Dynamic Competitive Retail Pricing Behavior with Uncertainty and Learning

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When competing retailers lack full information about rivals' decision processes, how will dynamic pricing behavior vary from patterns observed in more traditional static or full-information models? We investigate this question in a dynamic alternating-moves duopoly model. Retailers update (linear) conjectures about rivals' future prices in a Bayesian fashion. We show that as observed and expected prices converge, a pricing equilibrium is always achieved, whether or not the conjectured and actual values of the slope of the rival's best response function are consistent. Assuming specific parameter values, we compare equilibrium prices and associated profits in our Bayesian learning model with those obtained under the assumptions of static Nash behavior, collusive behavior and dynamically optimal behavior with full information. We apply the notions of strategic substitutability and strategic complementarity to the analysis and find that when products are strategic complements, conjectures of higher rival price responsiveness lead to higher steady-state prices and profits. The reverse is true for strategic substitutes. We also find that learning about a rival's behavior proceeds more quickly, the less intensely related in demand are products. We find, in general, that equilibrium pricing patterns and profits can vary considerably from those in full-information environments, but that even with grossly wrong beliefs about rival behavior, competing retailers are still attracted to an equilibrium. The analysis suggests not only the value of investigating less-than-full information situations but also the potential incremental value of signalling greater or less aggressiveness than truly characterizes one's behavior as a strategic option.

INTRODUCTION

The formation of competitive pricing strategies is a research topic of considerable interest to marketing managers and scholars. In a recent review of pricing research in marketing, Rao (1984) identifies two areas as important candidates for further analytical study: understanding how and when a firm reacts to a price change, and the dynamics of competitive pricing behavior. In this paper, we develop and analyze a model that addresses these issues in a retailing context.

Many local markets in the USA are characterized by oligopolistic retail competition, i.e. there are several retailers in each line of trade serving a fixed number of households in limited geographical areas of operations (Lusch, 1982). Price is a dominant, if not the only, competitive tool in such local retail markets (Davidson et al., 1988). A pricing action by one retailer affects its rivals' demand and typically triggers competitive pricing reactions (e.g. Allvine and Patterson, 1972; Nagle, 1987, Ch. 4). Therefore, it is reasonable to believe that an intelligent retailer would expect rivals to react to a price change, rather than 'stand pat' as assumed in Nash competition models, and would take into account its conjectures about rivals' price reactions when considering price changes. When a retailer ignores competitive pricing reactions in its price-setting, the results can be devastating. For example, the near-demise of the A&P supermarket chain, for over two decades the largest in America, can be
partially attributed to the unanticipated price wars precipitated by its own campaign of price cutting (Nagle, 1987).

The intensity of a rival’s response to a price change is influenced by its marginal costs of retailing and its perceptions of the market demand function. Such parameters of the rival’s decision are usually difficult to assess by the retailer considering the price change. Hence, there is likely to be some uncertainty present in any retailer’s conjectures about its rivals’ pricing reactions or response functions. Given such uncertainty with respect to competitive price response functions, how may retailers go about setting prices and achieve a pricing equilibrium? What is the nature of this equilibrium? How does this differ from competitive pricing behavior when there is no uncertainty? These are some of the research questions addressed in a duopoly model of competitive retail pricing behavior presented in this paper.

A retailer’s uncertainty with regard to its rival’s price response function may be reduced over time by observing and learning from the actual versus expected pricing actions of the rival. Cyert and DeGroot (1970a) advocate a Bayesian approach to the treatment of such learning in models of duopoly competition, as do Jeuland and Shugan (1988) and Eliashberg and Chatterjee (1985). As yet, however, there are few models that explicitly incorporate learning between competitors in the marketing literature. Specifically, Corstjens and Horowitz (1989) apply Bayesian updating to a bargaining problem in a bilateral monopoly channel situation. Some simulation work in this direction has been done by Jeck and Staelin (1989) in the context of a channel of distribution problem. Shugan (1985) and Coughlan and Mantrala (1992) model learning in duopoly pricing problems but do not explicitly treat uncertainty of the type we have described. In the economics literature, Kalai and Stanford (1985) use a conjectural model to examine Cournot (quantity-setting) behavior in a multiperiod model, and show that equilibrium quantities vary between the perfectly competitive level and the collusive level, depending on the value of conjectures about competitive responsiveness. Kalai and Stanford (1985), however, do not model a Bayesian learning mechanism.

Our objective here is to investigate competitive retail pricing strategies and the resulting profits when rivals are incompletely informed about each other’s decisions but engage in learning over time. To do this, we (1) Develop and solve a model of dynamic competitive pricing behavior involving Bayesian updating of conjectures; (2) Study the movement of prices over time, whether and what kind of pricing equilibrium is reached, and how this equilibrium varies with the nature of the conjectures that rivals make about each other; and (3) Compare the learning model-based equilibria and associated profits with those derived in a full-information (no uncertainty) setting as well as to those obtained under the assumptions of Nash or collusive behavior.

Insights into the above issues may inform us as well of the robustness of the results from models assuming Nash behavior in retail competition. If the results are not robust to this change in assumptions, then it suggests a role for the modeling of expectations of rival reactions in future analytical work in the retailing area. Such modeling might cover not only pricing reactions but also reactions involving other marketing-mix variables (e.g. advertising, promotions, or salesforce effort expenditures).

The rest of the paper is organized as follows. The next section presents and solves an ‘alternating moves’ model of retail price competition in which one rival retailer is not fully informed of the other's decision problem or of the parameters of the other's problem. Each retailer, however, has a conjecture about the rival's price reaction function with some uncertainty associated with one of its parameters. The retailers alternately set price after updating their prior estimates of the unknown parameter of the rival’s reaction function via a Bayesian approach. Some analytic properties of the dynamically optimal solution under this model are discussed in the third section. Because the model solution is not easily amenable to comparative-static analysis, we derive further results through numerical analysis of the model involving several different values of the parameters of the problem. In this analysis, our model solution is compared with three benchmarks: a myopic Nash solution, a dynamically optimal solution, and the collusive solution—all derived under the assumption of full competitive information—as described in the fourth section. The fifth section presents the results of a numerical analysis of all these scenarios. The concluding section summarizes the insights derived from these analyses, implications, and directions for future research.
MODEL DEVELOPMENT

Consider two risk-neutral single-product retailers, $i$ and $j$, who are uncertain about each other’s reactive behavior and interact over a long but finite multiperiod horizon. Suppose the retailers set price in alternating periods. Without loss of generality, assume the number of periods $n$ in the horizon is an even number and firm $i$ moves first in period zero and sets its price $p_{i,0}$ in this period in some arbitrary fashion (e.g. firm $i$ can set the monopoly price). Then, firm $j$ moves and sets price $p_{j,1}$ in period 1, while firm $i$ holds to its price $p_{i,0}$. Next, firm $i$ sets price $p_{i,2}$ in period 2, while firm $j$ holds its price at $p_{j,1}$ set in the preceding period. In the third period, firm $j$ chooses $p_{j,3}$ while firm $i$ holds to $p_{i,2}$ and so on (see Cyert and DeGroot, 1970b; Maskin and Tirole, 1988; Coughlan and Mantrala, 1992, for previous duopoly models involving such alternating moves).

Demand Functions

Suppose the demand functions facing the two retailers are stable over time. Let $q_{i,t}$ and $q_{j,t}$ denote the demands, and $p_{i,t}$ and $p_{j,t}$ the prices, in time period $t$ of firms $i$ and $j$, respectively. Specifically, in even periods, $s = 2, 4, \ldots$, let:

$$q_{i,s} = a - p_{i,s} + cp_{j,s-1} \tag{1}$$

$$q_{j,s} = a - p_{j,s} + cp_{i,s} \tag{2}$$

And in odd periods, $r = 1, 3, 5, \ldots$

$$q_{i,r} = a - p_{i,r-1} + cp_{j,r} \tag{3}$$

$$q_{j,r} = a - p_{j,r} + cp_{i,r-1} \tag{4}$$

We assume the demand parameter $a$ is positive while the parameter $c$ may be either positive (i.e. the two retailers' products or product lines are substitutes) or negative (implying complementary products). Further, assume that each retailer knows the values of $a$ and $c$, although one retailer may not be entirely sure about the other’s knowledge of these parameters.

Expectations

Now, retailer $i$’s behavior is myopic if it sets price in any even period taking into account only that period’s profits. Specifically, retailer $i$’s myopically optimal price is found by solving the problem:

$$\max_{p_{i,s}} E[\Pi_{i,s}] = (p_{i,s} - k)[a - p_{i,s} + cp_{j,s-1}],$$

$$s = 2, 4, \ldots \tag{5}$$

where $E[\Pi_{i,s}]$ denotes retailer $i$’s expected profit in period $s$ and $k$ denotes its marginal cost of retailing the product or product line (assumed to be constant over time and output levels). One would expect, however, that in any period in which it sets price, one retailer would consider the effects of its price on the other’s next price and, in turn, how that would affect its own subsequent price decision and so on. In other words, firms may aim to set dynamically optimal prices that consider the future effects of current decisions and associated profits.

As mentioned earlier, duopoly models frequently assume that the two players are endowed with full information and full rationality, i.e. each fully knows the structure of its own and the rival’s decision problem (full information) and is fully able to use all such information and make the optimal decision (full rationality). In reality, however, retailers are likely to be less knowledgeable about their rival’s pricing behavior. Such retailers might conjecture that the rival’s pricing behavior does indeed depend on own price and make some relatively simple assumption about the form of the response function.

Let us suppose that these conjectured response functions are linear in form. Specifically, assume retailer $i$ believes in some period $(r - 1)$ that $j$’s best response function in odd period $r$ is given by:

$$p_{j,r} = \alpha_j + \beta_j p_{i,r-1} + \epsilon_{j,r} \tag{6}$$

where $\alpha_j$ and $\beta_j$ are fixed parameters and $\epsilon_{j,r}$ denotes a random component, representing, say, environmental disturbances affecting $j$’s pricing behavior. Thus, there is some uncertainty present in $i$’s conjecture regarding $j$’s response to $i$’s current price.

Let $\epsilon_{j,r}$ be normally distributed with mean zero and variance given by $(1/\tau_j)$, where $\tau_j$ is known as the precision and, further, be independently and identically distributed across all periods $r = 1, 3, 5, \ldots, (n - 1)$. Assume retailer $i$ is fully aware of the distribution of $\epsilon_{j,r}$. Furthermore, let retailer $i$ have a firm (but not necessarily correct) belief about the value of the parameter $\beta_j$, i.e. the slope of the conjectured best response function of retailer $j$, but be uncertain about the true value of the intercept term, $\alpha_j$. This implies that retailer $i$ is
uncertain about the ‘base price’ that the rival will charge. For example, this uncertainty could arise because retailer $i$ does not exactly know $j$’s marginal cost of retailing the product. Finally, although the true value of $\alpha_j$ is not known, suppose retailer $i$ has a prior distribution for this parameter. Specifically, let retailer $i$’s belief about $\alpha_j$ for the period under consideration be represented by a normal distribution with mean $m_{i,r-1}$ and precision $h_{i,r-1}$. These beliefs about the distribution of $\alpha_j$ can be updated every alternate period as described later. Thus, retailer $i$’s expectation formed in period $(r-1)$ regarding $j$’s price in period $r$ is given by:

$$\hat{p}_{j,r} = m_{j,r-1} + \beta_j p_{i,r-1}$$

where $m_{j,r-1}$ is the updated estimate of $\alpha_j$ held by retailer $i$ at the beginning of period $(r-1)$.

Analogously to retailer $i$, let retailer $j$ also believe in period $r$ that $i$’s best response function in period $(r+1)$ depends linearly on the price $j$ selects in period $r$, i.e.

$$p_{i,r+1} = \alpha_i + \beta_i p_{j,r} + e_{i,r+1}$$

where the random disturbance term $e_{i,r+1}$ is independently and identically distributed $N(0, 1/\tau_i)$ across all even periods. Similar to retailer $i$, assume retailer $j$ is aware of the distribution of $e_{i,r+1}$, has an unwavering (but possibly wrong) belief about $\beta_i$, and is uncertain about the true value of $\alpha_i$. Retailer $j$’s prior distribution on $\alpha_i$ in period $r$ is assumed to be $N(m_{i,r}, 1/h_{i,r})$, where the mean and precision are updated every alternate period. Thus, retailer $j$ in period $r$ expects $i$’s price in period $(r+1)$ to be:

$$\hat{p}_{i,r+1} = m_{i,r} + \beta_i p_{j,r}$$

### Bayesian Updating

We assume retailer $j$ learns about the unknown parameter $\alpha_i$ via a Bayesian updating process (see e.g. Cyert and DeGroot, 1970a). The process is exactly analogous for retailer $i$ and is therefore omitted. At the beginning of any odd period $r$, retailer $j$ has observed retailer $i$’s actual price set in period $(r-1)$, denoted $p_{i,r-1}$, which will hold in period $r$. As discussed above, retailer $j$’s previous conjecture in period $(r-2)$ regarding $i$’s reaction in period $(r-1)$ is given by:

$$p_{i,r-1} = \alpha_i + \beta_i p_{j,r-2} + e_{i}$$

where we have dropped the time subscript on the error term, given the i.i.d. assumption.

Retailer $i$’s actual price in period $(r-1)$ observed by retailer $j$ is $p_{i,r-1}$. If this is different from what was expected by retailer $j$, we assume it attributes the discrepancy to the random component of its conjecture, specifically to inaccuracy in its previous estimate of $\alpha_i$, namely $m_{i,r-2}$. This estimate must therefore be updated. More precisely, retailer $j$’s current estimator for $\alpha_i$ is given by:

$$p_{i,r-1} - \beta_j p_{j,r-2} = \alpha_i + e_i$$

(11)

Now, for any given value of $\alpha_i$, the left-hand side of Eqn (11) has a normal distribution with mean $\alpha_i$ and precision $\tau_i$. Before observing $p_{i,r-1}$, retailer $j$’s prior distribution for $\alpha_i$ was $N(m_{i,r-2}, 1/h_{i,r-2})$. Given the observed value of the estimator $p_{i,r-1} - \beta_j p_{j,r-2}$ at the beginning of period $r$, and applying Bayes’ theorem, retailer $j$’s posterior distribution for $\alpha_i$ will again be normal with mean $m_{i,r}$ and precision $h_{i,r}$, specified as follows:

$$m_{i,r} = \frac{h_{i,r-2}m_{i,r-2} + \tau_i(p_{i,r-1} - \beta_j p_{j,r-2})}{h_{i,r-2} + \tau_i}$$

(12)

$$h_{i,r} = h_{i,r-2} + \tau_i$$

(13)

Thus, in every odd period retailer $j$ revises its estimate of $\alpha_i$ according to Eqns (12) and (13). The weight attached to the prior belief $(h_{i,r}, \tau_i)$, the precision of the distribution for $\alpha_i$) increases over time, while the coefficient weighting new information ($\tau_i$, the precision of the error in the conjectured response function) remains constant over time. The process of Bayesian updating therefore suggests ultimate convergence of $m_{i,r}$ to a steady-state value (assuming there is an unchanging true value of $\alpha_i$).

Considering a symmetric development for firm $i$’s conjecture, its prior with respect to $\alpha_j$ is updated every even period according to:

$$m_{j,s} = \frac{h_{j,s-2}m_{j,s-2} + \tau_j(p_{j,s-1} - \beta_j p_{i,s-2})}{h_{j,s-2} + \tau_j}$$

(14)

$$h_{j,s} = h_{j,s-2} + \tau_j$$

(15)

### Dynamic Evolution of Competitive Pricing with Bayesian Updating

Given the above model development, the appropriate approach to determine a firm’s dynamically optimal price for a particular period is via backward induction. We now outline this solution approach, considering firm $i$’s decisions to illustrate the method. Firm $j$’s problems, being analogous to those of firm $i$, are not explicitly discussed.
Suppose the duopoly interaction has reached the end of the horizon, i.e. period $n$ which is an even period when firm $i$ moves. At the beginning of period $n$, firm $i$ has observed $j$’s price set in period $(n-1)$, i.e. $p_{j,n-1}^*$, which holds in period $n$. As the game ends after period $n$, firm $i$ must choose its price $p_{i,n}^*$ that maximizes its expected profit in period $n$, i.e. $i$ must solve the problem:

$$\text{Max } E[\Pi_{i,n}] = (p_{i,n} - k)[a - p_{i,n} + cp_{j,n-1}^*]$$ \hspace{1cm} (16)

The solution to Eqn (16) is:

$$p_{i,n}^* = \frac{a + k}{2} + \frac{c}{2} p_{j,n-1}^*$$ \hspace{1cm} (17)

which is a linear function of $j$’s price in the preceding period. We shall rewrite Eqn (17) as

$$p_{i,n}^* = u_{i,n} + v_{i,n} p_{j,n-1}^*$$ \hspace{1cm} (18)

where

$$u_{i,n} = \frac{a + k}{2} ; \quad v_{i,n} = \frac{c}{2}$$

Now consider firm $i$’s decision in period $(n-2)$. It must set $p_{i,n-2}$, having observed $j$’s price in period $(n-3)$, $p_{j,n-3}^*$, which will continue to hold in period $(n-2)$. To introduce some notation that is useful in describing a general rule for optimal price setting, note that trivially:

$$p_{i,n-2} = w_{n-2} + x_{n-2} p_{i,n-2}$$ \hspace{1cm} (19)

where $w_{n-2}$ is zero and $x_{n-2}$ is 1. Firm $i$’s expectation regarding $j$’s price in period $(n-1)$ is given by:

$$\hat{p}_{j,n-1} = m_{j,n-2} + \beta_{j} p_{i,n-2}$$ \hspace{1cm} (20)

where $m_{j,n-2}$ is updated according to Eqn (14). We also have that:

$$p_{i,n}^* = w_{n} + x_{n} p_{i,n-2}$$ \hspace{1cm} (21)

where

$$w_{n} = u_{i,n} + v_{i,n} w_{n-1} \quad \text{and} \quad x_{n} = v_{i,n} x_{n-1}$$

The variables $w$ and $x$ are used here to represent the linear relationship between any future price and the current price being chosen (here, $p_{i,n-2}$). We can summarize these parametric identities in Table 1. Firm $i$ then determines the price that maximizes expected profits in the current and remaining periods of the game, i.e. periods $(n-2), (n-1)$, and $n$, by solving the problem:

$$\text{Max } E[\Pi_{i,n-2} + \Pi_{i,n-1}] + E[\Pi_{i,n}^*]$$ \hspace{1cm} (22)

| Table 1. Values of $w$ and $x$ When Setting $p_{i,n-2}$ |
|-----------------|-------|-------|
| $l = (n-2)$    | $w_{l}$ | $x_{l}$ |
| $l = (n-1)$    | $m_{j,n-2} + \beta_{j} w_{n-2}$ | $\beta_{j} x_{n-2}$ |
| $l = n$        | $u_{i,n} + v_{i,n} w_{n-1}$     | $v_{i,n} x_{n-1}$ |

Here, $E[\Pi_{i,n}^*]$ is the optimized profit function in period $n$ based on setting price $p_{i,n}^*$ according to Eqn (18), but with $p_{j,n-1}$ replaced by $\hat{p}_{j,n-1}$ given by Eqn (20), as firm $i$ can only use its expectation as of period $(n-2)$ regarding $j$’s decision in $(n-1)$ in making its decision. Thus, firm $i$’s problem in period $(n-2)$ is:

$$\text{Max } E[\Pi_{i,n-2}] = (p_{i,n-2} - k)[a - p_{i,n-2} + cp_{j,n-3}^*]$$

$$+ (p_{i,n-2} - k)(a - p_{i,n-2} + c\hat{p}_{j,n-1})$$

$$+ (p_{i,n-2}^* - k)(a - p_{j,n-2}^* + c\hat{p}_{j,n-1})$$ \hspace{1cm} (23)

where we substitute for $p_{i,n-2}, \hat{p}_{j,n-1},$ and $p_{j,n-2}^*$ from Eqns (19), (20), and (21), respectively, and where:

$$m_{j,n-2} = h_{j,n-4} m_{j,n-4} + \tau_{j}(p_{j,n-3}^* - \beta_{j} p_{j,n-4}^*)$$ \hspace{1cm} (24)

After making these substitutions, the first-order condition for $p_{i,n-2}$ can be written as:

$$p_{i,n-2} = a + k + c p_{j,n-3}^* - 2 p_{i,n-2} + Y_{1} + Z_{1} p_{i,n-2}$$ \hspace{1cm} (25)

where

$$Y_{1} = x_{n-2} [a + k - 2 w_{n-2} + c w_{n-1}]$$

$$+ x_{n-1} [c w_{n-2} - 2 c k + c w_{n}]$$

$$+ x_{n} [a + k - 2 w_{n} + c w_{n-1}]$$ \hspace{1cm} (26)

$$Z_{1} = -2 [(x_{n-2})^{2} - c x_{n-1} (x_{n-2} + x_{n}) + (x_{n})^{2}]$$ \hspace{1cm} (27)

Here, $Y_{1}$ is the sum of all constant (not dependent on $p_{i,n-2}$) terms in the first-order condition arising from $E(\Pi_{i,n-1} + \Pi_{i,n}^*)$. $Z_{1}$ is the sum of all coefficient (dependent on $p_{i,n-2}$) terms in the first-order condition arising from $E(\Pi_{i,n-1} + \Pi_{i,n}^*)$. Solving, we find again that $p_{i,n-2}^*$ is a linear function of $p_{j,n-3}^*$:

$$p_{i,n-2}^* = u_{i,n-2} + v_{i,n-2} p_{j,n-3}^*$$ \hspace{1cm} (28)

where

$$u_{i,n-2} = \frac{a + k + Y_{1}}{2 - Z_{1}} ; \quad v_{i,n-2} = \frac{c}{2 - Z_{1}}$$ \hspace{1cm} (29)
Moving back now to period \((n-4)\), firm \(i\) has observed \(p_{i,n-4}^*\) and must choose the \(p_{i,n-4}\) that solves the problem:

$$\text{Max } E[\Pi_{i,n-4} + \Pi_{i,n-3}^*] + E[\Pi_{i,n-2}^*]$$ \hspace{1cm} (30)

where \(E[\Pi_{i,n-2}^*]\) is the optimized objective function from period \((n-2)\) until the end of the horizon. Specifically, Eqn. (30) in expanded form is given by:

$$\text{Max } [(p_{i,n-4} - k)(a - p_{i,n-4} + cp_{i,n-5}^* - k) + (p_{i,n-4} - k)(a - p_{i,n-4} + cp_{i,n-3}^*) + (p_{i,n-2}^* - k)(a - p_{i,n-2}^* + cp_{i,n-1}^*) + (p_{i,n-2}^* - k)(a - p_{i,n-2}^* + cp_{i,n-1}^*)]$$ \hspace{1cm} (31)

where

$$p_{i,n-4} \equiv w_{n-4} + x_{n-4} p_{i,n-4}; \hspace{1cm} w_{n-4} = 0, \hspace{0.5cm} x_{n-4} = 1$$ \hspace{1cm} (32)

$$\hat{p}_{j,n-3} = m_{j,n-4} + \beta_j p_{i,n-4} \hspace{1cm} \equiv w_{n-3} + x_{n-3} p_{i,n-4} + \beta_j p_{i,n-4}$$ \hspace{1cm} (33)

$$p_{i,n-2} = u_{i,n-2} + v_{i,n-2} \hat{p}_{j,n-3} \hspace{1cm} \equiv (u_{i,n-2} + v_{i,n-2} w_{n-3} + v_{i,n-2} x_{n-3} p_{i,n-4} + \beta_j p_{i,n-4})$$ \hspace{1cm} (34)

$$\hat{p}_{j,n-1} = m_{j,n-4} + \beta_j p_{i,n-2} \hspace{1cm} \equiv w_{n-1} + x_{n-1} p_{i,n-4} + \beta_j p_{i,n-2}$$ \hspace{1cm} (35)

$$p_{i,n-3} = u_{i,n} + v_{i,n} \hat{p}_{j,n-1} \hspace{1cm} \equiv w_{n} + x_{n} p_{i,n-4} + \beta_j p_{i,n-2}$$ \hspace{1cm} (36)

In these expressions, \(m_{j,n-2} = m_{j,n-4} \) is the best estimate of \(x_j\) that can be used by firm \(i\) at the beginning of period \((n-4)\):

$$m_{j,n-4} = \frac{h_{j,n-6} m_{j,n-6} + \tau_j (p_{j,n-5}^* - \beta_j p_{j,n-6})}{h_{j,n-6} + \tau_j}$$ \hspace{1cm} (37)

As in the setting of \(p_{i,n-2}^*\) above, we can again represent the linear relationship between any future price and the current price being chosen (here, \(p_{i,n-4}\)) in Table 2.

<table>
<thead>
<tr>
<th>(l)</th>
<th>(w_l)</th>
<th>(x_l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n-4)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(n-3)</td>
<td>(m_{j,n-4} + \beta_j w_{n-4})</td>
<td>(\beta_j x_{n-4})</td>
</tr>
<tr>
<td>(n-2)</td>
<td>(u_{i,n-2} + v_{i,n-2} w_{n-3})</td>
<td>(v_{i,n-2} x_{n-3})</td>
</tr>
<tr>
<td>(n-1)</td>
<td>(m_{j,n-4} + \beta_j w_{n-2})</td>
<td>(\beta_j x_{n-2})</td>
</tr>
<tr>
<td>(n)</td>
<td>(u_{i,n} + v_{i,n} w_{n-1})</td>
<td>(v_{i,n} x_{n-1})</td>
</tr>
</tbody>
</table>

Table 2. Values of \(w\) and \(x\) When Setting \(p_{i,n-4}\)

The values of \(w_{n}, x_{n}, w_{n-1},\) and \(x_{n-1}\) are identical to those in Table 2 when setting \(p_{i,n-2}\), except for the substitution of \(m_{j,n-4}\) for \(m_{j,n-2}\) to reflect the current state of beliefs in the two different periods' decision times. Further, the form of the \(w\)'s and \(x\)'s is similar in \((n-3)\) and \((n-2)\) to those in \((n-1)\) and \((n)\), respectively. The current decision-making period (here, \((n-4)\)) has a \(w\) of zero and an \(x\) of 1, as before. These regularities will persist as firm \(i\) moves back increasingly further in its horizon.

Using these identities, we can write the first-order condition for \(p_{i,n-4}\) as:

$$p_{i,n-4} = a + k + cp_{i,n-5}^* - 2p_{i,n-4} + Y_1$$

$$+ Y_2 + (Z_1 + Z_2)p_{i,n-4}$$ \hspace{1cm} (38)

where

$$Y_2 = x_{n-4} [a + k - 2w_{n-4} + cw_{n-3}]$$

$$+ x_{n-3} [c w_{n-4} - 2ck + cw_{n-2}]$$

$$+ x_{n-2} [a + k - 2w_{n-2} + cw_{n-3}]$$ \hspace{1cm} (39)

$$Z_2 = -2 [(x_{n-4} - c x_{n-3} x_{n-4} + x_{n-2}) + (x_{n-3})^2]$$ \hspace{1cm} (40)

Analogously to the above example setting \(p_{i,n-2}\), here \(Y_2\) is the sum of all constant (not dependent on \(p_{i,n-4}\)) terms in the first-order condition arising from \(E[\Pi_{i,n-3} + \Pi_{i,n-2}^*]\). \(Z_2\) is the sum of all coefficient (dependent on \(p_{i,n-4}\)) terms in the first-order condition arising from \(E[\Pi_{i,n-3} + \Pi_{i,n-2}^*]\). \(Y_1\) and \(Z_1\) are as in Eqns (26) and (27), respectively. Solving for \(p_{i,n-4}^*\), we get:

$$p_{i,n-4}^* = u_{i,n} + v_{i,n} p_{i,n-4}$$ \hspace{1cm} (41)

where

$$u_{i,n} = \frac{a + k + Y_1 + Y_2}{2 - Z_1 - Z_2};$$

$$v_{i,n} = \frac{c}{2 - Z_1 - Z_2}$$ \hspace{1cm} (42)

By extension, firm \(i\)’s price-setting problem in any period \((n-\eta)\) of an \(n\)-period horizon produces an optimal price-setting policy given by:

$$p_{i,n-\eta} = \frac{a + k + \sum_{\lambda=1}^{n/2} Y_{\lambda}}{2 - \sum_{\lambda=1}^{n/2} Z_{\lambda}} + \frac{c}{2 - \sum_{\lambda=1}^{n/2} Z_{\lambda}} p_{i,n-\eta-1}$$ \hspace{1cm} (43)
where 

\[ Y_\lambda = (x_{n-\eta+2\lambda-2} + 2w_{n-\eta+2\lambda-2} + c) \]

\[ + x_{n-\eta+2\lambda-1} (cw_{n-\eta+2\lambda-2} + cw_{n-\eta+2\lambda-2} - 2ck) \]

\[ + x_{n-\eta+2\lambda} (a + k - 2w_{n-\eta+2\lambda} + cw_{n-\eta+2\lambda-1}) \]

(44)

\[ Z_\lambda = -2 \frac{[(x_{n-\eta+2\lambda-2})^2 - cw_{n-\eta+2\lambda-1}(x_{n-\eta+2\lambda-2} + x_{n-\eta+2\lambda}) + (x_{n-\eta+2\lambda})^2]} \]

(45)

The above backward-induction approach would also be applied in deriving firm \( j \)'s decisions from period \((n-1)\) (when \( j \) makes its last move) to period 1 (when it makes its first decision). The decision problems of the two firms must be solved alternately in order to determine the actual values of the optimal prices from periods 1 to \( n \). Further, as there is Bayesian updating of conjectures based on observed prices, we must play the game forward to derive values of the optimal prices for any specified values of the parameters of the problem. A program for computing numerical solutions to this duopoly problem has been developed using Mathematica (Wolfram, 1988). Before turning to some numerical analyses of interest, we examine analytically some properties of the solution.

**SOME ANALYTIC PROPERTIES OF THE SOLUTION**

While the nature of the dynamic programming problem modeled here does not permit the full range of comparative-static type results that are found in other models, we can verify some properties of the system. First, using the Tables 1 and 2 as templates, note that:

\[ x_{n-\eta+2\lambda-1} = (x_{n-\eta+2\lambda-2} + x_{n-\eta+2\lambda-2} + \beta_j x_{n-\eta+2\lambda-2} \]

\[ x_{n-\eta+2\lambda} = (x_{n-\eta+2\lambda} + x_{n-\eta+2\lambda-1} + \beta_j x_{n-\eta+2\lambda-2} \]

Using these expressions, we can rewrite \( Z_\lambda \) from Eqn (45) as:

\[ Z_\lambda = -2 \frac{[(1 - \beta v_{n-\eta+2\lambda})^2 - (1 + \beta v_{n-\eta+2\lambda})^2 - 2c\beta]} \]

(46)

Thus, the second-order conditions for a unique profit maximum in any period of the dynamic horizon are satisfied (since firm \( j \)'s decisions are analogous to those of firm \( i \), all that we say here for firm \( i \) also holds for firm \( j \)).

Further, since all the \( Z_\lambda \)'s are unambiguously negative, the best response function in Eqn (43) has the slope of \( c \), the parameter representing demand-relatedness between the two products. If the products are substitutes, \( c \) is positive and best-response functions slope upwards in any period. Thus, a price increase by firm \( j \) in one period can be expected to result in a price increase by firm \( i \) in the next. If the products are complements, \( c \) is negative, so that a price increase by \( j \) in one period will generate a price decrease by \( i \) in the next. Equivalently, a positive value for \( c \) means that the products are strategic complements and a negative \( c \) value means that the products are strategic substitutes (see Moorthy, 1988).

The denominator of the slope of the best-response function is positive, and increasing, the greater the number of periods intervening between \((n-\eta)\) and \( n \) itself. Thus, the responsiveness of \( p_{i,n-\eta} \) to \( \beta_{j,n-\eta} \) is greatest in the terminal period, and least at the beginning of the time horizon. This is sensible, since at the beginning of time there is still a lot of 'future' ahead of a retailer against which to price. Thus, the absolute value of the slope of the best-response function is monotonic and nondecreasing over time, and has a maximum of \( c/2 \) (its terminal value). This is itself a fraction, so that price matching is never 100%.

Given the combination of dynamic optimization and Bayesian updating in our model, we are unable to derive further analytic properties of the solution. To examine the model more fully, we now turn to specific parametric values and the comparison of various Bayesian updating scenarios with the more conventional naïve Nash, collusive, and full-information dynamic scenarios. We proceed by first deriving these benchmarks against which to compare our model.

**BENCHMARKS FOR COMPETITIVE PRICING BEHAVIOR**

It is useful to investigate the properties of our dynamic Bayesian duopoly model both absolutely and relative to some commonly used benchmarks of competitive behavior. Here, we develop three such benchmarks, all involving full-information competition: a myopic Nash solution, a dynamically optimal solution, and the collusive solution.
Myopic Nash Pricing Equilibrium (‘Nash Scenario’)

Suppose each of the two retailers is fully informed (knows all the parameters of its own as well as the rival’s pricing problem), but is myopic and assumes the rival will not change price as a result of a change in own price (has a zero conjectural variation; see Kamien and Schwartz, 1983). Then retailer i sets price by solving the following problem (time subscripts are dropped given the assumptions of Nash behavior):

$$\max_{p_i} (p_i - k)(a - p_i + cp_i) \tag{47}$$

The corresponding price-setting rule for retailer i is:

$$p_i = \left(\frac{a + k}{2}\right) + \frac{c}{2} p_j \tag{48}$$

Retailer j solves a symmetric problem and its price-setting rule is:

$$p_j = \left(\frac{a + k}{2}\right) + \frac{c}{2} p_i \tag{49}$$

Hence, the Nash equilibrium solution is:

$$p_{i,n}^* = p_{j,n}^* = \frac{a + k}{2 - c} \tag{50}$$

In the n-period alternating-move model, if retailer i chooses the price $p_{i,n}^*$ in every even period, then the optimal strategy for j is to choose $p_{j,n}^*$ in every odd period and vice versa. When, however, firms are far-sighted rather than myopic and each takes into account the pricing reaction of the other, then the Nash equilibrium is not the appropriate solution. We next investigate such a dynamically optimal solution.

Dynamically Optimal Pricing Given Full Information (‘Smart Scenario’)

This type of pricing assumes the retailers are fully informed and rational, and each expects its rival to optimally react to a change in its own price. In our finite-horizon formulation of the duopoly problem, retailer i moves last in the nth (even) period and sets price to maximize only that period’s profit, given retailer j’s price set in period (n − 1). Retailer i’s decision rule in period n is therefore:

$$p_{i,n}^* = \left(\frac{a + k}{2}\right) + \frac{c}{2} p_{j,n}^* \tag{51}$$

Moving back to period (n − 1), given the assumptions of full information and full rationality, firm j would know that Eqn (51) is i’s best-response function in period n, and would take this into account when setting its price in period (n − 1). Specifically, firm j’s dynamically optimal price for period (n − 1) is found by solving the problem:

$$\max_{p_j} (p_{j,n-1} - k)[(a - p_{j,n-1} + cp_{j,n-2})$$

$$+ (a - p_{j,n-1} + cp_{i,n}^*)] \tag{52}$$

with $p_{i,n}^*$ given by Eqn (51).

The solution to Eqn (52) is not the Nash equilibrium price but rather:

$$p_{j,n-1}^* = \left[\frac{4a + k(4 - c^2) + c(a + k)}{2(4 - c^2)}\right] + \frac{c}{4 - c^2} p_{i,n-2}^* \tag{53}$$

Now, in period (n − 2), firm i would know that Eqn (53) is j’s best-response function in period (n − 1) and Eqn (51) is its own best-response function in period n. Firm i would therefore take all this into account in setting its price in period (n − 2). Given the discussion to this point, it is apparent that firm i’s best price in period (n − 2) would ultimately be a linear function of j’s price in period (n − 3):

$$p_{i,n-2}^* = \bar{u}_{i,n-2} + \bar{v}_{i,n-2} p_{j,n-3} \tag{54}$$

Proceeding with this backward-induction approach, in every even period firm i’s dynamically optimal price-setting rule has the form:

$$p_{i,s}^* = \bar{u}_{i,s} + \bar{v}_{i,s} p_{j,s-1}^* \tag{55}$$

while in every odd period (s − 1), firm j’s dynamically optimal price is given by:

$$p_{j,s-1}^* = \bar{u}_{j,s-1} + \bar{v}_{j,s-1} p_{i,s-2}^* \tag{56}$$

The $\bar{u}$ and $\bar{v}$ coefficients that appear in the above equations can be determined recursively by the method of backward induction. Algebraic expressions for these recursive relations can be derived and it can be shown that as the time index moves back through successively smaller values, the values of these coefficients converge to limiting or stationary values. In order to conserve space, we do not go into these derivations, but convergence is demonstrated in the following numerical examples.

Let us denote the stationary values of the coefficients by $\bar{u}$ and $\bar{v}$ (these values are the same for the two firms due to symmetry). Then, in the stationary
or steady-state situation, we have:

\[ p_{i,s} = \bar{u} + \bar{v}p_{j,s-1} \]  \hspace{1cm} (57)

and

\[ p_{j,s-1} = \bar{u} + \bar{v}p_{i,s-2} \]  \hspace{1cm} (58)

In a process with a large number of periods, the firms will repeatedly choose their prices according to Eqns (57) and (58). It follows that the prices will also converge to a steady-state value which we will hereafter call the ‘Smart Scenario’ duopoly pricing equilibrium.

**Collusive Pricing Equilibrium (‘Collusive Scenario’)**

If there were to be perfect co-operation or collusion between the two retailers, then they would set prices so as to maximize the sum of their profit functions in each period. It can be seen that the collusive pricing solution in our duopoly problem formulation is:

\[ p_{i,c} = p_{j,c} = \frac{a + k(1 - c)}{2(1 - c)} \]  \hspace{1cm} (59)

**NUMERICAL ANALYSES OF THE MODEL**

In a numerical analysis of our dynamic Bayesian duopoly model, we assume the following three special values for \( \beta \):

1. Let \( \beta = \bar{v} \), where \( \bar{v} \) is the value defined in the ‘Smart Scenario’. Choosing this value for \( \beta \) implies an assumption that the retailers are fully rational, even though they are not fully informed about all the parameters of the rival’s problem (e.g. the rival’s marginal cost). Specifically, each retailer may set \( \beta = \bar{v} \) when it not only aims to set the dynamically optimal price each time it moves, but also assumes the rival is smart and holds the same information and objective function as it does. Each retailer solves the complete duopoly problem and concludes that the slope of the rival’s reaction function should be \( \bar{v} \), while acknowledging some uncertainty regarding the intercept of the rival’s best-response function (which depends on the unknown marginal cost parameter, \( k \)). We call this structure of beliefs the ‘Smart \( \beta \) Scenario’. Each retailer then attributes discrepancies between the rival’s expected and actual prices to error in its estimate of the intercept of the conjectured linear response function. This estimate is then revised using the Bayesian approach described above. The ‘Smart \( \beta \)’ belief structure assumes a level of retailer knowledge and sophistication that is unlikely to exist in reality. Therefore, we also consider other more inconsistent beliefs about the slope of the rival’s best-response function.

2. Suppose \( \beta = c/2 \). This choice of \( \beta \) represents a case where each retailer aims to set the dynamically optimal price, but assumes the other retailer is myopic in its pricing behavior. In particular, it assumes the other retailer sets price according to the myopic pricing rule given by Eqns (48) or (49). Alternatively, each retailer could be assuming its rival to be ‘reckless’, repetitively setting price as though the horizon is coming to an end.

3. Finally, suppose \( \beta = 0 \). This choice implies that although both retailers are dynamic optimizers, each makes Nash assumptions about its rival (that is, assumes that the rival has no propensity to respond to competitive price moves). Discrepancies between observed and expected prices in this model are attributed simply to measurement error.

All the cases examined here assume symmetric firms and the parameter values: \( a = 1; \ k = 0.01; \ h_i(0) = h_j(0) = 0.1; \ \tau_i = \tau_j = 2.5 \). We consider five values of the demand function parameter \( c \): \( c = +0.9, \ c = +0.1, \ c = 0, \ c = -0.1, \) and \( c = -0.9 \). The first two values imply substitutable products; the last two imply complementary products; and \( c = 0 \) implies demand independence. In each of the cases, we consider a 50-period horizon with retailer \( i \) moving in even periods and retailer \( j \) moving in odd periods. Further, we always assume that retailer \( i \)'s price at the beginning of the game, i.e. \( p_{i,0} \), is the optimal monopoly price (assuming that retailer \( j \) enters the market only in period 1). Given uncertainty of the kind we have incorporated into the learning model, retailer \( j \)'s initial estimate of the intercept of \( i \)'s reaction function is assumed to be \( m_i(1) = 1.0 \). To summarize, we consider six scenarios for competitive behavior: (1) the ‘Collusive Scenario’, (2) the ‘Nash Scenario’, (3) the ‘Smart Scenario’, (4) the ‘Smart \( \beta \) Scenario’, (5) the ‘\( \beta = c/2 \) Scenario’, and (6) the ‘\( \beta = 0 \) Scenario’. Within each scenario, we examine competitive pricing behavior,
profitability, and ability to learn (for scenarios (4) through (6)) for various degrees of product substitutability or complementarity.

**Steady-state values of u’s and v’s**

The parameters $u$ and $v$ are the intercept and slope terms, respectively, in the equation relating one retailer’s current pricing decision as a function of its rival’s previous-period price (e.g., in Eqn (43)) the general expression for $u$ is the first term on the right-hand side and the general expression for $v$ is the coefficient of $p_{n-1} - \eta - 1$. As $u$ and $v$ approach constant values, so do prices and profits for the competing retailers. Hence, Table 3 gives steady-state values (or functions) for $u$ and $v$ in the three Bayesian learning scenarios, as well as in the ‘Smart Scenario’. Entries in Table 3 pertain to firm $i$ (but analogous statements can be made about firm $j$). In the three Bayesian scenarios, the steady-state expression for $u_{i,t}$ is a function of $m_{j,t}$, retailer $i$’s expectation of the intercept of $j$’s pricing rule in the upcoming period. It is this $m_{j,t}$ that is the subject of Bayesian updating throughout the retailers’ horizon. The steady-state value of $v$ is simply a numeric expression.

In all four scenarios, expressions for $u$ and $v$ stabilize immediately (in period 1 for firm $j$ and in period 2 for firm $i$). Looking down the columns for the three Bayesian scenarios, the expressions for $u$ and $v$ are symmetric around the $c = 0$ axis. In particular, the steady-state value of $v$ for $c = -0.9$ is always the negative of that for $c = +0.9$. In the expressions for $u$, a similar symmetry is observed: while the intercept and coefficient terms of the expression are the same for $c = +0.9$ and $c = -0.9$, the coefficient term is added in the positive $c$ case but subtracted in the negative $c$ case.

Although not reported in Table 3, there is an end-of-horizon effect that causes these $u$ and $v$ values to be disturbed towards the end of the firms’ horizon. The end-of-horizon effect is more pronounced, the higher is the absolute value of $c$, that is, the more interrelated in demand the products are (whether substitutes or complements). When $c = +0.1$, only in the last three periods of our 50-period horizon does the end-of-horizon effect come into play. But for $c = +0.9$, the effect starts to show about 15 periods before the end of the horizon. In runs of the dynamic program with longer horizons (e.g., 100 periods and 200 periods), we discovered that the end-of-horizon effect lasts the same *absolute number of periods*, regardless of the total horizon length. Thus, in a 200-period horizon, the steady-state values of $u$ and $v$ are unchanged, and the end-of-horizon effect manifests itself in period 197 for $c = +0.1$.

Finally, when $c = 0$, the products are unrelated in demand; thus, there is no real role for learning. The steady-state value of $v$ is zero for every scenario.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$\bar{u}$</th>
<th>$\bar{v}$</th>
<th>$\bar{u}_t$</th>
<th>$\bar{v}_t$</th>
<th>$\bar{u}_t$</th>
<th>$\bar{v}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.9</td>
<td>0.92167</td>
<td>0.6536 + 0.290 + 0.25031</td>
<td>0.7534 + 0.3719 + 0.25031</td>
<td>0.55 + 0.225 + 0.25031</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>+0.1</td>
<td>0.56972</td>
<td>0.5509 + 0.0251</td>
<td>0.5518 + 0.0251</td>
<td>0.55 + 0.0251</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.55</td>
<td>0.55 + 0.0</td>
<td>0.55 + 0.0</td>
<td>0.55 + 0.0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-0.1</td>
<td>0.53744</td>
<td>0.5509 - 0.0251</td>
<td>0.5518 - 0.0251</td>
<td>0.55 - 0.0251</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-0.9</td>
<td>0.56039</td>
<td>0.6536 - 0.290</td>
<td>0.7534 - 0.3719</td>
<td>0.55 - 0.225</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. $\bar{u}$ in the ‘Smart Scenario’ is not a function of $m_{j,t}$, because in this scenario each retailer is assumed to have full information.
2. Thus, no Bayesian updating is necessary and the variable $m$ is absent.
3. In the other three cases, the formula for $u_{i,t}$ as well as the numeric value for $v_{i,t}$ hold from period 1 until the end-of-horizon effect appears. The time at which this effect appears varies, but, in general, is earlier, the more interrelated in demand are the products (i.e., the higher is the absolute value of $c$).
4. Because of Bayesian updating of the $m$’s in the last three cases, the actual value of $u_{i,t}$ will shift through time, even though the formula for $u_{i,t}$ does not. However, because Bayesian updating is a convergent process, the $u_{i,t}$’s also eventually converge.
5. All the statements made above for $u_{i,t}$ also hold for $v_{i,t}$, due to symmetry of the problem. The same is true of $v_{i,t}$ and $v_{j,t}$.
Bayesian Learning about $u$

Table 3 focuses on the steady state characterizing the formulae for $u$ and $v$. Table 4 goes on to show the actual progression of the $u$ parameter through time, as well as the discrepancy between actual and expected values of this parameter. Thus, Tables 4(a) through 4(c) show how productive Bayesian updating is in a dynamic retail pricing framework. Results are shown for $j$'s ability to predict $u_{t,t}$, but all comments made here apply similarly to firm $i$'s.

### Table 4(a). Bayesian Learning About $u$ in the ‘Smart $\beta$ Scenario’

<table>
<thead>
<tr>
<th>$t$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
</tr>
</thead>
<tbody>
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<tr>
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</tr>
<tr>
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<td>0.5651</td>
<td>-0.0016</td>
<td>0.5374</td>
<td>-0.0015</td>
<td>0.5108</td>
<td>-0.0082</td>
</tr>
<tr>
<td>30</td>
<td>0.9225</td>
<td>-0.0015</td>
<td>0.5651</td>
<td>-0.0014</td>
<td>0.5374</td>
<td>-0.0013</td>
<td>0.5104</td>
<td>-0.0075</td>
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<td>0.5651</td>
<td>-0.0010</td>
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<td>-0.0008</td>
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<td>-0.0058</td>
</tr>
<tr>
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<td>0.55</td>
<td>0.0356</td>
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</table>

### Table 4(b). Bayesian Learning About $u$ in the ‘$\beta = c/2$ Scenario’

<table>
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<tr>
<th>$t$</th>
<th>$u_{t,t}$</th>
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<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
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</tr>
</thead>
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</tr>
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<td>-0.3032</td>
<td>-0.55</td>
<td>-0.0021</td>
<td>0.55</td>
<td>-0.0018</td>
<td>0.55</td>
<td>-0.0703</td>
</tr>
</tbody>
</table>

### Table 4(c). Bayesian Learning About $u$ in the ‘$\beta = 0$ Scenario’

<table>
<thead>
<tr>
<th>$t$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
<th>$u_{t,t}$</th>
<th>$u_{t,t} - m_{t-t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.7617</td>
<td>-0.1954</td>
<td>0.5048</td>
<td>-0.0312</td>
<td>0.5368</td>
<td>-0.0054</td>
<td>0.4857</td>
<td>0.0121</td>
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<td>10</td>
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<td>-0.2107</td>
<td>0.5646</td>
<td>-0.0189</td>
<td>0.5369</td>
<td>0.0083</td>
<td>0.4754</td>
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<td>20</td>
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<tr>
<td>26</td>
<td>0.7719</td>
<td>-0.2174</td>
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<td>0.5369</td>
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<td>0.470</td>
<td>0.0706</td>
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<td>30</td>
<td>0.7722</td>
<td>-0.2180</td>
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<td>-0.0158</td>
<td>0.5369</td>
<td>0.0121</td>
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<tr>
<td>40</td>
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<td>-0.2195</td>
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<td>-0.0155</td>
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<td>46</td>
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<td>-0.2197</td>
<td>0.5645</td>
<td>-0.0153</td>
<td>0.5369</td>
<td>0.0122</td>
<td>0.4681</td>
<td>0.0754</td>
</tr>
<tr>
<td>50</td>
<td>0.55</td>
<td>-0.4431</td>
<td>0.55</td>
<td>-0.0298</td>
<td>0.55</td>
<td>0.0254</td>
<td>0.55</td>
<td>0.1581</td>
</tr>
</tbody>
</table>
prediction of $u_{i,t}$. Results for $c=0$ are omitted because learning about a competitor's pricing policies is irrelevant to one's own retail pricing decisions when products are unrelated in demand.

In any of the scenarios, predictive ability is better when products are less related in demand. This is because price responsiveness is lowest for less closely related products; thus, less learning about pricing behavior is necessary and it is easier to be 'right' sooner.

When using the full-information $\beta$'s (the 'Smart $\beta$ Scenario'), firm $j$ is always able to predict firm $i$'s $u_{i,t}$ very precisely (to within a 0.001 or less error margin in general) well before the end-of-horizon effect occurs. But the learning process is much less productive when $\beta = c/2$, that is, when each retailer believes its rival is acting naively. The error in predicting $u_{i,t}$ is about ten times as high as in the 'Smart $\beta$ Scenario', for low and negative values of $c$; for the high value of $c = +0.9$, the error is many times greater. Thinking one's rival is naive when in fact the rival is a dynamic optimizer is a relatively much worse assumption from the point of view of ability to learn. Approximately the same degree of error in learning is exhibited when retailers assume that rivals have no propensity to set price in response to one's own prior-period price ($\beta = 0$ scenario).

The actual value of $u$ to which the retailers tend over time in the 'Smart $\beta$ Scenario' is almost identical to the value in the full-information 'Smart Scenario'. Thus, while it takes time, Bayesian learning can produce behavior mimicking full-information behavior—i.e. the rival retailers have the 'right' guess about the rival's propensity to react to one's price changes.

Bayesian updating can also move the rivals to a $u$-value similar to that in the full-information 'Smart Scenario' even if their beliefs about rival reaction propensities are not right—as long as products are not too interactive in demand. Thus, in Tables 4(b) and Table 4(c) note how close $u$ approaches to the $u$ in the 'Smart Scenario' in Table 3, for $c$ values of $+0.1$ and $-0.1$. The Bayesian updating process is much less faithful to the full-information process for $c = +0.9$ or $c = -0.9$ in Tables 4(b) and 4(c).

Learning via Bayesian updating is thus a process that is sensitive to the initial assumption made about rival behavior rules. But it is less so for small $c$ values (i.e. products that are less related in demand).

**Price Forecasting Ability**

How does learning about $u$ translate into a retailer's ability to forecast its rival's price next period? Figures 1(a) through 1(d) consider this question for various values of $c$, and for the three scenarios with uncertainty about the rival's behavior: the 'Smart $\beta$ Scenario', the $\beta = c/2$ scenario, and the $\beta = 0$ scenario. These graphs focus on retailer $i$. Analogous graphs for retailer $j$ are in Figs 2(a) through 2(d).

In all cases, the retailer is able to forecast its rival's price remarkably accurately in a relatively short time period. The improvement in forecasts is speediest for low absolute values of $c$.

The graphs also show that there is very little difference in forecasting ability among the different $\beta$ scenarios, as long as products are not too interactive in demand (see Figs 1(b) and 1(d) and Figs 2(b) and 2(d)). The spread widens with more interrelated products (Figs 1(a) and 1(c) and Figs 2(a) and 2(c)). The $\beta = c/2$ and $\beta = 0$ scenarios 'flank' the 'Smart $\beta$ Scenario' in predictive ability for firm $i$. For substitutable products, a $\beta$ lower than the full-information level (e.g. $\beta$ equal to zero) leads to an underestimate of the rival's future prices; an overly high $\beta$ (e.g. $\beta = c/2$) leads to an overestimate of the rival's future prices. For complementary products, the converse is true.

In sum, incorrect beliefs about the rival's propensity to react to one’s own price hamper a retailer's ability to forecast the rival's actual prices, as one would expect. But this problem is eased when products are less interactive in demand.

**Price Paths Over Time**

Now we have an idea of how Bayesian updating affects retailers' ability to forecast rival behavior. Let's turn to an examination of price levels themselves. Figure 3 shows actual prices set by firm $i$ over the 50-period horizon for various values of $c$, across the four different dynamic scenarios (the 'Smart Scenario', the 'Smart $\beta$ Scenario', the $\beta = c/2$ scenario, and the $\beta = 0$ scenario). The values of the two static scenarios considered (the collusive and Nash scenarios) are noted in each graph. The graphs show firm $i$'s prices, but characterize the same qualitative conclusions from firm $j$'s price paths.
Figure 1. Differences between actual and expected prices of product $i$ for various scenarios and various values of $c$.

Figure 2. Differences between actual and expected prices of product $j$ for various scenarios and various values of $c$. 
In every graph, for every scenario, prices tend to flatten out quickly. The flat range is the analogue to a ‘steady-state’ pricing level, although prices never completely stabilize, due to the continual (albeit dampened over time) Bayesian updating of priors.

In the case of substitutable products (Figs 3(a) and 3(b)), the ‘Smart Scenario’, the ‘Smart β Scenario’, and the $β = c/2$ scenario all do better at getting the rival retailers to a collusive pricing level than do either the naive Nash scenario or the $β = 0$ scenario. But interestingly, among these three, the $β = c/2$ scenario does best at generating pricing patterns close to the collusive level (a $β$ of $c/2$ here means a belief that the rival retailer will react about twice as strongly to one’s pricing in the previous period than is really true).

For complementary products (Figs 3(c) and 3(d)), typically naive Nash behavior produces the closest pricing pattern to collusive behavior. The learning scenarios and the ‘Smart Scenario’ do worse at getting rivals close to a collusive pricing level. Interestingly, $β = 0$ beliefs produce price paths closest to the collusive level, followed by the ‘Smart Scenario’ and the ‘Smart β Scenario’. Overstating the rival’s propensity to respond to one’s price changes (the $β = c/2$ scenario) produces price paths furthest from the collusive level and the highest overall.

These results can be understood by remembering that substitutable products with price competition are strategic complements, while complementary products with price competition are strategic substitutes. (Moorthy, 1988). With strategic complements, an increase in the rival’s price elicits an increase in this firm’s equilibrium price. With strategic substitutes, an increase in the rival’s price elicits a decrease in this firm’s equilibrium price.

In our dynamic model with alternating moves, beliefs about the rival’s future pricing practices affect current pricing practice through the profit functions for future periods. Recall that in the substitutable-products case, the higher is $β$, the greater is the positive price response this retailer expects from its rival. This belief is incorporated into the dynamic profit function, via the expected demand function in future periods (which is itself a positive
function of the rival’s expected future price). A positive $\beta$ causes a flattening of the expected demand function, or in other words, an outward rotation of the function. This in turn creates an incentive to charge a higher price this period. Thus, a high $\beta$ leads to higher prices over time by this retailer. The symmetric behavior is followed by the other retailer, leading to a positive relationship between the level of $\beta$ and the relative height of price paths over time.

By comparison, in the complementary-products case, the higher the absolute value of $\beta$, the greater is the negative price response this retailer expects from its rival. Again, this belief is incorporated into the dynamic profit function via the expected demand function in future periods (which is itself a negative function of the rival’s expected future price). A negative $\beta$, multiplied by the negative cross-price coefficient in the demand function, leads to a positive overall effect on expected demand. This effect is, of course, greater, the larger is $\beta$ in absolute value. The resulting outward rotation of the expected demand function creates an incentive to charge a higher price this period. Thus, even in the complementary products cases, higher (absolute) values of $\beta$ lead to higher prices over time by the two retailers. Thus, in general, given price competition in our dynamic model, higher $\beta$ values lead to pricing patterns closer to collusive ones when products are demand substitutes (i.e. strategic complements); higher (absolute) $\beta$ values lead to pricing patterns further from collusive ones when products are demand complements (i.e. strategic substitutes).

It is visually clear that a steady state is approached in prices, even for highly demand-interrelated products. But the relative positioning of pricing patterns (relative to the collusive price) varies depending on the intensity and nature of demand.

**Price and Profit Comparisons**

Finally, we examine steady-state prices and profits in the different scenarios. Table 5(a) examines prices. The collusive and Myopic Nash prices are the static solutions to those pricing problems for our parameter values. The ‘Smart Scenario’ prices in Table 5(a) are the steady-state values of price to which the system converges and at which it remains

### Table 5(a). Price Comparisons, Different Competitive Scenarios for $a = +1.0$, $k = +0.1$, and $c = +0.9$, $+0.1$, $0$, $-0.1$, and $-0.9$

<table>
<thead>
<tr>
<th></th>
<th>Collusive</th>
<th>Myopic Nash</th>
<th>‘Smart Scenario’</th>
<th>$p_{i,20}$</th>
<th>$\beta = c/2$, $p_{i,20}$</th>
<th>$\beta = 0$, $p_{i,20}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = +0.9$</td>
<td>5.05</td>
<td>1.0</td>
<td>1.23761</td>
<td>1.2388</td>
<td>1.54</td>
<td>0.969101</td>
</tr>
<tr>
<td>$c = +0.1$</td>
<td>0.6056</td>
<td>0.5789</td>
<td>0.579579</td>
<td>0.5796</td>
<td>0.5803</td>
<td>0.579</td>
</tr>
<tr>
<td>$c = 0.0$</td>
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<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>$c = -0.1$</td>
<td>0.5045</td>
<td>0.5238</td>
<td>0.524315</td>
<td>0.5243</td>
<td>0.5248</td>
<td>0.523801</td>
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<tr>
<td>$c = -0.9$</td>
<td>0.3132</td>
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<td>0.403343</td>
<td>0.4092</td>
<td>0.4296</td>
<td>0.385927</td>
</tr>
</tbody>
</table>

### Table 5(b). Profit Comparisons, Different Competitive Scenarios for $a = +1.0$, $k = +0.1$, and $c = +0.9$, $+0.1$, $0$, $-0.1$, and $-0.9$

<table>
<thead>
<tr>
<th></th>
<th>Collusive</th>
<th>Myopic Nash</th>
<th>‘Smart Scenario’</th>
<th>$\beta = c/2$</th>
<th>$\beta = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = +0.9$</td>
<td>24.5025</td>
<td>8.1</td>
<td>9.96817</td>
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</tr>
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<td>2.300276</td>
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<td>2.29452</td>
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<td>$c = 0.0$</td>
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<td>2.025</td>
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<td>2.025</td>
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</tr>
<tr>
<td>$c = -0.1$</td>
<td>1.800225</td>
<td>1.796064</td>
<td>1.79593</td>
<td>1.79594</td>
<td>1.79572</td>
</tr>
<tr>
<td>$c = -0.9$</td>
<td>0.863289</td>
<td>0.780921</td>
<td>0.708757</td>
<td>0.718983</td>
<td>0.633062</td>
</tr>
</tbody>
</table>

**Note:**

All entries refer to 10 periods of profit for firm $i$. The collusive and myopic Nash entries simply multiply the one-period profit by 10. The ‘Smart Scenario’, ‘Smart $\beta$, $\beta = c/2$, and $\beta = 0$ entries refer to profits in the middle of the 50-period horizon (i.e. the sum of profits from period 21 through period 30, inclusive). This is done to give an idea of the 'average' profitability across cases when limiting values of the Bayesian parameters have been reached.
over, most of the time horizon (see Figures 3(a) through 3(d)). The prices for the ‘Smart β’, β = c/2, and β = 0 scenarios are those holding in period 26 of our 50-period horizon. As Figures 3(a) through 3(d) show, these prices are representative of prices over the majority of the horizon in all cases. All the insights drawn from Figures 3(a) through 3(d) can be verified here as well, regarding the implications of strategic substitutability and strategic complementarity. It can also be seen that the Myopic Nash price and the β = 0 price are virtually identical, as one would expect given the intuitive interpretation of β = 0 expectations.

Table 5(b) reports ten-period profit levels across the six scenarios, for the various values of product substitutability and complementarity. Collusive and myopic Nash profits are simply ten times the one-period profit levels. The latter four scenarios report the sum of profits in the middle of our 50-period horizon, i.e. from periods 21 through 30. Since pricing behavior has reached a virtual steady state by period 21, these figures are indicative of long-run profits for the various scenarios. As one would expect, the relative profitability of the different scenarios mirrors the pricing insights discussed above in relation to Figure 3. When pricing patterns are closer to collusive levels, profitability is higher; collusive profits are (by definition) the highest of all. The relative profit discrepancy among cases is greater, the more intensely related in demand are the products (whether substitutes or complements); in the limit, when products are completely unrelated (c = 0), profits are identical across all the scenarios. This is sensible, since a duopoly with unrelated products is tantamount to an ‘industry’ with two monopolists. Knowledge (or lack thereof) about other retailers’ behavior has no impact on profitability.

Summary of Results from Analysis of Scenarios

Several general insights emerge from our analysis of these scenarios of competitive dynamic retail pricing behavior:

1. Bayesian updating of priors on a rival retailer’s propensity to react to pricing decisions does, in fact, improve a retailer’s estimates of both the rival’s price-setting rules and the rival’s prices themselves. The ‘steady state’ of these expectations is reached very quickly, even when beliefs about β (the cross-price responsiveness term) are relatively far from true. Learning is both more productive (i.e. comes closer to the truth about the competitor) and quicker, the less related in demand are the products the retailers sell. This may seem a non-intuitive result (why, after all, should learning be better when one is dealing with a less meaningful competitor?), but reflects the fact that there is just less that is necessary to learn about a more remote competitor—and this learning can therefore be accomplished faster and more accurately.

2. Actual pricing behavior settles into a ‘steady-state’ pattern fairly quickly, even for products that are highly interrelated in demand and for beliefs that are fairly far wrong. Rivals may persist in blissfully ignorant (but stable) pricing patterns even when their beliefs about the rival’s behavior continue to be somewhat wrong.

3. Believing one’s rival to be more aggressive than the rival actually tends to improve the retailer’s pricing and profit positions if the products are demand substitutes (i.e. strategic complements). It tends to harm the retailers’ pricing and profits if the products are demand complements (i.e. strategic substitutes). This result puts into a dynamic framework the strategic competition insights derived originally in static models of Nash competition. It seems particularly intuitively appealing, since a retailer in our model can incorporate its beliefs about its rival’s future behavior, rather than having to rely on static Nash equilibrium arguments to get the strategic insights.

These results suggest that cultivating a reputation as a ‘hard bargainer’ may be the most productive when dealing with retail competition that is intense and filled with substitute products. This retail pricing strategy may backfire, however, if a retailer’s market has many complementary products or complementary retail formats in it. Then, a more conciliatory stance can be cultivated in an attempt to make productive use of the ignorance in the market among rival retailers. Our intuition often suggests to us that more information is preferred to less. Our analysis of a dynamic retail pricing model suggests that ignorance (tempered appropriately by beliefs about one’s rival) can be bliss-enhancing instead.
SUMMARY, IMPLICATIONS, AND CONCLUSIONS

Price is the dominant competitive tool in most local retail markets where there are usually a few retailers competing in each line of trade for a long period of time before differentiating themselves or moving to other product lines. Such retailers are likely to react to each other’s price changes in their competition over time. They are also likely to try to predict and account for the rival’s price reactions when making own-price decisions. In such circumstances, how will competitive pricing behavior evolve over time and what is the nature of pricing equilibria achieved? Answers to such questions would not only contribute to understanding observed pricing behavior in retail markets but would also have implications for the development and implementation of competitive retail pricing strategies. As yet, the marketing and retailing literatures contain few models that directly address these questions in retail pricing.

In this paper we advance the current level of understanding of the above competitive retail pricing issues and offer insights to retail pricing model-builders as well as managers. Specifically, we develop and analyze a model of retail price competition in a symmetric duopoly with linear demand functions and alternating decisions by expected-profit-maximizing (risk-neutral) retailers interacting over a long but finite multiperiod horizon. Unlike previous models (e.g., Cyert and DeGroot, 1970b; Coughlan and Mantrala, 1992), we explicitly allow for and incorporate uncertainty in the conjectured reaction functions.

We find in all the scenarios investigated, given our model assumptions, that a retailer’s optimal price-setting rule in any period that it moves is a linear function of the rival’s price set in the previous period. Hence, the linear form of the conjectured reaction functions is consistent with actual price-setting behavior. Next, we note that in the ‘Smart Scenario’ where there is no uncertainty, the optimal reaction functions are stationary for most of the time (although there are some shifts at the very beginning and toward the end of the horizon). Effectively, the stationary values of the coefficients of the reaction functions characterize a pricing equilibrium that is different from the static Nash and collusive equilibria.

In the Bayesian (the ‘Smart’ , ) scenarios the main finding of interest is that, given a sufficiently long horizon, the learning process always leads rivals to a pricing equilibrium regardless of whether their belief about the slope of the conjectured reaction function is consistent or inconsistent with its stationary value in the ‘Smart Scenario’. Indeed, when the rivals hold to inconsistent beliefs, an equilibrium is reached even though the conjectured slope value may be markedly different from the actual slope of the rival’s reaction function in any period of the interaction. The duopolists proceed to an equilibrium because neither sees any reason to modify its pricing behavior or beliefs about the rival due to the small and shrinking differences between observed and expected prices that occur over time. Any errors in predicting rival prices are attributed to inaccurate estimates of an intercept term, rather than to an incorrect belief about the slope parameter itself. The Bayesian estimates of the intercepts themselves tend to stabilize over time, even though they may be significantly different from the actual values of these parameters, reinforcing the retailers’ conviction that they are on the right track. In short, although the retailers are working with wrong beliefs, reality matches their expectations and so they happily persist in their ignorance (Kirkman, 1975, has addressed similar issues in a paper concerned with learning by firms who have the ‘wrong’ model in mind about the workings of the market. He calls such steady states ‘pseudo-equilibria’, and describes the rivals’ actions as ‘justifying-by-doing’ rather than learning-by-doing).

The learning equilibrium varies with the conjecture regarding the slope parameter. In the case of substitutable products (strategic complementarity), on the one hand, our results indicate that equilibrium prices and associated profits rise toward collusive levels as the conjectured slope value increases. A conjectured slope value of zero leads to static Nash equilibrium prices and profits. As long as conjectures are positive, learning over time always improves the duopolists’ profits. On the other hand, when products are demand complements (strategic substitutes), learning with zero conjectures in our model leads to Nash prices and profits that are higher than the ‘Smart Scenario’ levels. In short, there are circumstances when a certain amount of ignorance about each other’s response behavior may actually help rival retailers make more profits over time than when they are fully informed about each other.

Finally, the impact of uncertainty in our model becomes more important as the rival products’
interrelatedness in demand increases. Specifically, as the products become more interrelated, it takes a greater length of time to reach an equilibrium with Bayesian learning, and the differences between equilibrium prices and profits obtained with alternative conjectures are much more significant.

We see two significant implications of this research. First, in the practical arena, profits in competitive retail markets may be improved if rival retailers cultivate a reputation for aggressive price-matching. Ex ante retail investments that signal high pricing responsiveness may be one way to achieve this goal. When products are complements, however, it helps to appear less aggressive. Second, from the viewpoint of model-building, the robustness of results of models assuming naive Nash behavior in retail competition is questionable, especially once a dynamic perspective is taken. There is a role for the modeling of expectations and learning about rival reactions in future analytical work. Our results indicate, however, that analysts building such models must carefully specify the scenario describing competitive behavior to mirror actual conditions in the industry they are investigating.

The effect of uncertainty on dynamic competitive retail behavior can be fruitfully explored in future work. Different sources of uncertainty (such as the effectiveness of promotional campaigns, or the value of a retailer’s local reputation) can be examined, and different assumptions about the structure of demand may yield further insights. Assuredly, there is also a role for both econometric and experimental studies that will test the pricing and profitability implications of such models of competitive retail activity.

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NOTE

1. As will be seen below, the actual response function is always linear. Thus, we are permitting each retailer to have the correct form of expectation about its rival.

REFERENCES


