Chapter 13

Salesforce Compensation:
A Review of MS/OR Advances

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1. Introduction

A firm that has made the significant investment in a direct salesforce has an interest in the continuing motivation and sales performance of its salespeople. Sales management faces two fundamental problems in designing effective reward systems. First, salespeople typically have objectives that differ from those of the firm, necessitating compensation-plan design that aligns salesforce and sales-management objectives. Second, managers who set compensation often lack information that is crucial to setting the right compensation plan, such as how sales respond to the sales effort of different salespeople in different territories, or the relative preference a salesperson has for income versus leisure. While some array of monetary and non-monetary rewards may help solve both problems and result in effective selling performance, research shows that salespeople are much more highly motivated by monetary rewards than by non-monetary ones [Ford, Walker & Churchill, 1981]. This of course does not mean that sales management should abandon non-monetary rewards, since they too generate positive (albeit lower) utility for the salesforce. Nevertheless, the clear importance of monetary rewards for salesforce motivation justifies this chapter’s primary focus on management science/operations research approaches to salesforce compensation research.

The problem of designing an ‘optimal’ salesforce-compensation problem is very complex. On the most fundamental level, the manager must decide what ‘optimal’ means: is a plan optimal that maximizes the firm’s sales or profits? That maximizes salespeople’s income or utility? Is a plan optimal that is constrained by certain requirements of the firm, such as the maintenance of a particular form of plan?

Even after deciding what objectives a salesforce-compensation system is designed to attain, management still has to characterize some important aspects of its environment. It must do its best to characterize (a) the preferences and behavior of salespeople, (b) the nature of sales response, and (c) its own preferences and
behavior. Some examples will serve to illustrate the rich array of issues raised here. Salespeople may influence sales and profit outcomes only by the exertion of selling effort, or they may be responsible for other decisions as well, such as pricing.

Sales response may be a function of selling effort alone (as seems approximately true in some industrial-product markets), or may also be significantly influenced by product prices and other marketing-mix variables. Sales response for one product may depend critically on sales of another complementary product in the line, or may depend on the team-selling efforts of several people in the salesforce.

The firm itself may have several interrelated decisions to make, including not only the form of the compensation plan (e.g., whether it will include commission pay in addition to salaries), but also levels of incentives (e.g., commission rates) and whether to offer a tailored menu of plans to a heterogeneous salesforce. This set of decisions on the part of the firm abstracts from the greater environment in which it operates: how it responds to competition, what products it chooses to include in its product line, and the like.

By necessity, the literature on salesforce compensation takes many of these issues as given when attacking a particular aspect of the compensation-setting problem. But there is no doubt that all of these issues, and many more, have an impact on salesforce motivation and ensuing firm profits. The common theme pervading all the research in the area is the search for a compensation plan that makes the best use of available information (or elicits extra valuable information when needed) in designing a plan that comes the closest to aligning the incentives of sales management and the salesperson. Fortunately, the marketing literature on salesforce compensation has progressed far enough on some of these issues to merit review, although as we will see in the concluding section of the chapter, there is still a great deal left to accomplish.

Prior survey work in the salesforce-compensation area in marketing includes Coughlan & Sen [1986, 1989]. This chapter differs from both of these earlier papers in several ways. First, it is more up-to-date in the set of articles it surveys. It also presents material more analytically, to highlight MS/OR advances in modeling in the area. Further, it surveys three major approaches to salesforce-compensation modeling, whereas the other two papers concentrate on a subset of these methods: (i) a microeconomics-based approach assuming no uncertainty in sales response; (ii) an agency-theoretic approach; and (iii) a decision support system (DSS) approach. We also summarize empirical evidence for the first two areas as a guide to assessing the reliability and robustness of the theory.

In what follows, we summarize the modeling structures and results in the three major areas of the MS/OR salesforce-compensation literature. After the discussion of microeconomics-based models (i) and agency-theoretic models (ii), we summarize empirical tests of these theories before turning to a discussion of DSS models (iii). In each case, we provide tabular summaries of model structures as well as textual discussion. All models are characterized by their treatment of the salesforce, the sales-response function, and the firm itself. It turns out that in the microeconomics-based approach and the agency-theoretic approach, one can describe a base model from which later publications build and digress. The development of the DSS literature has been less orderly in some sense, but this is because of a focus on
solving different particular problems in salesforce compensation. We close with conclusions and directions for future research.

2. Model structure/techniques: Microeconomic approach

The first set of models is united by the common assumption of non-stochasticity of sales response to selling effort. This means that a given amount of selling effort exerted against a particular product always produces the same level of sales; there are no environmental or competitive ‘wild cards’ inducing randomness into the sales-response function. Despite this non-stochasticity, the firm may or may not know the exact form of a given salesperson’s sales-response function for a given product. The salesperson is always assumed to know the sales-response function for every product he sells, however.

All the models assume that the firm maximizes profits and the salesperson maximizes his utility. The firm’s problem is to design a compensation plan that causes the salesperson to make his decisions (time allocation across products, total selling-time allocation, and/or product price/discount setting) so as to maximize the firm’s profits. Essentially, the compensation plan is to be designed to align the salesperson’s objectives with those of the firm, despite the basic fact that the salesperson may seem to have different goals in mind than those of the firm.

We can characterize the general form of all the models in this section, and identify how each one is a special case of the general model. In this general model, the firm’s profit can be expressed as:

\[ \Pi = \sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} Q_{ij} - \sum_{i=1}^{n} C_i(Q_i) - \sum_{j=1}^{m} S_j, \tag{1} \]

where each of the \( m \) salespeople in the salesforce sells the firm’s line of \( n \) products and where

- \( P_{ij} \) = selling price of product \( i \) when sold by salesperson \( j \),
- \( Q_i = \sum_j Q_{ij} \) = number of units of product \( i \) sold by all salespeople,
- \( C_i(Q_i) \) = total variable cost of selling \( Q_i \) units of product \( i \),
- \( S_j \) = salesperson \( j \)’s total income.

The sales-response function for product \( i \) for salesperson \( j \) is

\[ Q_{ij} = f_i(t_{1j}, t_{2j}, \ldots, t_{nj}, P_{1j}, P_{2j}, \ldots, P_{nj}), \tag{2} \]

where \( t_{ij} \) is the selling time allocated by salesperson \( j \) to product \( i \) and \( P_{ij} \) is the price charged for product \( i \) by salesperson \( j \) (providing for the possibility that salespeople in different territories may charge different prices for the same product, although any one salesperson sets product \( i \)’s price just once for all his customers). Thus, in the most general form, sales response can include cross-price as well as cross-effort effects, and each salesperson may possibly control product prices as well as his own selling effort. All models assume that the firm acts as a Stackelberg leader relative to its salespeople, that is, that the firm knows the form of the
salesperson's reaction to any compensation plan it quotes to the salesforce. Note, however, that none of the articles explicitly considers the possibility of team-selling, where sales by salesperson \( j \) are a direct function of the effort exerted by other members of the salesforce. Nor do the articles focus attention on salesperson-specific differences in sales-response functions (hence the functional form \( f_i \) in Equation 2 typically applies to all \( m \) salespeople).

The salesperson's goal is to maximize utility. His decision variables include the allocation of selling time across the \( n \) products, and may also include the setting of product prices (or equivalently, discounts off list price) and the choice of total selling time. Formally, utility can be expressed as:

\[
U_j = g_j(S_j) - V_j(T_j) \quad \text{subject to} \quad \sum_{i=1}^{n} t_{ij} = T_j,
\]

where

- \( g_j(\cdot) \) = utility for income,
- \( V_j(\cdot) = \text{disutility for effort}, \)
- \( t_{ij} = \text{time allocated to selling product } i \text{ by salesperson } j, \)
- \( T_j = \text{total selling time of salesperson } j. \)

This summarizes the most generic form of the salesforce-compensation model, given certainty in the sales-response function. The goal of all the models in this class is to design a compensation plan (specifically, to set the form of \( S_j \)) that induces the salesperson to act so as to maximize the firm's profits. We can classify the papers referred to in this section by the ways in which they specify this general model. Tables 13.1, 13.2 and 13.3 summarize assumptions on the salesforce, the sales-response function, and the firm made by each article in the area, using the Farley [1964] paper as a base case.

Farley [1964] was the first paper to model salesforce compensation in this type of framework. His model makes the following four assumptions concerning the salesforce (expressed in our terminology):

\( \text{F1.} \) The salesforce has one person in it. (Equivalently, the model could represent a multi-person salesforce, where there are no interactions among salespeople's decisions.)

\( \text{F2.} \) The salesperson's only choice variable is sales-effort allocation across the \( n \) products sold.

\( \text{F3.} \) The salesperson's objective is to maximize income.

\( \text{F4.} \) There is one constraint on the salesperson's decisions: total selling time is limited to \( T. \)

The next three assumptions in Farley [1964] pertain to the sales response function:

\( \text{F5.} \) There are \( n \) products in the product line, with no demand- or cost-side interactions among them.
Table 13.1
Salesforce-compensation models, sales-response-function certainty: assumptions on the salesforce

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<tbody>
<tr>
<td>Base case:</td>
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<tr>
<td>Farley [1964]</td>
<td>One salesperson</td>
<td>Sales-effort allocation across</td>
<td>Maximize income</td>
<td>Total selling time of $T_j$ for</td>
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<tr>
<td></td>
<td>(generalizable to $n$ salespeople, no interactions)</td>
<td>products only</td>
<td></td>
<td>any salesperson $j$</td>
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<tr>
<td>Comparison of Farley [1964] with:</td>
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<tr>
<td>Davis &amp; Farley [1971]</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
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<tr>
<td>Weinberg [1975]</td>
<td>Same</td>
<td>Sales-effort allocation, and</td>
<td>Same</td>
<td>Same</td>
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<td></td>
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<td>discounts from list price,</td>
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<td></td>
<td></td>
<td>across products</td>
<td></td>
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</tr>
<tr>
<td>Tapiero &amp; Farley</td>
<td>$n$ salespeople, no interactions</td>
<td>Sales-effort allocation across $T$ time periods</td>
<td>Same</td>
<td>Total selling time of $T_j$ per</td>
</tr>
<tr>
<td>Weinberg [1978]</td>
<td>Same</td>
<td>Sales-effort allocation, and</td>
<td></td>
<td>salesperson per time period</td>
</tr>
<tr>
<td></td>
<td></td>
<td>discounts from list price,</td>
<td></td>
<td>(a) Time constraint; or</td>
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<td></td>
<td></td>
<td>across products</td>
<td></td>
<td>(b) income constraint; or</td>
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<td>(c) constraint on minimum</td>
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<td>marginal return for time</td>
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<td></td>
<td></td>
<td>spent on each product</td>
</tr>
<tr>
<td>Srinivasan [1981]</td>
<td>Same</td>
<td>(a) Sales effort and actual</td>
<td>(a) Maximize income s.t. time</td>
<td>(a) Time constraint; or</td>
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<tr>
<td></td>
<td></td>
<td>selling price across products; or</td>
<td>constraint; or</td>
<td>(b) 'fair' income constraint; or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(b) total selling time; or</td>
<td>(b) achieve 'fair' income; or</td>
<td>(c) unconstrained utility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(c) total selling time</td>
<td>(c) maximize utility</td>
<td>maximization</td>
</tr>
</tbody>
</table>

These assumptions are discussed more fully in Section 2 of the paper. Assumptions are noted for the Farley model, and deviations from the Farley assumptions are noted for the other papers in the table.
Table 13.2
Salesforce-compensation models, sales-response-function certainty: assumptions on the sales-response function

<table>
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<tr>
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<tbody>
<tr>
<td>Base case: Farley [1964]</td>
<td>( n ) products, no demand or cost interactions</td>
<td>Selling effort on own product only</td>
<td>( Q_i = f_i(t_i), \ i = 1, 2, \ldots, n, \ f_i &gt; 0 )</td>
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<tr>
<td>Comparison of Farley [1964] with:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Davis &amp; Farley [1971]</td>
<td>Same</td>
<td>Same</td>
<td>Same; but also, ( f_i'' &lt; 0 )</td>
</tr>
<tr>
<td>Weinberg [1975]</td>
<td>Same</td>
<td>Selling effort on own product and discount off list price</td>
<td>( Q_i = f_i(t_i, P_i), \ i = 1, 2, \ldots, n, \ ) \frac{\delta f_i}{\delta t_i} \frac{\delta f_i}{\delta P_i} &gt; 0; \ f_i \ \text{concave} )</td>
</tr>
<tr>
<td>Tapiero &amp; Farley [1975]</td>
<td>Same</td>
<td>Current and possibly past selling effort on own product</td>
<td>( Q_j(t) = F_j \left[ \int m(t,z)u_j(z)dz \right], \ \text{a function of current marginal effort productivity, a 'forgetting function', and effective units of selling effort} )</td>
</tr>
<tr>
<td>Weinberg [1978]</td>
<td>Same</td>
<td>Selling effort on own and other products; discounts on own and other products</td>
<td>( Q_i = f_i(t_1, t_2, \ldots, t_n, P_1, P_2, \ldots, P_n); \ ) \frac{\delta f_i}{\delta t_i} \frac{\delta f_i}{\delta P_i} &gt; 0 )</td>
</tr>
<tr>
<td>Srinivasan [1981]</td>
<td>Same</td>
<td>Selling effort on own and other products; discounts on own and other products</td>
<td>( Q_i = f_i(t_1, t_2, \ldots, t_n, P_1, P_2, \ldots, P_n); \ ) \text{cross-product effects not investigated throughout analysis} )</td>
</tr>
</tbody>
</table>

These assumptions are discussed more fully in Section 2 of the paper. Assumptions are noted for the Farley model, and deviations from the Farley assumptions are noted for the other papers in the table.
### Table 13.3
Salesforce-compensation models, sales-response-function certainty: assumptions on firm behavior

<table>
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<tbody>
<tr>
<td>Base case: Farley [1964]</td>
<td>Constant marginal cost, no economies of scale or scope</td>
<td>Knows salesperson’s utility function and sales-response function</td>
<td>Commission rates on gross margins of all products</td>
<td>Maximize profit</td>
</tr>
<tr>
<td>Comparison of Farley [1964] with Davis &amp; Farley [1971]</td>
<td>Total variable costs across product line are: $C = \sum C_i(Q_i)$, where $d^2C_i/dQ_i^2 &gt; 0$ (jointly decreasing returns to scale over product line)</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>Weinberg [1975]</td>
<td>Same</td>
<td>Same</td>
<td>Same (realized gross margins of all products)</td>
<td>Same (dynamic horizon)</td>
</tr>
<tr>
<td>Tapiéro &amp; Farley [1975]</td>
<td>General function of contemporaneous volume of own product; constant marginal cost for parametric results</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>Weinberg [1978]</td>
<td>Same</td>
<td>Same</td>
<td>Same (commission rates on total contribution)</td>
<td>Same</td>
</tr>
<tr>
<td>Srinivasan [1981]</td>
<td>Same</td>
<td>Same</td>
<td>Same (commission rates on total contribution)</td>
<td>Same</td>
</tr>
</tbody>
</table>

These assumptions are discussed more fully in Section 2 of the paper. Assumptions are noted for the Farley model, and deviations from the Farley assumptions are noted for the other papers in the table.
F6. The only argument of the sales-response function for product \( i \) is selling time on product \( i \).

F7. The form of the sales-response function is:

\[ Q_i = f_i(t_i), \quad i = 1, 2, \ldots, n, \ f'_i > 0. \]  

(4)

Finally, four assumptions characterize Farley’s assumptions on the firm:

F8. Each of the \( n \) products has a constant marginal cost of production, with no cross-product cost interactions (that is, there are no economies of scale or scope).

F9. The firm knows both the salesperson’s utility function and the form of the sales response function fully.

F10. The firm’s choice variables are commission rates paid on gross margins of each of the \( n \) products.

F11. The firm’s objective is to maximize profit.

Given these specific assumptions, Equations (1) and (3) become:

\[ \Pi = \sum_{i=1}^{n} (P_i - K_i)(1 - B_i)f_i(t_i). \]  

(5)

\[ U_j = S_j = \sum_{i=1}^{n} (P_i - K_i)B_if_i(t_{ij}). \]  

(6)

where \( B_i \) is the (fractional) commission rate on gross margin paid on product \( i \).

In this framework, Farley’s central result is a very simple rule about incentive-aligning compensation: the firm should compensate each salesperson with a commission-only compensation plan, where each commission rate is an equal percentage of the product’s gross margin. That is, \( B_i = B_j \) for all products \( i \) and \( j \). This plan maximizes the firm’s profit because it makes the salesperson’s utility function a fraction of total system profits, with the remaining fraction going to the firm. Incentives are aligned because the salesperson is in effect made a ‘residual claimant’ of the firm.

Farley’s result does not specify what exact commission rate is the ‘right’ one, although common sense suggests that total compensation should be at least equal to the salesperson’s opportunity cost of time (otherwise, he will leave the firm). Farley also establishes that in general, commissions paid on sales revenue are not optimal, because they cause the salesperson to maximize sales, rather than profits.

Davis & Farley [1971] examine the same problem as Farley’s [1964] model, with two differences in assumptions:

DF8. Total variable costs across the entire product line are given by:

\[ C = \sum_{i} C_i(Q_i), \quad d^2C_i/dQ_i^2 > 0 \]  

(there are decreasing returns to scale over the entire product line).
DF10. The firm sets commission rates on sales or on gross margins, or it sets quotas, on all products.

This change complicates matters significantly, since now each salesperson's sales of product \( i \) affect the marginal cost associated with selling another unit of \( i \), and hence the optimal sales of product \( i \) by the rest of the salesforce; the salespeople's actions are truly interdependent. Davis & Farley show that in this situation, neither a commission on sales nor a commission on gross margins solves the incentive incompatibility problem of the salesforce and the firm. They offer two solutions: the first involves centralized quota-setting by the firm, based on complete knowledge of the sales-response functions for each salesperson and each product.

The problem with this solution, Davis & Farley point out, is its significant information requirements for the firm. It is unlikely that the firm will possess all the relevant information about each salesperson's marginal productivity of selling effort for every product, for example. Thus, they sketch out an alternative solution. This involves an iterative procedure where the firm proposes a set of commission rates to the salesforce, and salespeople respond with desired quotas for each product; then commissions on products that are oversubscribed (i.e. that produce sales beyond the profit-maximizing level in total) are decreased, while those on products that are undersubscribed are increased, until a commission/quota system results that sells the optimal amount of each product. In such a system, salespeople implicitly inform the firm about their sales response functions when they pass information about preferred quotas back to the firm.

Weinberg's [1975] key extension is to examine how salesforce price-setting ability affects optimal commission-based compensation schemes. He thus extends the Farley framework by changing assumptions F2, F6 and F7 (we modify Weinberg's terminology to be consistent with our terminology above);

W5.2. The salesperson's choice variables are sales-effort allocation across all products and decimal discounts off the firm's list price for each product.

W5.6. Sales response for product \( i \) is a function of the selling effort and decimal discount off list price for product \( i \).

W5.7. Sales response is given formally by:

\[
Q_i = f_i(t_i, P_i), \quad i = 1, 2, \ldots, n
\]

\[
\delta f_i/\delta t_i > 0, \quad \delta f_i/\delta P_i < 0; \quad f_i \text{ concave.}
\]

(8)

Note that choosing a decimal discount is tantamount to choosing actual selling price; we therefore represent sales response as a function of selling time and actual price charged.

Weinberg shows that if the salesperson is setting both effort allocation among products and prices, a set of equal commission rates on gross margins of the products is optimal and will induce the profit-maximizing prices and quantities sold. If the salesperson is setting only price, and his effort allocation is dictated by the firm, Weinberg also shows that any commissions on gross margins (not necessarily equal rates across products) induce the right discounting by the
salesforce. The ability of the salesforce to set price could also be interpreted from a modeling point of view as any salesforce activity that increases demand, e.g., additional customer service or product features that the salesperson can include in order to close a sale, as long as the salesperson is informed of the cost of providing these services.

Tapiero & Farley [1975] also work with the basic Farley [1964] framework, but consider dynamic effects on salesforce compensation as well. They alter Farley’s assumptions F2, F4, F6 and F7 as follows:

TF2. Each salesperson chooses selling effort for the \( n \) products in the line across \( T \) time periods.

TF4. Each salesperson is constrained to spend not more than \( T_i \) time selling per period (the direct dynamic analogue to Farley’s assumption F4).

TF6. Sales response for product \( i \) is a function of current-period selling effort and may also be a function of past selling effort on product \( i \).

TF7. The rate of sales at any time \( t \) is given by:

\[
Q_{ij}(t) = F'_{ij} \left( \int_{-\infty}^{t} m(t, z)u_{ij}(z)dz \right),
\]

where

\[
F'_{ij} = \text{marginal productivity of an effective current unit of selling effort},
\]

\[
m(t, z) = \text{a ‘forgetting function’ representing residual effects of sales effort in some past period } z \text{ on current sales},
\]

\[
u_{ij}(t) = \text{effective units of selling effort in some time } z; \text{ a mapping between actual hours spent selling and sales response}{.^4}
\]

Further, while the authors model production costs for product \( i \) as a general function of the contemporaneous volume sold of product \( i \), their major results derive from an assumption of constant marginal cost, as in Farley [1984].

The complexity of this problem makes explicit solution in the general case impossible. However, an example with constant marginal costs and a very long planning period for sales performance results in an optimal compensation plan with commission rates proportional to gross margins of the products, with the constant of proportionality varying from product to product.\(^2\) Salespeople allocate more effort to products with larger marginal profit rates and smaller forgetting

\(^1\)The authors actually state in their Equation 2.1 [p. 979] that \( u_{ij}(t) \) is equal to the \text{int}
\text{egral} of past sales efforts, but then go on in Equation 2.2 to argue that \( F'_{ij} \) can have \( u_{ij} \) as an argument. This seems circular; the implication would be ‘double-counting’ of the effect of hours of effort in any time \( z \) on future sales of product. A more reasonable interpretation of \( u_{ij} \) seems to be the one above. This interpretation is also consistent with the authors’ example developed in Section 4 of their paper [pp. 982ff].

\(^2\)However, this result is derived under the assumption that the firm does not pay for commissions to salespeople but rather assigns quotas [cf. Equation 4.1, p. 982]. It is thus unclear whether the same result would prevail if the profit function were reformulated to include commission costs.
rates. However, more interesting cases with nonlinearities in either costs or sales-effectiveness generally require centralized solutions with optimal quota-setting and significant information acquisition by the firm.

Weinberg [1978] returns to Farley's static formulation and assumption, but examines both variations in the salesperson's objective function and demand interactions among products in price and sales-effort effects. He modifies Farley's assumptions F2, F3, F4, F6 and F7 as follows:

W8.2. The salesperson's choice variables are sales-effort allocation across all products and decimal discounts off the firm's list price for each product.

W8.3. The model investigates three possible objective functions for the salesperson: (a) to maximize income subject to a time constraint; (b) to minimize selling time subject to an income constraint; or (c) to maximize income subject to a minimum return per unit time.

W8.4. The constraint facing the salesperson varies with the objective function assumed: (a) the salesperson faces a total-selling-time constraint; or (b) he faces an income constraint; or (c) he faces a constraint on minimum return per unit time.

W8.6. Sales of product $i$ are a function of selling effort for all $n$ products and all $n$ prices.

W8.7. Formally (and using our terminology), the sales-response function is:

$$Q_i = f(t_1, t_2, \ldots, t_n, P_1, P_2, \ldots, P_n),$$

where

$$\frac{\partial f_i}{\partial t_i} > 0, \quad \frac{\partial f_i}{\partial P_i} < 0. \quad (10)$$

Under objective (a), (b) or (c) in W8.3. above, Weinberg shows that an equal-gross-margin commission system as derived in Farley [1964] is optimal. This is intuitively sensible, since (a) is just Farley's problem; (b) is the dual of Farley's problem; and (c) is tantamount to (a) due to the one-to-one relationship between return per unit time and total income.

Finally, Srinivasan [1981] points out that previous models neglect the fact that the compensation plan offered affects not only the allocation of effort across products, but also total sales effort expended by the salesperson. He builds the most comprehensive model to date by also including the ability of the salesperson to set product prices, and considering varying objective functions for the salesperson. His model differs from Farley's in assumptions F2, F3, F4, F6, F7 and F9 as follows (cases (a), (b) and (c) are consistently listed throughout):

S2. Under three different salesperson objective functions, the salesperson chooses, respectively: (a) sales effort and actual selling prices across products; (b) total selling time; and (c) total selling time.

S3. The salesperson's objective function can be: (a) to maximize income subject to a time constraint (the case of a 'fixed total time' salesperson); (b) to achieve 'fair income' (the case of a 'fair income' salesperson); or (c) to maximize utility (a 'utility-maximizing' salesperson).
S4. The constraint facing the salesperson varies with the objective function assumed: (a) the salesperson faces a total-selling-time constraint; or (b) he faces a 'fair income' constraint; or (c) he faces no constraints in the utility-maximization case.

S6. Sales of product \(i\) are a function of selling effort for all \(n\) products and all \(n\) prices.

S7. The sales response function is formally modeled as in Weinberg [1978] (see Equation (10)).

S9. The firm knows the salesperson's utility function, but not the sales-response function.

In fact, although Srinivasan is the first to express assumption S9, previous models' solutions and results would be the same with this assumption as well. This is because, given certainty in the sales response function, there is a one-to-one mapping between effort and sales: thus, observing sales made by a salesperson is equivalent to observing effort directly, and compensation can therefore be awarded on the basis of output (i.e. commission) to align the firm's and the salesperson's incentives.

Srinivasan shows, as do those before him, that an optimal compensation plan for objective functions (a) and (b) is the equal-gross-margin commission scheme. He uses the notion of opportunity cost of time to derive the exact commission rate that is optimal in case (a), of the fixed-total-time salesperson. For the utility-maximizing salesperson, however, an equal-commission-rate policy is generally suboptimal, since the salesperson is deciding both the allocation of selling time across products and total selling time itself; the equal-commission-rate policy induces too little total time allocation when it produces the right mix of time allocated across products. Srinivasan derives optimal unequal commission rates in at least one example, where the salesperson's opportunity cost of time is linear in total selling time and where sales response is given by:

\[
Q_i = d_i (t_i)^{\gamma_i}, \quad 0 < \gamma_i < 1. \tag{11}
\]

In this special case, optimal commission rates are equal to the corresponding elasticities of sales response to selling effort: \(B_i\) equals \(\gamma_i\) for all \(i\). This result makes intuitive sense, since salespeople should not be induced to aggressively sell products that have low responsiveness to selling effort.

Finally, through various examples, Srinivasan establishes that heterogeneous salesforces are not usually optimally compensated with equal-commission-rate policies, and that there is some profit loss when the firm is constrained to offer only one compensation plan to a heterogeneous salesforce.

**Summary of results in microeconomic models of salesforce compensation**

Farley's seminal paper shows that with a non-stochastic sales response function, an income-maximizing salesperson, constant marginal costs, and no across-product or across-salesperson interactions, an all-commission compensation plan with
commission rates set as an equal percentage of gross margins maximizes the firm's profits. The equal-commission-rate policy is also shown to be optimal when salespeople set prices as well as selling time across products [Weinberg, 1975], and under varying assumptions on the salesperson's objective function [Weinberg, 1978; Srinivasan, 1981]. When salespeople maximize utility and have a nonzero opportunity cost of time, however, unequal commission rates are generally optimal [Srinivasan, 1981]. Dynamic effects of selling effort in one period on sales in later periods also negate the equal-commission-rate policy in general [Tapiero & Farley, 1975], although not necessarily the optimality of all commission-rate policies. True interdependence among salespeople, as in Davis & Farley's [1971] assumption of non-constant marginal costs, also implies that an equal-commission-rate policy is suboptimal; the solution in such situations of interdependence may involve iterative methods with the salesforce to reveal optimal commission–quota combinations.

Thus, in general, the equal-commission-rate result holds only under fairly strict conditions, not the least of which is non-stochasticity of the sales-response function. Also crucially important is a lack of any fundamental interdependence among decisions about effort allocation or pricing. Thus, team-selling situations and situations where products in a line are closely interrelated in demand are outside the scope of this set of models.

Further, the articles generally do not question the underlying focus on commission-only salesforce-compensation plans. Srinivasan [1981] recognizes that his solutions are sufficient, but may be more than necessary, conditions for profit maximization. But in general, the authors do not consider the more basic question of the elements in the optimal salesforce-compensation plan, or indeed, the uniqueness of their optimum plan. Questions of the appropriate mix of commissions, bonuses and salaries are therefore ignored. As we will see below, the all-commission plan is never a unique optimum under certainty in the sales-response function. The optimal level of total compensation is also generally ignored (one exception being Srinivasan [1981]). Finally, because of the assumption of certainty in the sales-response function, the authors are unable to investigate the effects of differences in risk attitude of the salesforce or variance in the sales-response function on optimal compensation.

Despite these drawbacks, this literature is useful in defining the ultimate goal of salesforce compensation-setting: the aligning of incentives between the firm and its salesforce. The later literature using agency theory continues with this basic focus, and also seeks to answer some of these challenges posed by the first phase of analytical research in the area. We now turn to this literature.

3. Model structure/techniques: Agency-theoretic approach

Concurrent with the development of the microeconomics-based analytic approach to salesforce compensation, a new branch of economic analysis was also being developed: agency theory. Agency theory is designed to analyze problems where a principal (e.g. the firm) hires an agent (e.g. the salesperson) to perform some
action(s) for it (e.g. exert selling effort). In this framework, the responsiveness of output (e.g. sales) to the agent's input (e.g. effort) is assumed to be stochastic,\(^3\) and further, it is assumed that the principal can observe the agent's effort either imperfectly or not at all. Agency theory assumes that the principal and the agent have different attitudes toward risk: typically, the principal is assumed to be risk-neutral and the agent to be risk-averse. Henceforward, we will refer to the principal as the firm, and the agent as the salesperson.

Within the agency-theory literature, models make varying assumptions about the amount of information the firm has about the salesperson's response function. If the firm has complete information about the form of the response functions across various salespeople (e.g. differences in the productivity of their sales effort), but cannot observe effort itself, the firm is said to face a problem of *moral hazard*: there is the risk that the salesperson will shirk in the provision of effort, and that the firm will not be able to tell whether a particular outcome was due to the salesperson's (lack of) effort or to the stochastic element in the response function. If, in addition, the firm does not know a priori which salesperson has which level of productivity, the firm is said to face the problem of *adverse selection* as well: it may incorrectly classify a low-productivity salesperson as a high-productivity one because of a strongly positive draw from the error distribution in the sales-response function. Agency-theoretic models in economics, and their adaptations in marketing, have attacked the problem of optimal compensation under these various conditions.

The assumptions of uncertainty in the sales-response function, differences in risk attitudes, and unobservability of salesperson effort are key in differentiating the agency-theory approach from the microeconomic approach discussed in Section 2 above. In particular, one basic insight from agency theorists concerns the limiting case of certainty in the sales-response function. Under certainty, there is a simple and elegant alternative to the equal-commission-rate structures posited by earlier authors: the so-called 'forcing contract'. This contract promises compensation *exactly* equal to the salesperson's opportunity cost of time if he generates the profit-maximizing levels of sales for the firm (e.g. a strict quota), and zero otherwise. The salesperson will be just willing to do this job for the firm, since it generates (with certainty) income equal to his minimum acceptable level. Thus, the solution both satisfies the salesperson and maximizes the firm's profits. This implies that all the results in the certainty literature are feasible, but not unique, solutions to the firm's salesforce-compensation problem.

But in an agency-theory context, a forcing contract does not solve the incentive-incompatibility problem, because of the uncertainty in the sales-response function and the difference in risk attitudes of the salesperson and the firm. A risk-averse salesperson will be unwilling to undertake risky activities that a risk-neutral firm may find profit-maximizing, if the salesperson is forced to bear all the selling risk (as happens in a forcing-contract plan). The result will be inefficiently high total pay levels (to provide the necessary risk premium to meet the minimum-utility

\(^3\)Or at least, if the sales-response function is not stochastic, its true form is only known to the salesperson, and not to the firm (this means the firm sets compensation as if the sales-response function were stochastic). See Rao (1990) for an example of such a model.
requirement of the salesperson) and a misallocation of selling effort toward unduly low-risk selling activities. Agency-theoretic modeling seeks to solve this problem by developing optimal risk-sharing contracts that reduce the risk in the pay plan while still encouraging selling effort. In this section, we first elucidate the early models and then describe how later modeling efforts expand upon the initial contributions.

Berger [1972, 1975] develops a model that is a hybrid between the certainty approach profiled in the above section of this paper and the agency-theoretic approach. He posits a Farley-type [1964] model, but allows sales to be stochastic and varies the salesperson’s risk attitude. He assumes compensation is via commission only. He shows that Farley’s equal commissions on gross margins create compatible incentives when the salesforce is risk-neutral. He further shows that the Farley commission structure aligns the incentives of the firm and either risk-averse or risk-seeking salespeople, as long as the variance of the sales-response function is not a function of sales effort (that is, mean sales vary with effort, but not sales variance). But when sales-response variance is proportional to sales effort, for example, risk-averse salespeople should receive a lower commission rate on a product with a higher variance (holding profit margin constant). Conversely, risk-seeking salespeople should receive a higher commission rate on such a product.

The critical constraints in Berger’s approach, which are relaxed in true agency-theoretic approach, are (1) his assumption of commission-only pay; and (2) his restrictive choice of sales-response functions. The basic agency-theory literature in economics (see, for example, Harris & Raviv [1978, 1979], Holmstrom [1979], or Shavell [1979], among many others) focuses on the derivation of an optimal compensation contract form in a moral-hazard context. The first application of this approach in the marketing literature is in the Basu, Lal, Srinivasan & Staelin [1985] model, hereafter referred to as BLSS. The BLSS model is patterned after Holmstrom [1979], but applies the agency-theoretic framework directly to the salesforce-compensation context and develops many specific predictions based on distributional assumptions on the sales-response function and salesperson’s utility function. Because of its position as the first application in marketing, we summarize the form of the BLSS model here and use it as a basis for comparison with later models. Tables 13.4, 13.5 and 13.6 summarize the structure of the BLSS model and contrast its assumptions with those of later models in the marketing literature.

BLSS make several assumptions concerning the salesperson’s utility function and behavior, the sales-response function, and the firm. The salesperson is characterized by the first five assumptions below (see Table 13.4 for a contrast between these assumptions and those in later papers in the area):

B1. There is only one salesperson. (Equivalently, BLSS could have assumed many salespeople who are completely unrelated in their activities.)

B2. The salesperson is risk-averse.

B3. Selling effort is the salesperson’s only choice variable.

B4. The salesperson’s objective is to maximize expected utility. Utility is a separable function of income and the disutility for effort:

\[ W = U(s) - V(t), \]  

(12)
Table 13.4
Agency-theoretic models of salesforce compensation: assumptions on the salesforce

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td><strong>Base case:</strong> BLSS [1985]</td>
<td>Single salesperson</td>
<td>Risk-aversion</td>
<td>Sales effort only</td>
<td>Maximize expected utility</td>
<td>Single minimum acceptable utility level, $m$</td>
</tr>
<tr>
<td><strong>Comparison of BLSS [1985] with:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lal [1986]</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>Lal &amp; Staelin [1986]</td>
<td>Multiple salespeople, two possible types</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>Rao [1990]</td>
<td>Multiple salespeople, a continuum (beta-distributed) of types</td>
<td>Risk-neutral</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>Dearden &amp; Lilien [1990]</td>
<td>Same</td>
<td>Same</td>
<td>Sales effort only, in each of two time periods</td>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>Srinivasan &amp; Raju [1990]</td>
<td>Same</td>
<td>Same</td>
<td>Same</td>
<td>Same; but do not require specific functional form for results</td>
<td>Same ($m_1 = m_2 = m$)</td>
</tr>
<tr>
<td>Lal &amp; Srinivasan [1991]</td>
<td>Same</td>
<td>Constant absolute risk-aversion: $U = -\exp[(-rt - V(t))]$, $V$ = monetary disutility for effort</td>
<td>Sales effort only; but continuously adjustable over accounting period</td>
<td>Same</td>
<td>Same</td>
</tr>
</tbody>
</table>

These assumptions are discussed more fully in Section 3 of the paper. Assumptions are noted for the BLSS model, and deviations from the BLSS assumptions are noted for the other papers in the table.
Table 13.5
Agency-theoretic models of salesforce compensation: assumptions on the sales-response function

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Base case: BLSS [1985]</td>
<td>One product in line</td>
<td>Sales are a stochastic function of selling effort</td>
<td>Dollar sales are distributed either gamma or binomial</td>
</tr>
<tr>
<td><strong>Comparison of BLSS [1985] with:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lal [1986]</td>
<td>Same</td>
<td>Stochastic function of both selling effort and price</td>
<td>No specific parametric assumption on dollar sales distribution $x_t - \delta_t(1 + \xi_t)$</td>
</tr>
<tr>
<td>Lal &amp; Staelin [1986]</td>
<td>Same</td>
<td>Stochastic function of selling effort only, but $f_u$ is not equal to $f_u f_s$ exhibits first-order stochastic dominance over $f_s$</td>
<td></td>
</tr>
<tr>
<td>Rao [1990]</td>
<td>Same</td>
<td>Same</td>
<td>Two examples: $x_t = \theta t$ and $x_t = x_s(1 - \exp(-\theta t))$</td>
</tr>
<tr>
<td>Dearden &amp; Lilien [1990]</td>
<td>Same</td>
<td>Same (no lagged effect effects on sales)</td>
<td>Same</td>
</tr>
<tr>
<td>Srinivasan &amp; Raju [1990]</td>
<td>Same</td>
<td>Same</td>
<td>No specific functional form required</td>
</tr>
<tr>
<td>Lal &amp; Srinivasan [1991]</td>
<td>One product or multiple products; multi-product case assumes independence in demand and cost</td>
<td>Same</td>
<td>$E(x(t) = h + kt, \ h, k &gt; 0$; $x \sim N(h + kt, \sigma^2)$</td>
</tr>
</tbody>
</table>

These assumptions are discussed more fully in Section 3 of the paper. Assumptions are noted for the BLSS model, and deviations from the BLSS assumptions are noted for the other papers in the table.
<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Base case: BLSS [1985]</strong></td>
<td>Marginal cost is a constant fraction of price</td>
<td>Firm knows both salesperson's utility function and his sales-effort productivity</td>
<td>Both form and total size of compensation scheme, $s(x)$</td>
<td>Maximize expected profit (i.e. risk-neutral)</td>
</tr>
<tr>
<td><strong>Comparison of BLSS [1985] with:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lal [1986]</td>
<td>Same</td>
<td>Same</td>
<td>Firm sets $s(x)$ and also decides whether it or the salesperson sets price</td>
<td>Same</td>
</tr>
<tr>
<td>Lal &amp; Staelin [1986]</td>
<td>Dollar marginal cost is a constant (set equal to zero)</td>
<td></td>
<td>Firm chooses a menu of $s(x)$ plans and how many of $q$ total territories to leave unstaffed</td>
<td>Same</td>
</tr>
<tr>
<td>Rao (1990)</td>
<td>Same</td>
<td></td>
<td>Firm chooses a menu of $s(x)$ plans</td>
<td>Same</td>
</tr>
<tr>
<td>Dearden &amp; Lilien [1990]</td>
<td>Constant marginal cost in each period, $c_2$ is decreasing in $x_i$</td>
<td>Same</td>
<td>Firm chooses $s(x)$ in periods $i = 1, 2$</td>
<td>Maximize discounted expected profit stream over two-period horizon</td>
</tr>
<tr>
<td>Srinivasan &amp; Raju [1990]</td>
<td>Same; known for each salesperson</td>
<td>Same</td>
<td>Form of compensation plan is constrained to be salary plus commission if sales exceed quota; firm sets salary, commission and quota parameters</td>
<td>Same</td>
</tr>
<tr>
<td>Lal &amp; Srinivasan [1991]</td>
<td>Same</td>
<td>Same</td>
<td>Same; optimal scheme shown to be linear in total sales over accounting period</td>
<td>Same</td>
</tr>
</tbody>
</table>

These assumptions are discussed more fully in Section 3 of the paper. Assumptions are noted for the BLSS model, and deviations from the BLSS assumptions are noted for the other papers in the table.
where
\[ s = \text{total income to the salesperson}, \]
\[ t = \text{time spent selling}, \]
\[ U'(s) > 0, \ U''(s) < 0 \quad \text{(risk-aversion of salesperson)}, \]
\[ V'(t) > 0, \ V''(t) > 0 \quad \text{(increasing marginal disutility for effort)}. \]

B5. The salesperson requires a minimum level of utility equal to \( m \) to be willing to work for the firm.

Sales response is characterized by the next three assumptions in BLSS (see Table 13.5 for a listing of these assumptions and their contrast with later models):

B6. Only one product is sold. (Equivalently, many products could be sold as long as they are unrelated in demand or cost and their selling times do not make any total-selling-time constraint binding in the salesperson's effort-allocation decision.)

B7. Sales are affected only by selling time and a stochastic element; no other marketing-mix variables are modeled. The density function of dollar sales conditional on time spent selling is given by \( f(x|t) \).

B8. Dollar sales are assumed to be distributed either gamma or binomial. The firm's behavior is characterized by the last three assumptions (see Table 13.6 for a summary of these assumptions and their comparison with those in later papers):

B9. Marginal cost, \( c \), is a constant fraction of price.

B10. The firm knows the salesperson's utility functional form (Equation (12)), his minimum acceptable utility level, \( m \), and his sales-effort productivity, i.e. \( f(x|t) \).

B11. The firm's choice variable is \( s(x) \), the compensation plan for the salesperson. This includes both the form of the plan and the total compensation level.

B12. The firm's objective is to maximize expected profit. The profit function is given by,
\[
\Pi = \int_{0}^{\infty} [(1 - c)x - s(x)] f(x|t) dx. \tag{13}
\]

BLSS then use Holmstrom's [1979] model and solution technique to derive their basic result on optimal compensation. The philosophy of the model is similar to that in the certainty models discussed above: the firm sets compensation to maximize profits, constrained by the salesperson's control over selling effort and by a minimum utility constraint expressing the salesperson's opportunity cost of time. Because of the uncertainty inherent in the problem, however, the solution techniques are somewhat different.

Formally, the firm's optimization problem can be expressed as:

\[
\begin{align*}
\text{maximize} \quad & \int [(1 - c)x - s(x)] f(x|t) dx, \\
\text{subject to} \quad & \int [U(s(x))] f(x|t) dx - V(t) \geq m, \\
& \int [U(s(x))] f_{x}(x|t) dx - V'(t) = 0. \tag{14}
\end{align*}
\]
The first constraint ensures that the salesperson's expected utility is at least equal to \( m \), the minimum acceptable utility level. If this constraint were violated, the salesperson would leave the firm. The second constraint simply states that the salesperson chooses sales effort, \( t \), to maximize his utility, and the firm takes this utility-maximizing behavior into account in its compensation-setting problem.

Let \( \lambda \) be the Lagrange multiplier for the first constraint, on minimum utility, and let \( \mu \) be the Lagrange multiplier for the second constraint, on utility-maximizing sales-effort choice. Then Holmstrom's analysis shows that the optimum is characterized by four conditions which are functions of four unknowns: \( s(x) \), \( \lambda \), \( \mu \) and \( t \):

\[
\frac{1}{U'[s(x)']} = \lambda + \mu \frac{f_{x}(x|t)}{f(x|t)}, \tag{15a}
\]

\[
\int [(1 - c) x - s(x)] f_{x}(x|t) dx + \mu \left[ \int U[s(x)] f_{x}(x|t) dx - V'(t) \right] = 0, \tag{15b}
\]

\[
\int [U(s(x))] f_{x}(x|t) dx - V(t) = m, \tag{15c}
\]

\[
\int [U(s(x))] f_{x}(x|t) dx = V'(t). \tag{15d}
\]

Equation (15a) is derived from the condition \( \partial L/\partial s(x) = 0 \) (where \( L \) is the Lagrangian). Equation (15b) is the condition that \( \partial L/\partial t = 0 \). Equations (15c) and (15d) derive from the tightness of the minimum-utility constraint and the constraint on \( t \), respectively.

In order to get more specific insights, BLSS parametrize the problem by assuming a utility function exhibiting constant relative risk-aversion:

\[
U(s(x)) = \frac{[s(x)]^{\delta}}{\delta}, \quad \delta < 1. \tag{16}
\]

As \( \delta \) approaches 1, the salesperson's risk attitude approaches risk-neutrality. Then for sales distributed either gamma or binomial, BLSS show that the optimal sales-compensation scheme has the form:

\[
s(x) = [A + Bx]^{1/(1 - \delta)}, \quad A \geq 0, \quad B > 0. \tag{17}
\]

BLSS call \( A \) the 'salary parameter' and \( B \) the 'commission-rate parameter'. Both are nonlinear functions of the underlying parameters of the problem. They show how this can imply a number of real-world compensation-plan forms, including straight commission, progressive sliding commission, salary only, salary plus commission, salary plus progressive sliding commission, and salary plus commission beyond a sales target, depending on the values of \( A \) and \( B \).
This model structure produces many comparative-static results on optimal compensation (see Table 13.7 for a summary of the effects). As uncertainty, marginal cost, or minimum expected utility increase, the model predicts decreased optimal effort exertion; increased optimal salaries; decreased optimal commission rates; decreased profits at the optimum; and an increased ratio of salary to total pay. An increase in uncertainty or marginal cost causes expected salesperson income to fall, but an increase in minimum expected utility causes expected income to rise. Increased sales-effort productivity results in greater effort and greater firm profits, and an increased base sales rate (net of sales due to effort) increases the firm's profits as well. Basu & Kalyanaram [1990] verify these results through a numerical analysis, although their analysis is weakened by forcing a linear regression framework on the inherently nonlinear optimum of the BLSS model.

The BLSS/Holmstrom framework omits many important factors in setting optimal salesforce compensation. Only one salesperson is modeled, selling one product. Thus, the model does not consider any real interactions among salespeople or among products, such as team-selling, product complementarity, or economies of scale and scope. Effort produces concurrent sales; this approach abstracts away from situations of long lead times between initial salesperson contact and final sale. The firm is assumed to know the salesperson's utility function and minimum acceptable utility level, as well as the form of the sales response function; in a real setting, the firm may have to set compensation without this information. Price plays no role in generating sales in the BLSS model, nor do any marketing-mix variables other than selling effort. Later literature speaks to some of these limitations. Nevertheless, this paper makes a fundamental contribution to the literature on salesforce compensation because of its treatment of uncertainty and its specific results on optimal pay plans.

Lal [1986] uses the BLSS framework to examine when it is optimal to delegate pricing authority to the salesforce. He changes assumptions B3, B7, B8 and B11 of the BLSS framework, respectively, as follows (using BLSS terminology rather than Lal's terminology for consistency):

L3. The salesperson may decide how to price the product as well as the selling effort devoted to it.

L7. Stochastic sales are affected by both selling time and price charged. The density function for sales is f(x|t, p), where p is the product price.

L8. No specific parametric distribution is assumed for dollar sales.

L11. The firm sets s(x), the salesperson's compensation plan. It also decides whether to set p itself or to delegate the pricing decision to the salesperson.

This research question is also asked by Weinberg [1975] in the context of sales-response-function certainty, but Weinberg constrains the salesforce-compensation contract to be commission only. As discussed above, Weinberg's solution ignores the additional possibility of a forcing contract on both price and quantity sold. Neither solution is feasible in the agency-theory context.

Lal then considers two cases in which the information endowments of the firm vary: in the first, the firm and the salesperson have symmetric information about
Table 13.7  
Comparative-static effects in agency-theory models of salesforce compensation

<table>
<thead>
<tr>
<th>Effect of:</th>
<th>Effect* on optimal:</th>
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<th></th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Effort</td>
<td>Salary</td>
<td>Commission ratio</td>
<td>Profit</td>
<td>Expected income</td>
<td>Salary/expected income</td>
<td>Quota</td>
</tr>
<tr>
<td>Increased uncertainty</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>N/A</td>
</tr>
<tr>
<td>Increased marginal cost</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>N/A</td>
</tr>
<tr>
<td>Increased minimum expected utility</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>N/A</td>
</tr>
<tr>
<td>Increased sales-effort effectiveness</td>
<td>$(B, L)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B, L)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>N/A</td>
</tr>
<tr>
<td>Increased base sales</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>N/A</td>
</tr>
<tr>
<td>Increased risk-aversion</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>N/A</td>
</tr>
<tr>
<td>Increased disutility for effort</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>N/A</td>
</tr>
<tr>
<td>Increased number of plans offered (menu)</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>$(B)$</td>
<td>N/A</td>
</tr>
<tr>
<td>Increased production learning effects</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>$(B, L)$</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*↑ = goes up; ↓ = goes down; 0 = no change; N/A = no hypothesis available in literature; ? = ambiguous, depending on distributional assumption on sales response function. After each directional indicator is a notation for papers producing the comparative static result: B = Basu, Lal, Srinivasan & Staelin [1985], DL = Dearden & Lilien [1990], LS = Lal & Srinivasan [1991] and R = Rao [1990].
the sales-response function, while in the second, the salesperson’s information about the sales-response function is superior to that of the firm. In the symmetric-information case, the firm knows just as well as does the salesperson what is the best price to set. It can therefore write a compensation contract that rewards the salesperson based on sales output and the optimal price—no matter whether the salesperson or the firm itself sets that price. Via this logic, Lal shows that the firm is indifferent between delegating pricing authority to the salesperson and centralizing price-setting when information is symmetric. However, when the salesperson has superior information to that of the firm, Lal shows that delegating pricing authority to the salesperson cannot make the firm worse off than centralizing pricing decisions inside the firm, and in some examples, can make the firm better off. Intuitively, decisionmaking authority is best relegated to that decisionmaker with the greatest amount of information.

Lal & Staelin [1986] model optimal compensation-setting in a situation where salespeople may differ in their sales-effort productivity, and the firm does not know which are high- and which low-productivity salespeople. They alter assumptions B1, B5, B7, B8, B9, B10 and B11 of the BLSS framework, respectively, as follows:

LSt1. There are multiple salespeople, each of whom is one of two types, corresponding to his sales-effort productivity: h (‘high’) or l (‘low’).

LSt5. Salespeople of different types may have different minimum acceptable utilities, so that $m_t$ need not be equal to $m_h$.

LSt7. The density function of dollar sales conditional on time spent selling is given by $f_i(x_i|t_i)$, where $i = h$, $l$. The functions have the property that $f_h$ first-order stochastic dominates $f_l$. That is, for any $t^*_l = t^*_h$, the cumulative distribution function of $f_h$ is less than or equal to that of $f_l$ for all values of $x$, and is strictly less for at least one value of $x$.

LSt8. The sales-response function for salesperson $i$ is:

$$x_i = \bar{\alpha}_i t_i + \xi_i,$$

where $\bar{\alpha}_i > 0$, $\bar{\alpha}_l < 0$. Thus, sales effort is productive at a decreasing rate.

LSt9. Dollar marginal cost is constant and, without loss of generality, set equal to zero.

LSt10. The firm knows that all salespeople are either of type $l$ or type $h$, but does not know any individual salesperson’s type. However, the firm has a prior belief (represented by the probability density function $g(z)$) on $z$, the number of $h$-type salespeople in a group of $q$ salespeople. Then $P$, the expected number of $h$-types in the group of $q$ salespeople employed by the firm, is $\int z g(z)dz$.

LSt11. The firm’s choice variable is a set of $x(z)$’s, where the set may have more than member (the firm may offer a menu of plans). The firm also must choose $q$ salespeople to staff its $q$ sales territories; or it may choose to leave a subset of the $q$ territories unstaffed.

Given this set of assumptions, Lal & Staelin use a BLSS-style framework to analyze three strategies:
Strategy 1. Offer one contract only, such that only h-type salespeople work for the firm.

Strategy 2. Offer one contract only, such that only l-type salespeople work for the firm.

Strategy 3. Offer a menu of two contracts that are 'truth-revealing', that is, that cause each salesperson to self-select into the contract designed for his type. In this scheme, each salesperson opts for one of the contracts at the start of the sales period, and is paid at the end of the period based on sales outcomes.

Lal & Staelin show first that if $m_h \leq m_l$, and for sufficiently high $P$, Strategy 1 always dominates Strategy 2, because of the equal or less stringent minimum-utility requirements and better expected sales outcomes from the h-type salespeople. However, if h-types demand significantly higher minimum utilities (and hence pay), or if there are not likely to be many of them in the population, Strategy 2 can actually dominate Strategy 1. Intuitively, h-types may just be too expensive for the firm to hire if $m_h$ is significantly greater than $m_l$; or there may be too high a risk of leaving profitable sales territories unstaffed if $P$ is too low. Finally, for $m_h \leq m_l$ and for sufficiently low $P$, Strategy 3 (offering a menu of plans) is a superior choice to attracting just one type of salesperson via Strategies 1 or 2. Intuitively, the firm is better off attracting both types of salespeople when there are likely to be relatively few h-type salespeople in the population hired to staff and q sales territories.

Lal & Staelin also reinterpret their results with salespeople who vary not in their sales-effort productivity, but in their risk-aversion. Here, h-types are less risk-averse than l-types. The results go through analogously to those discussed above.

Rao’s [1990] model is conceptually very similar to that of Lal & Staelin [1986]. Rao looks at the problem of creating optimal compensation plans for a heterogeneous salesforce, but seeks conditions under which these plans are simple linear compensation plans (salary plus commission for achieving sales over quota). Rao’s assumptions differ from the BLSS assumptions B1, B2, B7/8 and B10, respectively, as follows:

R1. There are many salespeople, who differ by skill level (that is, marginal productivity of sales effort). The frequency distribution of the skill levels is a beta distribution.

R2. All salespeople are risk-neutral. Equivalently, if the sales-response function is non-stochastic, salespeople can have any risk attitude (because their knowledge of the non-stochastic sales-response function makes risk attitude a moot point).

R7/8. Sales are affected only by selling time and a stochastic element. For any sales response function $x = \psi(t;x;\theta)$ that can be inverted to yield $t = f(x;\theta)$, Rao assumes that: $\partial f/\partial x > 0$, $\partial f/\partial \theta < 0$, $\partial^2 f/\partial x \partial \theta < 0$, $\partial^2 f/\partial x^2 > 0$, $\partial^2 f/\partial \theta^2 > 0$, $\partial^2 f/\partial x^2 \partial \theta < 0$ (i.e., the marginal increase in effort required to achieve an incremental unit of sales decreases with increasing skill); and $\partial^3 f/\partial x \partial \theta^2 > 0$ (i.e. the
marginal effort required to achieve a unit sales increase is less for a more skilled person \(-\partial^2 f/\partial x \partial \theta < 0\) – and this effect is moderated, the higher the base skill-level). R10. The firm knows the form of the salespeople’s utility functions, but does not know any individual salesperson’s skill level. However, the firm knows the distribution of skill levels.

Under the above assumptions, Rao shows the existence of a *separating equilibrium*, where pay plans serve to separate lower-skill from higher-skill salespeople. In this optimal (nonlinear) compensation plan for a heterogeneous workforce, uncertainty induces two types of inefficiencies: first, every salesperson except the *most highly skilled* exerts less effort than if he were in a homogeneous workforce or if all skill levels were known. Second, every salesperson except the *least skilled* earns more money than if he were in a homogeneous workforce or if all skill levels were known. The optimal nonlinear plan described here can also be implemented by a menu of linear plans described graphically by the tangents to the optimal convex compensation scheme.

Dearden & Lilien [1990] look at a dynamic variation on Basu, Lal, Srinivasan & Staelin [1985]. Specifically, they model a two-period horizon for the firm and workforce, where production-learning effects make second-period marginal costs decline as first-period sales increase. Intuitively, the firm now has a reason to increase selling incentives in period 1, because of the positive externality on costs in period 2. The authors’ model varies from the BLSS model in assumptions B3, B9, B11 and B12 as follows:

**DL3.** While selling effort is the salesperson’s only choice variable, as in BLSS, now the salesperson has to decide in a two-period horizon how to allocate effort. However, Dearden & Lilien’s assumption of a competitive labor market implies that foresighted sales-effort allocation across periods is the same as myopic effort allocation, since the salesperson does not benefit from any of the positive cost externality accruing to the firm in period 2.

**DL9.** Marginal cost in each period is constant, but that in period 2, is lower, the higher are sales in period 1 \((c_2 = c_2(x_1)\) and \(c'_2\) is negative). This reflects production-learning economies.

**DL11.** The foresighted firm sets \(s_1\) and \(s_2\) in periods 1 and 2, respectively, taking into account the positive externality of period-1 sales on profitability in period 2.

**DL12.** The firm maximizes its discounted expected profit stream over the two-period horizon.

For the gamma distribution on sales, utility of income given by \(U(s) = 2\sqrt{s}\), and using BLSS’s terminology, the optimal compensation scheme has the same form as BLSS’s optimal compensation function (Equation (17) above), but the salary parameter \(A_t\) and the commission parameter \(B_t\) take into account the positive production externality:

\[
s(x_t) = \left[ \alpha_t + \frac{\mu g'(t_i) g}{g^2(t_i)} [x_t - g(t_i)] \right]^2, \tag{19}
\]
where \( g(t) \) is expected sales and \( g^2(t)/q \) is the variance of sales. \(^4\) This solution is exactly BLSS's solution, but with period-specific subscripts for sales, selling effort and the Lagrange multipliers. This time-dependency gives Dearden & Lilien their main result: the greater is the increase in period-2 discounted expected profit due to an increase in period-1 sales, the lower is \( A \), and the higher is \( B \). That is, the greater is the cross-period production cost externality, the greater should be the incentive to sell in period 1 to maximize profit margins in period 2.

Srinivasan & Raju [1990] build upon the basic BLSS model to show how optimal compensation changes when the firm constrains pay to be in the form of salary and commission beyond quota and when sales territories have unequal sales potentials. Thus, they modify assumptions B1, B8 and B11 in the basic BLSS model as follows:

SR1. There are two salespeople in the salesforce, each covering one sales territory.

SR8. No specific functional form of the sales-response function is required for the model's results. However, if the density function for sales conditional on effort is \( f_i(x|t)_0 \), \( i = 1, 2 \), then the authors assume \( f_1(x|t) = f_2(x + n|t) \). This implies that the marginal productivity of the salespeople's effort is equal across the two sales territories, but the means differ.

SR11. The firm is constrained to choose a particular form of compensation scheme, given by:

\[
s(x) = \begin{cases} 
A & \text{if } x \leq q, \\
A + B(x - q) & \text{if } x \geq q.
\end{cases}
\] (20)

Srinivasan & Raju also do not require any specific functional form on the utility function to derive their results, unlike BLSS. Solving the BLSS problem (Equation (14) above) subject to the constraint (20) on the form of the compensation plan, they derive their main result that the optimal quota-based compensation plan is characterized by:

\[
s^*_2(x + n) = s^*_n(x).
\] (21)

That is, both salespeople have the same salary, \( A \), and the same commission-rate parameter on sales beyond quota, \( B \), but the salesperson assigned to the higher potential territory has a correspondingly higher base quota.

However, Srinivasan & Raju point out that this solution generates less profit for the firm than does the BLSS result without the compensation-plan constraint. Why, then, would a firm want to use it? The authors respond by appealing to the plan's equity, flexibility and simplicity. They argue that cross-territory differences

\(^4\) This is a corrected version of Dearden & Lilien's Equation (3), p. 184. The original version has typographical errors.
would be compensated unfairly in the straight BLSS solution, with resulting
grumbling in the salesforce and reduced morale. Although beyond the scope
of their model, they argue intuitively that more-satisfied salespeople will generate
higher profits in the long run.

Lal & Srinivasan [1991] use a model first developed by Holmstrom & Milgrom
[1987] to examine optimal compensation when the salesperson can adjust his
effort decision more frequently than the firm can adjust the compensation plan.
Holmstrom & Milgrom describe a set of conditions under which optimal compensa-
tion is a linear function of total sales over the accounting period. We summarize
these as deviations from BLSS's assumptions B2, B3, B6 and B8:

LSr2. The utility function exhibits constant absolute risk-aversion, and disutility
for effort is expressed in monetary terms so that total utility is not separable in
the utility for income and the disutility for effort:

\[ W = -\exp[-r(s - V(t))], \quad (22) \]

where \( r \) is the constant absolute risk-aversion parameter and \( s - V(t) \) is the net
monetary value of income and disutility for effort.

LSr3. As in BLSS, the salesperson chooses only sales effort; but effort is
continuously adjustable over the accounting period.

LSr6. The major emphasis is on the single-product case, but the authors consider
multiple product lines where products are completely independent in demand and
cost (no complementary or substitutability in demand; no economies of scope or
scale).

LSr8. Sales at any instant of time (over which the salesperson can vary effort)
are distributed normal:

\[ E(x|t) = h + kt, \quad h, k > 0; \]
\[ x \sim N(h + kt, \sigma^2). \quad (23) \]

Thus, the mean (but not the variance) of sales is dependent on sales effort. The
authors assume \( h \) is large enough to guarantee positive sales virtually all the time.
With these assumptions, cumulative sales as a function of time follow Brownian
motion, where the drift is a function of sales effort but the variance is not.

Holmstrom & Milgrom show a very elegant result for this problem: optimal compen-
sation is a linear function of total sales over the accounting period, and
further, the problem can be treated as a static problem due to the lack of time-
dependence in the salesperson's effort decisions. This greatly simplifies the analysis
and permits Lal & Srinivasan to derive many comparative-static results on optimal
compensation, as summarized in Table 13.7. These results are broadly consistent
with those in BLSS, differing only in two cases: the effect of changes in minimum
utility \( m \) on optimal effort \( t \) and on the optimal commission rate \( B \) (both null
effects in this model, and both negative in BLSS).
Summary of results in agency-theory models of salesforce compensation

The agency-theory paradigm yields a rich set of hypotheses concerning optimal form of the salesforce-compensation plan (salary, commission, quota, menus of plans), relative emphasis on incentive components in the plan, total pay levels, and their effects on salesforce effort and firm profitability. Several hypotheses concerning the absolute form of the plan emerge. In general, some form of risk-sharing characterizes the optimal compensation plan, because of the salesperson’s risk-aversion relative to the firm and the stochastic nature of sales response. This risk-sharing typically takes the form of a combination plan, including both salary and some incentive component. It can be profitable to offer a menu of plans when the salesforce is made up of people of differing abilities and the firm cannot distinguish ability a priori [Lal & Staelin, 1986; Rao, 1990]. The firm may also be incompletely informed about demand responsiveness to price, in which case it can make sense to delegate pricing responsibility to the salesperson and alter compensation accordingly [Lal, 1986]. Constraining compensation to include quotas can increase equity across the salesforce, but may decrease short-term profitability [Srinivasan & Raju, 1990].

Comparative-static results predict the effect of a change in some parameter of the model on optimal pay parameters. These parameters, summarized in Table 13.7, include characteristics of the salesforce (minimum expected utility, sales-effort effectiveness, risk-aversion, disutility for effort), the firm (marginal cost, base sales rates, production-learning effects), and the environment (uncertainty). Interestingly, models tend to produce complementary rather than redundant results, and there is little contradiction among the models in their predictions.

Some issues remain unresolved. The plethora of results comes at the expense of specificity of assumptions about functional forms of the sales-response function and utility function. We have some comfort in the ‘convergent validity’ provided by the redundant results across some models, but a very general theoretical treatment has yet to generate rich predictions. The agency-theory approach in the marketing literature on salesforce compensation also does not attack such issues as when a commission is preferred to other forms of incentive pay, such as bonuses or contests. The differences in selling consumer and industrial products, or short-versus long-selling-cycle products, is also ill addressed in this literature. Product-line selling and team-selling are not explicitly modeled in the marketing area. Further, the firm and its salespeople are required to have extensive information about the salespeople’s utility functions and minimum acceptable utility levels—information that is unlikely to be available in many real-world situations.

Finally, as in any analytical area, we should question the validity of the models by confronting them with empirical evidence and statistical tests. In the next section, we focus on this task.

4. Empirical evidence on salesforce-compensation models

In the salesforce-compensation area, many empirical studies have of course been done. Space constraints prohibit our reviewing them all. Instead, we focus on those
studies that speak directly to tests of the theories advanced above. While the empirical literature testing management-science models of salesforce compensation is still rather limited, it gives some flavor for the applicability (and testability) of the theories' predictions.

The microeconomics-based models in Section 2 generally assume that the salesperson maximizes income, while a cornerstone of the agency-theory approach discussed in Section 3 is the assumption of utility maximization. Two early studies provide evidence that an income-maximization assumption is not representative of actual salesperson preferences. Winer's [1973] experimental work finds that salespeople at one company are not 'income maximizers' but rather 'quota achievers'. In another study, Darmon [1974] finds that salespeople seem to minimize effort subject to reaching an income threshold, or maximize 'satisfaction' subject to a time constraint.

These two studies provide some preliminary support for the assumption of utility maximization over income maximization. Later empirical work does not test this underlying 'maintained hypothesis', but instead investigates whether the comparative-static implications of agency-theory models are borne out in real-world practice. The papers differ in the datasets used as well as in the scope of their inquiries. John & Weitz [1988], Oliver & Weitz [1991] and Coughlan & Narasimhan [1992] use cross-sectional data. John & Weitz [1988] survey 161 manufacturing firms with annual sales of at least $50 million. Oliver & Weitz obtain a sample of 367 salespeople responding to a questionnaire on attitudes toward compensation plans. Coughlan & Narasimhan [1992] use secondary data collected by Dartnell Corporation from 286 firms in 39 different industry classifications. Eisenhardt [1988] collects data from 54 specialty stores in a single shopping center, thus focusing on retail salespeople exclusively. Finally, Lal, Outland & Staelin [1990] survey 77 sales manager/salesperson dyads within a single Fortune 500 firm selling computer equipment and services. There is some debate in the literature about the appropriateness of testing an individual-level theory (such as that in Basu, Lal, Srinivasan & Staelin [1985]) with aggregate, cross-sectional data; Lal, Outland & Staelin maintain that their approach is superior because it holds cross-firm variation constant. However, in some sense, none of these studies can test the BLSS or other agency-theory approaches purely, because all take measurements from multi-person, multi-product salesforces, while BLSS postulate a single salesperson selling a single product. We should thus take the results with some qualification in any case.

Table 13.8 summarizes the evidence these papers provide to test the comparative-static effects in the agency-theory literature. Each paper contains some other evidence as well (e.g. Coughlan & Narasimhan [1992] also tests some hypotheses from the executive-compensation and economics agency-theory literatures), but we will focus here on the evidence directly pertinent to the theories posited above. Some general observations deserve mention immediately. First, Table 13.8 has many empty cells; this is a sign both that the theories have been incompletely tested and that empirical tests of agency theory are rather difficult to accomplish. For example, it is difficult to operationalize measures of risk-aversion or disutility for effort in the salesforce. Second, many results are not statistically significant. This may not be a sign that we should reject the theory overall, but rather a sign
<table>
<thead>
<tr>
<th>Effect of:</th>
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<th>Salary</th>
<th>Commission</th>
<th>Profit</th>
<th>Expected income</th>
<th>Salary/expected income</th>
<th>Quota</th>
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<td>Profit</td>
<td>Expected income</td>
<td>Salary/expected income</td>
<td>Quota</td>
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<td>ns(JW)</td>
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</tr>
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<td>ns(JW)</td>
<td>√(JW)</td>
<td>ns(JW)</td>
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</tr>
<tr>
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<td>ns(JW)</td>
<td>√(JW)</td>
<td>ns(JW)</td>
<td>√(JW, CN)</td>
<td>n(JW)</td>
<td></td>
</tr>
<tr>
<td>Increased sales-effort effectiveness</td>
<td>√(JW)</td>
<td>√(JW)</td>
<td>√(JW)</td>
<td>√(JW)</td>
<td>n(JW, CN)</td>
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<td>Increased base sales</td>
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<td>√(JW)</td>
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<td>√(JW)</td>
<td>n(JW)</td>
<td>n(JW)</td>
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</tr>
<tr>
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<td>√(JW)</td>
<td>n(JW)</td>
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<td></td>
</tr>
</tbody>
</table>

*ns = not statistically significant at the 90% level; √ = significant at the 90% level and in the hypothesized direction; ∝ = significant at the 90% level and in the opposite direction from that hypothesized. A blank entry in a cell indicates that no empirical research has addressed that competitive-static effect. Parenthetical notations are references to empirical papers: JW = John & Weitz [1988]; LOS = Lal, Outland & Staelin [1990]; CN = Coughlan & Narasimhan [1992]; OW = Oliver & Weitz [1991]; and E = Eisenhardt [1988]. Thus, for example, the cell in the row for Increased sales-effort effectiveness and the column for Salary/expected income should be read. John & Weitz [1988] find significant evidence contrary to, while Lal, Outland & Staelin and Coughlan & Narasimhan find significant evidence consistent with, the hypothesis that Salary/expected income is negatively affected by increased sales-effort effectiveness.
of insufficient power of the statistical tests or insufficiently precise operationalizations of theoretical constructs.

On a more positive note, those results that are statistically significant tend to be consistent with the theory (the exceptions all being from the John & Weitz [1988] paper). There is considerable strength of support for hypotheses predicting the ratio of salary to total pay across several of the studies. Some support is also found for hypotheses predicting total expected income levels. Hypotheses concerning the effect of sales-effort effectiveness on compensation are not rejected in several studies.

In sum, while empirical results are inconclusive or incomplete in many cases, we certainly cannot reject agency-theory entirely as a predictive paradigm for salesforce compensation. We feel justified in reiterating the familiar refrain that more work is needed, with better empirical proxies for theoretical constructs and a wider range of hypotheses tested. It is unlikely that all the hypotheses will ultimately be tested, however, due to difficulties in measuring some predictive variables in the theory.

The economics-based approach to modeling salesforce-compensation problems profiled in this section of the paper provides directional, as well as some absolute, insights on optimal compensation practice. It typically does not produce point estimates of the right compensation amount or split among various components, however. Precise normative inputs to compensation-setting are instead provided by the literature on decision support systems. We turn in the next section to a brief survey of this literature, and refer the reader to Chapter 16 on decision support systems for a more in-depth treatment of their application not just to the salesforce-compensation issue, but to other marketing problems as well.

5. Decision support systems for salesforce compensation

Decision support systems (or DSSs) differ from analytical economics-based models or empirical tests in several ways. First, they are typically designed for direct managerial implementation: they have a distinctly normative focus, rather than a predictive focus. Second, they capitalize on expert judgments from decision-makers (in our case, the salesforce or sales managers themselves) to parametrize key aspects of the models—for example, the salesperson’s utility function or a territory-specific sales-response function. Third, one typically cannot do a statistical test for optimality or robustness of the model, given the case-specific managerial inputs to the problem. Nevertheless, improvements in profit and sales performance observed over the pre-DSS situation are indicators of their usefulness.

In the salesforce-compensation area, DSSs typically marry the economics-based approach with specific managerial inputs to try to get a parametric representation of an otherwise abstract model that is immediately useful and believable to the sales manager. The trend in these models is toward user-friendliness, via easy (usually computerized) interactions between the managers and salespeople and the model itself.
The content and structure of DSSs for salesforce compensation is summarized in Tables 13.9, 13.10 and 13.11. These differ from Tables 13.1–3 and 13.4–6 in several ways. First, there is no easily identified 'base-case' DSS against which to compare later advances in the area. This is because each DSS seeks to remedy a particular information shortage of the firm, and these shortages have no natural progression over time (however, as we will see below, subsets of the DSSs are strongly interrelated). Second, the use of managerial and salesforce inputs to parametrize the model is accounted for in a new column, D13, in Table 13.11 that summarizes how judgmental inputs are used in each particular DSS. Third, DSSs have focused primarily on the setting of quota-bonus plans, assuming salary to be fixed and exogenously given. Thus, in contrast to the general salary-plus-commission plan derivation in the agency-theory literature, the DSS literature to date does not focus at all on the optimal split between salary and incentive pay.

A model by Farley & Weinberg [1975] is an early application of these principles to the salesforce-compensation problem. They assume that the source of asymmetric information between the salesforce and the firm lies in the sales-response function. It takes the form:

$$q_{ij} = k_{ij} + b_{ij}f_{ij}^\alpha, \quad 0 < \alpha < 1. \quad (24)$$

While the salesperson knows the sales response function fully, the firm does not know the values of $k_{ij}$ or $b_{ij}$, either across products (indexed by $i$) or salespeople (indexed by $j$). By iteratively proposing sets of commission rates on the product line and asking salesperson $j$ his time allocation and predicted sales for all products, Farley & Weinberg are able to estimate the parameters of the sales-response function. Once these are known, it is relatively straightforward to set commission rates or quotas to maximize the firm's profit. This application assumes that the salesperson's objective is income maximization; thus, any plan that rewards the salesperson more for higher firm profits solves the incentive-incompatibility problem. Clearly, if the salesperson has a more complex set of incentives, this sort of compensation solution may not be optimal.

In Darmon's [1979] model, the firm lacks knowledge of the parameters of the salesperson's utility function, as well as of the sales-response function. Utility functions are estimated via a conjoint-analysis task where the salesperson ranks various combinations of quota and bonus, producing indifference curves relating the salesperson's willingness to trade off greater work time (via a higher quota) for greater income (via a higher bonus). Once the salesperson's indifference curves are estimated, the firm can offer the salesperson that quota–bonus combination that maximizes its profits.

Darmon [1987] extends his 1979 model in three major ways. First, the DSS is put on a personal computer, enhancing its ease of use for the sales manager as well as for the modeler. Second, he explicitly models the problem of heterogeneous salespeople by adding a module onto his earlier DSS to derive a 'consistent' quota–bonus plan that (a) keeps all salespeople at utility levels at least as great as before implementation of the new plan, and (b) minimizes the profit loss due
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<tbody>
<tr>
<td>Farley &amp; Weinberg [1975]</td>
<td>Multiple (but same plan for all)</td>
<td>N/A (no uncertainty)</td>
<td>Sales effort only</td>
<td>Maximize commission income</td>
<td>None</td>
</tr>
<tr>
<td>Darmon [1979]</td>
<td>Multiple (but same plan for all)</td>
<td>Major focus on risk-neutrality</td>
<td>Sales effort only</td>
<td>Maximize utility, a positive linear function of bonus and negative quadratic function of quota</td>
<td>Total time available for leisure and work</td>
</tr>
<tr>
<td>Darmon [1987]</td>
<td>Multiple (but same plan for all)</td>
<td>N/A (no uncertainty)</td>
<td>Sales effort only</td>
<td>Maximize utility, a positive linear function of bonus and negative quadratic function of quota</td>
<td>Total time available for leisure and work</td>
</tr>
<tr>
<td>Gonik [1978]</td>
<td>Multiple; each salesperson self-selects into one of menu of plans</td>
<td>N/A</td>
<td>Forecasted sales before start of monitoring period; sales effort during period</td>
<td>Not known</td>
<td>Not known</td>
</tr>
<tr>
<td>Mantrala &amp; Raman [1990]</td>
<td>Single or multiple</td>
<td>Major focus on risk-neutrality</td>
<td>Forecasted sales before start of monitoring period; sales effort during period</td>
<td>Maximize expected utility</td>
<td>None</td>
</tr>
<tr>
<td>Mantrala, Sinha &amp; Zeltner (1990)</td>
<td>Multiple; each salesperson self-selects into one of menu of plans</td>
<td>Risk-averse</td>
<td>Effort allocation in total and across multiple products</td>
<td>Maximize expected utility</td>
<td>None</td>
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<tr>
<td>Farley &amp; Weinberg [1975]</td>
<td>Multiple products; unrelated in demand and cost</td>
<td>Non-stochastic function of sales effort only</td>
<td>$q_{ij} = k_{ij} + b_{ij} r_{ij}^x$ $0 &lt; x &lt; 1$</td>
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</tr>
<tr>
<td>Darmon [1979]</td>
<td>Single product</td>
<td>Non-stochastic function of sales effort only</td>
<td>Only implicitly understood through salesperson's choice of quota-bonus pairs</td>
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<tr>
<td>Darmon [1987]</td>
<td>Single product</td>
<td>Non-stochastic function of sales effort only</td>
<td>Only implicitly understood through salesperson's choice of quota-bonus pairs</td>
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</tr>
<tr>
<td>Gonik [1978]</td>
<td>Single product</td>
<td>Not explicitly modeled; inference is that it is a function of sales effort only</td>
<td>Not modeled</td>
<td></td>
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</tr>
<tr>
<td>Mantrala &amp; Raman [1990]</td>
<td>Single product</td>
<td>Stochastic function of sales effort only</td>
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<td></td>
</tr>
<tr>
<td>Mantrala, Sinha &amp; Zoltmets [1990]</td>
<td>Multiple products; unrelated in demand or cost</td>
<td>Stochastic function of sales effort only</td>
<td>$q = g(u) + e$, $u = effort$, $g' &gt; 0$, $g'' &lt; 0$</td>
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<td></td>
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<td></td>
<td>$q_{ij} = \mu_{ij} + (M_{ij} - \mu_{ij})(1 - \exp(-b_{ij} r_{ij}))$, a function of maximum territory potential and selling effort</td>
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<tr>
<td>Farley &amp; Weinberg [1975]</td>
<td>Arbitrary differentiable variable-cost functions: $C_i = C_i(Q_i)$</td>
<td>Knows salesperson maximizes commission income; does not know sales-response function</td>
<td>Commission rates on sales volumes of all products</td>
<td>Maximize profit</td>
<td>Estimate sales response function</td>
</tr>
<tr>
<td>Darmon [1979]</td>
<td>Constant marginal cost</td>
<td>Knows neither territory-specific sales-response functions nor utility functions</td>
<td>Quotas and bonuses</td>
<td>Maintain utility levels of all salespeople; improve profit</td>
<td>Use conjoint analysis of bonus–quota tradeoff to estimate salesperson's utility function and (indirectly) the sales response function</td>
</tr>
<tr>
<td>Darmon [1987]</td>
<td>Constant marginal cost</td>
<td>Knows neither territory-specific sales-response functions nor utility functions</td>
<td>Quotas and bonuses</td>
<td>Maintain constant utility levels; minimize profit loss due to deviation from individual-plan optimum</td>
<td>Use conjoint analysis of bonus–quota tradeoffs to estimate salesperson's utility function and (indirectly) the sales response function; PC based for ease in implementation</td>
</tr>
<tr>
<td>Gonik [1978]</td>
<td>Not explicitly modeled; losses accrue when actual sales ≠ forecast</td>
<td>Not explicitly modeled</td>
<td>Quotas and bonuses as functions of both actual and forecasted sales</td>
<td>Maximize profit; minimize costly deviations of sales from forecast</td>
<td>Incentivize more accurate sales forecasts by salesforce</td>
</tr>
<tr>
<td>Mantrala &amp; Raman [1990]</td>
<td>Quadratic or asymmetric loss function for actual sales ≠ forecast</td>
<td>Must know utility function and sales-response function to implement</td>
<td>Quotas and bonuses as functions of both actual and forecasted sales</td>
<td>Maximize expected profit</td>
<td>Incentivize more accurate sales forecasts by salesforce</td>
</tr>
<tr>
<td>Mantrala, Sinha &amp; Zoltiers [1990]</td>
<td>Constant marginal cost</td>
<td>Knows neither territory-specific sales-response functions nor utility functions</td>
<td>Bonus plan, contingent on prospeced quotas</td>
<td>Maximize expected profit</td>
<td>Estimate utility function via conjoint analysis</td>
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</table>
to deviation from the optimum under salesperson-specific plans. Third, he allows
the sales manager to weight different salespeople and territories differently when
maximizing the firm's profits across all salespeople.

Despite these advances, however, Darmon's [1987] DSS has some limitations.
One is that the firm must commit to a specific bonus plan before the DSS opti-
mization is done; in fact, sales managers would also like to know what the optimal
bonus plan is. This model approach does not lend itself to easy solution of this
problem. Further, Darmon's model does not provide for a menu of plans to be
offered to the salesforce. The agency-theory literature dealing with this issue [Lal
& Staclin, 1986; Rao, 1990] indicates that it may be more profitable to design
a menu of plans with the self-selection feature.

A somewhat different managerial problem is raised when the salesforce's private
information about the sales-response function is used in sales forecasting. Salespeople
may suspect that their inputs to the forecasting process will be used 'against' them
when the next year's sales quotas are set. They thus may have an incentive to
systematically underrepresent their ability to sell in their sales forecasts. Other
salespeople may be overconfident of their selling ability, and may therefore
systematically overstate territory potential and sales forecasts. But this bias in
information can be costly to the firm, either in inventory holding costs (in the case
of overstated forecasts) or in lost sales due to insufficient production (in the case
of understated forecasts).

Gonik [1978] addresses this problem in a managerially oriented paper describing
IBM Brazil's experience. The company's solution to the problem is to create a
quota–bonus plan that first gives each salesperson a sales objective, or quota, for
his territory. The salesperson then turns in a forecast of his sales in the territory
(which may or may not equal the quota). The bonus plan always rewards the
salesperson more for achieving higher actual sales. But it also penalizes him for
under- or over-forecasting his sales potential. In effect, IBM Brazil has created a
menu of plans (differing according to the payouts promised for different forecasts
chosen by the salesperson), into which the salesperson self-selects.

Mantrala & Raman [1990] analyze the Gonik quota–bonus scheme more
formally. They show that the plan can be represented mathematically as

\[ B = \tilde{B} \{\hat{q}/\bar{q} \} \] 
for \( q = \hat{q} \),

\[ B = B_1 = \tilde{B} + \beta (q - \hat{q}) + \gamma (q - \bar{q}) \] 
for \( q < \hat{q} \),

\[ B = B_2 = \tilde{B} + \beta (q - \hat{q}) + \alpha (q - \bar{q}) \] 
for \( q > \hat{q} \),

where

- \( q \) = actual sales,
- \( \hat{q} \) = quota set by management,
- \( \hat{q} \) = sales forecast submitted by the salesperson,
- \( B \) = actual bonus awarded,
- \( \tilde{B} \) = fixed bonus offered upon exact fulfillment of \( \hat{q} \) when \( \hat{q} = \bar{q} \),
- \( \beta = (\tilde{B}/\hat{q}) \), \( \alpha = (B/2\bar{q}) \) and \( \gamma = (3\tilde{B}/2\bar{q}) \) in the Gonik scheme.
Clearly, Gonik's values for $\alpha$, $\beta$, and $\gamma$ are special cases, and need not hold in all situations. This way of expressing the bonus-payout scheme highlights the penalty/reward rate for choosing forecasts that differ from managerially set quotas ($\beta$), the penalty rate for underselling relative to forecast ($\gamma$), and the reward rate for overselling relative to forecast ($\alpha$). The parameters $\gamma$ and $\alpha$ need not be equal, because the cost to the firm of over-forecasting may not equal the cost of under-forecasting. Indeed, in the Gonik scheme, $\gamma > \beta > \alpha > 0$, so that underselling is penalized more heavily than overselling is rewarded.

Mantrala & Raman show that, when the salesperson can choose both selling effort and his sales forecast, the optimal quota level for the firm to set a priori is affected by the salesperson's effort decision. However, the firm can influence effort and thereby influence the salesperson's forecast, by manipulating the $\beta$ parameter in the bonus plan. This can be done while maintaining any desired probability of fulfillment of the forecast by further adjusting the $\beta$ and $\gamma$ parameters. The major limitation of this approach is the need for managerial information on specific territories' sales potentials and specific salespeople's utility functions. To fully implement the scheme, one must graft on a module like Darmon's conjoint-analysis process to provide these inputs to management.

Two shortcomings of earlier DSS's in the salesforce-compensation area are their inability to let heterogeneous salespeople self-select into the plans that maximize their productivity, and their focus on a single-product line. Mantrala, Sinha & Zoltners [1990] deal with both of these issues in a DSS similar to that of Darmon [1987] in its use of conjoint analysis to estimate utility functions, but using the agency-theoretic approach of offering a total quota–bonus plan to the salesforce, and letting them choose among different levels of effort and sales achievement within the plan. The authors apply their system to a pharmaceutical firm selling two products, 'Largex' and 'Smalllex' (names are disguised), through a direct salesforce. Interestingly, the application suggests that more aggressive bonus plans would increase sales and profits generated by several of the salespeople. The model also permits investigation of the profitability of different possible bonus schemes with relative computational ease. The authors show a new quota–bonus plan that increases firm profits from $4.003$ million to $4.41$ million, a significant improvement. However, profit improvement can be hampered considerably if management imposes uniformity on the quota–bonus plan across very heterogeneous salespeople.

**Summary of decision support systems for salesforce compensation**

One of the strongest criticisms of the economics-based approach to modeling salesforce-compensation problems, discussed in Sections 2 and 3 above, is its assumption that the firm knows the salesperson's utility function as well as the form of the sales response function. These assumptions are rarely verified in practice. The DSS literature profiled above helps answer this criticism by proposing salesforce survey techniques that provide information on both utility functions and sales response, sometimes through the same instrument (conjoint analysis). This collapsing of a two-stage informational problem into a one-stage information-
gathering procedure is valuable not only for its computational brevity, but also because it provides some concrete evidence in specific applications of the degree of heterogeneity in the salesforce. Later DSS approaches use this information to good advantage by creating quota–bonus schemes that let the salesperson self-select the quota – and thus sales achievement – to which he is willing to commit. DSSs and more theoretical analytical approaches can thus be valuable complements in any specific empirical application.

There is still work left to be done in the DSS area, however. All the applications surveyed here focus on designing quota–bonus schemes, and ignore the issue raised in the agency-theory literature of optimal risk-sharing in a combination plan. The optimal split between salary and incentive is not dealt with at all in the DSS literature.

It remains somewhat difficult to know when one has approached the optimum optimorum when applying a DSS as well. Since each application of a DSS is specifically designed for the company and salesforce in question, it is virtually impossible to generalize across different applications of the same DSS to determine optimality of the plans suggested. In many cases, it is up to the model user to evaluate the profits from different bonus formulations before deciding which one to use. If the user happens not to try a particularly advantageous formulation, the DSS will not be able to warn that the resulting solution is not optimal.

Despite these points, DSSs are enormously useful, both in economically generating information to calibrate an abstract problem, and also in guiding the theoretical modeler to sensible assumptions on parameters of his problem.

6. Conclusions and future research directions

The extensive MS/OR literature on salesforce compensation in marketing has attacked a great variety of problems facing sales managers, including how to set commission rates, whether to delegate pricing authority to a salesforce, what mix of salary and incentive compensation to provide to the salesforce, whether or not to offer a menu of compensation plans, and how to elicit valuable information from salespeople about their utility functions and sales response functions. In the case of theoretical models, some predictions have been tested statistically, but as we have noted above, these tests have been incomplete and, in some cases, inconclusive. Based on available empirical results, we cannot reject agency theory as a useful paradigm for compensation-setting, but further research is necessary to increase the level of confidence in the theory and its predictions. On the DSS front, few diagnostics have been presented for the models described in the literature, although those presented suggest the usefulness of the approach as well.

Some questions seem to have been relatively well researched. All approaches agree that salespeople's incentives to exert effort against a product should be greater when the profit results from effort exertion are higher. Increased profitability can of course arise from many sources: lower production costs, greater sales-effort effectiveness, and greater territory potential are some of the reasons. Similarly, it
is well understood that salespeople with a higher opportunity cost of time must be promised a greater expected total pay package. In cases of asymmetric information (i.e. where the salesperson is more completely informed than is sales management), it also makes sense to delegate decisionmaking authority to the salesperson— for example, in price-setting. In addition, when a salesforce is made up of people of differing abilities, or people assigned to territories with different sales potentials, it is generally agreed that the firm can do better by tailoring a menu of plans to the salesforce than by forcing all salespeople into one plan.

Some questions remain to be researched in depth. Further research into the dynamic nature of salesforce motivation is in order, particularly for understanding the sale of products with long selling cycles (such as some large-ticket industrial products). In the agency-theory literature in economics, some work has been done on this issue [see, for example, Fudenberg, Holmstrom & Milgrom, 1990], which suggests the value of tying pay horizons (such as bonus and commission award dates) to performance horizons (such as the closing of a sale) to avoid perverse salesforce incentives. But the modeling is presented at a level of abstraction that prohibits a great deal of detail for the salesforce manager. The empirical support for this line of modeling [Coughlan & Narasimhan, 1992] suggests that there may be value in extending the work to apply more closely to the salesforce context.

Another area in which marketing research has lagged is in the advisability of relative pay plans such as sales contests of various sorts. Again, the economics literature has developed some modeling insights in the area [see, for example, Lazear & Rosen, 1981; Holmstrom, 1982; Green & Stokey, 1983; Nalebuff & Stiglitz, 1983], but without much direct application in the salesforce area. Further modeling research could help suggest when such comparative compensation schemes perform better than salesperson-specific incentive schemes, and what intensity of use such schemes should have in the salesforce.

Finally, other theoretical paradigms could be integrated into the MS/OR approach to modeling salesforce compensation. John & Weitz [1989] empirically test some of the predictions of transaction-cost analysis (TCA) for compensation practice, but the theory (originally posited by Williamson [1975]) has eluded quantitative modelers. John & Weitz themselves note that TCA is concerned primarily with issues of salesforce control, while agency theory focuses on issues of salesforce motivation. It seems clear that a holistic approach, incorporating both aspects of effective salesforce management, could be very useful.

References


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