MARKETING SCIENCE
Vol. 28, No. 6, November–December 2009, pp. 1009–1026
ISSN 0732-2399 | EISSN 1526-548X | 09 | 2806 | 1009

informs®

DOI 10.1287/mksc.1090.0493

Optimal Sales Force Diversification and Group Incentive Payments

Fabio Caldieraro
Foster School of Business, University of Washington, Seattle, Washington 98195,
cfabio@u.washington.edu

Anne T. Coughlan
Marketing Department, Kellogg School of Management, Northwestern University,
Evanston, Illinois 60208, a-coughlan@kellogg.northwestern.edu

In this research, we show that the interaction between territory allocation and sales force compensation—two key drivers of sales productivity—strongly affects the firm's profitability. We analyze an agency-theoretic model that jointly considers the degree of negative or positive correlation across territory outcomes, differences in territories' sales potentials, the agency problem with risk-averse salespeople, and the availability of both own-territory compensation elements, such as commission, and elements dependent on the performance of others, such as group commissions or tournaments.

We find that allocating salespeople to negatively correlated sales territories beneficially diversifies each salesperson's portfolio of sales outcomes when this allocation includes a group commission pay component, and can improve profitability even with a decrease in average territory sales performance. In a larger sales force, a balanced allocation of salespeople, coupled with a group commission, dominates an imbalanced allocation. Comparing piece-rate compensation (with or without a group commission component) to tournaments alongside the allocation problem, we find that tournaments are favored over piece-rate plans when territories are highly positively correlated, territory sales potentials are similar, and salespeople have a low disutility for effort and are not very risk-averse. A piece-rate plan conversely dominates a tournament when these conditions are reversed.

Key words: agency theory; channels of distribution; compensation; sales force
History: Received: July 26, 2007; accepted: February 17, 2009; processed by Chakravarthi Narasimhan. Published online in *Articles in Advance* June 19, 2009.

1. Introduction

Territory allocation and sales force compensation are two key drivers of sales force productivity. Territory allocation refers to the choice the firm makes to assign its salespeople to specific areas of sales responsibility (be they delineated by geography, product focus, or vertical market target segment, as noted in Zoltners and Sinha 2005 or Johnston and Marshall 2006, p. 111, for example). Compensation refers to the reward structure offered to salespeople—including salary, own-territory commission or bonus, and any elements depending on factors other than the salesperson's own performance. Sales management research usually investigates these two problem areas separately.[1] Yet in this research, we show that the *interaction* between these two important sales management decisions affects the firm's profitability in terms of both costs (total compensation necessary to motivate sales effort) and demand results (sales productivity resulting from effort exertion). Our findings explain why a firm might consciously choose to expand into an apparently inferior sales territory, even given the ability to assign a salesperson to a better-performing territory. To study this problem, it is necessary to jointly consider several factors: the possibility of negatively or positively correlated sales outcomes across territories; differences in the inherent sales potential of various territory types; the agency problem the firm faces in employing salespeople; the availability of both own-territory compensation elements, such as commission, or elements dependent on the performance of others, such as group commissions or tournament payoffs; and risk aversion in the sales force.

The assortment of territories into which a firm allocates its salespeople can be quite varied, with allocation criteria including the salesperson's responsibility for the product(s) to be sold, the customer accounts allocated to each salesperson, the geography

[1] This is true not only in the academic research realm but also in textbook organization; Johnston and Marshall (2006) and Stanton and Spiro (1999) both treat territory allocation and sales force compensation in different chapters, within different sections, of their textbooks.

in which to operate, and/or the vertical market segment(s) to target. For example, Kodak used separate sales forces for consumer film products and business-to-business products such as blood analyzers and x-ray machines. Moreover, many pharmaceutical companies have their salespeople focus on selling particular lines of ethical drugs. Yet other firms allocate salespeople to specific end-user markets: Unisys Corporation allocates salespeople within its national subsidiaries by industry expertise (such as Financial Services, Communications, Transportation, Public Sector; Consumer and Industrial Products);[2] and computer equipment reseller CDW separates its government/educational sales arm from its corporate arm.

In any of these territories, sales are generally characterized by a variance—great or small—in the relationship between the exertion of effort and ultimate sales; our analysis shows that own-territory variances affect the joint allocation and compensation decision of the profit-maximizing firm. Furthermore, in any one of the allocation schemes, the resulting territory types may exhibit a positive or a negative correlation across outcomes. Positive cross-territory sales correlations may occur, for example, because of the common (but stochastic) value of the company's brand name in a competitive marketplace, or because of competitors' actions that commonly affect different territory types. Negative cross-territory sales correlations are also observed, however, as these examples demonstrate:

- CDW's dual sales efforts in government and corporate markets, mentioned above, represent negatively correlated opportunities; the present economic climate has led to a downturn in corporate sales, whereas a government stimulus package can herald a compensating trend upward in that part of the business.
- Gun manufacturers expanded into sales of products such as home alarm systems, safety glasses, sports clothing, stuffed toys, luggage, and bicycles to diversify beyond the nearly stagnant firearm market (which is subject to multiple legal threats); fuel revenue growth; and to improve the companies' images. Firearms manufacturer Colt also targeted the military small-arms market, which is a different market than that for consumer firearms (Barrett 1999). These represent market expansion efforts into countercyclical product lines, which are sold through different sales processes as well.
- Sales of tree-care services show negative cross-regional correlation because of two key factors: the difference in climactic conditions across regions that affect demand and sales effort productivity, and differences across regions in economic conditions that cause the demand for tree-care services to vary.[3]
- Mortgage lenders were advised to diversify their lines of business and geographies to better protect against negative results in any one area (Teitelbaum 2003).
- In a multi-industry study of firms selling industrial products and services, Cross et al. (2001) found that using different sales forces to reach new market segments (i.e., practicing sales force specialization) was listed as "extremely important" by 17% of all firms, and by over 30% of firms with sales up to \$499 million. Furthermore, 21% of all firms surveyed cited a diversification strategy (new markets and/or new products, suggesting the pursuit of negatively correlated business opportunities) as an "extremely important" part of the firm's overall marketing strategy, implying that diversification is a common growth strategy and that using different sales forces to accomplish this is common.

Just as territory allocation can proceed according to various criteria, sales force compensation can also encompass multiple elements. The main focus in compensation research has been on the use of salary and commission based on own-territory sales (see, for example, Basu et al. 1985, Coughlan and Sen 1989, Coughlan and Narasimhan 1992, Lal and Srinivasan 1993), but firms use compensation elements that offer rewards based on other salespeople's performance as well, such as group commissions (based on joint sales of multiple salespeople)[4] or tournament compensation (such as sales contests, which

[2] *Source*. http://www.unisys.com/industries/index.htm. In a personal interview with Francisco Mario Oliveira, former Customer Relationship Manager for Unisys (January 28, 2008), Oliveira reported that sales compensation included salary, commission on own sales, and commission on overall branch sales—thus, a group commission. He described the group commission as a good tool for cushioning sales compensation in the event of a bad outcome for a specific salesperson (consistent with our results in §3 on the role of the group commission as an insurance device).

[3] Indeed, the original inspiration for this paper came from the example of The Care of Trees, a national tree-care company that faced exactly this type of negative cross-territory variation in sales and which instituted a compensation plan redesign in response that involved group commission payments in addition to the standard salary plus own-territory commission compensation. Please see the Technical Appendix A, available at http://mktsci.pubs.informs.org, for more details on The Care of Trees.

[4] For example, Joseph and Kalwani (1998) found that 38% of survey respondents used division or company profit (not just own-territory performance) as a basis for awarding bonuses, and 20% reported this as the highest-weight dimension; an annual multi-industry sales force survey conducted by the Hay Group in 2006 found that in 28% of sales force compensation plans, incentive payout was based primarily on group performance at some level (Davenport et al. 2007), and one sales management/compensation consultant reported that because of high variance in the effort–sales relationship, group incentive payments are commonly offered (in about 30%–40% of cases) in the first 12 months after the launch

involve comparisons between different salepeople's performance and the granting of a larger award to a higher-selling salesperson). Group incentive payments are believed to have negative incentive effects because they can facilitate free-riding by some on the efforts of others, thus demotivating strong salespeople: "The question they [salespeople] ask themselves is 'why should I bust a gut to cover up for the incompetence of others?' " (Barry 2006, p. 14). Despite this concern, group compensation incentives clearly persist, suggesting that other factors must counterbalance the negative effect of free-riding under certain circumstances.

The risk aversion of the sales force also creates a challenge for sales force management. One research study calibrated a decision-support model in a small sales force and found that five out of six salespeople were indeed risk-averse (Darmon and Rouzies 2002). Stanton and Spiro (1999) list one of the strengths of a salary being the "assured regularity of income [that] gives the salesperson a considerable degree of security" and its provision of "stability of earnings, without the wide fluctuations often found in commission plans" (p. 290); similarly, Johnston and Marshall (2006) argue that salary is especially useful "when compensating new sales reps," while one of the disadvantages of commission is that "sales reps have little financial security" (p. 338).

Our investigation shows that territory allocation decisions, combined with the appropriate compensation choices, can mitigate the negative agency effects of managing risk-averse salespeople. Our analysis takes into account not only the risk aversion of the sales force and the sales potentials and variances of sales territories, but also the correlation between sales outcomes across territory types; the firm's territory-allocation decision (and the nature of the optimal compensation plan) is shown to depend jointly on all of these factors. Prior agency-theoretic literature (reviewed in §1.1) has considered the first three of these factors, given its focus on examining the impact of effort unobservability and, hence moral hazard, on compensation and profitability outcomes (see, for instance, Basu et al. 1985, Lal and Srinivasan 1993). Prior literature does not examine cross-territory covariances in sales outcomes, and our analysis therefore is able to shed light on the joint allocation-compensation decision in a way prior literature cannot.[5] To attack our research problem, we build a model wherein a firm allocates each salesperson to a specific territory and where territories may vary in their types. Concomitant with the allocation choice, the firm must set the compensation structure (piece-rate, piece-rate plus group commission, or tournament) as well as the specific amounts of pay to be awarded for various performance levels. Viewing one's own allocation to a particular type of sales territory as well as the compensation plan offered, each salesperson makes his effort decision, sales are realized, and compensation and profits are earned.

Our analysis generates some surprising results. We first analyze territory allocations when the firm chooses either a pure piece-rate or piece-rate plus group commission compensation plan. The results show that group commissions have no role to play when territory outcomes are positively correlated. However, with negatively correlated sales territories, (1) purposely choosing to allocate salespeople to obviously inferior territory types (thus diversifying the sales force) and (2) providing a group commission payment are both jointly optimal behaviors. This result holds because the lower average productivity of a diversified sales force is counterbalanced by the benefit of diversifying the portfolio of sales outcomes across the firm and the firm's ability to share the diversification benefits with the risk-averse sales force via group commissions, which serve to decrease the variability of the overall compensation plan. The logic is that the allocation of salespeople to different territory types, coupled with group incentives, effectively diversifies the reward the salesperson accrues in return for effort in his own territory and thus reduces the risk inherent in exerting high effort levels in a territory subject to variance in outcomes, which in turn increases the firm's sales and profits.

This result is reminiscent of the classic "portfolio diversification" insight from the finance literature.[6] However, financial portfolio theory is not sufficient to explain our approach to the problem. This is because in the diversification of a financial portfolio, the productivities (i.e., returns) of individual stocks chosen as elements of the portfolio themselves do not change because of their inclusion in the portfolio. In contrast, the salesperson's optimal effort level, and the total compensation necessary to motivate high effort, can be expected to change as a result of the

of a new product. Group incentive pay is also offered when the company faces a high-variance major event coming up in the near future, such as when the outcome of a pending patent dispute is uncertain (personal interview with Samrat Shenbaga of ZS Associates, January 2008).

[5] Other related factors can affect firms' territory-allocation decisions as well, for instance, differences in product margins across territories or heterogeneity in either salespeople's abilities or in minimum acceptable utilities. Our model does not explicitly consider these factors, but our results nevertheless provide some intuition into the effects such factors have on allocation and compensation outcomes; these are discussed later in the paper.

[6] The major insight is that a risk-averse investor should optimally diversify the assets held in his portfolio to maximize his risk-adjusted return (see Brealey et al. 2006, especially Chapter 8, for a summary).

compensation-plan "portfolio" offered to him, which in turn affects sales and profitability; such agency issues do not plague the standard finance portfolio-optimization problem.

We extend these insights from a 2-person to an N-person sales-force context as well. The analysis reveals that a piece-rate plus group commission plan, combined with the allocation of $N > 2$ salespeople to multiple territory types, leads to an optimal split of salespeople between territory types that is generally balanced rather than highly imbalanced.

We also examine optimal territory allocations when a tournament compensation plan (or sales contest) is used and what these results imply for the optimal allocations when any of the three compensation plans can be applied.[7] Salesperson allocation to highly positively correlated territories favors a tournament compensation plan over any type of piece-rate plan when territory sales potentials are similar enough, salespeople have a low disutility for effort, and the salespeople's risk aversion is low enough. These factors drive tournament success because they guarantee that the outcome is not already a foregone conclusion and that salespeople are willing to exert high effort levels to compete to win the tournament.

However, we show that a tournament is in fact not implementable (i.e., does not induce high sales force effort) in all situations. In particular, the tournament plan fails when salespeople are too risk-averse, the disutility for effort is too high, territories are too disparate in their sales potentials, or inter-territory sales outcomes are insufficiently correlated. In contrast, a piece-rate plan (either without or with a group commission component) is always feasible to implement, suggesting limited robustness for the tournament-plan solution. Noting that in the Care of Trees (COT) example, salespeople are characterized as reasonably highly risk-averse and cross-regional sales correlations are negative, our results suggest that the combination of territory allocation and application of a piece-rate plus group commission plan (versus a pure piece-rate plan or a tournament) is sensible under the conditions predicted in our model, whereas a tournament or sales contest would not be as profitable.

In what follows, we first summarize the relevant literature in economics and marketing, showing how our work extends existing research. Then we set up and solve a model of territory allocation and optimal sales force compensation, allowing for correlated sales outcomes between different territory types. We first consider the optimal allocation and compensation choices with two salespeople and three possible territory types, allowing for either a pure piece-rate plan or a piece-rate plus group incentive plan. A generalization of this model to allow more than two salespeople, with enough available territories of all three types to be able to choose any allocation among them, is discussed next. We then return to the two-salesperson model and consider a tournament compensation plan as an alternative to either a pure piece-rate plan or a piece-rate with group incentive plan. We close with a summary of our findings and a discussion of suggested future research avenues.

1.1. Related Literature

This research is closely related to the academic literature investigating sales force compensation and motivation (see Coughlan and Sen 1989, Coughlan 1993 for reviews of this literature). Basu et al. (1985) consider compensation for a single-product salesperson, and more recently, Lal and Srinivasan (1993) study sales force compensation plans for single- and multi-product sales forces: Mantrala et al. (1994) present a model-based methodology to specify a sales quota-bonus plan that provides incentives to heterogeneous salespeople, and Zhang and Mahajan (1995) consider the optimal compensation to a single salesperson who sells two products that may be complementary, substitutable, or independent in demand;[8] Joseph and Thevaranjan (1998) analyze compensation incentives coupled with monitoring of selling agents; Godes (2003) investigates selling tasks of high and low complexity and the relative importance of sales skills for each of these different selling complexity levels; Godes (2004) studies the optimality of risk-sharing contracts when agents' actions endogenously determine risks; and Caldieraro and Coughlan (2007) examine the role of spiffs in managing agency problems in independent sales force organizations.

Our research is also related to the sales force allocation substream of the sales force literature. This literature develops methods to optimally allocate selling resources to products and markets, typically with a deterministic view of the sales response function (e.g., Horsky and Nelson 1996; Lodish 1975, 1980; Mantrala et al. 1992, Montgomery et al. 1971; Rangaswamy et al. 1990; Skiera and Albers 1998; Zoltners and Sinha 1980, 1983). The main objective of these methods is to

[7] Some of the tournament results presented here are found in previous literature but are adapted to our framework for the purposes of comparison with the piece-rate plus group commission compensation plan, and for generating insights into the territory allocation problem; neither of these foci characterizes the existing literature on tournaments.

[8] With substitutable products, the optimal compensation is a salary plus a single commission awarded on the total sales of both products. While this compensation structure is similar to the group incentive we investigate here, its motivation is quite different, as it does not consider either a multiperson sales force or the "insurance" value of compensating a salesperson based on outcomes outside his own sphere of influence.

gain efficiency by allocating the right resources to the most responsive products or territories.

Each of the above research streams looks for efficiency in one of the key elements in sales force management, but the compensation literature ignores allocation issues while the allocation literature assumes away the agency problem by assuming that salespeople will work hard, given the optimal territory allocation. Each literature stream thus fails to account for the jointness of the territory allocation and sales force compensation decisions and abstracts away from the value of a diversified territory allocation as a means to reduce the risk inherent in exerting high effort levels in a territory subject to variance in outcomes. The fundamental insight of diversification as a means to reduce risk can be traced to the finance literature in portfolio theory. Seminal work developed by Markowitz (1952, 1991) identified that a decision maker facing the opportunity to invest in multiple assets can find a group of desirable investments, as a function of the expected risks and rewards of each type of asset and the covariances between the investment alternatives (for a full exposition of portfolio theory see Brealey et al. 2006). The concept of diversification can also be seen in the work of Holmstrom and Milgrom (1990) and Varian (1990), which recognized coinsurance as a beneficial factor that was later investigated as a possible explanation for partnerships of principals as a form of labor organization (Kandel and Lazear 1992, Gaynor and Gertler 1995, Lang and Gordon 1995). This stream of research, however, does not consider the purposeful allocation of agents (in our case salespeople) by a principal to tasks or territories with evident inferior expected productivity, nor does it consider how "balanced" this allocation should be.

Given our consideration of compensation plans including elements depending on another salesperson's performance, the tournament literature in economics is also of interest (see, for example, Lazear and Rosen 1981, Green and Stokey 1983, Nalebuff and Stiglitz 1983). A sales contest is an example of a tournament plan where the highest payoff is given to the salesperson with the highest sales performance. The tournament literature models a competition between agents (one of whom ultimately wins and the other(s) lose) and derives the optimal winning and losing payoffs to the players. Our analysis of a tournament sales compensation plan follows this literature while expanding on its assumptions concerning correlation of error structures across territories, the possibility of negatively correlated territory outcomes, the possible alternative of group commission in a piece-rate plan, and the jointness of the territory allocation decision along with the compensation decision. Prendergast (1999) notes that "relative performance" measurement, or "yardstick competition" (where one agent is punished [rewarded] if the other agent has a better [worse] result) can also improve a firm's profit under some conditions. The "relative compensation component" in such a plan is a negative commission based on the other agent's outcome. We consider a group commission or a tournament here rather than this yardstick compensation because they represent the comparative compensation examples summarized above better than the punitive yardstick concept does, and because group commissions and sales contests are known to be used in practice, while negative commissions based on other salespeople's sales are extremely rare.

In sum, several literature bases are of use in considering the joint territory allocation and sales force compensation problem we outline in §1. However, none fully accommodates all the elements of importance in our situation. Our modeling approach therefore includes components of the territory allocation literature in marketing, the agency-theoretic sales compensation literature, the portfolio diversification and coinsurance literature, and the tournament compensation literature to explain how allocation and compensation decisions like those discussed above can be sensible, given their conditions, whereas other territory allocation and compensation combinations are preferable under other marketplace and territorial characteristics. With this understanding of the positioning of our work relative to the literature, we continue with the model development below.

2. The Model

Consider an economy in which a risk-neutral firm employs two salespeople to sell its product; each salesperson can be assigned to one and only one of three possible sales territories, and a territory can accommodate at most one salesperson. We denote these territories by the set $T = \{\alpha, \beta, \gamma\}$. An allocation (or sales force deployment) denoted by $\mathcal{D}$ is a subset $\{t_1, t_2\} \in T$ such that $t_1 \neq t_2$. Hence, $\mathcal{D}$ can represent any of the three possible allocations $\{\alpha, \beta\}$, $\{\alpha, \gamma\}$, or $\{\beta, \gamma\}$. We index a territory in the subset $\mathcal{D}$ by $i \in \{1, 2\}$ and use the index j to be "the other territory" in the allocation (formally, $j = 3 - i$; thus if, for instance, $\mathcal{D} = \{\alpha, \beta\}$, $i = 2$ indexes territory β, and consequently, j assumes the value of 1 and indexes territory α).

The territories may have different sales-potential characteristics, and their sales outcomes may have some degree of correlation. Suppose that sales in territories α and β are positively correlated, and sales in territories α and γ are negatively correlated (consequently, sales in territories β and γ are likely to be negatively correlated). In our COT example, α and β might be distinct territories located in the Chicago region, whereas territory γ might be a territory in the California region.

Salespeople, however, are assumed to be homogeneous in their selling abilities, and each salesperson's selling effectiveness is assumed to be independent of the selling outcomes in other territories (i.e., there are no team selling effects). This assumption lets us focus on the incentive and coordination effects of the compensation plan without extraneous effects such as demand complementarity or substitutability altering the sales force's selling effort decisions. It also implies that the allocations $\{t_1, t_2\}$ and $\{t_2, t_1\}$ are equivalent.

A salesperson in territory i can exert effort $e_i \in \{L, H\}$ to affect the sales outcome in that territory, where L represents low effort and H represents high effort. High effort increases the probability of high sales; however, exerting effort implies a disutility $C(\cdot)$ for the salesperson. We normalize $C(L) = 0$ and $C(H) = c > 0$.

We assume that the salesperson's utility is separable in income (s_i) and effort: $u_i = v(s_i - C(e_i))$, where the function $v(\cdot)$ exhibits constant absolute risk-aversion (CARA): $v(\vartheta) = 1 - e^{-r\vartheta}$, where $r \geq 0$ is the risk aversion parameter. Moreover, the salesperson is assumed to have a minimum acceptable utility level of m, which represents his opportunity cost for taking another job, or simply his utility for leisure.

Sales by a single salesperson in each territory are characterized by the stochastic density function $f(x_i \mid e_i) = X_i(e_i) + \varepsilon_i$, where ε_i is a random shock-distributed Normal with mean zero and variance σ_i^2 ($\varepsilon_i \sim \mathbf{N}(0, \sigma_i^2)$), and the deterministic component is given by the functions $X_i(\cdot)$, where $X_i(L) = q_i$ and $X_i(H) = q_i + \Delta_i$. We can think of q_i as the base sales level for territory i and Δ_i as the selling-effort productivity for territory i.[9] This specification implies that x_i is a random variable distributed according to a density conditioned by the endogenous salesperson effort level e_i. We further assume that the sales covariance between territories is σ_{ij}, and we let the variable $\mathscr{X}$ capture the total sales by the firm (for example, if the firm has one salesperson in territory α and one salesperson in territory β, then $\mathscr{X} = x_\alpha + x_\beta$ and the sales covariance is $\sigma_{\alpha\beta}$).

As is usual in agency theory, we assume that the manufacturer's incentive compatibility constraint is always satisfied so that it is always optimal to induce high effort in both territories. This means that the sales return is high enough to compensate for the salesperson's risk-adjusted cost of effort. For expositional convenience, we assume the marginal cost of production is zero. Adding a positive marginal cost does not change our result; it would simply require us to multiply the deterministic part of our model by the margin factor.

Although sales are not known ex ante, all players have a common "belief" regarding the sales-response function. Therefore, even though the manufacturer does not directly observe the salesperson's effort choice, the manufacturer can infer it (imperfectly) from the relation $x_i \sim f(x_i \mid e_i)$.

This model follows a traditional moral hazard game, played according to the stages listed below:

- Stage 1—The firm allocates each salesperson to a territory.
- Stage 2—The firm offers a compensation contract to the salespeople (specifying the type of contract and the contract parameters).
- Stage 3—Each salesperson accepts or refuses the contract.
- Stage 4—Each salesperson chooses effort.
- Stage 5—Outcomes are realized and compensation is awarded.

The next section analyzes this model, first considering a discrete sales-response function with two salespeople and compensation via a piece-rate plan, to establish the conditions under which a group commission component is optimally included in the plan, in addition to the standard salary and own-territory commission components.

3. Territory Allocation Under Piece-Rate Compensation

Piece-rates are contracts awarded with respect to some absolute performance measure. In a typical piece-rate plan, a salesperson in territory i is compensated by the firm through contractual payments $s_i = b_i x_i + g\mathscr{X} + A_i$, where A_i is the salary, b_i is the sales commission on the salesperson's own-territory sales performance, and g is a group commission, based on the total sales for the organization.

Both linear and nonlinear compensation schemes are used in practice, but this research focuses on the linear plan because of its tractability and its consistency with previous literature that shows that linear plans may indeed be optimal (Holmstrom and Milgrom 1987). Furthermore, its wide use in the sales compensation and control literature (Holmstrom and Milgrom 1991, Hauser et al. 1994, Lal and Srinivasan 1993, Joseph and Thevaranjan 1998) allows a comparison of this paper's results with the extant literature.

Theoretically, an agent can be punished by the principal if an unwanted outcome is obtained, thus diminishing the cost of compensating high outcomes and maximizing the principal's profits. In practice, however, a salesperson cannot be punished with a negative reward because of low sales; hence, we assume that the salesperson enjoys limited liability for negative sales outcomes ($b_i \geq 0$, $g \geq 0$).

[9] A high Δ_i implies that changes in effort level are associated with great changes in the expected sales in territory type i.

Using Pratt's (1964) certainty-equivalent (CE) formulation, we can rewrite the expected utility of the salesperson in territory i as

$$u_i = b_i X_i(e_i) + g\mathscr{X} + A_i - \frac{r}{2} \cdot [(b_i+g)^2\sigma_i^2 + g^2\sigma_j^2 + 2(b_i+g)g\sigma_{ij}] - C(e_i). \quad (1)$$

The salesperson's optimal choice of effort that maximizes his utility is the same effort that maximizes the CE form of his expected utility.

Because the firm is risk-neutral, its profit function is given by

$$\pi^{\mathrm{PR}} = \sum_{i=1}^{2} [(1-b_i)X_i(e_i) - g\mathscr{X} - A_i]. \quad (2)$$

For $i = 1, 2$, the piece-rate compensation problem for the firm is therefore

$$(P^{\mathrm{PR}}): \max_{A_i\in\mathbb{R},\, b_i\in\mathbb{R}_+} \sum_{i=1}^{2}\big[(1-b_i)X_i(e_i) - g\mathscr{X} - A_i\big]$$

$$\text{s.t.} \quad PC: \; b_iX_i(e_i) + g\mathscr{X} + A_i - \frac{r}{2} \cdot \big[(b_i+g)^2\sigma_i^2 + g^2\sigma_j^2 + 2(b_i+g)g\sigma_{ij}\big] - C(e_i) \geq m,$$

$$IC: \; e_i^* \in \arg\max_{e_i\in\{L,H\}} b_iX_i(e_i) + g\mathscr{X} + A_i - \frac{r}{2}\big[(b_i+g)^2\sigma_i^2 + g^2\sigma_j^2 + 2(b_i+g)g\sigma_{ij}\big] - C(e_i).$$

The optimal compensation contract to offer the salespeople in this case, and the corresponding profits, are given in Lemma 1.

Lemma 1. *In the piece-rate compensation plan*:

If $\sigma_{ij} < 0$, offering both an own-sales and a group commission is optimal, and the optimal compensation plan is

$$b_i^{*\mathrm{PRG}} = \frac{c}{\Delta_i} - c\left(\frac{1}{\Delta_i} + \frac{1}{\Delta_j}\right)\left(\frac{-\sigma_{ij}}{\sigma_i^2+\sigma_j^2}\right),$$

$$g^{*\mathrm{PRG}} = c\left(\frac{1}{\Delta_i} + \frac{1}{\Delta_j}\right)\left(\frac{-\sigma_{ij}}{\sigma_i^2+\sigma_j^2}\right),$$

$$A_i^{*\mathrm{PRG}} = m + \frac{c}{2\Delta_i^2\Delta_j^2(\sigma_i^2+\sigma_j^2)^2} \cdot \{2\sigma_{12}\Delta_i\Delta_j(\Delta_i+\Delta_j)(q_j+\Delta_j)(\sigma_i^2+\sigma_j^2) + cr\sigma_{ij}^2(\Delta_i+\Delta_j)[\Delta_i\sigma_j^2 - \Delta_j(2\sigma_i^2+\sigma_j^2)] - \Delta_j^2(2\Delta_i q_i - cr\sigma_i^2(\sigma_i^2 + \sigma_j^2)^2\},$$

generating profits of

$$\pi^{*\mathrm{PRG}} = (q_i+\Delta_i+q_j+\Delta_j) - \frac{rc^2}{2} \cdot \left[\frac{\sigma_i^2}{\Delta_i^2} + \frac{\sigma_j^2}{\Delta_j^2} - \left(\frac{\Delta_i+\Delta_j}{\Delta_i\Delta_j}\right)^2\left(\frac{\sigma_{ij}^2}{\sigma_i^2+\sigma_j^2}\right)\right] - 2c - 2m.$$

If $\sigma_{ij} > 0$, the optimal group commission is zero and thus the optimal compensation is

$$b_i^{*\mathrm{PR,\,noG}} = \frac{c}{\Delta_i}, \quad A_i^{*\mathrm{PR,\,noG}} = m + c - \frac{c(q_i+\Delta_i)}{\Delta_i} + \frac{rc^2}{2}\frac{\sigma_i^2}{\Delta_i^2},$$

generating profits of

$$\pi^{*\mathrm{PR,\,noG}} = (q_i+\Delta_i+q_j+\Delta_j) - \frac{rc^2}{2}\left(\frac{\sigma_i^2}{\Delta_i^2} + \frac{\sigma_j^2}{\Delta_j^2}\right) - 2c - 2m.$$

Proof. See Technical Appendix B, available at http://mktsci.pubs.informs.org. □

Although one salesperson may be allocated to a more productive territory while the other salesperson may be allocated to a less-productive territory, Lemma 1 shows that both salespeople receive the minimum utility level m. Hence, the firm automatically achieves an outcome that is perceived as fair by both salespeople.

It is straightforward to verify the comparative-static effects on the own-sales commission rate, the group commission rate, and equilibrium profits for $\sigma_{ij} < 0$ or $\sigma_{ij} > 0$. A positive group commission g can be optimal only if $\sigma_{ij} < 0$, in other words, if the sales potentials in each territory are negatively correlated, as in The Care of Trees and other examples noted above. In this situation, more negatively correlated territories lead to a lower own-territory commission rate, a higher group commission rate, and a higher overall level of firm profitability. The comparative-static effects of the disutility parameter (c), the productivity of own-territory selling effort (Δ_i), and own-territory variance (σ_i^2) on commission rates and profitability are aligned with established game-theoretic results.

The results imply that a group commission is a more important compensation tool, the more negatively correlated are sales across territories. The reason for this is that when the risk-sharing benefits of group compensation are greater, the more different the territory types are.[10] The coinsurance literature also finds that when group incentives are more productive, the more correlated agents' outcomes are, although the coinsurance results are demonstrated only for territories of equal productivity. We find the group commission result even when the benefit of coinsurance is available only if the firm allocates a salesperson to a territory of inferior productivity.

[10] Note that the optimality of a group commission payment is well known in the analysis of team selling situations; in that literature, group commission payments are demonstrated to be optimal only when team selling yields complementarity effects (that is, one salesperson's effort has a positive effect on the productivity of the other salesperson's efforts). However, our result holds despite the absence of team-selling effects in our model, suggesting that group commission payments have a broader role to play.

Note that the case of two territories with identical characteristics is simply a special example of the $\sigma_{ij} > 0$ case (indeed, $\sigma_{ij} = \sigma_i^2 = \sigma_j^2$), where $q_i = q_j$ and $\Delta_i = \Delta_j$, and as such, the PR, noG results hold for this special case.

To understand the role of a group commission with negatively correlated territories, it is helpful to consider the effects of moving from a pure salary-plus-own-territory-commission plan (PR, noG) to a plan including group commission as well (PRG). Because effort is discrete in nature in our model (low or high), equilibrium effort levels in either the PR, noG, or the PRG case are the same: high effort is exerted in equilibrium by each salesperson. Thus, any profitability changes from including a group commission in the compensation plan will accrue from changes in the compensation necessary to induce high effort, not from changes in either effort levels or sales themselves.[11]

The following proposition summarizes the comparison of compensation elements with and without a group commission, when sales-territory outcomes are negatively correlated:

Proposition 1. *When sales-territory outcomes are negatively correlated*:

(a) *The equilibrium commission rate on own-territory sales under the PRG plan is lower than under the PR, noG plan*;

(b) *The sum of the own-territory commission rate and the group commission rate in the PRG plan is equal to that in the PR, noG plan without a group commission*;

(c) *Equilibrium salary in territory i is lower in the PRG plan than in the PR, noG plan whenever total pay in territory i falls with the move from PR, noG to PRG, and more generally, when*

$$\sigma_{ij} < \frac{2\Delta_i\Delta_j(q_j+\Delta_j)(\sigma_i^2+\sigma_j^2)}{cr[-\Delta_i\sigma_j^2+\Delta_j(2\sigma_i^2+\sigma_j^2)]};$$

(d) *Total pay to the salesperson in territory i is lower under PRG than under PR, noG if* $[-\Delta_i\sigma_j^2 + \Delta_j(2\sigma_i^2+\sigma_j^2)] > 0$; *and*

(e) *Total pay across the sales force is lower in the PRG plan than in the PR, noG plan*.

Proof. See Technical Appendix B, available at http://mktsci.pubs.informs.org. □

This proposition implies that total incentive pay to the salesperson facing the PRG plan is higher than under the PR, noG plan, because of the incremental group incentive pay awarded based on the other territory's sales outcome. It also demonstrates that equilibrium salary is lower in territory i with the PRG plan than with the PR, noG plan, under the conditions in Proposition 1(c) above. These conditions hold for any degree of negative covariance in sales between territories if $\Delta_j > \Delta_i$. But even when $\Delta_j < \Delta_i$, territory j merely has to be "good enough" relative to i's own territory ($\Delta_j/\Delta_i > \sigma_j^2/(2\sigma_i^2+\sigma_j^2)$) for salesperson i's equilibrium salary to fall, given negative sales-territory covariance.[12] The proposition thus implies that the ratio of salary to incentive pay is lower with a PRG plan than with a PR, noG plan under a broad range of conditions. Finally, when allocating the two salespeople to negatively correlated territories, the PRG compensation plan is shown to be less costly than the best possible PR, noG plan; this result reflects the coinsurance value of using a group commission element in the pay plan.

The next question addresses the optimal territory allocation itself. As above, suppose that sales in territories α and β are positively correlated, and sales in territories α and γ (and β and γ) are negatively correlated. Lemma 1 implies that if the firm allocates the two salespeople to $\{\alpha, \beta\}$, the optimal compensation plan is PR, noG: salary plus a commission on own-territory performance only. If the firm instead allocates the two salespeople to either $\{\alpha, \gamma\}$ or $\{\beta, \gamma\}$, the optimal compensation plan is PRG: salary plus an own-territory commission component plus a group commission component. Then the question of interest is this: What conditions favor an allocation to negatively correlated territories $\{\alpha, \gamma\}$ or $\{\beta, \gamma\}$ (with the PRG solution) over an allocation to positively correlated territories $\{\alpha, \beta\}$ (with the PR, noG solution)?

[11] We have also analyzed a model including a continuous sales-effort response function. In this case, a change in the compensation plan has demand-side (sales quantity) implications as well as the cost-side (cost of sales force compensation) implications shown here. When a group commission is optimal (which again occurs whenever sales territory outcomes are negatively correlated), this analysis shows that using this extra compensation component increases sales effort levels relative to the outcome when group commissions are omitted. Intuitively, even though a group commission leads to some free-riding (i.e., the tendency to reduce effort because one can "count on" the performance of the other salesperson), the coinsurance effect arising from negatively correlated sales outcomes overrides the free-riding effect, producing higher rather than lower effort exertion in equilibrium. This result is notable particularly when contrasted with results in the team-selling literature, which do not take into account any coinsurance effects and hence always find a pure free-riding effect leading to reduced effort exertion. Full details are available from the authors.

[12] For example, if the variance of sales is the same in both territories ($\sigma_i^2 = \sigma_j^2$), Δ_j only has to be at least one-third as large as Δ_i for i's salary to fall with a shift from PR, noG to PRG, no matter how negatively correlated sales outcomes are across territories. If the variance of sales is twice as large in the new territory as the original ($\sigma_j^2 = 2\sigma_i^2$), Δ_j only has to be at least one-half as large as Δ_i for i's salary to fall with a shift from PR, noG to PRG, no matter how negatively correlated sales outcomes are across territories.

The profitability difference between the $\{\alpha, \gamma\}$ allocation and the $\{\alpha, \beta\}$ allocation is (with the profitability difference between the $\{\alpha, \gamma\}$ allocation and the $\{\beta, \gamma\}$ allocation analogously written)

$$\begin{aligned} d_{\{\alpha\beta\}}^{\{\alpha\gamma\}} &\equiv \pi^{*\mathrm{PRG}\{\alpha\gamma\}} - \pi^{*\mathrm{PR,\,noG}\{\alpha\beta\}} \\ &= (q_\gamma + \Delta_\gamma - q_\beta - \Delta_\beta) \\ &\quad - \frac{rc^2}{2}\left[\frac{\sigma_\gamma^2}{\Delta_\gamma^2} - \frac{\sigma_\beta^2}{\Delta_\beta^2} - \left(\frac{\Delta_\alpha + \Delta_\gamma}{\Delta_\alpha \Delta_\gamma}\right)^2 \left(\frac{\sigma_{\alpha\gamma}^2}{\sigma_\alpha^2 + \sigma_\gamma^2}\right)\right]. \end{aligned}$$

By defining ∇ to be the difference in expected sales productivity between territories ($\nabla = (q_\gamma + \Delta_\gamma) - (q_\beta + \Delta_\beta)$), we can see that the reduction in expected total sales across the two territories because of diversification (represented by the magnitude of $|\nabla|$) means a lower average sales per salesperson, a factor working against this type of diversification as a profitable sales force allocation strategy. However, average sales per territory is not the only element of profit that changes with diversification: the firm also enjoys a savings in the risk premium payments to the sales force of

$$\frac{rc^2}{2}\left[\frac{\sigma_\gamma^2}{\Delta_\gamma^2} - \frac{\sigma_\beta^2}{\Delta_\beta^2} - \left(\frac{\Delta_\alpha + \Delta_\gamma}{\Delta_\alpha \Delta_\gamma}\right)^2 \left(\frac{\sigma_{\alpha\gamma}^2}{\sigma_\alpha^2 + \sigma_\gamma^2}\right)\right].$$

The following proposition details the factors affecting the optimality of an allocation to negatively correlated versus positively correlated sales territories:

Proposition 2. *A sales force allocation switch from positively correlated territories ($\{\alpha, \beta\}$) to negatively correlated territories ($\{\alpha, \gamma\}$) is incrementally more profitable*:

(a) *The smaller the absolute difference in baseline productivity between the newly adopted territory and the abandoned territory ($|q_\gamma - q_\beta|$) is,*

(b) *The greater the sales variance in the abandoned territory (σ_β^2) is,*

(c) *The smaller the sales variance in the newly allocated territory (σ_γ^2) is,*

(d) *The more negative the covariance between territory outcomes in the new allocation ($\sigma_{\alpha\gamma}$) is,*

(e) *The higher the salesperson risk aversion (r) or the cost of exerting high sales effort (c) [if the allocation shift generates a net savings in the risk premium paid to the sales force] is,*

(f) *The smaller the productivity of high effort in the retained territory (Δ_α) is,*

(g) *The smaller the productivity of high effort in the abandoned territory (Δ_β) is, or*

(h) *The higher the productivity of high effort in the newly adopted territory (Δ_γ) is.*

Proof. Follows directly from calculation of the relevant comparative-static expressions. □

Unlike the results in the previous literature, this proposition shows conditions under which it is optimal to purposefully allocate a salesperson to a lower-productivity territory type—even if it is possible to choose otherwise. This surprising result holds because by doing so, the firm gains the coinsurance or risk-sharing benefits of diversification of the sales force, but only if it also implements a group commission as part of the overall compensation structure.

Proposition 2(a) makes intuitive sense, because territory potential has a direct positive effect on profits. Proposition 2, factors (b) and (c) are similarly intuitive, given the risk aversion in the sales force and the resulting need of the firm to manage compensation risk. Proposition 2(d) favors the negative territory allocation, because of its ability to offer coinsurance when the PRG plan is used. Similarly, Proposition 2(e) (salespeople who are more risk-averse or who face a higher disutility of working hard) favors negatively correlated sales territories (and the PRG plan) when it saves on the risk premium paid to the sales force. This holds when

$$\left[\frac{\sigma_\gamma^2}{\Delta_\gamma^2} - \frac{\sigma_\beta^2}{\Delta_\beta^2} - \left(\frac{\Delta_\alpha + \Delta_\gamma}{\Delta_\alpha \Delta_\gamma}\right)^2 \left(\frac{\sigma_{\alpha\gamma}^2}{\sigma_\alpha^2 + \sigma_\gamma^2}\right)\right] < 0,$$

which is unambiguously true if ($\sigma_\gamma^2/\Delta_\gamma^2 < \sigma_\beta^2/\Delta_\beta^2$)—that is, when the productivity-adjusted riskiness of the new territory is less than that of the old territory. Also intuitive are Proposition 2, factors (f), (g), and (h), because they imply a lower opportunity cost of abandoning one of the positively correlated territories. Finally, although not explicitly stated in Proposition 2, inspection of the equation defining $d_{\{\alpha\beta\}}^{\{\alpha\gamma\}}$ shows that when either $\sigma_\gamma^2 \to 0$ (or "small enough") or $\Delta_\gamma^2 \to \infty$ (or "large enough"), the term defining the net change in the risk premium

$$\left(-\frac{rc^2}{2}\left[\frac{\sigma_\gamma^2}{\Delta_\gamma^2} - \frac{\sigma_\beta^2}{\Delta_\beta^2} - \left(\frac{\Delta_\alpha + \Delta_\gamma}{\Delta_\alpha \Delta_\gamma}\right)^2 \left(\frac{\sigma_{\alpha\gamma}^2}{\sigma_\alpha^2 + \sigma_\gamma^2}\right)\right]\right)$$

becomes strictly positive, meaning that the compensation awarded to control risk in the sales force decreases by shifting to the negatively correlated territories. This increases the favorability of shifting from $\{\alpha, \beta\}$ to $\{\alpha, \gamma\}$—even if doing so were to decrease average productivity across the sales force (i.e., even if $(q_\gamma + \Delta_\gamma - q_\beta - \Delta_\beta)$ were negative).

In sum, the analysis thus far shows the need to simultaneously consider territory-allocation decisions and sales-compensation decisions when compensation is given through piece-rate plans. A standard piece-rate plan, with salary and commission on own-territory performance, is optimal when salespeople are allocated to territories with positively correlated outcomes. However, when salespeople are deployed to negatively correlated territories, the optimal piece-rate compensation plan includes not only salary and own-territory commission components but also a group commission, because of its ability to manage the firm's total compensation costs by lessening the risk premium paid out to the salespeople. Our comparative-static results show the key effects of risk aversion, sales-territory variance, and territory covariance on both the size of the compensation components and the overall leverage in the compensation plan. We also show that, given a choice between positively correlated or negatively correlated sales territories and a piece-rate plan structure, the *PRG* plan with negatively correlated territories is relatively more preferred when the covariance becomes more and more negative and when salespeople's risk aversion increases. These insights are derived in a two-salesperson model; to check the robustness of the results with respect to sales force size, we next extend our model to allow for a sales force of size $N > 2$.

4. Territory Allocation Under Piece-Rate Compensation with Multiple Salespeople

In this section, we study the firm's sales force allocation problem when it has more than two salespeople ($N > 2$).[13] We consider the same model analyzed in §3 but now assume that there exist many territories of each type (α, β, and γ), enough that if the firm so chose, it could allocate all N salespeople to distinct territories of the same type. For notational simplicity, we redefine an allocation to be a set $\mathcal{D} = \{\mathbf{t}_1, \mathbf{t}_2\}$, where $\mathbf{t}_1$ and $\mathbf{t}_2$ are each a vector of territories, and we assume that the firm allocates a fraction θ of the salespeople to a territory type.[14]

The following lemma states the conditions under which a positive group commission is optimal when multiple salespeople are allocated to two different territory types.

Lemma 2. *A positive group commission is optimal when all the following conditions hold*:

(a) *The sales covariance* σ_{ij} *is negative*;

(b) *Territories in vector* $\mathbf{t}_2$ *are not too inferior to territories in vector* $\mathbf{t}_1$;

(c) *The number of salespeople N is not too large*; *and*

(d) θ *is strictly greater than* 0 *and strictly smaller than* 1.

Proof. See Technical Appendix B, available at http://mktsci.pubs.informs.org. □

The *first two* conditions echo those in the presence of just two salespeople in the sales force; the first point establishes the value of diversification as a risk-sharing or coinsurance mechanism, and the second point ensures that the resulting drop in average productivity is not so great as to overcome the benefits of coinsurance.

The intuition for the *third* condition above hinges on the trade-off between the benefit of diversification and the concomitant cost of free-riding by the sales force. In general, the larger the group size (N) is, the greater the possibility of free-riding is, for any given group commission rate, because one's own effort plays less and less of a role in the overall group-sales outcome. The proof of Lemma 2 establishes that the optimal group comission g^* is decreasing in N; in other words, as N increases, the optimal group commission rate decreases, precisely in a bid to minimize the free-riding effect. However, this tactic of lowering g^* for greater and greater N can only be so useful; for large enough N, the group commission would drop below zero, indicating that there is no positive group commission rate that optimally balances the benefits of diversifying the sales force versus the costs of free-riding under a group commission structure. Consider the Care of Trees example, where a higher percentage of group incentive pay is used for the higher-level (and fewer in number) regional vice presidents compared with the lower-level (and more numerous) district managers. The "number of salespeople" effect indicated here suggests why such a compensation variation might be optimal: when the group is smaller (fewer district managers), a group incentive component may be optimal where it is not optimal for the larger group of lower-level regional managers.

Notice that the *fourth* condition above implies that a group commission is not optimal without diversified allocations. The intuition for this is clear for the case of $\theta = 0$ or $\theta = 1$ exactly, as discussed above in the $N = 2$ case: when all salespeople are allocated to territory type 1 ($\theta = 1$) or all salespeople are allocated to territory type 2 ($\theta = 0$), the group comission would be a strictly negative value (which is impossible). The intuition for this result is that when all salespeople are in the same territory there is no coinsurance effect, but the negative free-riding effect is still present.

With this understanding of the role that a group commission plays in our sales force optimization

[13] As in the two-salesperson case, we have also investigated the optimal allocation of multiple salespeople when effort is a continuous decision variable. The results are qualitatively equivalent to those presented here with the discrete sales-response function. The analysis is available from the authors upon request.

[14] Of course, one cannot allocate a fractional number of salespeople to territories. In fact, one will allocate an integer number of salespeople closest to the optimal fractional number. For instance, if the sales force size is $N = 10$, and the optimal value for θ is $\theta^* = 0.32$, one will allocate three salespeople to a territory type and the remaining seven salespeople to the other territory types.

problem, we can now state our main result regarding sales force diversification:

Proposition 3. *If it is optimal for the firm to diversify the sales force across negatively correlated territory types, it will not be optimal to do so in an extremely imbalanced way.*

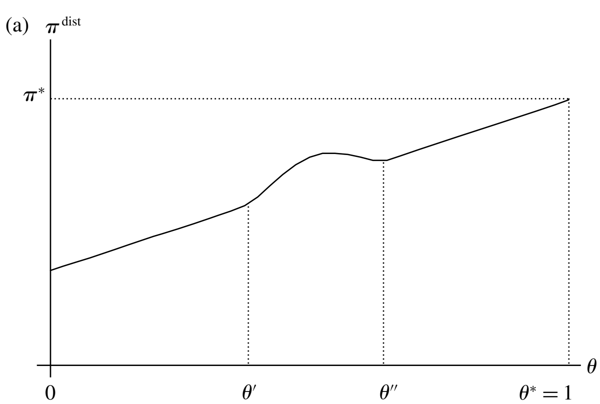
Proof. See Technical Appendix B, available at http://mktsci.pubs.informs.org. □

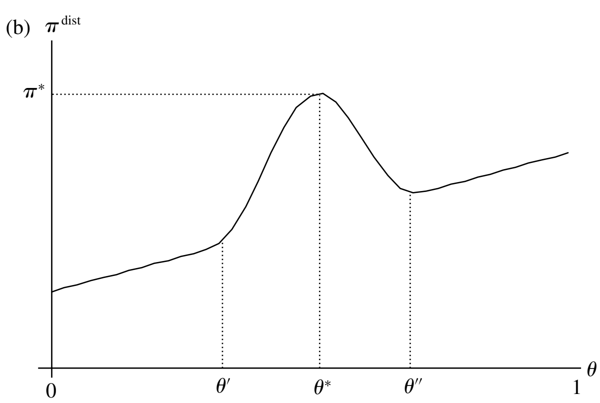
Given these results, Figure 1 below exemplifies the two possibilities for salesperson diversification across territory types, assuming that $\Delta_1 > \Delta_2$. Each of the panels of Figure 1 plots the upper envelope of the profitabilities of diversification with or without group commission. In the ranges $\theta \in [0, \theta']$ and $\theta \in [\theta'', 1]$, the upper-envelope profit values imply no group commission (because the nonnegativity constraint forces the optimal group commission to be the corner solution $g^* = 0$). In the range $\theta \in [\theta', \theta'']$, the upper-envelope profit values imply a positive group commission (because $g^* \geq 0$). Conditions (a)–(c) in Lemma 2 determine whether one is likely to face the scenario in Figure 1(a) or the scenario in Figure 1(b).

In Figure 1(a), even though there is a local profit maximum between the lower and upper bounds on the condition for g^* to be positive, it is not sufficiently profitable to outweigh allocating all salespeople to the more productive territory type 1; it is therefore never optimal to diversify the sales force across territory types in this situation. This happens when any of the conditions in Lemma 2 are not satisfied. In Figure 1(b), diversification is optimal because the local profit maximum at θ^* produces a profit level higher than that possible when failing to diversify the sales force by allocating all salespeople to territories of type 1. This happens when all of the conditions in Lemma 2 are satisfied. Because the optimum for the unconstrained objective function is necessarily at some θ value that is discretely different from either zero or one in Figure 1(b), a solution that involves diversifying the sales force will do so in a relatively "balanced" way, meaning that the firm will not allocate a large number of people to one type of territory and just a few people to the other type.

In the next section, we return to the two-salesperson baseline model but extend it to consider whether and when a different type of inter-salesperson compensation plan—the tournament (or sales contest)—could be better than the best piece-rate compensation plan.

Figure 1 **Optimal Distribution of Salespeople**





5. Territory Allocation Under Either Piece-Rate or Tournament Compensation with Two Salespeople

Tournaments are contracts based on relative selling performance, in which rivals compete with one another by expending effort to win prizes. A sales contest is an example of tournament compensation. In a contest with two salespeople, we denote the prize for the winner as W, while the prize for the loser is w. The winner is the salesperson who achieves the highest level of sales. We develop a contest model similar to the tournament model of Lazear and Rosen (1981), with the extension that we allow for any degree of positive or negative correlation across territories.

The probability that a salesperson allocated to some territory i wins the tournament is given by $P(X_i(e_i) + \varepsilon_i > X_j(e_j) + \varepsilon_j)$, which can be rewritten as $P(X_i(e_i) - X_j(e_j) > \varepsilon_j - \varepsilon_i)$. Since both ε_i and ε_j are distributed Normal with mean zero, the probability of winning is given by the Normal cumulative density function (c.d.f.) with mean zero and variance $\sigma^2_{j-i} = \sigma^2_i + \sigma^2_j - 2\sigma_{ij}$. We let $F(\cdot)$ denote the c.d.f. of this Normal distribution: $F(y) = F(X_i(e_i) - X_j(e_j))$, so that $F(y) \sim N(y, \sigma^2_{j-i})$. (A detailed discussion of how our tournament model compares with the classic tournament models in the literature is presented in Technical Appendix C, available at http://mktsci.pubs.informs.org.)

In this type of contract, the utility for the salesperson allocated to territory i can be expressed as

$$u_i = F(X_i(e_i)-X_j(e_j))v(W-C(e_i)) + [1-F(X_i(e_i)-X_j(e_j))]v(w-C(e_i)).$$

The firm's profit from running this tournament is

$$\pi^T = \sum_{i=1}^{2}[X_i(e_i)] - W - w. \quad (3)$$

5.1. Tournament Compensation: Analysis and Results

For $i = 1, 2$, the optimal tournament solves the problem

$$(P^T):\quad \max_{W,\, w\in\mathbb{R}_+} \sum_{i=1}^{2}[X_i(e_i)] - W - w \quad \text{s.t.}$$

$$PC:\quad F\big(X_i(e_i)-X_j[e_j]\big)u(W-C(e_i)) + \big[1-F(X_i(e_i)-X_j(e_j))\big]u(w-C(e_i)) \geq u(m),$$

$$IC:\quad e_i^* \in \arg\max_{e_i\in\{L,H\}} F\big(X_i(e_i)-X_j(e_j)\big)u(W-C(e_i)) + \big[1-F(X_i(e_i)-X_j(e_j))\big]u(w-C(e_i)).$$

Lemma 3 below characterizes the necessary parametric condition for a tournament solution to exist, optimal tournament compensation rewards, and resulting profits, assuming that the product indexed by $i=1$ exhibits the greater selling-effort productivity (that is, $\Delta_i > \Delta_j$).

Lemma 3. *The optimal tournament compensation arrangement is characterized by the following*:

If

$$e^{rc} \leq \frac{F(q_j-(q_i+\Delta_i))}{F((q_j+\Delta_j)-(q_i+\Delta_i)) - e^{rm}[F((q_j+\Delta_j)-(q_i+\Delta_i))-F(q_j-(q_i+\Delta_i))]},$$

the optimal tournament prizes are

$$w^* = \frac{1}{r}\ln\left(\frac{F((q_j+\Delta_j)-(q_i+\Delta_i)) - e^{r(c+m)}F(q_j-(q_i+\Delta_i))}{-F(q_j-(q_i+\Delta_i)) + e^{rc}F((q_j+\Delta_j)-(q_i+\Delta_i))}\right),$$

$$W^* = \frac{1}{r}\ln\left(\frac{F((q_j+\Delta_j)-(q_i+\Delta_i)) - e^{r(c+m)}F(q_j-(q_i+\Delta_i))}{1-F(q_j-(q_i+\Delta_i)) - e^{rc}[1-F((q_j+\Delta_j)-(q_i+\Delta_i))]}\right) \geq w^*,$$

generating profits of $\pi^{*T} = \pi^{*T1} = (q_i+\Delta_i+q_j+\Delta_j) - W^* - w^*$.

Conversely, if

$$e^{rc} > \frac{F(q_j-(q_i+\Delta_i))}{F((q_j+\Delta_j)-(q_i+\Delta_i)) - e^{rm}[F((q_j+\Delta_j)-(q_i+\Delta_i))-F(q_j-(q_i+\Delta_i))]},$$

the optimal prizes are

$$w^* = 0,$$

$$W^{*c} = \frac{1}{r}\ln\left(\frac{F(q_j-(q_i+\Delta_i)) - e^{rc}F((q_j+\Delta_j)-(q_i+\Delta_i))}{-1+F(q_j-(q_i+\Delta_i)) + e^{rc}[1-F((q_j+\Delta_j)-(q_i+\Delta_i))]}\right),$$

generating profits of $\pi^{*T} = \pi^{*T2} = (q_i+\Delta_i+q_j+\Delta_j) - W^{*c}$.

Furthermore, a tournament is implementable only if

$$e^{rc} < \frac{1-F(q_j-(q_i+\Delta_i))}{1-F((q_j+\Delta_j)-(q_i+\Delta_i))}.$$

Proof. See Technical Appendix B, available at http://mktsci.pubs.informs.org. □

Lemma 3 presents the optimal prizes of a tournament for both an "interior" and a "corner" situation. When

$$e^{rc} \leq \frac{F(q_j-(q_i+\Delta_i))}{F((q_j+\Delta_j)-(q_i+\Delta_i)) - e^{rm}[F((q_j+\Delta_j)-(q_i+\Delta_i))-F(q_j-(q_i+\Delta_i))]},$$

the firm does better by setting positive values for both prizes; this condition holds, for example, when risk aversion (r) is not too great, when the cost of exerting high effort (c) is not too high, or when the minimum acceptable payment (m) is high enough. In the reverse situation

$$\left(e^{rc} > \frac{F(q_j-(q_i+\Delta_i))}{F((q_j+\Delta_j)-(q_i+\Delta_i)) - e^{rm}[F((q_j+\Delta_j)-(q_i+\Delta_i))-F(q_j-(q_i+\Delta_i))]}\right),$$

the prize for losing a tournament is zero, and a salesperson receives compensation only if she wins the tournament.

An important result in Lemma 3 is that designing a tournament that implements high effort by both players is possible only under some specific parametric conditions (consistent with Dye 1984). In contrast, piece-rate plans (PR, noG, or PRG) are implementable across the entire parameter space. If the risk aversion coefficient r or the cost of effort c are sufficiently large, then a tournament is not implementable. Intuitively, the disutility of losing the tournament is too great when players are very risk-averse; no profitable set of prizes can then motivate high effort. Similarly, if effort exertion is too costly, there is no feasible set of winning and losing prizes that sufficiently motivates players to expend high effort to compete to win the tournament.

A tournament is likewise more limited in its parametric feasibility, the closer in value $F(q_j-(q_i+\Delta_i))$ and $F((q_j+\Delta_j)-(q_i+\Delta_i))$ are; this holds either when the sales productivity in territory j is very small ($\Delta_j \to 0$), or when the net variance σ^2_{j-i} is either very small ($\sigma^2_{j-i} \to 0$) or very large ($\sigma^2_{j-i} \to +\infty$). Thus, when either ($\Delta_j \to 0$), ($\sigma^2_{j-i} \to 0$) or ($\sigma^2_{j-i} \to +\infty$), $(1-F(q_j-(q_i+\Delta_i)))/(1-F((q_j+\Delta_j)-(q_i+\Delta_i))) \to 1$, requiring $e^{rc} < 1$ for tournament feasibility.[15] Clearly, either c or r must approach zero (i.e., the cost of effort approaches zero or the utility function approaches

[15] Please see Technical Appendix B for a demonstration that both $F(q_j-(q_i+\Delta_i)) \to 0$ and $F((q_j+\Delta_j)-(q_i+\Delta_i)) \to 0$ when $\sigma^2_{j-i} \to 0$, and that both $F(q_j-(q_i+\Delta_i)) \to 1/2$ and $F((q_j+\Delta_j)-(q_i+\Delta_i)) \to 1/2$ when $\sigma^2_{j-i} \to +\infty$.

Table 1 **Tournament Comparative Statics**

Parameter	$\partial W^*/\partial\cdot$	$\partial w^*/\partial\cdot$	$\partial \pi^{*T}/\partial\cdot$
c	$+$	≤ 0	$-$
r	$+$	≤ 0	$-$
σ^2_{j-i}	$+$	≤ 0	$-$
q_i	$+$	≥ 0	$-$
q_j	$-$	≤ 0	$+$
Δ_i	$+$	≥ 0	$-$
Δ_j	$-$	≥ 0	$+$

risk neutrality) to have an implementable tournament under these circumstances; in other words, the implementability of a tournament decreases under these conditions.

The effect of parametric changes in the optimal tournament prizes, and consequently on the tournament profitability, is presented in Table 1.[16]

The comparative statics are aligned with the general results in the tournament literature. Given these results, we now proceed to investigate how the compensation of the sales force influences the optimal sales force allocation across territories.

5.2. Tournament vs. Piece-Rate, and Implications for Territory Allocation

By using the results from Lemmas 1 and 3, we can compare the profitability of each compensation contract and determine the best salesperson allocation strategy. The profitability of each compensation system depends on the parameter values in the model. We examine how the results vary for different degrees of risk aversion of the sales force, and within these subcases we discuss the moderating effects of other parameters of the problem on the results.

5.2.1. Salespeople Are Extremely Risk-Averse. Consider first the case where

$$e^{rc} > \frac{1 - F(q_j - (q_i + \Delta_i))}{1 - F((q_j + \Delta_j) - (q_i + \Delta_i))},$$

which implies that

$$r > \frac{\ln((1 - F(q_j - (q_i + \Delta_i)))/(1 - F((q_j + \Delta_j) - (q_i + \Delta_i))))}{c};$$

that is, salespeople are very risk-averse. In this situation, Lemma 3 implies that a tournament that motivates high effort by the sales force is infeasible, and thus the tournament can never be optimal. In this case, only a piece-rate (without or with a group commission component) is feasible, and the results from §3 characterize both the optimal contract structure and territory allocation decisions.

[16] Please see Technical Appendix B, available at http://mktsci.pubs.informs.org, for the calculations and discussion of these effects.

5.2.2. Salespeople Are Risk-Neutral. The optimal allocation when the firm is risk-neutral is given by the following proposition:

Proposition 4. *When salespeople are risk-neutral*:

(a) *If territories have the same sales potential, the firm is indifferent among the possible territory allocation choices, and achieves the first-best result with either a piece-rate compensation plan or a tournament.*

(b) *If territories have different sales potentials, the optimal territory allocation deploys the salespeople to the two most productive territories and the firm achieves the first-best profit level by using the standard piece-rate compensation contract* ($g^* = 0$).

Proof. See Technical Appendix B, available at http://mktsci.pubs.informs.org. □

The rationale for this proposition is quite straightforward. Note first that π^{*T} converges to the first-best solution as the term

$$c\frac{1 - 2F((q_j + \Delta_j) - (q_i + \Delta_i))}{F((q_j + \Delta_j) - (q_i + \Delta_i)) - F(q_j - (q_i + \Delta_i))}$$

approaches zero. For any positive effort cost c, this only happens when $F((q_j + \Delta_j) - (q_i + \Delta_i)) = 1/2$, i.e., when territories are the same. Consequently, the optimal allocation with piece-rate compensation in this situation is to employ the standard piece-rate compensation contract ($g^* = 0$) and to deploy salespeople to the two best territories (regardless of the correlation between these territories). More generally, when salespeople are risk-neutral, the variance or uncertainty of outcomes is not a factor. The only issue that matters for salespeople is expected returns. Both plans therefore give the first-best result when territories have the same sales potential.

Although the piece-rate plan continues to give the first-best result when territory potentials are different (the group commission is still zero), the tournament loses effectiveness. The reason is that differences in the territories make one of the salespeople more likely to win the tournament, but the prizes are of the same form for both salespeople. Hence, to meet the disadvantaged salesperson's minimum-utility constraint and provide an incentive for him to work hard, the firm needs to increase the losing tournament prize. As a consequence, the advantaged salesperson necessarily ends up receiving more than his minimum utility to create an incentive for him to also exert high effort. This makes the tournament plan more costly than a piece-rate plan. Thus, the profit-maximizing solution for the firm when territories have different sales potentials is to deploy salespeople to the most profitable territories and to offer a standard ($g^* = 0$) piece-rate compensation plan.

5.2.3. Salespeople Are Moderately Risk-Averse. Now, consider what happens if the salesperson's risk aversion parameter is small enough that either a tournament compensation contract or a piece-rate contract can be optimal:

$$0 < r < \frac{1}{c}\ln\left(\frac{1-F(q_j-(q_i+\Delta_i))}{1-F((q_j+\Delta_j)-(q_i+\Delta_i))}\right).$$

The relative profitability levels of the PRG and tournament compensation contracts depend on the joint levels of the model's parameters, according to the proposition below.

Proposition 5. *When salespeople are moderately risk-averse*:

(a) *The PRG contract is relatively more profitable, the more risk-averse the salespeople are; the higher the cost of sales effort is; the more different expected sales across territories are; and the more negatively correlated the territories are.*

(b) *The tournament contract is relatively more profitable, the less risk-averse the salespeople are; the lower the cost of sales effort is; the less different expected sales across territories are; and the more positively correlated are the territories.*

Proof. See Technical Appendix B, available at http://mktsci.pubs.informs.org. □

The rationale is as follows. To allow the salespeople to achieve a specific level of minimum utility in the PRG contract, the higher the disutility of effort exertion is, the higher the reward offered to the salesperson must be, given any degree of riskiness in the sales process (i.e., variance in the sales-response function). In the tournament plan, this effect is also present, but the salesperson faces an additional risk as well: that of losing the tournament. The additional risk element $P(X_i > X_j)$, by which we must multiply the utility expression, accelerates the tournament reward that must be paid to the salesperson for him to find it worthwhile to work hard. This compounding effect makes it increasingly less worthwhile for the firm to use a contest as the disutility for effort increases.

Although the PRG contract does not suffer from the fact that territories are dissimilar (in a PRG contract, increases in selling productivity are always better regardless of the territory), the tournament suffers from the fact that territories are dissimilar (in a tournament, increases in selling productivity are better if they make territories more similar but are worse if they make territories more dissimilar). This occurs because similar territories are more likely to result in a contest in which both players exert high effort, whereas dissimilar territories are demotivating for both the disadvantaged salesperson (who thinks, "there is no way I can win this contest") and the advantaged salesperson (who thinks, "I have already won this contest, so why work hard?"). Hence, dissimilar territories require large prizes to be awarded if a contest is used (making the contest less profitable) to motivate the players.

Last, the effect of the covariances comes from the fact that in the PRG contract, the more negative the covariance, the higher the coinsurance effect, and thus the smaller the risk-premium the firm needs to pay to the salespeople. Conversely, in a tournament, the more positive the covariance is, the more the common uncertainty of the two territories is filtered out (since in the tournament, the net variance σ_{i-j} is a function of the difference of the two random components (a function of $\varepsilon_i - \varepsilon_j$)).

5.3. Overall Insights for Combined {Allocation, Compensation} Decisions

The above results show that, when contractual arrangements are taken into account, the optimal deployment of salespeople across territories is not a straightforward process.

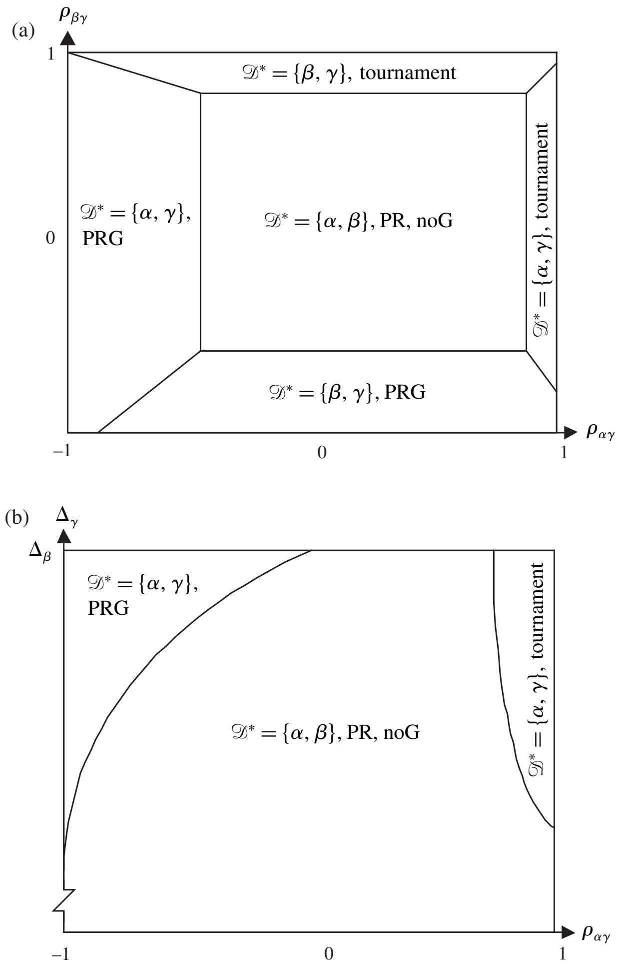
In line with common wisdom, there are situations in which the firm may prefer to allocate its salespeople to the two most productive territories ($\mathcal{D}^* = \{\alpha, \beta\}$). This is true, for instance, when the salespeople are risk-neutral or when the two most productive territories are considerably superior to the other territory. In this situation either the standard PR, noG, or tournament compensation plans could be optimal, depending on the model's parameters.[17]

However, there are conditions under which the firm does better by allocating a salesperson to the highest-productivity territory and the other salesperson to the lowest-productivity territory, thus bypassing the intermediate territory ($\mathcal{D}^* = \{\alpha, \gamma\}$). This could happen, for instance, when salespeople are risk-averse and experience great disutility for effort, territories are not very different from each other, and the highest- and the lowest-productivity territories have a negative covariance ($\sigma_{\alpha\gamma} < 0$). In this situation the PRG compensation plan is optimal.

It is even possible for the firm to optimize by completely bypassing the highest-productivity territory and allocating salespeople to the two least productive territories ($\mathcal{D}^* = \{\beta, \gamma\}$). This is optimal when salespeople are risk-averse, the highest-productivity territory is not much better than the other territories, and the (negative) covariance between territories β and γ is large in absolute value (more precisely, when $|\sigma_{\alpha\gamma}| < |\sigma_{\beta\gamma}|$). Under these conditions, the PRG contract is optimal if the covariance $\sigma_{\beta\gamma}$ is negative, and

[17] Here and in the next two paragraphs, the conditions in Proposition 5 govern what parametric scenarios favor piece rates over tournaments, and vice versa.

Figure 2 **Optimal Allocation and Compensation Contract Regions**



the tournament is optimal if the covariance $\sigma_{\beta\gamma}$ is positive.

We illustrate these possibilities in Figure 2. We consider parameter values for which tournaments are feasible (parameters r and c are not too high, and territories are not too different). In Figure 2(a) we vary the correlation between territories α and γ, $\rho_{\alpha\gamma}$, and between territories β and γ, $\rho_{\beta\gamma}$, and fix all other parameter values. It is easy to see that when the correlations are moderate, it is optimal to allocate salespeople to the two most productive territories, and the optimal compensation is PR, noG. However, there are parametric values that support allocation $\mathscr{D}^* = \{\alpha, \gamma\}$ (when $\rho_{\alpha\gamma}$ is very small or very large), as well as allocation $\mathscr{D}^* = \{\beta, \gamma\}$ (when $\rho_{\alpha\beta}$ is very small or very large). These results demonstrate that when salesperson compensation is taken into account, it is possible for the optimal allocation to call for bypassing the intermediate territory β, or even bypassing the most productive territory α.

In Figure 2(b) we vary the correlation between territories α and γ, $\rho_{\alpha\gamma}$, and the productivity of selling effort of the inferior territory Δ_γ, and fix all other parameter values. When the correlation $\rho_{\alpha\gamma}$ is negative and territory γ is not too inferior to territory β, it is optimal to bypass the intermediate territory β ($\mathscr{D}^* = \{\alpha, \gamma\}$) and the optimal compensation is *PRG*. When the correlation is very positive and territory γ is not too inferior to territory β, it is once again optimal to bypass the intermediate territory β ($\mathscr{D}^* = \{\alpha, \gamma\}$) but now the optimal compensation is a tournament. Conversely, when territory γ is significantly inferior to territory β or when territory γ is close (but still inferior) to territory β and $\rho_{\alpha\gamma}$ is moderate, it is optimal to allocate salespeople to the two most productive territories ($D^* = \{\alpha, \beta\}$) and the optimal compensation is PR, noG.

Notice that many of these insights could be applied to the firm's territory design choices as well. Our results imply that a firm could consider the intentional design of either negatively or positively correlated territories. The results in our lemmas and propositions support that the firm would find it optimal to design either strongly negatively correlated territories (and use group commission compensation contracts) or strongly positively correlated territories (and use tournaments as compensation contracts). These solutions are optimal only if the territory design thus chosen does not sacrifice baseline sales, or the productivity of sales effort of these territories, too much.

6. Discussion and Conclusion

Taking into account insights from multiple literatures, this paper considers a sales force allocation and compensation model in which a firm can allocate salespeople to territories of different sales potential characteristics to maximize profitability. Our research shows that taking into account inter-territory correlation in sales outcomes, and factoring this into sales force compensation, binds together the territory allocation decision and the compensation decision when managing risk-averse salespeople.

We find that allocating salespeople to negatively correlated sales territories beneficially diversifies each salesperson's own portfolio of sales outcomes when compensation includes a group incentive component, and can improve profitability even when this allocation involves decreasing the average territory sales performance across the sales force. Merely diversifying the sales force by allocating salespeople to territories of different types is insufficient to coordinate the channel and can actually decrease profitability for the firm. When group incentives are offered without diversifying the sales force into negatively correlated sales-territory types, the firm's profits also decrease.

The logic for this key result is that the allocation of salespeople to negatively correlated sales territories, combined with the aggregation of individual sales outcomes through a group incentive pay component, effectively diversifies the reward each individual salesperson accrues in return for effort in his own single territory. Thus, the risk inherent in exerting high effort levels in a territory subject to the possibility of adverse outcomes is reduced; the salary-based risk premium the firm needs to pay to the salespeople therefore falls, which increases the firm's profits.

From a methodological point of view, this insight is reminiscent of the finance literature on portfolio diversification. However, in the portfolio diversification literature, the productivity (i.e., returns) of individual stocks does not change with their inclusion in (or exclusion from) the portfolio itself; thus, the returns from any given stock are treated as parametric to the portfolio diversification process itself. By contrast, in the sales force allocation problem we attack here, the very offering of a group commission (which effectively diversifies the salesperson's portfolio) in fact can be expected to affect the salesperson's optimal effort level and therefore also can affect key firm-level outcomes of sales and profitability. In effect, the choice to diversify the portfolio itself affects the returns from individual "stocks" (salespeople's sales outcomes in our context) in the portfolio. This necessarily requires us to integrate the finance portfolio approach with an agency theory approach to appropriately solve the problem of optimal territory allocation and sales force compensation.

Our results also show that a "balanced" allocation of salespeople across multiple territories, coupled with a group commission, is profitable when salespeople are risk-averse, the sales potentials of candidate territories are not too different, the expected sales outcomes of different territories are negatively correlated, and the total number of salespeople is not large.

Comparing these results with the standard results from portfolio allocation theory we notice a number of differences summarized in Table 2.

Table 2 Portfolio of Financial Assets vs. Portfolio of Salespeople

Portfolio of assets	Portfolio of salespeople
Allocation to both negatively and positively correlated assets can be profitable	Only allocation to negatively correlated territories can be profitable
Profitability of portfolio diversification does not change purely with the number of assets in the portfolio	Profitability of portfolio diversification decreases with an increase in the number of salespeople in the portfolio
No restrictions in the split of assets (highly imbalanced split between assets are possible)	Restriction on the split of salespeople between territory types: either extreme split allocations (all salespeople in the same territory type) or balanced split allocations (similar number of salespeople in each territory type)

A firm may, in general, have access to territory allocations involving positively correlated territory sales outcomes as well as negatively correlated ones. Our findings indicate that a group incentive component has no diversification benefits when sales outcomes are positively correlated across territories. However, the existing literature on tournaments suggests that a tournament can be superior to a pure piece-rate plan when the territories involved have strong positive correlation in outcomes. We therefore also compare amongst the pure piece-rate, our combined piece-rate plus group incentive pay plans, and a tournament compensation structure.

We find that the tournament compensation structure is infeasible under a wide variety of conditions, specifically when salespeople are too risk-averse, when within-territory sales variances are too small, and when inter-territory differences in sales potential are too great (although an appropriate handicapping scheme could battle this latter problem). Furthermore, even when a tournament is implementable (i.e., causes salespeople to exert high effort), it can be outperformed by either a pure piece-rate plan when the correlation between territories' outcomes is weak enough, in addition to the above factors. In general, our results show that the conditions favoring a particular joint choice of territory allocation (between negatively correlated or positively correlated territory types) and compensation structure are as shown in Table 3.

In analyzing the scenarios in this research, we considered the main elements that are commonly studied in analytical models of sales force compensation: baseline sales, sales effort productivity, sales uncertainty (variance), disutility of sales effort, and risk aversion. We also considered the covariance in sales across territories and the type of compensation plan offered.

Table 3 Relative Optimality of Tournament and Piece-Rate Compensation Plans

Tournament is likely to be optimal when	Piece-rate is likely to be optimal when
Salespeople have low risk aversion	Salespeople have high risk aversion
Salespeople experience low disutility for exerting selling effort	Salespeople experience large disutility for exerting selling effort
Territories have similar sales potential and productivity of selling effort	Territories have dissimilar sales potential and productivity of selling effort
Territories have strong positive correlation	Territories have negative correlation (in which case the plan has a group incentive component) or some moderate positive correlation (in which case the plan is the pure piece-rate)

However, other elements such as differences in product margins across territories or heterogeneity either in salespeople's abilities or in mimimum utilities for different territories could also affect firms' territory allocation and compensation choices. Our results provide indications of the effects of these factors as well. For instance, we anticipate that if a territory supports the sales of products with higher margins, the impact on sales territory allocation should be similar to that of a territory that supports higher productivity of selling effort. Nevertheless, future reseach could scrutinize the effects of these other factors on a firm's choice of territory allocations in more detail.

In sum, this research shows that joint consideration of the territory alignment and sales force compensation decisions can improve the profitability of the firm, and can overturn the predictions one has for the profitability of either decision alone. Although the contributing literatures are all individually useful to our problem, none fully captures the opportunities open to the firm that capitalizes on the interaction between allocation and compensation decisions for the sales force. Despite these contributions of the current research, future investigations can move our understanding forward even further. For example, the management of very large sales forces may lead to insights about subclustering of salespeople into smaller groups; such practices may help curb the incentive to free-ride while retaining the benefits of coinsurance. Other research could investigate the implications of team selling or salespeople's perceptions of fairness of the joint territory allocation and compensation decisions. Finally, the development of more in-depth examples like our summary of The Care of Trees, and even full statistical examination across many sales forces, will add more insight to the options available for sales force managers.

Acknowledgments

The initial thinking on this research was supported in part by a Leavey grant from Santa Clara University. The authors thank Tat Chan, John Hendricksen, Scott Jameson, Kissan Joseph, Manish Kacker, Dmitri Kuksov, Mike Lewis, Chakravarthi Narasimhan, Mitchell Petersen, Ambar Rao, Rama Subramaniam, Harish Sujan, Mita Sujan, Ying Xie, the review team, and attendees at the Second Workshop on Game Theory in Marketing at HEC-Montréal, the 2007 Kellogg Marketing Camp, the 2008 Marketing Science Conference, and the Erin Anderson B2B Research Conference for helpful comments. All errors are the responsibility of the authors.

References

Barrett, P. M. 1999. Uneasy gun makers add gentler product lines. *Wall Street Journal* (March 25) B1.

Barry, F. 2006. Are you getting what you pay for? *Decision: Ireland's Bus. Rev.* **11**(3) 14.

Basu, A. K., R. Lal, V. Srinivasan, R. Staelin. 1985. Salesforce compensation plans: An agency theoretic perspective. *Marketing Sci.* **4**(4) 267–291.

Brealey, R. A., S. C. Myers, F. Allen. 2006. *Principles of Corporate Finance*, 8th ed. McGraw-Hill Irwin, Boston.

Caldieraro, F., A. T. Coughlan. 2007. Spiffed-up channels: The role of spiffs in hierarchical selling organizations. *Marketing Sci.* **26**(1) 31–51.

Coughlan, A. T. 1993. Sales compensation: A review of MS/OR advances. G. L. Lilien, J. Eliashberg, eds. *Handbooks in Operations Research and Management Science*, Vol. 5. Elsevier Science Publishers, North-Holland, Amsterdam, 611–651.

Coughlan, A. T., C. Narasimhan. 1992. An empirical analysis of sales-force compensation plans. *J. Bus.* **65**(1) 93–121.

Coughlan, A. T., S. K. Sen. 1989. Salesforce compensation: Theory and managerial implications. *Marketing Sci.* **8**(4) 324–342.

Cross, J., S. W. Hartley, W. Rudelius, M. J. Vassey. 2001. Sales-force activities and marketing strategies in industrial firms: Relationships and implications. *J. Personal Selling Sales Management* **21**(3) 199–206.

Darmon, R. Y., D. Rouzies. 2002. Optimal sales-force compensation plans: An operational procedure. *J. Oper. Res. Soc.* **53**(4) 447–456.

Davenport, B., C. Fisher, E. Rosner. 2007. Sales slip. *Pharmaceutical Executive* **27**(1) 68–74.

Dye, R. A. 1984. The trouble with tournaments. *Econom. Inquiry* **22**(1) 147–149.

Gaynor, M., P. Gertler. 1995. Moral hazard and risk spreading in partnerships. *RAND J. Econom.* **26**(4) 591–613.

Godes, D. 2003. In the eye of the beholder: An analysis of the relative value of a top sales rep across firms and products. *Marketing Sci.* **22**(2) 161–187.

Godes, D. 2004. Contracting under endogenous risk. *Quant. Marketing Econom.* **2**(4) 321–345.

Green, J. R., N. L. Stokey. 1983. A comparison of tournaments and contracts. *J. Political Econom.* **91**(3) 349–364.

Hauser, J. R., D. I. Simester, B. Wernerfelt. 1994. Customer satisfaction incentives. *Marketing Sci.* **13**(4) 327–350.

Holmstrom, B., P. Milgrom. 1987. Aggregation and linearity in the provision of intertemporal incentives. *Econometrica* **55**(2) 303–328.

Holmstrom, B., P. Milgrom. 1990. Regulating trade among agents. *J. Institutional Theoret. Econom.* **146**(1) 85–105.

Holmstrom, B., P. Milgrom. 1991. Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *J. Law, Econom., Organ.* **7**(Special Issue) 24–51.

Horsky, D., P. Nelson. 1996. Evaluation of salesforce size and productivity through efficient frontier benchmarking. *Marketing Sci.* **15**(4) 301–320.

Johnston, M. W., G. W. Marshall. 2006. *Churchill/Ford/Walker's Sales Force Management*, 8th ed. McGraw-Hill Irwin, Boston.

Joseph, K., M. U. Kalwani. 1998. The role of bonus pay in salesforce compensation plans. *Indust. Marketing Management* **27**(2) 147–159.

Joseph, K., A. Thevaranjan. 1998. Monitoring and incentives in sales organizations: An agency-theoretic perspective. *Marketing Sci.* **17**(2) 107–123.

Kandel, E., E. P. Lazear. 1992. Peer pressure and partnerships. *J. Political Econom.* **100**(4) 801–817.

Lal, R., V. Srinivasan. 1993. Compensation plans for single- and multi-product salesforces: An application of the Holmstrom-Milgrom model. *Management Sci.* **39**(7) 777–793.

Lang, K., P.-J. Gordon. 1995. Partnerships as insurance devices: Theory and evidence. *RAND J. Econom.* **26**(4) 614–629.

Lazear, E. P., S. Rosen. 1981. Rank-order tournaments as optimum labor contracts. *J. Political Econom.* **89**(5) 841–864.

Lodish, L. M. 1975. Sales territory alignment to maximize profit. *J. Marketing Res.* **12**(1) 30–36.

Lodish, L. M. 1980. A user-oriented model for sales force size, product, and market allocation decisions. *J. Marketing* **44**(3) 70–78.

Mantrala, M. K., P. Sinha, A. A. Zoltners. 1992. Impact of resource allocation rules on marketing investment-level decisions and profitability. *J. Marketing Res.* **29**(2) 162–175.

Mantrala, M. K., P. Sinha, A. A. Zoltners. 1994. Structuring a multiproduct sales quota-bonus plan for a heterogeneous sales force: A practical model-based approach. *Marketing Sci.* **13**(2) 121–144.

Markowitz, H. 1952. Portfolio selection. *J. Finance* **7**(1) 77–91.

Markowitz, H. 1991. Foundations of portfolio theory. *J. Finance* **46**(2) 469–477.

Montgomery, D. B., A. J. Silk, C. E. Zaragoza. 1971. A multiple-product sales force allocation model. *Management Sci.* **18**(4) P3–P24.

Nalebuff, B. J., J. E. Stiglitz. 1983. Prizes and incentives: Towards a general theory of compensation and competition. *Bell J. Econom.* **14**(1) 21–43.

Pratt, J. W. 1964. Risk aversion in the small and in the large. *Econometrica* **32**(1–2) 122–136.

Prendergast, C. 1999. The provision of incentives in firms. *J. Econom. Literature* **37**(1) 7–63.

Rangaswamy, A., P. Sinha, A. A. Zoltners. 1990. An integrated model-based approach for sales force structuring. *Marketing Sci.* **9**(4) 279–298.

Skiera, B., S. Albers. 1998. COSTA: Contribution optimizing sales territory alignment. *Marketing Sci.* **17**(3) 196–213.

Stanton, W. J., R. Spiro. 1999. *Management of a Sales Force*, 10th ed. McGraw-Hill Irwin, Boston.

Teitelbaum, D. 2003. The post-refinance landscape. *Mortgage Banking* **64**(1) 98–105.

Varian, H. R. 1990. Monitoring agents with other agents. *J. Institutional Theoret. Econom.* **146**(1) 153–174.

Zhang, C., V. Mahajan. 1995. Development of optimal salesforce compensation plans for independent, complementary and substitutable products. *Internat. J. Res. Marketing* **12**(4) 355–362.

Zoltners, A. A., P. Sinha. 1980. Integer programming models for sales resource allocation. *Management Sci.* **26**(3) 242–260.

Zoltners, A. A., P. Sinha. 1983. Sales territory alignment: A review and model. *Management Sci.* **29**(11) 1237–1256.

Zoltners, A. A., P. Sinha. 2005. The 2004 ISMS Practice Prize Winner—Sales territory design: Thirty years of modeling and implementation. *Marketing Sci.* **24**(3) 313–331.