Dynamic Investment and Product Market Rivalry: The Network Q Model

by Bustamante (Maryland) and Pellegrino (Columbia and NBER)

Discussion by Nicolas Crouzet (Kellogg)

This paper

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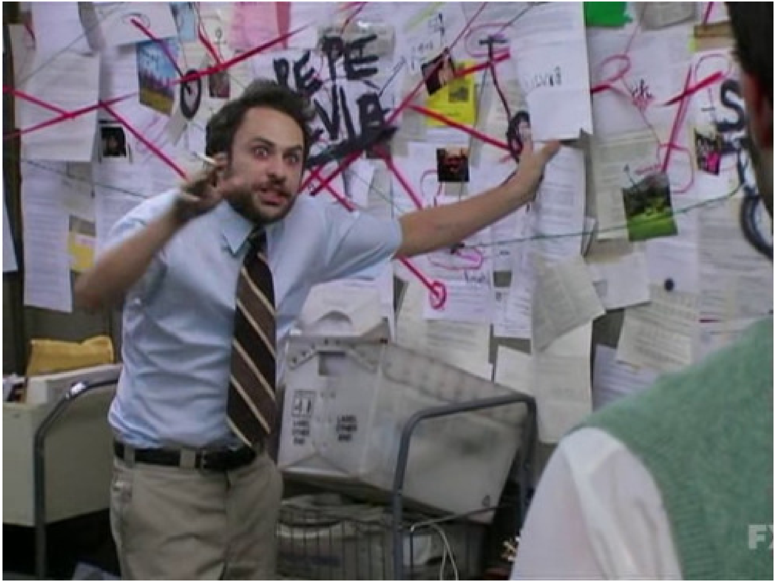
Counterfactuals: impact of mergers, collusion on investment

since 1995, mergers reduced the capital stock by 1% and increased markups by 10-20bps




PEPE

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   key for effect of shocks + counterfactuals; very difficult to tell apart in the data

   for the paper: provide more evidence substitutability assumption is right

Does the network actually matter?

A network profit function

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$$\pi_n(\mathbf{k}_t)$$

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$\mathcal{N} = (\mathcal{N}_{nm})_{N \times N}$ symmetric

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$\mathcal{N}_{nm} > 0 \leftrightarrow$ investment decisions are substitutes

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Next few slides: compare model with no interactions, $\mathcal{N} = 0$, to model with interactions, $\mathcal{N} \neq 0$

No interactions $(\mathcal{N} = 0)$

Steady-state capital:

$$\bar{k}_n = \frac{a_n - (r + \delta_n)\, p_n^k}{d_n}$$

Interactions $(\mathcal{N} \neq 0)$

No interactions $(\mathcal{N} = 0)$

Interactions $(\mathcal{N} \neq 0)$

Steady-state capital:

$$\bar{k}_n = \frac{a_n - (r + \delta_n)\, p_n^k}{d_n}$$

$$\bar{\mathbf{k}} = \text{diag}(\mathbf{d})^{-1} \left[\mathbf{a} - (r\mathbf{I} + \boldsymbol{\Delta})\, \mathbf{p}^k \right]$$

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Interactions ($\mathcal{N} \neq 0$)

Steady-state capital:

$$\bar{\mathbf{k}} = (\operatorname{diag}(\mathbf{d}) + \mathcal{N})^{-1} \left[\mathbf{a} - (r\mathbf{I} + \boldsymbol{\Delta}) \mathbf{p}^k \right]$$

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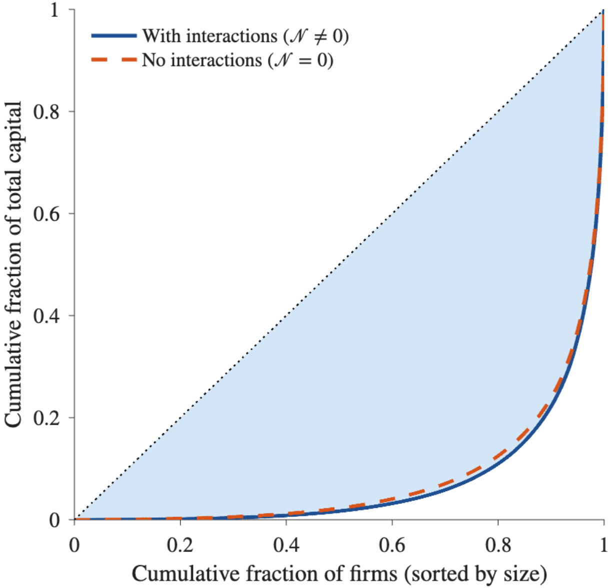
$\uparrow \mathcal{N} \implies$ more concentration



With interactions ($\mathcal{N} \neq 0$)
No interactions ($\mathcal{N} = 0$)
Cumulative fraction of total capital
Cumulative fraction of firms (sorted by size)
0
0.2
0.4
0.6
0.8
1

No interactions ($\mathcal{N} = 0$)

Steady-state capital:

$$\bar{k}_n = \frac{a_n - (r + \delta_n) p_n^k}{d_n}$$

$$\bar{\mathbf{k}} = \mathrm{diag}(\mathbf{d})^{-1} \left[\mathbf{a} - (r\mathbf{I} + \boldsymbol{\Delta}) \mathbf{p}^k \right]$$

Dynamics:

determined by own-firm curvature d_n only

independent across firms

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Dynamics:

determined by $\text{diag}(\mathbf{d}) + \mathcal{N}$

depends on other firms' investment choices

No interactions $(\mathcal{N}=0)$

Steady-state capital:

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Interactions $(\mathcal{N}\neq 0)$

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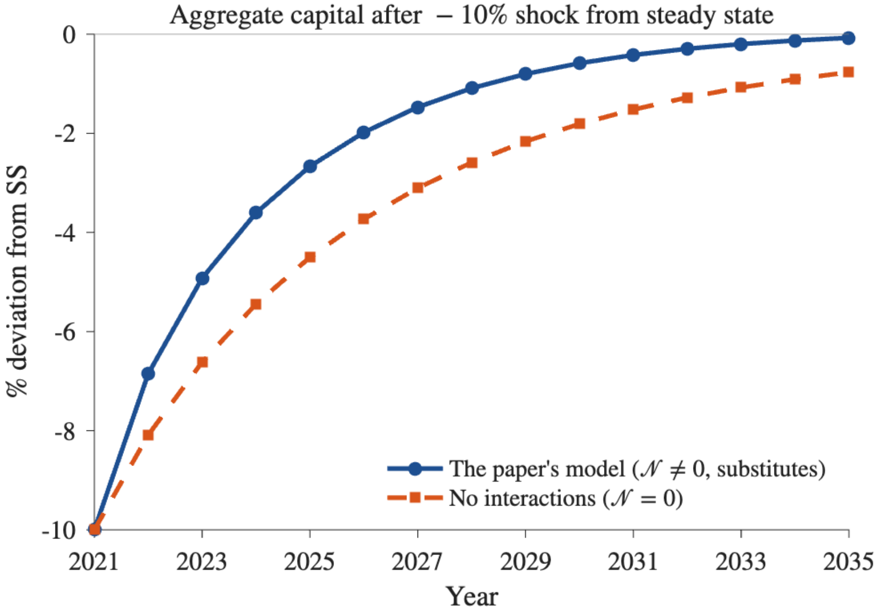
$\uparrow\mathcal{N}$: subtle!

aggregate shocks wash out more quickly

reallocation shocks wash out more slowly


Aggregate capital after − 10% shock from steady state
0
-2
-4
-6
-8
-10
% deviation from SS
2021
2023
2025
2027
2029
2031
2033
2035
Year
The paper's model ($\mathcal{N} \neq 0$, substitutes)
No interactions ($\mathcal{N} = 0$)

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Enterprise value of firm n, steady-state:

$$v_n = (1+r)\, p_n^k\, \bar{k}_n \qquad \text{[repl. cost]}$$

$$+ \frac{1+r}{r} \frac{1}{2} d_n \bar{k}_n^2 \qquad \text{[PV of rents]}$$

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Same formula — $\mathcal{N}$ enters only through $\bar{k}_n$

Why don't interactions show up in enterprise value?

$$\pi_n(\mathbf{k}_t) = \underbrace{a_n k_{nt} - \frac{1}{2} d_n k_{nt}^2}_{\text{standalone profit}} - \overbrace{\left(\sum_{m \neq n} \mathcal{N}_{nm} k_{mt}\right) k_{nt}}^{\text{network interactions}}$$

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Because average-marginal product gap is independent of network interactions $\mathcal{N}$

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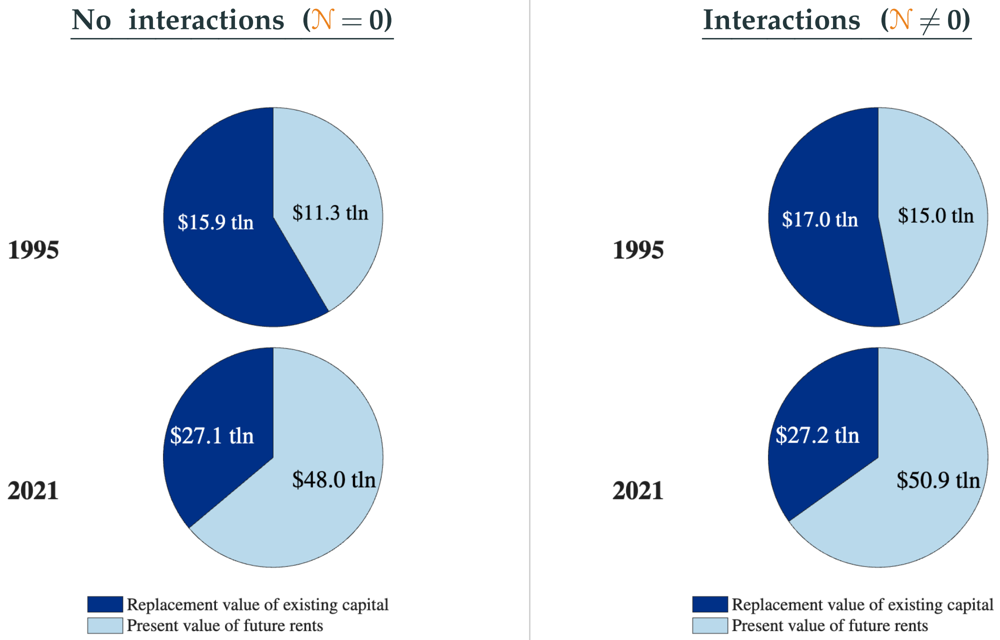
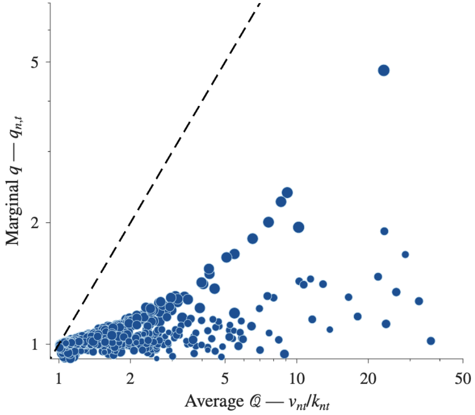
When other firms change investment, my average and marginal products change equally


No interactions ($\mathcal{N} = 0$)
Interactions ($\mathcal{N} \neq 0$)
1995
2021
$15.9 tln
$11.3 tln
$27.1 tln
$48.0 tln
$17.0 tln
$15.0 tln
$27.2 tln
$50.9 tln
Replacement value of existing capital
Present value of future rents

No interactions ($\mathcal{N} = 0$)



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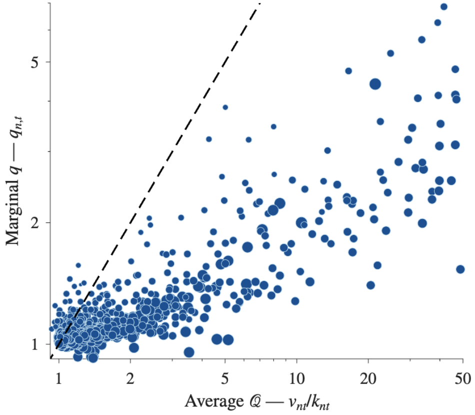

Smaller investment gap for the typical firm

But more cross-sectional dispersion in Q, q, and $Q - q$

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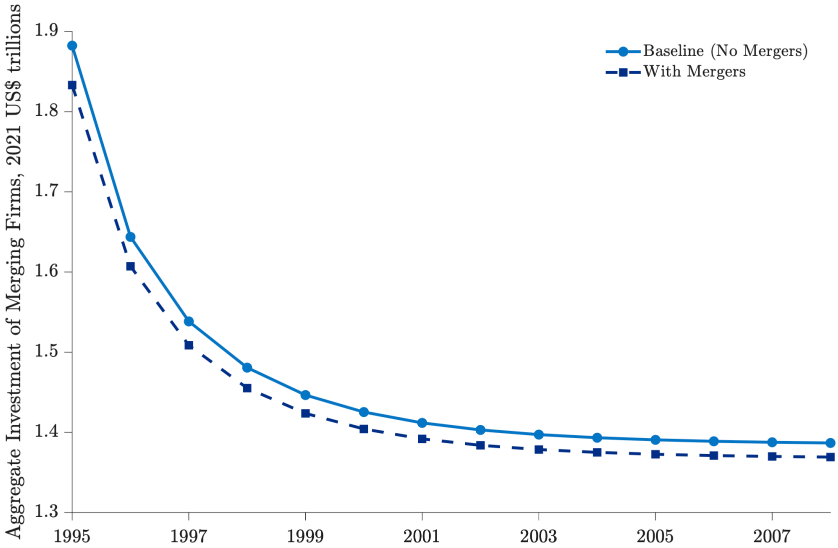
For counterfactuals that alter firm interactions: essential

- effects of mergers, cartels on corporate investment
- under $\mathcal{N} = \mathbf{0}$, these counterfactuals are impossible
- this is the paper's distinctive contribution

FIGURE 14: MERGERS SIMULATION: INVESTMENT



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Suggestion 1: spend more time shock transmission + competitive counterfactuals

- effect of lower α (degree to which firms internalize the network)?
- perfectly competitive limit? effect of entry/exit of large competitors?

If the network matters, what kind of network should we focus on?

Back to the general network profit function

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This is not required, i.e. could have $\mathcal{N}_{nm} < 0$, complements

depends on the nature of economic interactions between firms

Where could interactions come from?

Microfoundation	Sign($\mathcal{N}_{nm}$)	Source of d_n	Source of $\mathcal{N}_{nm}$
Product market competition	> 0	Own-price demand	Product substitution σ_{nm}
Labor market competition	> 0	Own labor demand	Shared labor market
Input-output linkages	$\gtrless 0$	Own-price + input costs	Trade vs. input competition
Demand complementarities	< 0	Own-price demand	Cross-product demand
Knowledge spillovers	< 0	Diminishing returns	Bilateral knowledge flows

■ substitutes ■ complements ■ either

Product market: [Pellegrino 2025]. Labor: [Jarosch et al. 2024; Azar et al. 2022]. I/O: [Barrot, Sauvagnat 2016; Acemoglu et al. 2012]. Demand compl.: [Lee et al. 2024]. Knowledge: [Bloom et al. 2013; Zacchia 2020].

Why does complements vs. substitutes matter?

Consider the symmetric duopoly case: $N = 2, \quad \mathcal{N}_{12} = \mathcal{N}_{21} = \nu$ [Fudenberg and Tirole, 1984]

$\nu > 0$: substitutes

$\nu < 0$: complements

Positive predictions: Firm 1 gets a 10% shock to their capital stock. What happens to Firm 2?

$\nu > 0$: firm 2 contracts

$\nu < 0$: firm 2 expands

[Bernstein, Nadiri 1989; Bloom et al. 2013]

Normative predictions: Firm 1 and Firm 2 merge. What happens to their capital stock?

$\nu > 0$: $k_{\text{merged}} < \bar{k}$, merged entity shrinks (anti-competitive)

$\nu < 0$: $k_{\text{merged}} > \bar{k}$, merged entity expands (pro-competitive)

[Williamson 1968; Farrell, Shapiro 1990]

Substitutes vs. complements: can we tell from the data?

Is it possible to directly recover $\mathcal{N}_{nm}$ from the data, without committing to a microfoundation?

- using only data on capital, investment, sales, profits and Q?
- no: $N(N-1)/2$ parameters, but only NT obs, with $T << N$ in firm-level panel data

So we have to commit to a microfoundation, as the paper does. But can we test it?

- given $\mathcal{N}$, many possible overidentifying restrictions
- pick those that might best distinguish substitutes from complements

I'll give an example of a test. Bottom line is it cannot distinguish between:

- the data favors complements over substitutes; and
- firms that are similar in HP space are subject to common shocks

∴ a version of the reflection problem [Manski, 1983]

An example: comovement in profits

$$\text{Model}: \quad \overbrace{ROA_{nt} + \frac{1}{2} d_n k_{nt}}^{\text{adjusted ROA}} = a_n - \sum_{m \neq n} \overbrace{\sigma_{nm} z_n z_m}^{\mathcal{N}_{nm}} k_{mt}$$

$$\text{Data}: \quad ROA_{nt} + \frac{1}{2} d_n k_{nt} = \delta_{j(n),t} + \alpha_n - \beta \sum_{m \neq n} \sigma_{nm} z_n z_m k_{mt} + \varepsilon_{nt}$$

Under the paper's model, $\beta > 0$ (substitutes)

	(1)	(2)
$\hat{\beta}$	−0.0094	−0.0099
s.e.	(0.0001)	(0.0001)
Firm FE	✓	✓
Year FE	✓	✗
Industry × year FE	✗	✓
Obs.	88,863	88,863

Complements? Or common shocks among HP-similar firms? Cannot tell without an instrument

So what?

The demand system is a structural assumption on $\mathcal{N}$ that imposes substitutability

this assumption drives the sign of counterfactuals

Imposing structure on $\mathcal{N}$ is the right approach

but want to "kick the tires" on whether substitutability is right

Suggestion 2: Provide more over-identification tests, particularly in the cross-section

Comovement in investment across firms [$\text{cov}(\iota_{m,t}, \iota_{n,t})$]; comovement in Q across firms [$\text{cov}(Q_{m,t}, Q_{n,t})$]; cross-comovement between capital and Q [$\text{cov}(Q_{m,t}, k_{n,t})$]; ...

Suggestion 3: Compare to other microfounded $\mathcal{N}$ interaction matrices

$\mathcal{N}$ from customer-supplier data (I/O), cross-firm patent citations (knowledge spillovers)

probably for future work!

Conclusion

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This is two fantastic papers wrapped into one

a model to think about interactions between corporate investment decisions

an application to interactions driven by product market competition

The theoretical framework is more general than the paper lets on

many possible microfoundations w/ very different implications for policy-relevant questions

Interpreting the product competition application requires care

show $\mathcal{N} = 0$ counterfactual + provide more over-identifying tests for subs. vs comps.

I hope people will read this paper and build on it

I certainly plan to!