

Online Appendix to “Aggregate Implications of Corporate Debt Choices”

Nicolas Crouzet

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1 Data sources not mentioned in the main text

The Figure for the decline of investment during the Great Recession cited in the introduction section corresponds to the percent decline in capital expenditures of private nonfinancial corporations reported in the Flow of Funds (Table F.103, line series FA105050005.Q) between 2007Q4 and 2009Q4. The fractions of investment accounted for by firms with access to public debt markets reported in the introduction is constructed using Compustat data. I take firm-year observations from the domestic segment of the fundamentals annual North America file, and merge them with S&P credit ratings. Firm-year observations for the year 2007 are kept in sample if they satisfy the following criteria: (i) assets (at) are non-missing and greater than or equal to \$1m; (ii) sales (sale) are non-missing and greater than or equal to \$1m; (iii) liabilities (lt) are non-missing and greater than or equal to 0; (iv) debt in current liabilities (dlc) is non-missing and greater than or equal to 0; (v) debt in long-term liabilities (dltt) is non-missing and greater than or equal to 0; (vi) capital expenditures (capxv), sales of property, plant and equipment (sppe) and acquisitions (aqc) are non-missing; (vii) the 2-digit GICS code is not equal to 40 (corresponding to the financial industry). This screen leaves 4838 unique observations for fiscal year 2007. Among these observations, 945 (or 19.5%) have an S&P credit rating. Total investment is defined as capital expenditures plus acquisitions, less sales of property, plants and equipment ($capxv + aqc - sppe$). The ratio reported is equal to the sum of investment across all rated Compustat firms, divided by capital expenditures of nonfinancial corporate businesses (Table F.103, line series FA105050005.Q). The Flow of Fund value is taken for 2007Q3. Finally, the decline in investment among rated firms mentioned in section 3 is computed by constructing selecting a sample of firms according to the same criteria as above, but for the fiscal year 2010. The value $\chi = 0.38$ for the fraction of assets available in bankruptcy is from Table III, p. 1265 of Bris et al. (2006). This number adjusts for the value of collateralized assets that creditors may have seized outside of the formal bankruptcy proceedings. Bond market costs are calibrated using the results of Fang (2005) and Altinkilic and Hansen (2000). In both paper, the authors report the “underwriting spread,” that is, the ratio of proceeds to underwriters to total proceeds from the issuance, which matches the definition of γ_m in the model.

2 Characterization of the lending contracts

This section describes the characterization of the bank and market lending contracts. Throughout, the following notation is used:

$$\begin{aligned}\gamma &\equiv (1 - \delta)(1 - \chi_P) \\ \delta_R(\chi_P) &\equiv 1 - \frac{1 - \chi_P}{1 - \chi}(1 - \delta) \\ \delta_K(\chi_P) &\equiv \chi\delta + (1 - \chi)\delta_R(\chi_P) \\ G(x; z) &= (1 - F(x|z))x + \int_0^x \phi dF(\phi|z) \\ I(x; k, z) &= (1 - F(x|z))x - F(x|z)(1 - \delta_R(\chi_P))k^{1-\zeta} \\ M(x; k, z) &= \chi G(x; z) + (1 - \chi)I(x; k, z).\end{aligned}$$

For clarity, all the derivations in this appendix omit the time index t . Moreover, they are conducted using the change of variables:

$$g = k - e,$$

so that the formulas reported in the appendix to the main text can all be obtained by replacing $k = e + g$ in what follows. Finally, these derivations also treat the case in which the firm is allow to borrow at the risk-free rate.

2.1 Payment, renegotiation and liquidation sets

Let $V^c(.,.)$ be the continuation value function of the firm. Regardless of the particular form of the financial contracts, $V^c(.,.)$ is increasing in its first argument, and satisfies $V^c(n, z') \geq n \geq 0, \forall z' \in \mathcal{Z}$ and $n \geq 0$; see the proof of proposition 1 in the appendix to the main text. Let $\mathcal{P}(k, s, e, z, z')$ denote the set of realizations of the idiosyncratic productivity shocks ϕ such that the firm chooses to repay. This set is given by:

$$\mathcal{P}(k, s, e, z, z') = \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + \chi_P(1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z')) \geq 0 \\ V^c(\phi k^\zeta + (1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z')), z') \geq \\ V^c(\phi k^\zeta + (1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)), z') \\ V^c(\phi k^\zeta + (1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z')), z') \geq \\ V^c(\max(0, \chi(\phi k^\zeta + (1 - \delta)k) - (R_m(k, e, z, z') + R_b(k, e, z, z'))), z') \end{cases} \right\}$$

The first inequality indicates that repayment must be feasible. The second and third inequalities indicate that the value of repayment must exceed that of renegotiation and liquidation, respectively. Given that $V^c(n, z')$ is increasing in its first argument:

$$\begin{aligned} \mathcal{P}(k, s, e, z, z') &= \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + \chi_P(1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z')) \geq 0 \\ \phi k^\zeta + (1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z')) \geq \\ \phi k^\zeta + (1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)) \\ \phi k^\zeta + (1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z')) \geq \\ \max(0, \chi(\phi k^\zeta + (1 - \delta)k) - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z'))) \end{cases} \right\} \\ &= \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + (1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z') + \gamma k_t) \geq 0 \\ \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)) \geq R_b(k, s, e, z, z') \end{cases} \right\} \\ &= \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + (1 - \delta)k \geq (R_m(k, s, e, z, z') + R_b(k, s, e, z, z') + \gamma k_t) \\ \phi k^\zeta + (1 - \delta)k \geq \frac{R_b(k, s, e, z, z')}{\chi} \end{cases} \right\} \end{aligned}$$

Here, going from the first to the second line uses the fact that, when repayment is feasible, it necessarily dominates liquidation (since equity is the residual claimant in liquidation, and liquidation involves deadweight losses), so that the third inequality is redundant.

Similarly, let $\mathcal{R}(k, s, e, z, z')$ denote the set of realizations of the idiosyncratic productivity shocks ϕ such that the firm chooses to repay. This set is given by:

$$\mathcal{R}(k, s, e, z, z') = \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + \chi_P(1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)) \geq 0 \\ V^c(\phi k^\zeta + (1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)), z') \geq \\ V^c(\phi k^\zeta + (1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z')), z') \\ V^c(\phi k^\zeta + (1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)), z') \geq \\ V^c(\max(0, \chi(\phi k^\zeta + (1 - \delta)k) - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z'))), z') \end{cases} \right\}$$

Using the same properties of $V^c(.,.)$ as above, the renegotiation set can be rewritten as:

$$\begin{aligned}
\mathcal{R}(k, s, e, z, z') &= \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + \chi_P(1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)) \geq 0 \\ \phi k^\zeta + (1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)) \geq \\ \phi k^\zeta + (1 - \delta)k - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z')) \\ \phi k^\zeta + (1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)) \geq \\ \max(0, \chi(\phi k^\zeta + (1 - \delta)k) - (R_m(k, s, e, z, z') + R_b(k, s, e, z, z'))) \end{cases} \right\} \\
&= \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + \chi_P(1 - \delta)k - R_m(k, s, e, z, z') - \min(R_b(k, s, e, z, z'), \chi(\phi k^\zeta + (1 - \delta)k)) \geq 0 \\ \frac{R_b(k, s, e, z, z')}{\chi} \geq \phi k^\zeta + (1 - \delta)k \end{cases} \right\} \\
&= \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + \chi_P(1 - \delta)k - R_m(k, s, e, z, z') - \chi(\phi k^\zeta + (1 - \delta)k) \geq 0 \\ \frac{R_b(k, s, e, z, z')}{\chi} \geq \phi k^\zeta + (1 - \delta)k \end{cases} \right\} \\
&= \left\{ \phi \in \Phi(z) \text{ s.t. } \begin{cases} \phi k^\zeta + (1 - \delta)k \geq \frac{R_m(k, s, e, z, z') + \gamma k_t}{1 - \chi} \\ \phi k^\zeta + (1 - \delta)k \leq \frac{R_b(k, s, e, z, z')}{\chi} \end{cases} \right\}
\end{aligned}$$

Finally, the liquidation set is given by:

$$\mathcal{L}(k, s, e, z) = \Phi(z) \setminus (\mathcal{P}(k, s, e, z) \cup \mathcal{R}(k, s, e, z))$$

2.2 Debt pricing

Given the payment, renegotiation and liquidation sets, debt prices are given by:

$$\begin{aligned}
R_m(k, s, e, z, z') &= \begin{cases} (1 + r)(1 - s)(k - e) & \text{if } 0 \leq k \leq e \\ \min \{ R_m \geq 0 \text{ s.t. } (1 + r_m)(1 - s)(k - e) = \\ \sum_{z'} \pi(z|z') \left(\int_{\mathcal{D}(k, s, e, z, z')} \min(R_m, \max(0, \chi(\phi k^\zeta + (1 - \delta)k) - R_b(k, s, e, z, z'))) dF(\phi|z) \right. \\ \left. + \left(\int_{\mathcal{P}(k, s, e, z, z') \cup \mathcal{R}(k, s, e, z, z')} dF(\phi|z) \right) R_m \right) \} & \text{if } k > e \end{cases} \\
R_b(k, s, e, z, z') &= \begin{cases} (1 + r)s(k - e) & \text{if } 0 \leq k \leq e \\ \min \{ R_b \geq 0 \text{ s.t. } (1 + r_b)s(k - e) = \\ \sum_{z'} \pi(z|z') \left(\int_{\mathcal{D}(k, s, e, z, z') \cup \mathcal{R}(k, s, e, z, z')} \min(R_b, \chi(\phi k^\zeta + (1 - \delta)k)) dF(\phi|z) \right. \\ \left. + \left(\int_{\mathcal{P}(k, s, e, z, z')} dF(\phi|z) \right) R_b \right) \} & \text{if } k > e \end{cases}
\end{aligned}$$

This formulation of the debt contracts is more general than in the main text, in that it also includes cases in which the firm is borrowing. Since the action regions are independent of z' except through debt prices, and conversely, debt prices are independent of z' except through the action regions, it is easy to see that neither

depends on z' . For example,

$$\begin{aligned}
R_b(k, s, e, z, z') &= \begin{cases} (1+r)s(k-e) & \text{if } 0 \leq k \leq e \\ \min \{ R_b \geq 0 \text{ s.t. } (1+r_b)s(k-e) = \\ \sum_{z'} \pi(z'|z') \left(\int_{\mathcal{D}(k,s,e,z,z') \cup \mathcal{R}(k,s,e,z,z')} \min(R_b, \chi(\phi k^\zeta + (1-\delta)k)) dF(\phi|z) \right. \\ \left. + \left(\int_{\mathcal{P}(k,s,e,z,z')} dF(\phi|z) \right) R_b \right) \} & \text{if } k > e \end{cases} \\
&= \begin{cases} (1+r)s(k-e) & \text{if } 0 \leq k \leq e \\ \min \{ R_b \geq 0 \text{ s.t. } (1+r_b)s(k-e) = \\ \underbrace{\left(\sum_{z'} \pi(z'|z) \right)}_{=1} \left(\int_{\mathcal{D}(k,s,e,z) \cup \mathcal{R}(k,s,e,z)} \min(R_b, \chi(\phi k^\zeta + (1-\delta)k)) dF(\phi|z) + \left(\int_{\mathcal{P}(k,s,e,z)} dF(\phi|z) \right) R_b \right) \} & \text{if } k > e \end{cases} \\
&= \begin{cases} (1+r)s(k-e) & \text{if } 0 \leq k \leq e \\ \min \{ R_b \geq 0 \text{ s.t. } (1+r_b)s(k-e) = \\ \int_{\mathcal{D}(k,s,e,z) \cup \mathcal{R}(k,s,e,z)} \min(R_b, \chi(\phi k^\zeta + (1-\delta)k)) dF(\phi|z) + \left(\int_{\mathcal{P}(k,s,e,z)} dF(\phi|z) \right) R_b \} & \text{if } k > e \end{cases}
\end{aligned}$$

Thus:

$$\begin{aligned}
\mathcal{P}(k, s, e, z, z') &= \mathcal{P}(k, s, e, z) \\
\mathcal{R}(k, s, e, z, z') &= \mathcal{R}(k, s, e, z) \\
\mathcal{L}(k, s, e, z, z') &= \mathcal{L}(k, s, e, z) \\
R_m(k, s, e, z, z') &= R_m(k, s, e, z) \\
R_b(k, s, e, z, z') &= R_b(k, s, e, z)
\end{aligned}$$

2.2.1 Bank lending contract

The bank contract can be solved independently of the market lending contract (it only depends on it via s and k). Define:

$$\tilde{r}_b(s) = \frac{(1+r_b)s}{\chi} - 1$$

Rewrite the contract using the expression of the lending sets:

$$R_b(k, s, e, z) = \begin{cases} (1+r)s(k-e) & \text{if } 0 \leq k \leq e \\ \min \{ R_b \geq 0 \text{ s.t. } (1+r_b)s(k-e) = \\ R_b \int_{\phi k^\zeta + (1-\delta)k \geq \frac{R_b}{\chi}} dF(\phi|z) + \chi \int_{\phi k^\zeta + (1-\delta)k < \frac{R_b}{\chi}} (\phi k^\zeta + (1-\delta)k) dF(\phi|z) \} & \text{if } k > e \end{cases}$$

For $k > e$, define the threshold $\bar{\phi}_R(k, s, e, z)$ by:

$$\bar{\phi}_R(k, s, e, z) = \begin{cases} (\tilde{r}_b(s) + \delta)k^{-\zeta} \left(k - \frac{1+\tilde{r}_b(s)}{\delta+\tilde{r}_b(s)} e \right) & \text{if } \tilde{r}_b(s) + \delta \leq 0 \text{ and } e < k \\ (\tilde{r}_b(s) + \delta)k^{-\zeta} \left(k - \frac{1+\tilde{r}_b(s)}{\delta+\tilde{r}_b(s)} e \right) & \text{if } \tilde{r}_b(s) + \delta > 0 \text{ and } e < k \leq \frac{1+\tilde{r}_b(s)}{\delta+\tilde{r}_b(s)} e \\ \min \left\{ \phi^d \geq 0 \text{ s.t. } (\tilde{r}_b(s) + \delta)k^{-\zeta} \left(k - \frac{1+\tilde{r}_b(s)}{\delta+\tilde{r}_b(s)} e \right) = G(\phi^d; z) \right\} & \text{if } \tilde{r}_b(s) + \delta > 0 \text{ and } k > \frac{1+\tilde{r}_b(s)}{\delta+\tilde{r}_b(s)} e \end{cases}$$

This the same characterization of the renegotiation threshold as given in the appendix to the main text, except that it is here expressed in (k, s) space, instead of (g, s) space. The terms of the bank lending contract can

then more compactly be expressed as:

$$R_b(k, s, e, z) = \begin{cases} (1+r)s(k-e) & \text{if } 0 \leq k \leq e \\ \chi(\bar{\phi}_R(k, s, e, z)k^\zeta + (1-\delta)k) & \text{if } k > e \end{cases}$$

Given (s, e, z) , define the borrowing limit implied by the bank contract, $k^b(s, e, z)$, as the maximum level of k for which $\bar{\phi}_R(k, s, e, z)$ is well-defined. When $\tilde{r}_b(s) + \delta \leq 0$, the borrowing limit is $+\infty$. When $\tilde{r}_b(s) + \delta > 0$, it's the unique solution to:

$$(\tilde{r}_b(s) + \delta)k^{-\zeta} \left(k - \frac{1 + \tilde{r}_b(s)}{\delta + \tilde{r}_b(s)} e \right) = \mathbb{E}(\phi|z).$$

The borrowing limit solves an equation of the form $Ak^\zeta = k - B$, $A > 0$, $B \geq 0$, where:

$$A(s, e, z) = \frac{\mathbb{E}(\phi|z)}{\delta + \tilde{r}_b(s)}, \quad B(s, e, z) = \frac{1 + \tilde{r}_b(s)}{\delta + \tilde{r}_b(s)} e.$$

2.2.2 Market contract with renegotiation

Define:

$$\tilde{r}_m(s) = \frac{(1+r_m)(1-s)}{1-\chi} - 1$$

Note:

$$1 + \tilde{r}(s) = (1-\chi)(1 + \tilde{r}_m(s)) + \chi(1 + \tilde{r}_b(s))$$

Renegotiation can only occur in equilibrium when:

$$\frac{R_m(k, s, e, z) + \gamma k}{1-\chi} \leq \frac{R_b(k, s, e, z)}{\chi}.$$

Moreover, the renegotiation is unsuccessful whenever:

$$\phi k^\zeta + (1-\delta)k \leq \frac{R_m(k, s, e, z) + \gamma k}{1-\chi}.$$

Define:

$$\delta_R(\chi_P) = 1 - \frac{1-\chi_P}{1-\chi}(1-\delta).$$

Since $\gamma = (1-\chi_P)(1-\delta) = (1-\chi)(\delta_R(\chi_P) - \delta)$, the conditions above can be written as:

$$\frac{R_m(k, s, e, z)}{1-\chi} + (\delta_R(\chi_P) - \delta)k \leq \frac{R_b(k, s, e, z)}{\chi}$$

$$\phi k^\zeta + (1-\delta_R(\chi_P))k \leq \frac{R_m(k, s, e, z)}{1-\chi}$$

If renegotiation occurs in equilibrium, then market lenders obtain either R_m or 0. The terms of the market contract solve:

$$R_m(k, s, e, z) = \begin{cases} (1+r)(1-s)(k-e) & \text{if } 0 \leq k < e \\ \min \left\{ R_m \geq 0 \text{ s.t. } (1+r_m)(1-s)(k-e) = R_m \int_{\phi k^\zeta + (1-\delta_R(\chi_P))k \geq \frac{R_m}{1-\chi}} dF(\phi|z) \right\} & \text{if } k \geq e \end{cases}$$

For $k > e$, define the threshold $\phi_R(k, s, e, z)$ by:

$$\phi_R(k, s, e, z) = \begin{cases} (\tilde{r}_m(s) + \delta_R(\chi_P))k^{-\zeta} \left(k - \frac{1 + \tilde{r}_m(s)}{\delta_R(\chi_P) + \tilde{r}_m(s)} e \right) & \text{if } \tilde{r}_m(s) + \delta_R(\chi_P) \leq 0 \text{ and } e < k \\ (\tilde{r}_m(s) + \delta_R(\chi_P))k^{-\zeta} \left(k - \frac{1 + \tilde{r}_m(s)}{\delta_R(\chi_P) + \tilde{r}_m(s)} e \right) & \text{if } \tilde{r}_m(s) + \delta_R(\chi_P) > 0 \text{ and } e < k \leq \frac{1 + \tilde{r}_m(s)}{\delta_R(\chi_P) + \tilde{r}_m(s)} e \\ \min \left\{ \phi^d \geq 0 \text{ s.t. } (\tilde{r}_m(s) + \delta_R(\chi_P))k^{-\zeta} \left(k - \frac{1 + \tilde{r}_m(s)}{\delta_R(\chi_P) + \tilde{r}_m(s)} e \right) = I(\phi^d; k, z) \right\} & \text{if } \tilde{r}_m(s) + \delta_R(\chi_P) > 0 \text{ and } k > \frac{1 + \tilde{r}_m(s)}{\delta_R(\chi_P) + \tilde{r}_m(s)} e \end{cases}$$

This the same characterization of the liquidation threshold after renegotiation as given in the appendix to the main text, except that it is here expressed in (k, s) space, instead of (g, s) space. The terms of the market lending contract can then more compactly be expressed as:

$$R_b(k, s, e, z) = \begin{cases} (1+r)s(k-e) & \text{if } 0 \leq k \leq e \\ (1-\chi)(\underline{\phi}_R(k, s, e, z)k^\zeta + (1-\delta_R(\chi_P))k) & \text{if } k > e \end{cases}$$

Given (k, s, e, z) given, the maximum default threshold admissible under the market contract is the unique solution to:

$$f(\underline{\phi}_R(k, z)|z) \left(\hat{\phi}_m^d(k, z) + (1-\delta_R(\chi_P))k^{1-\zeta} \right) - (1-F(\underline{\phi}_R(k, z)|z)) = 0.$$

When $F(\cdot|z)$ is Weibull with location and scale parameters $a(z)$ and $b(z)$, the maximum default threshold is the unique solution to:

$$\left(\underline{\phi}_R(k, z) + (1-\delta_R(\chi_P))k^{1-\zeta} \right) = \frac{a(z)}{b(z)} \left(\frac{\hat{\phi}_R(k, z)}{b(z)} \right)^{1-b(z)}.$$

It satisfies:

$$0 \leq \underline{\phi}_R(k, z) \leq a(z)^{-\frac{(1-b(z))}{b(z)}} \left(\frac{b(z)}{a(z)} \right)^{-\frac{1}{b(z)}}.$$

For (s, e, z) given, define the borrowing limit implied by the market contract, $k^m(s, e, z)$, as the maximum level of k for which $\underline{\phi}_R(k, s, e, z)$ is well-defined. if $\tilde{r}_m(s) + \delta_R(\chi_P) > 0$, the borrowing limit $k = k^m(s, e, z)$ is the unique solution to:

$$(\tilde{r}_m(s) + \delta_R(\chi_P))k^{-\zeta} \left(k - \frac{1 + \tilde{r}_m(s)}{\delta_R(\chi_P) + \tilde{r}_m(s)} e \right) = I(\hat{\phi}_R(k, z); k, z).$$

When $\zeta > \frac{1}{2}$, the borrowing constraint can be bounded by $k^m(s, e, z) \in [0, \bar{k}^m(s, e, z)]$. The upper bound solves an equation of the form $\bar{A}(s, e, z)k^\zeta = k - \bar{B}(s, e, z)$, $A > 0$, $B \geq 0$, where:

$$\begin{aligned} \bar{A}(s, e, z) &= \frac{\hat{\phi}(z)(1-F(\hat{\phi}(z)))}{\underline{\phi}_R(k, z) + \tilde{r}_m(s)}, & \bar{B}(s, e, z) &= \frac{1 + \tilde{r}_m(s)}{\underline{\phi}_R(k, z) + \tilde{r}_m(s)} e \\ \hat{\phi}(z) &= a(z)^{-\frac{(1-b(z))}{b(z)}} \left(\frac{b(z)}{a(z)} \right)^{-\frac{1}{b(z)}} \end{aligned}$$

2.2.3 Debt structures with renegotiation

Using the expression for $\mathcal{R}(k, s, e, z)$, there is renegotiation ex-post, if and only if:

$$\begin{aligned} \frac{R_m(k, s, e, z)}{1-\chi} + (\delta_R(\chi_P) - \delta)k &\leq \frac{R_b(k, s, e, z)}{\chi} \\ \exists \phi \geq 0 \text{ s.t. } \frac{R_b(k, s, e, z)}{\chi} &\geq \phi k^\zeta + (1-\delta)k \end{aligned}$$

Given the expression of $R_b(k, s, e, z)$, this is equivalent to:

$$\begin{aligned} \frac{R_m(k, s, e, z)}{1-\chi} + (\delta_R(\chi_P) - \delta)k &\leq \frac{R_b(k, s, e, z)}{\chi} \\ k &\geq \frac{1 + \tilde{r}_b(s)}{\delta + \tilde{r}_b(s)} e \end{aligned}$$

It's then easy to see that $\mathcal{R}(k, s, e, z) \neq \emptyset$, if and only, (k, s, e, z) satisfy:

$$\begin{aligned}
(\tilde{r}_b(s) + \delta)k^{-\zeta} \left(k - \frac{1 + \tilde{r}_b(s)}{\delta + \tilde{r}_b(s)} e \right) &\leq \mathbb{E}(\phi|z) \\
(\tilde{r}_m(s) + \delta_R(\chi_P))k^{-\zeta} \left(k - \frac{1 + \tilde{r}_m(s)}{\delta_R(\chi_P) + \tilde{r}_m(s)} e \right) &\leq I(\hat{\phi}_R(k, z); k, z) \\
\underline{\phi}_R(k, s, e, z) &\leq \bar{\phi}_R(k, s, e, z) \\
\bar{\phi}_R(k, s, e, z) &\geq 0
\end{aligned} \tag{1}$$

Note in particular that, when a debt structure allows for renegotiation, the payment, renegotiation and liquidation sets are given by:

$$\begin{aligned}
\mathcal{P}(k, s, e, z) &= \{ \phi \in \Phi(z) \quad \text{s.t.} \quad \bar{\phi}_R(k, s, e, z) \leq \phi \} \\
\mathcal{R}(k, s, e, z) &= \left\{ \phi \in \Phi(z) \quad \text{s.t.} \quad \underline{\phi}_R(k, s, e, z) \leq \phi < \bar{\phi}_R(k, s, e, z) \right\} \\
\mathcal{D}(k, s, e, z) &= \left\{ \phi \in \Phi(z) \quad \text{s.t.} \quad \phi < \underline{\phi}_R(k, s, e, z) \right\}
\end{aligned}$$

Let:

$$\begin{aligned}
s^{r,1} &= \frac{1}{1 + \frac{1 - \chi}{1 + r_m} \frac{1 + r_b}{\chi}} \\
s^{r,2} &= \left(1 + \frac{1 - \chi}{1 + r_m} (\delta_R(\chi_P) - \delta) \right) s^{r,1} > s^{r,1} \\
s^{r,3} &= \frac{1}{1 - \frac{\delta(\chi_P) - \delta}{1 - \delta} (1 - s^{r,1})} s^{r,1} > s^{r,2}
\end{aligned}$$

A necessary condition for the non-emptiness of the renegotiation set, which follows from the penultimate inequality in (1) and the fact that $G \geq I$, is:

$$s \in [s^{r,1}, 1] \quad \text{and} \quad k \geq \underline{k}_1(s, e) \equiv \frac{s - s^{r,1}}{s - s^{r,2}} e$$

This constraint is loosed than the last inequality in (1) whenever $s \geq s^{r,3}$. The set of debt structures with renegotiation can then be described parametrically as:

$$\mathcal{S}^R(e, z) = \{ (k, s) \in \mathbb{R}_+ \times]s^r(e, z), 1] \quad \text{s.t.} \quad \underline{k}(s, e) \leq k \leq \bar{k}(s, e, z) \}$$

$$\begin{aligned}
\underline{k}(s, e, z) &= \begin{cases} \frac{s - s^{r,1}}{s - s^{r,2}} e & \text{if } s^r(e, z) \leq s \leq s^{r,2} \\ \frac{s}{s - \frac{1 - \delta}{1 + r_b} \chi} e & \text{if } s^{r,2} \leq s \leq 1 \end{cases} \\
\bar{k}(s, e, z) &= \begin{cases} k^f(s, e, z) & \text{if } s^r(e, z) \leq s \leq s^{fm}(e, z) \\ k^m(s, e, z) & \text{if } s^{fm}(e, z) \leq s \leq s^{mb}(e, z) \\ k^b(s, e, z) & \text{if } s^{mb}(e, z) \leq s \leq 1 \end{cases}
\end{aligned}$$

$$k^f(s^r(e, z), e, z) = \underline{k}(s^r(e, z), e, z)$$

$$k^f(s^{fm}(e, z), e, z) = k^m(s^{fm}(e, z), e, z)$$

$$k^m(s^{mb}(e, z), e, z) = k^b(s^{mb}(e, z), e, z)$$

The function $k^f(s, e, z)$ characterizes the capital level at which the renegotiation and liquidation thresholds coincide: $\underline{\phi}_R(k, s, e, z) = \bar{\phi}_R(k, s, e, z)$. Specifically, $k^f(s, e, z)$ is the unique solution to:

$$G \left(\underline{\phi}_R(k, s, e, z); z \right) k^\zeta - (\tilde{r}_b(s) + \delta)k + (1 + \tilde{r}_b(s))e = 0.$$

The threshold $s^r(e, z)$ is the bank/market debt ratio for which the lower bound on k and the frontier $k^f(s, e, z)$ coincide. When $\chi_P = 1$ and $\delta(\chi_P) = \delta$, we have $s^{r,1} = s^{r,2} = s^{r,3}$, and $s^r(e, z) = s^{r,1}$, regardless of (e, z) . The threshold $s^{fm}(e, z)$ is the bank/market debt ratio for which the market borrowing limit coincides with the frontier $k^f(s, e, z)$, or, using the notation above, $k^f(s^{fm}(e, z), e, z) = k^m(s^{fm}(e, z), e, z)$. This is equivalent to:

$$\begin{cases} k^{fm}(e, z)^{-\zeta} ((\hat{r} + \delta + (1 - s^{r,1})(\delta(\chi_P) - \delta)) k^{fm}(e, z) - (1 + \hat{r})e) &= (1 - s^{r,1})I(\hat{\phi}_R(k^{fm}(e, z), z); k^{fm}(e, z), z) + s^{r,1}G(\hat{\phi}_R(k^{fm}(e, z), z); z)) \\ s^{fm}(e, z) &= \frac{\chi}{1+r_b} \frac{1}{k^{fm}(e, z) - e} \left(G(\hat{\phi}_R(k^{fm}(e, z), z); z) k^f(e, z)^\zeta + (1 - \delta) k^{fm}(e, z) \right) \end{cases}$$

The threshold $s^{mb}(e, z)$ is the bank/market debt ratio for which the bank and market borrowing limit are equal. The relationship $k^m(s^{mb}(e, z), e, z) = k^b(s^{mb}(e, z), e, z)$ is then equivalent to:

$$\begin{cases} k^{mb}(e, z)^{-\zeta} ((\hat{r} + \delta + (1 - s^{r,1})(\delta(\chi_P) - \delta)) k^{mb}(e, z) - (1 + \hat{r})e) &= (1 - s^{r,1})I(\hat{\phi}_R(k^{mb}(e, z), z); k^{mb}(e, z), z) + s^{r,1}\mathbb{E}(\phi|z) \\ s^{mb}(e, z) &= \frac{\chi}{1+r_b} \frac{1}{k^{mb}(e, z) - e} \left(\mathbb{E}(\phi|z) k^{mb}(e, z)^\zeta + (1 - \delta) k^{mb}(e, z) \right) \end{cases}$$

2.2.4 Debt structures without renegotiation

Ex-ante, renegotiation is not possible whenever:

$$\begin{aligned} & \frac{R_m(k, s, e, z)}{1-\chi} + (\delta_R(\chi_P) - \delta)k > \frac{R_b(k, s, e, z)}{\chi} \\ \text{or} \\ \forall \phi \geq 0, & \frac{R_b(k, s, e, z)}{\chi} < \phi k^\zeta + (1 - \delta)k \end{aligned}$$

Given the expression of $R_b(k, s, e, z)$, this is equivalent to:

$$\begin{aligned} & \frac{R_m(k, s, e, z) + \gamma k}{1-\chi} > \frac{R_b(k, s, e, z)}{\chi} \\ \text{or} \\ k & < \frac{1 + \tilde{r}_b(s)}{\delta + \tilde{r}_b(s)} e \end{aligned}$$

It's straightforward to see that in this case:

$$\begin{aligned} \mathcal{P}(k, s, e, z) &= \{ \phi \in \Phi(z) \quad \text{s.t.} \quad \phi k^\zeta + (1 - \delta_K(\chi_P))k \geq R_m(k, s, e, z) + R_b(k, s, e, z) \} \\ \mathcal{R}(k, s, e, z) &= \emptyset \\ \mathcal{D}(k, s, e, z) &= \{ \phi \in \Phi(z) \quad \text{s.t.} \quad \phi k^\zeta + (1 - \delta_K(\chi_P))k < R_m(k, s, e, z) + R_b(k, s, e, z) \} \end{aligned}$$

where:

$$\delta_K(\chi_P) = 1 - \chi_P(1 - \delta) = (1 - \chi)\delta_R(\chi_P) + \chi\delta$$

In that case, the two forms of debt are equivalent, save for r_b and r_m , and the firm effectively faces a joint borrowing constraint. It only cares about $R(k, s, e, z) = R_m(k, s, e, z) + R_b(k, s, e, z)$ (total repayments) and $\phi_K(k, s, e, z)$, the natural default threshold. Define:

$$1 + \tilde{r}(s) = 1 + r_m + (r_b - r_m)s.$$

The derivation of the lending contract is the same as it would be in a one-instrument model. Specifically, total repayments are given by:

$$R(k, s, e, z) = \begin{cases} (1 + \tilde{r})(k - e) & \text{if } 0 \leq k \leq e \\ \phi_K(k, s, e, z)k^\zeta + (1 - \delta_K(\chi_P))k & \text{if } k > e \end{cases}$$

where the default threshold when $k > e$ is given by:

$$\underline{\phi}_K(k, s, e, z) = \begin{cases} (\tilde{r}(s) + \delta_K(\chi_P))k^{-\zeta} \left(k - \frac{1+\tilde{r}(s)}{\delta_K(\chi_P) + \tilde{r}(s)} e \right) & \text{if } e < k \leq \frac{1+\tilde{r}(s)}{\delta_K(\chi_P) + \tilde{r}(s)} e \\ \min \left\{ \phi^d \geq 0 \text{ s.t. } (\tilde{r}(s) + \delta_K(\chi_P))k^{-\zeta} \left(k - \frac{1+\tilde{r}(s)}{\delta_K(\chi_P) + \tilde{r}(s)} e \right) = M(\phi^d; k, z) \right\} & \text{if } k > \frac{1+\tilde{r}(s)}{\delta_K(\chi_P) + \tilde{r}(s)} e \end{cases}$$

The action sets are given by:

$$\begin{aligned} \mathcal{P}(k, s, e, z) &= \left\{ \phi \in \Phi(z) \quad \text{s.t.} \quad \phi \geq \underline{\phi}_K(k, s, e, z) \right\} \\ \mathcal{R}(k, s, e, z) &= \emptyset \\ \mathcal{D}(k, s, e, z) &= \left\{ \phi \in \Phi(z) \quad \text{s.t.} \quad \phi < \underline{\phi}_K(k, s, e, z) \right\} \end{aligned}$$

For (k, z) given, the maximum default threshold is defined as the unique solution to:

$$f(\underline{\phi}_K(k, z)|z)(1 - \chi) \left(\hat{\underline{\phi}}_K(k, z) + (1 - \delta_R(\chi_P))k^{1-\zeta} \right) - (1 - F(\hat{\underline{\phi}}_K(k, z)|z)) = 0.$$

When $F(\cdot|z)$ is Weibull, the maximum default threshold is the unique solution to:

$$(1 - \chi) \left(\hat{\underline{\phi}}_K(k, z) + (1 - \delta_R(\chi_P))k^{1-\zeta} \right) = \frac{a(z)}{b(z)} \left(\frac{\hat{\underline{\phi}}_K(k, z)}{b(z)} \right)^{1-b(z)}.$$

It satisfies:

$$0 \leq \hat{\underline{\phi}}_K(k, z) \leq a(z)^{-\frac{(1-b(z))}{b(z)}} \left((1 - \chi) \frac{b(z)}{a(z)} \right)^{-\frac{1}{b(z)}}.$$

For (s, e, z) given, the borrowing limit $k = k^j(s, e, z)$ is the unique solution to:

$$(\tilde{r}(s) + \delta_K(\chi_P))k^{-\zeta} \left(k - \frac{1 + \tilde{r}(s)}{\delta_K(\chi_P) + \tilde{r}(s)} e \right) = M(\hat{\underline{\phi}}_K(k, z); k, z).$$

When $\zeta > \frac{1}{2}$, the borrowing limit can be bounded by $k^j(s, e, z) \in \left[\underline{k}^j(s, e, z), \bar{k}^j(s, e, z) \right]$. Both bounds solve equations of the form $Ak^\zeta = k - B$, $A > 0$, $B \geq 0$, where:

$$\begin{aligned} \underline{A}(e, z) &= \frac{\chi \mathbb{E}(\phi|z)}{\delta_K(\chi_P) + \tilde{r}(s) + (1 - \chi)(1 - \delta_R(\chi_P))}, & \underline{B}(e, z) &= \frac{1 + \tilde{r}(s)}{\delta_K(\chi_P) + \tilde{r}(s)(1 - \chi)(1 - \delta_R(\chi_P))} e. \\ \bar{A}(e, z) &= \frac{\mathbb{E}(\phi|z)}{\delta_K(\chi_P) + \tilde{r}(s)}, & \bar{B}(e, z) &= \frac{1 + \tilde{r}(s)}{\delta_K(\chi_P) + \tilde{r}(s)} e. \end{aligned}$$

3 Transmission of aggregate shocks

This section reports further results on the effects of aggregate shocks to the supply of credit on investment and output in the model.

3.1 Response to an aggregate increase in deadweight liquidation losses

Figure 1 reports the perfect foresight response of the model to a 10% fall in the parameter χ , which captures deadweight losses in liquidation. Results are reported in a version of the model where $\chi_P = 1$, calibrated to match the 2007Q3 bank share documented from the QFR. Initial deadweight losses ($\chi = 0.38$) are identical to the baseline calibration of the model reported in the main text.

The top left panel reports the path of χ_t . the top right panel reports response of the aggregate bank share. The bottom left panel reports the path of aggregate investment, in percentage terms relative to the initial

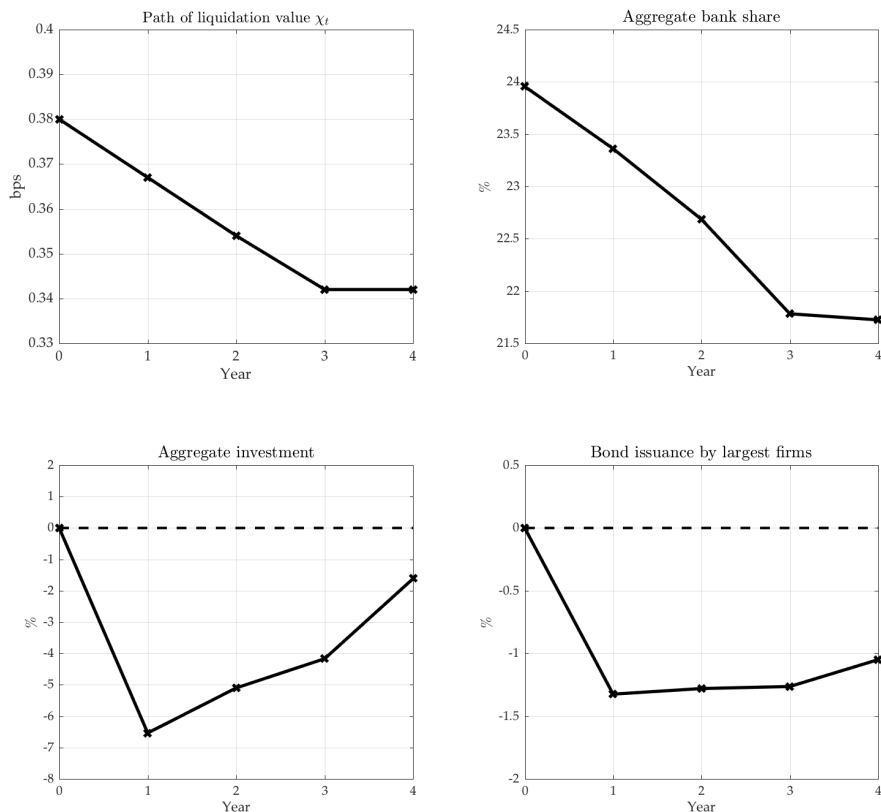


Figure 1: The aggregate response to an increase in deadweight liquidation losses.

steady-state. The bottom right panel reports the path of bond issuance by large firms, expressed in percentage terms relative to steady-state. Large firms are defined as those having assets above a cutoff computed to match the fraction of total assets held by firms with more than 250m\$ in assets in the QFR in 2007Q3.

3.2 Stochastic shocks to the intermediation wedge

Figure 2 reports the results from an alternative approach to modelling the shock to the intermediation wedge $\gamma_{b,t} - \gamma_{m,t}$. In the main text, the analysis centers around the perfect foresight response of firms in the model to the shock. In this section, I instead assume that $\gamma_{b,t}$ follows a discrete-state Markov chain.

The Markov chain used to construct the results of figure 2 has three states. In the lowest state, the intermediation cost $\gamma_{b,t}$ is such that the required return on bank loans is the same as in the baseline calibration of the paper: $r_{b,t} = \underline{r}_b = r_{b,\text{baseline}}$.¹ In the highest state, total bank lending costs are 5% larger: $r_{b,t} = \bar{r}_b = 1.05 \times \underline{r}_b$. In the intermediate state, they are 2.5% higher. The low state is the most persistent of the three; in the ergodic distribution of the Markov chain, $r_{b,t} = \underline{r}_b$ with probability 0.67, and the probability to stay in the aggregate state $r_{b,t} = \underline{r}_b$ is 0.80.

Assuming that $\gamma_{b,t}$ follows a stochastic process first affects firms' policy functions. The top left panel of figure 2 reports conditional policy functions for total assets. The policy functions are plotted for firms with the highest productivity, $z_t = z_H$; the black line shows policies conditional on the aggregate bank lending cost being low ($r_{b,t} = \underline{r}_b$), while the grey line shows them conditional on the aggregate bank borrowing cost being high ($r_{b,t} = \bar{r}_b$). In both states, the policy functions display the same key properties as in the baseline model: in particular, they are sharply discontinuous as firms switch from a mixed-debt to a market-debt structure. The switching threshold is lower when aggregate bank borrowing costs are high. Thus, the predictions of

¹The baseline model used here is calibrated to match the bank share documented in the QFR in 2007Q3, under the assumption that $\chi_P = 1$. Thus, leverage in this version of the model is higher than in the baseline calibration of the main text; as a result, the effects of the shock on investment are larger.

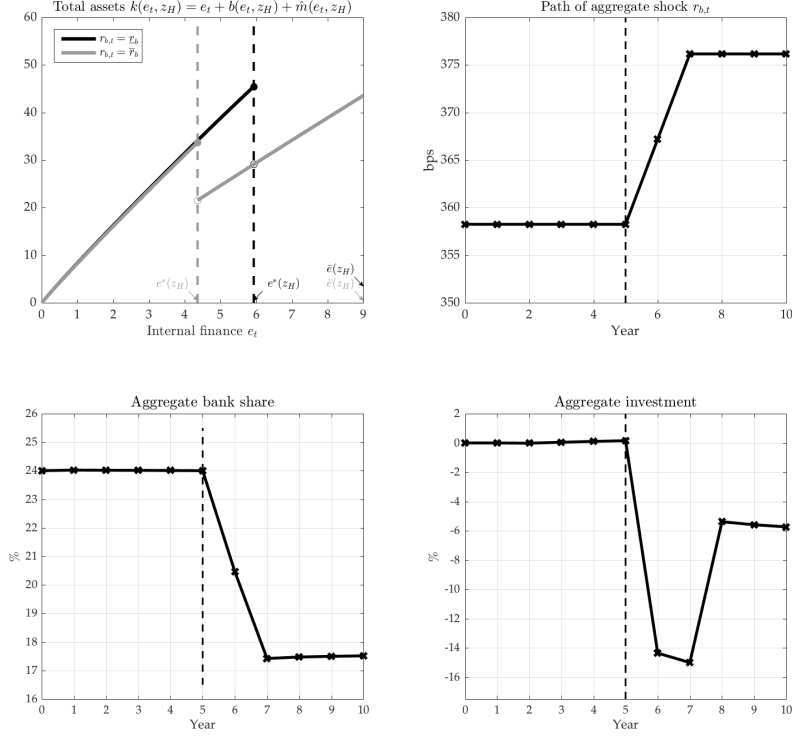


Figure 2: The model with stochastic bank intermediation costs $\gamma_{b,t}$. Top left panel: conditional policy functions for total assets $\hat{k}(e_t, z_H)$, for the highest productivity level $z_t = z_H$. Top right panel: path of total required return on bank loans, $r_{b,t}$. The economy is first simulated for 1000 periods, using the aggregate law of motion for the bank intermediation cost $\gamma_{b,t}$. The economy then receives the sequence of shocks reported in the top right panel. Bottom left panel: response of the aggregate bank share. Bottom right panel: response of aggregate investment.

the model with stochastic shocks to credit supply are overall consistent with the predictions of the baseline, perfect-foresight model.

4 The Italian tax reform

In this appendix, I first briefly describe the introduction of tax shields into the model. I then illustrate the effects of the Italian reform described in section 5 of the main text on the borrowing and investment choices of firms. Throughout, I maintain the assumption that $\chi_P = 1$, so that resources available for debt repayment are equal to:

$$\pi_t^L = \pi_t = \phi_t k_t^\zeta + (1 - \delta)k_t.$$

Introducing tax shields As mentioned in section 5 of the main text, when gross income and debt payments are subject to differential taxation, the cash on hand of firm that repays its creditors can generally be written as:

$$n_t^R = (1 - \tau)\pi_t - (1 - \tau_b)R_{b,t} - (1 - \tau_m)R_{m,t}.$$

One must specify how the firm and its creditor's income are taxed under payment, restructuring and liquidation. I make two key assumptions in this regard:

Assumption 1 (Tax treatment of restructuring and liquidation).

- *Income tax liabilities are senior to bank and market debt payments in liquidation;*
- *There are no tax shields for debt payments that have been restructured.*

The first assumption is innocuous, and simply guarantees that firms will not find it beneficial to default in order to avoid the payment of tax liabilities. The second assumption guarantees that, when tax shields are identical ($\tau_b = \tau_m$), the restructuring choices of the firm are similar to the baseline model; it therefore helps to focus the discussion on the effects of *asymmetric* tax treatment of debt.

With these assumptions, the payoffs to stakeholders in liquidation are given by:

$$\begin{aligned}\tilde{R}_{b,t} &= \min(R_{b,t}, (1-\tau)\chi\pi_t) && \text{(bank lenders)} \\ \tilde{R}_{m,t} &= \min(\max(0, (1-\tau)\chi\pi_t - R_{b,t}), R_{m,t}) && \text{(market lenders)} \\ n_t^L &= \max(0, (1-\tau)\chi\pi_t - R_{b,t} - R_{m,t}) && \text{(firm)}\end{aligned}\tag{2}$$

Moreover, in restructuring the firm will drive the bank down to its reservation value, $(1-\tau)\chi\pi_t$. In that case, given the second assumption, the cash on hand of the firm after restructuring will be given by:

$$n_t^R = (1-\tau)\pi_t - (1-\tau)\chi\pi_t - (1-\tau_m)R_{m,t}.$$

Given this, the following lemma is straightforward to establish.

Lemma 1 (Debt settlement outcomes). *Assume that $V^c(\cdot)$ is increasing, and $V^c(0) \geq 0$. Then, there are two types of debt settlement outcomes:*

- **When** $\frac{(1-\tau_b)R_{b,t}}{\chi} \geq \frac{(1-\tau_m)R_{m,t}}{1-\chi}$, *the firm chooses to repay its creditors in full, if and only if, $\pi_t \geq \frac{(1-\tau_m)R_{b,t}}{\chi}$. It successfully restructures its debt, if and only if, $\frac{(1-\tau_m)R_{m,t}}{1-\chi} \leq \pi_t < \frac{(1-\tau_b)R_{b,t}}{\chi}$, and it is liquidated when $\pi_t < \frac{(1-\tau_m)R_{m,t}}{1-\chi}$.*
- **When** $\frac{(1-\tau_b)R_{b,t}}{\chi} < \frac{(1-\tau_m)R_{m,t}}{1-\chi}$, *the firm repays its creditors in full if and only if $\pi_t \geq \frac{(1-\tau_b)R_{b,t} + (1-\tau_m)R_{m,t}}{(1-\tau)}$, and it is liquidated otherwise.*

Moreover, in any successful restructuring offer, the bank obtains its reservation value $(1-\tau)\chi\pi_t$, and in all debt settlement outcomes resulting in liquidations, $n_t^L = 0$.

Thus, under the two assumptions above, the structure of the debt settlement outcomes is similar to the baseline model: when the firm's bank liabilities are large enough, it will sometimes restructure debt contracts conditional on its productivity realizations; otherwise, it never uses the restructuring option. The difference is that, when $\tau_b \neq \tau_m$, the firm's decision to restructure debt contracts also depends on the relative values of the tax shield on bank and market debt, and not simply on liquidation losses (associated with the parameter χ).

Effects of the reform Lemma 1 can be used to fully characterize the set of feasible debt contracts, and therefore the general formulation of firms' optimal debt structure problem when there are differential tax treatments of debt, in a similar way to the derivations of section 2 of this appendix.

I next turn to the effects of the policy experiment described in section 5 of the main text. Namely, I compare firm-level borrowing and investment in an economy without and with the tax reform. In the baseline economy, all firms enjoy tax shields for bank debt, but only large firms enjoy a tax shield for market debt issuance:

$$\begin{aligned}\tau_b(e_t) &= \tau \quad \forall e_t, \\ \tau_m(e_t) &= \begin{cases} 0 & \text{if } e_t \leq e_{sm} \\ \tau & \text{if } e_t > e_{sm} \end{cases}\end{aligned}$$

In the reformed economy, however, the tax shield applies to all debt issuances of all firms:

$$\tau_b(e_t) = \tau_m(e_t) = \tau \quad \forall e_t.$$

So as to clarify the exposition of the effects of the subsidy, I set the threshold for the reform at $e_{sm} = e^*$. Moreover, I focus on the borrowing policies of firms in the static version of the model, with a single level of productivity: $z_t = z, \forall t$. Indeed, the results of section 5.1 suggest that the bulk of the effects of this type of policy is mediated by firms' borrowing decisions, rather than by long-run changes in the firm size distribution.

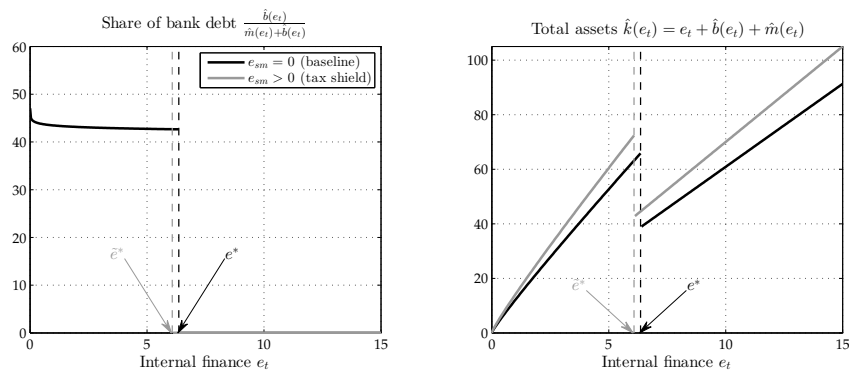


Figure 3: The effect of the tax reform on borrowing and investment.

Figure 3 reports the results of the experiment, when the tax rate is $\tau = 2.5\%$. The reform has little incidence on the debt composition of firms, but induces a measure of firms to switch to pure market finance (left panel). In doing so, these firms operate at a lower scale than they otherwise would have (right panel). The investment of all other firms, however, gains from the introduction of the tax shield. The results are thus broadly analogous to those of the reform analyzed in section 5.1 of the main text.

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