursive competitive equilibrium of this economy.*

The proof of Proposition 3 is given in Appendix B. It uses two key insights mentioned in the exposition of the model: (1) the structure of the feasible set of debt contracts for a firm *only* depends on its state variables (e_t, z_t) , and not on its value function V ; (2) the feasible set can be partitioned into two subsets, $S_R(e_t, z_t)$ and $S_K(e_t, z_t)$, associated with the two types of debt settlement outcomes.

The first step of the proof is to establish the existence of a unique solution to problem (7), and to verify that the dividend issuance policy described in Section 2.2 is the optimal one. In general, problem (7) is a double fixed point problem, where the value function V and the constraint correspondence $\mathcal{S}: B \rightarrow \mathbb{R}_+^2$ must be simultaneously determined. However, the first insight implies that it reduces to a fixed point problem in V only; that is to say, the debt pricing functions can be obtained independently of the computation of V . Still, an additional difficulty is that the resulting value function $V(\cdot, z_t)$ need not be globally concave for each z_t , so that it is not immediate that the dividend issuance policy described in Section 2.2 is optimal. The lack of global concavity arises because, as suggested by the second insight, the value function V is the upper envelope of two functions: $V(e_t, z_t) = \max_{K, R} (V_K(e_t, z_t), V_R(e_t, z_t))$, where $V_K(e_t)$ denotes the continuation value of a firm restricted to using debt structures that are in $S_K(e_t, z_t)$, and $V_R(e_t, z_t)$ is analogously defined. V does not inherit the concavity of V_K and V_L , and instead has a downward kink where these restricted value functions intersect. The proof of Proposition 3 establishes that the dividend issuance policy of Section 2.2 is still optimal under weaker conditions than global concavity. It then shows that these conditions are met by the firm's value function, using the generalized envelope theorems of Milgrom and Segal (2002).

The second step of the proof of Proposition 3 is to derive the expression of the transition mapping M and the entry measure ν , and show that there is a solution to the functional equation $\mu = M(\mu) + \nu$. This part of the proof also uses the partition of the feasible set of debt structures between $S_R(e_t, z_t)$ and $S_K(e_t, z_t)$, because transition probabilities between levels of internal finance depend on the firm's current debt structure. Given the expression constructed for M , standard approaches can be used to prove that there is an invariant firm measure in this model.

2.9. Discussion of key assumptions

2.9.1. Liquidation and seniority. The first key assumption about liquidation (assumption 1) is that it involves deadweight losses: $\chi < 1$. This assumption is common to many models in which the underlying financial friction is limited liability. It embodies the notion that bankruptcy

and liquidation are costly processes, and is supported by evidence on changes in asset values of firms that go through bankruptcy proceedings (see, *e.g.*, [Bris et al. \(2006\)](#)).

The second key assumption is the seniority of bank lenders in liquidation. This assumption is motivated by two considerations. First, empirically, bank loans tend to be either senior or secured by liens on assets, as documented by [Rauh and Sufi \(2010\)](#). Second, in the model, this priority structure may be optimal for firms. There are two opposing forces that affect how desirable firms find bank seniority in this model, both having to do with how much firms value the flexibility offered by banks. On the one hand, seniority increases the reservation value of the bank during restructuring negotiations. As a result, it is more difficult for firms to obtain large concessions from the bank, which implies that bank debt is effectively less flexible. This tends to make seniority less attractive for firms. On the other hand, if liquidation actually occurs, the bank's payoff is larger when it is senior. The debt pricing condition (6) then implies that the "liquidation risk premium" it charges is lower, so that it is cheaper and easier for firms to issue bank debt. This makes bank seniority more attractive to firms. While it cannot be established analytically that seniority is optimal in the context of this model, the logic above suggests that bank seniority can improve the overall ability of firms to issue bank debt and use its flexibility.

The last assumption is that outside of formal bankruptcy, firms might incur a loss when selling capital early to pay down debt. This loss is captured by the parameter $\chi_P \in [\chi, 1]$. Direct evidence for discounts in asset sales for financially distressed firms can be found in [Brown et al. \(1994\)](#) and, in the context of the aircraft industry, in [Pulvino \(1998\)](#). More broadly, the discount $\chi_P < 1$ can be interpreted as capturing the fact that assets are industry- or firm-specific, and therefore difficult to redeploy when they are sold, as argued by [Shleifer and Vishny \(1992\)](#). A closely related interpretation of the $\chi_P < 1$ assumption is that it stems from irreversibility of investment in capital goods, which has been widely documented and studied in the investment literature (see, *e.g.*, [Cooper and Haltiwanger \(2006\)](#)). From the standpoint of external financing, this assumption helps moderate the level of leverage under which firms can operate, because it limits the fraction of revenue not subject to productivity risk (the depreciated capital stock).¹⁴

2.9.2. Bank flexibility. The assumption that banks are more flexible than markets when borrowers are in financial distress (Assumption 2) receives considerable support in the data. A number of empirical studies have shown that firms that rely more heavily on bank credit are more likely to successfully restructure debt and avoid liquidation when facing difficult financial conditions. [Gilson et al. \(1990\)](#) show that, in a sample of 169 financially distressed firms, the single best predictor of restructuring success is the existence of bank loans in the firm's debt structure. [Asquith et al. \(1994\)](#) find that firms with a large number of public debt issues are less likely to restructure debt out of court. [Denis and Mihov \(2003\)](#), in a sample of 1,560 new debt financings by 1,480 public companies, show that bank debt issuances offer more flexibility in the timing of borrowing and payment, and that firms with higher revenue volatility tend to issue more bank debt.

[Bolton and Scharfstein \(1996\)](#) provide a theoretical rationale for bank flexibility, by noting that ownership of market debt tends to be more dispersed than ownership of bank debt. This creates a free-rider problem, as market creditors have little individual incentive to participate in debt renegotiations. To the extent that they have an informational advantage over other creditors, banks may also be more flexible in restructuring negotiations because they are more precisely

14. It is straightforward to see that the maximum risk-free debt to asset ratio that a firm can operate under is given by $\chi_P \frac{1-\delta}{1+r}$. When $\chi_P = 1$, under typical calibrations of the rate of depreciation of capital, this number is close to 0.8; firms in the model thus take on relatively high leverage, compared to the data.

aware of the going concern value of the firm, as in [Rajan \(1992\)](#). The precise cause of banks' greater flexibility in distress is not explicitly modelled in this article; rather, the focus is on the implications of this difference in flexibility for firms' investment choices.

In the model, bank flexibility in times of financial distress is captured by assuming that a distressed firm can renegotiate outstanding debt with its bank. Crucially, bargaining takes place over the surplus generated by avoiding liquidation. Firms with low income realizations are able to obtain concessions from their banks in the form of reduced repayments, because banks have an incentive to avoid the deadweight losses associated with the asset transfers that would occur in liquidation. As in [Hart and Moore \(1998\)](#), the threat of liquidation therefore plays a central role in determining the nature and outcome of debt renegotiations. This role of the threat liquidation, and in particular the fact that creditors of distressed firms can concede to reductions in interest and principal payments, has also been empirically documented; see, *e.g.*, [Benmelech and Bergman \(2008\)](#).

It is important to emphasize that, in reality, debt renegotiations need not involve a reduction in principal or interest payments. As documented by [Roberts and Sufi \(2009b\)](#), debt renegotiations often aim to help firms weather a period of temporarily low revenue by conceding to an extension of the maturity in the loan, sometimes with a change in covenants, in exchange for a fee paid upfront—as opposed to a reduction in interest and principal payments.¹⁵ This renegotiation outcome is made possible by the dynamic nature of the relationship between creditor and debtor. Since, in the model, debt contracts are one-period, renegotiations involving a restructuring of *future* debt payments are not possible. However, these dynamic loan modifications share some of the features of the debt restructurings in the model. First, they are typically associated with violations of a financial ratio (the ratio of net income, π_t , to senior debt, $R_{b,t}$). Second, and more fundamentally, the concessions agreed by creditors are intended to allow the firm to continue operating in the face of low income realizations, and mitigate the risk of default and liquidation. A conservative interpretation of the model is that it studies how *current* concessions by creditors can help avoid *current* liquidation risk; this is an important part, but not the full picture of how debt renegotiations can be conducted, and for what purposes.

2.9.3. Intermediation costs. The fact that financial intermediation is costly ($r_b > r$ and $r_m > r$) is not controversial: [Philippon \(2015\)](#) provides recent and comprehensive evidence that overall intermediation costs in the U.S. financial sector have averaged approximately 2% between 1870 and 2012. The assumption specific to this model is that $r_b > r_m$. This assumption can be thought of as capturing two main differences between bank-based and market-based financial intermediation:

First, the intermediation spread $r_b - r_m$ can be thought of as resulting from these differences in regulatory treatment of banks and market-based financial intermediaries. Appendix C spells out this argument formally in a stylized model. In particular, the difference between r_b and r_m is given by $r_b - r_m = \xi(\psi_b - r)$, where ξ is the regulatory constraint on bank leverage (the ratio of equity to assets that banks must hold), ψ_b represents equity issuance costs for banks, and r is rate of return that both banks and market-based intermediaries offer to depositors. Thus, provided that bank capital requirements are positive ($\xi > 0$), and that equity issuance costs of bank exceed the rate of return on safe assets, an intermediation wedge will naturally arise. The latter assumption,

15. See [Smith \(1993\)](#), [Chava and Roberts \(2008\)](#), [Nini et al. \(2009\)](#) and [Roberts and Sufi \(2009a\)](#) for evidence on financial and real outcomes of firms after renegotiations triggered by covenant violations. I thank an anonymous referee for pointing this out.

TABLE 1
Baseline calibration of the model

Parameter	Description	Value	Parameter	Description	Value
ζ	Degree of returns to scale	0.975	ξ	Net worth consumed on exit	0.500
δ	Depreciation rate	0.200	χ	Liquidation efficiency	0.380
β	Discount rate	1/1.0200	χ_P	Capital fungibility	0.675
γ_e	Initial equity issuance cost	0.322	γ_m	Market intermediation costs	0.0100
η	Exogenous exit rate	0.040	γ_b	Bank intermediation costs	0.0174

in particular, seems plausible, given the high degree of stickiness of the equity base of financial intermediaries (see [Adrian and Shin \(2011\)](#) for a review of this evidence).

Second, bank borrowing requires active management *after* the initiation of the loan. Firms must share private information with their bank lenders, as part of contractual arrangements that allow them to verify that loan covenants are satisfied. [Demiroglu and James \(2010\)](#) presents evidence on the information-intensive nature of debt covenants, while [Rauh and Sufi \(2010\)](#) show that bank lending contracts tend to involve more covenants than straight bond issuances. The costs associated with implementing and verifying these covenants, and more generally “relationship management” costs associated with bank lending, can also contribute to the intermediation wedge $r_b - r_m$. Appendix C makes this point more formally. Specifically, if banks and markets have the same opportunity costs of funds $1 + r$ and intermediation costs γ , but if loans require firms to incur costs of v_b per unit of bank debt issued with their bank once output has been realized (*e.g.*, to share information with the bank about output realizations), then the equilibrium bank lending contract would be identical to the one described in Sections 2.3 and 2.4, with the intermediation wedge given by $r_b - r_m = \chi v_b$. Intuitively, debt management costs reduce the firms’ net income outside of renegotiation states, and thus expands the size of the renegotiation region. But because the zero-profit condition of banks must hold, this increase has to be offset by an increase in the repayment promised to the bank outside of the renegotiation region, $R_{b,t}$. Thus, the intermediation wedge can be thought of as a reduced-form way of capturing the costs that firm must incur to manage their relationship with bank lenders.

3. FINANCIAL POLICIES IN STEADY STATE

3.1. Baseline calibration

The model has no closed-form solution. Instead, the discussion focuses on the properties of a numerical solution, using a baseline calibration of the model to the U.S. before the Great Recession. This calibration is summarized in Table 1. The frequency of the model is annual. All data sources used in the calibration are described in more detail in Appendix A.

3.1.1. Production technology. The parameter ζ implies close to constant returns to scale, consistent with the evidence provided by [Basu and Fernald \(1997\)](#), [Basu et al. \(2006\)](#) and [Foster et al. \(2008\)](#) for the U.S. The rate of depreciation is close to the average ratio of depreciation expenses to book assets for Compustat firms, as documented, *e.g.*, by [Hennessy and Whited \(2007\)](#).

The productivity processes z_t and ϵ_t are calibrated using the total factor productivity (TFP) data constructed by [Imrohoroglu and Tüzel \(2014\)](#) for the population of Compustat firms. Specifically, I select the sub-sample of rated firms active between 1995 and 2007, and I calibrate z_t and ϵ_t

so as to match three moments of the log TFP distribution for this subset of firms: (1) the cross-sectional standard deviation, equal to 0.406 in the data; (2) the autocorrelation, equal to 0.77 in the data; (3) the ratio of average book assets of firms in the bottom decile of the TFP distribution, relative to that of firms in the top decile of the TFP distribution, equal to 0.083 in the data. These moments are matched by constructing a discrete-state Markov chain for z_t using the method of Kopecky and Suen (2010), which allows to target exact unconditional moments of the Markov chain. The shocks ϵ_t are assumed to follow a Weibull distribution with mean 1 and variance chosen so that the total variance of ϕ_t matches that of the data. Appendix A describes the calibration of the productivity process in more detail.

3.1.2. Entry, exit, and liquidation. To calibrate the magnitude of deadweight losses in liquidation, I use evidence reported by Bris *et al.* (2006). They analyse a sample of 61 chapter 7 liquidations in Arizona and New York between 1995 and 2001. Their median estimate of the ratio of assets post- to pre-chapter 7 liquidation is 38%; I therefore use $\chi = 0.38$. I choose an exogenous exit frequency of $\eta = 0.04$. Together with the magnitude of deadweight liquidation losses, this implies a total exit rate of 4.34% in steady-state, consistent with the estimates of Duffie *et al.* (2007), who find that default intensities for U.S. industrial firms between 1980 and 2004 range between 3.5% and 5% at the one-year horizon. Entry costs γ_e are set to 0.322. This parameter controls the size of new entrants; it implies that, in steady-state, new entrants account for 1.85% of total net worth. In contrast, in 2006, new Compustat firms accounted for 1.28% of total net worth of firms, with net worth defined as the difference between book assets and book liabilities.¹⁶ Finally, I set the fraction of net worth consumed upon exogenous exit, ξ , to 0.5. Together with the exogenous exit rate, it implies expected net worth losses due to exogenous exit of 2%. This is analogous to expected losses implied by typical parametrizations of “disaster” risk shocks (see, *e.g.*, the calibration of the disaster risk model of Gourio (2013)). I discuss the role of this parameter in more detail in Section 3.5.¹⁷

3.1.3. Borrowing costs and debt composition. As a proxy for market-specific lending costs γ_m , I use existing estimates of underwriting fees for corporate bond issuances. Fang (2005) studies a sample of bond issuances in the U.S., and finds an average underwriting fee of 0.95%, while Altinkilic and Hansen (2000), in a sample including lower-quality issuances, find an average underwriting fee of 1.09%. Given this evidence, I set market-specific intermediation costs to $\gamma_m = 0.0100$. Finally, I choose the intermediation costs of banks, γ_b , and the parameter controlling the fungibility of depreciated capital with output, χ_P , to match the aggregate net worth to assets, and the aggregate bank share of total debt of medium and large U.S. manufacturing firms in 2007Q3, as reported in the Quarterly Financial Report of manufacturing firms (QFR). Given the model’s focus on firms with access to public debt markets, I only use data on firms with more than \$250m in assets. At that date, the ratio of aggregate net worth (defined as assets net of liabilities) to aggregate assets is 44.0% for these firms. Moreover, aggregate bank borrowing accounts for 24.0% of total outstanding debt. Given other parameters, the model generates a net worth to asset ratio of 44.0% and a bank share of debt of 24.0% for $\chi_P = 0.675$ and $\gamma_b = 0.0174$, respectively.

16. This Figure is computed with the same sample of firms as the one used for the calibration of the productivity process; see Appendix A for details.

17. The number of potential entrants is normalized to $N_e = 1$, and the distribution G of sunk entry costs is chosen to be exponential with parameter 1. These choices do not affect the results of this section; $N_e = 1$ is a normalization, and the choice of G does not affect the properties of the mode, since in steady-state, entry and exit rates must be equal.

Using the QFR data to calibrate the model has two advantages. First, the QFR provides an accurate measure of the bank share: it is one of the few balance-sheet datasets in which firms are asked to report separately bank debt from other debt outstanding. Second, the QFR covers the universe of all medium and large manufacturing firms at a quarterly frequency. This will be useful in Section 4, when I compare the business-cycle implications of the model with the data. However, two limitations of the QFR could introduce bias in the measurement of the bank share: the restriction to manufacturing firms and the inability to explicitly separate firms with access to public debt markets from others. In Appendix A, however, I show that the QFR bank share is similar to what can be obtained using two alternative sources of evidence: [Rauh and Sufi \(2010\)](#) and [Colla et al. \(2013\)](#). The small difference is likely due to by composition: QFR firms are smaller, on average, than in either of these datasets.

3.2. Optimal debt structure

The firm's optimal debt structure, $\hat{b}(e_t, z_t)$ and $\hat{m}(e_t, z_t)$, as well as the total investment policy $\hat{k}(e_t, z_t)$, under the calibration described above, are reported in Figure 5a and b. These Figure graph policies as a function of internal finance e_t , for two separate levels of productivity $z_L < z_H$.

These graphs indicate that, conditional on a particular level of productivity, the optimal debt structure chosen by firms fall into two broad categories, depending on their level of internal finance. Firms with internal funds below the threshold $e^*(z_t)$ choose a “mixed” debt structure, involving a combination of bank and market debt, while firms with internal funds strictly above $e^*(z_t)$ choose a “market-only” debt structure. As firms grow, they will therefore switch from a mixed debt structure to a market-only debt structure. This shift in debt structures is also illustrated in Figure 5a, which displays the optimal fraction of bank debt to total debt, $\frac{\hat{b}(e_t, z_t)}{\hat{b}(e_t, z_t) + \hat{m}(e_t, z_t)}$, for the two different levels of productivity z_L and z_H .¹⁸

The intuition for the dependence of debt structure on internal finance is as follows. Because of decreasing returns, for a given productivity level, firms with a small stock of internal finance have a larger financing gap, and tend to be more leveraged than firms with a large stock of internal finance. This also implies that they have a higher probability of financial distress. Since, all other things equal, borrowing more from banks reduces the expected losses associated with financial distress, these firms, seeking high leverage, have a strong incentive to use bank debt. Their debt composition is therefore more tilted towards bank loans. At the other end of the spectrum of net worth, firms with a lower financing gap (a higher stock of internal finance) are less likely to be financially distressed; for these firms, the benefits of using bank debt are more limited.¹⁹ More generally, the trade-off between flexibility and costs changes with the level of the firm's stock of internal resources e_t , and this is reflected in the optimal share of bank debt.

This intuition does not explain why firms switch in a discrete manner to pure market finance at the boundary $e^*(z_t)$. To understand this, it is useful to think back to Proposition 1. This proposition indicates that, when a firm is predominantly market-financed, bank debt restructuring is never sufficient to avoid liquidation *ex-post*. Bank creditors are still flexible, but since bank liabilities are small relative to market liabilities, no concession they could make *ex-post* (after the realization of ϕ_t) would be sufficient to stave off liquidation. On the other hand, *ex-ante*, the lending wedge

18. For a proof of the existence and unicity of the threshold $e^*(z_t)$ in a static version of this model, see [Crouzet \(2015\)](#).

19. In fact, when the financing gap is sufficiently small, firms in the model can reach their desired size by issuing risk-free debt. In this case, since renegotiation never occurs *ex-post*, bank loans are strictly dominated by market financing. I thank an anonymous referee for pointing this out.

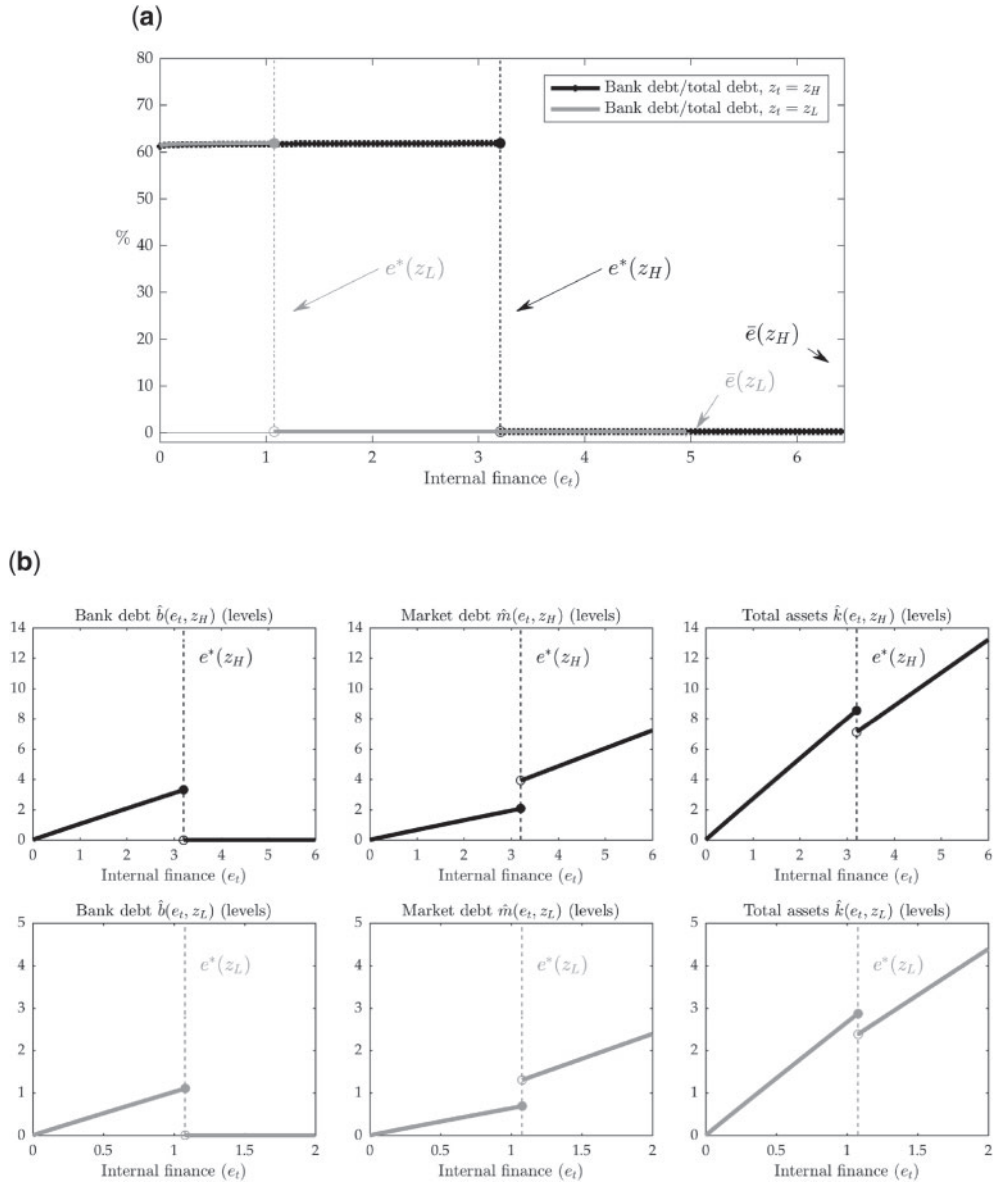


FIGURE 5

Optimal debt structure in the baseline calibration of Table 1. The top graph reports the optimal share of bank debt. The bottom graphs reports, respectively, optimal bank borrowing, optimal market borrowing, and optimal total assets. Grey lines represent the policies for $z_t = z_L$, while black lines represent the policies for $z_t = z_H$, where $z_H > z_L$. (a) Optimal composition of debt; (b) Optimal level of bank and market debt, and total assets.

$\gamma_b - \gamma_m$ is strictly positive, making bank lending costly. For predominantly market-financed firms, bank debt is thus costly *without* offering flexibility benefits. For this reason, these firms choose a corner solution: $\hat{b}(e_t, z_t) = 0$.

Apart from the decline of the bank share with internal finance, the second key aspect of firms' optimal debt choices is that they imply non-monotonicities in the borrowing and investment policies of firms. This is illustrated in Figure 5b, which reports bank borrowing $\hat{b}(e_t, z_t)$ (left panels), market borrowing $\hat{m}(e_t, z_t)$ (middle panels), and total assets $\hat{k}(e_t, z_t) = e_t + \hat{b}(e_t, z_t) + \hat{m}(e_t, z_t)$ (right panels) as a function of internal funds e_t , for the two levels of productivity $z_t = z_L$ and $z_t = z_H$. In each of the two regions $e_t \leq e^*(z_t)$ and $e_t > e^*(z_t)$, the amounts borrowed from banks and markets are increasing (or equal to 0 for bank borrowing when $e_t > e^*(z_t)$). However, when a firm crosses the threshold $e^*(z_t)$, total assets and total borrowing fall: bank debt is not replaced one for one by market debt.

This effect can be interpreted as a precautionary response of firms that migrate from a mixed to a market debt structure. At the point $e_t = e^*(z_t)$, firms are exactly indifferent between mixed and market debt structures. Imagine that a firm were to choose the same overall leverage under the market debt structure as under the mixed debt structure. Since, under the market debt structure, the firm has no option to restructure debt in bad times, its ex-ante likelihood of liquidation would be much higher than under the mixed debt structure. The firm can offset this higher “financial fragility” by operating under a lower overall leverage. This imperfect substitutability between types of debt plays a role in the transmission of credit shocks to corporate investment in the model; I come back to it in the following section.

3.3. Cross-sectional implications

The left panel of Figure 6 reports the steady state marginal distribution of firms across levels of internal finance, $\mu^e(e_t) = \sum_{z_t \in \mathcal{Z}} \mu(e_t, z_t)$. The key property of this distribution is that it is strongly

skewed to the left. Firms are born with a relatively small stock of internal finance. They initially take on a substantial amount of leverage, which exposes them to high liquidation risk. Few of them therefore reach the dividend issuance threshold $\bar{e}(z_t)$.²⁰ Because of this left-skewness, the share of bank loans as a fraction of total borrowing is high for most firms and only declines for the largest firms. This is reported in the right panel of Figure 6. Each point in this graph corresponds to the median bank share of firms within a particular decile of the distribution of internal finance. As a result of the left-skewness of the distribution, in the baseline U.S. calibration, 57.1% of firms use a mixed debt structure; only the top 32.9% of firms are purely market financed.

Are these predictions about variation in debt structure consistent with the data? [Rauh and Sufi \(2010\)](#) provide evidence on the composition of debt in a sample of publicly traded and rated U.S. firms between 1996 and 2006, combining data from firms' 10-K filings with data on issue origination. They have two key findings. First, a majority of firms simultaneously have outstanding issues of different types. Out of all firm-year observations, 68.3% use at least two types of debt; among firms using bank debt (52.6% of the total), 70.5% also use straight or convertible bond debt. Second, the degree to which the debt structure of firms is “spread out” across types is strongly related to firms' credit ratings. Investment-grade firms (with ratings of BBB and above) mostly use senior unsecured debt (bonds), while speculative grade firms (with ratings of BB

20. This is with the exception of the lowest average productivity level in the baseline calibration, z_{\min} . For $z_t = z_{\min}$, the dividend issuance threshold is small; this accounts for the point mass close to $e_t = 2.5$ in the left panel of Figure 6. This scale is reached by about 2% of firms in steady-state; these firms are those that are born with higher productivity, have accumulated large stocks of internal finance, but whose productivity eventually declined to the lowest level. These firms bunch at the threshold $\bar{e}(z_{\min})$. They are “cash-rich”, in the sense that they face negligible liquidation risk and are able to borrow at close to risk-free rates. There is similar bunching, but for a much more limited mass of firms, at the dividend issuance thresholds associated with higher productivity levels.

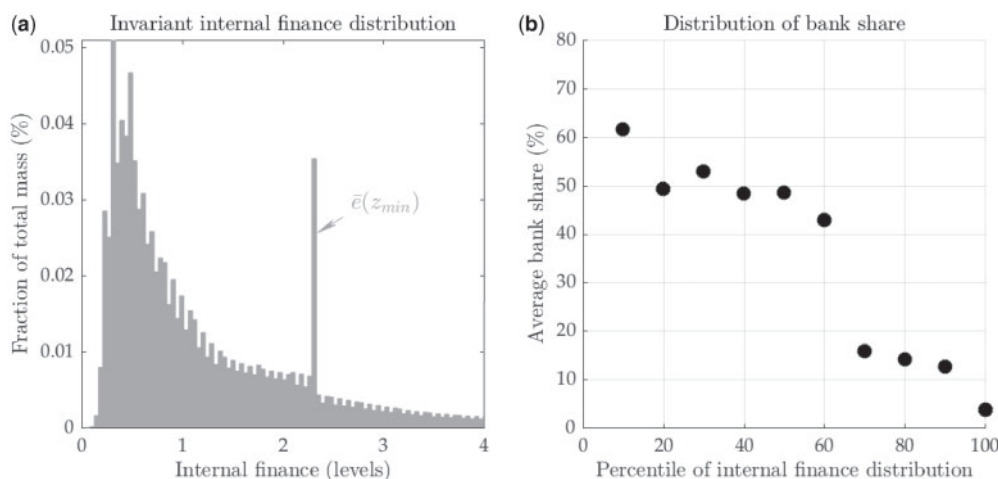


FIGURE 6

The distribution of internal finance and of the bank share in steady-state.

and below) use a combination of secured debt, senior unsecured debt, and subordinated bonds. Moreover, their results indicate that “the increase in secured debt as credit quality deteriorates is driven almost exclusively by an increase in secured bank debt, and the increase in subordinated debt is driven almost exclusively by an increase in subordinated bonds and convertibles”.

Colla *et al.* (2013) provide additional evidence on the debt structure of both rated and unrated U.S. firms, using information on debt structure gathered from 10-K filings. Their findings support the evidence of Rauh and Sufi (2010) for rated firms. Specifically, they find that 36.6% of firm-year observations of rated firms concentrate 90% or more of their debt within a single type (mostly senior bonds and notes); the remaining 63.4% are diversified across debt types. They also find that firms that move from investment to speculative grade tend to increase their reliance on term loans and junior bonds and notes, instead of using exclusively senior bonds and notes: for example, drawn credit lines and term loans together account for over 30% of total debt for firms with a credit rating of BB and below, whereas they account for less than 6% of total debt for firms with a credit rating of AA and above.

The cross-sectional variation of debt structure in the model is strongly consistent with these two facts. First, in the model, a majority of firms use a mixed debt structure, combining bank with market debt—the first of the two key findings of Rauh and Sufi (2010) and Colla *et al.* (2013). The fraction of firms using a mixed debt structure in the model (60.0%) is close to that documented in both Rauh and Sufi (2010) (68.3%) and Colla *et al.* (2013) (63.4%), despite the fact that this moment is not a target of the calibration. This suggests that the mechanisms at work in the model can capture both the intensive and extensive margin components of debt choices. The reason why most firms use both bank and market debt instead of specializing is that, by increasing market debt issuance, firms increase the scale of total investment. They therefore increase their value in liquidation. This in turn loosens the borrowing constraint of firms with respect to banks, who are senior in the priority structure, and allows firms to increase bank debt issuance. In this manner, the endogenous investment scale decision creates a complementarity between types of debt, which accounts for the existence of an interior solution, consistent with the data.

Second, the model is qualitatively consistent with the empirical finding that as credit quality improves, firms move away from mixed debt structures and use mostly bond and program debt.

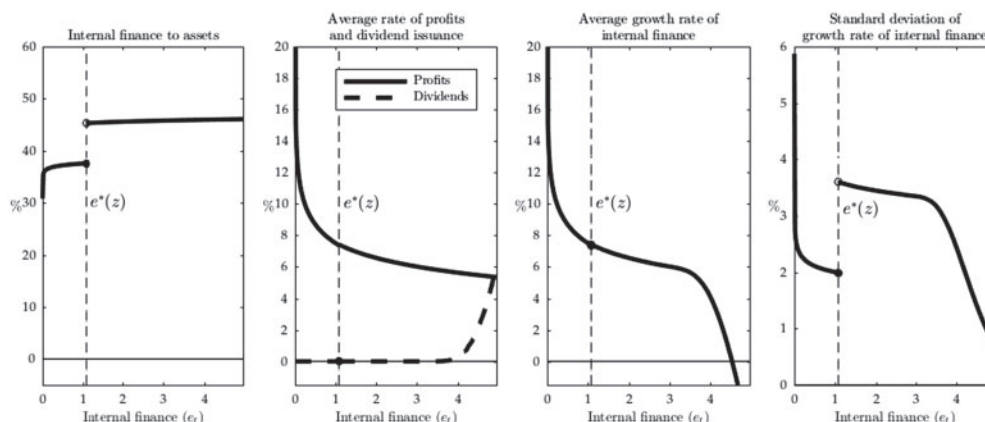


FIGURE 7

Other aspects of firms' financial policies and implied growth dynamics. The Appendix gives the exact definition of each moment reported. All policies and implied moments are reported for an intermediate current productivity level, $z_t = z_M$.

Indeed, in the model, the key driver of the shift from a mixed to a market debt structure is that, as firms' internal finance increases and their leverage falls, the probability that they will face financial distress *ex-post* declines, making the use of bank debt less attractive. Firms with market debt structures, in the model, have generically lower *ex-ante* liquidation probabilities than firms with mixed debt structures. Thus, to the extent that credit ratings proxy for a firm's credit or liquidation risk, the model captures well the empirical correlation between credit ratings and the concentration of debt structure.

3.4. Other aspect of firms' policies

Does the model's ability to account for cross sectional variation in debt structure come at the expense of other predictions? Figure 7 addresses this question, by reporting further characteristics of the financial and real policies of firms. In particular, with respect to financial policies, the model predicts that, conditional on productivity, firms with low internal resources have a higher rate of profit, but distribute less dividends, consistent with the evidence of in Fazzari *et al.* (1988) and Gilchrist and Himmelberg (1998), among others.²¹ With respect to firm dynamics, the key predictions of the model can be summarized as follows: (1) smaller firms experience a higher rate of growth; (2) smaller firms are more volatile. These facts also align well with empirical evidence on the relationship between size and growth dynamics.²² Note that, as was the case for financial policies, the switch between financial regimes is associated with discontinuities in the

21. Conditional on productivity, Figure 7 suggests that book leverage is broadly declining with net worth. However, the unconditional predictions of the model are more nuanced: the unconditional relationship between book assets, k_t , and total book leverage, $\frac{b_t + m_t}{e_t}$, is -0.17 , while the unconditional relationship between book assets and market leverage, $\frac{m_t}{e_t}$, is positive, and equal to 0.10 . This is for two reasons: first, firms with new growth options—*i.e.*, firms that have received a positive shock to their productivity level z_t —will choose to increase leverage, as an increase in productivity reduces default risk and loosens borrowing constraints. Second, leverage is not exactly monotonic in net worth, since there is an upward discontinuity at the threshold between mixed and bank-finance regime. Generating a conditionally positive relationship between leverage and book assets, as documented by Rajan and Zingales (1995) and analysed by Gomes and Schmid (2010), would require introducing either fixed costs of production, or fixed default costs, both of which would increase the cost of leverage for smaller firms. I thank an anonymous referee for pointing this out.

22. See, *e.g.*, Evans (1987) or more recently, Davis *et al.* (2006).

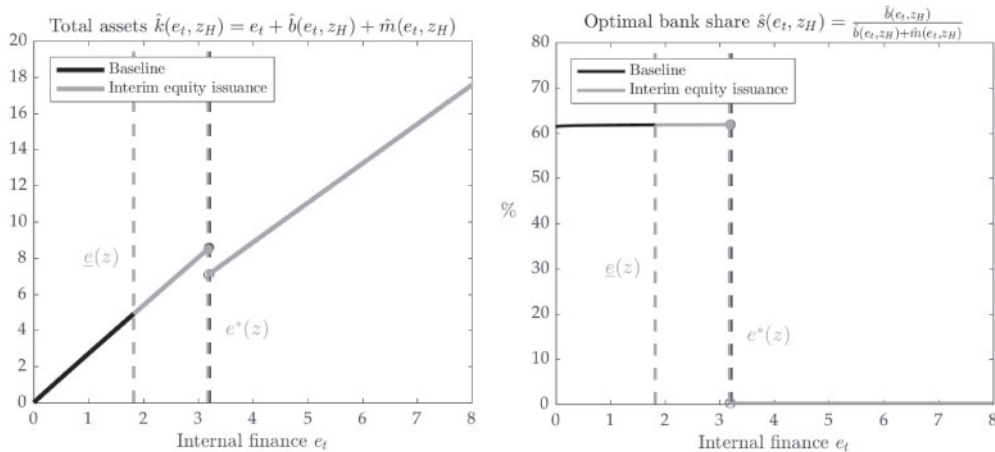


FIGURE 8

Optimal policies in a model with interim equity issuance, compared to those in the baseline model. Policies are reported for the same level of productivity. Except for the possibility of interim equity issuance, the calibrations for the two models are identical.

growth dynamics of firms. In particular, the volatility of firms that switch to market-only debt also increases, in line with the intuition that switching to market debt increases their liquidation risk.

3.5. Robustness

3.5.1. Interim equity issuance. An important assumption of the model is that firms only issue equity at birth; seasoned equity offerings are left out. This assumption is consistent with the fact that equity issuance plays a small role in financing capital expenditures of U.S. firms.²³ A natural question is whether the key qualitative properties of the model would be changed if firms could issue equity after birth. In this case, the problem of a firm that survives the debt settlement stage can be written as:

$$V^c(n_t, z_{t+1}) = \max_{e_{t+1}} n_t - e_{t+1} - \mathbf{1}_{\{n_t \leq e_{t+1}\}} \gamma_{e,int} (e_{t+1} - n_t) + (1 - \eta) \beta V(e_{t+1}, z_{t+1}) + \eta \xi e_{t+1}.$$

Here, $\gamma_{e,int}$ represents interim marginal equity issuance costs, which are assumed to be constant, as in [Cooley and Quadrini \(2001\)](#). In this case, it is straightforward to establish that, analogously to Lemma (1), the firm's optimal equity/dividend issuance policy will be to issue dividends when $n_t > \bar{e}(z_{t+1})$; reinvest all retained earnings when $\underline{e}(z_{t+1}) \leq n_t \leq \bar{e}(z_{t+1})$; and issue equity so that net worth is replenished up to $e_{t+1} = \underline{e}(z_{t+1})$ when $n_t < \underline{e}(z_{t+1})$. The resulting optimal policies are depicted in Figure 8. Aside for the interim equity issuance cost, which is set to $\gamma_{e,int} = 0.1$, the calibration of the model used for these graphs is identical to the baseline. The resulting policy functions are virtually identical (and difficult to distinguish visually), save for the fact that the

23. In fact, [Corbett and Jenkinson \(1996\)](#) document that aggregate net equity issuance in the U.S. is *negative*, on average, between 1970 and 1989, reflecting the “mergers and acquisition process”. More recently, [Hackethal and Schmidt \(2004\)](#) show that gross flows from equity issuance of U.S. corporations represent only about one-tenth of the gross flows from debt issuance.

relevant state-space for net worth is $[\underline{e}(z_t), \bar{e}(z_t)]$, for each level of productivity z_t . This does not alter the aggregate implications of the model substantially: the aggregate bank share falls to 20.4% (compared to the targeted 24.0% in the baseline calibration), while the share of firms with a mixed debt structure falls to 49.5% (compared to 57.9% in the baseline calibration). Thus, the introduction of interim equity issuance does not alter the key properties of the model.

3.5.2. Saving. Finally, in the baseline model, while firms accumulate net worth *across* periods, they cannot save *within* periods. That is, all net worth e_t at the beginning of the period must be invested in productive capital, so that $k_t \geq e_t$, or equivalently, $b_t + m_t \geq 0$. One can relax this assumption and allow firms to deposit any portion $0 \leq l_t \leq e_t$ of its stock of internal finance with financial markets. Productive capital is given by $k_t = e_t - l_t \leq e_t$, and the firm's gross expected returns on its deposits are $(1+r)l_t$. In that case, a firm making deposits then never defaults; its value is thus given by:

$$V^S(e_t, z_t) = \max_{0 \leq k_t \leq e_t} \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1}|z_t) \int_{\phi_t \geq 0} V^c(\phi_t k_t^\zeta - (r+\delta)k_t + (1+r)e_t, z_{t+1}) dF(\phi_t|z_t), \quad (11)$$

where V^c is the same continuation value function as the one given in problem 7.²⁴ The dynamic problem of the firm can then be thought of as choosing between saving during the current period (with value $V^S(e_t, z_t)$) and borrowing during the current period, with value given as in the baseline model. Appendix D gives a complete description of the firm's problem with intra-period savings.

In a model with the same calibration as in Section 3.1, however, the introduction of savings does not change the equilibrium policies of the firms. That is, the optimal within-period saving policy in the baseline calibration extend to allow for savings as in (11) is $\hat{l}(e_t, z_t) = 0$, for all $e_t \in [0, \bar{e}(z_t)]$, $z_t \in \mathcal{Z}_t$. This result can be understood as follows. Consider two choices for the same firm: saving a small amount and operating at scale $k_t^- = e_t - \epsilon_t$, and borrowing a small amount and operating at scale $k_t^+ = e_t + \epsilon_t$ where ϵ_t is a small number. If the firm chooses to save, it operates at a smaller scale, but has zero liquidation risk. If it chooses to borrow, it operates at a larger scale, but it could potentially also face liquidation losses. However, if ϵ_t is small enough, because of the lower bound on idiosyncratic shocks $\phi_t \geq 0$, the firm can borrow risk-free.²⁵ Provided that the marginal value of capital at $k_t = e_t$ is strictly larger than r_m , the firm will therefore strictly prefer borrowing. This discussion suggests that saving could be optimal in alternative versions of the model, with sufficiently high intermediation costs, so long as there exist firms which cannot issue any debt without incurring strictly positive liquidation risk.

4. DEBT CHOICES AND THE TRANSMISSION OF AGGREGATE SHOCKS

This section focuses on the transmission of aggregate shocks to investment when firms endogenously choose debt composition. The discussion centers mostly on shocks to the intermediation cost of banks, γ_b . From the standpoint of the model, such shocks offer the best account of observed changes in debt structure in the U.S. during the Great Recession.

The central finding of this section is that debt choices introduce a new channel of propagation of aggregate shocks, which operates through substitution between bank and market finance and

24. This formulation of the savings assumes that firms with positive deposits do not borrow at the same time. Because of the intermediation spreads $\gamma_m, \gamma_b > 0$, this assumption is without loss of generality; that is, a firm never wants to simultaneously borrow and hold deposits. This is established in Appendix D.

25. Specifically, so long as $\epsilon_t \leq \chi_P \frac{1-\delta}{r+\delta} e_t$.

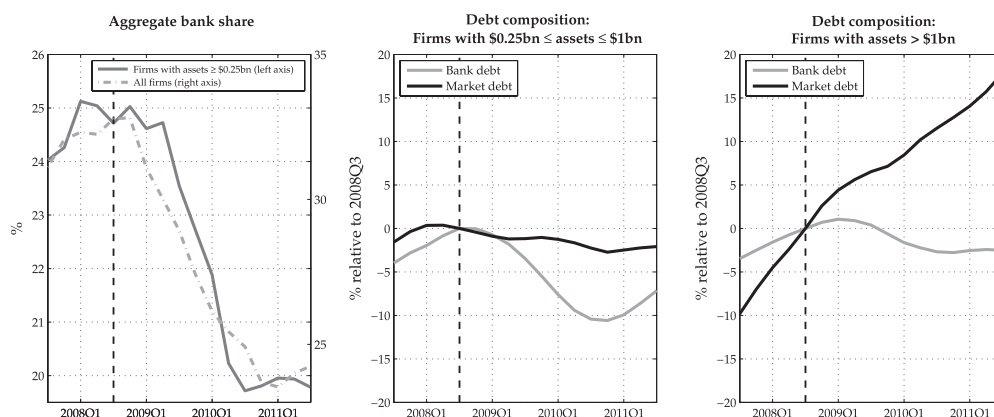


FIGURE 9

Changes in debt structure in the U.S. during the Great Recession. Data from the Flow of Funds and the QFR. Small firms are those with assets below \$1bn; large firms are those with assets above 1bn\$. See Appendix A for details on the construction of the series.

affects both the scale and composition of aggregate borrowing. In response to an increase in the relative cost of intermediation of banks, this channel is quantitatively important: it accounts for approximately one-third of the decline in investment in the numerical example analyzed in this section.

4.1. Debt structure during the Great Recession

In the U.S., the 2007–9 recession was accompanied by large and persistent changes in debt composition. Figure 9 illustrates these changes. First, the aggregate bank share fell substantially. For the universe of nonfinancial firms, the bank share fell from 31.1% in 2007Q3 to 24.3% in 2011Q3 (left panel, right axis). A similar pattern can be documented for manufacturing firms with more than \$250m in assets in the QFR: their bank share fell from 24.0% in 2007Q3 to 19.8% in 2011Q3 (left panel, left axis). Second, in the cross section of firms, changes in debt composition differed substantially: debt substitution was salient only for the largest firms. The middle and right panels of Figure 9 show the changes, relative to 2008Q3, in bank and non-bank liabilities of firms with assets between \$250m and \$1bn, and firms with assets above \$1bn dollars, respectively. In the former group, both bank and non-bank liabilities fell, with bank debt showing the largest drop relative to peak: debt substitution was limited. In the latter group, on the other hand, the decline in bank liabilities was accompanied by an increase in non-bank liabilities. Thus, the decline in the aggregate bank share reflected substantially different patterns of debt choices across firms. These patterns echo other evidence on the 2007–9 recession, most notably, [Adrian *et al.* \(2012\)](#). They are also consistent with evidence from other periods or other countries on the effects of shocks to the supply of bank credit, such as [Becker and Ivashina \(2014\)](#) and [Baumann *et al.* \(2003\)](#).

4.2. The effects of a bank credit supply shock

In the model, the decline in the aggregate bank share along with the patterns of debt choices across firms emerge naturally in response to an increase in γ_b , the marginal cost of bank lending. To illustrate this, I compute the perfect foresight response of the model to an exogenous increase in γ_b . At date 0, the economy starts from the steady state described in Section 3. At date 1, the bank

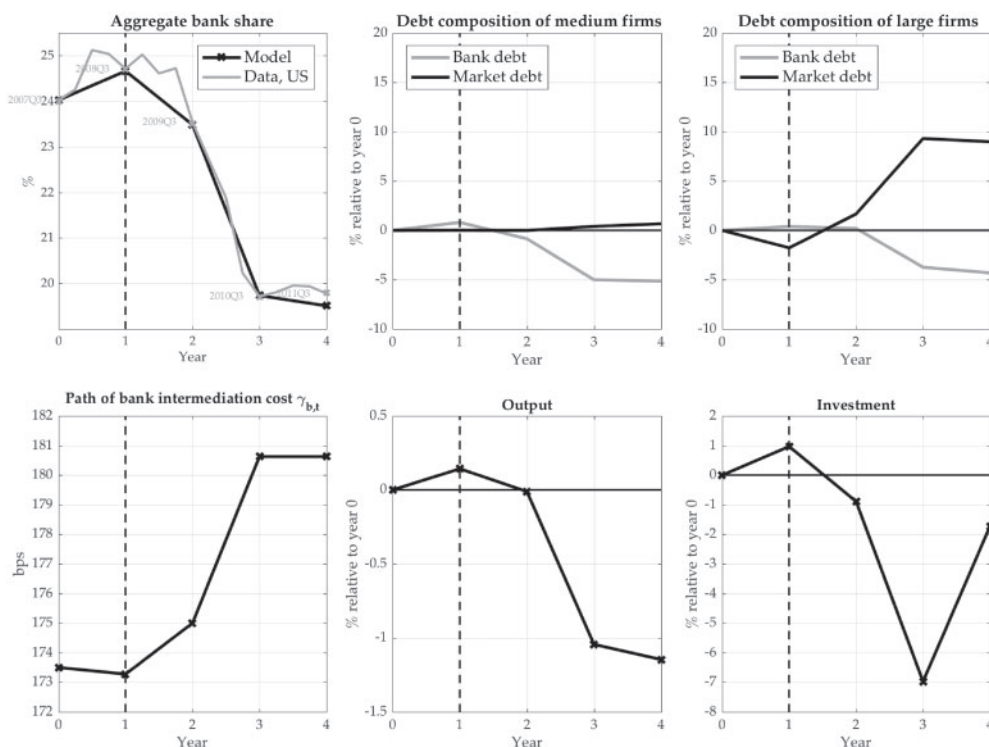


FIGURE 10

Response of the model to an increase in bank intermediation costs.

intermediation cost $\gamma_{b,t}$ increases. At date 4 and in subsequent periods, it reaches its new long-run value. The current and future path of $\gamma_{b,t}$ is known to all agents in the economy as of time 1. This path is reported in the bottom left panel of Figure 10. It is chosen to match, quantitatively, the fall in the bank share of firms with more than \$250m in assets documented in Figure 9 between 2008Q3 and 2011Q3. The model-implied path of the bank share is reported in the top left panel of Figure 10, along with its data counterpart. The required increase in $\gamma_{b,t}$ in the long-run is only about 9 bp, or 12% of the initial gap between the intermediation costs of banks and bond markets.

Figure 10 indicates that the increase in γ_b is associated with a 1.2% fall in output and a 7.9% fall in investment over the first three years. In contrast, in the data, the investment of rated firms fell by 21.1% between 2007 and 2010. After year 3, investment starts recovering, having overshot relative to its long-run value, while aggregate output remains low.²⁶

The effect of the shock on borrowing in the cross section is also consistent with the evidence on the response of debt composition of medium and large firms documented in the middle and right panels of Figure 9. To compare the model's predictions with this evidence, I compute the change in bank and market liabilities of firms of two size groups in the model, using a threshold for internal finance to define the two groups.²⁷ Figure 10 reports the changes in borrowing of these two firm groups, relative to year 1. For small firms, bank and market borrowing both decline,

26. Aggregate investment is defined as $I_t = K_t - (1 - \delta)K_{t-1}$, with $K_t \equiv \int_{z_t, e_t} (e_t + \hat{b}(e_t, z_t) + \hat{m}(e_t, z_t)) d\mu_t(e_t, z_t)$. Aggregate output is defined as $Y_t = \int_{z_t, e_t} (\int_{\phi_t} \phi_t(e_t + \hat{b}(e_t, z_t) + \hat{m}(e_t, z_t))^\xi dF(\phi_t|z_t)) d\mu_t(e_t, z_t)$.

27. See Appendix E for details on the definition of the groups.

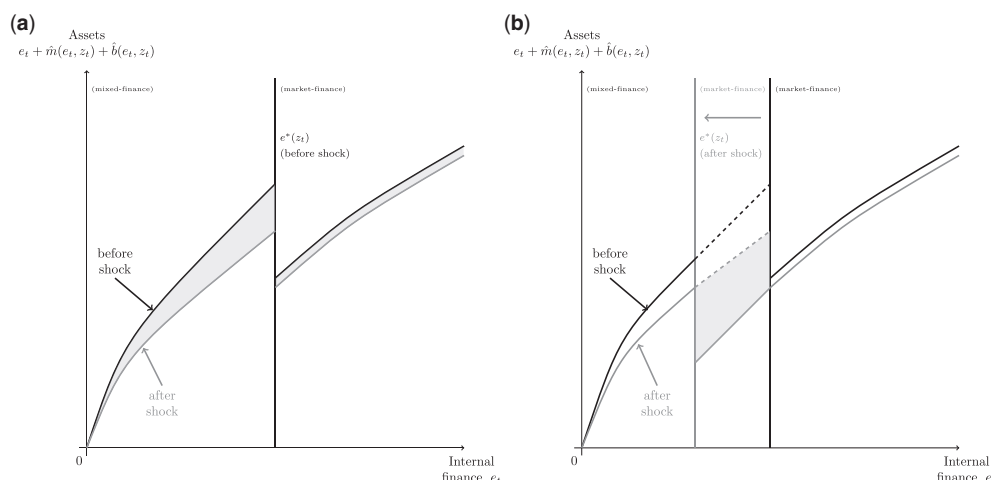


FIGURE 11

“Credit” and “substitution” channels in the response of firms to the lending wedge shock. (a) “credit” channel; (b) “Substitution” channel.

while for large firms, an increase in market borrowing accompanies the decline in bank liabilities. “Debt substitution” in the model, as in the data, is a phenomenon concentrated among larger firms.

4.3. Propagation mechanisms

To understand the results, it is useful to first analyse the short-run response of firms’ policy functions for total assets. Figure 11 illustrates the two key mechanisms through which the shock affects these policies in the short run.

The first mechanism is illustrated in the top panel of Figure 11: the increase in the lending wedge results in a higher cost of bank borrowing outside of financial distress and makes bank lending less attractive for mixed finance firms. Those firms issue less bank debt and reduce their scale of operation. This is the traditional “bank lending” channel, present in models with a single borrowing constraint. The model however makes two additional predictions about this channel. First, for mixed-debt firms, the shock also leads to a fall in market borrowing. This is a manifestation of the fact that, when firms are in the mixed finance regime, market and bank debt display a degree of complementarity. As discussed in Section 3, this complementarity arises because issuing market liabilities can partially relax a firm’s bank borrowing constraint, since it increases its scale of operation, and therefore its value in liquidation. The second additional prediction of the model is that, although market financed firms are not directly affected by the increase in γ_b , they still somewhat deleverage. These firms anticipate the fact that, given a sufficiently bad sequence of productivity shocks, they will have to revert to now costlier bank borrowing. They seek to avoid this by reducing their leverage.

The second mechanism by which the shock affects output and investment is that it changes the threshold at which firms switch from a mixed debt structure to a market financed debt structure. This is illustrated in the bottom panel of Figure 11. This threshold shifts to the left, and all firms between the pre- and post-shock threshold switch to a market financed debt structure. A central prediction of the model is that this switch has real effects: it affects the scale at which switching firms choose to operate. Firms that switch lose the flexibility associated with bank finance and become more exposed to liquidation risk. Put differently, these firms

would face excessively high liquidation premia, under a market financed structure, to continue operating at the same scale. As a result, their total borrowing, and therefore their output and investment, drop below what they would be under a mixed debt structure. The additional fall in borrowing due to the substitution channel corresponds to the shaded area in the bottom panel of Figure 11.

Quantitatively, the contribution of the substitution channel to the total decline in borrowing can be large. One way to gauge this contribution is to compute what total investment would have been in period 3 (once $\gamma_{b,t}$ has declined), had switching firms been constrained to keep using a mixed debt structure. In that counterfactual case, the drop in aggregate investment relative to year 0 would have been 5.0%, instead of 7.9%. The substitution channel thus accounts for a little less than a third of the decline in aggregate investment in the model.

The previous discussion helps explain the short-run response of the economy to the shock. In the long run, the shock has protracted effects on firms' ability to accumulate internal finance, both because the lending terms of banks worsen and because the shock induces firms to leverage less. As a result, the steady state distribution of firms across levels of internal finance shifts to the left, and the total stock of internal finance of firms in the economy falls. The slow adjustment of firms' internal funds is what drives the endogenous persistence in the response output and investment, since, after the lending wedge has stabilized to a new, higher level, the thresholds $e^*(z_t)$ and $\bar{e}(z_t)$ do not change.

4.4. Extensions

4.4.1. Can other shocks account for the behaviour of bank and market debt issuance? The previous discussion focused on the effects of an increase in bank lending costs; such a shock generates a deep recession accompanied by a fall in the aggregate bank share. Shocks to average productivity and shocks that increase γ_b and γ_m jointly (and thus do not affect relative debt costs) also generate large recessions, but have limited effects on debt composition at the aggregate and firm levels. A recession driven by an increase in idiosyncratic volatility, on the other hand, is accompanied by an *increase* in the aggregate bank share. This reflects the greater demand for debt flexibility when firm-level uncertainty is high. The response of large firms also involves debt substitution, but in the *opposite* direction, away from market debt and into bank debt. Thus, in this model, dispersion-driven recessions are associated with changes in corporate debt structure that are at odds with the patterns documented in Figure 9.

Finally, aggregate shocks to the liquidation value χ can also alter debt composition, which captures the fraction of current output that creditors can claim in liquidation. The intuition is that when banks are senior, they are the main claimants of the liquidation proceeds if a mixed firms default. As a result, higher deadweight liquidation losses would make bank borrowing more costly, and result in heavier use of market debt. One limit of this intuition is that, when firms do not rely on bank debt (*i.e.* when they are large enough), this shock also increases the cost of market debt, as it reduces claims of market creditors in liquidation. Thus, it should also reduce market debt issuance. Which effect dominates is a quantitative question.

The Online Appendix to the article reports the response of the predictions of the model following a 10% decline in χ , the parameter capturing the extent of liquidation losses. Quantitatively, the substitution effect dominates the response of aggregate debt composition, and the bank share falls. However, relative to the data, a shortcoming of this approach is that it leads to a decline in bond issuance, in particular by large firms. In the data, bond issuance among the largest firms was stable or increasing during the Great Recession, as has been documented,

by, among others, [Adrian *et al.* \(2012\)](#), as well as in Section 4.1 in the particular context of manufacturing and retail firms.

4.4.2. Potential general equilibrium implications. Given its focus on the response of debt composition in the cross-section, the model analysed does not incorporate a complete analysis of the general equilibrium effects which the shock to the intermediation costs of banks may have on aggregate output and investment. Indeed, in the model, the supply of deposits to financial intermediaries is infinitely elastic, at the fixed interest rate r . In a more general version of the model, with interest-elastic deposits, the rate of return r would need to adjust along the transition path to equate the supply of deposits to total lending to firms. It is therefore reasonable to expect that the interest rate would fall, thereby mitigating the reduction in overall demand for credit by firms, and the decline in investment. Thus, relative to a full-fledged general equilibrium version of the model, the results of this section should be viewed as an upper bound on the potential effects of the bank-specific credit shock on the real decision of rated firms.

Summarizing, this section has shown that lending wedge shocks can generate large recessions accompanied by substitution towards market borrowing, both at the firm and aggregate levels. However, debt substitution also forces some firms to reduce leverage, and therefore investment. Deleveraging is optimal from the standpoint of switching firms: relying only on market debt deprives them from the flexibility offered by banks in bad times. But it amplifies the response of investment and aggregate activity to the shock. The following section turns to the potential policy implications of this mechanism.

5. CORPORATE FINANCE REFORMS AND THEIR REAL EFFECTS

As a result of the contraction in bank lending that followed the Great Recession, some countries have considered encouraging market-based intermediation as a remedy to lagging investment. The Bank of England, *e.g.*, included corporate bonds in the Asset Purchase Facility that was set up in January 2009 to provide liquidity to credit markets, with the explicit goal of stimulating primary market issuance. This section discusses the potential real effects of efforts to stimulate the use of market-based intermediation, in the context of two specific examples: a new exchange for bond issuances by German firms of the “Mittelstand”, and an Italian tax reform improving the tax deductibility of interest payments on bonds.

5.1. *The Bondm market*

The Bondm exchange was launched by Boerse Stuttgart in 2010 with the goal of providing a more favourable environment for bond issuance for midsize companies than existing markets. Bondm provides firms with a primary market for new issuances and also operates a secondary exchange in which private and retail investors trade existing issues. Participation is restricted to firms with less than €250m in assets and to issuances between €50m and €150m. Crucially, it aims at making bond issuances more attractive to midsize firms by reducing intermediation costs. Bondm issuances do not need to be individually rated, and do not need to be underwritten by a bank. Underwriting requirements are particularly costly, as few European investment banks specialize in underwriting small issuers.²⁸

28. For more details on the German corporate bond market during the recession and the Bondm exchange, see the case study of the German mid-cap Dürr in [Hillion *et al.* \(2012\)](#).

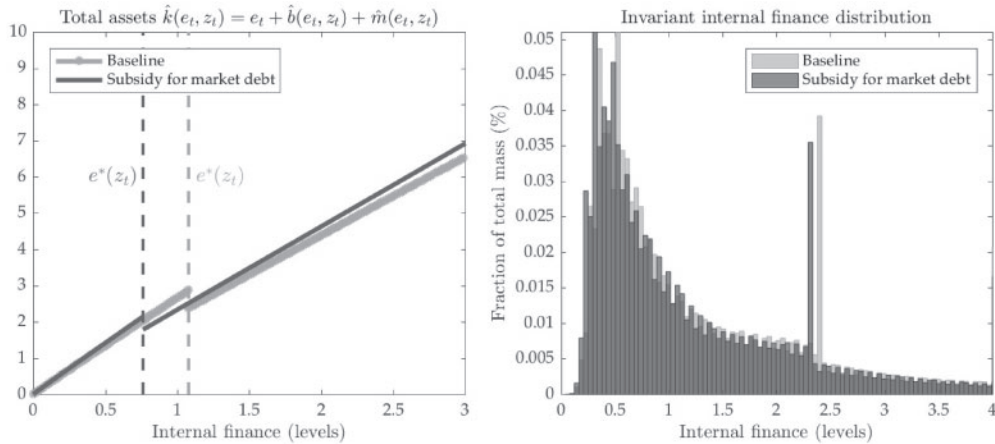


FIGURE 12

The effect of lower intermediation costs targeted to small firms.

In the context of the model described in Section 2, a simple way to illustrate the potential effects of this type of policy is to let market intermediation costs depend on internal funds e_t :

$$\gamma_m(e_t) = \begin{cases} \tilde{\gamma}_m & \text{if } e_t \leq e_{sm} \\ \gamma_m & \text{if } e_t > e_{sm} \end{cases}, \quad \tilde{\gamma}_m < \gamma_m.$$

One can then contrast two economies: a baseline economy in which $e_{sm} = 0$, and an economy with $e_{sm} > 0$. $\tilde{\gamma}_m \leq \gamma_m$ denotes the lower intermediation costs for midsize firms ($e_t \leq e_{sm}$) associated with markets that resemble Bondm.

Figure 12 compares an economy without low market intermediation costs for mid-sized firms, to an economy with low market intermediation costs for midsize firms. In the baseline economy, $\gamma_m = 0.100$, as in Section 3. In the economy with size-dependent intermediation costs, I assume that $\tilde{\gamma}_m = 0.080$, so that market intermediation costs are 20 basis points lower for mid-size firms.

The right panel of Figure 12 displays the size distribution without and with the subsidy; they are relatively similar. The effects of the subsidy on investment are best understood by looking at changes in firms' total assets $\hat{k}(e_t, z_t) = e_t + \hat{b}(e_t, z_t) + \hat{m}(e_t, z_t)$, which are reported in the right panel of Figure 12, for a specific level of productivity z_t .

This response depends on internal funds e_t . There are three cases. First, the lower intermediation costs create a new threshold $\tilde{e}^*(z_t)$. Firms with internal funds $e_t \leq \tilde{e}^*(z_t)$ keep relying on bank loans even when there are lower market intermediation costs. However, these firms increase total borrowing: lower intermediation costs indeed encourage them to increase market debt issuance, which additionally relaxes their bank borrowing constraint because of the complementarity between forms of borrowing in the mixed finance regime.

The second case is that of firms with internal funds $e_t > e^*(z_t)$, where $e^*(z_t)$ is the threshold above which firms cannot take direct advantage of low market intermediation costs (*i.e.* they cannot issue on the Bondm market because they are too large). These firms nevertheless increase bond issuance in response to the lower intermediation costs. This is because they anticipate that, now that market borrowing is cheaper at small scales, they will be able to operate more profitably if a string of bad shocks pushes them back to a mixed debt structure.

The third case relates to intermediate firms, those with $\tilde{e}^*(z_t) < e_t \leq e^*(z_t)$. These firms are large enough that the policy will induce them to switch to an entirely market financed debt structure,

but small enough that they will still benefit from the Bondm intermediation costs for their bond issuances. The switch makes the debt structure of these firms more fragile, in the sense that they lose the option to restructure debt in bad times. As a result, some of these firms deleverage: their total borrowing is *smaller* than in the world with higher intermediation costs, and they operate at a *smaller* scale. Surprisingly, the lower intermediation costs therefore result in a reduction in total debt issuance and total investment by these firms.

How do these two effects—the “lending channel” effect that stimulates borrowing and investment by firms with $e_t \leq \tilde{e}^*(z_t)$ and $e_t > e^*(z_t)$, and the “substitution channel” effect that depresses borrowing and investment by firms with $\tilde{e}^*(z_t) < e_t \leq e^*(z_t)$ —measure up against each other? Overall, the effect of the policy $e_{sm} > 0$ on total investment is positive: it increases by 5.3% under the policy of lower issuance costs for midsize firms. However, the negative effect on investment by medium-sized firms is sizeable, because the mass of firms that are in this region is large. Specifically, these firms account for a -1.7% decline in aggregate investment, relative to the initial steady state.

5.2. An Italian tax reform

To the extent that they discriminate between bank and market liabilities, taxes are an alternative tool that can be used as an instrument to influence the choice of debt structure by Italian firms. A 2012 Italian tax reform meant to incentivize access to bond markets for private companies includes the tax treatment of interest payments to corporate bonds.²⁹ Specifically, the reform allows private firms to deduct interest paid on bonds in the same way as interest paid on other debt, in line with the tax rules imposed on large firms.

The model of Section 2 does not explicitly incorporate tax deductibility of interest payments on debt, but it can easily be generalized to include it.³⁰ Let cash on hand of the firm in repayment be given by

$$n_t^P = (1 - \tau)\pi_t - (1 - \tau_b)R_{b,t} - (1 - \tau_m)R_{m,t},$$

where τ denotes the marginal corporate tax rate. Then, $\tau_b \leq \tau$ and $\tau_m \leq \tau$ reflect preferential tax treatments of different types of debt instruments.

The two tax regimes that existed in Italy prior to the reform were, respectively, $\tau_b = \tau$ and $\tau_m = 0$ (only bank debt is interest-deductible) and $\tau_b = \tau_m = \tau$ (both types of debt are treated identically for tax purposes). The policy experiment then consists of comparing an economy where only sufficiently large firms enjoy preferential tax treatment for market debt ($\tau_b = \tau_m = \tau$ for $e_t > e_{sm}$; $\tau_b = \tau$ and $\tau_m = 0$ for $e_t \leq e_{sm}$) with an economy in which all firms enjoy the same tax shields for market and bank debt ($\tau_m = \tau_b = \tau$ for all e_t).

The impact of this change in tax policy on borrowing and investment is qualitatively similar to the previous experiment. Intuitively, a differential treatment of bank and market debt ($\tau_m < \tau_b$) directly affects the relative cost of market and bank debt outside of liquidation. Introducing a tax shield on market debt issuance for midsize firms will therefore have similar effects on borrowing as lowering market intermediation costs. Namely, the tax shield boosts total borrowing by the smallest and the largest firms, because of the “credit channel” effect discussed above. However,

29. The reform also relaxes the requirements to find a sponsor to guarantee the issuance of the bond and eliminates existing limits on indebtedness as a fraction of net worth for private firms. See <http://www.paulhastings.com/Resources/Upload/Publications/2351.pdf> for more details on the contents of the reform.

30. The Online Appendix to the article contains the extension of the model to include tax deductibility of interest payment on debt, and reports these results described below.

it reduces total borrowing by the mass of intermediate firms that switch entirely to a market financed debt structure in response to the introduction of the tax shield.

6. CONCLUSION

The composition of corporate borrowing between bank loans and market debt exhibits substantial variation across firms and over time. This article starts from the view that while banks are costly financial intermediaries, they also specialize in providing a flexible form of borrowing to firms. Embedding this trade-off between flexibility and cost into a simple model of firm dynamics leads to cross-sectional predictions that align well with firm-level evidence on the composition of debt. In response to an aggregate shock to bank intermediation costs, the model generates a drop in output and investment, accompanied by changes in debt composition which, both at the aggregate and firm level, are close to those experienced by rated firms in the U.S. during the 2007–9 recession.

A key finding of the article is that, aside from the traditional “lending channel” effects of financial shocks or policies, debt heterogeneity introduces a new “substitution channel”: in response to aggregate shocks affecting the relative cost of bank debt, some firms substitute towards bond issuances, but do so less than one-for-one and, as result, experience a deleveraging episode. Firms respond in this fashion because switching from a mixed debt structure to a purely bond-financed debt structure exposes them to more liquidation risk, precisely because bond and bank financing do not offer the same degree of flexibility in distress. The deleveraging that accompanies debt substitution by certain firms in the model can thus be interpreted as a precautionary response to their increases in financial fragility, when more of external financing is obtained via public debt markets.

An important question, left unaddressed in this analysis, is whether changes in the regulatory regime of banks—in particular if they affect banks’ risk-bearing capacity—would have similar effects on corporate debt choices and investment. Because of the simplicity of the description of financial intermediation adopted in this model, the results of this article cannot directly speak to this question. It is an interesting avenue to explore in future research.

APPENDIX

A. DATA APPENDIX

A description of data sources not mentioned in this Appendix can be found in the Online Appendix to the article.

Main measures of the bank share and debt composition. The bank share of all nonfinancial corporate businesses mentioned in the introduction, and reported in Figure 9, is constructed using Table L.103 of the Flow of Funds, the balance sheet of the non-financial corporate sector. It is defined as the ratio of the sum of depository institution loans (line 27, series FL103168005.Q) and other loans and advances (line 28, series FL103169005.Q), to total credit market instruments outstanding (line 23, series FL104104005.Q).

The bank share used in the calibration of the model is constructed using data from the QFR for manufacturing firms. This dataset contains information on firms’ balance sheets and income statements, and is reported in semi-aggregated form (by asset size categories). To focus on firms with potential access to public debt markets, I use only the two top asset size groups: firms with between \$250m and \$1bn dollars in assets, and firms with more than \$1bn in assets (asset size groups 9 and 10 in the QFR data). I also focus only on manufacturing firms (industry group TMFG in the QFR data), since the absence of size brackets for wholesale and retail firms do not allow me to narrow down the sample to firms in those industries with more than \$250m in assets.

For each size group i , bank debt $b_{i,t}$ is defined as the sum of short-term bank debt (*stbank* in the QFR data), currently due long-term bank debt (*instbank*), and bank debt due in more than one year (*ltbnkdebt*). Total debt $d_{i,t}$ is defined as the sum of bank debt and commercial paper (*compaper*), total bond debt (the sum of *instbonds* and *ltbnddebt*), and total other debt (the sum of *stdebttho*, *instother* and *ltothdebt*). The bank share reported in the left panel of Figure 9 is then given by $BS_t = \frac{\sum_i b_{i,t}}{\sum_i d_{i,t}}$.

The middle and right panels of Figure 9 report changes in the levels of bank and non bank debt for each of the two groups of firms (firms with \$250m – \$1bn in assets, and firms with more than \$1bn in assets). I define market debt for each group as: $m_{i,t} = d_{i,t} - b_{i,t}$. The series reported are normalized relative to 2008Q3, and are weighted so that the sum of market and bank debt adds up to the total change in debt. Specifically, the change in bank debt for group i is defined as: $\gamma_{b,i,t} = \frac{b_{i,t_0}}{b_{i,t_0} + m_{i,t_0}} \left(\frac{b_{i,t}}{b_{i,t_0}} - 1 \right)$, where t_0 is 2008Q3. The series for the change in market debt is defined analogously. All series in the middle and right panels of Figure 9 are smoothed by a 2 by 4 MA smoother.

Alternative measures of the bank share. While the QFR data can be used to construct a time series for the bank share in a transparent and consistent manner, it has two main limitations: (1) it is restricted to manufacturing firms and (2) it may include data on firms that do not have access to public debt markets.

An alternative data source is the firm-level dataset created by [Rauh and Sufi \(2010\)](#).³¹ It combines data from Compustat on capital structure, from SDC and Dealscan data on debt issuances, and from companies' 10K data, in order to provide a breakdown of firms' debt by instrument. The data made available by the authors are composed of a random sample of approximately 220 firms per year, between 1996 and 2006. The sample is limited to rated firms (as measured by the existence of an S&P credit rating), thus narrowing the focus to firms with access to public debt markets. Industries other than manufacturing are represented: while manufacturing firms make up 44.3% of the sample, utilities, services, and wholesale and retail account for 18.0%, 17.2%, and 11.4% of the sample.

I look at the subset of firm-year observations with debt (d) above \$1 m. I denote the total debt of a firm-year observation by $d_{i,t}$. The data break down debt instruments into seven categories. To compute the bank share, I define bank debt $b_{i,t}$ as the sum of three instruments: outstanding bank debt (*bankout*), privately placed debt (*pp*), and mortgage or equipment debt (*mgeq*). Following the definitions of debt categories provided by [Rauh and Sufi \(2010\)](#), these are the debt instruments most likely to be held by a bank or by a concentrated group of creditors. Additionally, I keep only observations for which the discrepancy between total debt $d_{i,t}$ and the sum of all seven debt instrument groups are within 1% (or \$1 m) of each other. These criteria narrow down the sample to 2,322 firm-year observations (out of the initial 2,453). I then define the bank share for each year as: $BS_t = \frac{\sum_i b_{i,t}}{\sum_i d_{i,t}}$, to maintain comparability with the QFR data.

With this measure of the bank share, the average annual bank share in the sample is 21.2% (the median is 20.9%), close to the 24.0% documented in the QFR. The somewhat lower bank share may be driven the fact that firms are, on average, larger in this sample than in the QFR (the assets of the median firm are \$1.4bn). The average bank share when restricting the sample to firms with assets between \$250m and \$1bn is 42.6% (that share is 51.3% in the QFR in 2007Q3), but these firms only account for 4.6% of total debt in the dataset (whereas they account for 8.6% of total debt in the QFR in 2007Q3).

The other data source for the measurement of debt composition is the work of [Colla et al. \(2013\)](#), who study debt composition by instrument in a sample of 16,115 observations of non-financial U.S. firms between 2002 and 2009. Of these 16,115 firm-year observations, 9,968 (or approximately 60%) correspond to rated firms (again measured by the existence of an S&P rating). They report a systematic breakdown of debt in seven instrument classes, two of which (drawn credit lines and term loans) can be used to measure outstanding bank debt. Among rated firms, the average share of bank debt to total debt, as measured using these two instruments, is 19.3%. This number is close to the aggregate bank share computed above in the [Rauh and Sufi \(2010\)](#) data, and somewhat smaller than the QFR bank share. Since this number is computed by averaging unweighted means of different rating classes, the discrepancy may be driven by variation in the number of firms in different rating groups.³²

Overall, both the data compiled by [Rauh and Sufi \(2010\)](#) and the evidence reported by [Colla et al. \(2013\)](#) indicate that the bank share of firms with access to public debt markets is close to the Figure of 24.0% obtained using the two largest asset size classes from the 2007Q3 QFR. Thus, biases in the measurement of the bank share introduced by the fact that the QFR is limited to manufacturing, and the fact that firms without access to public debt markets may be included in the QFR totals, are likely to be small.

Calibration of the productivity process. The productivity process is calibrated in two steps. First, firm-level TFP provided [Imrohoroglu and Tüzel \(2014\)](#) is combined with the annual fundamentals Compustat files and with S&P credit ratings.³³ Firm-year observations with fiscal years between 1995 and 2006 are selected if they satisfy the same criteria as those used for computing total investment above; this leaves a total of 60,090 unique firm-year observations. Among

31. The dataset is available at http://faculty.chicagobooth.edu/amir.sufi/data-and-appendices/RauhSufi_RFSdebtstructure_20100216.zip.

32. A direct computation of the aggregate bank share, as measured in the QFR and in the [Rauh and Sufi \(2010\)](#) data, is not feasible, since the firm-level data used by [Colla et al. \(2013\)](#) is not available publicly. Instead, the Figure of 19.3% is computed using the information from their Table VI, p. 2130.

33. The TFP data is available at <http://www-bcf.usc.edu/~tuzel/>.

those, only the 13,600 observations for which the firm has an S&P credit rating (or 22.6% of the total) are kept. The following three moments are then computed. First, the unconditional standard deviation of log TFP is computed by taking the average cross-sectional standard deviation across the twelve years of the sample; it is equal to 0.407, slightly higher than the value reported by Imrohoroglu and Tüzel (2014) for the overall sample, but lower than the value of 0.506 that obtains for the population of all Compustat firms (rated and unrated) over the same period. This is due to the fact that, as reported in that paper, the standard deviation of log productivity is larger in latter parts of the sample. Second, the autocorrelation of log-TFP at the firm level is equal to 0.77, slightly higher than the value of 0.72 reported by Imrohoroglu and Tüzel (2014). Finally, the ratio of average book assets of firms in the bottom decile of the productivity distribution, relative to book assets of firms in the top decile of the productivity distribution, is 8.3%. This is again slightly higher than the 5.69% ratio of the size of firms in the bottom decile of the TFP distribution to those of the top of the TFP distribution reported in Table 1 of Imrohoroglu and Tüzel (2014). Overall, the moments obtained for the sample of rated firms suggest that there is slightly less productivity and size dispersion among those firms than in the population of Compustat firms, consistent with the intuition that rated firms tend to be larger and more mature than average public firms (see Faulkender and Petersen (2006) for more evidence on this point).

The second step is to choose the state-space $\mathcal{Z} = \{z_i\}_{i=1}^{N_z}$, the transition matrix $p(z_j|z_i)_{i,j}$, and the distribution F_ϵ to match these moments. The state-space \mathcal{Z} and the matrix $p(z_j|z_i)$ are constructed using the Rouwenhorst method, described in detail in Kopecky and Suen (2010). This approach has the advantage of providing closed-form solutions for the unconditional moments of the resulting Markov chain, which can then be used to match their data counterparts. Specifically, the transition matrix $p(z_j|z_i)$ is chosen to be symmetric and to match the autocorrelation of $\rho = 0.77$ documented in the data. Values for the state $\{z_i\}_{i=1}^{N_z}$ are defined as $z_i = \exp(\tilde{z}_i + \mu_z)$, where $\tilde{z}_i = -\psi + \frac{2\psi}{N_z-1}(i-1)$, $i = 1, \dots, N_z$ represents the state-space for log-productivity. The parameter ψ is chosen to match relative optimal size in the bottom decile relative to the top decile of the TFP distribution in the model, $\frac{k^*(z_1)}{k^*(z_{N_z})}$; this requires $\psi = -\frac{1-\zeta}{2} \log(0.083)$, where ζ is the degree of returns to scale. The parameter μ_z is then picked to ensure that $k^*(N_z) = 100$, a normalization. Finally, given this calibration, one can derive the unconditional variance of z , σ_z^2 using the results reported in Kopecky and Suen (2010). The cross-sectional standard deviation of ϵ_t (the last remaining parameter, since the mean of ϵ_t is normalized to 1) is then chosen to match the remaining cross-sectional standard variance of TFP; that is, $\sigma_\epsilon^2 = 0.406^2 - \sigma_z^2$. In the baseline calibration of the model, I use $N_z = 3$; the resulting predictions of the model for the cross-sectional composition of debt structure are not sensitive to the choice of the number of states.

B. PROOFS

The proof of Propositions 1 and 2—regarding the solutions to the debt settlement game and the debt pricing problem—proceed by assuming that the value function V satisfies two properties: first, it is continuous in its first argument; second, it satisfies $V(0, z_t) \geq 0$ for any $z_t \in \mathcal{Z}$. The proof of Proposition 3 below then formally establishes that the unique fixed point of the functional operator corresponding to the firm's problem satisfies these two conditions.

B.1. Proof of Proposition 1

Let $z_{t+1} \in \mathcal{Z}$. First, note that when $V(\cdot, z_{t+1})$ is continuous, the continuity of $V^c(\cdot, z_{t+1})$ follows from the theorem of the maximum. Additionally, when $n_t = 0$ the feasible set contains only $d_t = 0, e_{t+1} = 0$. So $V^c(0) = (1 - \eta)\beta V(0)$. Therefore, when $V(0, z_{t+1}) \geq 0$, $V^c(0, z_{t+1}) \geq 0$.

The proof starts by establishing two useful properties of $V^c(\cdot, z_{t+1})$. First, it is necessarily increasing in net worth. Let $(n_t^a, n_t^b) \in \mathbb{R}_+^2$ such that $n_t^a > n_t^b$, and let e_{t+1}^b be a value for next period net worth that solves problem (1), when $n_t = n_t^b$. We have $e_{t+1}^b \leq n_t^a < n_t^a$, so e_{t+1}^b is also feasible when $n_t = n_t^a$. Therefore, $V^c(n_t^a, z_{t+1}) \geq n_t^a - e_{t+1}^b + (1 - \eta)\beta V(e_{t+1}^b) + \eta \xi e_{t+1}^b > n_t^b - e_{t+1}^b + (1 - \eta)\beta V(e_{t+1}^b) + \eta \xi e_{t+1}^b = V^c(n_t^b, z_{t+1})$. This proves that $V^c(\cdot, z_{t+1})$ is strictly increasing. Second, for any $z_{t+1} \in \mathcal{Z}$, if $V(0, z_{t+1}) \geq 0$, then $V^c(n_t, z_t) \geq n_t$. This is established by noting that the dividend policy $e_{t+1} = 0$ is always feasible at the dividend issuance stage, and that the value of this policy is $n_t + (1 - \eta)\beta V(0, z_{t+1}) \geq n_t$.

Fix $z_{t+1} \in \mathcal{Z}$. Assume, first, that $\frac{R_{b,t} + \gamma k_t}{1 - \chi} \leq \frac{R_{b,t}}{\chi}$. Then, $\frac{R_{m,t}}{1 - \chi} \leq R_{b,t} + R_{m,t} + \gamma k_t \leq \frac{R_{b,t}}{\chi}$. Additionally, recall that:

$$\pi_t^L = \chi \pi_t, \quad \pi_t^P = \pi_t - \gamma k_t.$$

The proof proceeds by comparing V_t^L, V_t^R and V_t^P , the values of the firm under liquidation, restructuring or repayment, for each realization of π_t . There are five possible cases:

- when $\pi_t \geq \frac{R_{b,t} + R_{m,t} + \gamma k_t}{\chi}$, one has $V_t^L = \chi \pi_t - R_{b,t} - R_{m,t} < \pi_t - R_{b,t} - R_{m,t} \leq V^c(\pi_t - R_{b,t} - R_{m,t} - \gamma k_t, z_{t+1}) = V_t^P$. Moreover, since $\pi_t \geq \frac{R_{b,t} + R_{m,t} + \gamma k_t}{\chi} \geq \frac{R_{b,t}}{\chi}$, the reservation value of the bank is $R_{b,t}$, so the best restructuring offer for the firm is $R_{b,t}^{(r)} = R_{b,t}$. Therefore $V_t^P = V_t^R$; I will assume the firm chooses repayment.

- when $\frac{R_{b,t}+R_{m,t}+\gamma k_t}{\chi} > \pi_t \geq \frac{R_{b,t}}{\chi}$, one has $V_t^L = 0 \leq V^c(\pi_t - R_{b,t} - R_{m,t} - \gamma k_t, z_{t+1}) = V_t^P$, since $\pi_t \geq \frac{R_{b,t}}{\chi} \geq R_{b,t} + R_{m,t} + \gamma k_t$, $V^c(0, z_{t+1}) \geq 0$ and $V^c(\cdot, z_{t+1})$ is strictly increasing. ($V_t^L < V_t^R$ so long as $\pi_t > R_{b,t} + R_{m,t} + \gamma k_t$). Moreover, $V_t^R = V_t^P$ for the same reason as above. Again, the firm chooses repayment.
- when $\frac{R_{b,t}}{\chi} > \pi_t \geq R_{b,t} + R_{m,t} + \gamma k_t$, the reservation value of the bank is $\chi \pi_t$. The restructuring offer at which the participation constraint of the bank binds, $R_{b,t}^{(r)} = \chi \pi_t$, is feasible because $\pi_t - R_{b,t}^{(r)} - R_{m,t} - \gamma k_t = (1 - \chi)\pi_t - R_{m,t} - \gamma k_t \geq 0$. So $V_t^R \geq V^c((1 - \chi)\pi_t - R_{m,t} - \gamma k_t, z_{t+1})$. This implies $V_t^R > V^c(\pi_t - R_{b,t} - R_{m,t} - \gamma k_t, z_{t+1}) = V_t^P$, since $V^c(\cdot, z_{t+1})$ is strictly increasing and $(1 - \chi)\pi_t - R_{m,t} - \gamma k_t > \pi_t - R_{b,t} - R_{m,t} - \gamma k_t$. For the same reasons as above, $V_t^R \geq V_t^L$. So the firm chooses to restructure. Because $V^c(\cdot, z_{t+1})$ is increasing, the optimal restructuring offer makes the participation constraint of the bank bind: $R_{b,t}^{(r)} = \chi \pi_t$.
- when $R_{b,t} + R_{m,t} + \gamma k_t \geq \pi_t \geq \frac{R_{m,t} + \gamma k_t}{1 - \chi}$, one has $V_t^L = 0 < V^c((1 - \chi)\pi_t - R_{m,t} - \gamma k_t, z_{t+1}) = V_t^R$, where again the properties of $V^c(\cdot, z_{t+1})$ were used. Moreover, $V_t^P = V_t^L$, since the firm does not have enough funds to repay both its creditors. So the firm chooses to restructure, again with $R_{b,t}^{(r)} = \chi \pi_t$.
- when $\pi_t < \frac{R_{m,t} + \gamma k_t}{1 - \chi}$, the firm is liquidated because any restructuring offer consistent with the participation constraint of the bank will leave the firm unable to repay market creditors. Since in that case, $0 > \chi \pi_t - R_{b,t} - R_{m,t} = \gamma k_t$, the liquidation value for the firm is $V_t^L = 0$.

This shows that when $\frac{R_{m,t} + \gamma k_t}{1 - \chi} \leq \frac{R_{b,t}}{\chi}$, the firm repays when $\pi_t \geq \frac{R_{b,t}}{\chi}$, restructures when $\frac{R_{b,t}}{\chi} \geq \pi_t \geq \frac{R_{m,t} + \gamma k_t}{1 - \chi}$, and is liquidated otherwise. Moreover, this also establishes the two additional claims of the proposition, in the case $\frac{R_{m,t} + \gamma k_t}{1 - \chi} \leq \frac{R_{b,t}}{\chi}$: $V_t^L = 0$ whenever liquidation is chosen, and the restructuring offer always makes the participation constraint of the bank bind: $R_{b,t}^{(r)} = \chi \pi_t$. The claims of the proposition when $\frac{R_{m,t} + \gamma k_t}{1 - \chi} > \frac{R_{b,t}}{\chi}$ can similarly be established, by focusing on the three sub-cases $\pi_t \geq \frac{R_{b,t} + R_{m,t} + \gamma k_t}{\chi}$, $\frac{R_{b,t} + R_{m,t} + \gamma k_t}{\chi} > \pi_t \geq R_{b,t} + R_{m,t} + \gamma k_t$ and $R_{b,t} + R_{m,t} + \gamma k_t > \pi_t$. \parallel

B.2. Proof of Proposition 2

In the case of $\chi_P = 1$, given the results of Proposition 1, the debt settlement outcomes yield the same conditional return functions for banks and market lenders, $\tilde{R}_{b,t}(\phi_t, k_t, R_{b,t}, R_{m,t})$ and $\tilde{R}_{m,t}(\phi_t, k_t, R_{b,t}, R_{m,t})$, as those reported in the appendix to Crouzet (2015). In that case, Proposition 2 follows from Proposition 2 in that paper. The case $\chi_P < 1$ is analogous; derivations are reported in the Online Appendix to this article. \parallel

B.3. Proof of Proposition 3

The proof of the existence of a recursive competitive equilibrium of the economy of Section 2 proceeds by first establishing the existence and unicity of a solution to the individual firm problem, then by characterizing the optimal dividend policy, and finally by establishing the existence of a steady state distribution.

Firm problem

Throughout, the firm's problem is restated in terms of the variables $g_t = b_t + m_t$ and $s_t = \frac{b_t}{b_t + m_t}$. g_t denotes total borrowing by a firm, and s_t denotes the share of borrowing that is bank debt. Note that $(g_t, s_t) \in \mathbb{R}_+ \times [0, 1]$.

Preliminaries: With some abuse of notation, the set of feasible debt structures (g_t, s_t) is still denoted by $\mathcal{S}(e_t, z_t)$, and its partition established in Proposition 2 as $(\mathcal{S}_K(e_t, z_t), \mathcal{S}_R(e_t, z_t))$. Note first that the conditional payoffs to bank and market lenders are independent of the realization of z_{t+1} . This is because the firm's decision to liquidate, renegotiate or continue only depends on k_t and the realization of ϕ_t , and not on the continuation value function $V^c(n_t, z_t)$, as emphasized in Proposition 2. The conditional payoff function for bank lenders, for example, satisfies $\tilde{R}_b(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) = \tilde{R}_b(\phi_t, k_t, R_{b,t}, R_{m,t})$, so that the zero-profit condition can be written as:

$$\begin{aligned} (1 + r_b)b_t &= \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t \geq 0} \tilde{R}_b(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t | z_t) \right) \\ &= \int_{\phi_t \geq 0} \tilde{R}_b(\phi_t, k_t, R_{b,t}, R_{m,t}) dF(\phi_t | z_t), \end{aligned}$$

and a similar relationship applies to the market lending contract. It is then straightforward to see that the equilibrium lending contracts are identical to the case in which z_t is *i.i.d* over time and across firms. (This is not the case for borrowing policies, which depend on the firm's expected continuation values, and therefore on the realization of z_{t+1}).

The Online Appendix contains complete derivations of the contract space and the contract terms in the general case $\chi_P \leq 1$. Here, we briefly describe their main characteristics. Define:

$$\delta_R(\chi_P) = 1 - \frac{\chi_P - \chi}{1 - \chi}(1 - \delta), \quad \delta_K(\chi_P) = 1 - \chi_P(1 - \delta).$$

Note that $\gamma = (1 - \chi_P)(1 - \delta) = (1 - \chi)(\delta_R(\chi_P) - \delta) = \delta_K(\chi_P) - \delta$. Moreover, $\delta_R(1) = \delta_K(1) = \delta$. For $(z, k) \in \mathcal{Z} \times \mathbb{R}_+$, define the following mappings from \mathbb{R}_+ to \mathbb{R}_+ :

$$\begin{aligned} G(x; z) &= x(1 - F(x|z)) + \int_0^x \phi dF(\phi|z), \\ I(x; k, z) &= x(1 - F(x|z)) - F(x|z)(1 - \delta(\chi_P))k^{1-\zeta}, \\ M(x; k, z) &= x(1 - F(x|z)) + \chi \left(\int_0^x \phi dF(\phi|z) \right). \end{aligned}$$

These mappings can be used to express the conditional payoffs to banks and market lenders. They have inverses $G^{-1}(\cdot; z)$, $I^{-1}(\cdot; k, z)$ and $M^{-1}(\cdot; k, z)$, defined, respectively, on $[0, \mathbb{E}(\phi|z)]$, $[0, \hat{I}(k, z)]$ and $[0, \hat{M}(k, z)]$, where $\hat{I}(k, z)$ is the global maximum of $I(\cdot; k, z)$ and similarly $\hat{M}(k, z)$ is the global maximum of $M(\cdot; k, z)$. These inverses can be used to express the terms of debt contracts $(R_{b,t}, R_{m,t})$ as a function of (g_t, s_t, e_t, z_t) explicitly. For example, the terms of bank contracts when $(g_t, s_t) \in \mathcal{S}_R(e_t, z_t)$ are given by:

$$R_{b,t} = R_b(g_t, s_t, e_t, z_t) = \begin{cases} (1 + r_b)g_t s_t & \text{if } \frac{(1 + r_b)g_t s_t}{\chi(e_t + g_t)^\zeta} < (1 - \delta)(e_t + g_t)^{1-\zeta} \\ \chi(1 - \delta)(e_t + g_t) & \text{if } (1 - \delta)(e_t + g_t)^{1-\zeta} \leq \frac{(1 + r_b)g_t s_t}{\chi(e_t + g_t)^\zeta} \\ + \chi(e_t + g_t)^\zeta G^{-1}\left(\frac{(1 + r_b)g_t s_t - \chi(1 - \delta)(e_t + g_t)}{\chi(e_t + g_t)^\zeta}; z_t\right) & \leq \mathbb{E}(\phi|z_t) + (1 - \delta)(e_t + g_t)^{1-\zeta}. \end{cases}$$

Expressions for $R_b(g_t, s_t, e_t, z_t)$ and $R_m(g_t, s_t, e_t, z_t)$ for other cases are reported in the Online Appendix to the paper. Finally, define the following thresholds for productivity:

$$\begin{aligned} \underline{\phi}_R(g_t, s_t, e_t, z_t) &\equiv \frac{R_m(g_t, s_t, e_t, z_t) - (1 - \chi)(1 - \delta_R(\chi_P))(e_t + g_t)}{(1 - \chi)(e_t + g_t)^\zeta} & (\text{liquidation threshold when } (g_t, s_t) \in \mathcal{S}_R(e_t, z_t)) \\ \bar{\phi}_R(g_t, s_t, e_t, z_t) &\equiv \frac{R_b(g_t, s_t, e_t, z_t) - \chi(1 - \delta)(e_t + g_t)}{\chi(e_t + g_t)^\zeta} & (\text{restructuring threshold when } (g_t, s_t) \in \mathcal{S}_R(e_t, z_t)) \\ \underline{\phi}_K(g_t, s_t, e_t, z_t) &\equiv \frac{R_b(g_t, s_t, e_t, z_t) + R_m(g_t, s_t, e_t, z_t) - (1 - \delta_K(\chi_P))(e_t + g_t)}{(e_t + g_t)^\zeta} & (\text{liquidation threshold when } (g_t, s_t) \in \mathcal{S}_K(e_t, z_t)). \end{aligned}$$

These thresholds can be expressed as direct functions of (g_t, s_t, e_t, z_t) as:

$$\begin{aligned} \underline{\phi}_R(g_t, s_t, e_t, z_t) &= \begin{cases} \frac{(1 + r_m)g_t(1 - s_t) - (1 - \chi)(1 - \delta_R(\chi_P))(e_t + g_t)}{(1 - \chi)(e_t + g_t)^\zeta} & \text{if } (1 + r_m)g_t(1 - s_t) < (1 - \chi)(1 - \delta_R(\chi_P))(e_t + g_t) \\ I^{-1}\left(\frac{(1 + r_m)g_t(1 - s_t) - (1 - \chi)(1 - \delta_R(\chi_P))(e_t + g_t)}{(1 - \chi)(e_t + g_t)^\zeta}; e_t + g_t, z_t\right) & \text{if } (1 - \chi)(1 - \delta_R(\chi_P))(e_t + g_t) \leq (1 + r_m)g_t(1 - s_t) \\ & \leq (1 - \chi)(1 - \delta_R(\chi_P))(e_t + g_t) + (1 - \chi)(e_t + g_t)^\zeta \hat{I}(e_t + g_t) \end{cases} \\ \bar{\phi}_R(g_t, s_t, e_t, z_t) &= \begin{cases} \frac{(1 + r_b)g_t s_t - \chi(1 - \delta)(e_t + g_t)}{\chi(e_t + g_t)^\zeta} & \text{if } (1 + r_b)g_t s_t < \chi(1 - \delta)(e_t + g_t) \\ G^{-1}\left(\frac{(1 + r_b)g_t s_t - \chi(1 - \delta)(e_t + g_t)}{\chi(e_t + g_t)^\zeta}; z_t\right) & \text{if } \chi(1 - \delta)(e_t + g_t) \leq (1 + r_b)g_t s_t \\ & \leq \chi(1 - \delta)(e_t + g_t) + \chi(e_t + g_t)^\zeta \mathbb{E}(\phi|z_t) \end{cases} \\ \underline{\phi}_K(g_t, s_t, e_t, z_t) &= \begin{cases} \frac{(1 + r_m(1 - s_t) + r_b s_t)g_t - (1 - \delta_K(\chi_P))(e_t + g_t)}{(e_t + g_t)^\zeta} & \text{if } (1 + r_m(1 - s_t) + r_b s_t)g_t < (1 - \delta_K(\chi_P))(e_t + g_t) \\ M^{-1}\left(\frac{(1 + r_m(1 - s_t) + r_b s_t)g_t - (1 - \delta_K(\chi_P))(e_t + g_t)}{(e_t + g_t)^\zeta}; e_t + g_t, z_t\right) & \text{if } (1 - \delta_K(\chi_P))(e_t + g_t) \leq (1 + r_m(1 - s_t) + r_b s_t)g_t \\ & \leq (1 - \delta_K(\chi_P))(e_t + g_t) + (e_t + g_t)^\zeta \hat{M}(e_t + g_t; z_t). \end{cases} \end{aligned}$$

With these thresholds, one can express net income n_R when $(g_t, s_t) \in \mathcal{S}_R(e_t, z_t)$ as an explicit function of (g_t, s_t, e_t, z_t) , and likewise, net income n_K when $(g_t, s_t) \in \mathcal{S}_K(e_t, z_t)$ as an explicit function of (g_t, s_t, e_t, z_t) :

$$n_R(\phi_t, g_t, s_t, e_t, z_t) = \begin{cases} \phi_t(e_t + g_t)^\zeta + (1 - \delta)(e_t + g_t) - (1 - r_m)g_t s_t - (1 + r_b)g_t(1 - s_t) & \text{if } \underline{\phi}_R(g_t, s_t, e_t, z_t) = 0 \\ (1 - \chi) \left(\phi_t - \underline{\phi}_R(g_t, s_t, e_t, z_t) \right) (e_t + g_t)^\zeta + \gamma(e_t + g_t) & \text{if } 0 < \underline{\phi}_R(g_t, s_t, e_t, z_t) \leq \phi_t \leq \bar{\phi}_R(g_t, s_t, e_t, z_t) \\ \left(\phi_t - \chi \bar{\phi}_R(g_t, s_t, e_t, z_t) - (1 - \chi) \underline{\phi}_R(g_t, s_t, e_t, z_t) \right) (e_t + g_t)^\zeta + \gamma(e_t + g_t) & \text{if } \bar{\phi}_R(g_t, s_t, e_t, z_t) \leq \phi_t \end{cases}$$

$$n_K(\phi_t, g_t, s_t, e_t, z_t) = \begin{cases} \phi_t(e_t + g_t)^\zeta + (1 - \delta)(e_t + g_t) - (1 - r_m)g_t s_t - (1 + r_b)g_t(1 - s_t) & \text{if } \underline{\phi}_K(g_t, s_t, e_t, z_t) = 0 \\ \left(\phi_t - \underline{\phi}_K(g_t, s_t, e_t, z_t) \right) (e_t + g_t)^\zeta + \gamma(e_t + g_t) & \text{if } 0 < \underline{\phi}_K(g_t, s_t, e_t, z_t) \leq \phi_t. \end{cases}$$

The proof then proceeds in three steps:

Step 1: Reformulate the problem as the combination of a discrete choice problem, between using a debt structure belonging to $\mathcal{S}_R(e_t, z_t)$ and one belonging to $\mathcal{S}_K(e_t, z_t)$, and continuous maximization problem, the maximization of the present value of dividends under each option.

Step 2: Show that the functional mapping T associated with this new formulation maps the space $C(E)$ of real-valued functions on $[0, E] \times \mathcal{Z}$ that are continuous with respect to their first argument endowed with the sup norm $\|\cdot\|_s$, onto itself, where $E > 0$ is an arbitrarily large upper bound for equity. Additionally, show that $T(C_0(E)) \subseteq C_0(E)$, where $C_0(E) = \{V \in C(E) \text{ s.t. } \forall z \in \mathcal{Z}, V(0, z) \geq 0\}$. $(C(E), \|\cdot\|_s)$ is a complete metric space and $C_0(E)$ is a closed subset of $C(E)$ under $\|\cdot\|_s$ which is additionally stable through T . So, if T is a contraction mapping, then its fixed point must be in $C_0(E)$.

Step 3: Check that T satisfies Blackwell's sufficiency conditions, so that it has a unique fixed point V .

Note that step 2 is crucial because Lemma 1 requires the continuity of V in its first argument and the fact that $V(0, z) \geq 0$ for $V^c(\cdot, z)$ to be continuous, strictly increasing, and to satisfy $V^c(0, z) \geq 0$. In turn, these three conditions are necessary for characterizing of the set of feasible debt structures, that is, for Proposition 2 to hold.

Step 1: Define the mapping T on $C(E)$ as:

$$\forall e_t \in [0, E] \times \mathcal{Z}, \quad TV(e_t, z_t) = \max_{R, K} \left(T_R V(e_t, z_t), T_K V(e_t, z_t) \right) \quad (\text{A1})$$

where the mappings T_R and T_K , also defined on $C(E)$, are given by:

$$\forall (e_t, z_t) \in [0, E] \times \mathcal{Z}, \quad T_R V(e_t, z_t) = \max_{(g_t, s_t) \in \mathcal{S}_R(e_t, z_t)} \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t \geq \underline{\phi}_R(g_t, s_t, e_t, z_t)} V^c(n_R(\phi_t; g_t, s_t, e_t, z_t), z_{t+1}) dF(\phi_t | z_t) \right) \quad (\text{A1-R})$$

$$\text{s.t.} \quad V^c(n_t, z_{t+1}) = \max_{0 \leq e_{t+1} \leq n_t} n_t - e_{t+1} + (1 - \eta)\beta V(e_{t+1}, z_{t+1}) + \xi \eta e_{t+1}$$

$$\forall (e_t, z_t) \in [0, E] \times \mathcal{Z}, \quad T_K V(e_t, z_t) = \max_{(g_t, s_t) \in \mathcal{S}_K(e_t, z_t)} \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t \geq \underline{\phi}_K(g_t, s_t, e_t, z_t)} V^c(n_K(\phi_t; g_t, s_t, e_t, z_t), z_{t+1}) dF(\phi_t | z_t) \right) \quad (\text{A1-K})$$

$$\text{s.t.} \quad V^c(n_t, z_{t+1}) = \max_{0 \leq e_{t+1} \leq n_t} n_t - e_{t+1} + (1 - \eta)\beta V(e_{t+1}, z_{t+1}) + \xi \eta e_{t+1}.$$

Consider a solution V to problem (7) and a particular value of $(e_t, z_t) \in [0, E] \times \mathcal{Z}$. Since $(\mathcal{S}_K(e_t, z_t), \mathcal{S}_R(e_t, z_t))$ is a partition of $\mathcal{S}(e_t, z_t)$, the optimal policies (\hat{d}_t, \hat{s}_t) (there may be several) must be in either $\mathcal{S}_R(e_t, z_t)$ or $\mathcal{S}_K(e_t, z_t)$. Assume that they are in $\mathcal{S}_R(e_t, z_t)$. Then the contracts $R_{b,t}, R_{m,t}$ associated with the optimal policies satisfy $\frac{R_{b,t}}{\chi} \geq \frac{R_{m,t} + \gamma k_t}{1 - \chi}$. Given the results of Proposition 2, the constraints and objectives in problem (7) can be rewritten as in (A1-R). Since, by assumption, V solves (7), it's straightforward to check that it also solves $V(e_t, z_t) = T_R V(e_t, z_t)$. Moreover, in that case $T_R V(e_t, z_t) = V(e_t, z_t) \geq T_K V(e_t, z_t)$, by optimality of (\hat{d}_t, \hat{s}_t) . Thus, $TV(e_t, z_t) = V(e_t, z_t)$. The same equality obtains if $(\hat{d}_t, \hat{s}_t) \in \mathcal{S}_K(e_t, z_t)$. Any solution to V to problem (7) must thus satisfy $TV = V$. The rest of the proof therefore focuses on the properties of the operators T, T_K and T_R .

Step 2: Let $V \in C(E)$. By Lemma 1, the associated continuation value V^c is continuous on \mathbb{R}_+ . Moreover, since I^{-1} and G^{-1} are continuous functions of e_t, g_t and s_t , the functions $n_R, \underline{\phi}_R$ and $\bar{\phi}_R$ are continuous in their (e_t, g_t, s_t) arguments.

Fix $z_t \in \mathcal{Z}$. Define the mapping $O_R: [0, \bar{d}(E, z_t)] \times [0, 1] \times [0, E] \rightarrow \mathbb{R}_+$ by:

$$O_R(g_t, s_t, e_t) = \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{\phi_t \geq \phi_R(e_t, g_t, s_t)} V^c(n_t^R(\phi_t; e_t, g_t, s_t), z_{t+1}) dF(\phi_t | z_t) \right).$$

Here, $\bar{d}(E, z_t)$ denotes the upper bound on borrowing for the maximum level of equity E when current productivity is equal to z_t . By continuity of V^c, n_t^R, ϕ_R and $\bar{\phi}_R$, the integrand in O_R is continuous on the compact set $[0, E] \times [0, \bar{d}(E, z_t)] \times [0, 1]$, and therefore uniformly continuous. Hence, O_R is continuous on $[0, E] \times [0, \bar{d}(E, z_t)] \times [0, 1]$. The constraint correspondence $\Gamma_R: e_t \rightarrow S_R(e_t, z_t)$ maps $[0, E]$ into $[0, \bar{d}(E, z_t)] \times [0, 1]$. The characterization of the set $S_R(e_t, z_t)$ reported in the Online Appendix moreover shows that the graph of the correspondence Γ_R is closed and convex. Theorems 3.4 and 3.5 in [Stokey et al. \(1989\)](#) then indicate that Γ_R is continuous. Given that O_R is continuous and Γ_R compact-valued and continuous, the theorem of the maximum applies, and guarantees that $T_R V(\cdot, z_t)$ is real-valued and continuous. Since this is true for each $z_t \in \mathcal{Z}$, one has that $T_R V \in C(E)$.

By analogously defining a mapping $O_K(e_t, g_t, s_t)$ the problem [A1-K](#), one can also prove that $T_K V \in C(E)$. Therefore, $TV = \max(T_R V, T_K V) \in C(E)$. Moreover, let $V \in C_0(E)$. Then $\forall z_{t+1} \in \mathcal{Z}$, $V^c(0, z_{t+1}) \geq 0$ and $V^c(\cdot, z_{t+1})$ is strictly increasing, by Lemma 1. Moreover, $S_R(0, z_{t+1}) \neq \emptyset$, so one can evaluate O_R at some $(d_{t,0}, s_{t,0}) \in S_R(0, z_t)$. Since $n_R \geq 0$, $V^c(0, z_t) \geq 0$ and V^c is increasing, $O_R(d_{t,0}, s_{t,0}, 0, z_{t+1}) \geq 0$. Therefore $T_R V(0, z_{t+1}) \geq 0$, so $TV(0, z_{t+1}) \geq 0$. Since this is true for all $z_{t+1} \in \mathcal{Z}$, one has that $TV \in C_0(E)$.

Step 3: First, fix z_t and let $(V, W) \in C(E)$ such that $\forall e_t \in C(E)$, $V(e_t, z_t) \geq W(e_t, z_t)$. Pick a particular $e_t \in [0, E]$. By an argument similar to the proof of Lemma 1, $\forall n_t \geq 0$, $V^c(n_t, z_t) \geq W^c(n_t, z_t)$, where W^c denotes the solution to the dividend issuance problem when the continuation value is W (and analogously for V). Since the functions $\phi_R, \bar{\phi}_R$ and n_R are independent of V , this inequality implies $O_R^V(e_t, g_t, s_t) \geq O_R^W(e_t, g_t, s_t)$ for any $(g_t, s_t) \in S_R(e_t, z_t)$, where the notation O_R^W designates the objective function in problem [\(A1-R\)](#) when the continuation value function is W and current productivity is z_t (and analogously for V). Thus $T_R V(e_t, z_t) \geq T_R W(e_t, z_t)$. Similarly, one can show that $T_K V(e_t, z_t) \geq T_K W(e_t, z_t)$. Therefore, $TV(e_t, z_t) \geq TW(e_t, z_t)$. We have established this for any $(e_t, z_t) \in [0, E] \times \mathcal{Z}$, so T has the monotonicity property.

To establish the discounting property, it is sufficient to note that $(V+a)^c(n_t, z_t) = V^c(n_t, z_t) + \beta a$. Fix z_t , then for any $e_t \in [0, E]$ and $(g_t, s_t) \in S_R(e_t, z_t)$, $O_R^{V+a}(g_t, s_t, e_t) = O_R^V(g_t, s_t, e_t) + (1 - F(\phi_R(g_t, s_t, e_t, z_t)))\beta a \leq O_R^V(g_t, s_t, e_t, z_t) + \beta a$. This shows that $T_R(V+a)(e_t, z_t) \leq T_R V(e_t, z_t) + \beta a$. A similar claim can be made for T_K . Therefore, the operator $T(V+a)(e_t, z_t) \leq TV(e_t, z_t) + \beta a$. Since this holds for any $(e_t, z_t) \in [0, E] \times \mathcal{Z}$, the operator has the discounting property. Therefore, the Blackwell sufficiency conditions hold, so that T is a contraction mapping. As a contracting mapping on a complete metric space, it has a unique fixed point, which I denote V in what follows.

B.4. The dividend issuance policy

We start by establishing conditions on the value function $V(\cdot, z_t)$ weaker than global concavity under which the dividend issuance problem still has a threshold solution. These weaker conditions are needed because of V is not globally concave; instead, it is the upper envelope of two concave and differentiable functions, and so may have regions of non-concavity. Recall that the dividend issuance problem is:

$$\begin{aligned} \tilde{V}^c(n_t, z_{t+1}) &= \max_{d_t, e_{t+1}} d_t + (1 - \eta)\beta \tilde{V}(e_{t+1}, z_{t+1}) + \eta \xi e_{t+1} \\ \text{s.t.} \quad & d_t + e_{t+1} \leq n_t, \quad d_t \geq 0, \end{aligned} \quad (\text{B.1})$$

Here, the notation \tilde{V}^c and \tilde{V} is used to refer to any potential value function, not necessarily the solution to $TV = V$.

Lemma 1. (Dividend policy: Cooley and Quadrini (2001)). Let $z_{t+1} \in \mathcal{Z}$ be given. If $\tilde{V}(\cdot, z_{t+1})$ is (1) continuous on \mathbb{R}_+ , and if (2) $\tilde{V}(\cdot, z_{t+1})$ is left- and right-hand differentiable for all $e > 0$, and (3) there exists $\bar{e}(z_{t+1}) > 0$ such that

$$\forall e \in [0, \bar{e}(z_{t+1})[, \quad (1 - \eta)\beta \frac{\partial \tilde{V}^{(-)}}{\partial e}(e, z_{t+1}) + \eta \xi > 1 \quad \text{and} \quad \forall e \in [\bar{e}(z_{t+1}), +\infty[, \quad (1 - \eta)\beta \frac{\partial \tilde{V}^{(+)}}{\partial e}(e, z_{t+1}) + \eta \xi \leq 1,$$

then, the solution to problem [\(B.1\)](#) is given by:

$$\hat{d}(n_t, z_{t+1}) = \begin{cases} 0 & \text{if } 0 \leq n_t < \bar{e}(z_{t+1}) \\ n_t - \bar{e}(z_{t+1}) & \text{if } n_t \geq \bar{e}(z_{t+1}) \end{cases}.$$

Proof of Lemma 1. Define the function $F(e) \equiv (1 - \eta)\beta \tilde{V}(e, z_{t+1}) + \eta \xi e - e$ on \mathbb{R}_+ . F is continuous whenever $\tilde{V}(\cdot, z_{t+1})$ is continuous. Since $\tilde{V}(\cdot, z_{t+1})$ is also assumed to be left and right-hand differentiable everywhere, F has directional derivatives, given by $F'_+(e) = (1 - \eta)\beta \frac{\partial \tilde{V}^{(+)}}{\partial e}(e, z_{t+1}) + \eta \xi - 1$ and $F'_-(e) = (1 - \eta)\beta \frac{\partial \tilde{V}^{(-)}}{\partial e}(e, z_{t+1}) + \eta \xi - 1$. Assumption

(3) above implies that, $\forall e \in]0, \bar{e}(z_{t+1})[, F'_e(e) > 0$. As a continuous function with at least one strictly positive Dini differential, F is thus strictly increasing on $[0, \bar{e}(z_{t+1})]$, so that $\forall e < \bar{e}(z_{t+1}), F(e) < F(\bar{e}(z_{t+1}))$ (condition (a)). Likewise, because of assumption (3), F is weakly decreasing on $[\bar{e}(z_{t+1}), +\infty[$. As a result, $\forall e > \bar{e}(z_{t+1}), F(\bar{e}(z_{t+1})) \geq F(e)$ (condition (b)). It is straightforward to check that conditions (a) and (b) are equivalent to the optimality of the dividend policy above. \parallel

The next lemma establishes the fact that the value function of the firm (the unique fixed point of the operator T) satisfies these technical conditions. This lemma uses results from Milgrom and Segal (2002), who establish left- and right-hand side differentiability of functions defined as upper envelopes of families differentiable functions.

Lemma 2. (Optimality of the dividend policy). *The value function of the firm satisfies the conditions of Lemma 1.*

Proof of Lemma 2. First, for any z_t , $T_K V(., z_t)$ is differentiable at any $e_t \in [0, E]$. Let $z_t \in \mathcal{Z}$ and $e_t \in [0, E]$. For any (g_t, s_t) such that $\phi_K(g_t, s_t, e_t, z_t) > 0$, using the change of variable $n_t = n_K(\phi_t, g_t, s_t, e_t, z_t)$ one can rewrite:

$$O_K(g_t, s_t, e_t, z_t) = \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \left(\int_{n_t \geq 0} V^c(n_t, z_{t+1}) dG_K(n_t; g_t, s_t, e_t, z_t) \right), \quad (B.2)$$

$$G_K(n_t; g_t, s_t, e_t, z_t) \equiv F \left(\phi_K(g_t, s_t, e_t, z_t) + \frac{n_t - (\delta_K(\chi_P) - \delta)(e_t + g_t)}{(e_t + g_t)^\zeta} \right).$$

Given the continuous differentiability of ϕ_K , this expression establishes that $O_K(e_t, g_t, s_t)$ is continuously differentiable in (g_t, s_t) . This implies that $T_K V(., z_t)$ is differentiable. Similarly, one can establish that $T_R V(., z_t)$ is differentiable. Theorem 3 of Milgrom and Segal (2002) then applies to the family of functions $(T_K V(., z_t), T_R V(., z_t))$ (this family is equidifferentiable because it is finite, and each of the two functions is continuously differentiable), so that $TV(., z_t)$ has left and right-hand derivatives for all $e > 0$. Moreover, except at points e_t where $TV_R(e_t, z_t) = TV_K(e_t, z_t)$, the left and right hand side derivative of TV are both given by $\frac{\partial TV}{\partial e_t}^{(+)}(e_t, z_t) = \frac{\partial TV}{\partial e_t}^{(-)}(e_t, z_t) = T_i V'(e_t, z_t)$, where $i = R$ if $T_R V(e_t, z_t) > T_K V(e_t, z_t)$, and $i = K$ if $T_R V(e_t, z_t) < T_K V(e_t, z_t)$. Since $TV(., z_t) = V(., z_t)$, these properties also hold for $V(., z_t)$.

Fix z_t . For sufficiently large e_t , firms can operate at the unconstrained optimal scale,

$$k^*(z_{t+j}) \equiv \left(\frac{\mathbb{E}(\phi | z_{t+j}) \zeta}{\delta + r_m} \right),$$

for any $j \geq 0$. In this case, their flow profits in period $t+j$ are given by $(1+r_m)e_{t+j} + \mathbb{E}(\phi | z_{t+j})(k^*(z_t))^\zeta - (r_m + \delta)k^*(z_{t+j})$. Thus, for sufficiently large e_t , the marginal value of internal finance is $\frac{\partial T_K V}{\partial e_t}(e_t, z_t) = \frac{\partial T_R V}{\partial e_t}(e_t, z_t) = (1+r_m)$. Thus, $(1-\eta)\beta \frac{\partial TV}{\partial e_t}^{(-)}(e_t, z_t) = (1-\eta)\beta \frac{\partial TV}{\partial e_t}^{(+)}(e_t, z_t) = (1-\eta)\beta(1+r_m) < 1-\eta\zeta$ for sufficiently large e_t , provided that $(1+r_m)(1-\eta)\beta < 1-\eta\zeta$, which is assumed to hold.

For sufficiently small e_t , the lower bound $V^c(n_t, e_t) \geq n_t$, along with expression B.2, implies that:

$$O_K(e_t, g_t, s_t) \geq (e_t + g_t)^\zeta \left(\mathbb{E}(\phi | z_t) - \phi_K(g_t, s_t, e_t, z_t) + \gamma(e_t + g_t)^{1-\zeta} \right) \geq (e_t + g_t)^\zeta \left(\mathbb{E}(\phi | z_t) - \phi_K(g_t, s_t, 0, z_t) + \gamma(e_t + g_t)^{1-\zeta} \right),$$

for small (e_t, g_t) , where the second inequality follows from the fact that $\phi_K(g_t, s_t, e_t, z_t)$ is decreasing in e_t . The derivative of the right-hand side with respect to e_t is proportional to $\zeta(e_t + g_t)^{\zeta-1}$, which becomes infinitely large for $(e_t, g_t) \rightarrow 0$ when $\zeta < 1$. Thus in turn implies that $\frac{\partial T_K V}{\partial e_t}(e_t, z_t)$ becomes infinitely large for small e_t . A similar reasoning holds for $\frac{\partial T_R V}{\partial e_t}(e_t, z_t)$, and therefore, for $\frac{\partial V}{\partial e_t}^{(+)}(e_t, z_t)$. Thus, for small enough e_t , $(1-\eta)\beta \frac{\partial V}{\partial e_t}^{(+)}(e_t, z_t) > 1-\eta\zeta$. This establishes the existence of the threshold $\bar{e}(z_t)$ described in Lemma 1. \parallel

The dividend issuance policy of the firm then implies that no surviving firm will enter the period with productivity z_t and internal finance $e > \bar{e}(z_t)$. This is useful to derive the transition mapping for the density of firms, and then to establish existence and unicity of the invariant distribution, which are considered below.

The mapping M and the measure of entering firms ν . Let $\mathcal{M}(B)$ denotes measures defined on the measurable set (B, \mathcal{B}) , where $B = [0, \bar{e}(\bar{z})] \times \mathcal{Z}$, $\bar{e}(\bar{z})$ is the dividend issuance threshold for the largest level of productivity $\bar{z} = \max_{z \in \mathcal{Z}} z$, and \mathcal{B} is the σ -algebra generated by subsets of B of the form $[0, e] \times z$, $0 \leq e \leq \bar{e}(\bar{z})$ and $z \in \mathcal{Z}$. It is clear that an element of \mathcal{B} can always be written as $X_{\mathcal{E}} \times Y$, where $X_{\mathcal{E}}$ is a Borel subset of $[0, \bar{e}(\bar{z})]$ and Y is an arbitrary subset of \mathcal{Z} . For any $X_{\mathcal{E}} \times Y \in \mathcal{B}$, define the mapping $M: \mathcal{M}(B) \rightarrow \mathcal{M}(B)$ as:

$$M(\mu)(X_{\mathcal{E}} \times Y) = (1-\eta) \int_{(e_t, z_t) \in B} \left(\sum_{z_{t+1} \in Y} p(z_{t+1} | z_t) \mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z_{t+1}) \in X_{\mathcal{E}} | z_t) \right) d\mu(e_t, z_t).$$

Here, $\mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z_{t+1}) \in X_{\mathcal{E}} | z_t)$ denotes the probability that a firm with current net worth e_t and productivity z_t will survive until the following period, and have net worth $\hat{e}(e_t, z_t, \phi_t, z_{t+1})$ in the measurable set $X_{\mathcal{E}} \times Y$ at that stage. This

function depends on the optimal policies of the firm. For example, if the optimal debt structure involves no bank financing ($\hat{s}(e_t, z_t) = 0$), and if the firm's implied liquidation threshold in the current period is given by $\hat{\phi}_K(e_t, z_t) \equiv \phi_K(\hat{d}(e_t, z_t), \hat{s}(e_t, z_t), e_t, z_t)$, then next-period net worth is given by:

$$\begin{aligned} & \hat{e}(e_t, z_t, \phi_t, z_{t+1}) \\ &= \begin{cases} (\phi - \hat{\phi}_K(e_t, z_t))(e_t + \hat{d}(e_t, z_t))^\zeta + \gamma(e_t + \hat{d}(e_t, z_t)) & \text{if } \hat{\phi}_K(e_t, z_t) \leq \phi \leq \hat{\phi}_K(e_t, z_t) + (e_t + \hat{d}(e_t, z_t))^{-\zeta}(\bar{e}(z_{t+1}) - \gamma(e_t + \hat{d}(e_t, z_t))) \\ \bar{e}(z_{t+1}) & \text{if } \phi \geq \hat{\phi}_K(e_t, z_t) + (e_t + \hat{d}(e_t, z_t))^{-\zeta}(\bar{e}(z_{t+1}) - (e_t + \hat{d}(e_t, z_t))) \end{cases} \end{aligned}$$

As a result,

$$\begin{aligned} & \mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z_{t+1}) \in [0, e_{t+1} | z_t]) \\ &= \begin{cases} \left(F(\hat{\phi}_K(e_t, z_t) + (e_t + \hat{d}(e_t, z_t))^{-\zeta}(e_{t+1} - \gamma(e_t + \hat{d}(e_t, z_t))) | z_t) - F(\hat{\phi}_K(e_t, z_t) | z_t) \right) & \text{if } e_{t+1} \in [0, \bar{e}(z_{t+1})[\\ \left(1 - F(\hat{\phi}_K(e_t, z_t) | z_t) \right) & \text{if } e_{t+1} = \bar{e}(z_{t+1}) \end{cases} \end{aligned}$$

The derivation of the expression of $\mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z_{t+1}) \in [0, e_{t+1} | z_t])$ for cases when $\hat{s}(e_t, z_t) > 0$ is straightforward. Finally, the measure of entering firms ν is defined on (B, \mathcal{B}) by:

$$\nu(X_{\mathcal{E}} \times Y) = \sum_{z' \in Y} \mathbf{1}_{\{e'_e(z) \in X_{\mathcal{E}}\}} N_e p(z') G(E(z')),$$

where, $\forall z_t \in \mathcal{Z}$, the entry size $e_e(z_t) \geq 0$ is defined by $e_e(z_t) = \arg \max_{e_t} \beta V(e_t, z_t) - (1 + \gamma_e)e_t$, and $\pi(\cdot)$ is the ergodic distribution of the Markov chain governing z_t .

Existence of an invariant firm measure. I next prove that, given a solution to problem (7), an invariant measure of firms across levels of e_t exists. By invariant measure, I mean a measure $\mu \in \mathcal{M}(B)$ such that: $\mu = T(\mu) + \nu$. To do this, one can define transition kernel N such that, if a probability measure ψ is invariant under this transition kernel, then a (properly rescaled) version of ψ is invariant under M and ν .

Define the transition kernel N on $B \times \mathcal{B}$ by:

$$\begin{aligned} N((e_t, z_t), (X_{\mathcal{E}} \times Y)) &= (1 - \eta) \sum_{z' \in Y} p(z' | z_t) \mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z') \in X_{\mathcal{E}} | z_t) + \frac{1}{m_e} \nu(X_{\mathcal{E}} \times Y) \\ m_e &= \frac{\sum_{z \in \mathcal{Z}} N_e p(z) G(E(z))}{\eta} = \frac{\nu(B)}{\eta}. \end{aligned}$$

Given that \mathbf{P} is a probability measure and that $\pi(\cdot | \cdot)$ is a Markov transition probability, for any $(e_t, z_t) \in \mathcal{B}$, $N((e_t, z_t), \cdot)$ defines a probability measure on (B, \mathcal{B}) (the normalization by m_e ensures that $N((e_t, z_t), B) = 1$ for any $(e_t, z_t) \in \mathcal{B}$). Moreover, for any $(X_{\mathcal{E}} \times Y) \in \mathcal{B}$, the function $(e_t, z_t) \rightarrow N((e_t, z_t), (X_{\mathcal{E}} \times Y))$, which maps B to $[0, 1]$, is measurable with respect to \mathcal{B} . Thus, N defines a transition kernel. Suppose that one can find a probability measure ψ on (B, \mathcal{B}) such that:

$$\forall (X_{\mathcal{E}} \times Y) \in \mathcal{B}, \quad \psi(X_{\mathcal{E}} \times Y) = \int N((e_t, z_t), X_{\mathcal{E}} \times Y) d\psi(e_t, z_t). \quad (\text{B.3})$$

Then, define $\mu = m_e \psi$. Multiplying the equation above by m_e , one obtains:

$$\begin{aligned} \forall (X_{\mathcal{E}} \times Y) \in \mathcal{B}, \quad \mu(X_{\mathcal{E}} \times Y) &= m_e \psi(X_{\mathcal{E}} \times Y) \\ &= \int m_e N((e_t, z_t), X_{\mathcal{E}} \times Y) d\psi(e_t, z_t) \\ &= (1 - \eta) \int_{(e_t, z_t) \in B} \left(\sum_{z_{t+1} \in \mathcal{Y}} p(z_{t+1} | z_t) \mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z_{t+1}) \in X_{\mathcal{E}} | z_t) \right) d(m_e \psi)(e_t, z_t) + \nu(X_{\mathcal{E}} \times Y) \\ &= (1 - \eta) \int_{(e_t, z_t) \in B} \left(\sum_{z_{t+1} \in \mathcal{Y}} p(z_{t+1} | z_t) \mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z_{t+1}) \in X_{\mathcal{E}} | z_t) \right) d\mu(e_t, z_t) + \nu(X_{\mathcal{E}} \times Y) \\ &= M(\mu)(X_{\mathcal{E}} \times Y) + \nu(X_{\mathcal{E}} \times Y). \end{aligned}$$

That is, the firm distribution $\mu = m_e \psi$ is invariant in the sense define above. Thus, to find an invariant firm measure, it is sufficient to establish that N has an invariant probability measure ψ . A sufficient condition for this is that N satisfy the Feller property, as indicated by Theorem 12.10 of [Stokey et al. \(1989\)](#).

To establish that the transition kernel N has the Feller property, it is sufficient to show that $\forall (e_t, z_t) \in B$ and $\forall (e_{n,t}, z_{n,t})_n \in B^{\mathbb{N}}$ such that $(e_{n,t}, z_{n,t}) \rightarrow e_t$, $N((e_{n,t}, z_{n,t}), \cdot) \Rightarrow N((e_t, z_t), \cdot)$. Here, \Rightarrow denotes weak convergence, and \rightarrow

denotes convergence in the metric space (B, d_∞) where $d_\infty((e_{t,1}, z_{t,1}), (e_{t,2}, z_{t,2})) = \max(|e_{t,1} - e_{t,2}|, |z_{t,1} - z_{t,2}|)$. To establish this, one can use the convergence-determining class $\mathcal{A} = \{[0, e[\times z \text{ s.t. } 0 \leq e \leq \bar{e}(\bar{z}) \text{ and } z \in \mathcal{Z}\}$ (see [Stokey et al. \(1989\)](#), Theorem 12.5). That is, it is sufficient to show that $N((e_{n,t}, z_{n,t}), [0, e_{t+1}[\times z_{t+1})) \rightarrow N((e_t, z_t), [0, e_{t+1}[\times z_{t+1}))$ for each $(e_{t+1}, z_{t+1}) \in B$. The following establishes this result when (e_t, z_t) are such that $\hat{s}(e_t, z_t) = 0$; the proof for the other case ($\hat{s}(e_t, z_t) = 0$) is similar. Note that:

$$N((e_{n,t}, z_{n,t}), [0, e_{t+1}[\times z_{t+1})) = (1 - \eta)p(z_{t+1} | z_{n,t})\mathbf{P}(\hat{e}(e_{n,t}, z_{n,t}, \phi_t, z_{t+1}) \leq e_{t+1} | z_{n,t}) + \frac{1}{m_e}v([0, e_{t+1}[\times z_{t+1})),$$

Note that the second term, $\frac{1}{m_e}v([0, e_{t+1}[\times z_{t+1}))$, is independent of $(e_{n,t}, z_{n,t})$, so it is sufficient to establish convergence of $p(z_{t+1} | z_{n,t})\mathbf{P}(\hat{e}(e_{n,t}, z_{n,t}, \phi_t, z_{t+1}) \leq e_{t+1} | z_{n,t})$ to $p(z_{t+1} | z_t)\mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z') \leq e_{t+1} | z_t)$. Clearly, under the metric d_∞ , if $z_{n,t} \rightarrow z_t$, then for n sufficiently large, $z_{n,t} = z_t$. So it is sufficient to show that:

$$\lim_{n \rightarrow +\infty} \mathbf{P}(\hat{e}(e_{n,t}, z_t, \phi_t, z') \leq e_{t+1} | z_t) = \mathbf{P}(\hat{e}(e_t, z_t, \phi_t, z') \leq e_{t+1} | z_t) \text{ if } e_{n,t} \rightarrow e_t.$$

This latter result follows from the expression of \mathbf{P} reported above, along with the fact that $\hat{\phi}_K(e_t, z_t)$ and $\hat{d}(e_t, s_t)$ are both continuous in their first argument.

C. THE INTERMEDIATION WEDGE $R_B - R_M$

This Appendix describes more precise micro-foundations for the assumption that $r_b > r_m$.

C.1. Regulatory environment

The following, highly stylized model, illustrates how differences in regulatory treatment can generate an intermediation wedge. Consider a risk-neutral bank with access to an infinitely elastic supply of projects offering an average rate of return r_b . Each dollar of lending also involves a unit intermediation cost γ . The bank takes deposits D ; deposits are safe and offer the risk-free rate of return r , which is exogenously given, as in the model of the main text. Expected bank lending profits (or, equivalently, the notional value of equity) are given by:

$$E^N = (1 + r_b)A - (1 + r)D - \gamma A.$$

Assets purchases are financed using a combination of deposits and bank equity $A = E + D$. Moreover, the bank is subject to the regulatory constraint:

$$\xi A \leq E,$$

where $\xi \in [0, 1]$ is the fraction of equity capital that the bank must hold against its assets. The value of bank equity, $V(E)$, is then given by:

$$V(E) = \left((1 + r) + \frac{r_b - r - \gamma}{\xi} \right) E.$$

The second term, in particular, reflects the leveraged returns on the investment opportunities to which the bank has access. Finally, assume that banks face a cost ψ_b per dollar of equity issued. If there is free-entry in the banking sector, it must be the case that:

$$V(E) = (1 + \psi_b)E,$$

or alternatively,

$$r_b = (1 - \xi)r + \xi\psi_b + \gamma.$$

Thus, the required average rate of return r_b on the banks' project is equal to the intermediation cost γ , plus the banks' weighted average cost of capital, where the weight is the required capital ratio ξ . When $\xi = 0$, there are no capital requirements, projects can be entirely funded by deposits, and $r_b = r + \gamma$. In contrast, when $\xi = 1$, projects are all equity-funded, and so banks require a return on assets $r_b = \psi_b + \gamma$.

In contrast, market-based financial intermediaries face no such capital requirements. They also offer the risk-free rate of return to depositors, and incur the same intermediation cost γ per unit of debt intermediated. The required rate of return on their bond portfolio is then:

$$r_m = r + \gamma.$$

Therefore,

$$r_b - r_m = \xi(\psi_b - r).$$

Thus, the intermediation wedge $\gamma_b - \gamma_m = r_b - r_m$ used in the main text captures the effect of regulatory constraints on banks' required rate of return ($\xi > 0$), in combination with the fact that bank equity issuance is costly ($\psi_b > r$).

The intermediation wedge could arise even if one factors in the fact that depositors may require a higher rate of return from market-based financial intermediaries because these intermediaries are not covered by deposit insurance, whereas banks are. Let γ_d denote the premium that depositors may require because of the lack of deposit insurance among market-based intermediaries; then, the difference in required rates of return is given by:

$$r_b - r_m = \xi(\psi_b - r) - \gamma_d,$$

So long as the premium γ_d satisfies $\gamma_d \leq \xi(\psi_b - r)$, the intermediation wedge $r_b - r_m$ will remain positive.

C.2. Information-sharing costs

The intermediation spread $\gamma_b - \gamma_m > 0$ can also be thought of as capturing information disclosure costs, and more generally contractual costs the firm must incur as part of borrowing from a bank. To see why the two are isomorphic, consider a model in which, after output has been realized, the firm must pay a disclosure cost v_b per unit of bank debt initially issued. Otherwise, both bank and market intermediaries have identical opportunity cost of funds r and unit intermediation costs γ . Additionally, assume that $\chi_P = 1$. Let $\tilde{r} = r + \gamma$. The zero-profit condition of the bank is then:

$$(1 + \tilde{r})b = (1 - F(\phi_b))R_b + \int_0^{\phi_b} \min(R_b, \chi(\pi(k, \phi) - v_b b)) dF(\phi|z),$$

where b denotes bank loan issuance, R_b denotes the promised repayment to the bank, $\pi(k, \phi) = \phi k^\zeta + (1 - \delta)k$ is gross firm output, and $v_b b$ denotes the disclosure costs. Since these costs are incurred by the firm after output has been realized, but before debt settlement, the renegotiation threshold ϕ_b is now given by:

$$\chi(\phi_b k^\zeta + (1 - \delta)k - v_b b) = R_b.$$

Substituting this into the zero-profit condition of the bank, one obtains:

$$(1 + \tilde{r})b = \chi \left((1 - F(\phi_b))\phi_b + \int_0^{\phi_b} \phi dF(\phi|z) \right) k^\zeta + \chi(1 - \delta)k - \chi v_b b$$

or, equivalently,

$$(1 + \tilde{r} + \chi v_b)b = \chi \left((1 - F(\phi_b))\phi_b + \int_0^{\phi_b} \phi dF(\phi|z) \right) k^\zeta + \chi(1 - \delta)k.$$

This zero-profit condition is the same as the one that would obtain in the model with an intermediation wedge, $\gamma_b - \gamma_m$, under the assumption that:

$$\gamma_b = \gamma + \chi v_b.$$

Thus, one can think of the wedge $\gamma_b - \gamma_m$ as capturing the costs v_b incurred by bank-dependent firms in order to share information and maintain their relationship with their bank.

D. INTRODUCING SAVINGS IN THE BASELINE MODEL

D.1. Firm problem

When firms are allowed to deposit a fraction of their net worth with financial intermediaries, the dynamic problem of a firm can be written as $V(e_t, z_t) = \max(V^B(e_t, z_t), V^S(e_t, z_t))$, where V^S and V^B are given by:

$$V^S(e_t, z_t) = \max_{0 \leq k_t \leq e_t} \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1}|z_t) \left(\int_{\phi_t \geq 0} V^c(\phi_t k_t^\zeta - (r + \delta)k_t + (1 + r)e_t, z_{t+1}) dF(\phi_t|z_t) \right),$$

$$V^B(e_t, z_t) = \max_{(b_t, m_t) \in \mathcal{S}(e_t, z_t)} \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1}|z_t) \left(\int_{\phi_t \geq \phi^l(k_t, R_{b,t}, R_{m,t})} V^s(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t|z_t) \right)$$

$$\begin{aligned}
\text{s.t.} \quad k_t &= e_t + b_t + m_t && \text{(Capital structure)} \\
\pi(\phi_t, k_t) &= \phi_t k_t^\zeta + (1-\delta)k_t && \text{(Production)} \\
(1+r_b)b_t &= \sum_{z_t \in \mathcal{Z}} p(z_{t+1}|z_t) \left(\int_{\phi_t} \tilde{R}_b(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t|z_t) \right) && \text{(ZPC bank)} \\
(1+r_m)m_t &= \sum_{z_t \in \mathcal{Z}} p(z_{t+1}|z_t) \left(\int_{\phi_t} \tilde{R}_m(\phi_t, k_t, R_{b,t}, R_{m,t}, z_{t+1}) dF(\phi_t|z_t) \right) && \text{(ZPC market)} \\
V^s(\phi, k_t, R_{b,t}, R_{m,t}, z_{t+1}) &= V^c(\max(n_t^P, n_t^R), z_{t+1}) && \text{(Debt settlement)} \\
n_t^P &= \pi(\phi_t, k_t) - R_{b,t} - R_{m,t} && \text{(Repayment)} \\
n_t^R &= \pi(\phi_t, k_t) - \chi \pi(\phi_t, k_t) - R_{m,t} && \text{(Restructuring)} \\
\pi(\phi^l(k_t, R_{b,t}, R_{m,t}), k_t) &= \min\left(\frac{R_{m,t} + \gamma k_t}{1-\chi}, R_{b,t} + R_{m,t} + \gamma k_t\right) && \text{(Liquidation)} \\
V^c(n_t, z_{t+1}) &= \max_{0 \leq e_{t+1} \leq n_t} n_t - e_{t+1} + (1-\eta)\beta V(e_{t+1}, z_{t+1}) + \eta \xi e_{t+1}. && \text{(Dividend issuance)}
\end{aligned}$$

The dividend issuance policy of the firm takes the same form as before; therefore, active firms in equilibrium will be those with $0 \leq e_t \leq \bar{e}(z_t)$, where $\bar{e}(z_t)$ is the dividend issuance threshold.

D.2. Simultaneous saving and borrowing

The main text notes that assuming no simultaneous borrowing and lending is without loss of generality. To see why, consider the case where $\chi_P = 1$ and productivity z_t is constant. Let e_t denote the net worth of a firm, and assume it is borrowing only from market lenders, but simultaneously making deposits. The following shows that, for any operating scale k_t that can be achieved using some combination of borrowing m_t and deposit-making l_t , the *same* scale k_t is feasible under the alternative borrowing level $\hat{m}_t = k_t - e_t$, with no deposits ($\hat{l}_t = 0$), and at a *lower* implied default probability.

The balance sheet constraint of such a firm is $k_t = e_t + m_t - l_t$, with $l_t > 0$. The zero-profit condition of market lenders for this firm can be written as:

$$(1+r_m)m_t = (1-F(\phi_{m,t}))(\phi_{m,t}k_t^\zeta + (1-\delta)k_t + (1+r)l_t) + \chi \int_0^{\phi_{m,t}} (\phi k_t^\zeta + (1-\delta)k_t + \frac{1}{\chi}(1+r)l_t) dF(\phi),$$

where $R_{m,t} \equiv \phi_{m,t}k_t^\zeta + (1-\delta)k_t + (1+r)l_t$, and where it was assumed that deposits are not subject to deadweight losses in liquidation. Substituting $m_t = k_t - e_t + l_t$ into the zero-profit conditions of lenders, we obtain:

$$(1+r_m)\left(k_t - \left(e_t - \frac{r_m - r}{1+r_m}l_t\right)\right) = (1-F(\phi_{m,t}))(\phi_{m,t}k_t^\zeta + (1-\delta)k_t) + \chi \int_0^{\phi_{m,t}} (\phi k_t^\zeta + (1-\delta)k_t) dF(\phi).$$

Note that, *so long as* $r_m > r$, this is identical to the zero-profit condition of a firm with net worth $e_t - \frac{r_m - r}{1+r_m}l_t < e_t$, operating at the same scale k_t , but making no deposits. Because borrowing limits are increasing in net worth, if the scale k_t is feasible under $l_t > 0$ (*i.e.*, there is a solution to the zero-profit condition above), then it is also feasible under $\hat{l}_t = 0$ (for the borrowing level $\hat{m}_t = k_t - e_t$). That is, the firm could also operate at scale k_t , without making deposits. Moreover, the liquidation threshold associated with such a contract would satisfy $\hat{\phi}_{m,t} < \phi_{m,t}$, since liquidation thresholds decline with net worth. Thus, the firm could achieve the same scale k_t and lower liquidation risk, by setting $\hat{l}_t = 0$ and $\hat{m}_t = k_t - e_t$. Note that this argument relies on the existence of an intermediation wedge, $r_m > r$, so that the effective net worth of a firm making deposits is strictly smaller than that of a firm without deposits.

E. DEFINITION OF MODEL MOMENTS NOT REPORTED IN THE MAIN TEXT

A number of moments of real and financial policies of firms are reported in Section 3; they are defined as follows. The ratio of net worth to assets is defined as $\frac{e}{e_t + \hat{b}(e_t, z_t) + \hat{m}(e_t, z_t)}$. The expected rate of profits is defined as:

$$\Pi^e(e_t, z_t) = \frac{1}{e_t} \int_{\phi \geq \hat{\phi}(e_t, z_t)} \hat{n}(\phi; e_t, z_t) dF(\phi),$$

where the liquidation threshold $\hat{\phi}(e_t, z_t) \equiv \underline{\phi}(e_t, \hat{b}(e_t, z_t), \hat{m}(e_t, z_t))$ and the net worth function $\hat{n}(\phi; e_t, z_t) \equiv n(\phi; e_t, \hat{b}(e_t, z_t), \hat{m}(e_t, z_t))$ both depend on the type of debt structure (mixed or market-financed) chosen by the firm, given its internal finance level e_t . Expected dividend issuance, the expected growth rate of internal finance are analogously

defined as:

$$D^e(e_t, z_t) = \frac{1}{e_t} \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \int_{\hat{n}(\phi; e_t) > \tilde{e}(z_{t+1})} (\hat{n}(\phi; e_t, z_t) - \tilde{e}(z_{t+1})) dF(\phi | z_t)$$

$$E^e(e_t, z_t) = \frac{1}{e_t} \sum_{z_{t+1} \in \mathcal{Z}} p(z_{t+1} | z_t) \int_{\phi} (\hat{e}(\phi, e_t, z_t, z_{t+1}) - e_t) dF(\phi | z_t),$$

where $\hat{e}(\phi, e_t, z_t, z_{t+1}) \equiv \mathbb{1}_{\{\hat{n}(\phi; e_t, z_t) > \tilde{e}(z_{t+1})\}} \tilde{e}(z_{t+1}) + \mathbb{1}_{\{\hat{n}(\phi; e_t, z_t) \leq \tilde{e}(z_{t+1})\}} \hat{n}(\phi; e_t, z_t)$ denotes next period internal finance, given current average and realized productivity z_t and ϕ_t , current internal finance e_t and next-period productivity z_{t+1} .

In Section 4, the path of bank and market borrowing for small and large firms in response to the shock to the bank intermediation wedge is reported. The size groups are defined as follows. I first determine a cutoff for internal finance, $e_{S/L}$, such that, in year 0, firms with $e_0 > e_{S/L}$ account for a fraction $s_{S/L}$ of the total stock of internal finance in the economy. For $t \geq 1$, I define borrowing by small and large firms, in bank and market debt, as

$$BS_t = \int_{z_t \in \mathcal{Z}, 0 \leq e_t \leq e_{S/L}} \hat{b}_t(e_t, z_t) d\mu_t(e_t, z_t), \quad MS_t = \int_{z_t \in \mathcal{Z}, 0 \leq e_t \leq e_{S/L}} \hat{m}_t(e_t, z_t) d\mu_t(e_t, z_t);$$

$$BL_t = \int_{z_t \in \mathcal{Z}, e_{S/L} \leq e_t} \hat{b}_t(e_t, z_t) d\mu_t(e_t, z_t), \quad ML_t = \int_{z_t \in \mathcal{Z}, e_{S/L} \leq e_t} \hat{m}_t(e_t, z_t) d\mu_t(e_t, z_t),$$

where, along the perfect foresight path, $\hat{b}_t(\cdot)$, $\hat{m}_t(\cdot)$ characterize firms' policies, and μ_t is the distribution of firms. This definition is analogous to that of the data because it uses fixed cutoffs on successive cross sections to define firm groups. Qualitative results on the borrowing of these groups are similar if one uses a cutoff for assets instead of internal finance. I choose $s_{S/L} = 55.7\%$. This is the ratio of internal finance of manufacturing firms with more than \$1bn in assets, to internal finance of all firms with more than \$250m in assets in the QFR in 2007Q3. Internal finance is computed as the difference between non-financial assets and total debt, as in the model.

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

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